My homework:

Unitarity constraints on TGCs and QGCs

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Basics:

- Higher dimension operators lead to unitarity violation in PT
- Unitarity of the S matrix $S^\dagger S = 1$ implies

$$\mathrm{Im}\mathcal{M}_{\mathrm{elastic}}(k_1 k_2 \to k_1 k_2) = 2E_{cm} p_{cm} \sigma_{total}(k_1 k_2 \to \text{ anything})$$

- using just the elastic channel

$$\mathcal{M}(s,t,u) = 32\pi \sum_\ell P_\ell(\cos\theta) a_\ell \longrightarrow |a_\ell| \leq \frac{1}{2}$$

with

$$a_\ell = \frac{1}{64} \int_{-1}^{1} d\cos\theta P_\ell(cos\theta)\mathcal{M}$$

Quartic couplings play in important role: $W_L^+W_L^- \to W_L^+W_L^-$

[Cornwall; Lee-Quigg-Thacker;etc]

- J=0 partial wave

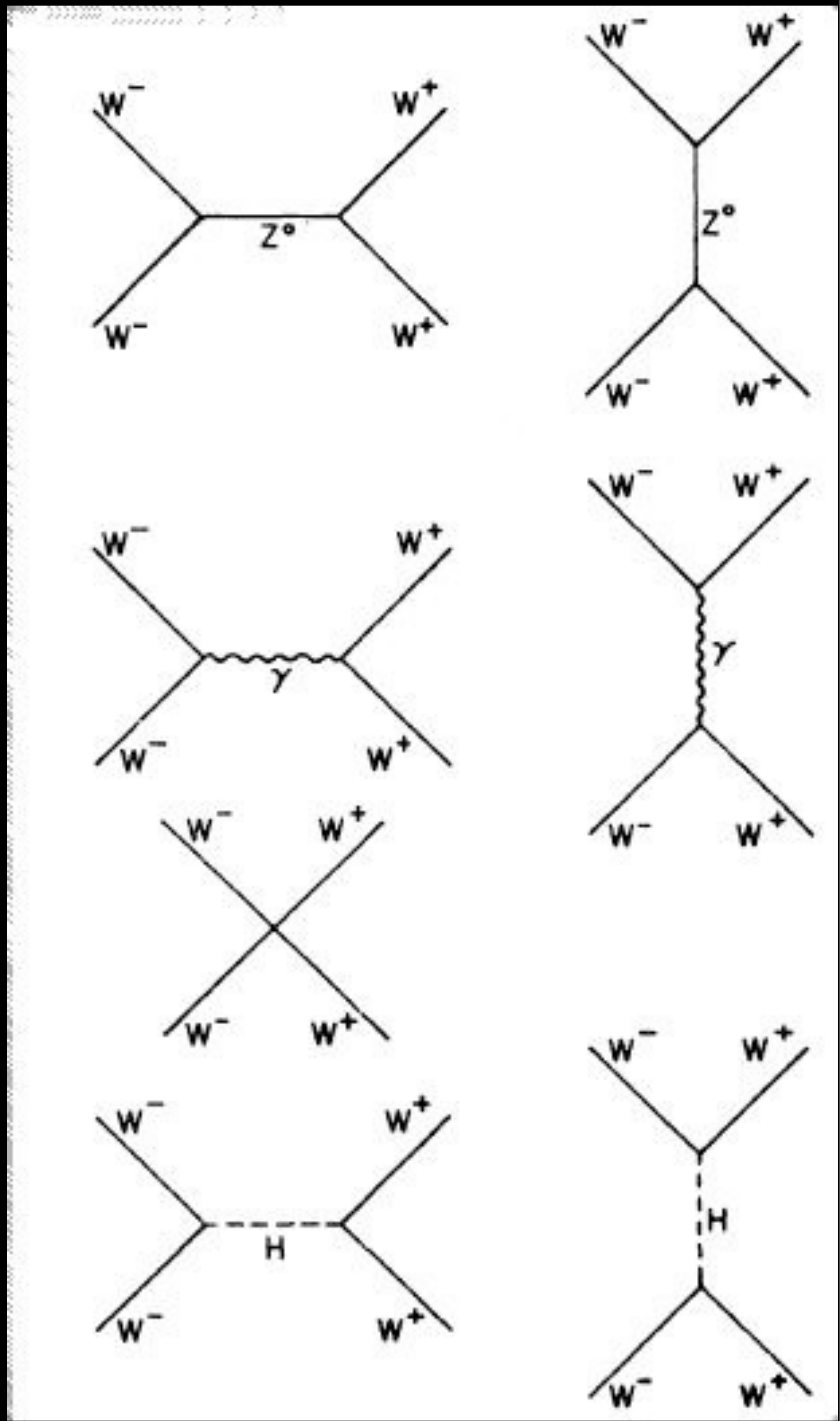
$$A = A_4 \frac{E^4}{M_W^4} + A_2 \frac{E^2}{M_W^2} + \cdots$$

