Garfield++ simulations for electronegative gases.

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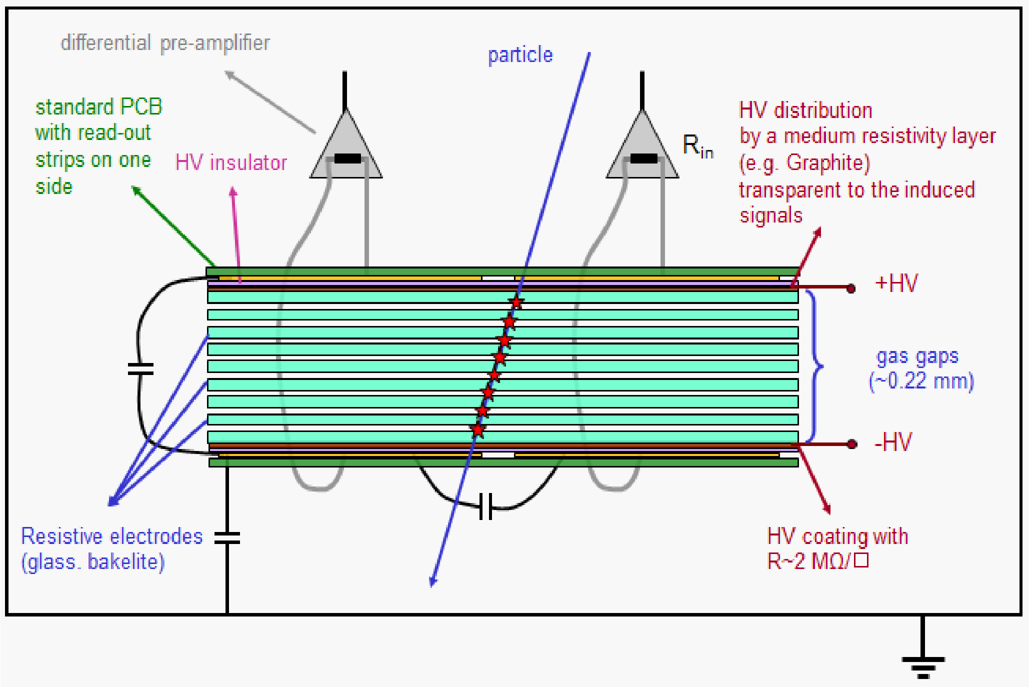
Outline

- Motivation
- Avalanche equations
- Garfield++ simulations of electron swarm coefficients
- Avalanche statistics
- Efficiency estimate for RPC
- Summary and outlook

Motivation

Increase the realism of RPC simulations

- Better understanding of the electron swarm parameters in electron-negative gases (Freon/iso-butane/SF6 mixture)
- Precise Avalanche statistics in a realistic way
- Transmission-line characteristics



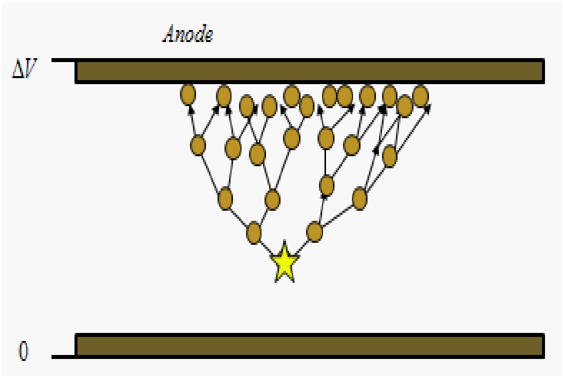
Time resolution: 20 - 100 ps
Efficiency: >90%
Rate capability: ~ 50 kHz/cm^2
Potential position resolution: <50 μm

- Motivation
- **Avalanche equations**
  - ✓ Huxley's formula
  - ✓ Brambring's formula
  - ✓ Description from Townsend's theory
  - ✓ Relation between Huxley's and Brambring's formulas
- Garfield++ simulations of electron swarm coefficients
- Avalanche statistics
- Efficiency estimate for RPC
- Summary and outlook

Huxley's formula

Huxley writes the avalanche equation as [1]

$$\frac{\partial n}{\partial t} + div\vec{J} = \alpha W n \qquad (1)$$



$$-\frac{\partial n}{\partial t} + \alpha W n + D\left(\frac{\partial^2}{\partial x^2} n + \frac{\partial^2}{\partial y^2} n\right) + D_L \frac{\partial^2}{\partial z^2} n - W \frac{\partial}{\partial z} n = 0 \qquad (2)$$

n_0, initial electron number
e, electron charge
h, gap width
W, drift velocity
α, effective Townsend coeffecient
D_L, longitudinal diffusion coeffecient
D, transverse diffusion coefficient

$$n(r,z,t) = \frac{n_0 \exp\left(-r^2/4Dt\right)}{(4\pi D t)(4\pi D_L t)^{1/2}} \frac{1}{Wt} \exp[\alpha W t] \times \left\{ z \exp\left[-\frac{(z-Wt)^2}{4D_L t}\right] + (z-2h)\exp\left(\frac{hW}{D_L}\right)\exp\left[-\frac{(z-2h-Wt)^2}{4D_L t}\right] \right\} \qquad (3)$$

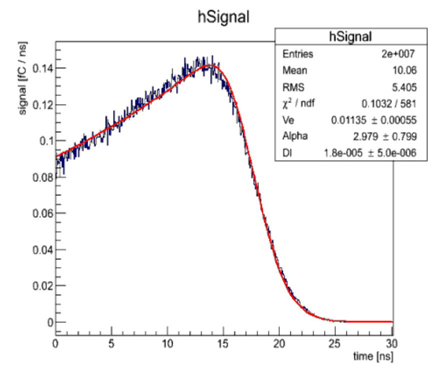
Boundary condition:
n(h)=0

[1] L. G. H. Huxley, The diffusion and drift of electrons in gases, 1972

Huxley's formula

Induced current at the anode

$$i(t) = \frac{N(t)eW}{h} = \frac{eW\int_0^h n(z,t)dz}{h} = i_1(t) + i_2(t) + i_3(t) + i_4(t) \qquad (4)$$



where

$$i_1(t) = \exp[\alpha Wt]\frac{n_0 eW}{2h}\left[1 + Erf\frac{h - Wt}{(4D_L t)^{1/2}}\right]$$

$$i_2(t) = \exp[\alpha Wt]\frac{n_0 eW}{2h}\left[Erf\frac{2h + Wt}{(4D_L t)^{1/2}} - Erf\frac{h + Wt}{(4D_L t)^{1/2}}\right]\exp\left(\frac{hW}{D_L}\right)$$

$$i_3(t) = \exp[\alpha Wt]\frac{n_0 e}{h}\left(\frac{D_L}{\pi}\right)^{1/2} t^{-1/2}\left\{\exp\left(-\frac{W^2 t}{4D_L}\right) - \exp\left[-\frac{(h - Wt)^2}{4D_L t}\right]\right\}$$

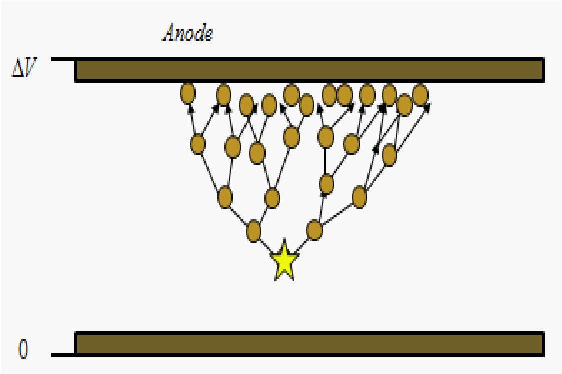
$$i_4(t) = \exp[\alpha Wt]\frac{n_0 e}{h}\left(\frac{D_L}{\pi}\right)^{1/2} t^{-1/2}\left\{\exp\left[-\frac{4h^2 + W^2 t^2}{4D_L t}\right] - \exp\left[-\frac{(h - Wt)^2}{4D_L t}\right]\right\}$$

A simplified formula can be obtained by making the integral in Eq. (4) from minus infite.

