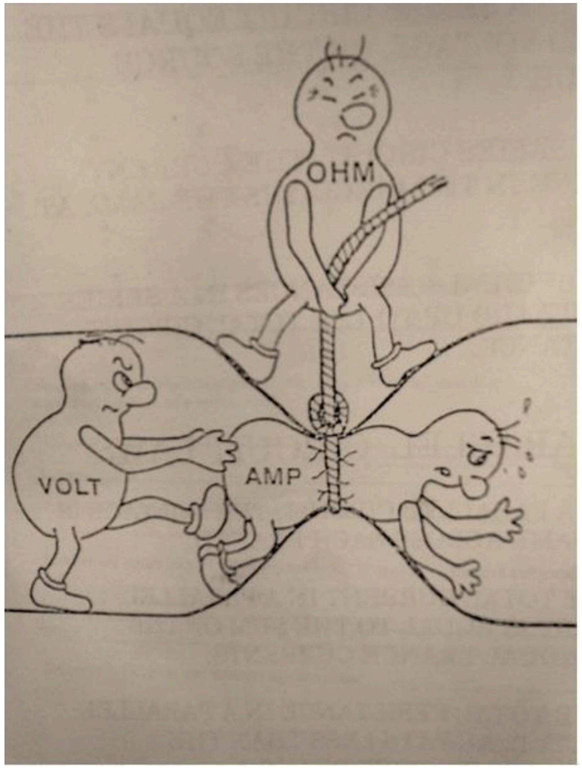
CALCULATION OF
TIME DEPENDENT WEIGHTING POTENTIALS
FOR MICROMEGAS.



Introduction:

- Motivation
- The principle of Signal induction
- The Shockley-Ramo theorem
- An example: AGATA

Latest developments:

- Time dependent weighting potentials
- First results: Clas12 / Astrobox
- (Laplace transforms)
- Dixit vs. Riegler

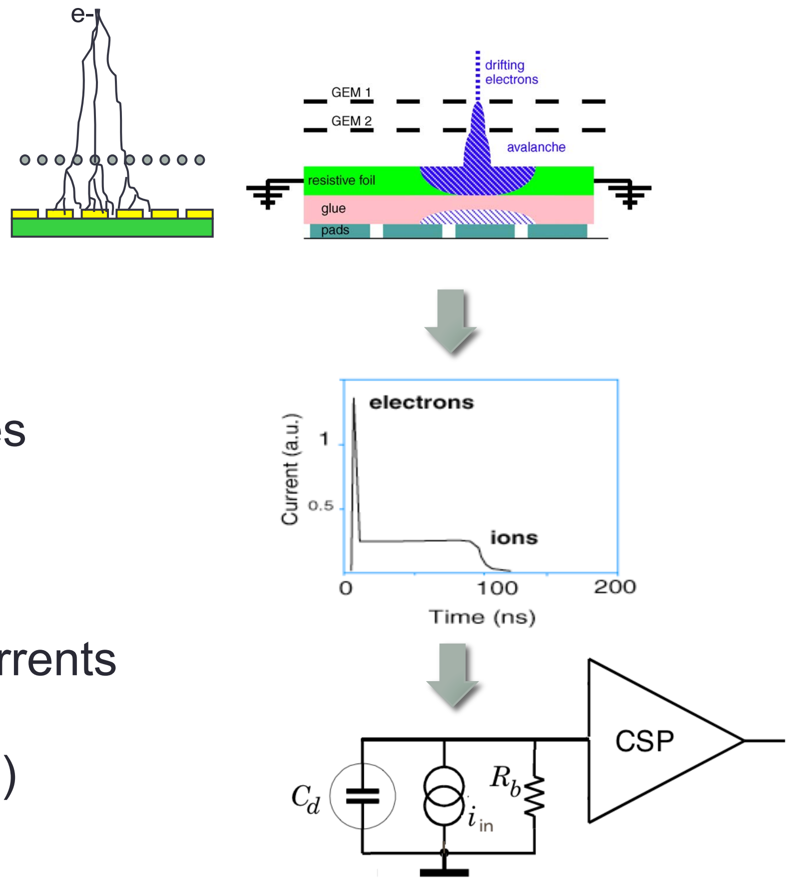
Prospects

Motivation: the missing link

- Garfield
  calculates the charge collection
  and charge breeding process

- Commercial codes (Multisim / Spice…)
  Allow to solve general electronics schemes

- Link between output of Garfield
  and input to Spice is missing:
  - Transformation of charge cloud into currents
  - Calculation of detector's impedance
    (capacity, series & parallel resistivity,…)



Principle of Signal induction

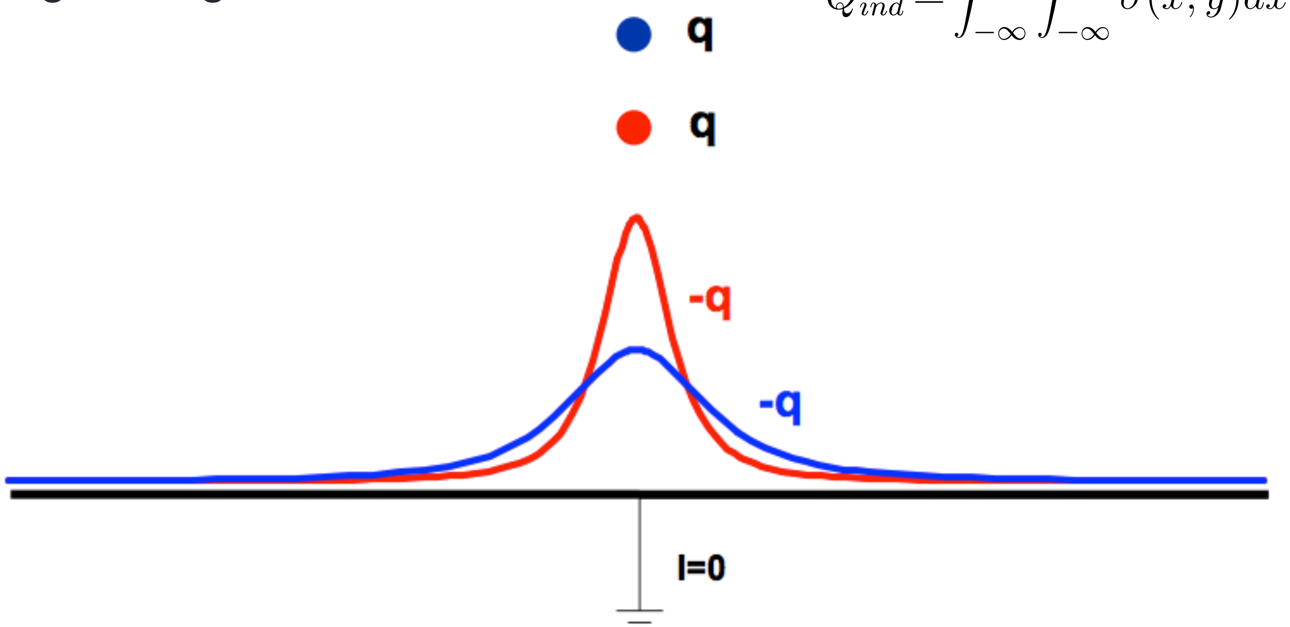
- A point **charge q at distance z_0** above a **grounded metal plate** induces a surface charge
- Different positions of q yield different charge distributions
- Here image charges can be used

$$E_z(x,y) = -\frac{q z_0}{2\pi\varepsilon_0 (x^2+y^2+z_0^2)^{3/2}}$$

$$E_x, E_y = 0$$

$$\sigma(x,y) = \varepsilon_0 E_z(x,y)$$

$$Q_{ind} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sigma(x,y)\,dx\,dy = -q$$