lead to unitarity violation at tree level

- In the SM

$$A_4 \propto g_{WWWW} - g^2_{WWZ} - g^2_{WW\gamma} \equiv 0$$

- H takes part into $A_2 = 0$



Unitarity bounds on dimension-six operators:

$$\mathcal{O}_B = (D_\mu\Phi)^\dagger \hat{B}^{\mu\nu}(D_\nu\Phi) \;\; , \quad \mathcal{O}_W = (D_\mu\Phi)^\dagger \hat{W}^{\mu\nu}(D_\nu\Phi)$$

$$HVV \;\; ; \;\; VVV \;\; ; \;\; VVVV \quad \cdots$$

$$\mathcal{O}_{BB} = \Phi^\dagger \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\Phi \;\; , \quad \mathcal{O}_{BW} = \Phi^\dagger \hat{B}_{\mu\nu}\hat{W}^{\mu\nu}\Phi \;\; , \quad \mathcal{O}_{WW} = \Phi^\dagger \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\Phi$$

$$\mathcal{O}_{WWW} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\nu\rho}\hat{W}_\rho^\mu]$$ [VVV,VVVV]

most stringent bounds: [Gounaris et al, hep-ph/9409260;]

$$\left|\frac{f_B}{\Lambda^2}\right| \leq \frac{98}{s} \quad \text{for } \sqrt{s} = 2 \text{ TeV} \quad \left|\frac{f_B}{\Lambda^2}\right| \leq 24.5 \text{ TeV}^{-2}$$

$$\left|\frac{f_W}{\Lambda^2}\right| \leq \frac{31}{s} \quad \text{for } \sqrt{s} = 2 \text{ TeV} \quad \left|\frac{f_W}{\Lambda^2}\right| \leq 7.8 \text{ TeV}^{-2}$$

$$\left|\frac{f_{WWW}}{\Lambda^2}\right| \leq \frac{38}{3g^2 s} \quad \text{for } \sqrt{s} = 2 \text{ TeV} \quad \left|\frac{f_{WWW}}{\Lambda^2}\right| \leq 7.5 \text{ TeV}^{-2}$$

$$\left|\frac{f_{WW}}{\Lambda^2}\right| \leq \frac{35.2}{3g^2 s} + \frac{4.86}{\sqrt{s} M_W} \quad \text{for } \sqrt{s} = 2 \text{ TeV} \quad \left|\frac{f_{WW}}{\Lambda^2}\right| \leq 39.2 \text{ TeV}^{-2}$$

in principle no form factor will be needed

Higgs physics already leads to contraints:

[arXiv:1207.1344; 1211.4580;1304.1151]

$$\frac{f_W}{\Lambda^2} \in [-6.7, 8.3] \quad \Longrightarrow \sqrt{s} \leq 1.9 \text{ TeV}$$

$$\frac{f_B}{\Lambda^2} \in [-23., 6.5] \quad \Longrightarrow \sqrt{s} \leq 2.1 \text{ TeV}$$

$$\frac{f_{WW}}{\Lambda^2} \in [2.6, 3.2] \text{ or } [0.4, 0.13] \quad \Longrightarrow \sqrt{s} \leq 3 \text{ TeV}$$