Brambring's formula

Brambring writes the avalanche equation as [2]

$$\frac{\partial n}{\partial t} + div\vec{J} = \alpha_T \vec{J} \qquad (5)$$



In one-dimensional case

$$-\frac{\partial n}{\partial t} + \alpha_T\left(W_T - D_L\frac{\partial}{\partial z}\right)n + D_L\frac{\partial^2}{\partial z^2}n - W_T\frac{\partial}{\partial z}n = 0$$

n_0, initial electron number
e, electron charge
h, gap width
W_T, drift velocity
α_T, effective Townsend coeffecient
D_L, longitudinal diffusion coeffecient

Solution for n_o electrons emitted from the cathode

$$n(z,t) = \frac{n_0}{(4\pi D_L t)^{1/2}}\exp\left[\alpha_T W_T t - \frac{(W_T+\alpha_T D_L)^2}{4D_L}t + \frac{W_T+\alpha_T D_L}{2D_L}z\right] \times\left\{\exp\left[-\frac{z^2}{4D_L t}\right] - \exp\left[-\frac{(2h-z)^2}{4D_L t}\right]\right\} \qquad (6)$$

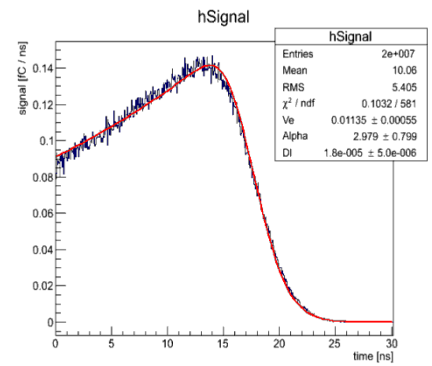
Boundary condition:
n(h)=0

[2] J. Brambring, Z. Physik 179, 532 1964

Brambring's formula

Induced current at the anode

$$I(t)=\frac{N(t)eW}{h}=\frac{eW\int_{-\infty}^{h}n(z,t)dz}{h}=I_1(t)+I_2(t) \quad (7)$$



where

$$I_1(t)=\exp[\alpha_T W_T t]\frac{n_0 eW_T}{2h}\left[1-Erf\,\frac{(W_T+\alpha_T D_L)t-h}{(4D_L t)^{1/2}}\right]$$

$$I_2(t)=\exp[\alpha_T W_T t]\frac{n_0 eW_T}{2h}\exp\left[\frac{(W_T+\alpha_T D_L)h}{D_L}\right]\left[Erf\,\frac{(W_T+\alpha_T D_L)t+h}{(4D_L t)^{1/2}}-1\right]$$

The fomula of the induced current is adopted by J. De Urquijo for obtaining the electron swarm parameters. [3-5]

[3] J. de Urquijo, et al., Eur. Phys. J. D **51**, 241–246 (2009)
[4] J. de Urquijo, et al., 1999 J. Phys. D: Appl. Phys. 32 41
[5] J.L. Hern´andez-´Avila, E. Basurto, J. de Urquijo, J. Phys. D **35**, 2264 (2002), and references therein

Description from Townsend's theory

In 1910 John Sealy Townsend measured an approximately exponential increase of the current between two parallel electrodes (with gas in between), when a sufficiently high voltage difference exists between them.

If we ignore the boundary conditions, the total number of electrons that cross the a certain plane z=constant is

$$N(z) = \int_0^\infty J(z,t)dt = \left(W - D_L \frac{\partial}{\partial z}\right)\int_0^\infty n(z,t)dt$$

From Huxley's equation

$$N(z) = n_0 \frac{\lambda + \eta}{2\eta} \exp\left[\left(\frac{W - \sqrt{W^2 - 4\alpha D_L W}}{2D_L}\right)z\right] \quad (8)$$

$$2\lambda = W / D_L, \eta = \left(\lambda^2 - 2\lambda\alpha\right)^{1/2}$$

From Brambring's equation

$$N(z) = n_0 \frac{\lambda' + \eta'}{2\eta'} \exp\left(\alpha_T z\right) \quad (9)$$

$$2\lambda' = \frac{W_T + \alpha_T D_L}{D_L}, \eta' = \lambda' - \alpha_T$$

α_T represents the Townsend coefficient in the current growth

Relation between Huxley's and Brambring's formulas

$$\vec{J} = \left(-D\frac{\partial}{\partial x}n, -D\frac{\partial}{\partial y}n, W_T n - D_L \frac{\partial}{\partial z}n \right)$$

Huxley

$$\frac{\partial n}{\partial t} + div\vec{J} = \alpha W n$$

Brambring

$$\frac{\partial n}{\partial t} + div\vec{J} = \alpha_T \vec{J}$$

Urquijo's measurement

$$\begin{cases} \alpha = \alpha_T - \alpha_T^2 \dfrac{D_L}{W} \\ W = W_T + \alpha_T D_L \end{cases} \quad (10)$$

$$\begin{cases} \alpha_T = \dfrac{W - \sqrt{W^2 - 4\alpha D_L W}}{2D_L} \\ W_T = W - \alpha_T D_L \end{cases}$$

The coefficient has two valid definitions:

a) α is the microscopic coefficient, defined as the number of new electrons created by each electron in drifting 1 cm.

b) α_T is the coefficient in the exponent of the growth of current relationship (as in Eq. (9)), coinciding with α_{SST}.

In the region where the first-order theory is applicable, Huxley's equation is correct [6]

[6] R. W. Crompton, Comments on some recent analyses of the pulsed Townsend discharge,

- Motivation
- Avalanche equations
- **Garfield++ simulations of electron swarm coefficients**
  - ✓ Simulations in Magboltz
  - ✓ Simulations in Garfield++
  - ✓ Drift velocity, effective Townsend coefficient, longitudinal diffusion coefficient
  - ✓ Attachment coefficient
  - ✓ Transverse diffusion coefficient
  - ✓ Comparison
- Avalanche statistics
- Efficiency estimate for RPC
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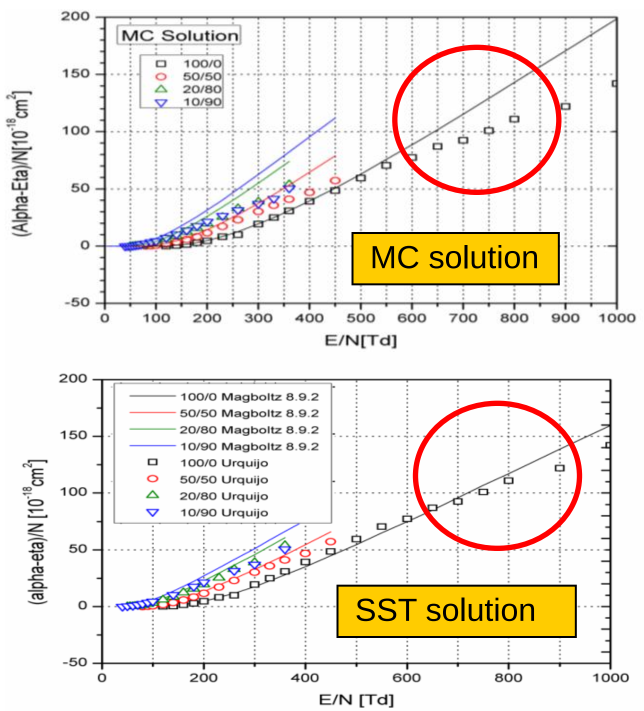
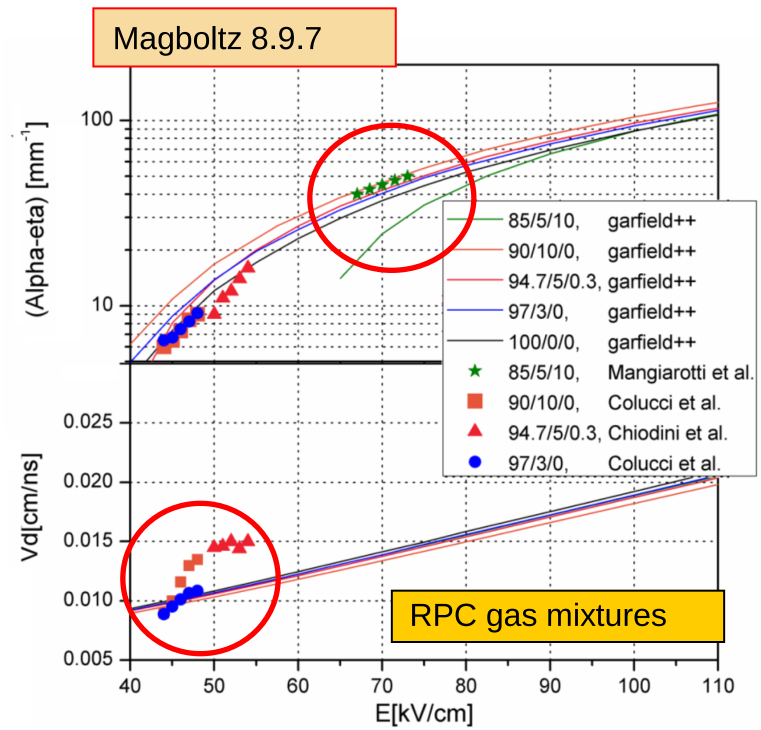
Simulation in Magboltz

See my talk in the 10th RD51 collaboration meeting
https://indico.cern.ch/contributionDisplay.py?sessionId=11&contribId=3&confId=179611

- At recent, we have theoretically understood α_{SST}
- For electron negative gases (Freon/iso-butane/SF6), we have observed a disagreement between measurement and simulation(right figures).



