PDFs @ LO, NLO, NNLO with correlated uncertainties between orders.

arXiv:1404.4234

Misha Lisovyi (DESY)

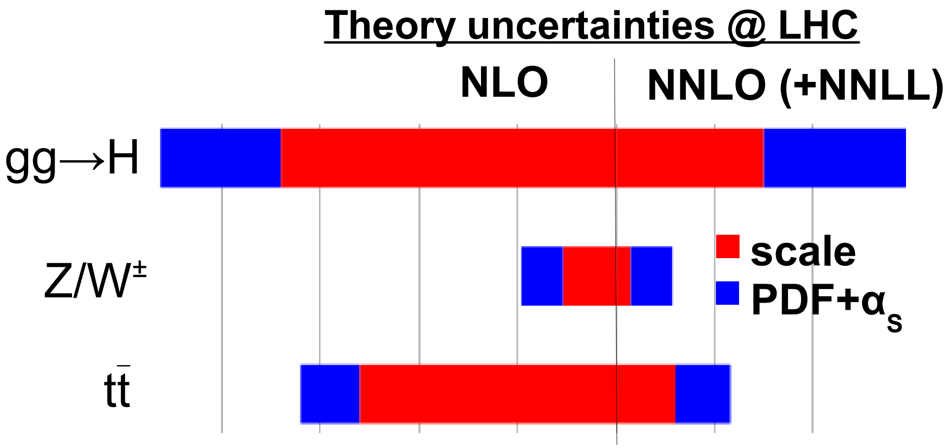
and HERAFitter developers' team

DIS2014, Warsaw

29/04/2014

Introduction.

- Predictions for various processes @ LHC are available at LO, NLO, NNLO in pQCD. Each should be paired with PDFs extracted at appropriate order.
- Theoretical uncertainties arise from PDFs (PDF uncertainties) and missing higher orders (typically, μ_R and μ_F variations).
- LO: used with parton showers.
- NLO: scale variations often exceed PDF uncertainties.
- NNLO: sensitivity to scales is reduced → PDF uncertainties often exceed size of scale variations.



** see backup for references*

Motivation.

- Ratios of cross sections are used to reduce uncertainties (both theoretical and experimental).
- Precision of LHC data is comparable to accuracy of predictions. beneficial to use state-of-the-art predictions:

$$\frac{NLO_X}{NLO_Y}$$

☺cancellation of PDF unc.

☹large scale unc.

$$\frac{NLO_X}{NNLO_Y}$$

☹PDF unc. do not cancel

☺improved scale unc.

$$\frac{NLO_X + NNLO\ PDF}{NNLO_Y + NNLO\ PDF}$$

☺PDF unc. cancel

☺improved scale unc.

☹unclear definition in pQCD

$$\frac{NLO_X + NLO\ PDF_{corr}}{NNLO_Y + NNLO\ PDF_{corr}}$$

☺PDF unc. cancel

☺improved scale unc.

Require knowledge about correlation of PDF uncertainties at different orders!

In most cases: treat scale uncertainties uncorrelated between processes

QCD analysis.

arXiv:1404.4234

- QCD fit to the combined HERAI data using the HERAFitter framework (talk by H. Pirumov).
- 13-parameter HERAPDF-style parametrisation.

$$
\begin{aligned}
xg(x) &= A_g x^{B_g}(1-x)^{C_g} - A'_g x^{B'_g}(1-x)^{C'_g}\,; \\
x\bar{U}(x) &= A_{\bar{U}} x^{B_{\bar{U}}}(1-x)^{C_{\bar{U}}}(1+D_{\bar{U}}x+E_{\bar{U}}x^2)\,; \\
x\bar{D}(x) &= A_{\bar{D}} x^{B_{\bar{D}}}(1-x)^{C_{\bar{D}}}\,; \\
xu_v(x) &= A_{u_v} x^{B_{u_v}}(1-x)^{C_{u_v}}(1+E_{u_v}x^2)\,; \\
xd_v(x) &= A_{d_v} x^{B_{d_v}}(1-x)^{C_{d_v}}(1+D_{d_v}x)\,.
\end{aligned}
$$

