(bosonic)

Open String Field Theory - status update

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What is String Field Theory?

- Field theoretic description of all excitations of a string (open or closed) at once.
- Useful especially for physics of backgrounds: tachyon condensation or instanton physics, etc.
- Single Lagrangian field theory which around its various critical points should describe physics of diverse D-brane backgrounds, possibly also gravitational backgrounds.

Open String Field Theory as the fundamental theory ?

- Closed string field theory (Zwiebach 1990) —presumably the fundamental theory of gravity— is given by a technically ingenious construction, but it is hard to work with, and it also may be viewed as too perturbative and `effective'.

- Open string field theory (Witten 1986, Berkovits 1995) might be more fundamental (Sen). (cf. holography.)

First look at OSFT

Open string field theory uses the following data

$$\mathcal{H}_{BCFT}, \quad *, \quad Q_B, \quad \langle . \rangle .$$

Let all the string degrees of freedom be assembled in

$$|\Psi\rangle = \sum_i \int d^{p+1}k \, \phi_i(k) |i, k\rangle ,$$

Witten (1986) proposed the following action

$$S = -\frac{1}{g_o^2} \left[\frac{1}{2} \langle \Psi * Q_B \Psi \rangle + \frac{1}{3} \langle \Psi * \Psi * \Psi \rangle \right] ,$$

First look at OSFT

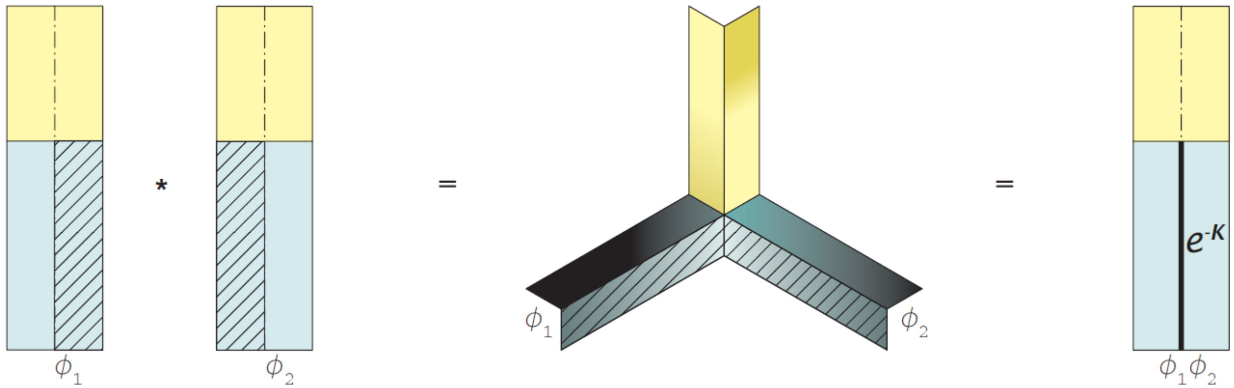
This action has a huge gauge symmetry

$$\delta\Psi = Q_B\Lambda + \Psi * \Lambda - \Lambda * \Psi,$$

provided that the star product is associative, Q_B acts as a graded derivation and < . > has properties of integration.

Note that there is a gauge symmetry for gauge symmetry so one expects infinite tower of ghosts — indeed they can be naturally incorporated by lifting the ghost number restriction on the string field.

Witten's star product



- The elements of string field star algebra are states in the BCFT, they can be identified with a piece of a worldsheet.
- By performing the path integral on the glued surface in two steps, one sees that in fact:

$$|\phi_1\rangle * |\phi_2\rangle = |\phi_1 e^{-K} \phi_2\rangle.$$

Witten's star product as operator multiplication

We have just seen that the star product obeys

$$|\phi_1\rangle * |\phi_2\rangle = |\phi_1 e^{-K} \phi_2\rangle.$$

And therefore states $\hat{\phi} = e^{K/2} \phi e^{K/2}$ obey

$$|\hat{\phi}_1\rangle * |\hat{\phi}_2\rangle = |\widehat{\phi_1 \phi_2}\rangle$$

The star product and operator multiplication are thus isomorphic!

Simple subsector of the star algebra

- The star algebra is formed by vertex operators and the operator *K*. The simplest subalgebra relevant for tachyon condensation is therefore spanned by *K* and *c.* Let us be more generous and add an operator *B* such that *QB=K.*

- The building elements thus obey

$$c^2 = 0, \quad B^2 = 0, \quad \{c, B\} = 1$$
$$[K, B] = 0, \quad [K, c] = \partial c$$

- The derivative *Q* acts as $Q_B K = 0, \quad Q_B B = K, \quad Q_B c = cKc.$

Classical solutions

This new understanding lets us construct solutions to OSFT equations of motion $Q_B\Psi + \Psi * \Psi = 0$ easily.

