

# $\gamma/\phi_3$ measurements at LHCb

CKM2014, Vienna

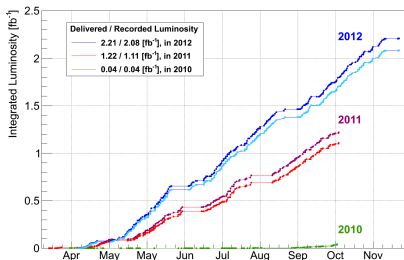
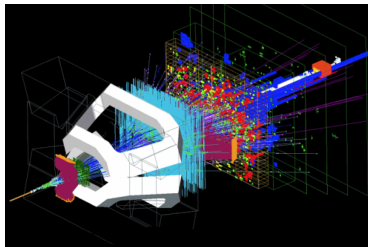
Till Moritz Karbach  
(on behalf of the LHCb collaboration)

CERN

11.09.2014

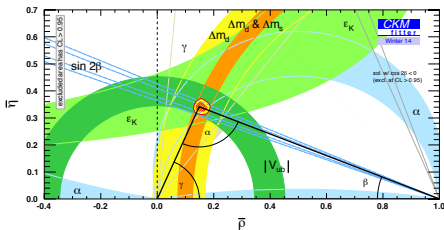
# Introduction

- ▶ LHCb is a forward spectrometer operated in collider mode.
- ▶ precision measurements of  $b$ - and  $c$ -hadron decays
- ▶ rare decays,  $CP$  violation,  $\gamma$
- ▶ very good  $K^\pm/\pi^\pm$  separation: two RICH detectors
- ▶ very good time resolution ( $B_s^0$  oscillation): VELO
- ▶ Dataset:
  - 2011:  $1 \text{ fb}^{-1}$
  - 2012:  $2 \text{ fb}^{-1}$
  - total:  $3 \text{ fb}^{-1}$



# Introduction

- ▶  $\gamma$  is the least well known angle of the unitarity triangle
- ▶ it can be measured entirely from tree decays
- ▶ the residual uncertainty is negligible:  $\delta\gamma/\gamma \approx 10^{-7}$   
see Brod on Tuesday!  
Brod, Zupan 2013 [1]



- ▶ **however:** the branching ratios are small, typically at the  $10^{-7}$  level
- ▶ measure  $\gamma$  in different “methods”

GLW  $CP$  eigenstates, e.g.  $D \rightarrow KK$

ADS flavored states, e.g.  $D \rightarrow K\pi$

GGSZ self-conjugate 3-body decays, e.g.  $D \rightarrow K_S^0 \pi \pi$

GLS singly Cabibbo suppressed states, e.g.  $D \rightarrow K_S^0 K \pi$

time-dep. e.g.  $B_s^0 \rightarrow D_s^\mp K^\pm$

## Introduction

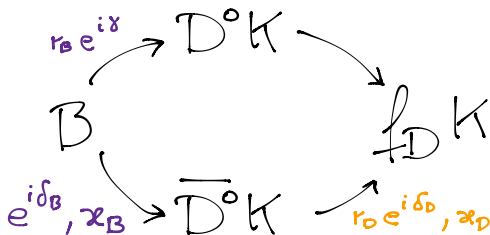


Figure: Example interference diagram. Coherence factors appear with multibody states.

Example observable:

$$R_+^{DK, K3\pi} = \frac{\Gamma(B^+ \rightarrow [\pi K \pi \pi]_D K^+)}{\Gamma(B^+ \rightarrow [K \pi \pi \pi]_D K^+)},$$

$$R_+^{DK, K3\pi} = \frac{(r_B^{DK})^2 + (r_D^{K3\pi})^2 + 2\kappa_D^{K3\pi} r_B^{DK} r_D^{K3\pi} \cos(\delta_B^{DK} + \delta_D^{K3\pi} + \gamma)}{1 + (r_B^{DK})^2 (r_D^{K3\pi})^2 + 2\kappa_D^{K3\pi} r_B^{DK} r_D^{K3\pi} \cos(\delta_B^{DK} - \delta_D^{K3\pi} + \gamma)}$$

## Introduction

Our new results, LHCb-CONF-2014-004, supersede the previous LHCb combinations.

**Table:** Summary of results for  $\gamma$  from the  $B$  factories BaBar and Belle, and from LHCb, and combiners. Errors correspond to 68% confidence or credibility.

experiment	result	date
BaBar [2]	$(69^{+17}_{-16})^\circ$	Jan 2013
Belle [3]	$(68^{+15}_{-14})^\circ$	Jan 2013
LHCb $3 \text{ fb}^{-1}$ preliminary [4]	$(67 \pm 12)^\circ$	Apr 2013 (superseded!)
LHCb $1 \text{ fb}^{-1}$ [5]	$(72.6^{+9.7}_{-17.2})^\circ$	Aug 2013 (superseded!)
UTfit	$(68.3 \pm 7.5)^\circ$	post Moriond 2014
CKMfitter	$(70.0^{+7.7}_{-9.0})^\circ$	Moriond / Jun 2014
<b>CKMfitter</b>	<b><math>(73.2^{+6.3}_{-7.0})^\circ</math></b>	<b>CKM2014</b>

# Inputs

new LHCb results and changes

Two combinations:

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**robust**  $B \rightarrow DK$ -like

full  $B \rightarrow DK$ -like and  $B \rightarrow D\pi$

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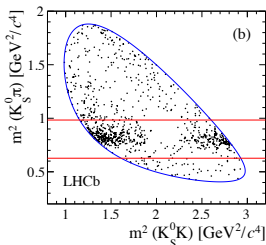
- ▶ **updated:** Update the auxiliary inputs
- ▶  $B^+ \rightarrow Dh^+$ ,  $D \rightarrow hh$ , GLW/ADS,  $1 \text{ fb}^{-1}$  LHCb [6]
- ▶  $B^+ \rightarrow Dh^+$ ,  $D \rightarrow K\pi\pi\pi$ , ADS,  $1 \text{ fb}^{-1}$  LHCb [7]
- ▶ **updated:**  $B^+ \rightarrow DK^+$ ,  $D \rightarrow K_s^0 hh$ , model-ind. GGSZ,  $3 \text{ fb}^{-1}$  LHCb [8]
- ▶ **new:**  $B^+ \rightarrow DK^+$ ,  $D \rightarrow K_s^0 K\pi$ , GLS,  $3 \text{ fb}^{-1}$  LHCb [9]
- ▶ **new:**  $B^0 \rightarrow D^0 K^{*0}$ ,  $D \rightarrow hh$ , GLW/ADS,  $3 \text{ fb}^{-1}$  LHCb [10]
- ▶ **new:**  $B_s^0 \rightarrow D_s^\mp K^\pm$ ,  $1 \text{ fb}^{-1}$  LHCb [11]

new:  $B^+ \rightarrow DK^+$ ,  $D \rightarrow K_S^0 K \pi$ , GLS,  $3 \text{ fb}^{-1}$ 

- ▶ GLS method

Grossman, Ligeti, Soffer 2002 [12]

- ▶ use singly Cabibbo suppressed  $D$  decays to non- $CP$  eigenstates
- ▶ first GLS measurement! LHCb [9]
- ▶ no significant  $CP$  violation was observed



we restrict the phase space to the more sensitive  $D \rightarrow K^{*+} K^-$  region

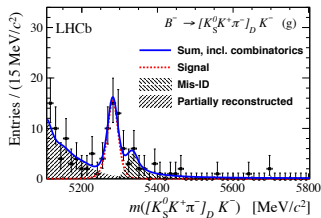
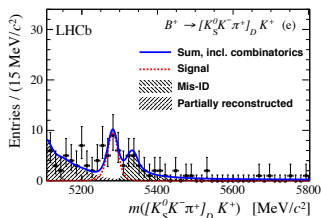
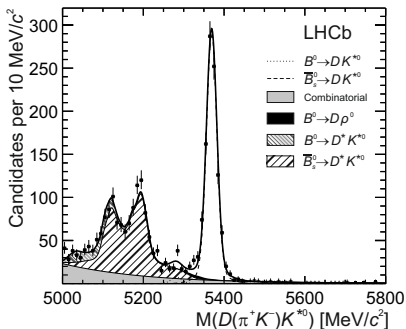
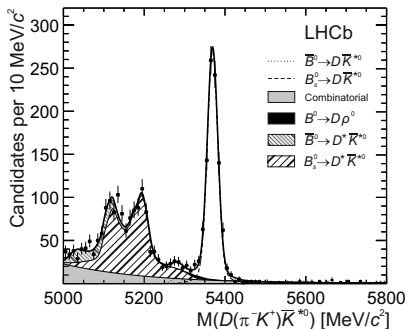


Figure: "opposite-sign" decays

new:  $B^0 \rightarrow D^0 K^{*0}$ ,  $D \rightarrow hh$ , GLW/ADS,  $3 \text{ fb}^{-1}$ 

- ▶ this mode has great potential exploiting the full  $D^0 K \pi$  phase space  
Gershon 2009 [13]
- ▶ the  $K^{*0}$  decay tags the  $B$  flavour at decay—no time dependent measurement
- ▶ significance for suppressed decay  $B^0 \rightarrow D^0 K^{*0}$ ,  $D^0 \rightarrow \pi^- K^+$ :  $2.9\sigma$

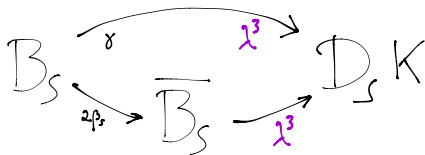


Phys. Lett. B712 (2012) 203 [10]



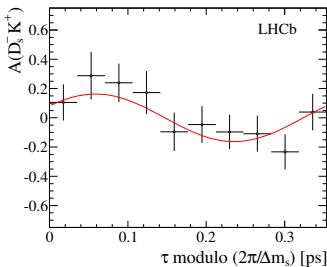
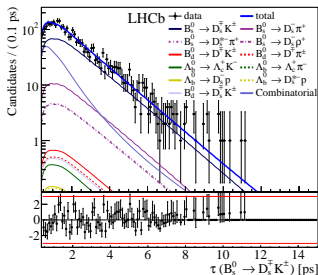
new:  $B_s^0 \rightarrow D_s^\mp K^\pm$ , time-dep.,  $1 \text{ fb}^{-1}$ 

- ▶ This new technique gains sensitivity to  $\gamma$  through  $B_s^0$  mixing.



- ▶ first measurement! Different set of systematics, pure “LHCb only”
- ▶  $B$  tagging power of  $\epsilon_{\text{eff}} = 5.07\%$
- ▶ Lots of hadronic backgrounds: use multidimensional fit to identify the signal
- ▶  $1770 \pm 50$  signal events

See also talk by Vava Gligorov!