+ additional constraints & assumptions, e.g. momentum sum rules, $x\bar{s} = r_s x\bar{d}$

QCD analysis.

arXiv:1404.4234

Settings	LO	NLO	NNLO
HF scheme	RT Opt	RT Opt	RT Opt
r_s	1.0	1.0	1.0
m_c [GeV]	1.38	1.38	1.32
m_t [GeV]	14317.82	14317.82	14317.82
α_S	0.13	0.1184	0.1184
Q^2_0 [GeV^2]	1.7	1.7	1.7
Q^2_{min} [GeV^2]	7.5	7.5	7.5

> Vary model parameters and parametrisation following HERAPDF-like prescription to provide model and parametrisation uncertainties.

> Treat model and parametrisation uncertainties correlated between orders

Extraction of correlated PDFs.

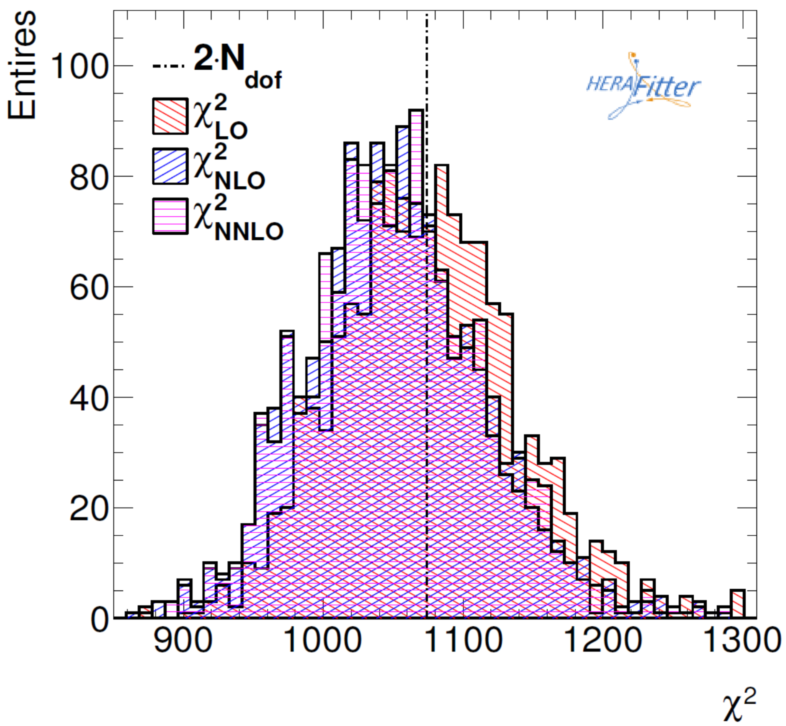
arXiv:1404.4234

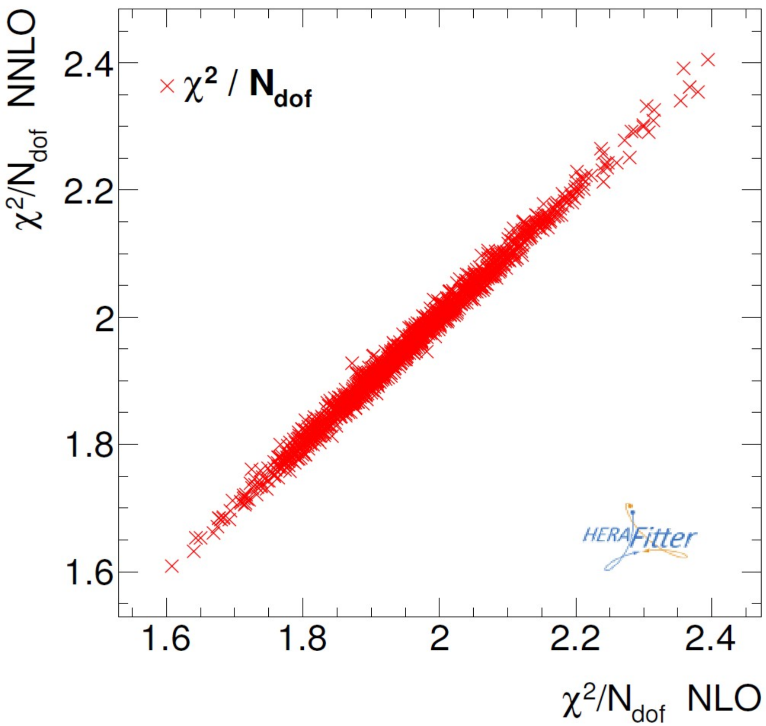
Use MC replica method to preserve correlations between PDFs at different orders:

- prepare 1337 MC replicas of the data fluctuating the inputs within uncertainties using Gaussian probability densities;
- do a consistent fit of PDFs at different orders to each replica;
- covariance matrix/ratios can be built using individual PDF replicas;
- central PDFs: average over replicas;
- 68% CL uncertainties: RMS.

QCD fit quality.

arXiv:1404.4234





> Mean of the χ^2 distributions agrees with the expectation $2{\cdot}N_{dof}$.

> Clear correlation of the quality of the fits at NLO and NNLO.

Eigenvector representation.

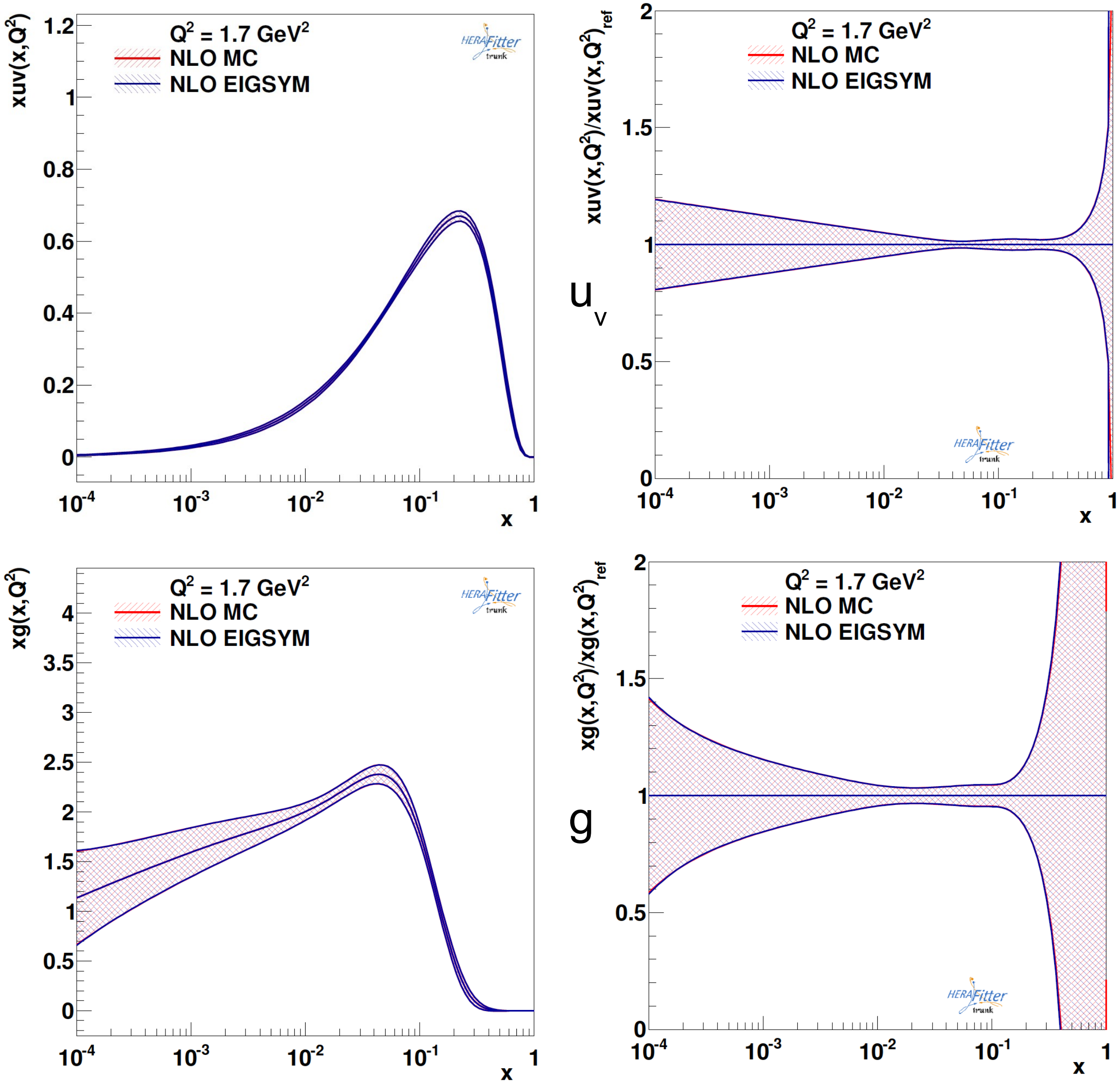
arXiv:1404.4234

Eigenvector representation is often more convenient:

- use approach suggested for extraction of META PDFs (arXiv:1401.0013)
- extract full covariance matrix for PDFs at the starting scale on the QCDNUM x-grid;
- diagonalise the covariance matrix and keep leading eigenvalues to capture all features of correlations;
- do DGLAP evolution of the tabulated central and eigenvectors using QCDNUM.

Eigenvector representation.

