comments on ...

Merging the Fixed and Variable Flavor Number PDF schemes

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Would you buy a one-gear car?

2

- car with single gear (FFNS)

✓ don't have to shift

✓ drive around city

✗ won't work on highway

- automatic trans. changing at fixed speed: 30, 40 ... (VFNS)

✓ shifts automatically

✓ can go to highway

✗ somethimes can shift when you don't want

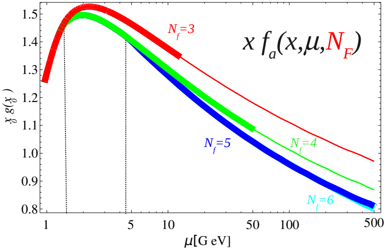
- manual transmition (Hybrib VFNS)

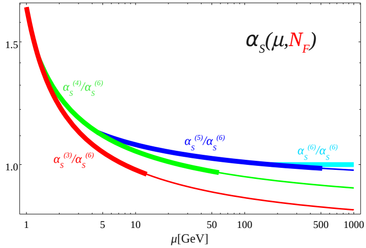
✓ you decide what to do

✗ do it responsible

OUR SOLUTION: N_F-dependent PDFs & α_S

$$f_i(x,\mu) \to f_i(x,\mu,N_F)$$
$$\alpha_S(\mu) \to \alpha_S(\mu,N_F)$$



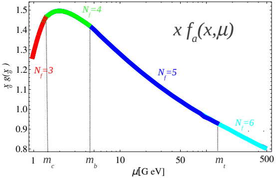


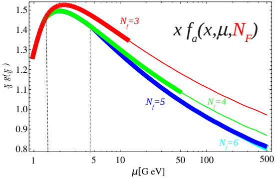
- coexisting N_F-dependent PDFs & α_S
- user can choose when to switch between N_F and N_F+1 scheme

Similarities and differences with VFNS

✗ VFNS has no overlap between schemes with different N_F

✗ In VFNS *historically* switching/transition **always** at $\mu = m_Q$





✓ both use the same matching conditions

$$f_i(x,\mu,N_F+1) = A^{ij} \otimes f_j(x,\mu,N_F)$$

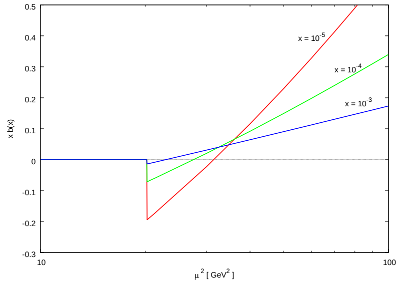
$$\alpha_S(\mu,N_F+1) = B \times \alpha_S(\mu,N_F)$$

✓ both features

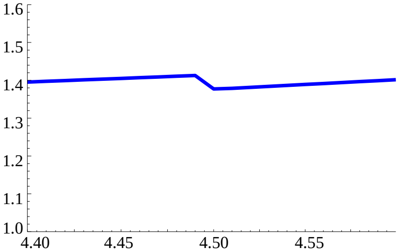
- resummation of logarithms $\log \mu/m_{c,b}$
- can be reliably extended to high scales $1 \lesssim \mu/m_{c,b} \lesssim \infty$
- in the low scales reduces to FFNS