Principle of Signal induction

If we segment the metal plate
and keep individual strips grounded:

- Surface charge does not change compared to continuous plate
- The charge on each segment is now depending on position of q
- The **movement** of charge q **induces a current**

Method for image charges created for irregular geometries is required

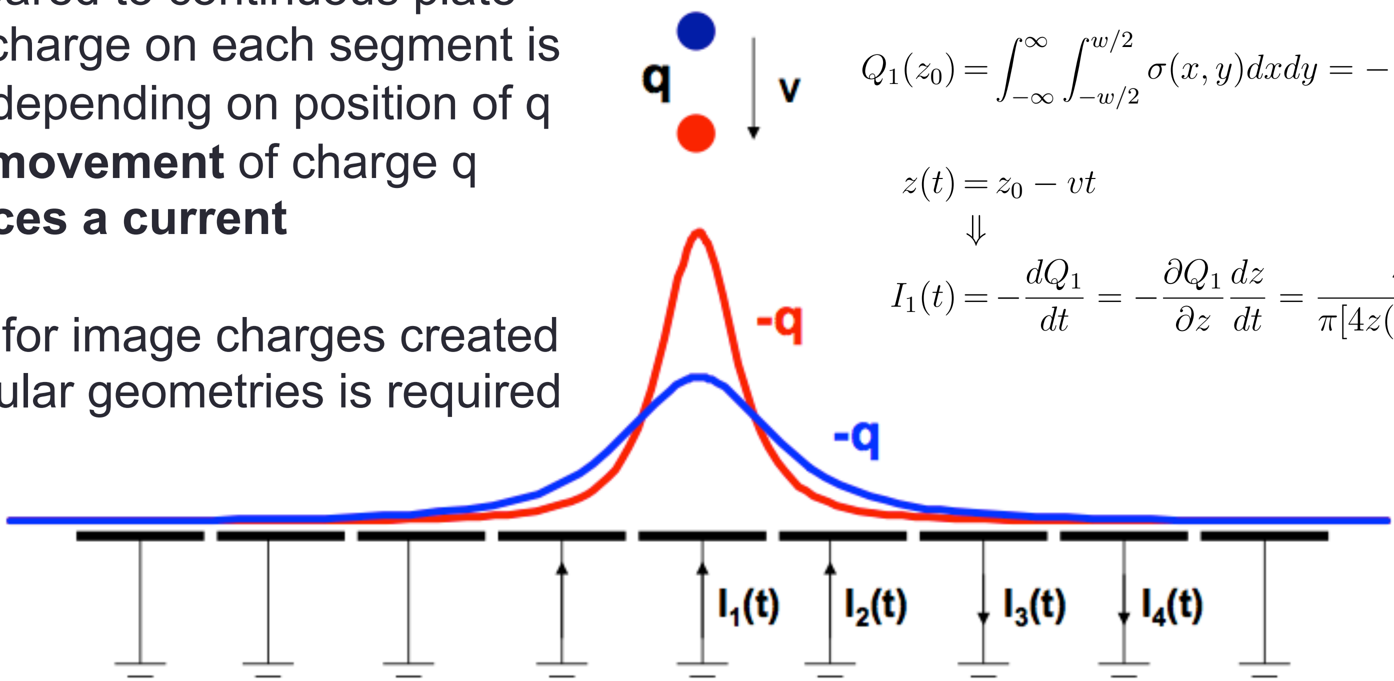
$$E_z(x,y) = -\frac{q z_0}{2\pi\varepsilon_0 (x^2+y^2+z_0^2)^{3/2}}$$

$$Q_1(z_0) = \int_{-\infty}^{\infty}\int_{-w/2}^{w/2} \sigma(x,y)\,dx\,dy = -\frac{2q}{\pi}\arctan\frac{w}{2z_0}$$

$$z(t) = z_0 - vt$$

$$\Downarrow$$

$$I_1(t) = -\frac{dQ_1}{dt} = -\frac{\partial Q_1}{\partial z}\frac{dz}{dt} = \frac{4qw}{\pi[4z(t)^2+w^2]}\cdot v$$