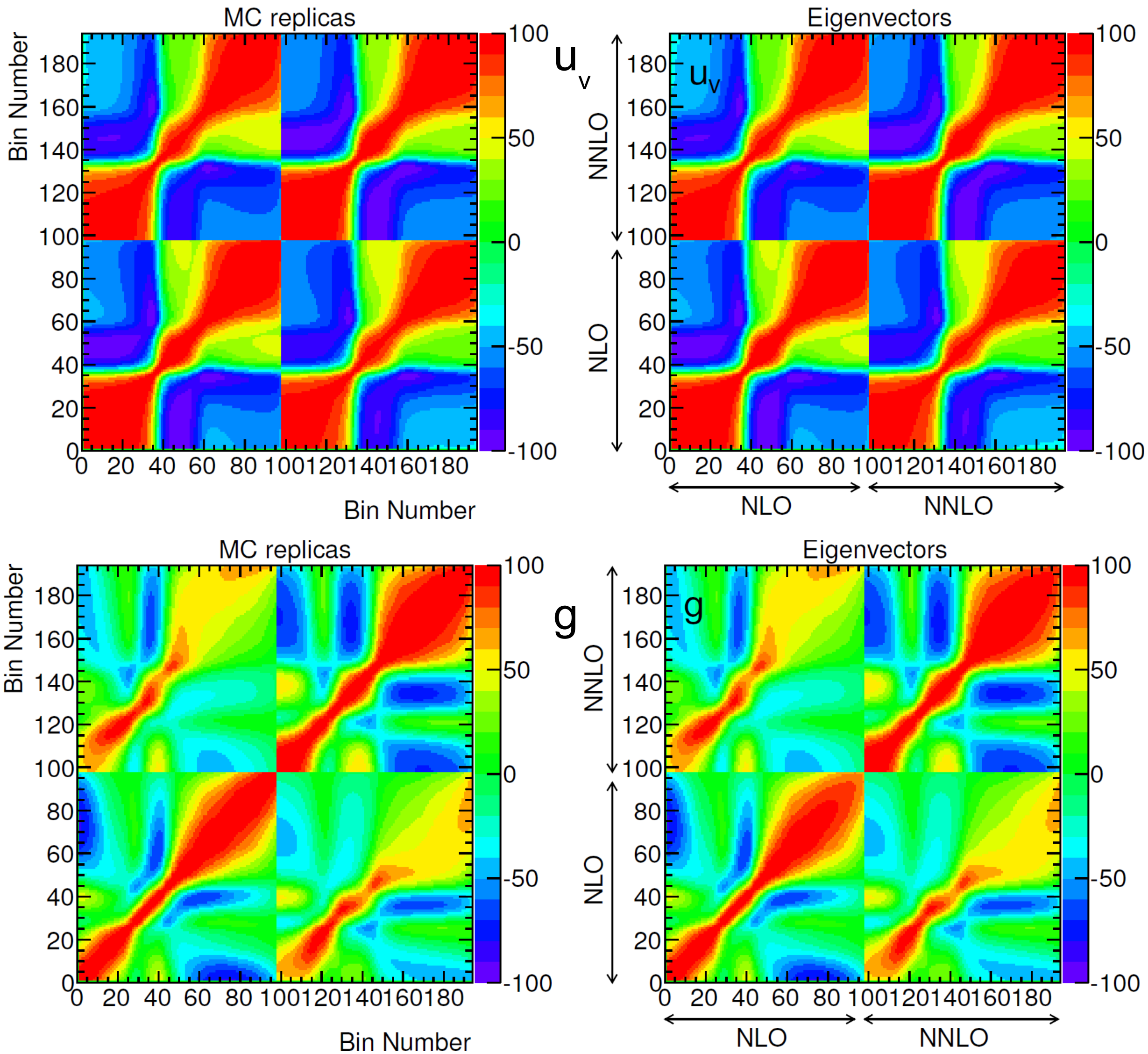
arXiv:1404.4234



- The eigenvectors reproduce the original uncertainties extracted from MC PDF replicas very good.

Correlation of PDFs at different orders.

arXiv:1404.4234



- The eigenvectors reproduce the correlation patterns very good.
- High degree of correlation between neighboring x points: smooth parametrisation and constraints from similar data.
- A strong correlation between NLO and NNLO PDFs (mild for high-x gluon).

Anchor points: 10^{-5}, 0.01, 0.1, 0.4, 0.7

Use case example : W^+W^- / Z.