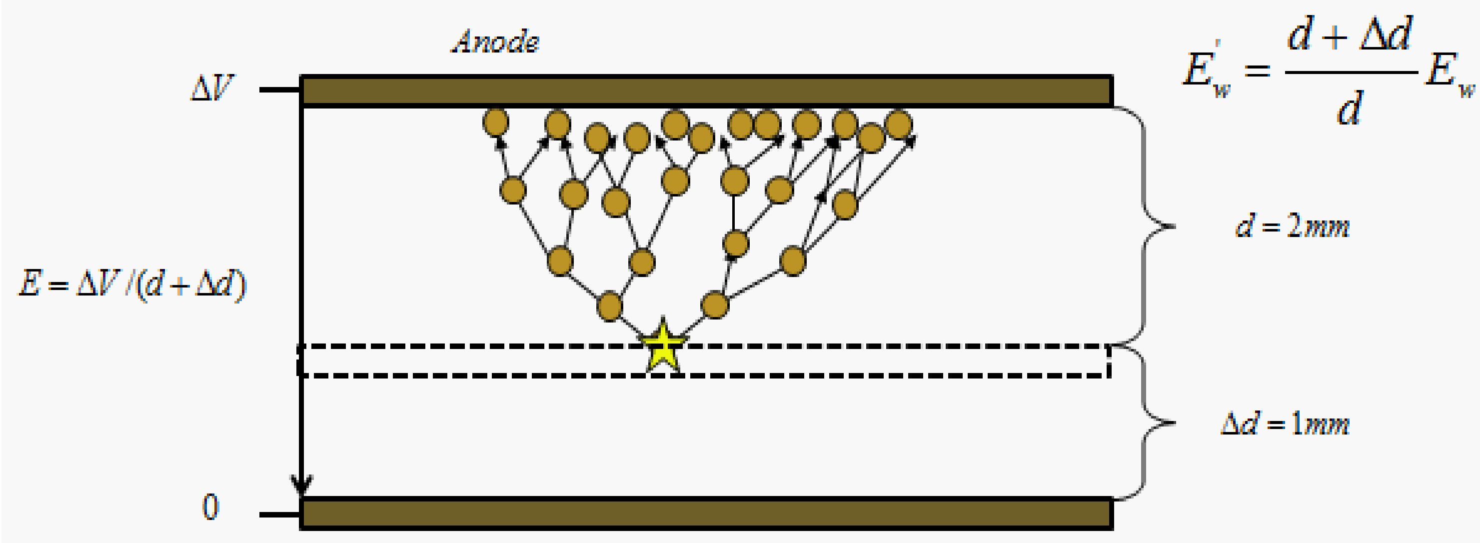


Simulation in Garfield++

See my presentation in RD51 Miniweek (3-5 Dec 2012)
https://indico.cern.ch/getFile.py/access?contribId=19&sessionId=7&resId=0&materialId=slides&confId=218341

Garfield++ provides a microscopic simulation for the avalanche evolution. The electron swarm coefficients can be obtained through the similar approach as Urquijo's measurement.

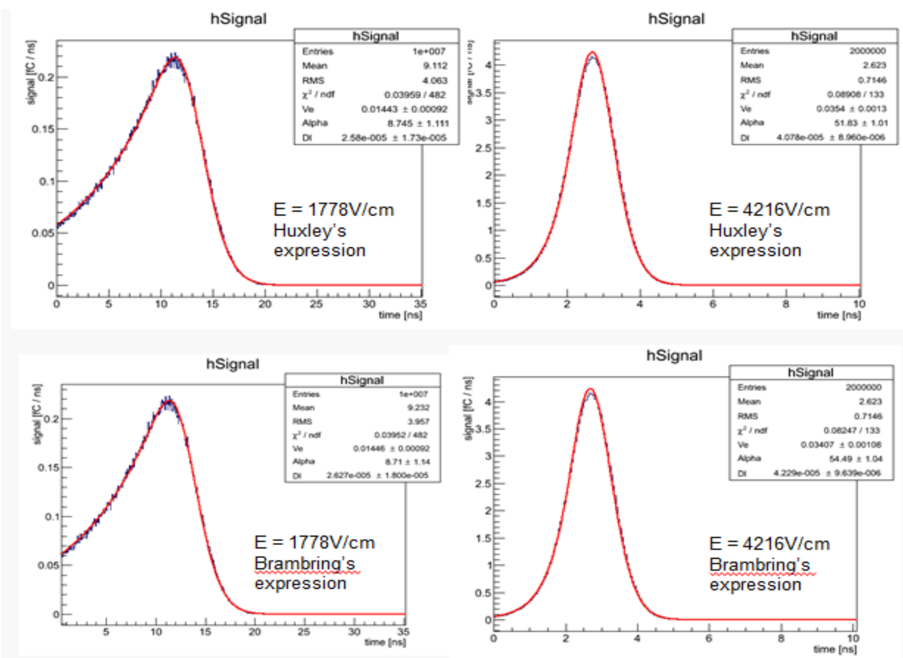
a) From the waveform of the induced current, we have the effective Townsend, drift velocity, and longitudinal diffusion coefficients
b) From the avalanche size distribution ,we have the attachment coefficient
c) From the electron distribution at the anode, we have the transverse diffusion coefficient.



Effective Townsend coefficient, drift velocity and longitudinal diffusion coefficient

- The waveforms of the induced current are simulated by the class *AvalancheMicroscopic*.
- Fitting the waveforms with Huxley's formula, we have the microscopic electron swarm coefficients.