It does not take much trying to find the simplest solution is $\Psi = \alpha c - cK$

$$Q\Psi = \alpha(cKc) - (cKc)K$$

$$\Psi * \Psi = \alpha^2 c^2 - \alpha c^2 K - \alpha cKc + (cK)(cK)$$

More general solutions are of the form

$$\Psi = Fc\frac{KB}{1-F^2}cF,$$

Here *F=F(K)* is arbitrary

M.S. 2005, Okawa, Erler 2006

Classical solutions

- What do these solutions correspond to?
- In 2011 with Murata we succeeded in computing their energy

$$E = \frac{1}{2\pi^2}\oint_C \frac{dz}{2\pi i}\frac{G'(z)}{G(z)}$$

  in terms of the function $G(z) = 1 - F^2(z)$

- For simple choices of G, one can get perturbative vacuum, tachyon vacuum, or exotic multibrane solution. At the moment the multibrane solutions appear to be a bit singular. (see also follow-up work by Hata and Kojita)

OSFT = physics of backgrounds

- So far all the discussion concerned background independent solutions and aspects of OSFT.
- The new theme of the past year or two, is that OSFT can be very efficient in describing BCFT backgrounds and their interrelation.

Outline

- Introduction
- Review of D-branes:

  - in Ising model

  - in $(\text{Ising})^2 =$ Free boson in S^1/Z_2
- Numerical solutions in OSFT

  - 1, ε branes from σ

  - σ brane from 1, ε → Positive energy solutions !!

  - $1\otimes1$, $\varepsilon\otimes\varepsilon$, $1\otimes\varepsilon$, $\varepsilon\otimes1$,…. branes from $\sigma\otimes\sigma$ → Fractional branes !!

  - Double branes in the universal sector
- Conclusion

Boundary states

- The nicest problems in theoretical physics are ones which are: easy to state, difficult to solve and with many relations to other branches to physics

- One such problem is classification of the boundary states in a given CFT or equivalently admissible open string vacua or D-branes.

Boundary states

- Describe possible boundary conditions from the closed string channel point of view.

- Conformal boundary states obey:

1) the gluing condition $(L_n - \bar{L}_{-n})|B\rangle = 0$

2) Cardy condition (modular invariance)

3) sewing relations (factorization constraints)

See e.g. reviews by Gaberdiel or by Cardy

Boundary states

- The gluing condition is easy to solve:
  For any spin-less primary $|V_\alpha\rangle$ we can define

$$||V_\alpha\rangle\rangle = \sum_{IJ} M^{IJ}(h_\alpha) L_{-I} \bar{L}_{-J} |V_\alpha\rangle$$

where M^{IJ} is the inverse of the real symmetric Gram matrix

$$M_{IJ}(h_\alpha) = \langle V^\alpha | L_I L_{-J} | V_\alpha \rangle$$

where $L_{-X} \equiv L_{-n_k} ... L_{-n_1}$
(with possible null states projected out).

Ishibashi 1989

Boundary states – Cardy's solution

- By demanding that

$$\langle\langle\alpha\|q^{\frac{1}{2}(L_0+\bar{L}_0-\frac{c}{12})}\|\beta\rangle\rangle = \mathrm{Tr}_{\mathcal{H}^{\mathrm{open}}_{\alpha\beta}}\left(\tilde{q}^{L_0-\frac{c}{24}}\right)$$

and noting that RHS can be expressed as

$$\sum_i n^i_{\alpha\beta}\chi_i(\tilde{q})$$

Cardy derived integrality constraints on the boundary states. Surprisingly, for certain class of rational CFT's he found an elegant solution (relying on Verlinde formula)

$$\|\tilde{k}\rangle\rangle = \sum_j \sqrt{\frac{S^j_{\ k}}{S^j_{\ 0}}}\,|j\rangle\rangle$$

where $S^j_{\ k}$ is the modular matrix.

Ising model CFT

- Ising model is the simplest of the unitary minimally models with c = ½.
- It has 3 primary operators

  1 (0,0)

  ε (½,½)

  σ (1/16, 1/16)
- The modular S-matrix takes the form

$$\mathbf{S} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

Boundary states – Cardy's solution

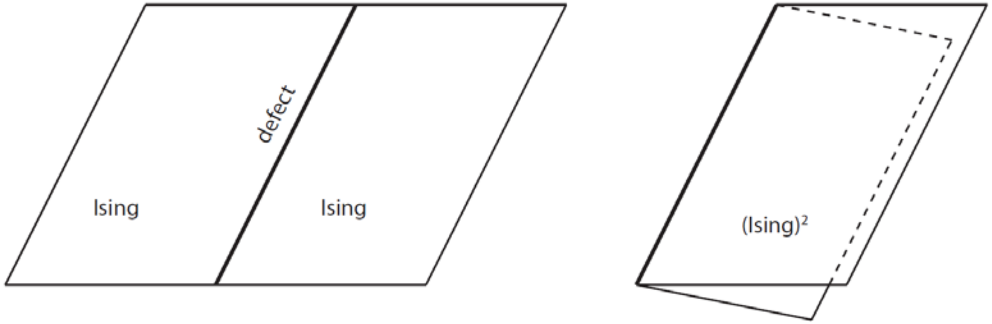
- And thus the Ising model conformal boundary states are

$$\begin{aligned} \|\tilde{0}\rangle\rangle &= \frac{1}{\sqrt{2}}|0\rangle\rangle + \frac{1}{\sqrt{2}}|\varepsilon\rangle\rangle + \frac{1}{\sqrt[4]{2}}|\sigma\rangle\rangle \\ \|\tilde{\varepsilon}\rangle\rangle &= \frac{1}{\sqrt{2}}|0\rangle\rangle + \frac{1}{\sqrt{2}}|\varepsilon\rangle\rangle - \frac{1}{\sqrt[4]{2}}|\sigma\rangle\rangle \\ \|\tilde{\sigma}\rangle\rangle &= |0\rangle\rangle - |\varepsilon\rangle\rangle \end{aligned}$$