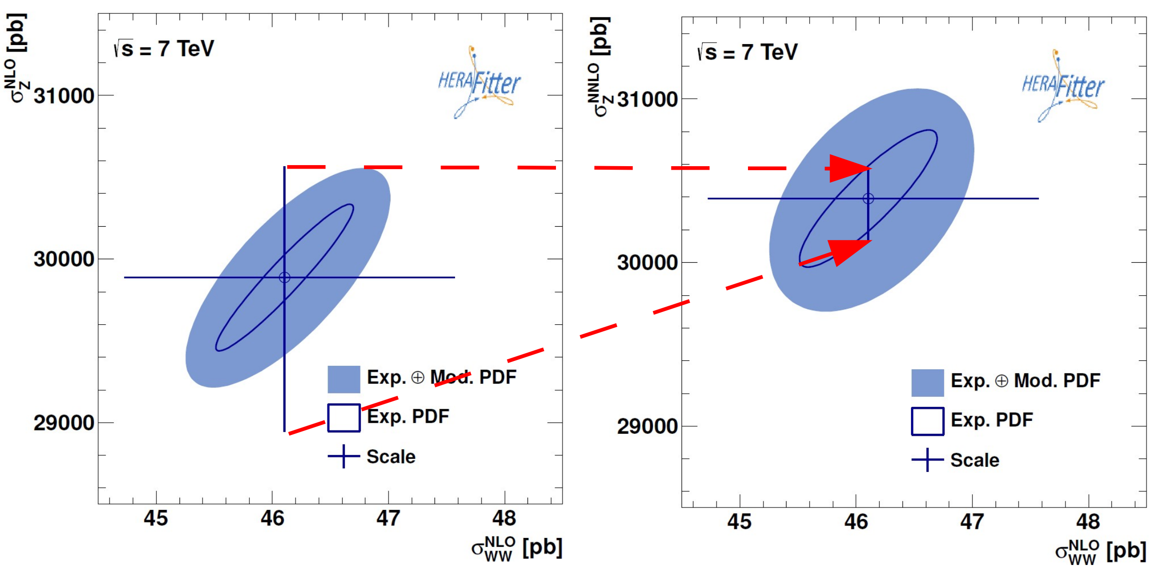
arXiv:1404.4234

- WW measurements @ LHC are 1-2·σ away from the NLO QCD predictions. Higher orders? EW corrections? New physics?
- Build ratio to the Z cross sections: cancel part of experimental and theoretical (same initial state at LO) uncertainties. But no rigorous path to benefit from existing NNLO QCD Z predictions.
- Use correlated PDFs to make the optimal predictions for WW/Z ratio.
- Compare predictions to the recent WW/Z measurement by CMS (arXiv:1306.1126).
- Use MCFM 6.6 for WW and FEWZ for Z.