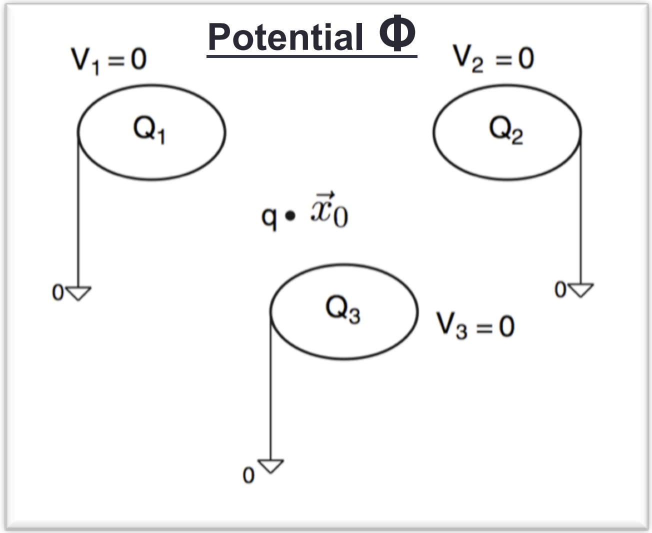


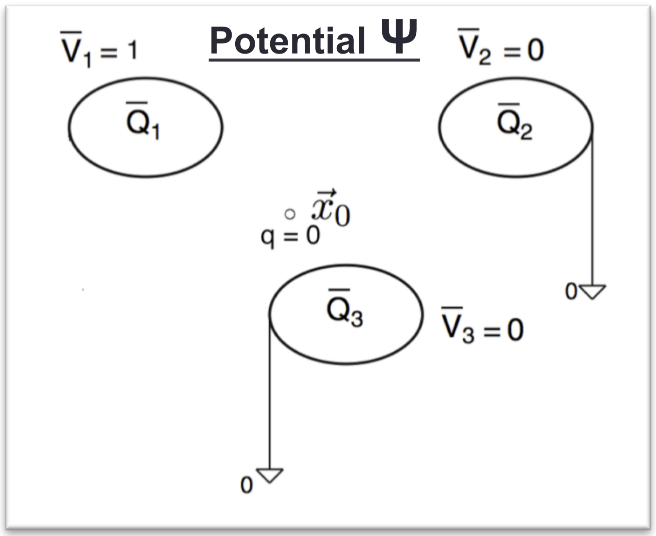
The Shockley-Ramo theorem

- Consider the potentials Φ, Ψ corresponding to:
  - **Φ** the "original problem" potential by charge q
    with all electrodes grounded.
    Of interest: the total induced charge $Q_{ind,i}$ on electrode i
  - **Ψ** the potential without charge q
    with all electrodes grounded,
    except for electrode i, put to 1V.
- What have both potentials in common?
  use **Greens 2nd identity**:

$$q\Psi(\vec{x}_0) = -\oint_{S_i} \sigma_{q,i}\, dS = -Q_{ind,i}$$

- Advantage:
  - **Ψ** needs to be calculated only once, while
    Φ needs recalculation + integral for every position of q.





The Shockley-Ramo theorem

- The induced charge Q_{qi} on electrode i
  by a point charge q located at position x_0 is

$$Q_{qi} = -q \cdot \psi_i(\vec{x}_0)$$

- With **weighting potential** ψ_i defined by

$$\nabla^2 \psi_i(\vec{x}) = 0 \qquad \phi|_{S_j} = \delta_{i,j}$$

- The current $I_{qi}(t)$ to electrode i is then given by

$$I_{qi} = \frac{dQ_{qi}}{dt} = -q \cdot \left(\frac{\partial \Psi_i}{\partial x_0} \frac{dx_0}{dt} + \frac{\partial \Psi_i}{\partial y_0} \frac{dy_0}{dt} + \frac{\partial \Psi_i}{\partial z_0} \frac{dz_0}{dt} \right)$$
$$= q \; \vec{E}_{\Psi i}(\vec{x}_0) \cdot \vec{v}_{drift}$$

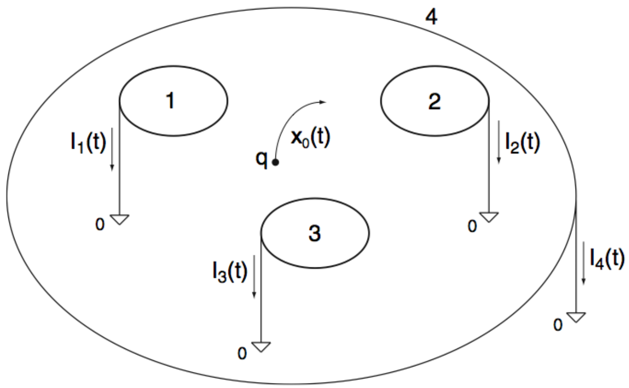
- The function $\vec{E}_{\Psi i} = -\nabla \Psi_i$ is called the **weighting field**

Weighting field properties

For a set of electrodes completely enclosing the detector volume V:

- The sum of weighting potentials is 1 everywhere on V

$$\Psi(\vec{x}) = \sum_i \Psi_i(\vec{x}) \equiv 1$$



- The total current is 0 at any time

$$I_{tot}(t) = \sum_i I_{q,i} \propto \nabla\Psi \equiv 0$$

- The total induced charge is 0 at any time

$$Q_{tot}(t) = \sum_i Q_{q,i} \equiv 0$$

Extended Ramo theorem

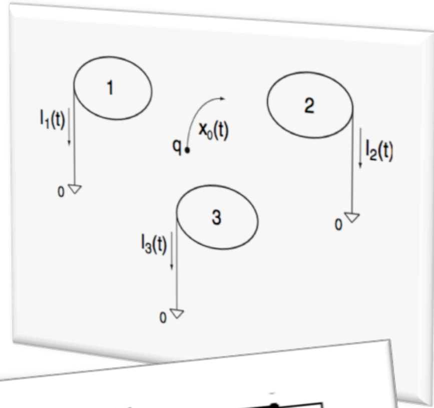
- Describes detectors in a **realistic** electronic network.

- In 3 steps:
  - 1) Apply the Ramo theorem:
    Calculate the ideal induced currents in each electrode
  - 2) Equivalent electronics scheme:
    Proof: see Gatti and Padivini, NIM 193 (1982) 651-653
    -Determine the capacitances of your detector,
    -Add the current sources found from 1)
  - 3) Realistic electronics scheme:
    Change the above simplified scheme
    into a realistic model.

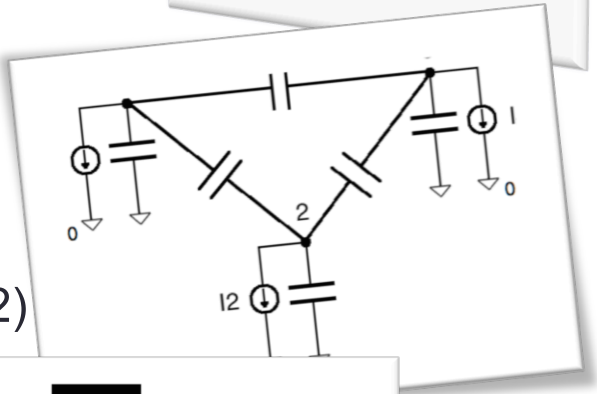
- Result = realistic signals,

  (e.g. including cross talk)

1)



2)



