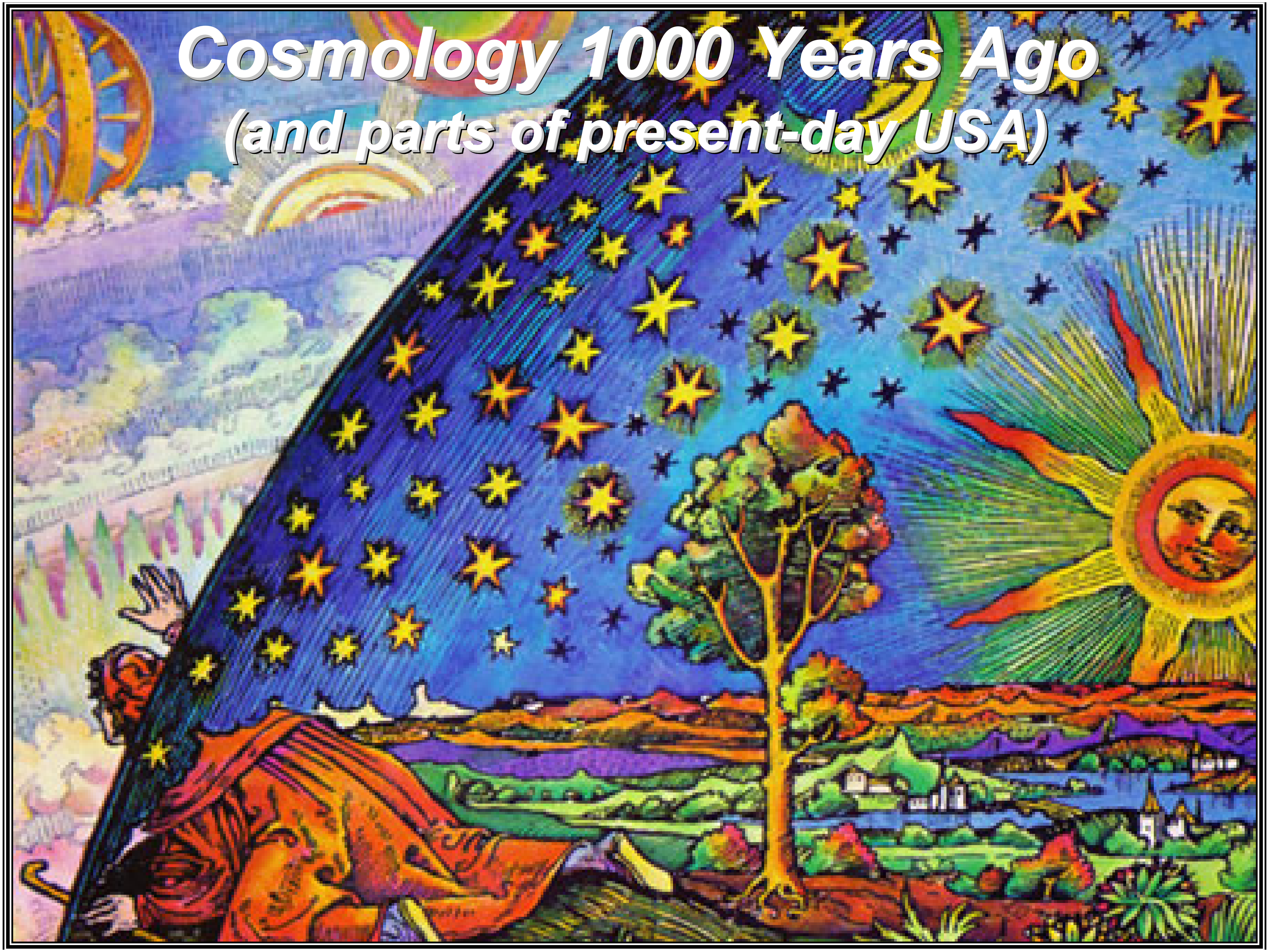
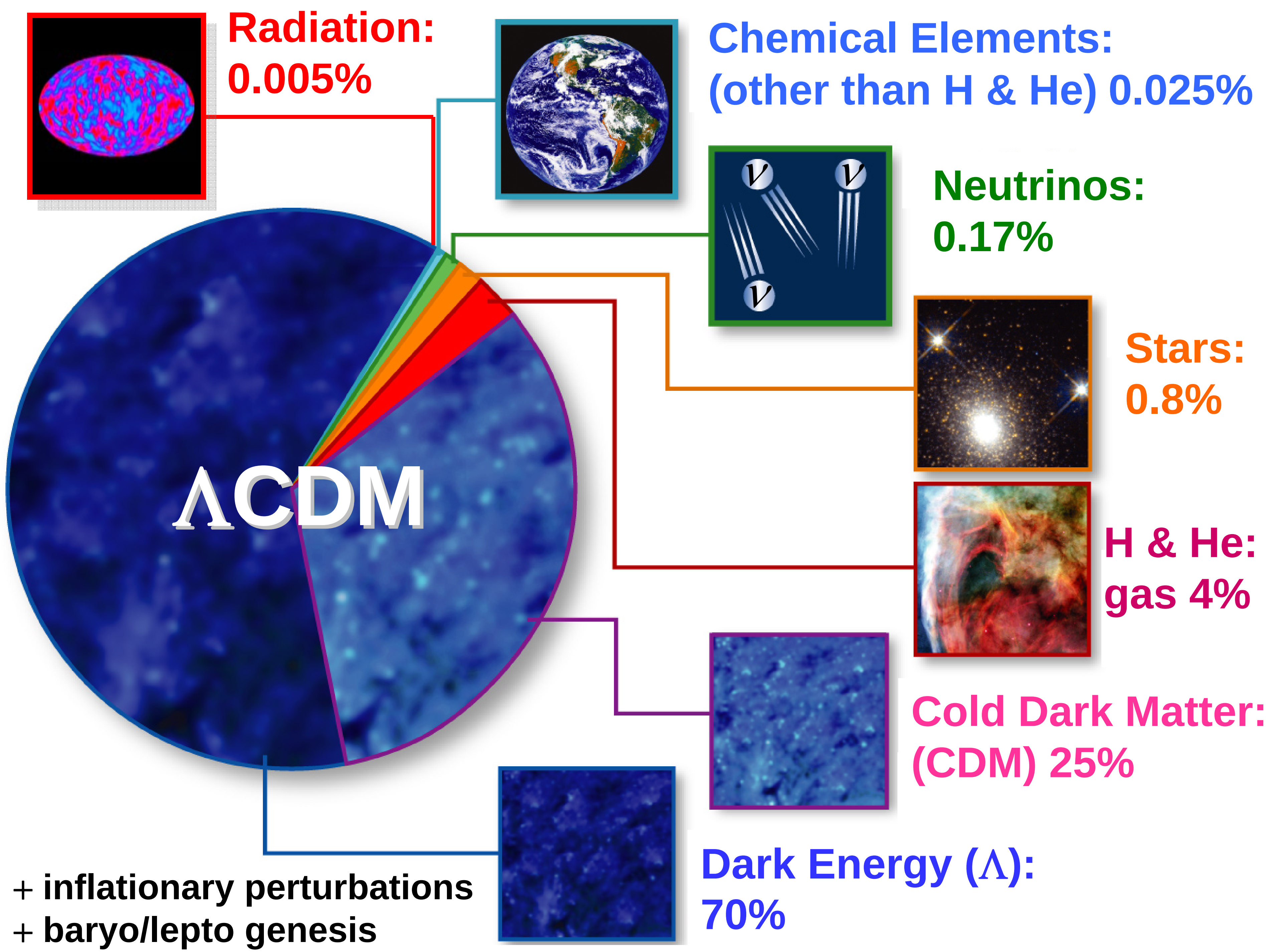
The Dark Universe:
Dark Matter and Dark Energy

Rocky I:	The Universe Observed	Monday
Rocky II:	Inflation	Tuesday
Rocky III:	Dark Matter	Wednesday
Rocky IV:	Dark Energy	Thursday

CERN Academic Training Lectures **January 2008**

Rocky Kolb ***The University of Chicago***


Cosmology 1000 Years Ago
(and parts of present-day USA)

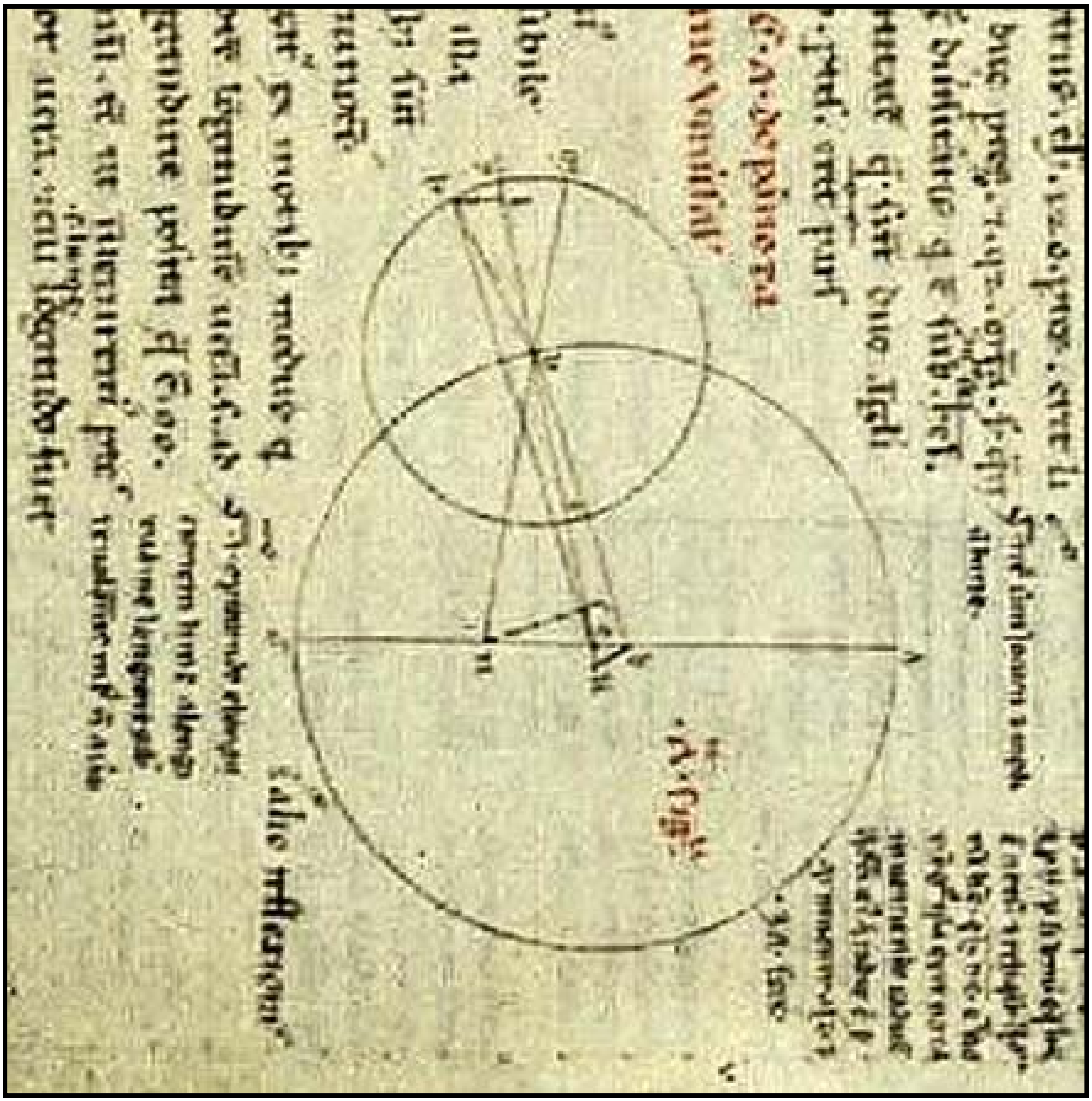

Radiation:
0.005%
Chemical Elements:
(other than H & He) 0.025%
Neutrinos:
0.17%
Stars:
0.8%
H & He:
gas 4%
Cold Dark Matter:
(CDM) 25%
Dark Energy (Λ):
70%
ΛCDM
+ inflationary perturbations
+ baryo/lepto genesis

ΛCDM: Reality Or A Substitute?

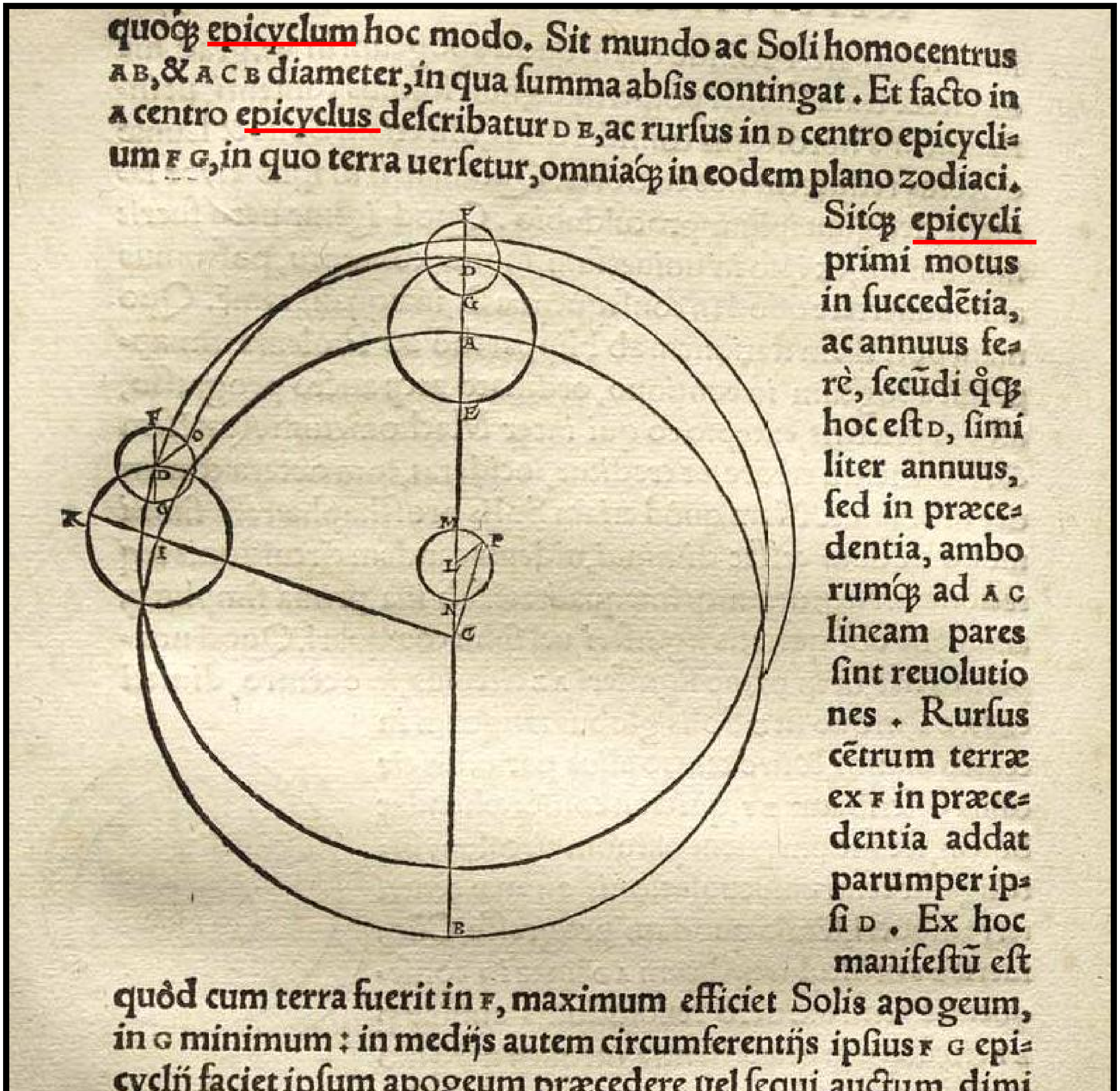
The construction of a model … consists of snatching from the enormous and complex mass of facts called reality a few simple, easily managed key points which, when put together in some cunning way, becomes for certain purposes a substitute for reality itself.

Evsey Domar
Essays on the Theory of Economic Growth

Ptolemaic System From *The Almagest*



From Book III of *De Revolutionibus*



quocꝫ epicyclum hoc modo. Sit mundo ac Soli homocentrus A B, & A C B diameter, in qua ſumma abſis contingat. Et facto in A centro epicyclus deſcribatur D E, ac rurſus in D centro epicyclium F G, in quo terra uerſetur, omniaq́ꝫ in eodem plano zodiaci.