W^+W^- / Z: correlations.

arXiv:1404.4234

Cross section	Value pb	Exp. PDF pb	Mod. PDF pb	Scale pb	Correlation coefficient σ_Z^{NLO}	σ_Z^{NNLO}	σ_{WW}^{NLO}
σ_Z^{NLO}	29890	± 450	$^{+490}_{-490}$	$^{+680}_{-940}$	1	0.697	0.736
σ_Z^{NNLO}	30390	± 420	$^{+520}_{-540}$	$^{+190}_{-260}$	0.697	1	0.451
σ_{WW}^{NLO}	46.1	± 0.6	$^{+0.7}_{-0.6}$	$^{+1.5}_{-1.4}$	0.736	0.451	1



σ_Z^{NLO} vs. σ_{WW}^{NLO} σ_Z^{NNLO} vs. σ_{WW}^{NLO}

- Experimental (Exp.) PDF uncertainties are strongly correlated between Z@NNLO and WW@NLO.
- Some model and parametrisation (Mod.) PDF uncertainties are anti-correlated.

DESY

W^+W^- / Z: model and parametrisation uncertainties

arXiv:1404.4234

Variation	σ^{NLO}_{WW} %	σ^{NLO}_{Z} %	σ^{NNLO}_{Z} %
$r_s(-0.3)$	1.00	-0.29	-0.33
$r_s(+0.3)$	-0.81	0.39	0.42
$M_c(-0.06$ GeV$)$	-0.81	-0.89	-0.76
$M_c(+0.06$ GeV$)$	0.55	0.66	0.61
$M_b(-0.45$ GeV$)$	0.13	0.11	-0.02
$M_b(+0.25$ GeV$)$	-0.07	-0.07	0.00
$\alpha_S(M_Z)(-0.002)$	-0.54	-1.27	-1.17
$\alpha_S(M_Z)(+0.002)$	0.52	1.23	1.17
$Q^2_{min}(-2.5$ GeV$^2)$	-0.25	-0.35	0.23
$Q^2_{min}(+2.5$ GeV$^2)$	0.75	0.73	-1.06
$Q^2_0(-0.2$ GeV$^2)$	-0.21	-0.19	-0.14
$+D_{u_v}$	-0.03	-0.32	0.97
$+D_{\bar{U}}$	-0.04	-0.02	-0.01
$+E_{\bar{U}}$	0.01	0.00	0.00

Variation	σ^{NLO}_{WW} / σ^{NLO}_{Z} $\times 10^{-3}$	σ^{NLO}_{WW} / σ^{NNLO}_{Z} $\times 10^{-3}$
$r_s(-0.3)$	0.020	0.020
$r_s(+0.3)$	-0.018	-0.019
$M_c(-0.06$ GeV$)$	0.001	-0.001
$M_c(+0.06$ GeV$)$	-0.002	-0.001
$M_b(-0.45$ GeV$)$	0.000	0.002
$M_b(+0.25$ GeV$)$	0.000	-0.001
$\alpha_S(M_Z)(-0.002)$	0.011	0.010
$\alpha_S(M_Z)(+0.002)$	-0.011	-0.010
$Q^2_{min}(-2.5$ GeV$^2)$	0.002	-0.007
$Q^2_{min}(+2.5$ GeV$^2)$	0.000	0.028
$Q^2_0(-0.2$ GeV$^2)$	0.000	-0.001
$+D_{u_v}$	0.005	-0.015
$+D_{\bar{U}}$	0.000	-0.001
$+E_{\bar{U}}$	0.000	0.000

- r_s is anti-correlated between Z and WW.
- M_c, M_b, α_s are correlated and mostly cancel in the ratio.
- The low-Q^2 DIS data have anti-correlated effect on Z at NLO and at NNLO

W^+W^- / Z: predictions vs. data.