Discontinuities in higher orders

$xb(x)$



$F_L(x = 10^{-5})$

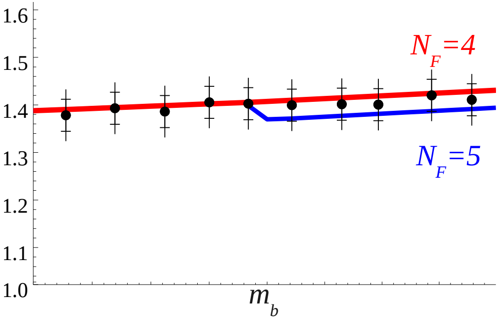


RESULTS

Practical problem for VFNS

If we want to analyze HERA F_2^{charm} data (1106.1028): $Q \sim [2, 10]$GeV

✗ flavor threshold in the middle of a (precision) data set



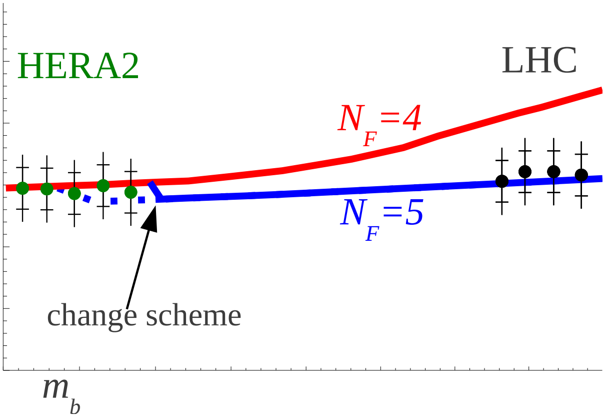
- at higher orders PDFs/α_S discontinuous
- discontinuities in perturbatively calculated observables

✓ In principle the matching can be done wherever for $\mu \sim m_Q$

✗ Historically it is always done for $\mu = m_Q$

Why do we want overlap?

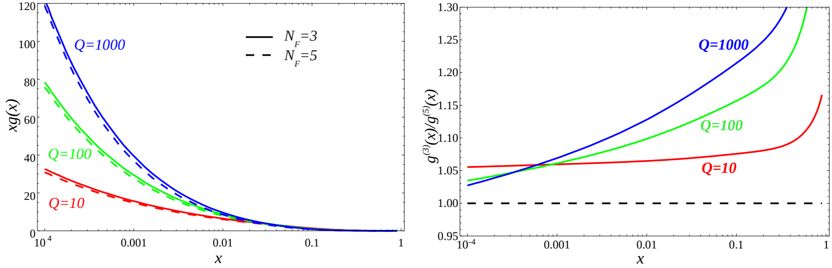
Switch between N_F and $N_F + 1$ when convenient



- remember about uncancelled $\log(Q/m_Q)$ in FFNS

Charm production at HERA

Estimate effect of $N_f = 3$ vs. $N_f = 5$ by comparing gluons



1. HVQDIS (hep-ph/9706334), used in HERA analyses, uses $N_F = 3$ FFNS. We can use gluon ratio

$$g^{(N_F=5)}(x) \simeq g^{(N_F=3)}(x) \left[\frac{g^{(N_F=5)}(x)}{g^{(N_F=3)}(x)} \right]$$

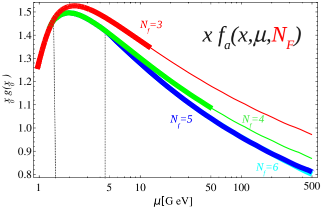
to judge the effect of using $N_F = 5$ PDFs.

2. at low Q (10GeV) the shift is $\sim 6\%$ and *flat* along x

BACKUP SLIDES

Sketch of a PDF fit in HFNS

1. Parametrize PDFs at $\mu_0 \sim 1$ GeV, and generate family of N_F dependent PDFs:



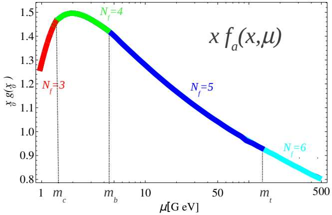
2. Fit HERA F_2^{charm} data using $N_F = 3$ PDFs and α_S.
3. Fit high-scale LHC data using $N_F = 5$ PDFs and α_S.
4. Minimize χ^2 by adjusting PDF parameters at $\mu = \mu_0$.
5. Regenerate new family of PDFs (for $N_F = 3, 4, 5$) and repeat steps 2-4.