- The first two boundary states describe fixed (+/-) boundary condition, the last one free boundary condition

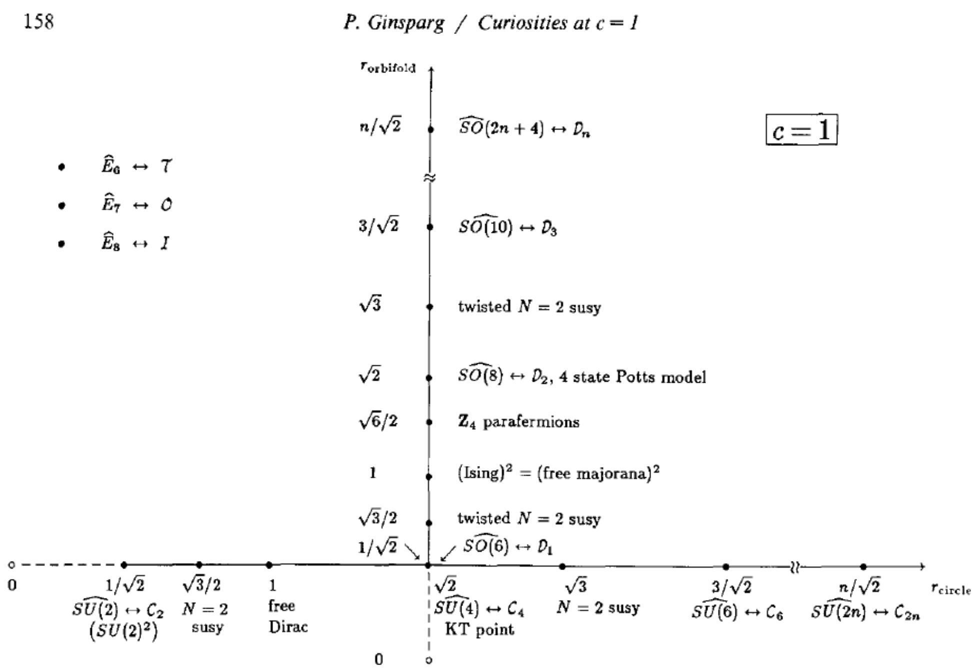
(Ising)^2

- This model naturally arises when one considers Ising model on a plane with a defect line and employs the folding trick.



(Ising)^2

- (Ising)^2 model is well known point on the orbifold branch of the moduli space of c=1 models



(Ising)^2

- Even though Ising model itself has only 3 bulk primaries, (Ising)^2 has infinite number of them (Yang 1987)

$\Delta = \bar{\Delta}$	Multiplicity	(Ising)^2 Examples	Orbifold Examples
$n^2 = 0, 1, 4, \ldots$	1	$1 \otimes 1, \varepsilon \otimes \varepsilon$	$1,\ \partial X \bar{\partial} X$
$\frac{(n+1)^2}{2} = \frac{1}{2}, 2, \frac{9}{2}, \ldots$	2	$1 \otimes \varepsilon, \varepsilon \otimes 1$	$\cos(\sqrt{2}X),\ \cos(\sqrt{2}\tilde{X})$
$\frac{(2n+1)^2}{8} = \frac{1}{8}, \frac{9}{8}, \frac{25}{8}, \ldots$	1	$\sigma \otimes \sigma$	$\sqrt{2}\cos(\frac{X}{\sqrt{2}})$
$\frac{(2n+1)^2}{16} = \frac{1}{16}, \frac{9}{16}, \frac{25}{16}, \ldots$	2	$1 \otimes \sigma, \sigma \otimes 1, \varepsilon \otimes \sigma, \sigma \otimes \varepsilon$	twist fields, excited twist fields

- It is precisely equivalent to a free boson on an orbifold S^1/Z_2 with radius $R_{orb} = \sqrt{2}$ (in our units $\alpha' = 1$.)

Boundary states in $(\text{Ising})^2$

- Some boundary states are readily available
- However, in general the problem of constructing D-branes in tensor product of simple CFT's is rather difficult since we get always infinitely many new primaries, and hence potentially many new exotic boundary states.