$D^0-\bar{D}^0$  mixing correction

- ▶  $D^0-\bar{D}^0$  mixing has an effect on  $\gamma$   
Meca, Silva, Atwood, Grossman  
[14, 15, 16, 12, 17]
- ▶ see also Matteo Rama on Tuesday!
- ▶  $\mathcal{O}(1^\circ)$  on  $\gamma$  for  $B \rightarrow DK$
- ▶ The effect can be larger for  $B \rightarrow D\pi$   
[18, Rama 2013]:

$$\sqrt{x_D^2 + y_D^2}/r_B^{D\pi} \approx \mathcal{O}(1)$$

- ▶ The topic was revisited following  
FPCP2013 Rama; Bondar et al. [19, 18].
- ▶ We correct for the effect of  $D^0-\bar{D}^0$  mixing
  - ▶ in leading order in  $x_D, y_D$
  - ▶ taking into account individual  $D^0$   
decay time acceptances

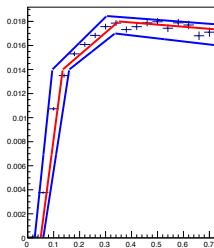
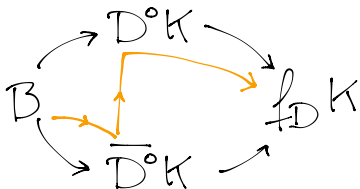


Figure: LHCb  $D^0$  decay time acceptance for  $B^+ \rightarrow DK^+$ ,  $D \rightarrow hh$ .

# Auxiliary input for $D^0 \rightarrow K^\pm \pi^\mp \pi^+ \pi^-$

## updated:

- ▶ The  $D^0 \rightarrow K^\pm \pi^\mp \pi^+ \pi^-$  ADS measurement needs input on the charm system.
- ▶ amplitude ratio:  $r_D^{K3\pi}$
- ▶ eff. strong phase diff.:  $\delta_D^{K3\pi}$
- ▶ coherence factor:  $\kappa_D^{K3\pi}$
- ▶ We use the histogrammed likelihood for  $\delta_D^{K3\pi}$ ,  $\kappa_D^{K3\pi}$  from CLEO legacy.
- ▶ We use a CLEO [20] and a Belle measurement [21] on  $\Gamma(D^0 \rightarrow K^- \pi^+ \pi^- \pi^+)$  and  $\Gamma(D^0 \rightarrow \pi^- K^+ \pi^- \pi^+)$  to constrain  $r_D^{K3\pi}$ .

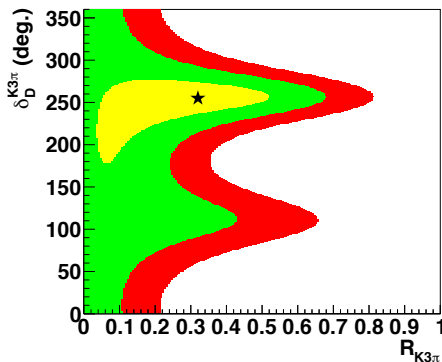


Figure: CLEO legacy likelihood for  $\delta_D^{K3\pi}$  and  $\kappa_D^{K3\pi}$  (labeled " $R_{K3\pi}$ ")  $D \rightarrow K3\pi$ . Histogram is used in the combination.

## Other auxiliary input

**updated:** The GLW/ADS measurements need inputs on the charm system:

- ▶ for the  $D^0 \rightarrow K^- \pi^+$  and  $D^0 \rightarrow \pi^- K^+$  decays:  $r_D^{K\pi}$ ,  $\delta_D^{K\pi}$
- ▶ for charm mixing:  $x_D$ ,  $y_D$
- ▶ for direct  $CP$  violation in  $D^0 \rightarrow h^+ h^-$  decays:  $A_{CP}^{\text{dir}}(KK)$ ,  $A_{CP}^{\text{dir}}(\pi\pi)$
- ▶ We use the recent HFAG charm fit (FPCP 2014).

**new:** Input for  $D \rightarrow K_s^0 K \pi$  **GLS**

- ▶ We use a CLEO measurement for  $r_D^{K_S K \pi}$ ,  $\delta_D^{K_S K \pi}$ ,  $\kappa_D^{K_S K \pi}$  CLEO [23].

**new:** Input for  $B^0 \rightarrow D^0 K^{*0}$  **GLW/ADS**

- ▶ We use a MC-based estimate for  $\kappa_B^{D K^{*0}} = 0.95 \pm 0.03$  LHCb [10].

**new:** Input for  $B_s^0 \rightarrow D_s^\mp K^\pm$  **time-dep.** to interpret  $\gamma - 2\beta_s$

- ▶ We use the LHCb measurement of  $-2\beta_s \approx \phi_s = 0.01 \pm 0.07 \pm 0.01$  rad LHCb [24].