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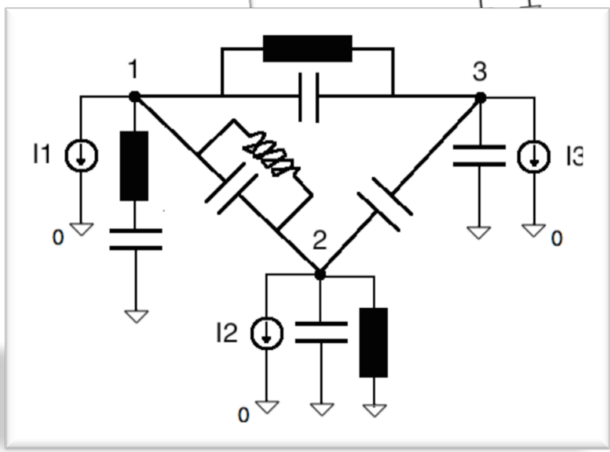
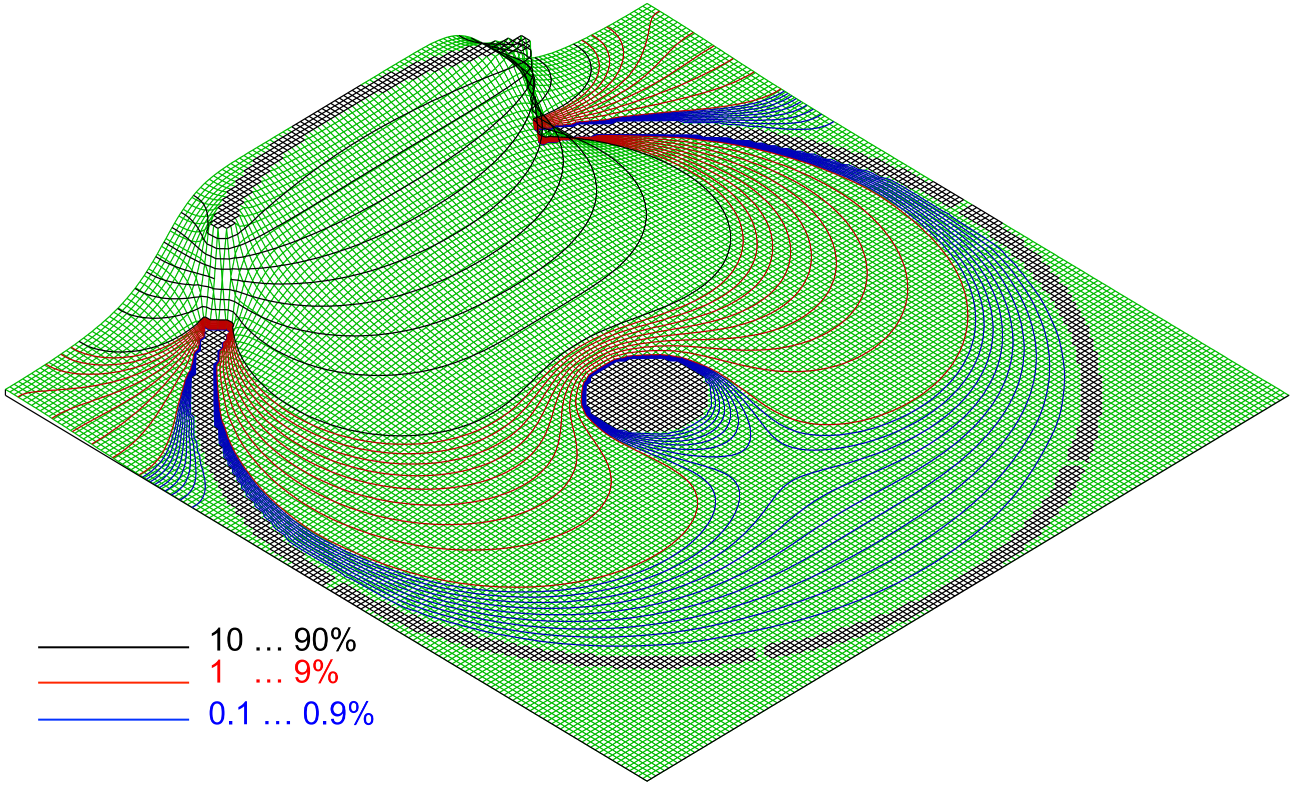
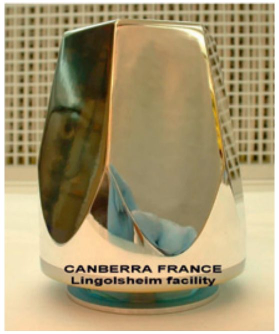


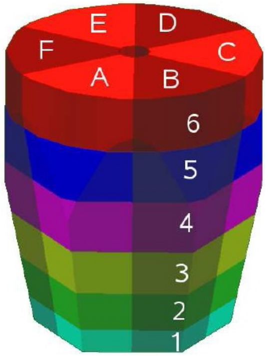
THE example: AGATA

- AGATA = Advanced Gamma Tracking Array
- Detector Simulation Software "ADL"
- Weighting potential - solution to Laplace equation