Sitq́ꝫ epicycli primi motus in ſuccedẽtia, ac annuus ferè, ſecũdi quoqꝫ hoc eſt D, ſimiliter annuus, ſed in præcedentia, amborumq́ꝫ ad A C lineam pares ſint reuolutiones. Rurſus cẽtrum terræ ex F in præcedentia addat parumper ipſi D. Ex hoc manifeſtũ eſt quòd cum terra fuerit in F, maximum efficiet Solis apogeum, in G minimum: in medijs autem circumferentijs ipſius F G epicyclij faciet ipſum apogeum præcedere uel ſequi, auctum dimi

Precision Cosmology

$\Omega_{tot} = 1.02^{+0.02}_{-0.02}$

$w < -0.78$ (95% CL)

$\Omega_\Lambda = 0.73^{+0.04}_{-0.04}$

$\Omega_b h^2 = 0.0224^{+0.0009}_{-0.0009}$

$\Omega_b = 0.044^{+0.004}_{-0.004}$

$n_b = 2.5 \times 10^{-7}{}^{+0.1\times10^{-7}}_{-0.1\times10^{-7}}$ cm^{-3}

$\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$

$\Omega_m = 0.27^{+0.04}_{-0.04}$

$\Omega_\nu h^2 < 0.0076$ (95%CL)

$m_\nu < 0.23$ eV (95% CL)

$T_{cmb} = 2.725^{+0.002}_{-0.002}$ K

$n_\gamma = 410.4^{+0.9}_{-0.9}$ cm^{-3}

$\eta = 6.1 \times 10^{-10}{}^{+0.3\times10^{-10}}_{-0.2\times10^{-10}}$

$\Omega_b \Omega_m^{-1} = 0.17^{+0.01}_{-0.01}$

$\sigma_8 = 0.84^{+0.04}_{-0.04}$ Mpc

$\sigma_8 \Omega_m^{0.5} = 0.44^{+0.04}_{-0.05}$

$A = 0.833^{+0.086}_{-0.083}$

$n_s = 0.93^{+0.03}_{-0.03}$

$dn_s/d \ln k = -0.031^{+0.016}_{-0.018}$

$r < 0.71$ (95% CL)

$z_{dec} = 1089^{+1}_{-1}$

$\Delta z_{dec} = 195^{+2}_{-2}$

$h = 0.71^{+0.04}_{-0.03}$

$t_0 = 13.7^{+0.2}_{-0.2}$ Gyr

$t_{dec} = 379^{+8}_{-7}$ kyr

$t_r = 180^{+220}_{-80}$ Myr (95% CL)

$\Delta t_{dec} = 118^{+3}_{-2}$ kyr

$z_{eq} = 3233^{+194}_{-210}$

$\tau = 0.17^{+0.04}_{-0.04}$

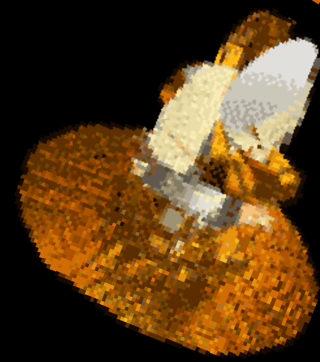
$z_r = 20^{+10}_{-9}$ (95% CL)

$\theta_A = 0^\circ.598^{+0.002}_{-0.002}$

$d_A = 14.0^{+0.2}_{-0.3}$ Gpc

$l_A = 301^{+1}_{-1}$

$r_s = 147^{+2}_{-2}$ Mpc



WMAP

Precision Cosmology

The cosmological standard model ΛCDM

Dark Matter

+

Dark Energy

+

Seed Perturbations (Inflation)

+

Baryo/Leptogenesis

The Universe seems to demand *NEW* PHYSICS!

Astronomy is helpful to physics!

Precision Cosmology

"How helpful to us is astronomy's pedantic accuracy, which I used to secretly ridicule!"

Einstein's statement to Arnold Sommerfeld on December 9, 1915 (regarding measurements of the advance of the perihelion of Mercury)

The Universe Observed

- Big-bang primer (just theory)

- Cosmological parameters (just numbers)

  $H_0 \rightarrow$ the present expansion rate (Hubble's constant)

  $\Omega_i \rightarrow$ the present cosmic food chain
  (Ω_{TOTAL}, Ω_M, Ω_B, Ω_Λ, Ω_γ, Ω_ν, ...)

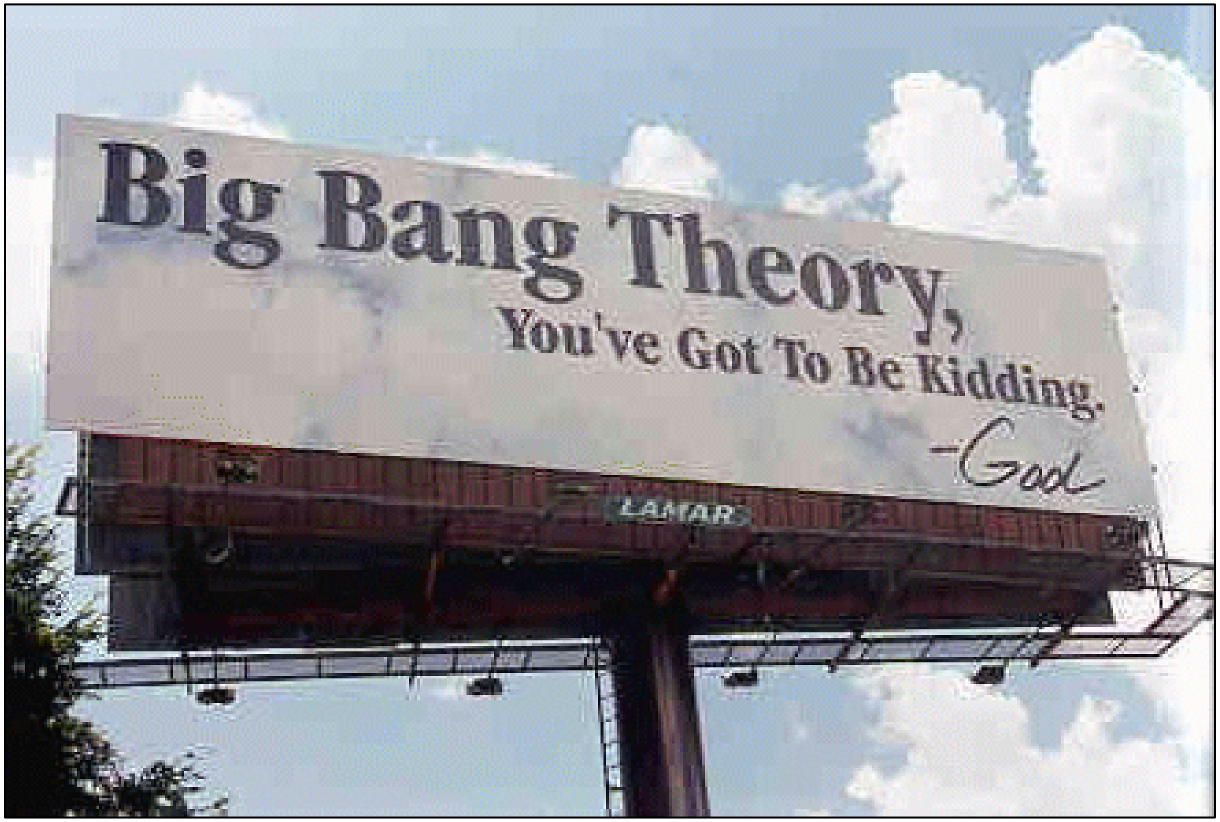
  $T_0 \rightarrow$ the present temperature of the Universe

  $t_0 \rightarrow$ the present age of the Universe

- Power spectra–characterization of perturbations:
  Galaxies: $P(k)$ Radiation: C_ℓ

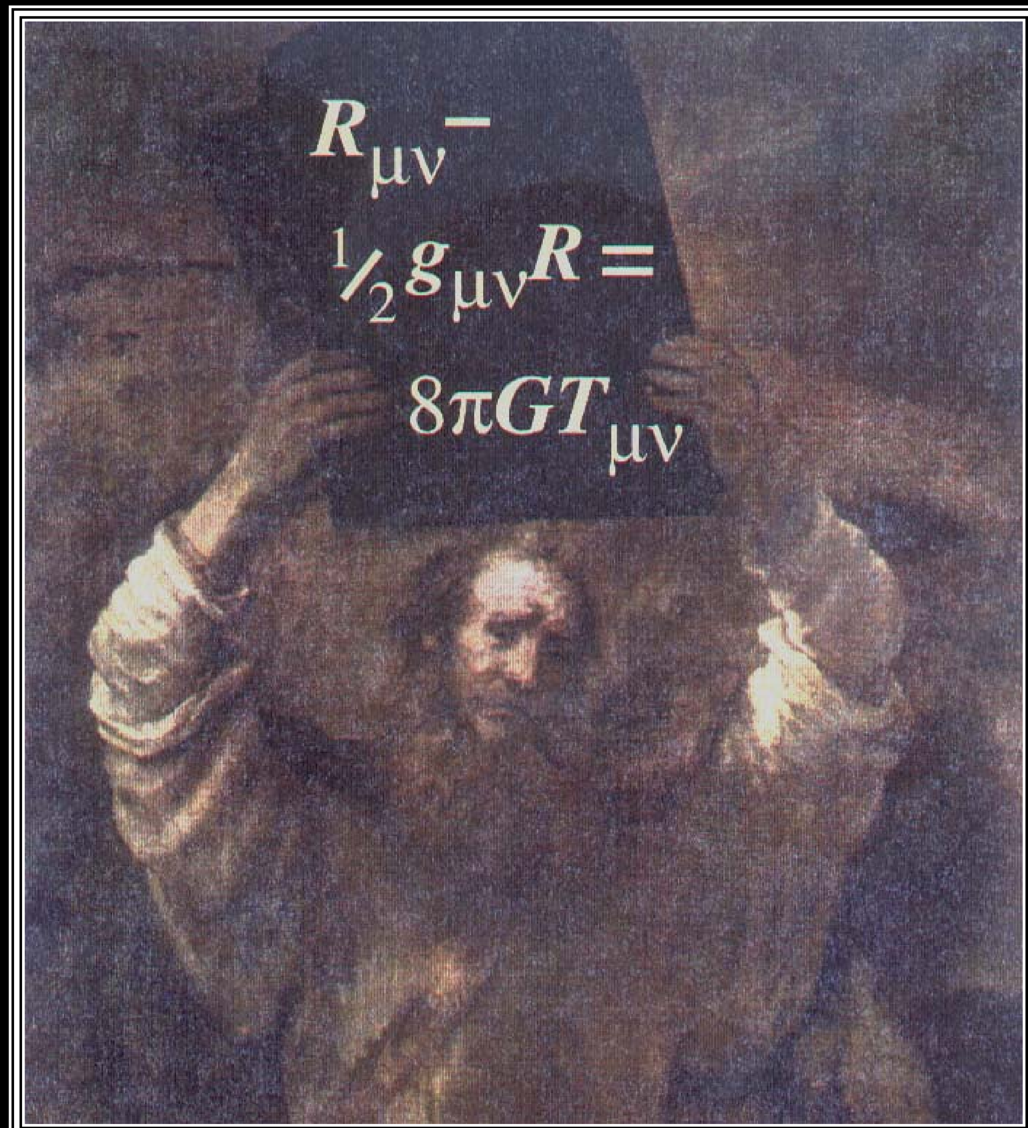
- "Standard model": ΛCDM Dark Energy and Dark Matter

Big-Bang (Just A Theory)



Meet two brainiacs with a lot to learn. Leonard and Sheldon can tell their quarks from their quantum physics, but have no clue how women add up. Leave it to their pretty new neighbor, just off a messy breakup, to teach them a thing or two in THE BIG BANG THEORY.

Modern Commandments of Genesis



(10 nonlinear partial differential equations)

Big-Bang Theory

Robertson-Walker metric

$a(t) = \text{cosmic scale factor}$

$k = 0, \pm 1$

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Perfect-fluid stress tensor

$\rho =$ energy density

$p =$ pressure

$T^{\mu}{}_{\nu} = \mathrm{diag}(\rho, p, p, p)$

Robertson-Walker Metric

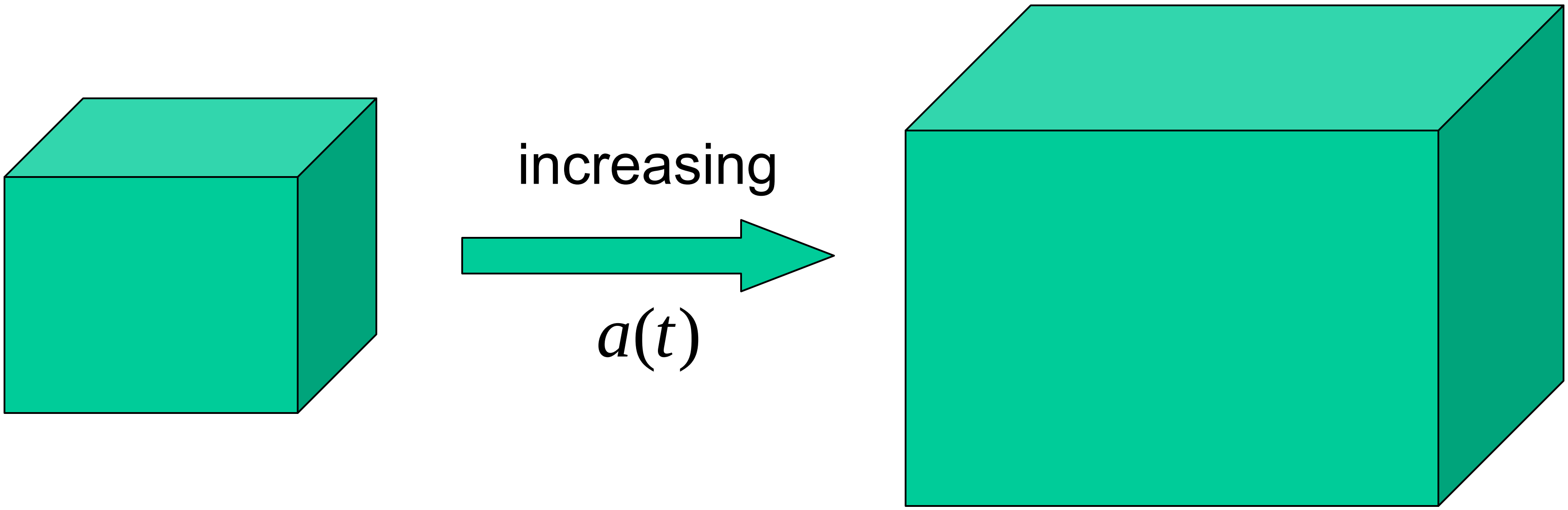
$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

$k = 0$ (spatially flat)

$$ds^2 = dt^2 - a^2(t)\left(dr^2 + r^2 d\Omega^2\right) = dt^2 - a^2(t)\left(d\vec{x}^2 + d\vec{y}^2 + d\vec{z}^2\right)$$

(comoving coordinates: $\vec{x}, \vec{y}, \vec{z}$)

(physical distance: $d\vec{l}^{\,2} = a^2(t)\left(d\vec{x}^2 + d\vec{y}^2 + d\vec{z}^2\right)$



Perfect Fluid Stress-Energy Tensor

- Must specify energy and pressure content of the Universe
- Assume pressure is related to energy density: $p_i = w_i\, \rho_i$
- Conservation of stress-energy tensor, $T^{\mu\nu}{}_{;\nu} = 0 \rightarrow \rho_i \propto a^{-3(1+w_i)}$
- If $w = w(a)$ then $a^{-3(1+w)} \rightarrow \exp\left(-3\int_a^1 \frac{da'}{a'}\left[1+w\left(a'\right)\right]\right)$

$T^{\mu\nu}$: fluids with different w

Radiation:	$p_R = \rho_R/3$	$w = 1/3$	$\rho_R \propto a^{-4}$
Matter:	$p_M = 0$	$w = 0$	$\rho_M \propto a^{-3}$
Curvature:	$p_k = -\rho_k/3$	$w = -1/3$	$\rho_k \propto a^{-2}$
Vacuum:	$p_V = -\rho_V$	$w = -1$	$\rho_V \propto a^{0}$

Dynamics From Field Equations

(00) $$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho$$ Friedmann Equation

$(00)-(ii)$ $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right)$$ Deceleration equation

$$H \equiv \frac{\dot{a}}{a} = \text{expansion rate}$$

$$q \equiv -\frac{\ddot{a}}{a}\frac{1}{H^2} = \text{deceleration parameter}$$

Dynamics → Evolution

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \qquad \rho = \rho_M(a) + \rho_R(a) + \rho_\Lambda + \ldots$$

- $a(t)$ & $H(t)$ depend on matter/energy content
- $a(t)$ measurable via redshift
- Redshift z is a proxy for time or scale factor: $1 + z = a_0/a(t)$

Expansion Rate Is A Key Quantity

Friedmann equation ($G_{00} = 8\pi G T_{00}$) : Expansion rate $H(z)$

$\Omega_i \equiv \rho_i / \rho_C \qquad \rho_C \equiv 3H_0^{\,2} / 8\pi \mathrm{G}$

Hubble const. radiation matter curvature dark energy

$$H^2(z) = H_0^2\left[\Omega_R(1+z)^4 + \Omega_M(1+z)^3 + \left(1-\Omega_{\mathrm{TOTAL}}\right)(1+z)^2 + \Omega_w(1+z)^{3(1+w)}\right]$$

CMB LSS CMB $H(z)$

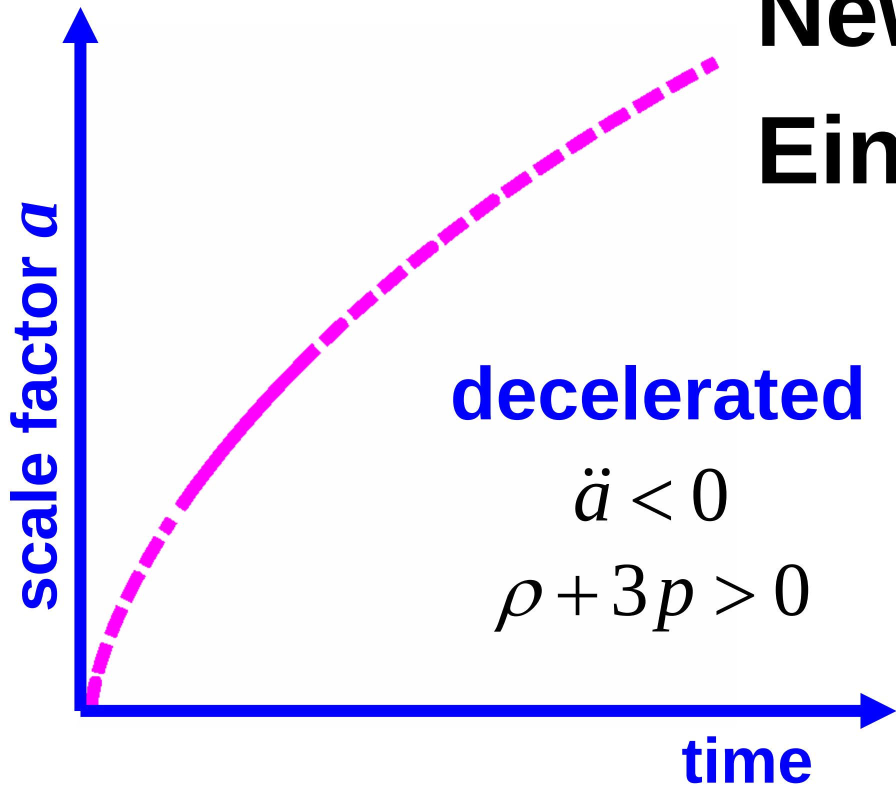
Equation of state parameter: $w = p/\rho$