# Evolution from the previous preliminary result

## 1. previous result

LHCb preliminary [4]

## 2. Update the auxiliary inputs

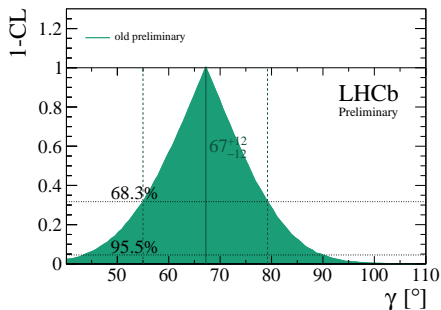
3. Add new  $B^+ \rightarrow DK^+$ ,  
 $D \rightarrow K_S^0 hh$  GGSZ measurement

4. Add new  $B^+ \rightarrow DK^+$ ,  
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5. Add new  $B^0 \rightarrow D^0 K^{*0}$ ,  
 $D \rightarrow hh$  GLW/ADS  
 measurement

6. Add new  $B_s^0 \rightarrow D_s^\mp K^\pm$   
 time-dependent measurement

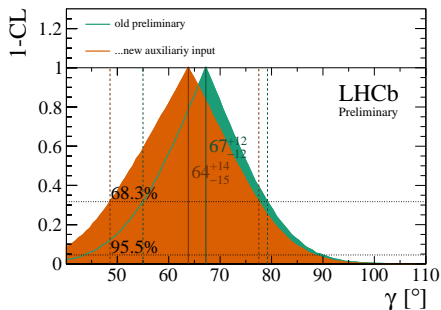
## 7. robust nominal



**Figure:** Making one change at a time.  
 Robust combination.

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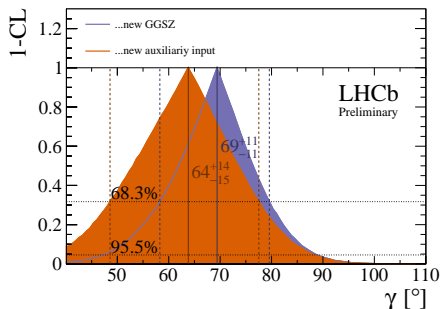
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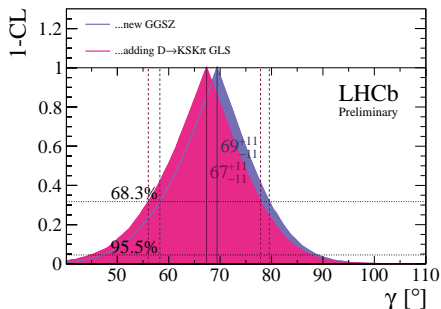


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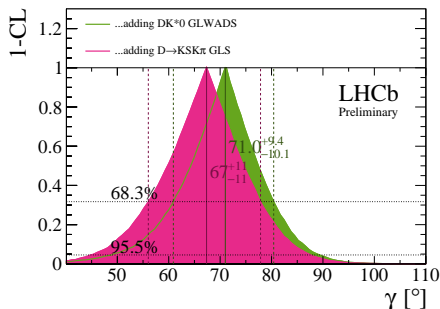


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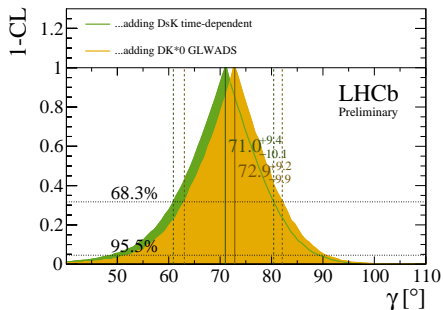


Figure: Making one change at a time.  
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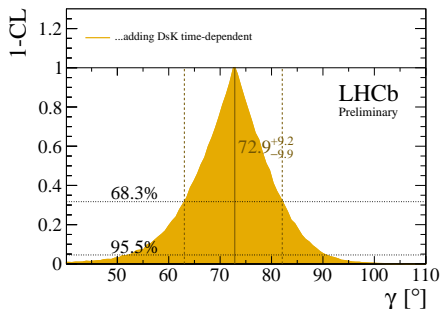


Figure: Making one change at a time.  
Robust combination.

# Statistical procedure

The **method is unchanged** from previous combinations.

- ▶ We assume (almost) all observables to be Gaussian distributed.
- ▶ We assume Gaussian systematic fluctuations.
- ▶ We don't allow non-physical parameter values.
- ▶ Use the **plugin method** for nominal results (Feldman-Cousins based frequentist method nuisances at their best-fit values).
- ▶ Use the profile likelihood method for 2D illustration.
- ▶ Estimate the frequentist coverage.
- ▶ Additional **Bayesian interpretation** with uniform priors.

# Result of the robust combination

**Table:** Credibility regions and most probable values for the hadronic parameters extracted from the robust combination. The second part of the table repeats the frequentist results for comparison.

Observable	Central value	Intervals	
		68%	95%
Frequentist			
$\gamma [^\circ]$	<b>72.9</b>	<b>[63.0, 82.1]</b>	[52.0, 90.5]
$\delta_B^{DK} [^\circ]$	126.8	[115.3, 136.7]	[101.6, 145.2]
$r_B^{DK}$	0.0914	[0.0826, 0.0997]	[0.0728, 0.1078]
Bayesian		68%	95%
$\gamma [^\circ]$	<b>71.9</b>	<b>[61.9, 81.8]</b>	[50.9, 91]
$\delta_B^{DK} [^\circ]$	127.4	[115.6, 137.3]	[100.3, 147.1]
$r_B^{DK}$	0.091	[0.0826, 0.0984]	[0.0740, 0.106]

The agreement of frequentist and Bayesian results is quite good.