Boundary states in (Ising)^2

- Here is the list found by Affleck and Oshikawa (1996)

$(\text{Ising})^2$ D-brane	Interpretation	Energy $= \langle 1 \rangle$	$\frac{\langle \partial X \bar{\partial} X \rangle}{\langle 1 \rangle}$	Position
$1 \otimes \varepsilon$	fractional D0	$\frac{1}{2}$	$+1$	πR
$\varepsilon \otimes 1$	fractional D0	$\frac{1}{2}$	$+1$	πR
$1 \otimes 1$	fractional D0	$\frac{1}{2}$	$+1$	0
$\varepsilon \otimes \varepsilon$	fractional D0	$\frac{1}{2}$	$+1$	0
$1 \otimes \sigma$	fractional D1	$\frac{1}{\sqrt{2}}$	-1	$-$
$\sigma \otimes 1$	fractional D1	$\frac{1}{\sqrt{2}}$	-1	$-$
$\varepsilon \otimes \sigma$	fractional D1	$\frac{1}{\sqrt{2}}$	-1	$-$
$\sigma \otimes \varepsilon$	fractional D1	$\frac{1}{\sqrt{2}}$	-1	$-$
$\sigma \otimes \sigma$	centered bulk D0	1	$+1$	$\frac{\pi R}{2}$
$\sum_i a_i(\phi) \vert \text{AT}, i\rangle\rangle$	generic bulk D0	1	$+1$	ϕR
$\sum_i b_i(\tilde{\phi}) \vert \text{AT}, i\rangle\rangle$	generic bulk D1	$\sqrt{2}$	-1	$-$

Now we would like to find all this from OSFT !?!

Numerical solutions in OSFT

- To construct new D-branes in a given BCFT with central charge *c* using OSFT, we consider strings 'propagating' in a background $\mathrm{BCFT}_c \otimes \mathrm{BCFT}_{26\text{-}c}$ and look for solutions which do not excite any primaries in $\mathrm{BCFT}_{26\text{-c}}$.

Numerical solutions in OSFT

- To get started with OSFT, we first have to specify the starting BCFT, i.e. we need to know:

  - spectrum of boundary operators

  - their 2pt and 3pt functions

  - bulk-boundary 2pt functions (to extract physics)

- The spectrum for the open string stretched between D-branes *a* and *b* is given by boundary operators which carry labels of operators which appear in the fusion rules

$$\phi_a \times \phi_b = \sum_c N_{ab}^c \, \phi_c$$

Numerical solutions in OSFT

- In the case of Ising the boundary spectrum is particularly simple

D-brane	Energy	Boundary spectrum
$\|\tilde{1}\rangle\rangle$	$\frac{1}{\sqrt{2}}$	1
$\|\tilde{\varepsilon}\rangle\rangle$	$\frac{1}{\sqrt{2}}$	1
$\|\tilde{\sigma}\rangle\rangle$	1	1, ε

Ellwood conjecture

- Solutions to OSFT e.o.m. are believed to be in 1-1 correspondence with consistent boundary conditions.

- The widely believed (and tested, but unproven) Ellwood conjecture states that for every on-shell $\mathcal{V}_{cl}$

$$\langle \mathcal{V}_{cl} | c_0^- | B_\Psi \rangle = -4\pi i \langle I | \mathcal{V}_{cl}(i) | \Psi - \Psi_{\rm TV} \rangle ,$$

Here Ψ is a solution of the e.o.m., $\Psi_{\rm TV}$ is the tachyon vacuum and $|B_\Psi\rangle$ is the boundary state we are looking for.

Ellwood (2008)

Generalized Ellwood invariants

- The restriction to an on-shell state can be bypassed. Any solution built using reference BCFT_0 can be written as

$$\Psi = \sum_j \sum_{\substack{I=\{n_1,n_2,\ldots\} \\ J=\{m_1,m_2,\ldots\}}} a^j_{IJ}\, L^{\text{matter}}_{-I} |\mathcal{V}_j\rangle \otimes L^{\text{ghost}}_{-J} c_1 |0\rangle$$

and uplifted to $\text{BCFT}_0 \otimes \text{BCFT}_{\text{aux}}$, where BCFT_{aux} has *c=0* and contains free boson *Y* with Dirichlet b.c. One can then compute Ellwood invariant with $\tilde{\mathcal{V}}^\alpha = c\bar{c}V^\alpha e^{2i\sqrt{1-h}\,Y} w$

Trick inspired by Kawano, Kishimoto and Takahashi (2008)

Generalized Ellwood invariants

- Since $|B_\Psi\rangle^{\mathrm{CFT}_0\otimes\mathrm{CFT}_{\mathrm{aux}}} = |B_\Psi\rangle^{\mathrm{CFT}_0} \otimes |B_0\rangle^{\mathrm{CFT}_{\mathrm{aux}}}$
  we find

That the lift leads to the factorization is our little assumption !

$$\langle\, c\bar{c}V^\alpha |\, c_0^- \,|B_\Psi\,\rangle = -4\pi i\langle\, E[\tilde{\mathcal{V}}^\alpha]|\tilde{\Psi} - \tilde{\Psi}_{TV}\,\rangle$$