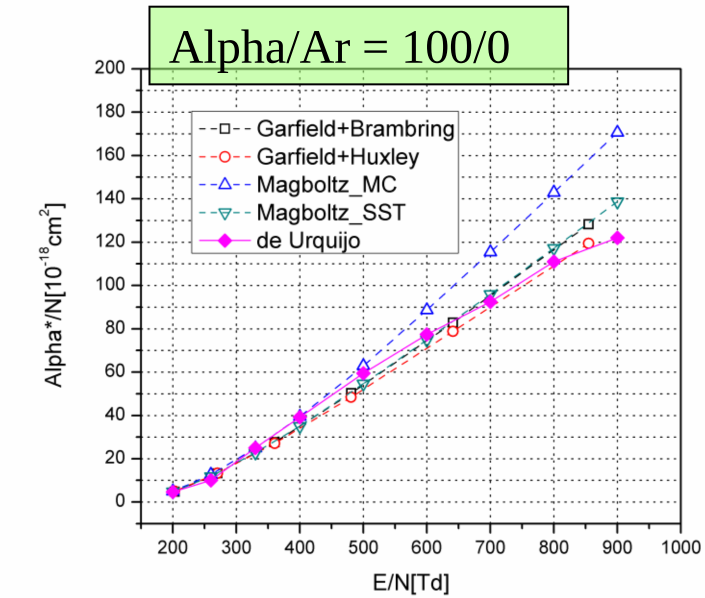
Waveform fit of pure freon

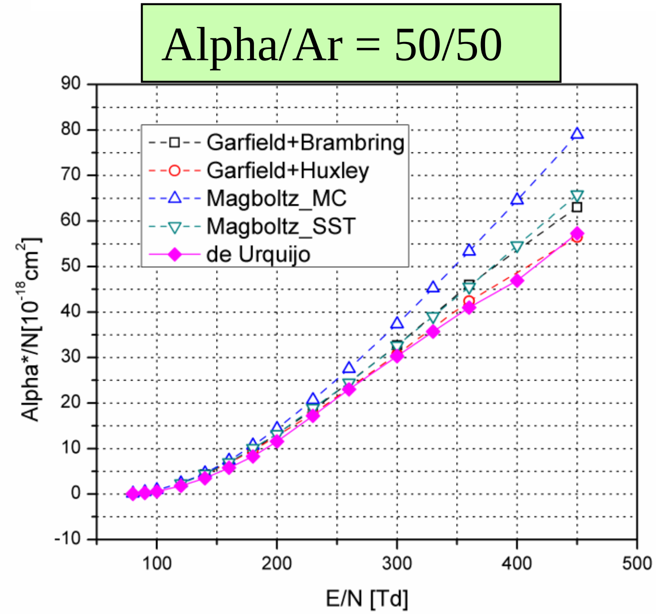


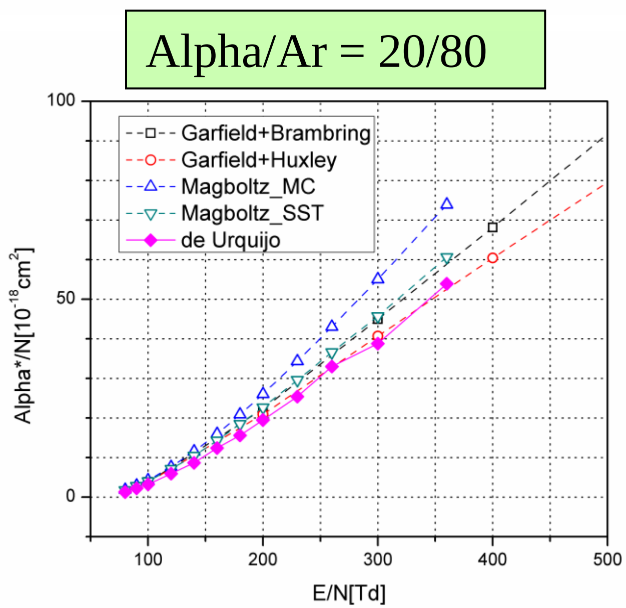
Fit with Huxley's formula → Microscopic coefficients

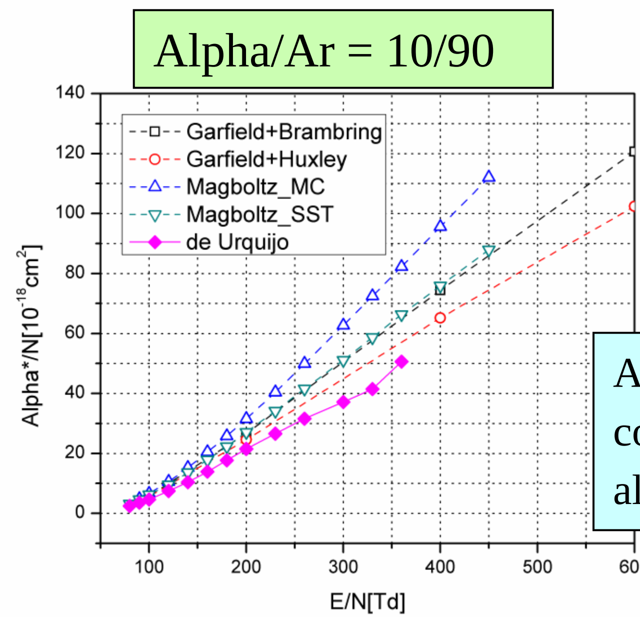
Fit with Brambring's formula (Urquijo) → Coefficients in a current growth experiment

Effective Townsend coefficient





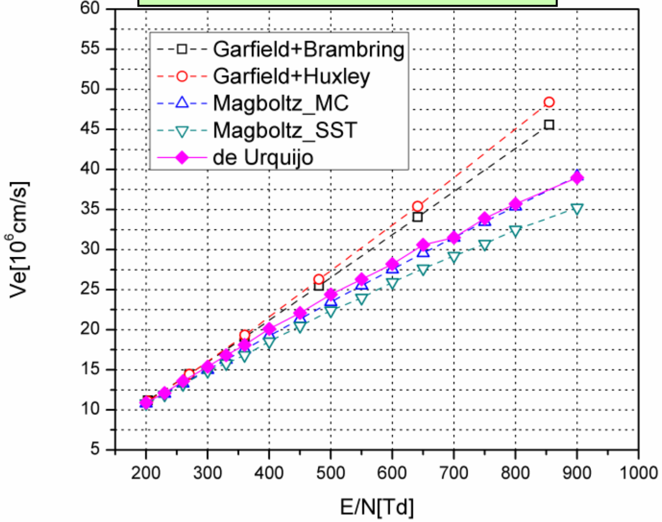




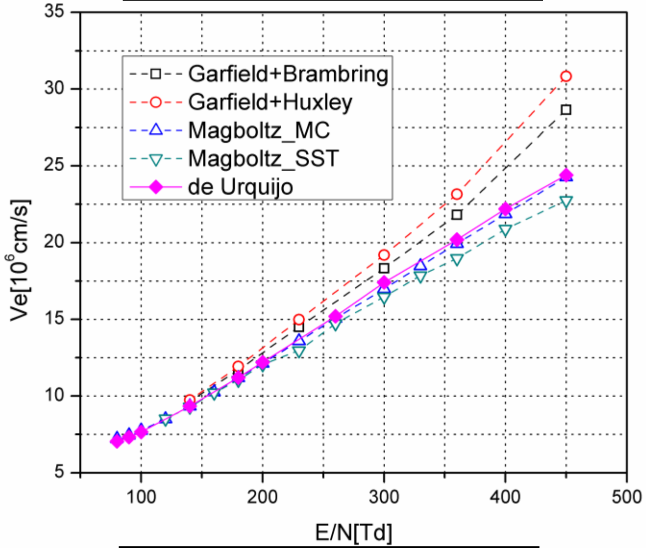
Alpha_Brambring coincides with alpha_SST