## Result of the robust combination

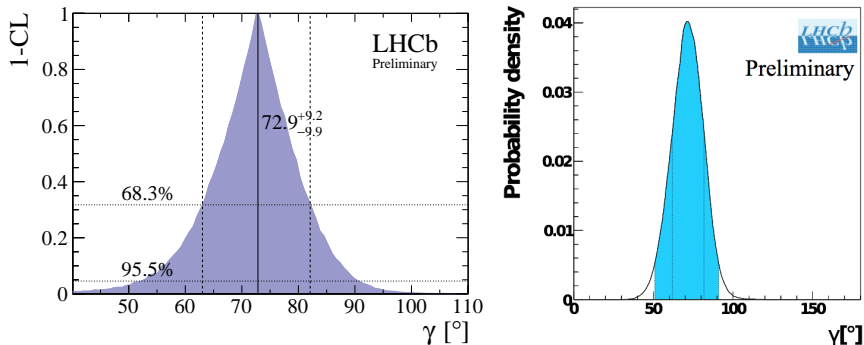
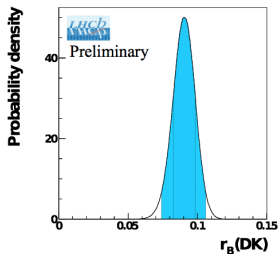
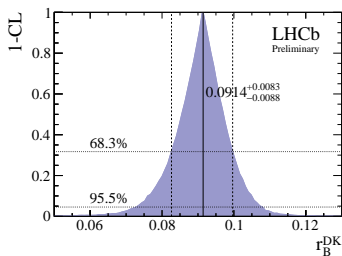
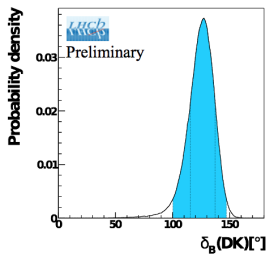
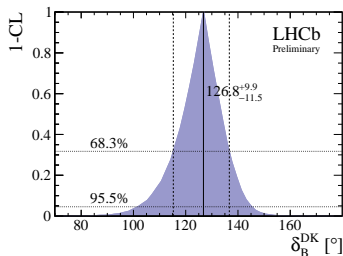


Figure: Graphs for the robust combination (left: frequentist 1 – CL curves, right: Bayesian posterior).

## Result of the robust combination



# Lets add $B^+ \rightarrow D\pi^+$ information

Table: Confidence intervals and central values for the full combination.

quantity	full	
$\gamma$ ( $^\circ$ )	78.9	72.8
68% CL ( $^\circ$ )	[71.5, 84.7]	
95% CL ( $^\circ$ )	[54.6, 91.4]	
$r_B^{DK}$	0.0928	
68% CL	[0.0845, 0.1008]	
95% CL	[0.0732, 0.1085]	
$\delta_B^{DK}$ ( $^\circ$ )	128.9	
68% CL ( $^\circ$ )	[118.9, 137.9]	
95% CL ( $^\circ$ )	[102.0, 145.9]	
$r_B^{D\pi}$	0.027	0.006
68% CL	[0.016, 0.034]	[0.005, 0.007]
95% CL	[0.001, 0.040]	
$\delta_B^{D\pi}$ ( $^\circ$ )	341.8	215.6
68% CL ( $^\circ$ )	[328.7, 351.4]	[210.2, 231.5]
95% CL ( $^\circ$ )	no constraint	



Lets add  $B^+ \rightarrow D\pi^+$  information

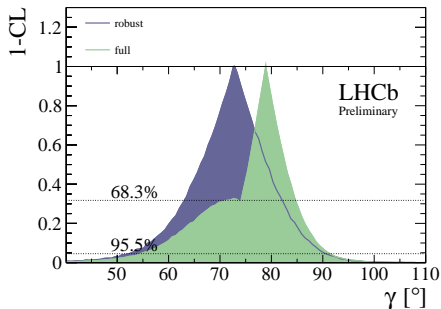
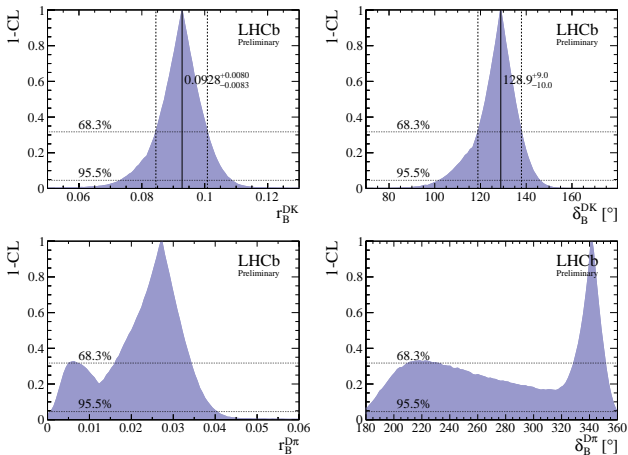


Figure: 1 – CL curve for the robust and full combinations.