- This is gauge invariant even w.r.t. the gauge symmetry of the original OSFT based on BCFT_0

$$\mathrm{Lift}\circ(\mathrm{Gauge\ Transf})_\Lambda = (\mathrm{Gauge\ Transf})_{\mathrm{Lift}(\Lambda)}\circ\mathrm{Lift}$$

Boundary state from Ellwood invariants

- The coefficients of the boundary state

$$|B_\Psi\rangle = \sum_\alpha n_\Psi^\alpha ||V_\alpha\rangle\rangle$$

can be computed from OSFT solution via

$$n_\Psi^\alpha = 2\pi i \langle I|\mathcal{V}^\alpha(i)|\Psi - \Psi_{\mathrm{TV}}\rangle^{\mathrm{BCFT}_0 \otimes \mathrm{BCFT}_{\mathrm{aux}}}$$

$$\mathcal{V}^\alpha = c\bar{c}V^\alpha \, e^{2i\sqrt{1-h_\alpha}\,Y} \, w$$

See: Kudrna,Maccaferri, M.S. (2012)
Alternative attempt:
Kiermaier, Okawa, Zwiebach (2008)

Tachyon condensation on the σ-brane

- The computation proceeds along the similar line as for (Moeller, Sen, Zwiebach)
- String field truncated to level 2:

$$|\psi\rangle = tc_1|0\rangle + ac_1|\epsilon\rangle + uc_{-1}|0\rangle + vc_1L^I_{-2}|0\rangle + wc_1L^R_{-2}|0\rangle$$

- The action is

$$\begin{aligned}\mathcal{V}(t,a,u,v,w) &= -\frac{1}{2}t^2 - \frac{1}{4}a^2 - \frac{1}{2}u^2 + \frac{1}{8}v^2 + \frac{51}{8}w^2 + \frac{27}{64}va^2 - \frac{255}{64}wa^2 + \frac{11}{16}ua^2 + \\ &- \frac{165\sqrt{3}}{3456}tuv - \frac{8415\sqrt{3}}{3456}tuw + \frac{1049\sqrt{3}}{9216}tv^2 + \frac{256423563\sqrt{3}}{746496}tw^2 + \frac{66\sqrt{3}}{128}ut^2 - \\ &- \frac{15\sqrt{3}}{256}vt^2 - \frac{765\sqrt{3}}{256}wt^2 + \frac{19\sqrt{3}}{192}tu^2 + \frac{27\sqrt{3}}{64}t^3 + \frac{425\sqrt{3}}{1536}tvw + \frac{27}{16}ta^2.\end{aligned}$$

Tachyon condensation on the σ-brane

- Going to higher levels, we should properly take care of the Ising model null-states
- It turns out that we can effectively remove them by considering only Virasoro generators:

  - in the Verma module of 1:

$$L_{-2}, L_{-3}, L_{-4}, L_{-5}, L_{-11}, L_{-12}, L_{-13}, L_{-14}, L_{-18}, L_{-19}, L_{-20}, L_{-21}, \ldots$$

  - in the Verma module of ε :

$$L_{-1}, L_{-4}, L_{-6}, L_{-7}, L_{-9}, L_{-10}, L_{-12}, L_{-15}, L_{-17}, L_{-20}, L_{-20}, L_{-22}, \ldots$$

The patterns repeats modulo 16!

- Had we needed Verma module of σ only, L_{odd} would be needed only!

Tachyon condensation on the σ-brane

- Already in the lowest truncation levels we see two solution corresponding to 1- and ε-branes

Level	0.5	2.0	2.5
$2\pi^2\mathcal{V}(\psi)$	-0.16971	-0.24579	-0.26454
Percentage	57.9 %	83.9 %	90.3 %
$c_1\|0\rangle$	0.14815	0.20553	0.21454
$c_1\|\epsilon\rangle$	±0.24348	± 0.27818	±0.29230
$c_{-1}\|0\rangle$		0.07382	0.07305
$c_1 L^I_{-2}\|0\rangle$		-0.09006	-0.10418
$c_1 L^R_{-2}\|0\rangle$		0.02750	0.02643
$c_{-1}\|\epsilon\rangle$			±0.02764
$c_1 L^I_{-2}\|\epsilon\rangle$			±0.02178
$c_1 L^R_{-2}\|\epsilon\rangle$			±0.00915

Level	$2\pi^2\mathcal{V}(\psi)$	$n^{\mathbb{1}}_\psi$	n^{ϵ}_ψ	n^{σ}_ψ
1	-0.169718	0.767289	-0.767289	0.643203
2	-0.250828	0.733703	0.893387	0.739416
3	-0.261047	0.725226	0.945626	0.76589
4	-0.273442	0.722133	0.487621	0.778236
5	-0.276177	0.719333	0.500237	0.796483
6	-0.280671	0.715848	0.721123	0.801822
7	-0.281747	0.714764	0.730309	0.80727
8	-0.284039	0.714011	0.629844	0.810113
9	-0.284577	0.713460	0.631591	0.814922
10	-0.285964	0.712159	0.704802	0.816787
∞	-0.294334	0.705668	0.700167	0.839425
Expected	-0.292893	0.707106	0.707106	0.840896

Positive energy solutions on the 1-brane

- On the 1-brane we expect to find the usual tachyon vacuum, but can we find also something else ???