Drift velocity

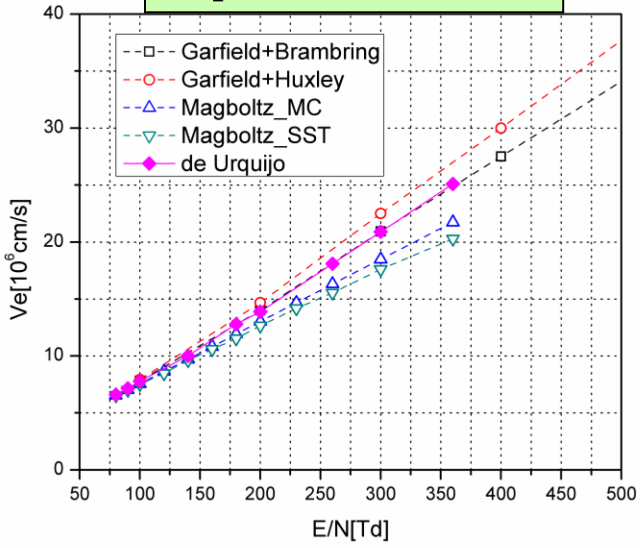
Alpha/Ar = 100/0



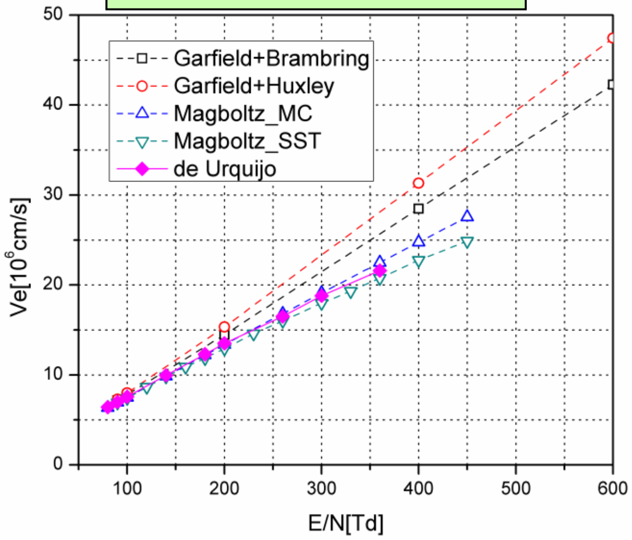
Alpha/Ar = 50/50



Alpha/Ar = 20/80



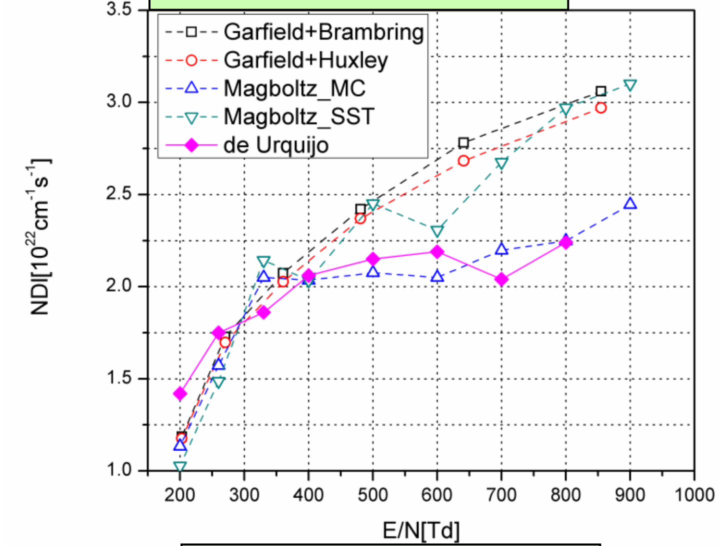
Alpha/Ar = 10/90



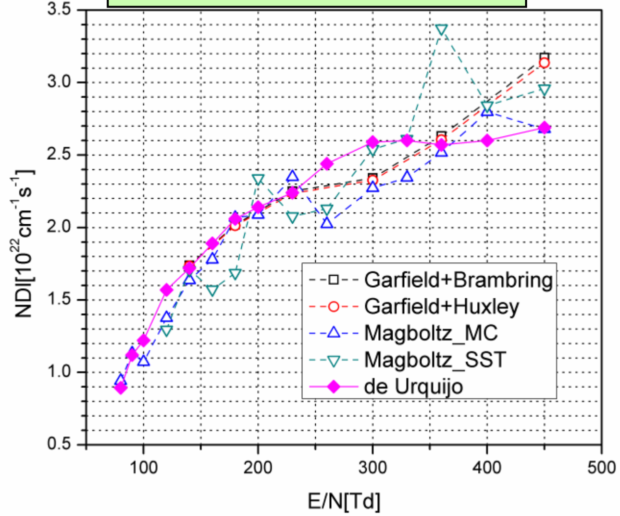
Definition problem?

Longitudinal diffusion coefficient

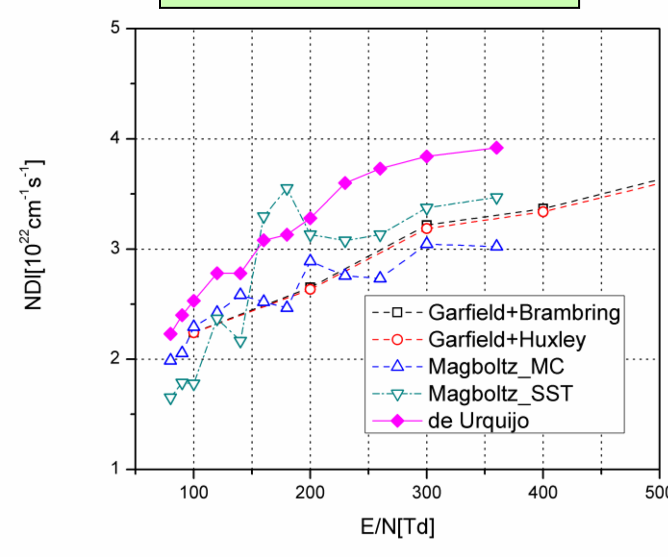
Alpha/Ar = 100/0



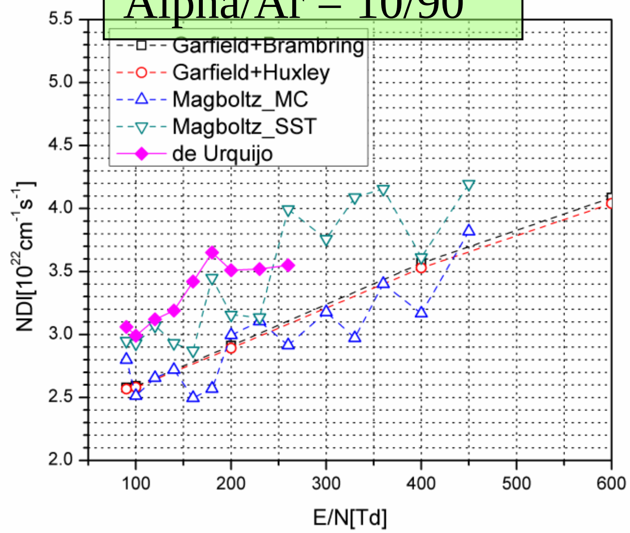
Alpha/Ar = 50/50



Alpha/Ar = 20/80



Alpha/Ar = 10/90



Large error

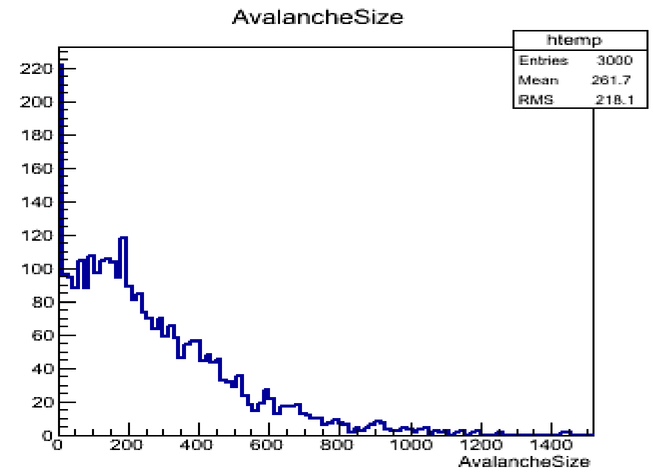
Attachment coefficient

The general solution of Legler's model [7] gives the probability to find n electrons at position z:

$$P(n,z)=\begin{cases} K\dfrac{\bar{n}(x)-1}{\bar{n}(x)-K}, & n=0 \\ \bar{n}(x)\left(\dfrac{1-K}{\bar{n}(x)-K}\right)^2\left(\dfrac{\bar{n}(x)-1}{\bar{n}(x)-K}\right)^{n-1}, & n>0 \end{cases} \quad (11)$$

where

$$\bar{n}(x)=e^{(\alpha_i-\eta_a)z}=e^{\alpha z}, K=\frac{\eta_a}{\alpha_i}$$



[7] Werner Legler, Z. Naturforschung. 16a (1961) 253.,

Transverse diffusion coefficient

We take Huxley's formula. the number density (integral over time) of electrons that pass the anode (z=h) is

$$N(r,h)=\int_0^{\infty} j(r,z,t)dt\Big|_{z=h} \approx \frac{n_0}{2\pi\sigma'^2}\exp\left(-\frac{1}{2\sigma'^2}r^2\right) \qquad (12)$$

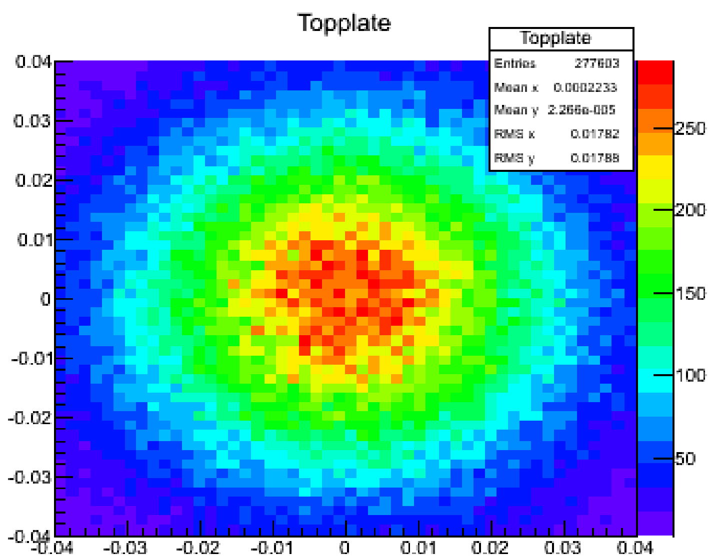
where

$$\tilde{W} = \left(W^2 - 4\alpha D_L W\right)^{1/2}$$

$$\sigma'_L = \sigma_L\sqrt{\frac{W}{\tilde{W}}} = \sqrt{\frac{2D_L z}{\tilde{W}}}$$