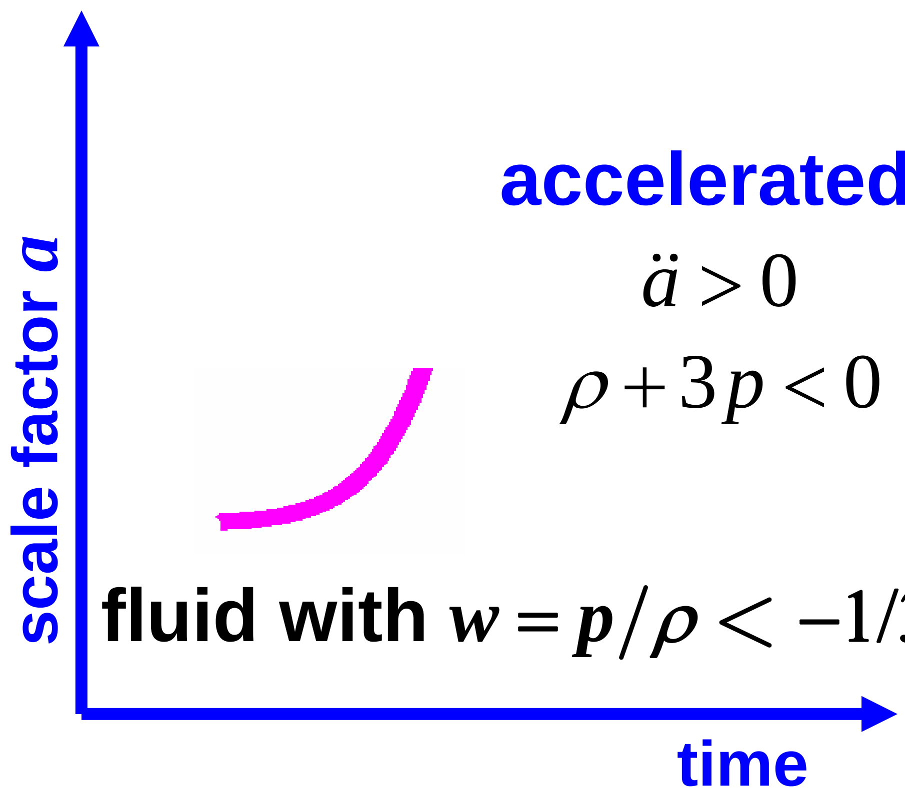
$$w = \begin{cases} 1/3 & \text{radiation} \\ 0 & \text{matter} \\ -1/3 & \text{curvature} \\ -1 & \Lambda \end{cases}$$

Newton

Einstein

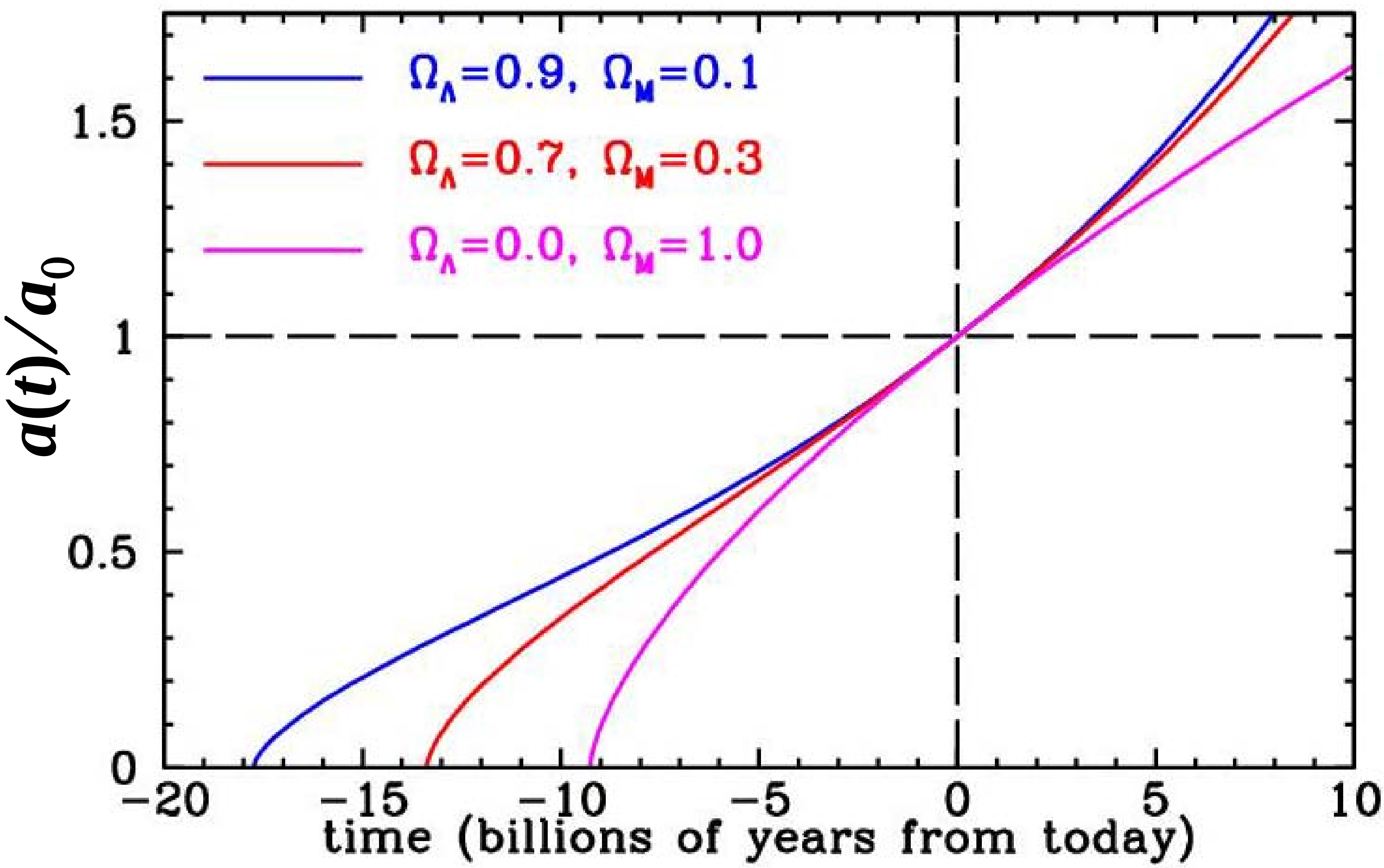
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$





Evolution of *a*(*t*) For Flat Models

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left[\rho_{M0}\left(\frac{a_0}{a}\right)^3 + \rho_\Lambda\right]$$

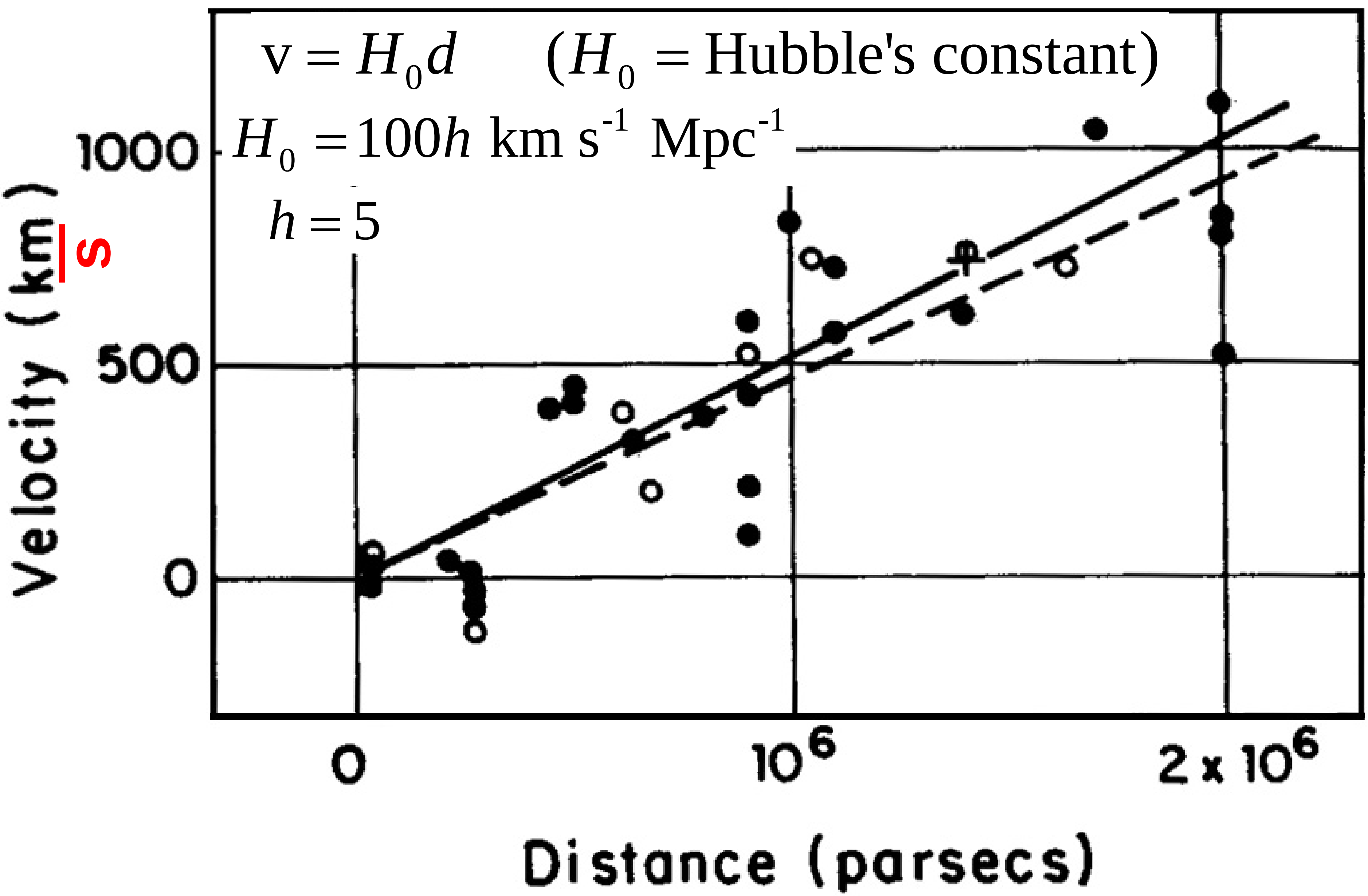


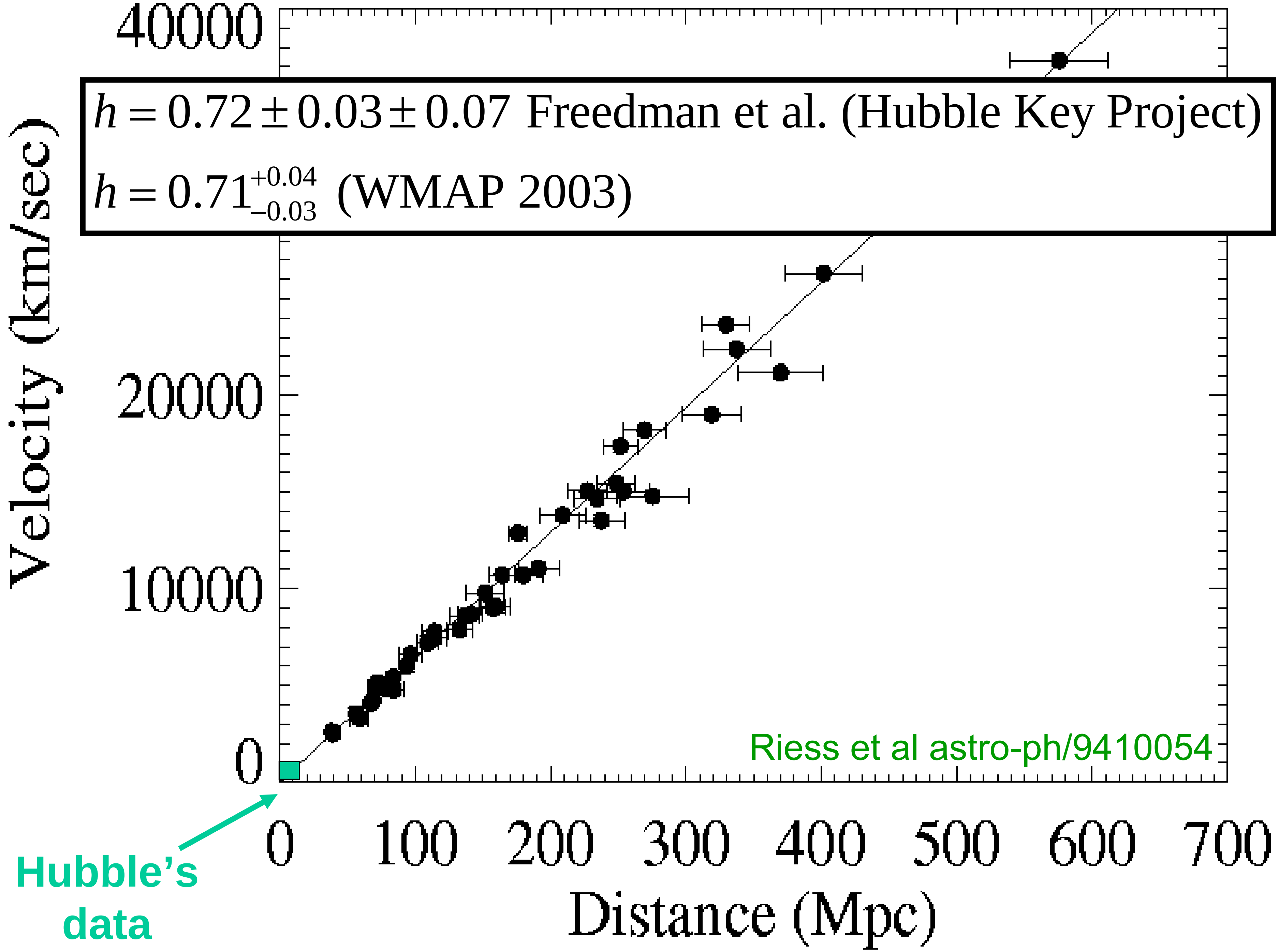
Description of Famous Chicago Alumnus

"… physically, he is a splendid specimen…"
"… magnificent physique …."
"… manly …"
"… loveable character …"

University of Chicago 1909 National Champions

Hubble's Discovery Paper – 1929





$h = 0.72 \pm 0.03 \pm 0.07$ Freedman et al. (Hubble Key Project)
$h = 0.71^{+0.04}_{-0.03}$ (WMAP 2003)
Velocity (km/sec)
40000
20000
10000
0
0
100
200
300
400
500
600
700
Distance (Mpc)
Riess et al astro-ph/9410054
Hubble's data

$$h^2 = 1/2$$

Distance–Redshift Relation

$$F = \frac{L}{4\pi d_L^2}$$ defines *luminosity distance* – "know" L, measure F

$4\pi\, d_L{}^2 =$ area of ^2S centered on source at time of detection t_0

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right) \rightarrow \text{area} = 4\pi\, a_0{}^2\, r^2$$

Energy redshifted: $(1 + z)$
Time interval redshifted: $(1 + z)$
Flux redshifted: $(1 + z)^2$

$$d_L{}^2 = a_0{}^2\, r^2\, (1 + z)^2$$

Light emitted from comoving coordinate r at time t reaches us now (t_0) redshifted by an amount $1 + z = a_0 / a(t)$

Distance–Redshift Relation

$$d_L = a(t_0)\, r\, (1+z)$$

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

light travels on geodesics

$$ds^2 = 0$$

$$\int \frac{dr}{\sqrt{1-kr^2}} = \int \frac{dt}{a(t)} = \int \frac{da}{H(a)a^2}$$

$$\int_0^r \frac{dr'}{\sqrt{1-kr'^2}} = \int_0^z \frac{a^{-1}(t_0) H_0^{-1} \quad dz'}{\sqrt{(1-\Omega_0)(1+z')^2 + \Omega_M (1+z')^3 + \Omega_w (1+z')^{3(1+w)} + \ldots}}$$

Distance–Redshift Relation

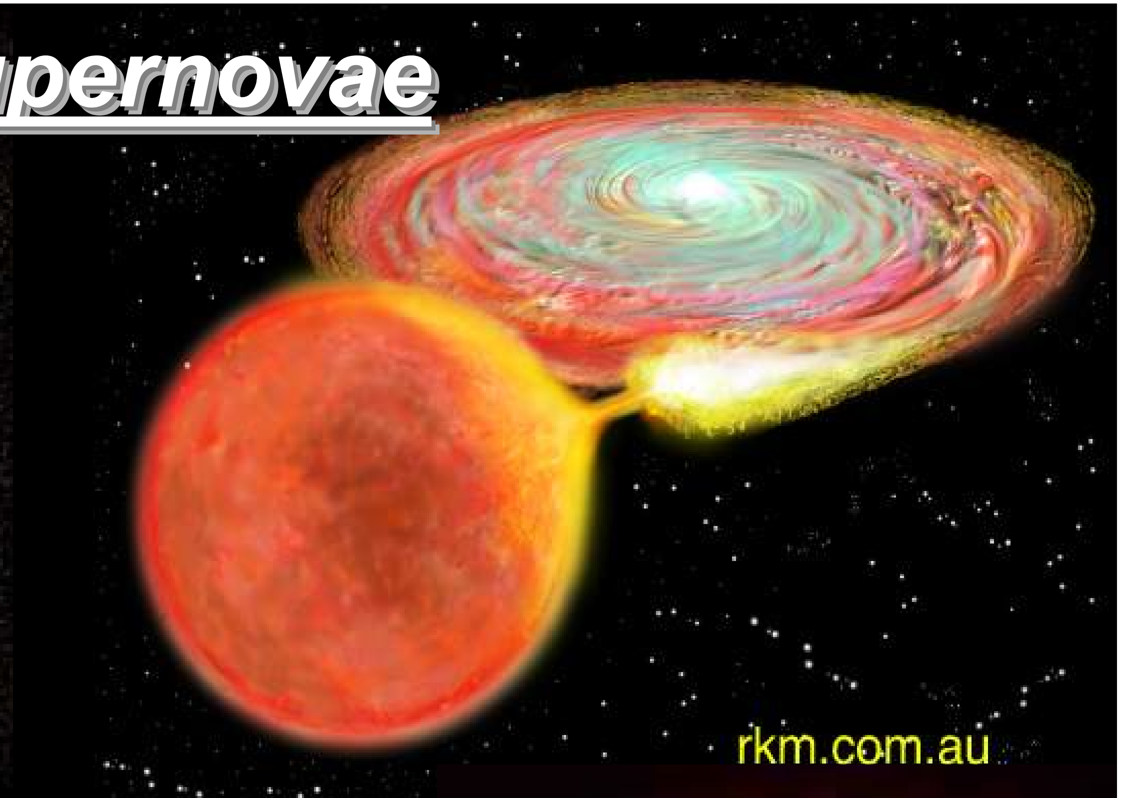
$$\boxed{d_L = a_0 r (1+z)}$$

$$a_0 r \text{ from } \int_0^r \frac{dr'}{\sqrt{1-kr'^2}} = \int_0^z \frac{a_0^{-1} H_0^{-1} \quad dz'}{\sqrt{(1-\Omega_{\text{TOTAL}})(1+z')^2 + \Omega_M (1+z')^3 + \ldots}}$$

Program:

- measure d_L (via $d_L{}^2 = L/4\pi F$) and z
- input a model cosmology (Ω_i) and calculate $a_0\, r$
- compare to data
- need bright "standard candle"

Supernovae



Monastic Chronicles re: Supernova 1006:

"in 1006 there was a very great famine and a comet appeared for a long time …."