Variable Flavor Number Scheme (VFNS)

- Different schemes for different number of flavors connected by matching conditions

$$f_i(x,\mu,N_F+1) = A^{ij} \otimes f_j(x,\mu,N_F)$$

$$A^{ij} = \delta^{ij} + \frac{\alpha_s}{2\pi}\left(a_1^{ij} + b_1^{ij}\ln\left[\frac{\mu}{m}\right]\right) + \left(\frac{\alpha_s}{2\pi}\right)^2\left(a_2^{ij} + b_2^{ij}\ln\left[\frac{\mu}{m}\right] + c_2^{ij}\ln^2\left[\frac{\mu}{m}\right]\right) + \ldots$$



  - resummation of logarithms $\log \mu/m_{c,b}$
  - can be reliably extended to high scales $1 \lesssim \mu/m_{c,b} \lesssim \infty$
  - in the low scales reduces to FFNS

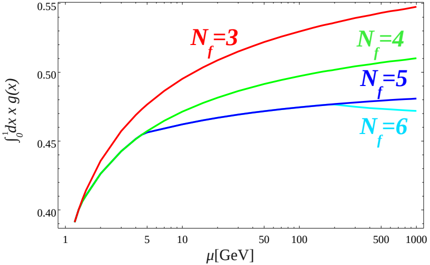
☺ Can be used for description of different data in broad kinematic range, like in PDF global analyses

Effect of N_f channels in PDFs

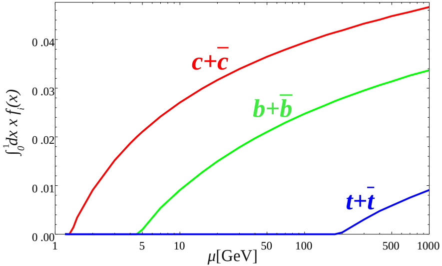
Opening of the new channels (c, b) is compensated by the decreasing gluon

$$g \longrightarrow c\bar{c}$$

Momentum Fraction: g



Momentum Fraction: c, b, t



$$f_i(x,\mu,N_F+1) = A^{ij} \otimes f_j(x,\mu,N_F)$$

$$A^{ij} = \delta^{ij} + \frac{\alpha_s}{2\pi}\left(a_1^{ij} + b_1^{ij}\ln\left[\frac{\mu}{m}\right]\right) + \left(\frac{\alpha_s}{2\pi}\right)^2\left(a_2^{ij} + b_2^{ij}\ln\left[\frac{\mu}{m}\right] + c_2^{ij}\ln^2\left[\frac{\mu}{m}\right]\right) + \ldots$$

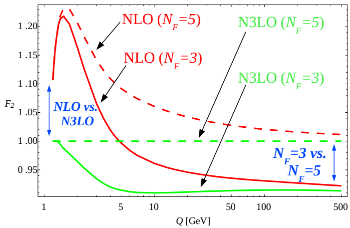
at NLO $a_1^{ij} = 0$

at NNLO $a_2^{ij} \neq 0$

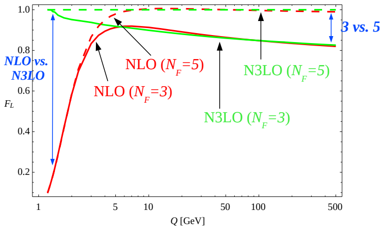
N_f dependence of structure functions (N3LO: 1203.0282)

- $Q < m_Q$:
  - higher order corrections are crucial;
  - $N_F = 3$ and $N_F = 5$ results coincide
- $Q \gg m_Q$:
  - higher order corrections not important;
  - $N_F = 3$ and $N_F = 5$ results diverge due to uncancelled logs $\log(Q/m_q)$
- $Q \geq m_Q$:
  - $N_F = 3$ result can be extended to $Q \sim$ (a few) $\times m_c$