$$\sigma' = \sigma\sqrt{\frac{W}{\tilde{W}}} = \sqrt{\frac{2Dz}{\tilde{W}}}$$

$$r = \sqrt{x^2 + y^2}$$



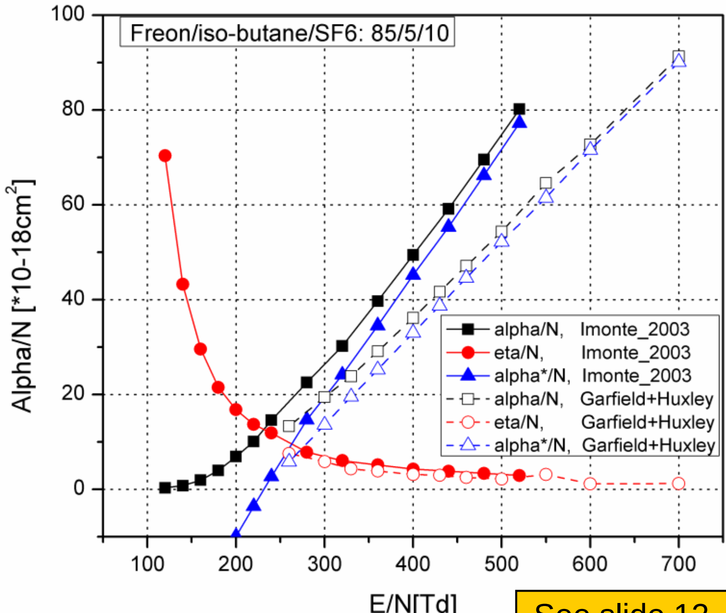
The transverse diffution coefficient can be obtained by fitting the electron distribution at the anode with Eq. (12)

Comparison: Freon/iso-butane/SF6 mixture

Comparison between different approaches:

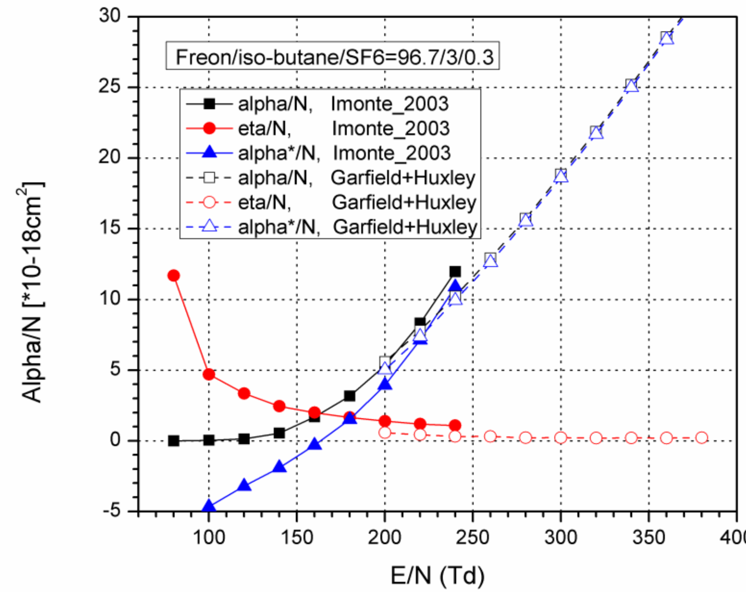
- ✓ Solid symbols are results simulated by Werner Riegler in 2003, with Imonte
- ✓ Open symbols are results simulated with Garfield+Huxley approach

Freon/iso-butane/SF6 = 85/5/10



See slide 12 - right

Freon/iso-butane/SF6 = 96.7/3/0.3



[8] Werner Riegler, Nucl. Instr.and Meth. A 500 (2003) 144–162

- Motivation
- Avalanche equations
- Garfield++ simulations of electron swarm coefficients
- **Avalanche statistics**
  - ✓ Avalanche size distribution for electron-negative gases
  - ✓ Relative variance
- Efficiency estimate for RPC
- Summary and outlook

Avalanche size distribution

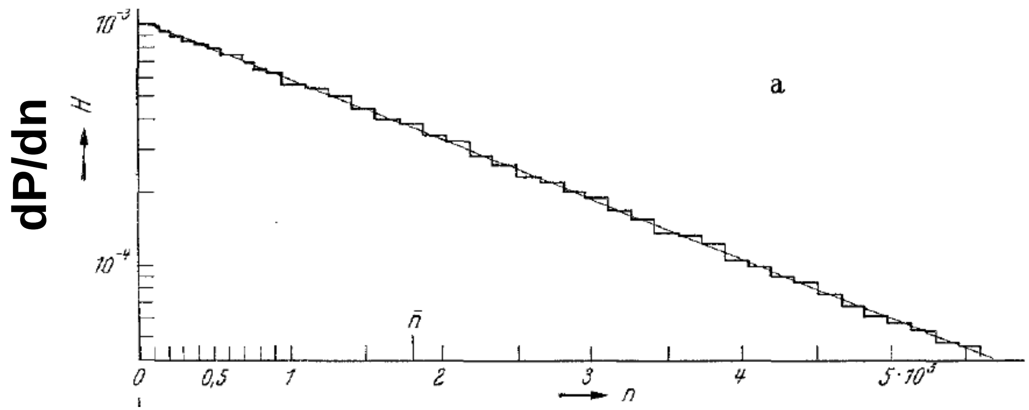
The average number of electrons created by a single electron (N_o=1) over a distance h is:

$$\bar{n} \sim e^{\alpha z}$$

The avalanche size distribution can be obtained (in statistical equilibrium) through **Legler model** that predicts an exponential distribution of the final number of electrons in the avalanche (previously known as the **Furry law**)

avalanche size distribution

$$\frac{dP}{dn} = \frac{1}{\bar{n}} e^{-n/\bar{n}} \qquad (13)$$



$$f = \frac{\sigma^2}{\bar{n}^2} = 1$$

f: relative variance

[9] H. Schlumbohm, Z. fur Physik, 511(1958)563

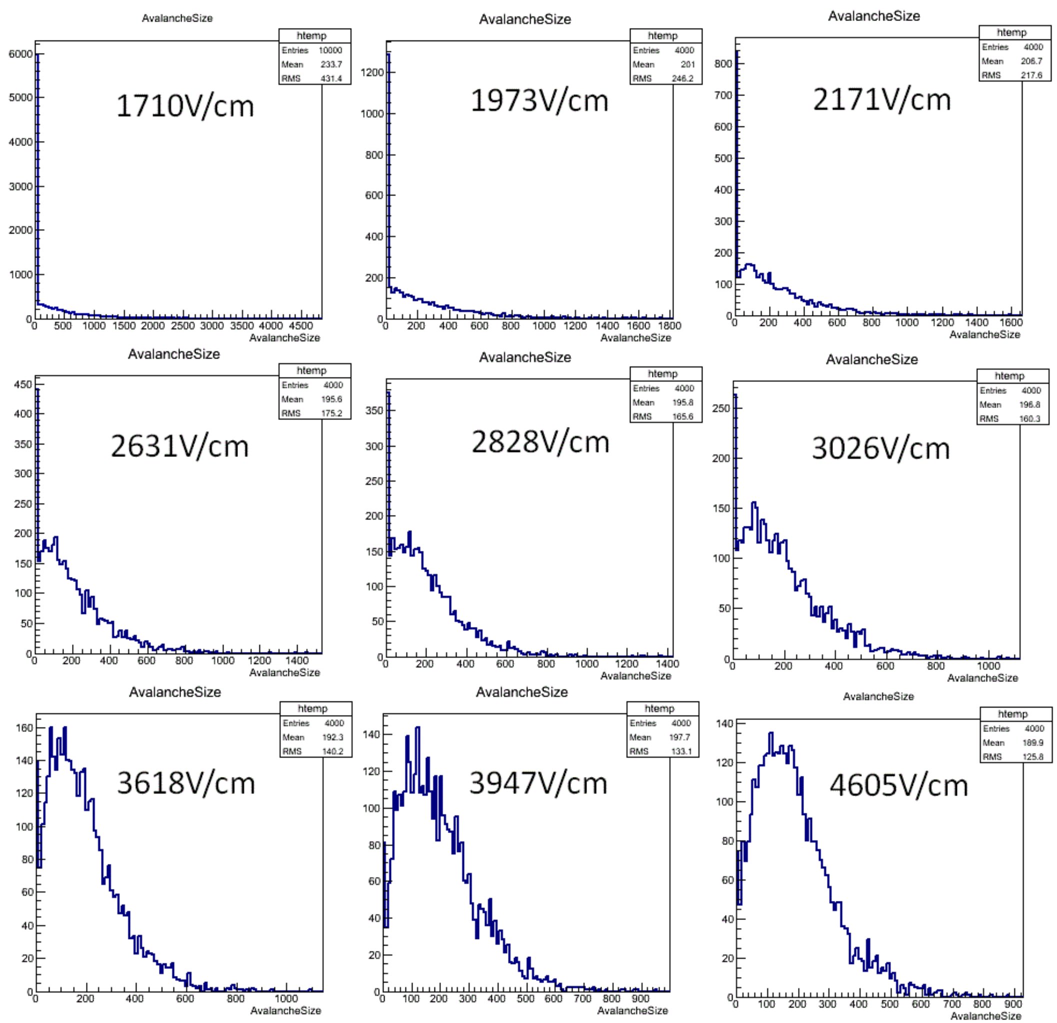
Two important practical effects have been ignored in the discussion, that would imply a deviation of the number of electrons created in an avalanche from an exponential distribution. They are both characteristic of high fields

(1) The electron keeps memory of the previous interaction, so the **electrons do not reach statistical equilibrium** between two ionizing collisions.