Unitarity bounds on dimension-eight operators:

$$\mathcal{L}_{M,0} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right]$$
$$\mathcal{L}_{M,1} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right]$$
$$\mathcal{L}_{M,2} = \left[B_{\mu\nu}B^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right]$$
$$\mathcal{L}_{M,3} = \left[B_{\mu\nu}B^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right]$$
$$\mathcal{L}_{M,4} = \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu}D^\mu\Phi\right] \times B^{\beta\nu}$$
$$\mathcal{L}_{M,5} = \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu}D^\nu\Phi\right] \times B^{\beta\mu}$$
$$\mathcal{L}_{M,6} = \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu}\hat{W}^{\beta\nu}D^\mu\Phi\right]$$
$$\mathcal{L}_{M,7} = \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu}\hat{W}^{\beta\mu}D^\nu\Phi\right]$$

$$\mathcal{L}_{T,0} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \mathrm{Tr}\left[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}\right]$$
$$\mathcal{L}_{T,1} = \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}\right]$$
$$\mathcal{L}_{T,2} = \mathrm{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}\right]$$
$$\mathcal{L}_{T,5} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times B_{\alpha\beta}B^{\alpha\beta}$$
$$\mathcal{L}_{T,6} = \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right] \times B_{\mu\beta}B^{\alpha\nu}$$
$$\mathcal{L}_{T,7} = \mathrm{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right] \times B_{\beta\nu}B^{\nu\alpha}$$
$$\mathcal{L}_{T,8} = B_{\mu\nu}B^{\mu\nu}B_{\alpha\beta}B^{\alpha\beta}$$
$$\mathcal{L}_{T,9} = B_{\alpha\mu}B^{\mu\beta}B_{\beta\nu}B^{\nu\alpha}$$

	WWWW	WWZZ	ZZZZ	WWAZ	WWAA	ZZZA	ZZAA	ZAAA
$\mathcal{L}_{S,0}$, $\mathcal{L}_{S,1}$	X	X	X	O	O	O	O	O
$\mathcal{L}_{M,0}$, $\mathcal{L}_{M,1}$, $\mathcal{L}_{M,6}$, $\mathcal{L}_{M,7}$	X	X	X	X	X	X	X	O
$\mathcal{L}_{M,2}$, $\mathcal{L}_{M,3}$, $\mathcal{L}_{M,4}$, $\mathcal{L}_{M,5}$	O	X	X	X	X	X	X	O
$\mathcal{L}_{T,0}$, $\mathcal{L}_{T,1}$, $\mathcal{L}_{T,2}$	X	X	X	X	X	X	X	X
$\mathcal{L}_{T,5}$, $\mathcal{L}_{T,6}$, $\mathcal{L}_{T,7}$	O	X	X	X	X	X	X	X
$\mathcal{L}_{T,9}$, $\mathcal{L}_{T,9}$	O	O	X	O	O	X	X	X

There are few results easily available:

- New

$$\frac{f_{T8}}{\Lambda^4}\frac{s_w^4}{60\pi}\frac{s^4}{M_Z^4} \leq \frac{1}{2} \qquad \text{for } \sqrt{s} = 2 \text{ TeV} \implies \left|\frac{f_{T8}}{\Lambda^4}\right| \leq 5\times 10^{-4} \text{ TeV}^{-4}$$

a form factor is needed to enforce unitarity

- "In the literature" for LEP couplings [hep-ph/0009262]

$$a_0 \text{ and } a_c \simeq F_{M0},\ F_{M1},\ F_{M2},\ F_{M3}$$

$$\left(\frac{\alpha\beta s}{16}\right)^2 \left(1 - \frac{4M_W^2}{s}\right)^{1/2} \left(3 - \frac{s}{M_W^2} + \frac{s^2}{4M_W^4}\right) \leq N \text{ for } V = W\,, \tag{17}$$

$$\left(\frac{\alpha\beta s}{16c_W^2}\right)^2 \left(1 - \frac{4M_Z^2}{s}\right)^{1/2} \left(3 - \frac{s}{M_Z^2} + \frac{s^2}{4M_Z^4}\right) \leq N \text{ for } V = Z\,, \tag{18}$$

where $\beta = \beta_0$ or β_c and $N = 1/4$ (4) for β_0 (β_c). For instance, unitarity is violated for $\gamma\gamma$ invariant masses above 240 GeV for $\beta_0 = 5.6 \times 10^{-3}$ GeV^{-2} (one of the present LEP bounds).

- Theory home work:
  - obtain the unitarity bounds for dim8 operators
  - get the EWPT limits on dim8 operators