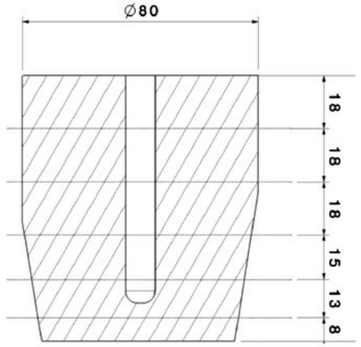




AGATA germanium crystal

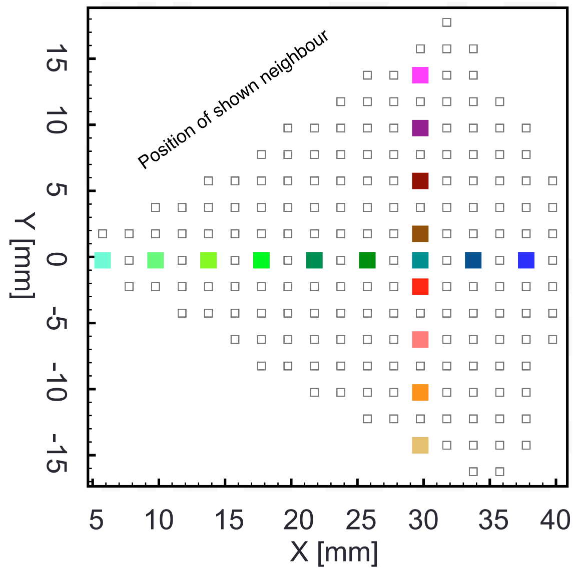


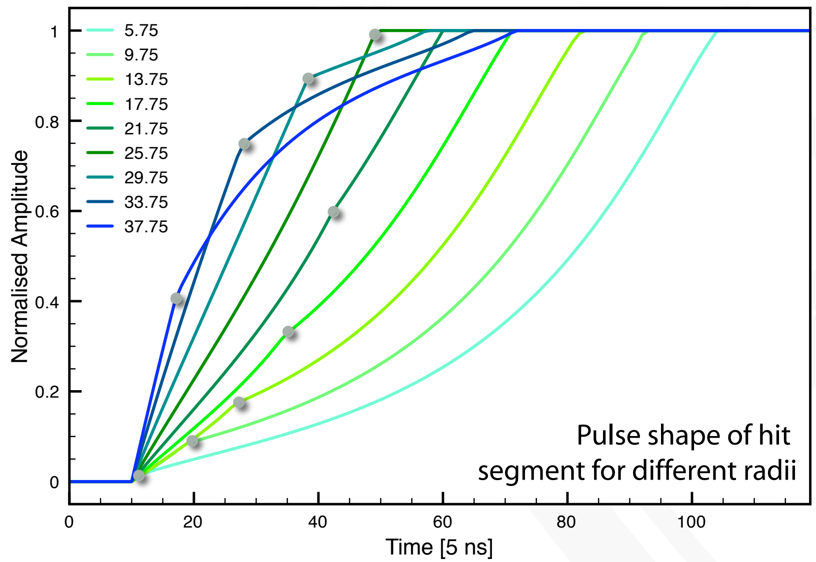
AGATA 36-fold segmented

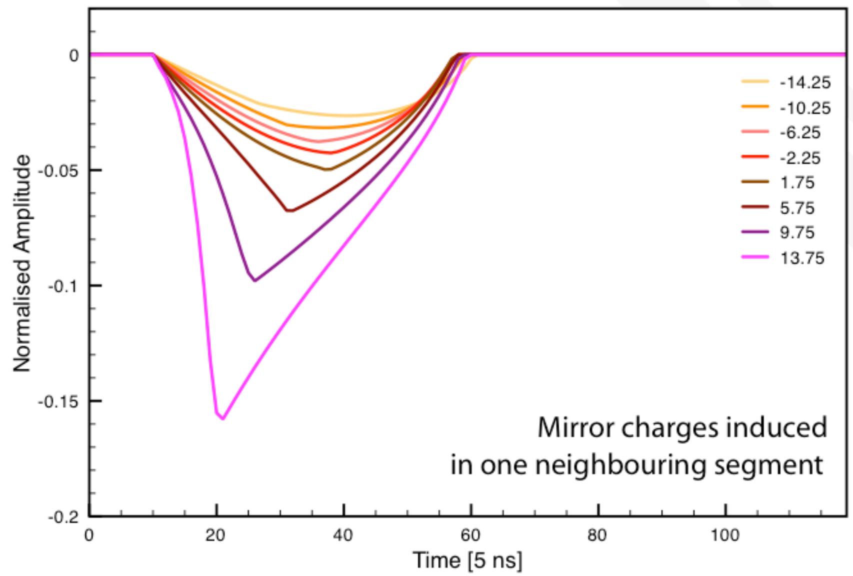


Example: AGATA

- Signal shapes from an AGATA detector as function of position
- Simulation using ADL
  using weighting potentials and
  drift velocities of electrons and holes







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So far:
Homogeneous dielectric media only

Introduction:
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- The principle of Signal induction
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Now:
Inhomogeneous media, including **resistive** layers

Latest developments:
- Time dependent weighting potentials
- First results: Clas12 / Astrobox
- (Laplace transforms)
- Dixit vs. Riegler

Prospects

Time dependent weighting potentials

Equation to solve:

- Homogeneous dielectric medium: $$\nabla^2 \phi(x) = 0$$
- Inhomogeneous medium: $$\nabla \varepsilon(x) \nabla \phi(x) = 0$$
- Including resistivity : $$\nabla (\varepsilon(x) + \sigma(x)/s) \nabla \phi(x, s) = 0$$

$\updownarrow$ Laplace Transform

= Time dependent weighting potential $$\phi(x, t)$$

The Laplace Transform

- Laplace Transform of f(t): $F(s) = \int_0^\infty f(t) e^{-st}\, dt$ with complex frequency $s \in \mathbf{C}$

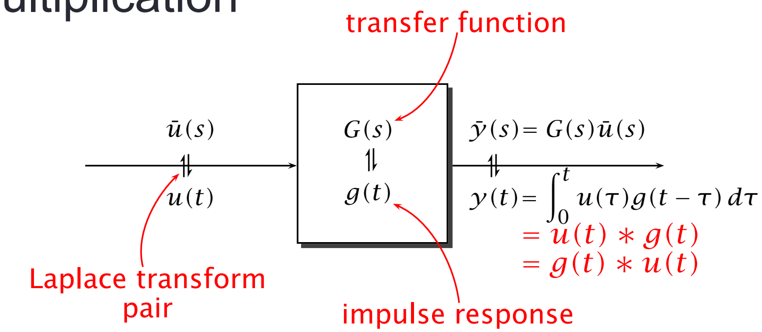
- Fourier transform is for periodic functions, while
  Laplace transform for signals “switching on” at time t=0 : f(t<0) = 0

- Transforms differential equations into algebraic equations:
  L, R, C circuits easily solved with **impedances**

$$i(t) = C\frac{du(t)}{dt} \rightarrow I(s) = C \cdot s \cdot U(s) \quad \text{or} \quad \frac{U(s)}{I(s)} = \frac{1}{sC}$$

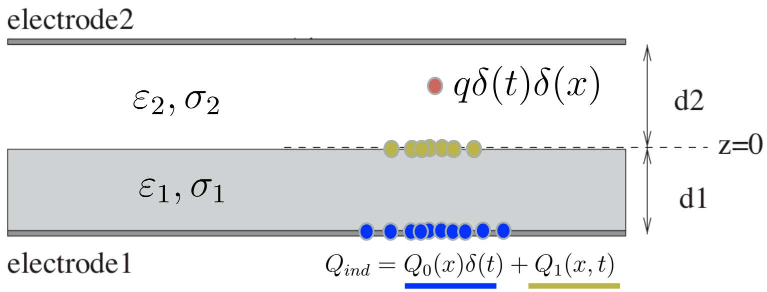
X	$Z_X(s)$
R	R
C	1/sC
L	sL

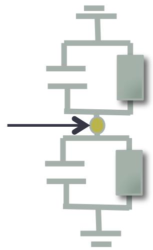
- Convolution becomes multiplication



Time dependent weighting potentials

An example:





- Infinite short times:
  directly induced charges,
  time independent fraction of weighting potential
  $$\lim_{s\to\infty} \nabla(\varepsilon(x) + \sigma(x)/s)\nabla\phi(x,s) = 0$$
- q also charges the interface at z=0 :
  indirectly induces charges
  time evolution of the charges at interface
- Total Weighting potential
  $$\phi = \phi_0(x)\delta(t) + \phi_1(x,t)$$
- Time integrated charge:
  $$\phi_{int} = \int_0^\infty \phi \, dt \quad \leftrightarrow \lim_{s\to 0} \nabla(\varepsilon(x) + \sigma(x)/s)\nabla\phi_{int}(x,s) = 0$$