(2) The electric field of the avalanche itself is comparable to the applied field, (**self space-charge is important)**.

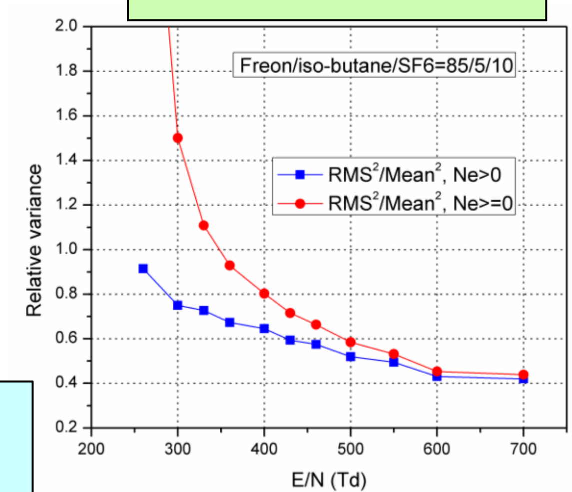
The first one has been solved in Garfield++ by electron tracking.

Avalanche distributions for PPC with Freon/iso-butane/SF6=85/5/10 mixture



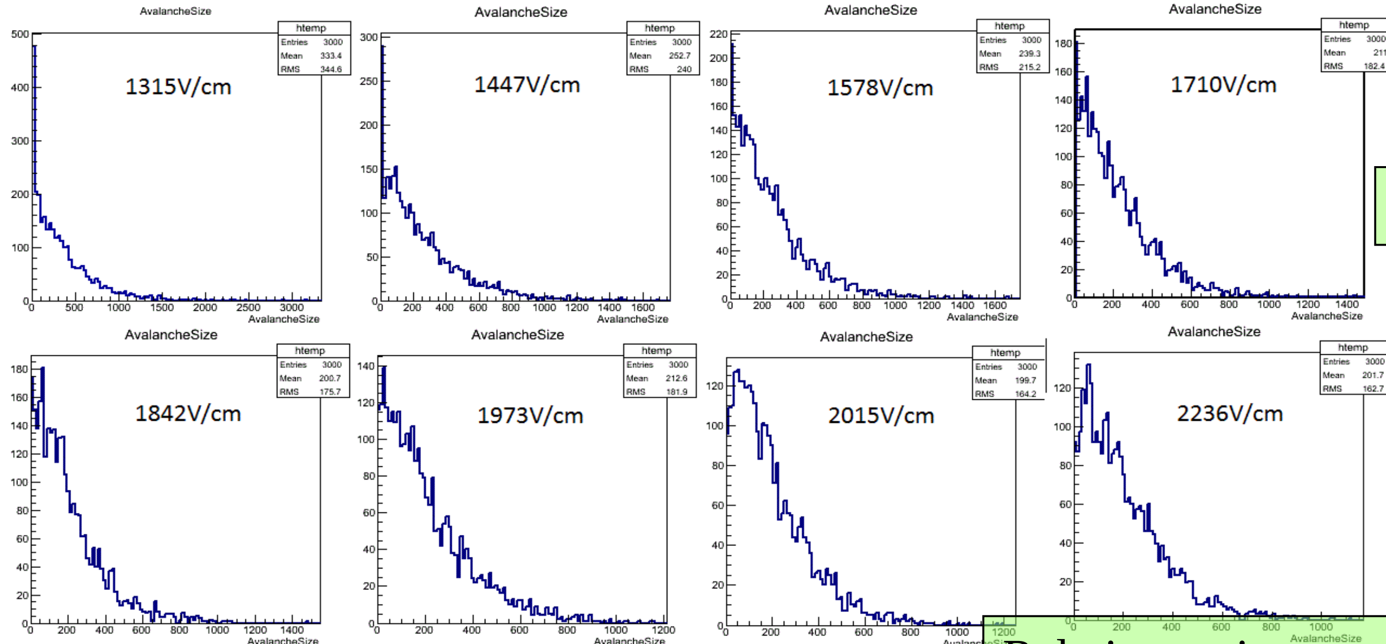
P = 20Torr

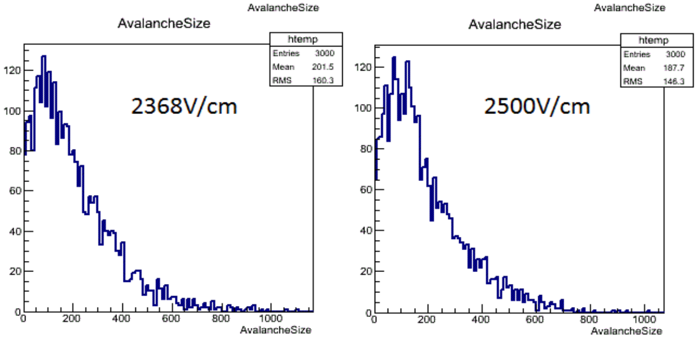
Relative variance



The transition from Furry law to Polya function can be seen when the field increases.

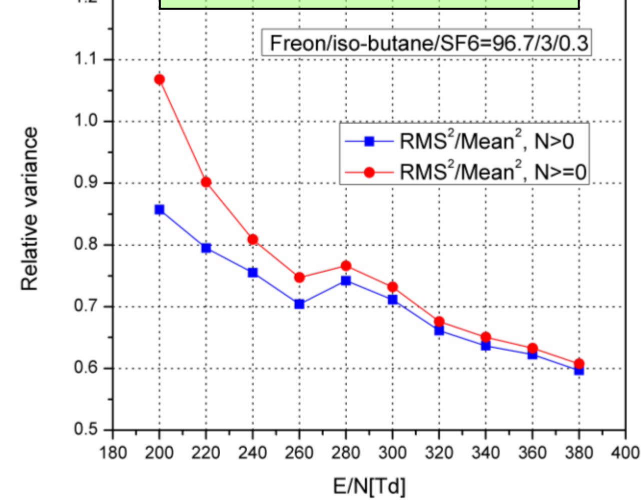
Avalanche distributions for PPC with Freon/iso-butane/SF_6=96.7/3/0.3 mixture