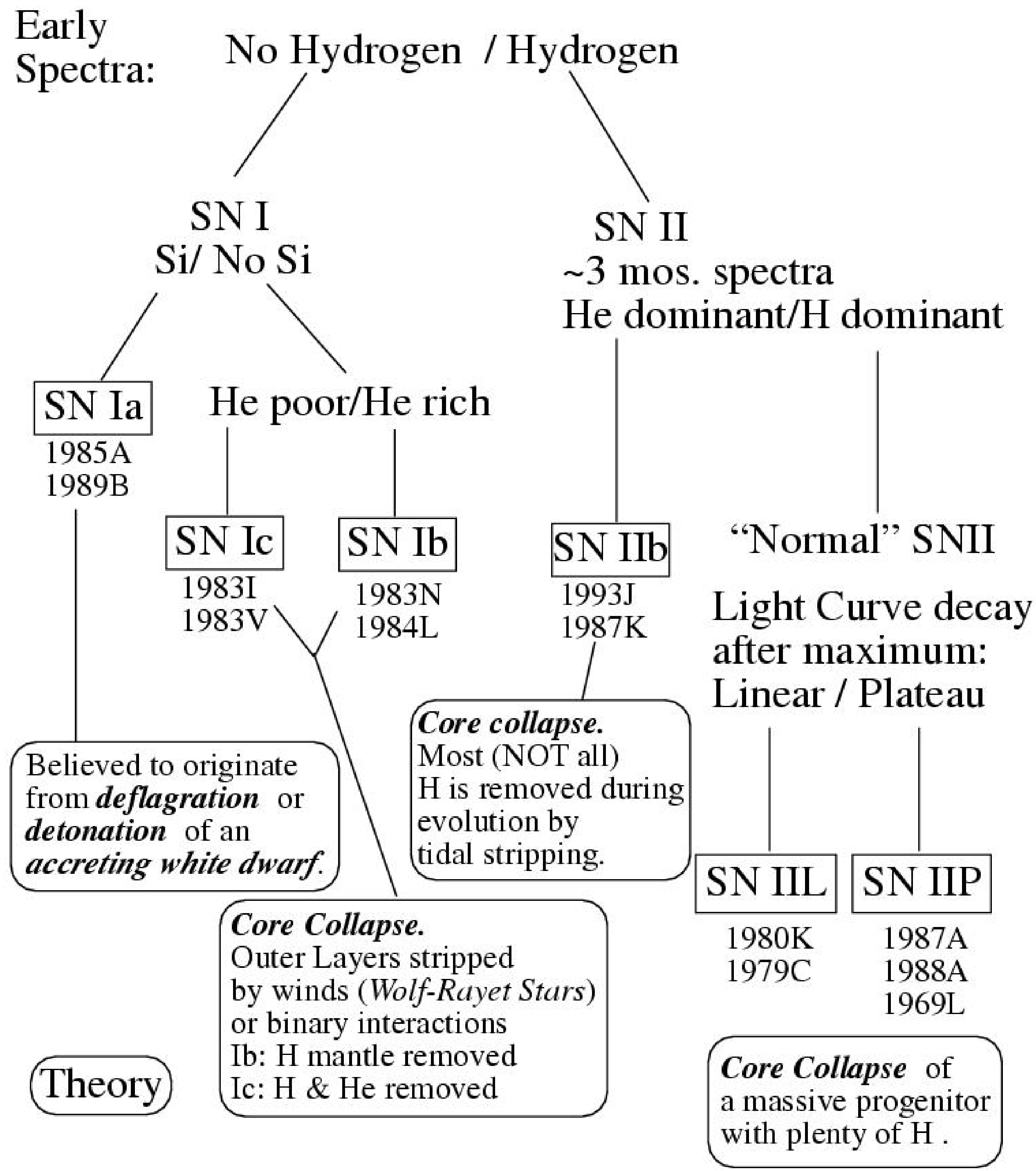
"at the same time a comet, which always announces human shame, appeared in the southern regions, which was followed by a great pestilence…"

"three years after the king was raised to the throne, a comet with a horrible appearance was seen in the southern part of the sky, emitting flames this way and that…"

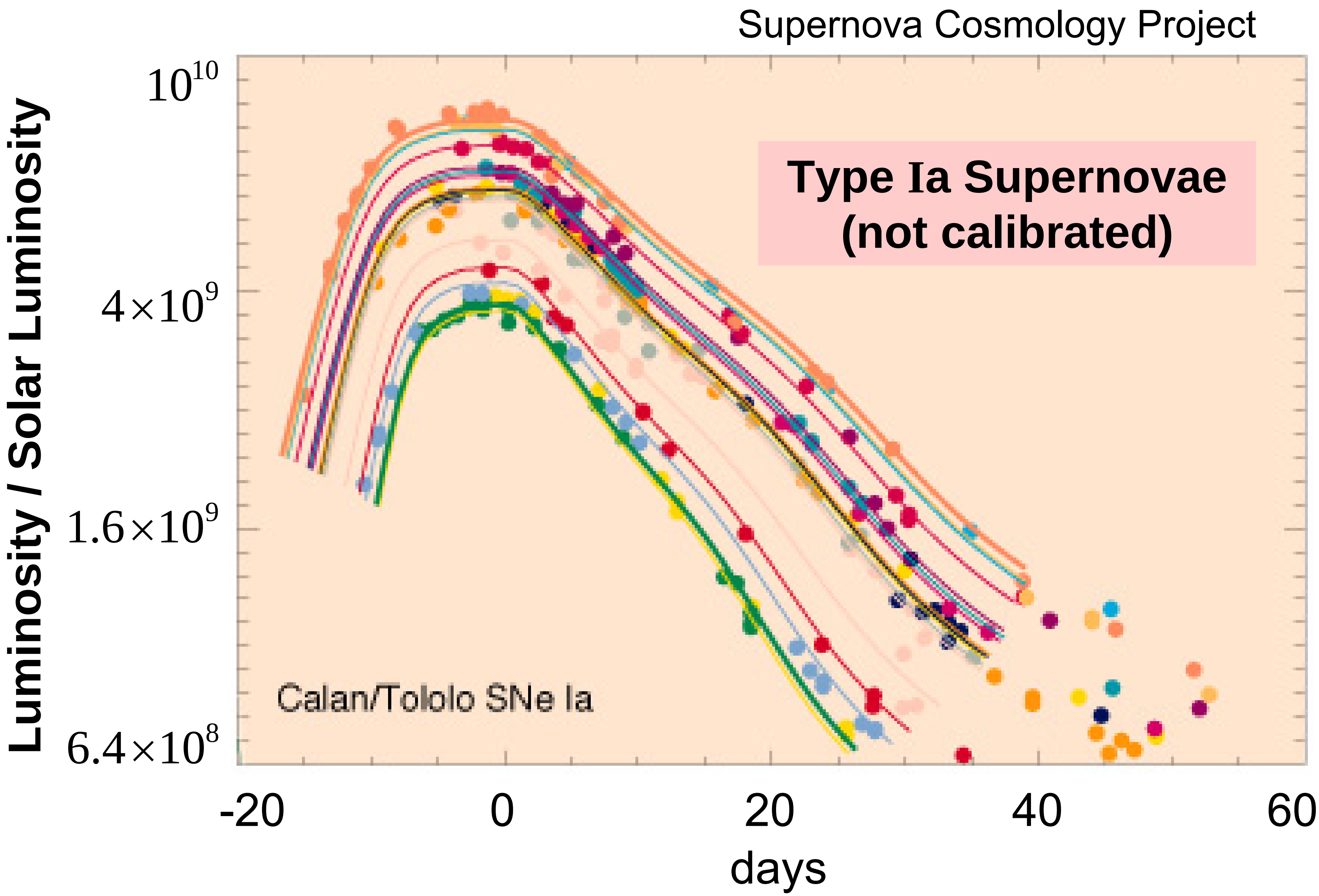
Georg Busch (German painter) in 1572:

"It is a sign that we will be visited by all sorts of calamities such as inclement weather, pestilence, and Frenchmen."

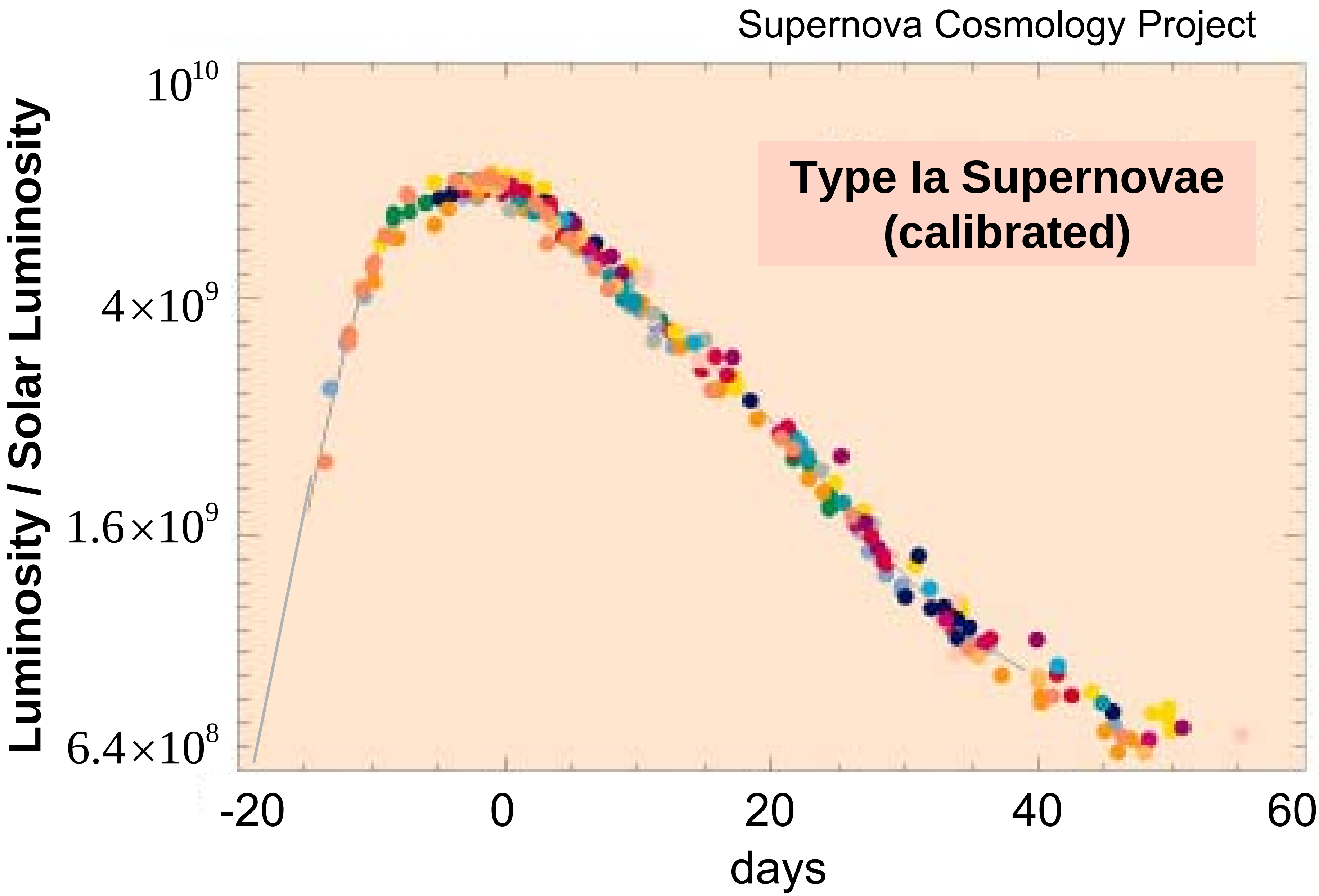
Supernova Taxonomy



Type Ia Supernovae



Type Ia Supernovae



Evolution of $H(z)$ Is A Key Quantity

Robertson–Walker metric

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$$

Many observables based on $H(z)$ through coordinate distance $r(z)$

$$r(z) = \left.\begin{matrix}\sin \\ 1 \\ \sinh\end{matrix}\right\}\left(\int_0^z \frac{dz'}{H(z')}\right)$$

- Luminosity distance

  Flux = (Luminosity / $4\pi\, d_L{}^2$)

$$d_L(z) \propto r(z)(1+z)$$

- Angular diameter distance

  $\alpha =$ Physical size / d_A

$$d_A(z) \propto \frac{r(z)}{(1+z)}$$

- Volume (number counts)