Example: Astrobox

Explicit solution for x<d3:

see e.g. W. Riegler, NIM A 491 (2002) 258

define

$$a = \varepsilon_1\varepsilon_2 d_3 + \varepsilon_2\varepsilon_3 d_1 + \varepsilon_3\varepsilon_1 d_2$$

$$b = \varepsilon_3\sigma_2 d_1 + \varepsilon_1\sigma_2 d_3$$

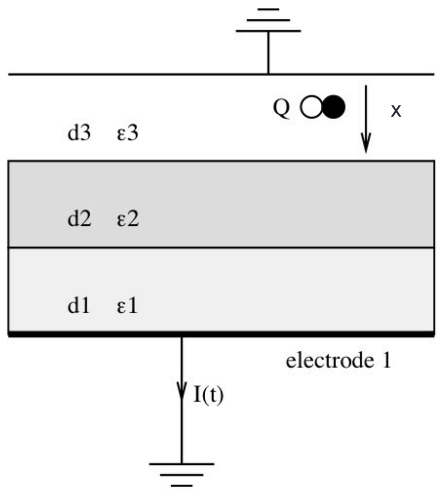
$$c(x) = \varepsilon_1\varepsilon_2 x/a$$

$$\tau_2 = \varepsilon_2/\sigma_2$$

$$\tau = a/b$$

$$V_3(x,s) = c(x)\frac{s+1/\tau_2}{s+1/\tau}$$

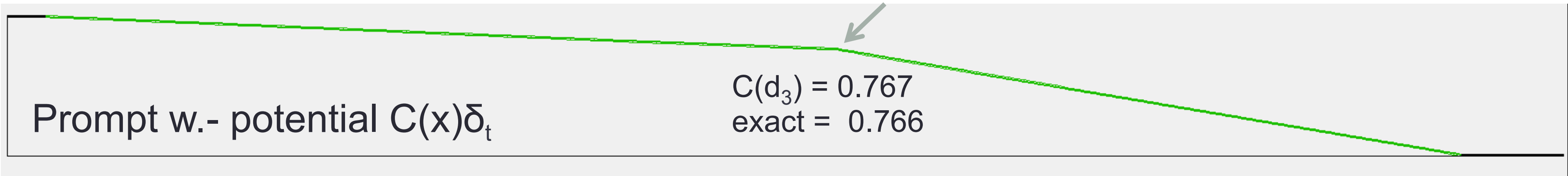
$$V_3(x,t) = c(x)\left[\delta_t + (1/\tau_2 - 1/\tau)\exp(-t/\tau)\right]$$

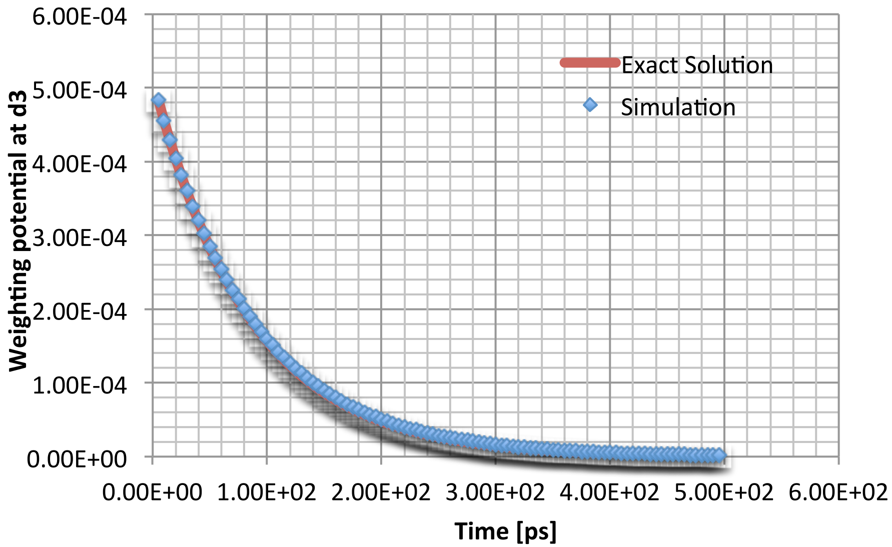


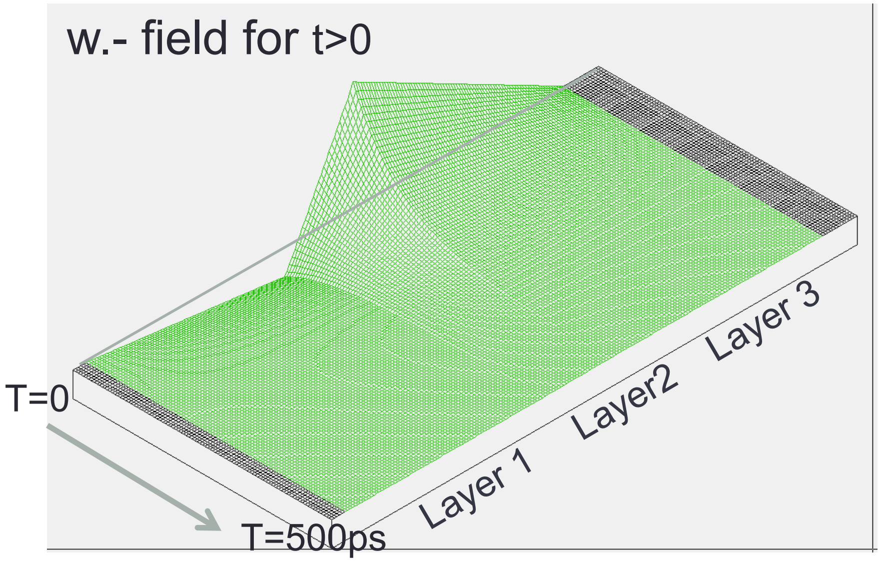
Astrobox	d [µm]	εr	σ [1/Ωm]
layer 3	80	1	0
layer 2	25	4.5	0.5
layer 1	75	4	0