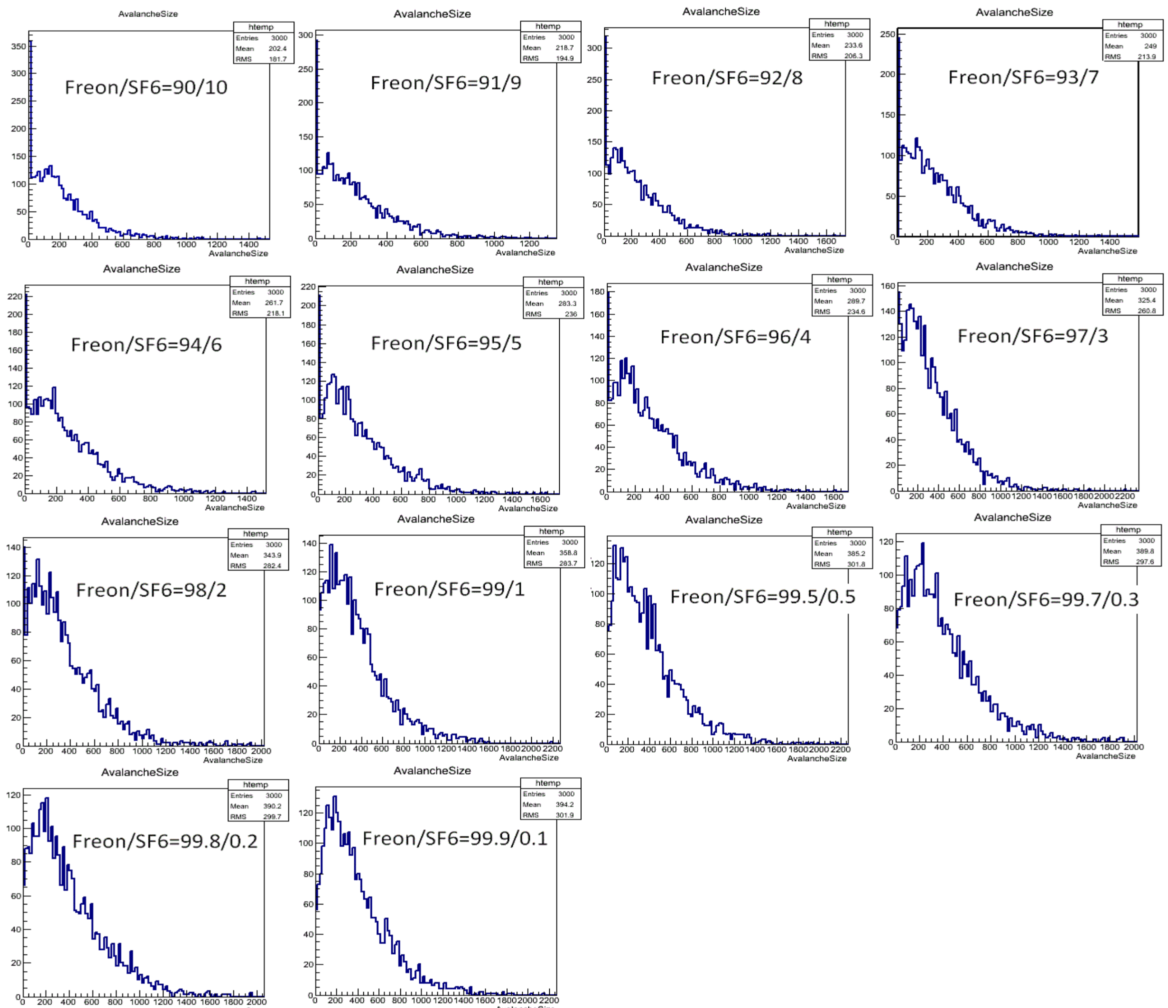


P = 20Torr

Relative variance



Avalanche distributions for PPC with Freon/SF6 mixtures



P = 20Torr

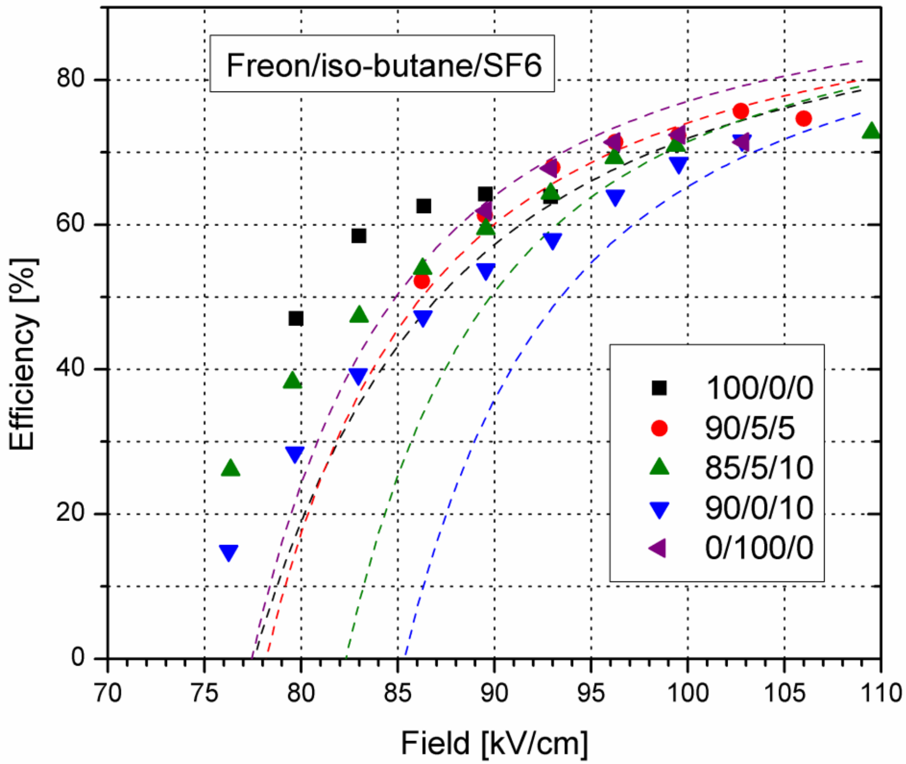
The primary avalanche distributions can be taken as the input parameter for the following hydrodynamic model for RPC simulation.

- Motivation
- Avalanche equations
- Garfield++ simulations of electron swarm coefficients
- Avalanche statistics
- **Efficiency estimate for RPC**
- Summary and outlook

Efficiency estimate with analytical formula

Very preliminary!

$$\varepsilon = 1 - e^{-(1-\eta/\alpha)d/\lambda}\left[1 + \frac{\alpha - \eta}{E_W e_0} Q_{th}\right]^{1/\alpha\lambda} \qquad (14) \quad [10]$$



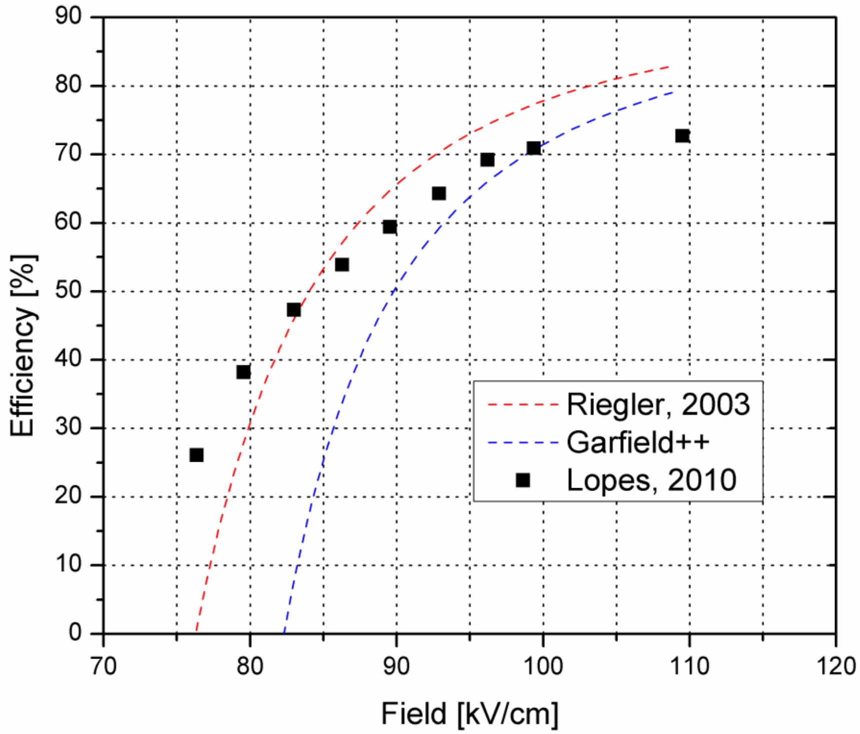
[11]

- ✓ Cross-section of SF6 has been changed?
- ✓ Alpha for pure Freon is underestimated?

[10] W. Riegler, et al., Nucl. Instr. and Meth. A 500 (2003) 144–162
[11] L. Lopes, et al., Nucl. Instr. and Meth. A (2010), doi:10.1016/j.nima.2010.08.073

Analytical description for 85/5/10 mixture

Freon/iso-butane/SF6 = 85/5/10



See slide 12 - right

Solid square: Measurements of Lopes, 2010
Red line: analytical formula with parameters simulated by Imonte 2002
Blue line: analytical formula with parameters simulated by Garfield++

Summary and outlook

- α_{SST} is theoretically understood.
- Electron swarm parameter simulation in Garfield++
- Cross-sections of electron negative gases
- Precise simulation of Avalanche statistics is possible

Next

- Hydrodynamic model including the space charge effect

Thanks for your attention