Positive energy solutions on the 1-brane

- On the 1-brane we expect to find the usual tachyon vacuum, but can we find also something else ???

- Yes !

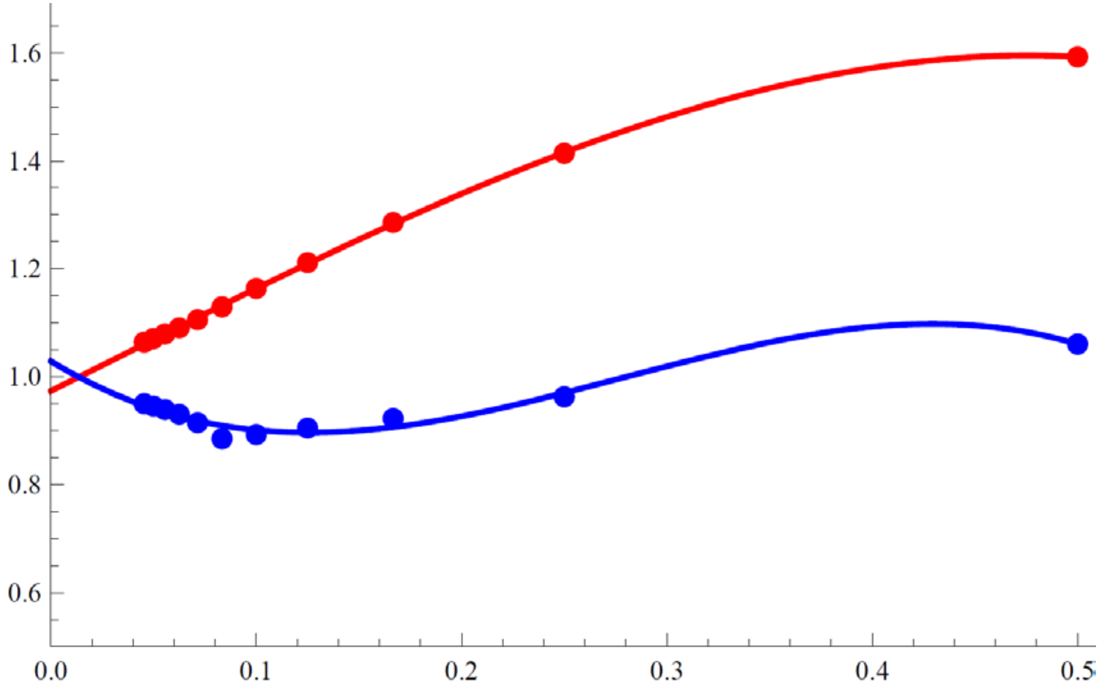
Positive energy solutions on the 1-brane

- Starting with a complex solution at level 2 we find a real solution at level 14 and higher!

L	Energy	n_ψ^1	n_ψ^ε	n_ψ^σ	Im/Re	Its.	Time(s)
2	$1.59267 + 0.726878i$	$1.06048 - 0.184547i$	$-9.73471 - 5.23904i$	$-0.343579 - 0.970819i$	0.788398	3	0
4	$1.41414 + 0.201521i$	$0.962899 - 0.142672i$	$-0.66854 + 1.99191i$	$-0.369755 - 0.564227i$	0.438384	6	0
6	$1.28579 + 0.0766818i$	$0.922618 - 0.113783i$	$-3.86207 - 0.373757i$	$-0.389334 - 0.394362i$	0.307463	5	0
8	$1.2116 + 0.0305389i$	$0.904803 - 0.0868545i$	$-0.575138 + 0.822662i$	$-0.372165 - 0.281935i$	0.221002	5	0
10	$1.16345 + 0.0100715i$	$0.892563 - 0.0617353i$	$-2.48552 + 0.00261068i$	$-0.376292 - 0.192323i$	0.152223	5	2
12	$1.12943 + 0.00122565i$	$0.885097 - 0.0310941i$	$-0.569512 + 0.245609i$	$-0.368913 - 0.0939861i$	0.0748655	6	8
14	1.10568	0.914693	-1.93951	-0.266065	0	9	92
16	1.09045	0.930444	-0.950873	-0.206326	0	5	326
18	1.07936	0.939178	-1.69824	-0.174965	0	5	4037
20	1.07084	0.945384	-1.04849	-0.15003	0	5	33258
22	1.06405	0.949943	-1.55407	-0.13398	0	4	230589

Positive energy solutions on the 1-brane

- Cubic extrapolations of energy and Ellwood invariant (boundary entropy) to infinite level