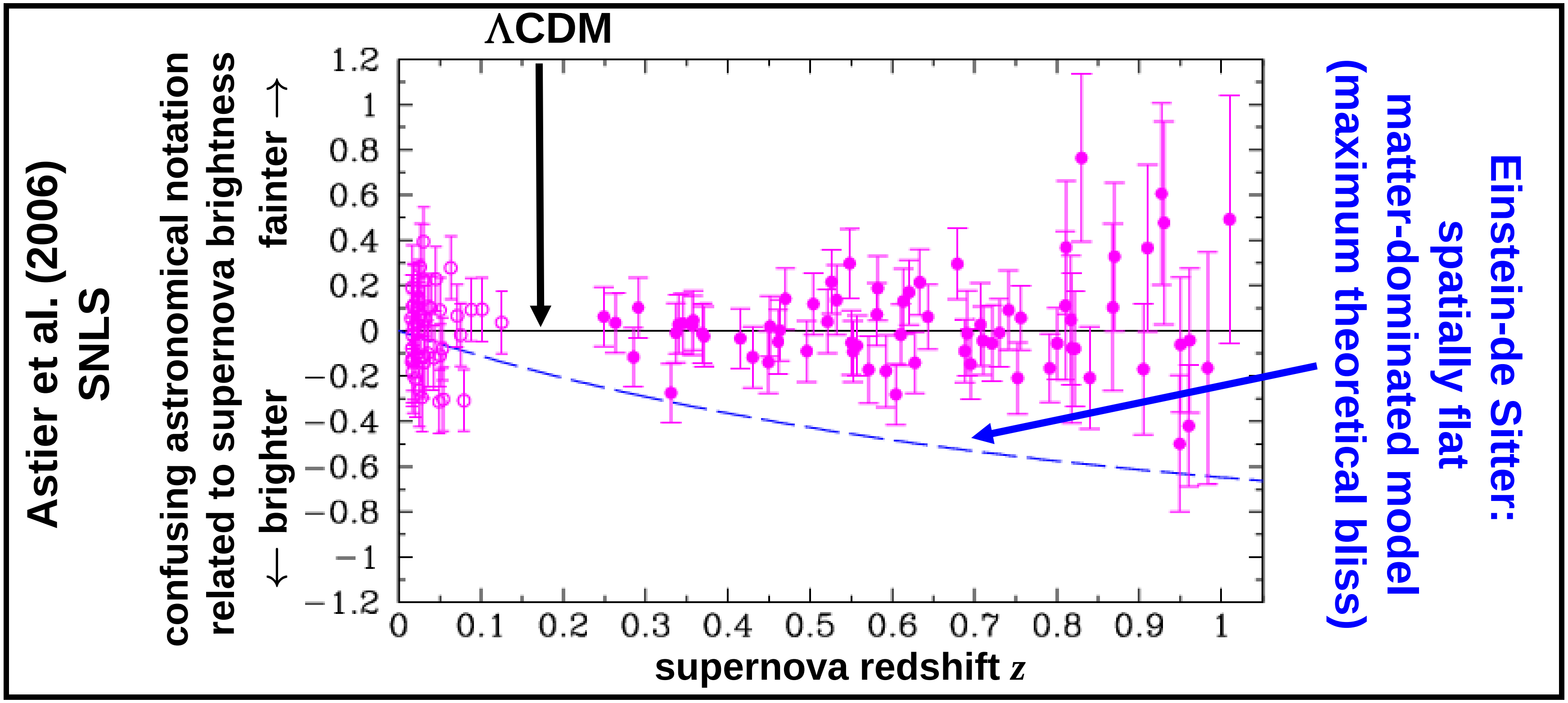
  $N \propto V^{-1}(z)$

$$dV = \frac{r^2(z)}{\sqrt{1-kr^2(z)}}\, dr\, d\Omega$$

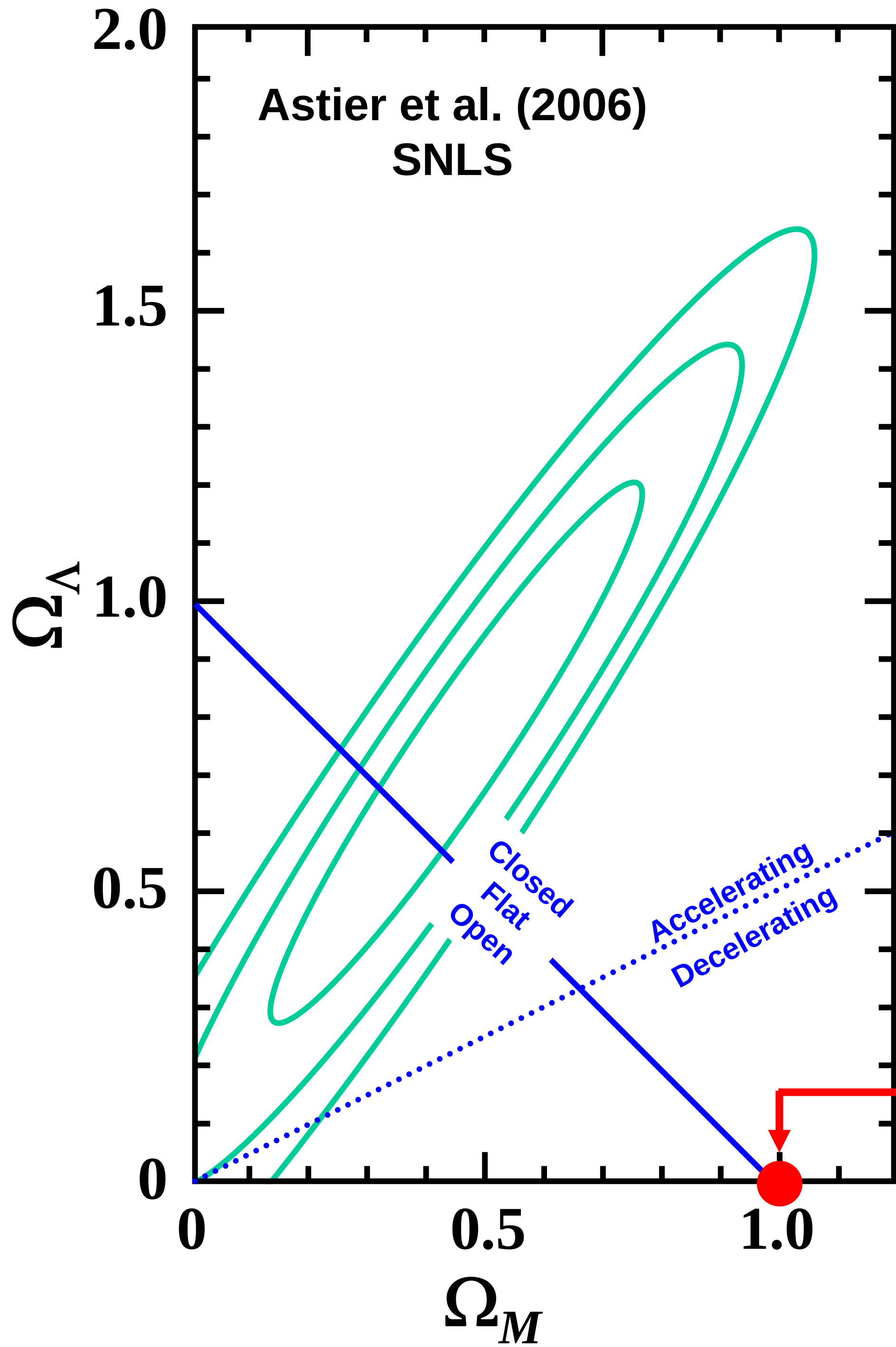
- Age of the universe

$$t(z) \propto \int_z^\infty \frac{dz'}{(1+z')H(z')}$$

Evidence For Dark Energy



Evidence For Dark Energy



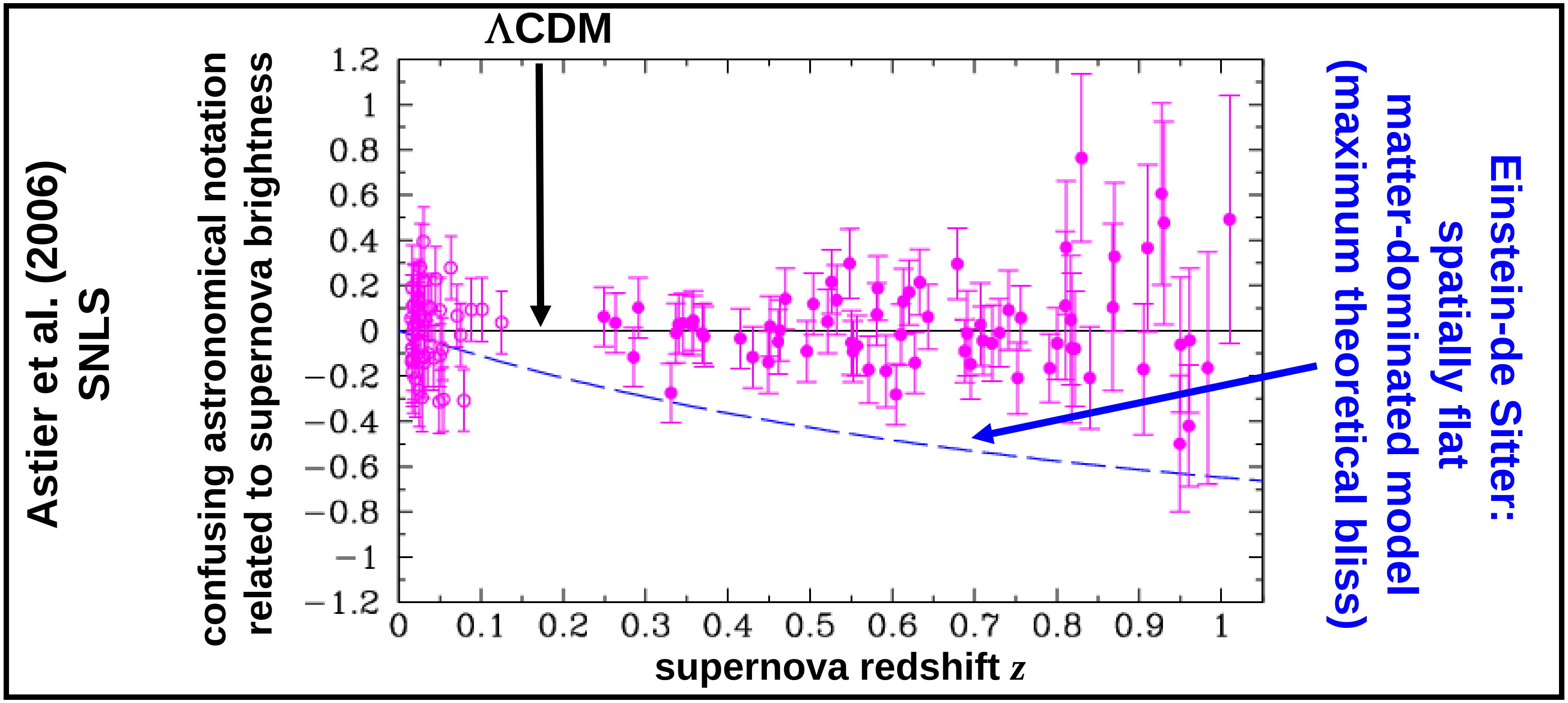
1. Observe magnitude & redshift
2. Assume a cosmological model
   - Assumes $w = -1$ (i.e., Λ)
   - Assumes priors on H_0, etc.
3. Compare observations & model
4. Fit needs cosmoillogical constant

$$\rho_V \sim 10^{-30}\ \mathrm{g\ cm}^{-3}$$

	ρ_V	$\Lambda = 8\pi G \rho_V$
length scale	10^{-3} cm	10^{+29} cm
mass scale	10^{-4} eV	10^{-33} eV

Einstein-de Sitter model
flat, matter-dominated
(maximum theoretical bliss)

Evidence For Dark Energy

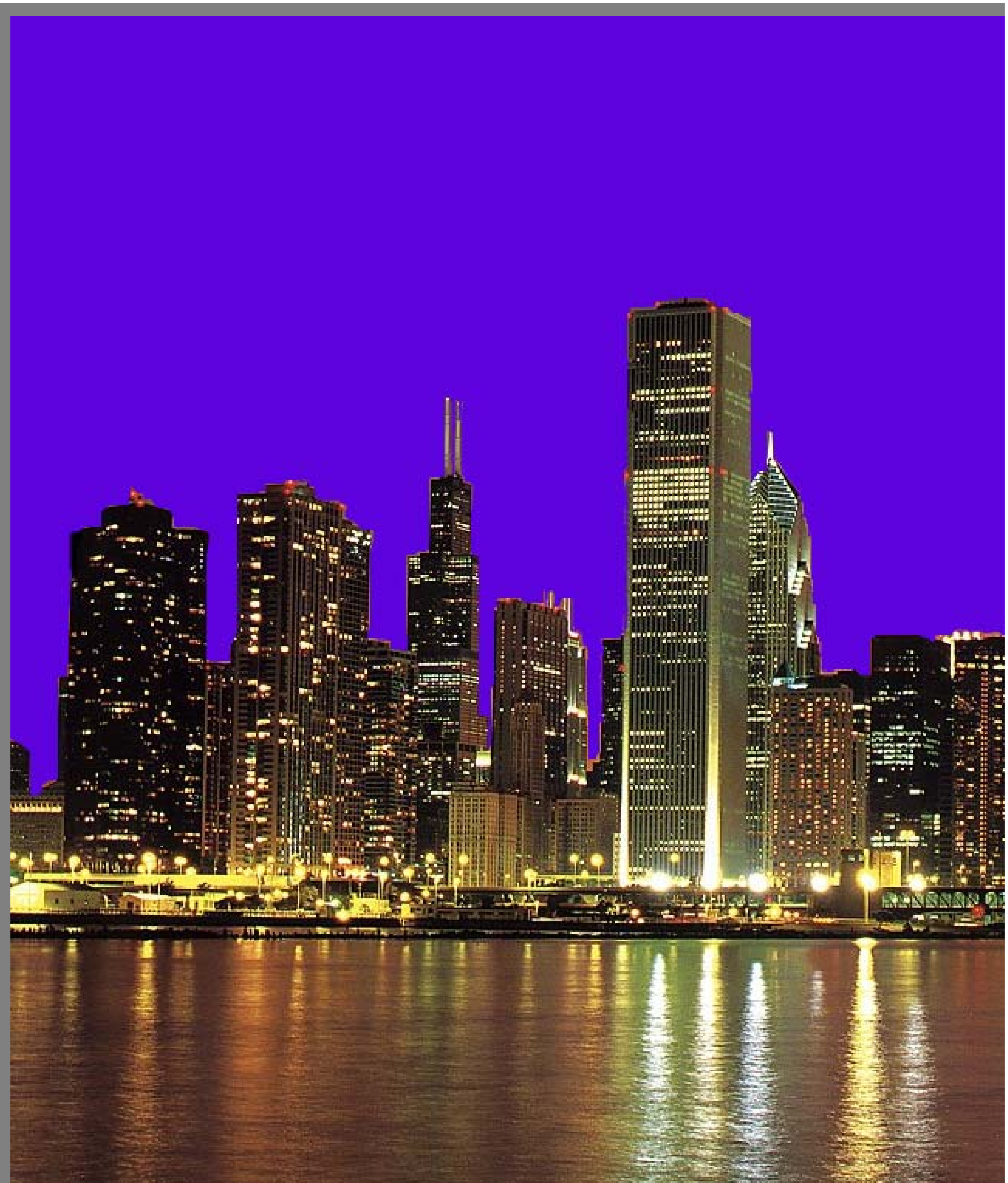


The case for Λ:

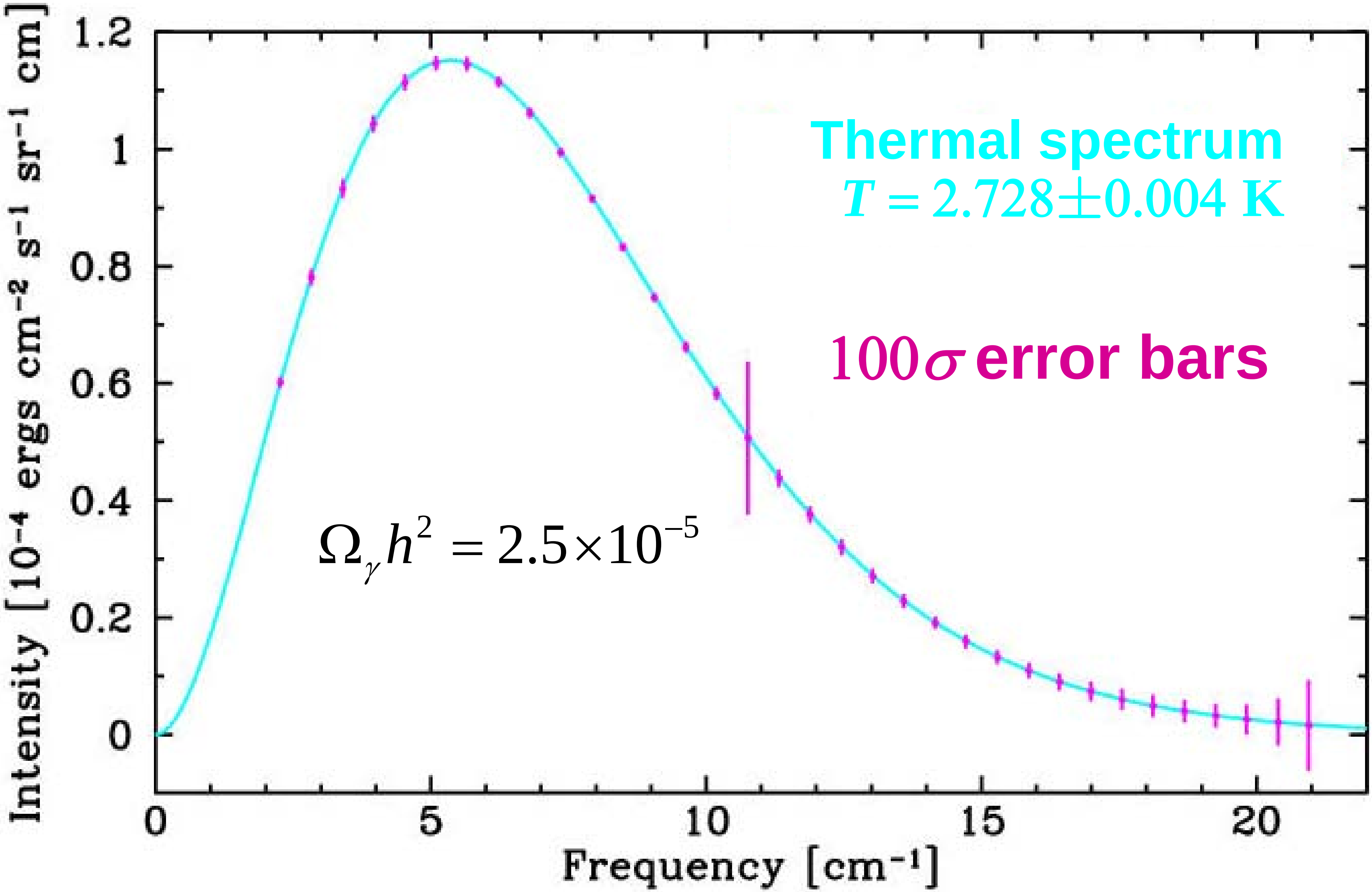
1) Hubble diagram (SNe)
2) Cosmic Subtraction
3) Baryon acoustic oscillations
4) Weak lensing
5) Galaxy clusters
6) Age of the universe
7) Structure formation