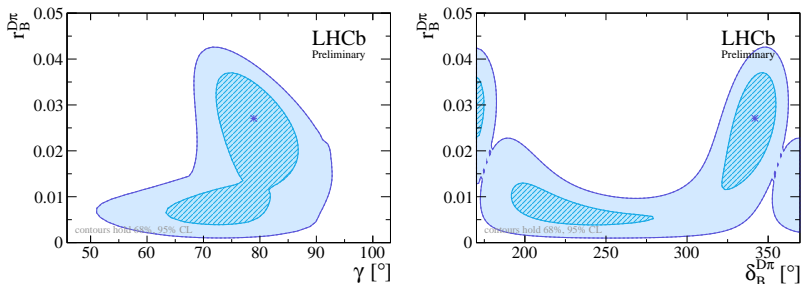
- ▶ A second minimum appears.
- ▶ The  $1\sigma$  errors are **misleadingly small**: highly non-Gaussian!
- ▶ At  $2\sigma$ , the agreement with the robust combination is very good.

## Result of the full combination



At  $2\sigma$ ,  $r_B^{D\pi}$  is compatible with 0.

## Result of the full combination



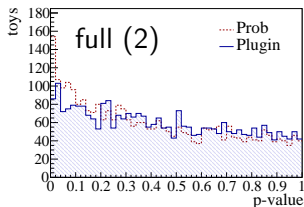
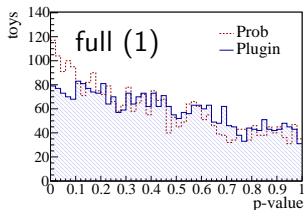
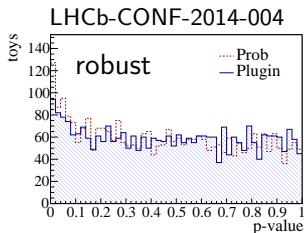
**Figure:** Profile likelihood contours, full combination. The contours are the two-dimensional  $1\sigma$  and  $2\sigma$  contours.

The high observed value of  $r_B^{D\pi}$  also pulls up  $\gamma$ .

# Coverage test

- ▶ We test the frequentist coverage at the minima of the combinations.
- ▶ We find that the profile likelihood construction undercovers quite a bit.
- ▶ The robust plugin method has good coverage.
- ▶ The coverage of the full combination is worse than of the robust. Expected due to the low value of  $r_B^{D\pi}$ .

$\eta = 0.683$	$\alpha$ (prof. LH.)	$\alpha$ (plugin)
robust	0.6158	0.6494
full (1), $r_B^{D\pi} = 0.027$	0.5593	0.6154
full (2), $r_B^{D\pi} = 0.006$	0.5454	0.6120



## Conclusion

- ▶ We updated the LHCb  $\gamma$  combination of tree-level measurements.
- ▶ The effect of  $D^0-\bar{D}^0$  mixing is taken into account, the frequentist coverage was tested.
- ▶ robust Bayesian results are available and agree.
- ▶ We explore  $B^+ \rightarrow D\pi^+$  channels: they show up early all obstacles on the way to high precision.
- ▶ The robust coverage is good.
- ▶ Robust frequentist combination:

$$\gamma = (72.9^{+9.2}_{-9.9})^\circ \quad @ 68\% \text{ CL},$$

$$\gamma \in [63.0, 82.1]^\circ \quad @ 68\% \text{ CL},$$

$$\gamma \in [52.0, 90.5]^\circ \quad @ 95\% \text{ CL}.$$

- ▶ More precise than the  $B$  factory legacy.

Backup

# Auxiliary input for $D \rightarrow K_S^0 K \pi$

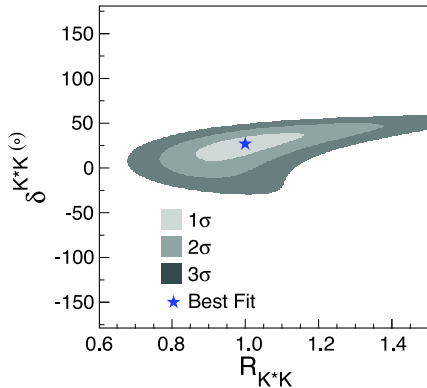
## new:

- ▶ Similar input is needed as in the case of  $D \rightarrow K \pi \pi \pi$ .
- ▶ rate ratio:  $R_D^{K_S K \pi}$
- ▶ eff. strong phase diff.:  $\delta_D^{K_S K \pi}$
- ▶ coherence factor:  $\kappa_D^{K_S K \pi}$
- ▶ We use a single CLEO measurement [23].

$$R_D^{K_S K \pi} = 0.356 \pm 0.034 \pm 0.007,$$

$$\delta_D^{K_S K \pi} = 0.46 \pm 0.28,$$

$$\kappa_D^{K_S K \pi} = 1.00 \pm 0.16.$$



**Figure:** CLEO likelihood for  $\delta_D^{K_S K \pi}$  and  $\kappa_D^{K_S K \pi}$  (labeled “ $R_{K^* K}$ ”) [23]. Histogram is **not** used in the combination.