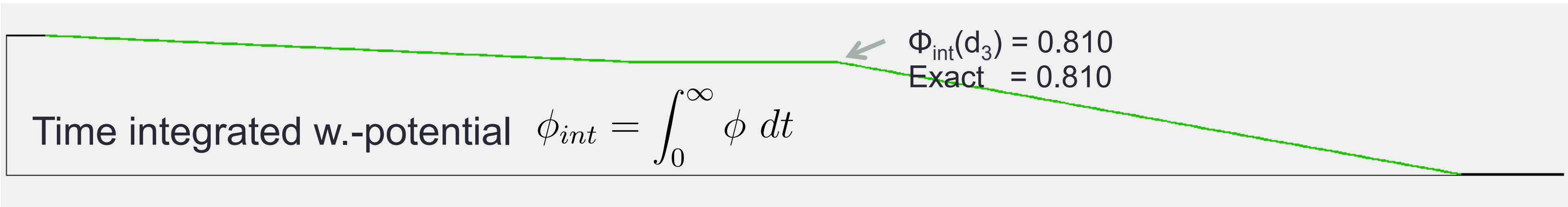
Ex: Astrobox

$$V_3(x,t) = c(x)\,[\delta_t + (1/\tau_2 - 1/\tau)\exp(-t/\tau)]$$



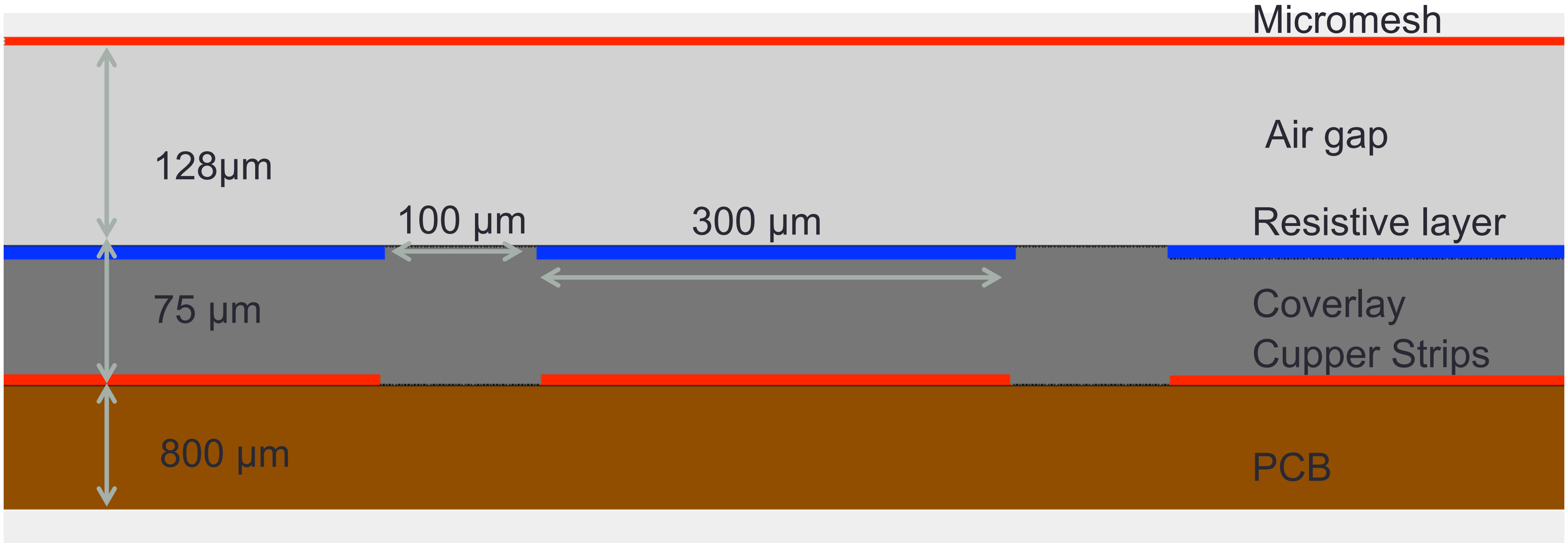






Example: CLAS12

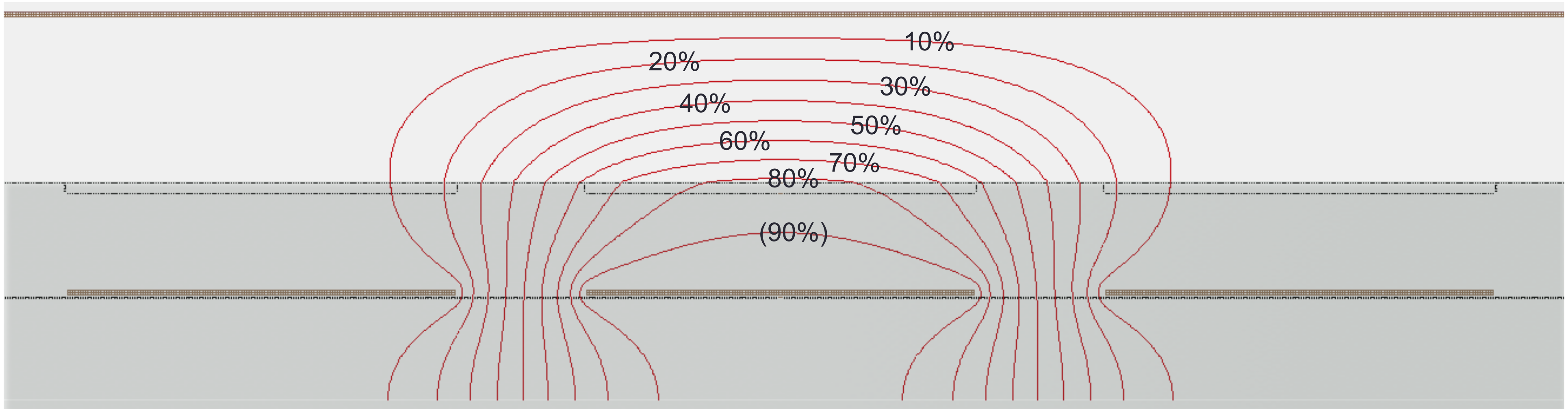
CLAS12	d [μm]	εr	σ [1/Ωm]
air gap	128	1	0
resistive layer	7	3.1	8.5
coverlay	75	4	0
cupper strips	5	inf	inf
pcb	800	4.6	0



Example: CLAS12

$$\phi = \phi_0(x)\delta(t) + \phi_1(x,t)$$

- Prompt weighting potential $\Phi_0(x)$:



- Charge sharing!

The Inverse Laplace Transform

$$f(t) = \frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty} e^{st} F(s) ds$$

A line integral in the complex plane

Good numerical approximations exist of the form:

$$f(t) = \frac{1}{t}\sum_i K_i \cdot F\left(\frac{z_i}{t}\right)$$

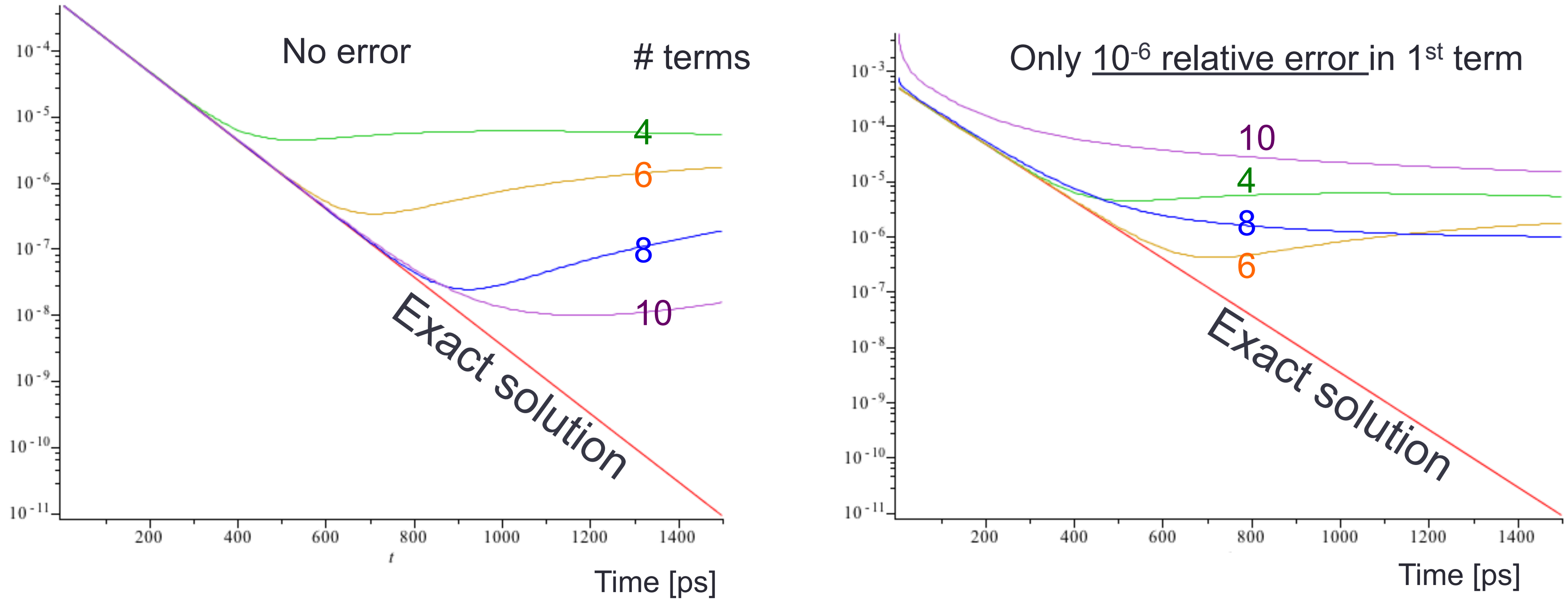
With K_i, Z_i complex constants

e.g.: Zakian's method, Stehfest's method, Talbot's method, Fourier series method,…

- Simple implementation
- Allows easy inversion of fields F(s,x)
- Does not work for t=0
- Goes wrong as t grows.
- High numerical precision required (see next slide)

The Inverse Laplace Transform

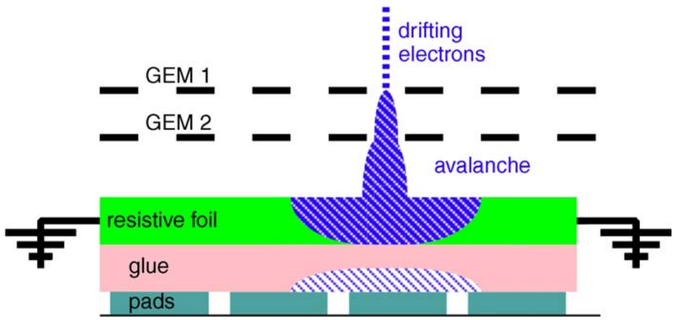
- Error sensitivity: Astrobox problem solved using Zakian's method:



- Adding more terms allows to calculate further in time
- But solution becomes more sensitive to calculation error

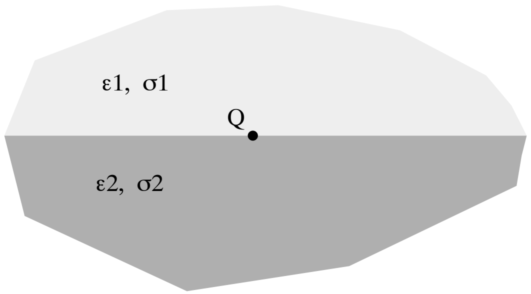
$$f(t) = \frac{1}{t} \sum_i K_i \cdot F\left(\frac{z_i}{t}\right)$$

Dixit vs. Riegler



- M.S. Dixit, A. Rankin - NIM A 566 (2006) 281–285
  Telegraph equation in the limit L->0:
  charges spread but are conserved

$$\rho(x,t) = \sqrt{\frac{1}{4\pi th}} \exp(-x^2/4th).$$



- W. Riegler – NIM A 491 (2002) 258–271
  Quasi static approximation for weak conductive media
  immobile charge fades away exponentially

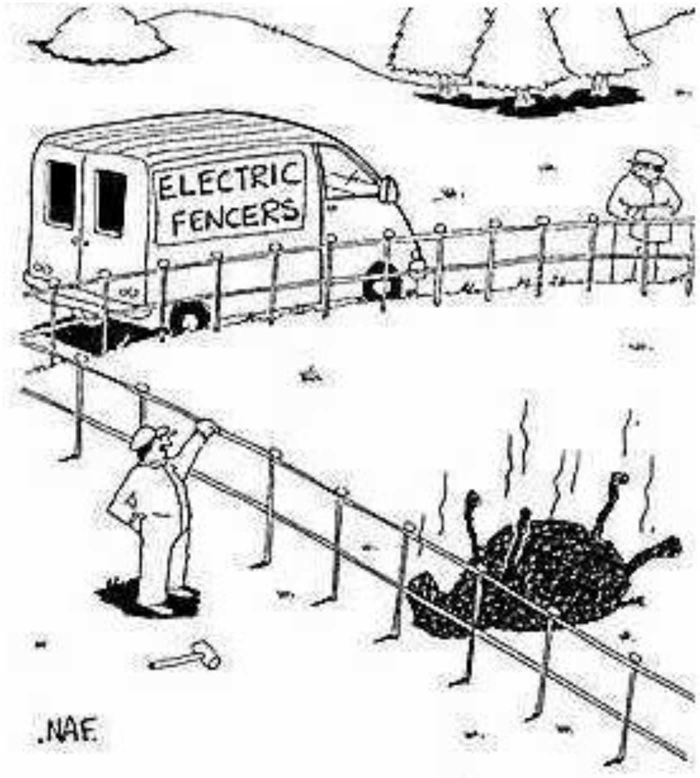
$$\Phi(\vec{r},t) = \frac{2Q}{4\pi(\varepsilon_1+\varepsilon_2)|\vec{r}|} e^{-t/\tau}, \qquad \tau = \frac{\varepsilon_1+\varepsilon_2}{\sigma_1+\sigma_2}.$$

…unification of both theories needed…

(see also **F. Rapetti** : ***Electromagnetic quasi-static models applied to transmission lines***

Prospects

- Get the theory straight.
- Multi grid methods for acceleration of calculation
- Different length scales: Cubic ➔ rectangular grid
- Laplace inversion limitations: Time stepping method?



"Down a few volts yet Harry!"