Theorist's view of the universe

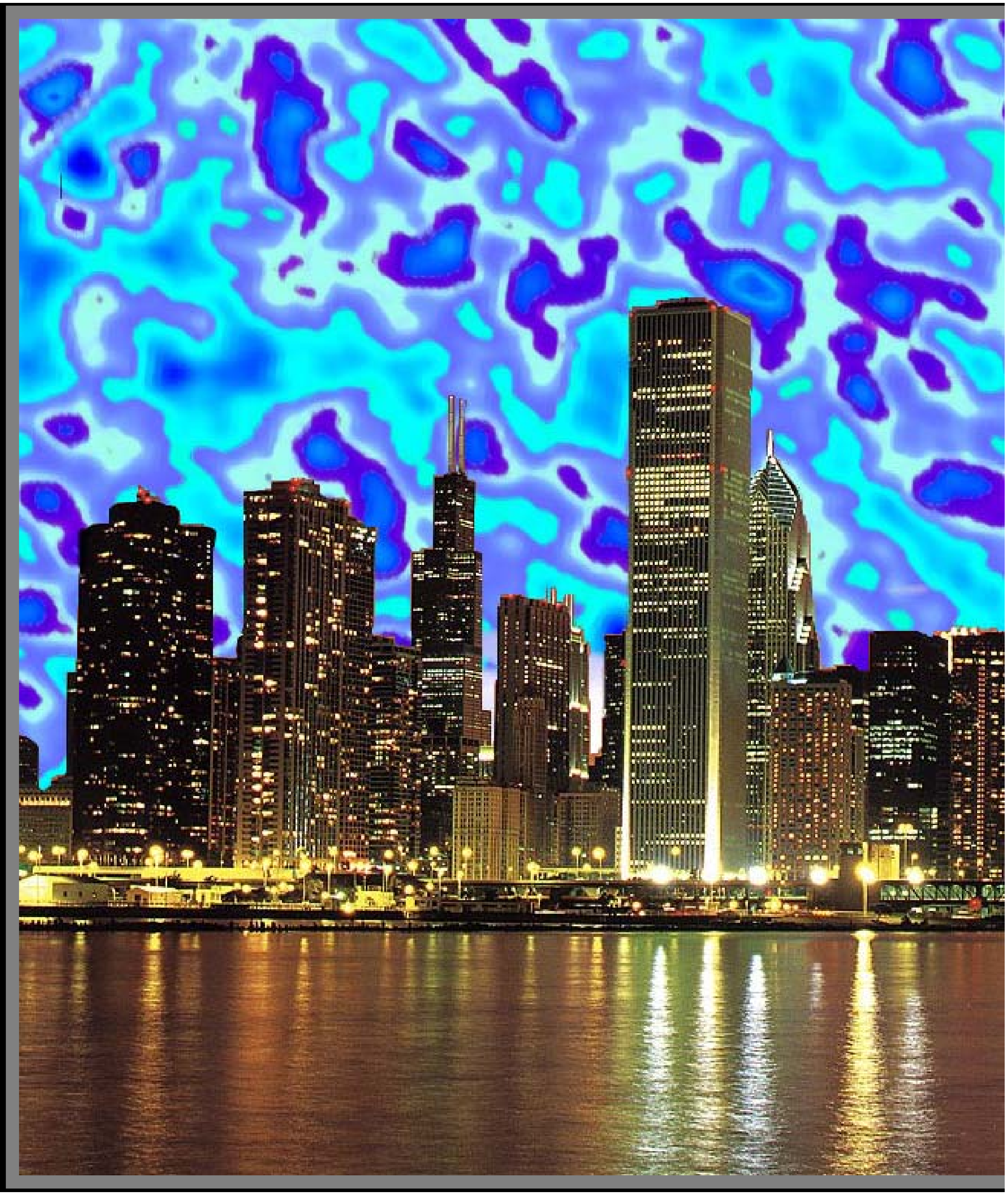
(isotropic)



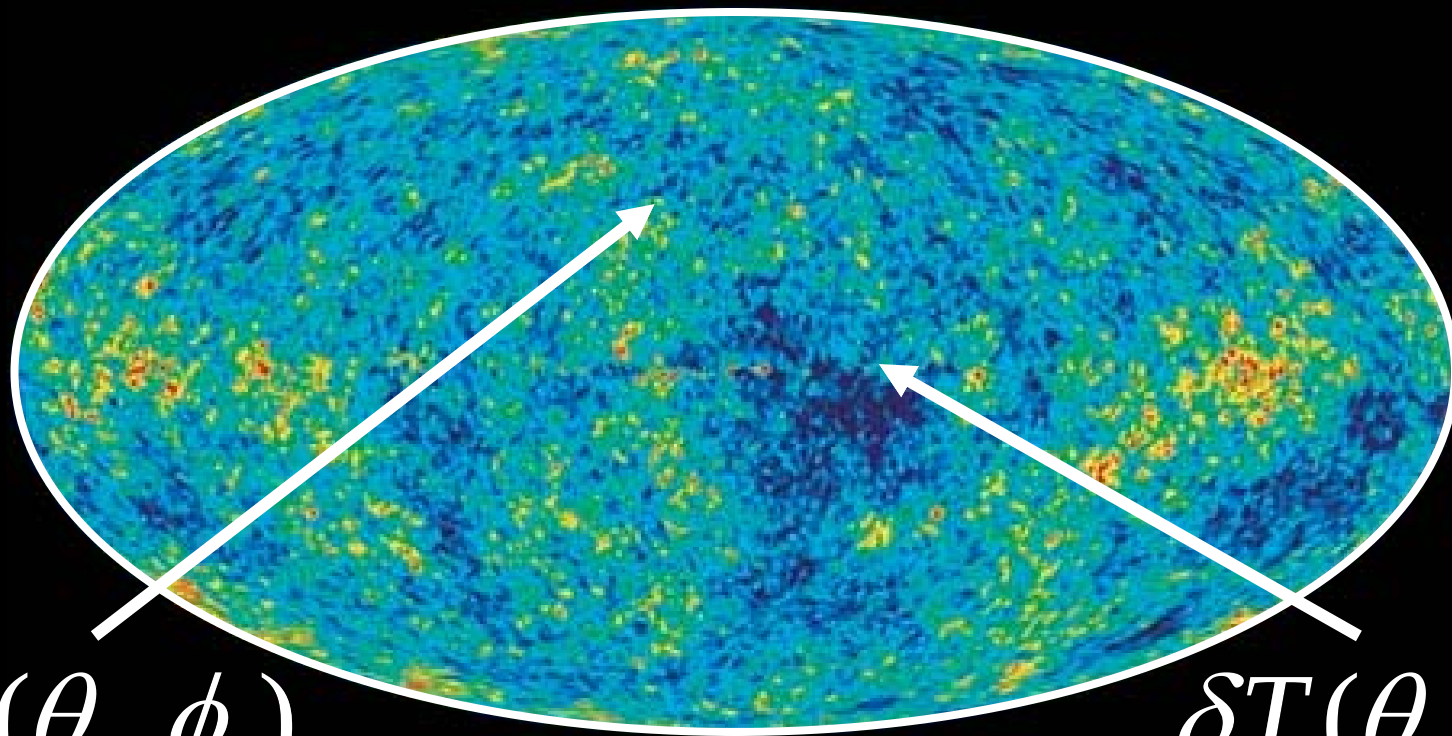
Temperature Of The Universe (COBE)



Observer's view of the universe (fluctuations)



Angular Power Spectrum



$\delta T(\theta_1, \phi_1)$ $\delta T(\theta_2, \phi_2)$

$$\delta T(\theta, \phi) = \sum a_{lm} Y_{lm}(\theta, \phi)$$

$$C_l \equiv \left\langle \left| a_{lm} \right|^2 \right\rangle$$

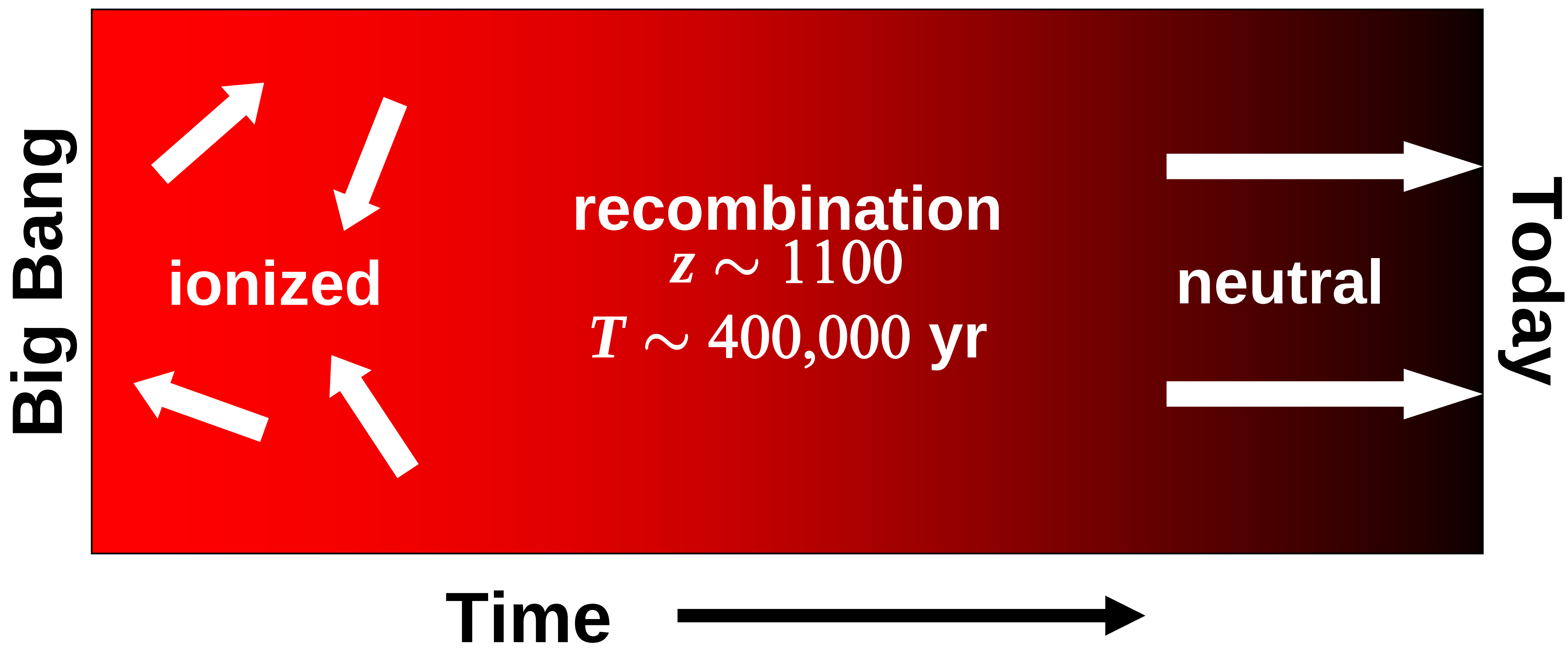
Baryon Acoustic Oscillations

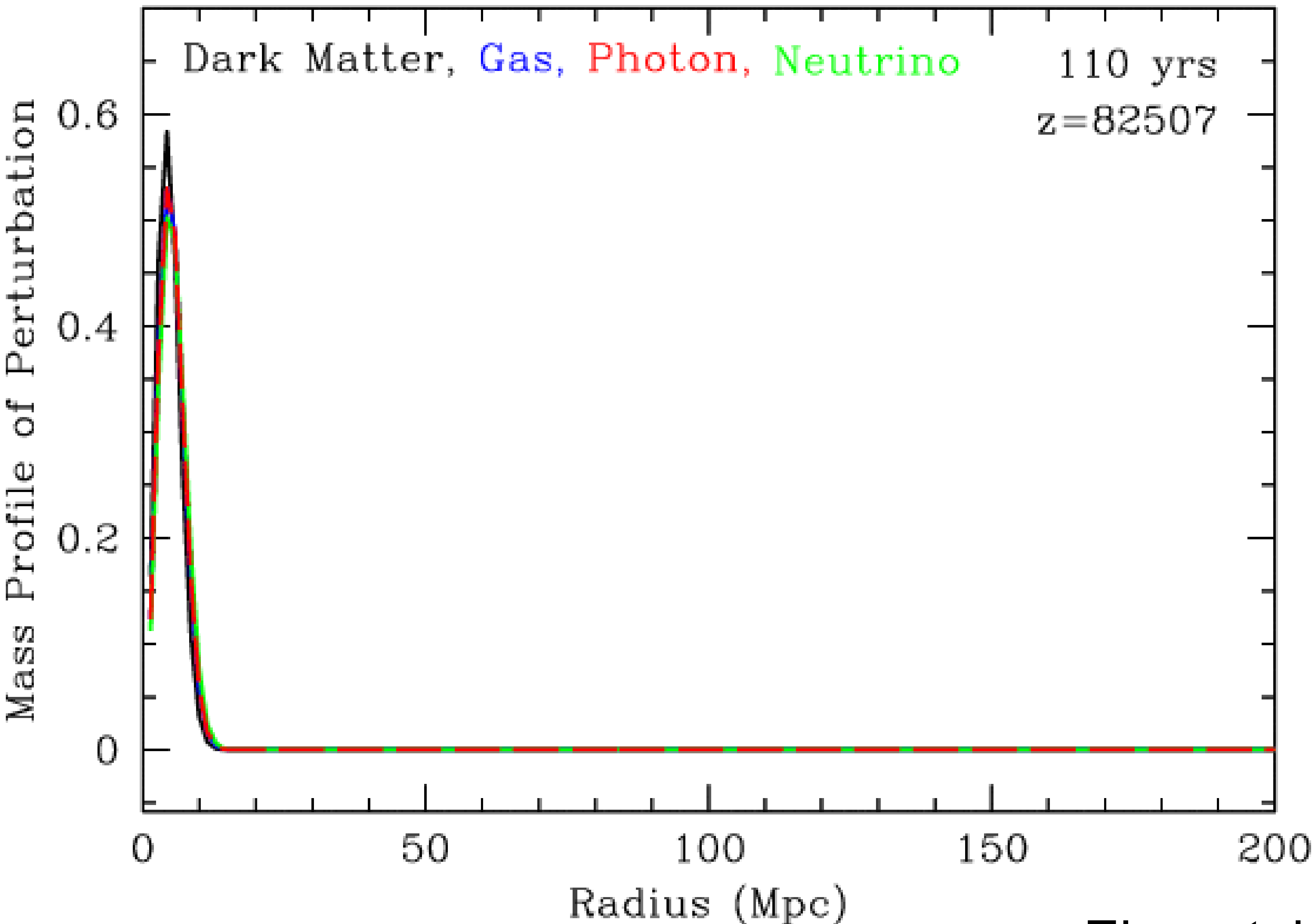
Pre-recombination

- universe ionized
- photons provide enormous pressure and restoring force
- perturbations oscillate (acoustic waves)

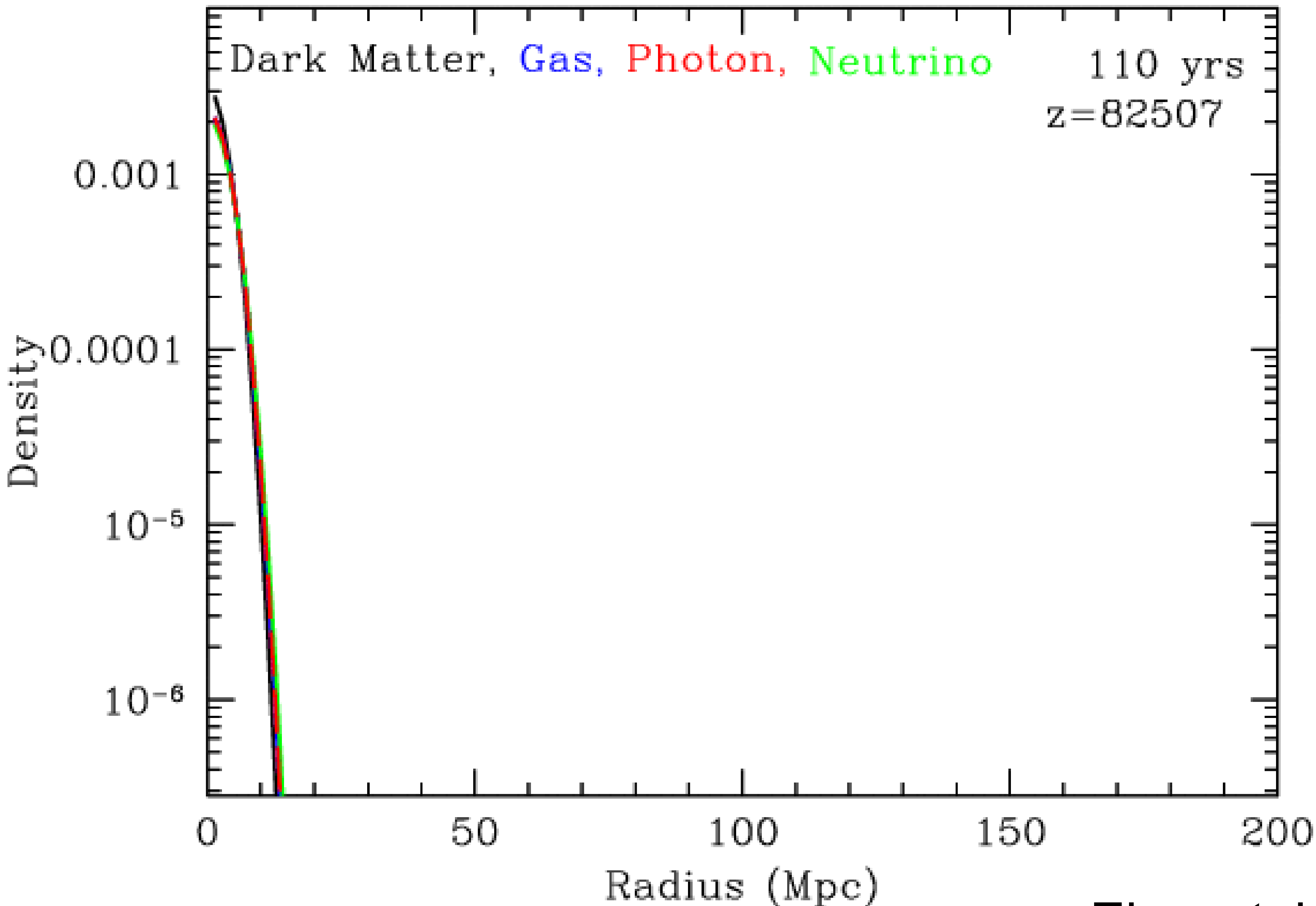
Post-recombination

- universe neutral
- photons travel freely (decouple from baryons)
- perturbations grow (structure formation)