# Auxiliary input from HFAG

taking a step back

Comparing the old auxiliary inputs to the new ones:

**old:**

- ▶ we needed input on the  $D \rightarrow K\pi\pi\pi$  system from CLEO
- ▶ that measurement had some influence on the charm mixing parameters  $x_D, y_D$
- ▶ it was obtained using the HFAG status as of 2009 as input
- ▶ ... so we took the 2009 CLEO input and added the new LHCb  $D^0$  wrong sign mixing measurement
- ▶ we also added “ $\Delta A_{CP}$ ”



# Auxiliary input from HFAG

taking a step back

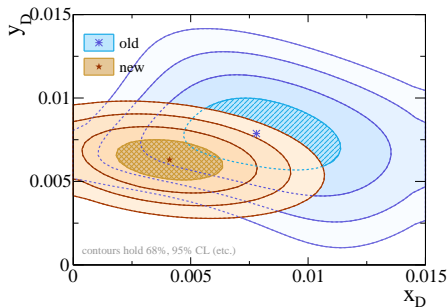
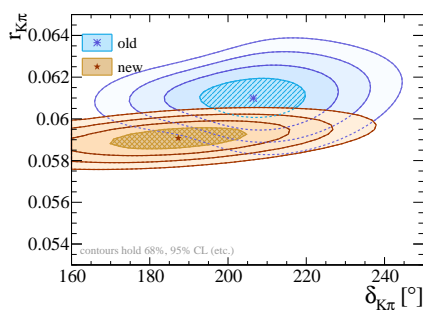
Comparing the old auxiliary inputs to the new ones:

**new:**

- ▶ the input for  $D \rightarrow K\pi\pi\pi$  was updated using more CLEO-c data
- ▶ the new measurement no longer impacts  $x_D, y_D$ , as the world average is much stronger now
- ▶ it was obtained using the HFAG status as of 2014 as input
- ▶ ... so now we can use HFAG directly, including up-to-date charm mixing, and " $\Delta A_{CP}$ "

# Auxiliary input from HFAG

comparing old and new



**Figure:** Profile likelihood contours: The “old” contour corresponds to what was used in the previous (2013) combination (the 2009 CLEO input [25] together with the 2013 LHCb charm mixing measurement [26]). The “new” contour is what is used in this combination (HFAG 2014). The contours are two-dimensional 1–4 $\sigma$  contours.

## Auxiliary input from HFAG

The parameter  $R_D^{K\pi}$  is the squared ratio of the doubly-Cabibbo-suppressed amplitude  $D^0 \rightarrow \pi^- K^+$  to the favored one  $D^0 \rightarrow K^- \pi^+$ . It is not the ratio of branching ratios. It gets often measured in time-dependent wrong-sign  $D^0$  mixing measurements:

$$R_{WS} = R_D^{K\pi} + \sqrt{R_D^{K\pi}} (x \cos(\delta_D^{K\pi}) \pm y \sin(\delta_D^{K\pi})) \frac{t}{\tau} + \frac{x_D^2 + y_D^2}{4} \left(\frac{t}{\tau}\right)^2$$

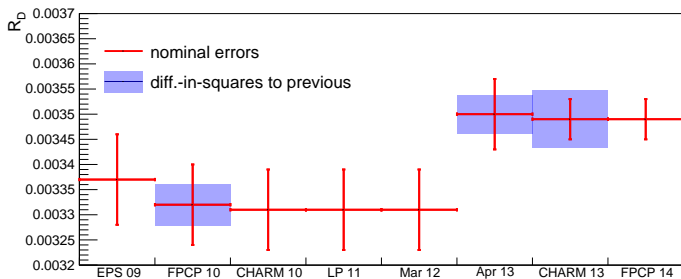


Figure: Evolution of HFAG results on  $R_D^{K\pi}$ .

## Auxiliary input for $D^0 \rightarrow K^\pm \pi^\mp \pi^+ \pi^-$

**updated:** We use an updated CLEO-c measurement of  $\kappa_D^{K3\pi}$  and  $\delta_D^{K3\pi}$  (histogrammed likelihood). For the amplitude ratio

$$r_D^{K3\pi} = |A(D^0 \rightarrow \pi^- K^+ \pi^- \pi^+) / A(D^0 \rightarrow K^- \pi^+ \pi^- \pi^+)| \quad (1)$$

we use

- ▶ a CLEO measurement [20] of  $\Gamma(D^0 \rightarrow K^- \pi^+ \pi^- \pi^+)$ ,

$$\Gamma(D^0 \rightarrow K^- \pi^+ \pi^- \pi^+) = 0.08290 \pm 0.00043 \pm 0.00200, \quad (2)$$

- ▶ a Belle measurement [21] of  $\Gamma(D^0 \rightarrow \pi^- K^+ \pi^- \pi^+)$ ,

$$\Gamma(D^0 \rightarrow \pi^- K^+ \pi^- \pi^+) = 0.00026 \pm 0.00001 \pm 0.00001. \quad (3)$$

- ▶ These are related to  $r_D^{K3\pi}$  through the equation

$$R_{WS}(D \rightarrow K\pi\pi\pi) = \quad (4)$$

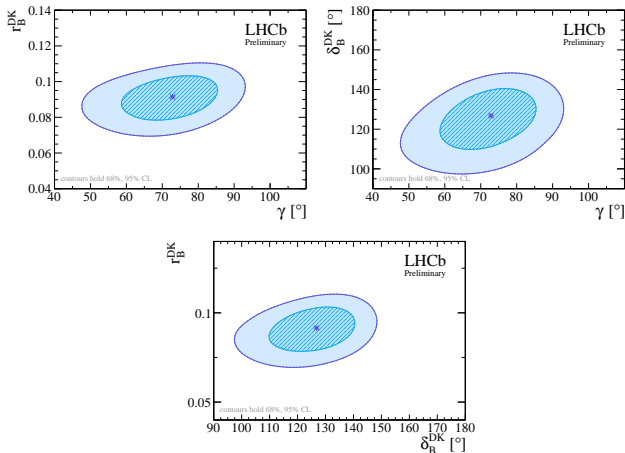
$$(r_D^{K3\pi})^2 - \kappa_D^{K3\pi} r_D^{K3\pi} (y_D \cos \delta_D^{K3\pi} - x_D \sin \delta_D^{K3\pi}) + \frac{1}{2}(x_D^2 + y_D^2). \quad (5)$$

# Goodness of fit

Table: Fit probabilities of the best fit values of the two combinations.