Boundary states in (Ising)^2 from OSFT

- In (Ising)^2 we have focused so far on the tachyon condensation of the $\sigma\otimes\sigma$ –brane. To lowest level the string field takes the form

$$|\psi\rangle = tc_1|0\rangle + ac_1|\epsilon^{(1)}\rangle + bc_1|\epsilon^{(2)}\rangle + cc_1|\epsilon^{(1)}\epsilon^{(2)}\rangle$$

and the action to this order

cf. Longton & Karczmarek

$$-\frac{1}{2}t^2 - \frac{1}{4}a^2 - \frac{1}{4}b^2 + \frac{1}{3}K^3t^3 + \frac{4}{3\sqrt{3}}K^3a^2t + \frac{4}{3\sqrt{3}}K^3b^2t + \frac{3\sqrt{3}}{2}abc + \frac{3\sqrt{3}}{4}tc^2$$

There are four interesting solutions :

$\|B\rangle_\psi$	$\|\mathbb{1}\rangle\otimes\|\mathbb{1}\rangle$	$\|\mathbb{1}\rangle\otimes\|\epsilon\rangle$	$\|\epsilon\rangle\otimes\|\mathbb{1}\rangle$	$\|\epsilon\rangle\otimes\|\epsilon\rangle$
$c_1\|0\rangle$	0.23926	0.23926	0.23926	0.23926
$c_1\|\epsilon^{(1)}\rangle$	-0.16828	0.16828	-0.16828	0.16828
$c_1\|\epsilon^{(2)}\rangle$	-0.16828	-0.16828	0.16828	0.16828
$c_1\|\epsilon^{(1)}\epsilon^{(2)}\rangle$	-0.11836	0.11836	0.11836	-0.11836

Boundary states in $(\text{Ising})^2$ from OSFT

- For example the $1 \otimes 1$ brane solution has the following invariants:

Level	$2\pi^2\mathcal{V}(\psi)$	$n_\psi^{\mathbb{1}\mathbb{1}}$	$n_\psi^{\mathbb{1}\epsilon}$	$n_\psi^{\mathbb{1}\sigma}$	$n_\psi^{\epsilon\mathbb{1}}$	$n_\psi^{\epsilon\epsilon}$	$n_\psi^{\epsilon\sigma}$	$n_\psi^{\sigma\mathbb{1}}$	$n_\psi^{\sigma\epsilon}$	$n_\psi^{\sigma\sigma}$
1.0	-0.28149	0.62417	-0.62417	0.44455	-0.62417	0.62417	-0.44455	0.44455	-0.44455	0.52585
2.0	-0.39683	0.58024	0.28858	0.48785	0.28858	-1.15741	-0.48785	0.48785	-0.48785	0.61593
2.5	-0.43040	0.54753	0.43339	0.53440	0.43339	-1.41439	0.71872	0.53440	0.71872	0.64164
3.0	-0.43544	0.54367	0.48059	0.53102	0.48059	-1.50484	0.74643	0.53103	0.74643	0.62219
4.0	-0.45553	0.53344	0.26231	0.53601	0.26231	1.54248	0.81851	0.53601	0.81851	0.63243
4.5	-0.46222	0.52735	0.27837	0.55040	0.27837	1.74282	0.29745	0.55040	0.29745	0.63174
5.0	-0.47130	0.52629	0.28106	0.54984	0.28106	1.80851	0.29662	0.54984	0.29662	0.65732
6.0	-0.47130	0.51879	0.41792	0.55168	0.41792	-0.7563	0.29982	0.55168	0.29982	0.66158
6.5	-0.47397	0.51657	0.43234	0.55874	0.43234	-0.84041	0.60779	0.55874	0.60779	0.66244
7.0	-0.47397	0.51614	0.43788	0.55856	0.43788	-0.87424	0.61640	0.55856	0.61640	0.66078
8.0	-0.47476	0.51333	0.37867	0.55949	0.37867	1.25240	0.62768	0.55949	0.62768	0.66279
∞	-0.49473	0.49752	0.44967	0.58356	0.44967	0.50129	0.51098	0.58356	0.51098	0.72564
Expected	-0.5	0.5	0.5	0.59460	0.5	0.5	0.59460	0.59460	0.59460	0.70711

I.e. $2^{-3/4}$

Comments on double branes

- Since we have developed quite an efficient code for solving e.o.m. in level truncation, it is natural to look for other solutions in the universal basis besides the tachyon vacuum.

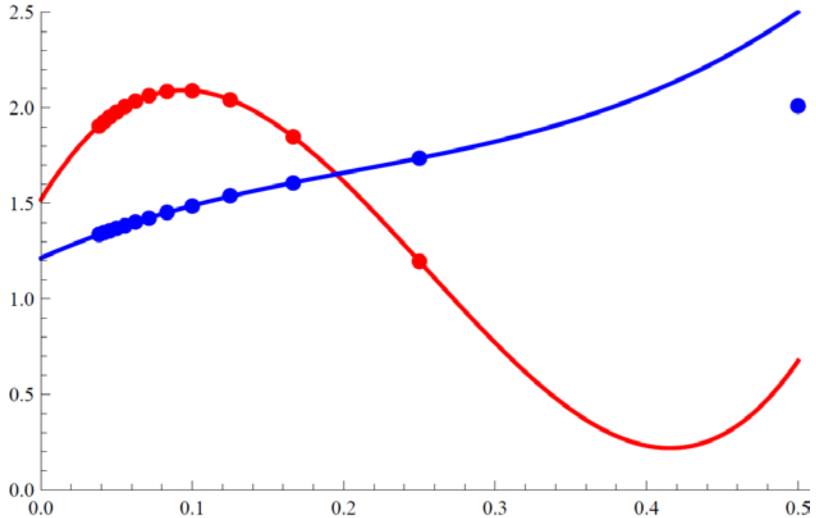
- We start with a promising complex solution found easily at level 2 and improve it via the Newton's method to higher levels.