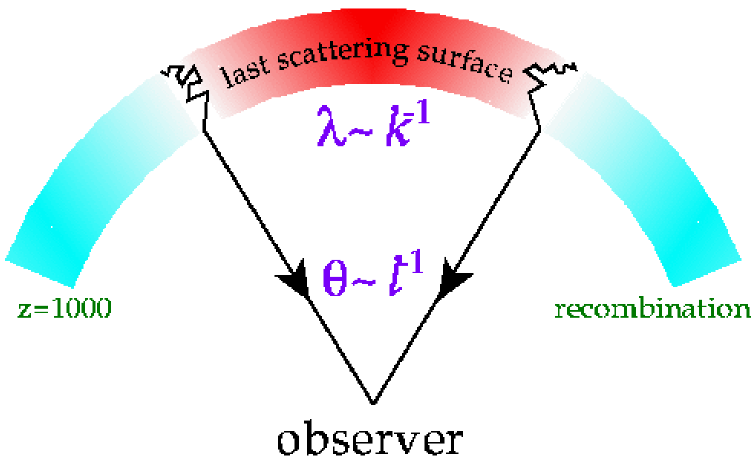

Dark Matter, Gas, Photon, Neutrino
110 yrs
z=82507
Mass Profile of Perturbation
0.6
0.4
0.2
0
0
50
100
150
200
Radius (Mpc)

Eisenstein


Dark Matter, Gas, Photon, Neutrino
110 yrs
z=82507
Density
0.001
0.0001
10^{-5}
10^{-6}
0
50
100
150
200
Radius (Mpc)

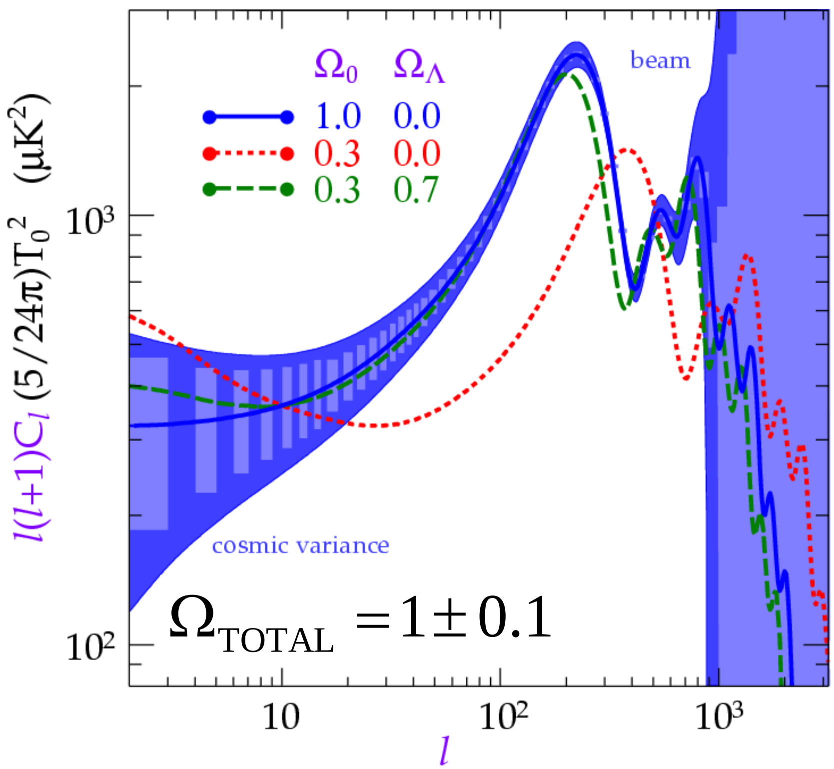
Eisenstein

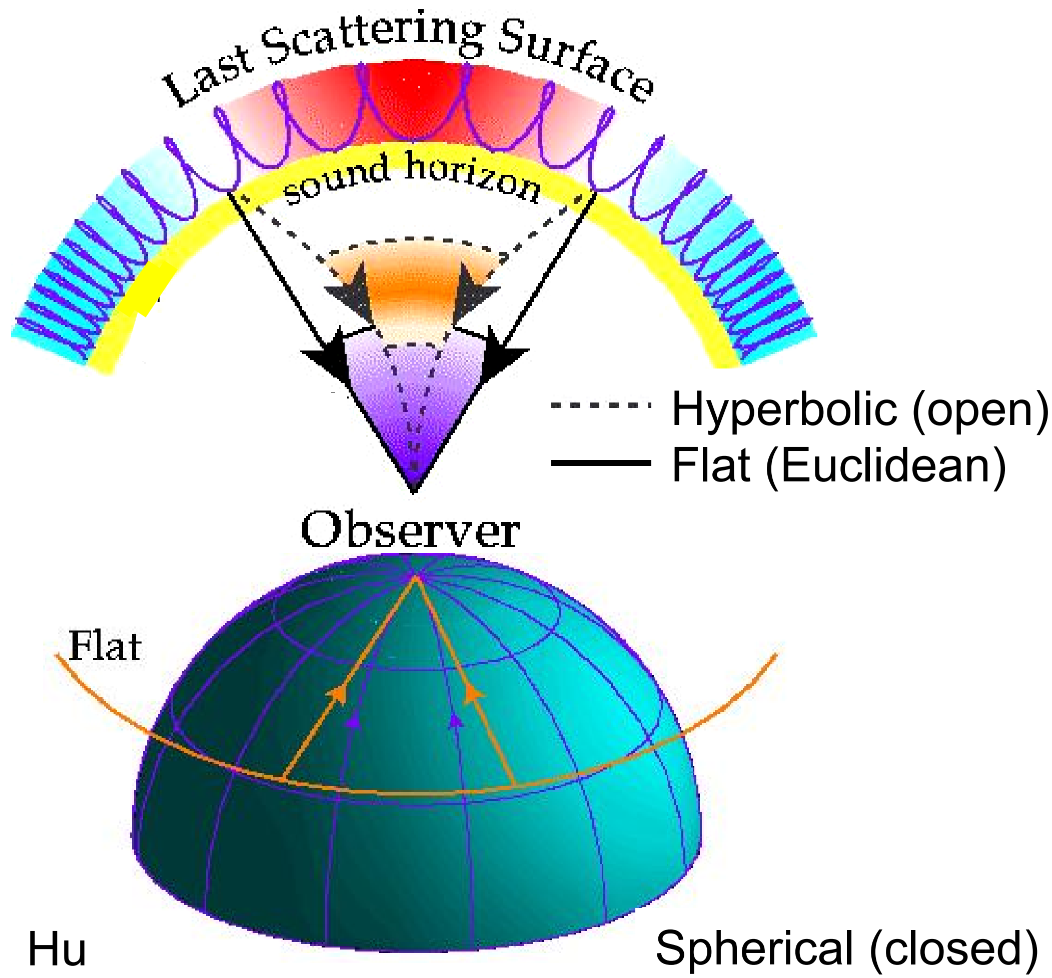
Acoustic Peaks



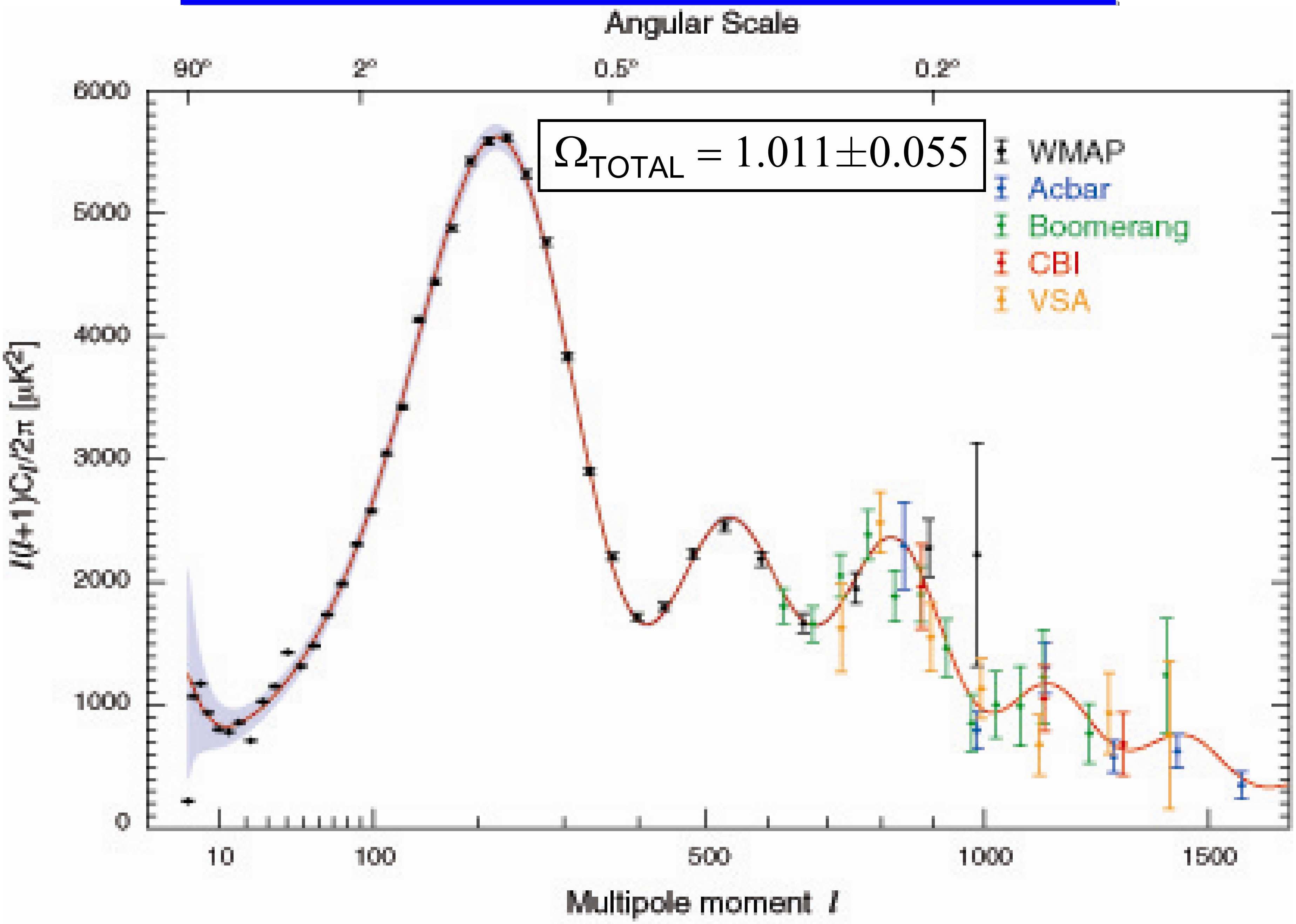
Sound travel distance known

Observed l_{peak} ~ geometry



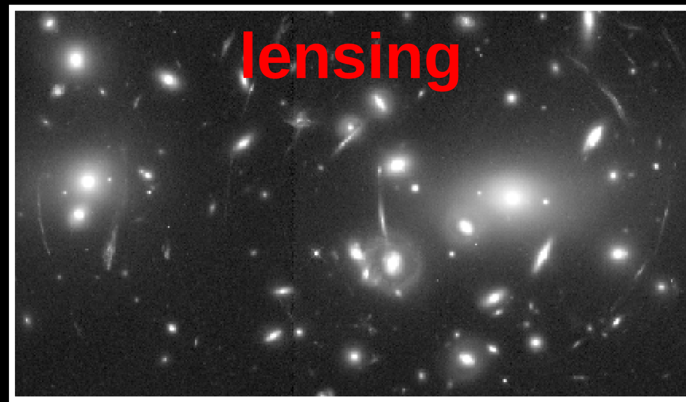


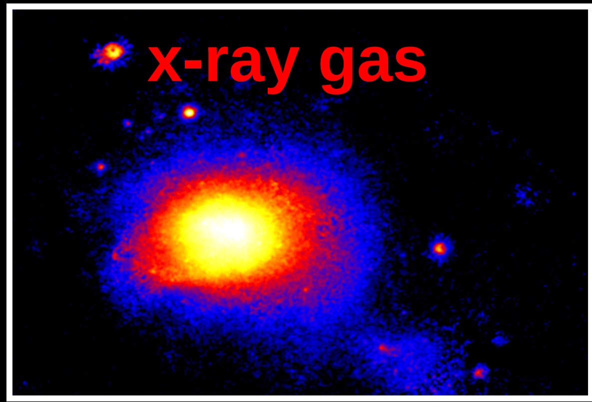
Baryon Acoustic Oscillations

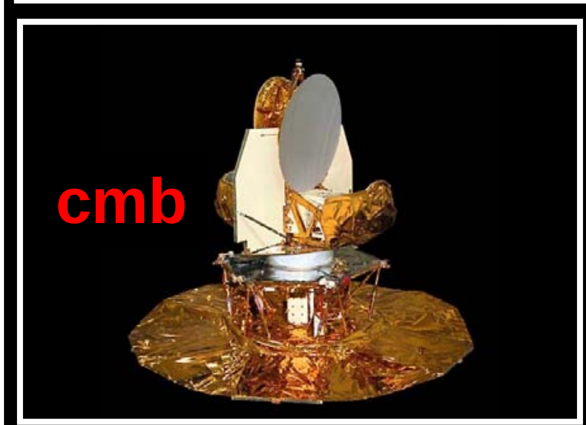


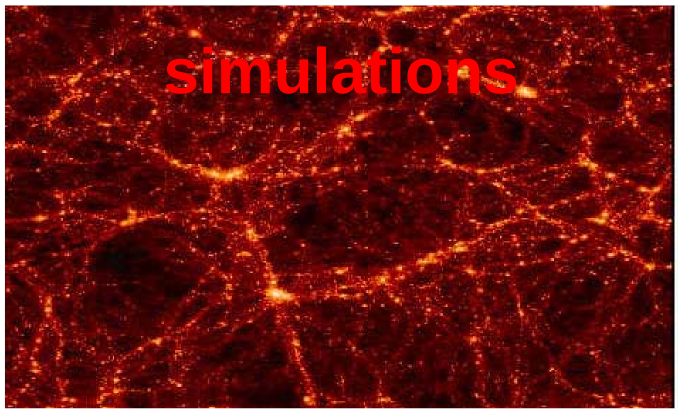
ESA Mission Planck - Launch In 2008?

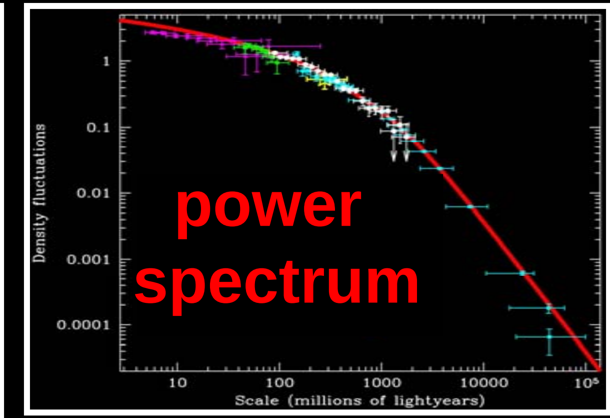
Cosmic Subtraction











$\Omega_{\mathrm{TOTAL}} = 1$

$\Omega_M \sim 0.3$

$1.0 - 0.3 = 0.7$

CMB

many methods

subtraction

Observer's View of the Universe

lumpy (inhomogeneous and anisotropic)
full of stars, galaxies, clusters, ….

Theorist's View of the Universe

Actual image of dark matter

smooth (homogeneous and isotropic)
full of dark matter (and dark energy)

Power Spectrum

- Assume there is an average density $\bar{\rho}$
- Expand density contrast $\delta(\vec{x})$ in Fourier modes

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}} = \int \delta_{\vec{k}} \exp\left(-i\vec{k}\cdot\vec{x}\right) d^3k$$

- Autocorrelation function defines power spectrum

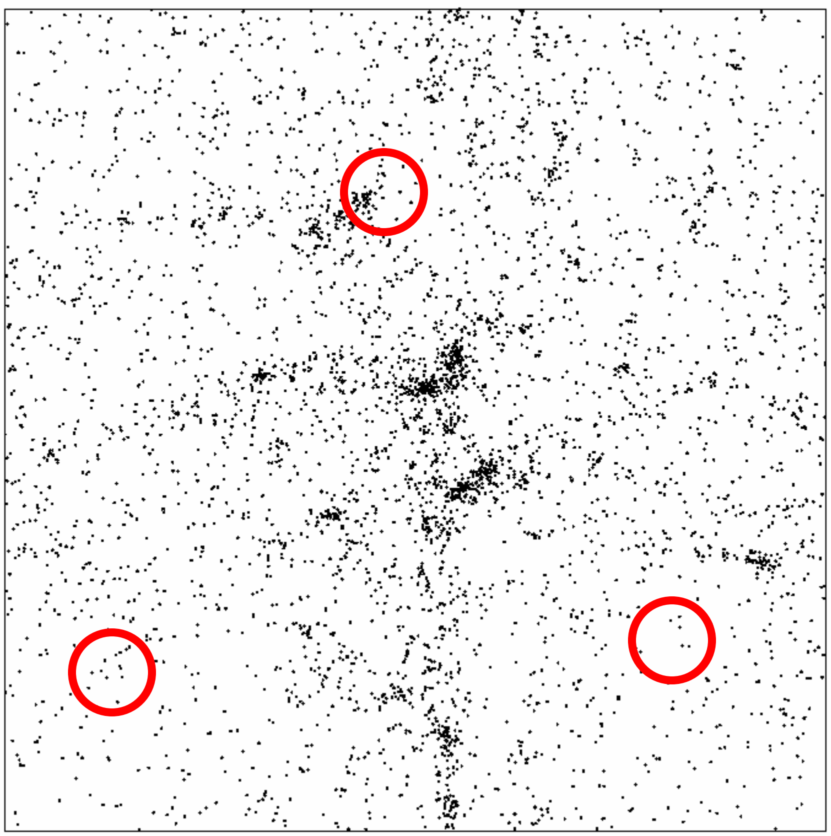
$$\left\langle \frac{\delta\rho(\vec{x})}{\rho} \right\rangle^2 = \left\langle \delta(\vec{x})\delta(\vec{x}) \right\rangle = \int_0^\infty \frac{dk}{k} \, \frac{k^3 \left|\delta_{\vec{k}}^2\right|}{2\pi^2}$$

$$\Delta^2(k) \equiv \frac{k^3 \left|\delta_{\vec{k}}^2\right|}{2\pi^2} \qquad\qquad P(k) \equiv \left|\delta_{\vec{k}}^2\right|$$