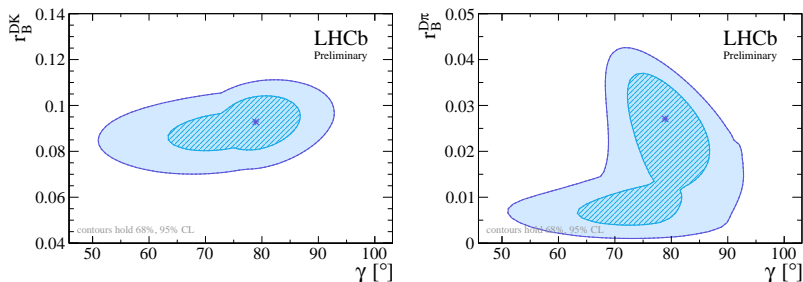
Combination	$P$ (PROB)	$P$ (toy-based)
robust	89.7%	$(89.4 \pm 0.5)\%$
full	97.2%	$(96.8 \pm 0.3)\%$

# Result of the robust combination



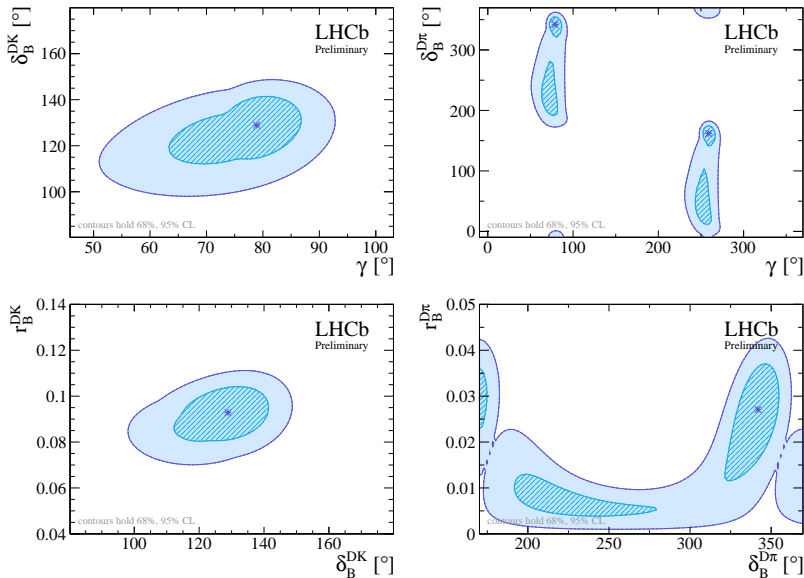
**Figure:** Profile likelihood contours, robust combination. The contours are the two-dimensional  $1\sigma$  and  $2\sigma$  contours.

# Result of the full combination



**Figure:** Profile likelihood contours, full combination. The contours are the two-dimensional  $1\sigma$  and  $2\sigma$  contours.

## Result of the full combination





## Coverage test

**Table:** Measured coverage  $\alpha$  of the confidence intervals for  $\gamma$ . The expected coverage is denoted as  $\eta$ .

robust	$\alpha$ (profile likelihood)	$(\alpha - \eta)$	$\alpha$ (PLUGIN)	$(\alpha - \eta)$
$\eta = 0.6827$	<b>0.6158</b> $\pm 0.0089$	-0.0669	<b>0.6494</b> $\pm 0.0087$	-0.0333
$\eta = 0.9545$	0.9200 $\pm 0.0050$	-0.0345	0.9358 $\pm 0.0045$	-0.0187
$\eta = 0.9973$	0.9923 $\pm 0.0016$	-0.0050	0.9919 $\pm 0.0016$	-0.0054
full, $r_B^{D\pi} = 0.027$	$\alpha$ (profile likelihood)	$(\alpha - \eta)$	$\alpha$ (PLUGIN)	$(\alpha - \eta)$
$\eta = 0.6827$	<b>0.5593</b> $\pm 0.0091$	-0.1234	<b>0.6154</b> $\pm 0.0089$	-0.0673
$\eta = 0.9545$	0.9160 $\pm 0.0051$	-0.0385	0.9402 $\pm 0.0043$	-0.0143
$\eta = 0.9973$	0.9929 $\pm 0.0015$	-0.0044	0.9929 $\pm 0.0015$	-0.0044
full, $r_B^{D\pi} = 0.006$	$\alpha$ (profile likelihood)	$(\alpha - \eta)$	$\alpha$ (PLUGIN)	$(\alpha - \eta)$
$\eta = 0.6827$	<b>0.5454</b> $\pm 0.0091$	-0.1373	<b>0.6120</b> $\pm 0.0089$	-0.0707
$\eta = 0.9545$	0.9026 $\pm 0.0054$	-0.0519	0.9314 $\pm 0.0046$	-0.0231
$\eta = 0.9973$	0.9906 $\pm 0.0018$	-0.0067	0.9933 $\pm 0.0015$	-0.0040

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