Comments on double branes

- We found the following dependence on the level:

L	$Energy$	W
2	$-1.42791 - 3.40442i$	$2.00934 - 0.0545341i$
4	$1.19625 - 2.25966i$	$1.73651 + 0.117637i$
6	$1.84813 - 1.58507i$	$1.60634 + 0.195442i$
8	$2.04207 - 1.14971i$	$1.53973 + 0.217911i$
10	$2.08908 - 0.866428i$	$1.48598 + 0.22801i$
12	$2.08515 - 0.674602i$	$1.4521 + 0.227059i$
14	$2.06302 - 0.53887i$	$1.42232 + 0.224184i$
16	$2.03499 - 0.439057i$	$1.40194 + 0.218266i$
18	$2.00593 - 0.363272i$	$1.38304 + 0.212332i$
20	$1.9778 - 0.304197i$	$1.36942 + 0.205378i$
22	$1.95135 - 0.257139i$	$1.35632 + 0.198765i$
24	$1.92679 - 0.218971i$	$1.34654 + 0.191784i$
26	$1.90411 - 0.187545i$	$1.33691 + 0.185169i$

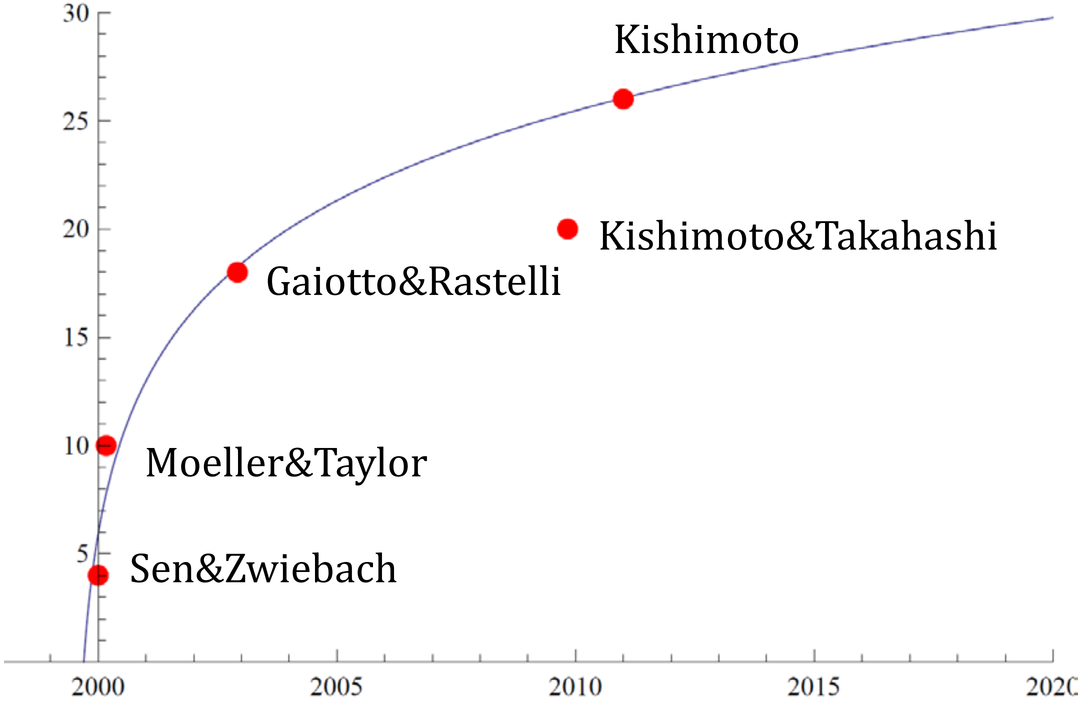
Real part of energy and W-invariant as a function of 1/L :



- Should we dismiss the solution, or hope for a oscillation or a cusp at higher levels?

Future of level truncation

- Hopefully soon level 30 should be reached



Required tools:

-universal basis

- conservation laws

- C++

-SU(1,1) singlet basis

-Parallelism

- ???

N.B.: Level 30 is interesting, as we should see the oscillation for the tachyon vacuum energy predicted by Gaiotto and Rastelli.

Conclusions

- High level numerical computations in OSFT have the potential to discover new boundary states (e.g. they could have predicted existence of fractional D-branes) .
- The key tool for physical identification are the boundary states obtained from the generalized Ellwood invariants.
- First well behaved positive energy solution discovered ! (describing σ-brane on a 1-brane, or perhaps double branes)
- We are coming to an era of possible computer exploration of the OSFT landscape – stay tuned!