Power Spectrum

- Power spectrum related to *rms* fluctuations

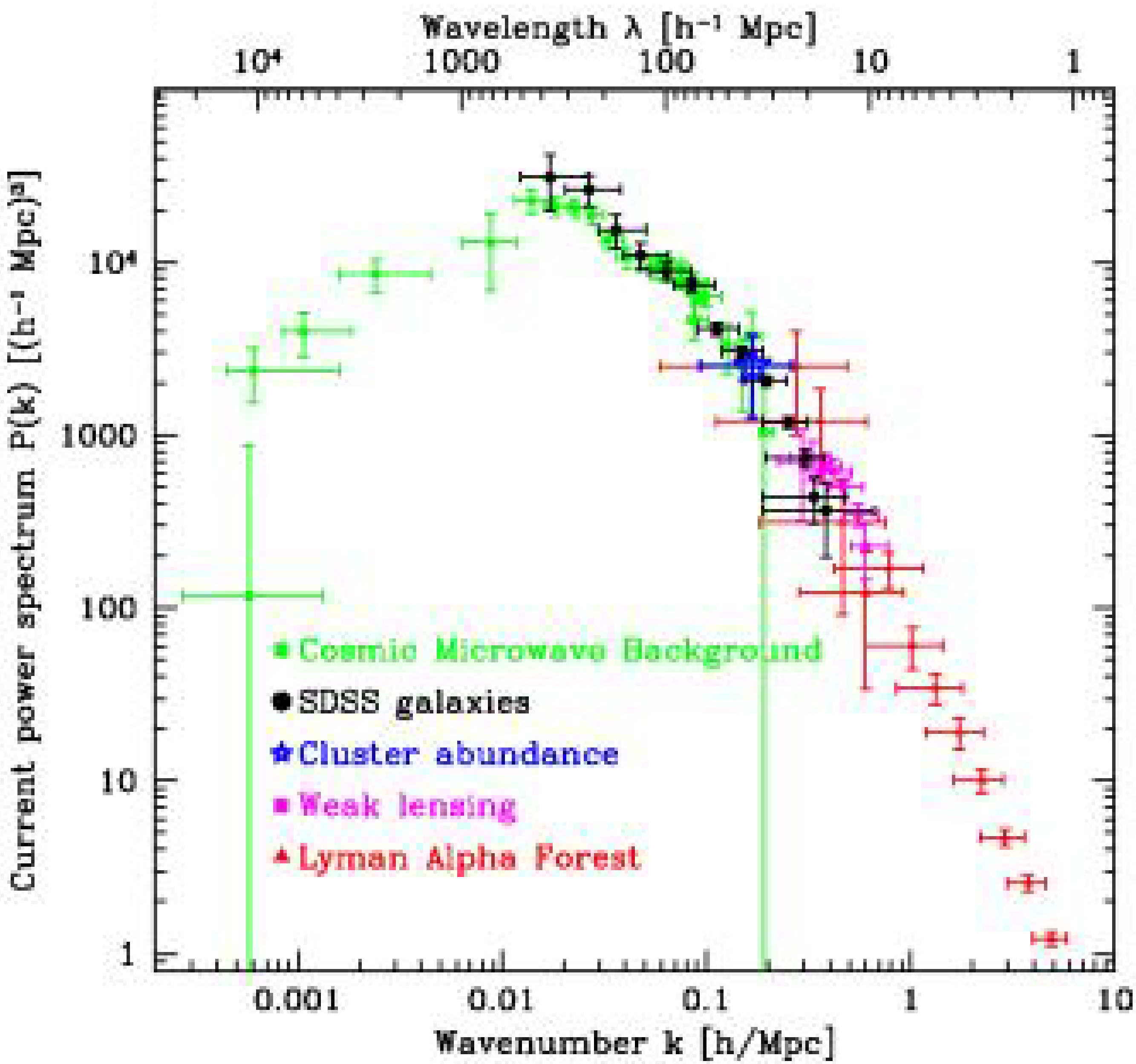
$$\left\langle \frac{\delta\rho(\vec{x})}{\rho} \right\rangle_R \approx \Delta(k = R^{-1})$$



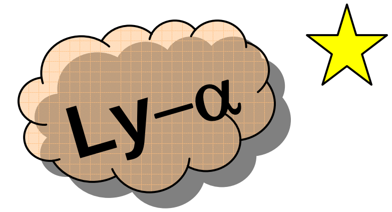
○ sphere of radius *R*

trial	N	variance
1	N_1	$N_1 - \bar{N}$
2	N_2	$N_2 - \bar{N}$
3	N_3	$N_3 - \bar{N}$
⋮	⋮	⋮
	$\bar{N}$	$\sqrt{\sum (N_i - \bar{N})^2}$

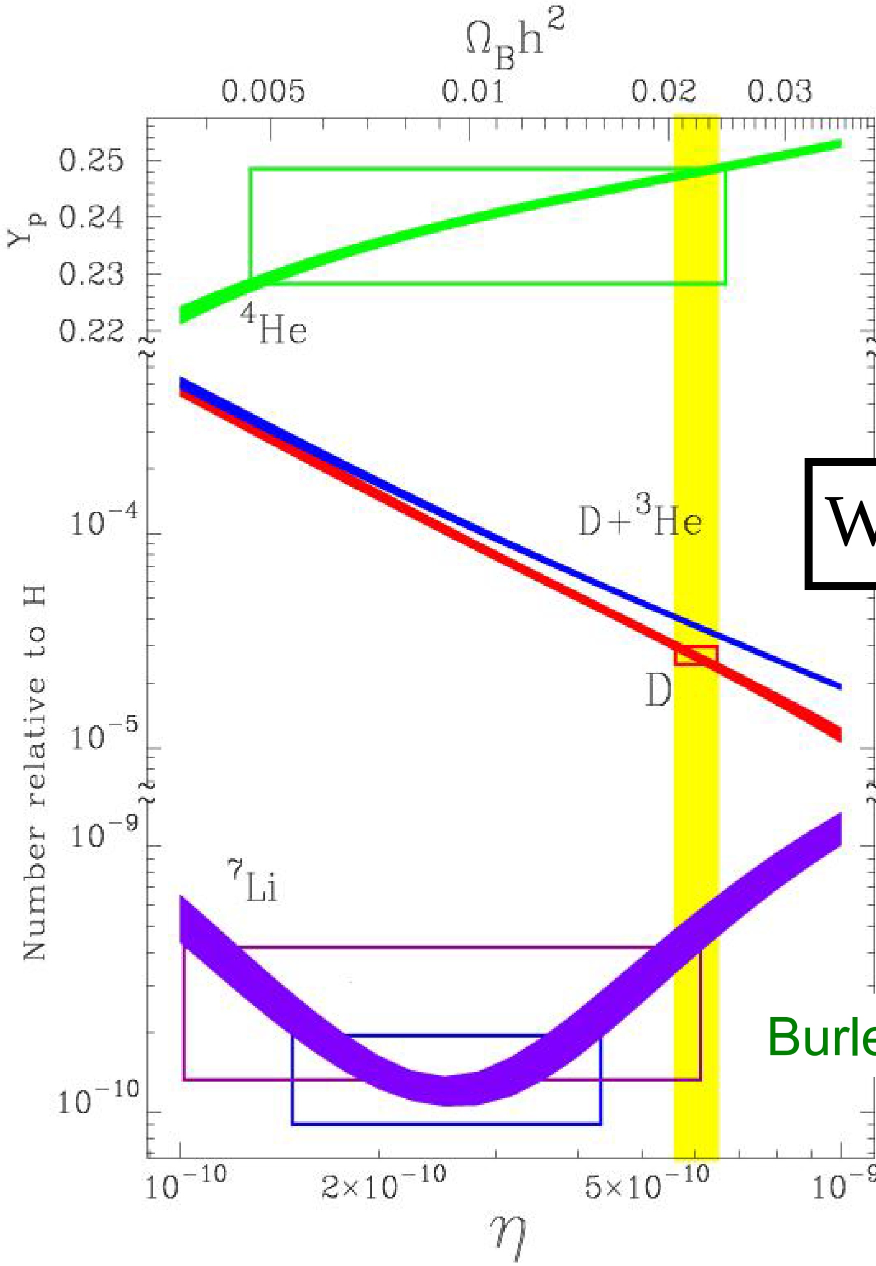
Power Spectrum

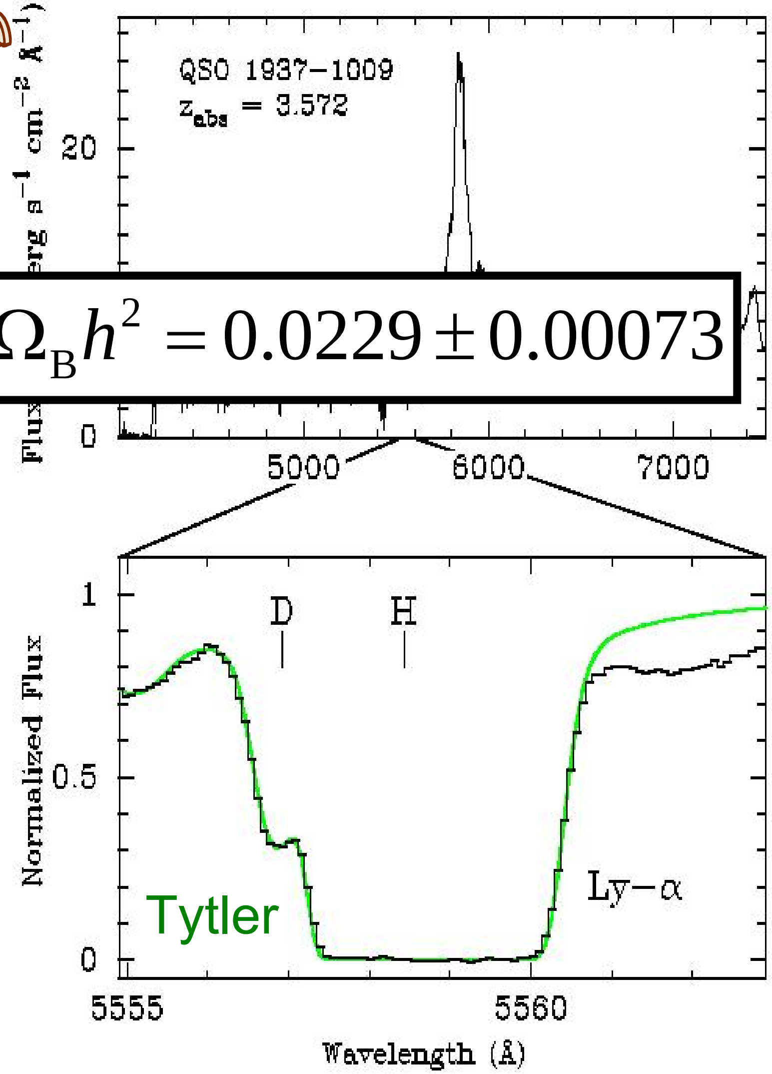


<u>Baryons</u> $\Omega_B h^2 \sim 0.02$



QSO 1937-1009





$$\text{WMAP: } \Omega_B h^2 = 0.0229 \pm 0.00073$$

Burles et al.

Neutrinos $\Omega_\nu h^2 \sim 0.001$

- Neutrinos decouple earlier than photons (about 1 second AB)
- Their number density relative to photons is reduced because of $e^\pm$ annihilation.
- Today $T_\nu = 1.9$ K (rather than 2.7 K for photons)
- $\Omega_\nu h^2 = m_\nu / 93$ eV
- Of course have never detected directly background νs, … but strong circumstantial evidence that they exist

Age Of The Universe: t_0

- **white dwarf star cooling** 11 ± 2 Gyr
- **nucleocosmochronology** 12.6 ± 3 Gyr
- **globular cluster evolution** 13.5 ± 2 Gyr

$H_0 = 70$	Ω_M	Ω_Λ	t_0 **(*Gyr*)**
Flat	1.0	0	9.3
Open	0.3	0	12
Open	0.2	0	14
Flat	0.3	0.7	13.5
Flat	0.2	0.8	15

The Universe Observed

- Big-bang primer (just a little theory)

- Cosmological parameters (after all, they are just numbers):

  $H_0 \rightarrow$ the present expansion rate (Hubble's constant)

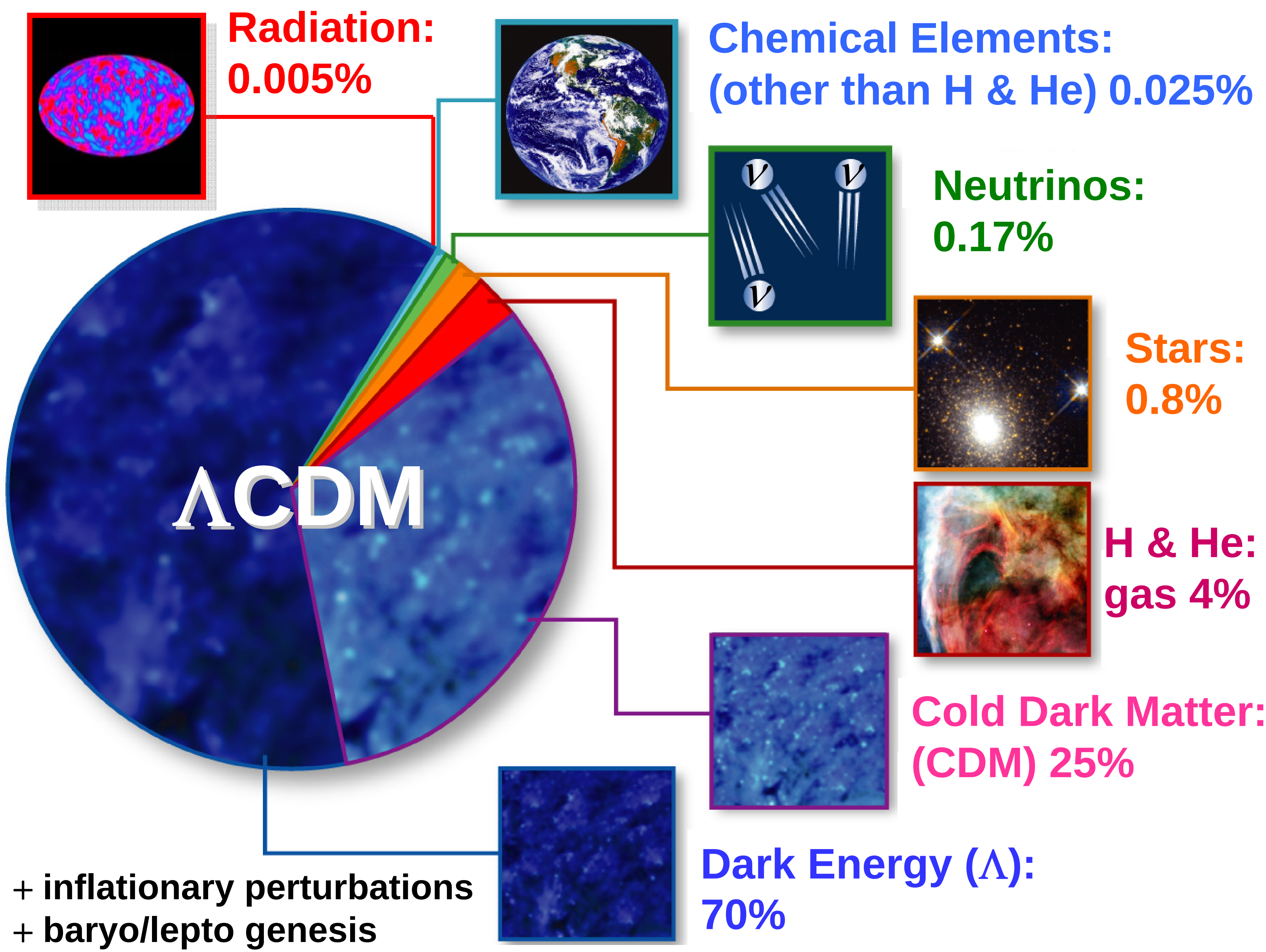
  $\Omega_i \rightarrow$ the present cosmic food chain
  (Ω_{TOTAL}, Ω_M, Ω_B, Ω_Λ, Ω_γ, Ω_ν, …)

  $T_0 \rightarrow$ the present temperature of the Universe

  $t_0 \rightarrow$ the present age of the Universe

- Power spectra–characterization of perturbations:
  Galaxies: $P(k)$ Radiation: C_ℓ

- "Standard model": ΛCDM Dark Energy and Dark Matter


Radiation:
0.005%
Chemical Elements:
(other than H & He) 0.025%
Neutrinos:
0.17%
Stars:
0.8%
H & He:
gas 4%
Cold Dark Matter:
(CDM) 25%
Dark Energy (Λ):
70%
ΛCDM
+ inflationary perturbations
+ baryo/lepto genesis

Suggested reading:

Particle orientation:
The Early Universe, Kolb & Turner

Structure formation, closer to astronomy:
Cosmological Physics, Peacock

Inflation and structure formation:
Cosmological Inflation and large-Scale Structure, Liddel & Lyth

Cosmic Background Radiation:
Modern Cosmology, Dodelson

The Dark Universe: Dark Matter and Dark Energy

Rocky I:	The Universe Observed	Monday
Rocky II:	Inflation	Tuesday
Rocky III:	Dark Matter	Wednesday
Rocky IV:	Dark Energy	Thursday

CERN Academic Training Lectures **January 2008**

Rocky Kolb *The University of Chicago*