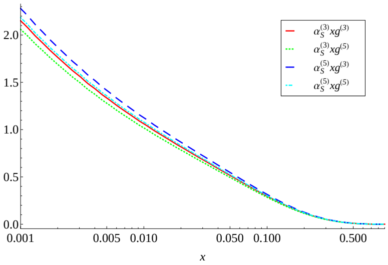
F_2 Ratio

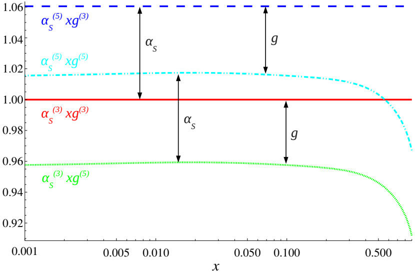


F_L Ratio



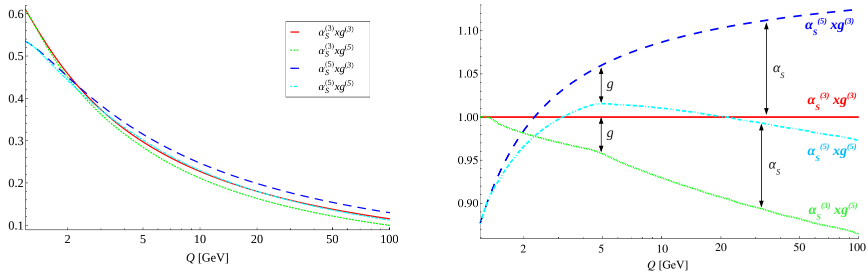
Interplay between $\alpha_S(\mu_R, N_f)$ and $g(x, \mu, N_f)$





- LO ~ light quark PDFs (not sensitive to change $N_F \to N_F + 1$)
- combination $\alpha_S g$ enters many NLO corrections (leading component)
- for $Q \sim 10$GeV consistent 3-flavor (FFNS) or 5-flavor (VFNS) schemes are comparable: flavor dependence in α_S and g partly cancel

Interplay between $\alpha_S(\mu_R, N_f)$ and $g(x, \mu, N_f)$



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Matching conditions for PDFs

$$f_i(x, \mu, N_F + 1) = A^{ij} \otimes f_j(x, \mu, N_F)$$

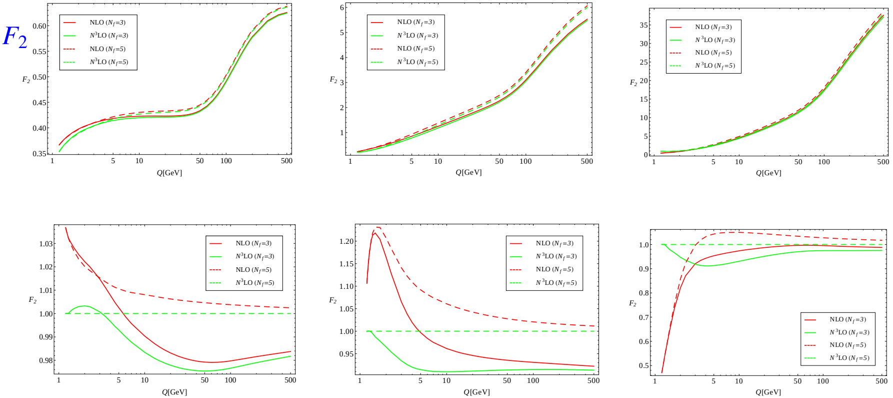
where A^{ij} can be computed perturbatively as (hep-ph/9601302, hep-ph/9612398)

$$\begin{aligned} A^{ij} &= \delta^{ij} + \frac{\alpha_s}{2\pi} \left(a_1^{ij} + P_0^{ij} \ln\left[\frac{\mu^2}{m^2}\right] \right) \\ &+ \left(\frac{\alpha_s}{2\pi}\right)^2 \left(a_2^{ij} + \left\{ P_1^{ij} + P_0^{ij} \otimes a_1^{ij} - \beta_0 a_1^{ij} \right\} \ln\left[\frac{\mu^2}{m^2}\right] \right. \\ &\left. + \frac{1}{2} \left\{ P_0^{ij} \otimes P_0^{ij} - \beta_0 P_0^{ij} \right\} \ln^2\left[\frac{\mu^2}{m^2}\right] \right) + \ldots \end{aligned}$$

N_f dependence of structure functions: F_2

(N3LO calculation: `1203.0282`)

$x = 10^{-1}$ $x = 10^{-3}$ $x = 10^{-5}$



N_f dependence of structure functions: F_L

$x = 10^{-1}$ | $x = 10^{-3}$ | $x = 10^{-5}$

F_L