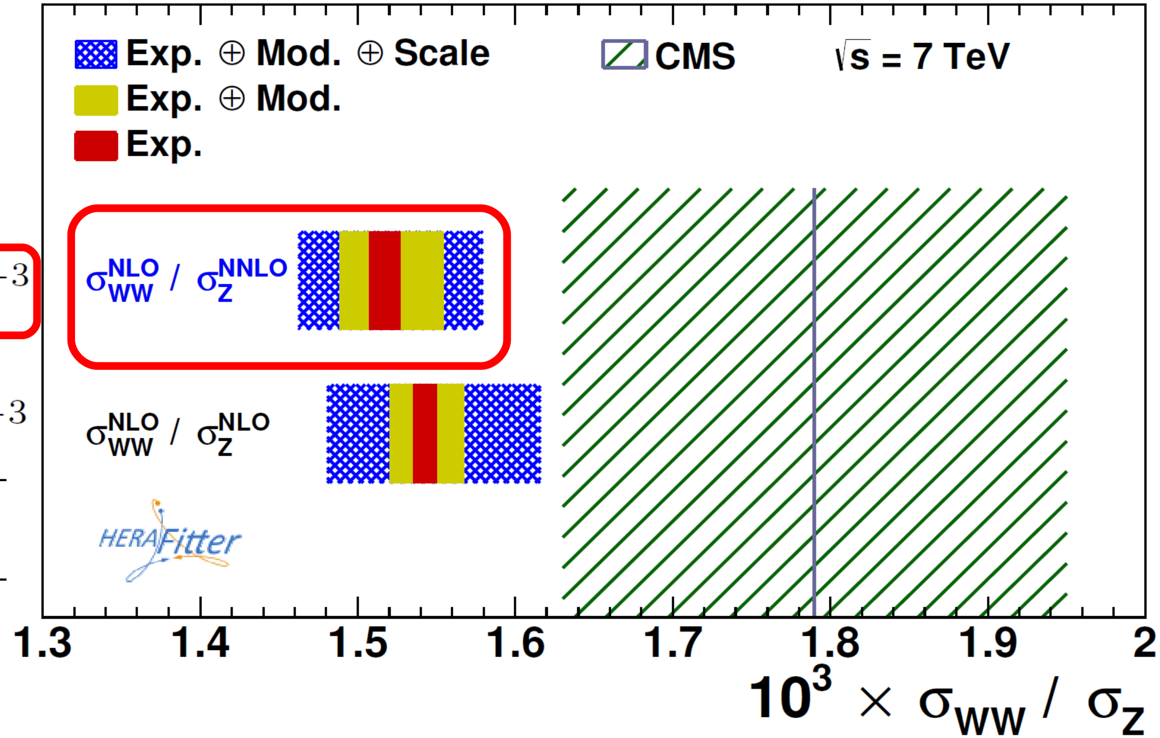
arXiv:1404.4234

±0.066 total uncertainty for NLO/NNLO assuming uncorrelated total PDF unc.

$$\sigma_{WW}^{NLO}/\sigma_{Z}^{NNLO} = [1.517^{+0.051}_{-0.047}] \times 10^{-3}$$

$$\sigma_{WW}^{NLO}/\sigma_{Z}^{NLO} = [1.543^{+0.073}_{-0.062}] \times 10^{-3}$$

Ratio	Value $\times 10^{-3}$	Exp. PDF $\times 10^{-3}$	Mod. PDF $\times 10^{-3}$	Scale $\times 10^{-3}$
$\frac{\sigma_{WW}^{NLO}}{\sigma_{Z}^{NLO}}$	1.543	± 0.008	$^{+0.023}_{-0.021}$	$^{+0.069}_{-0.058}$
$\frac{\sigma_{WW}^{NLO}}{\sigma_{Z}^{NNLO}}$	1.517	± 0.010	$^{+0.036}_{-0.027}$	$^{+0.050}_{-0.046}$



- Predictions agree with the data within 1-2σ.
- The total theoretical uncertainty is reduced by 30-40%.
- The usage of the mixed-order calculations with new PDFs with correlations between orders allow to benefit from cancellation of experimental PDF uncertainties and reduction of the scale uncertainties.

Summary.

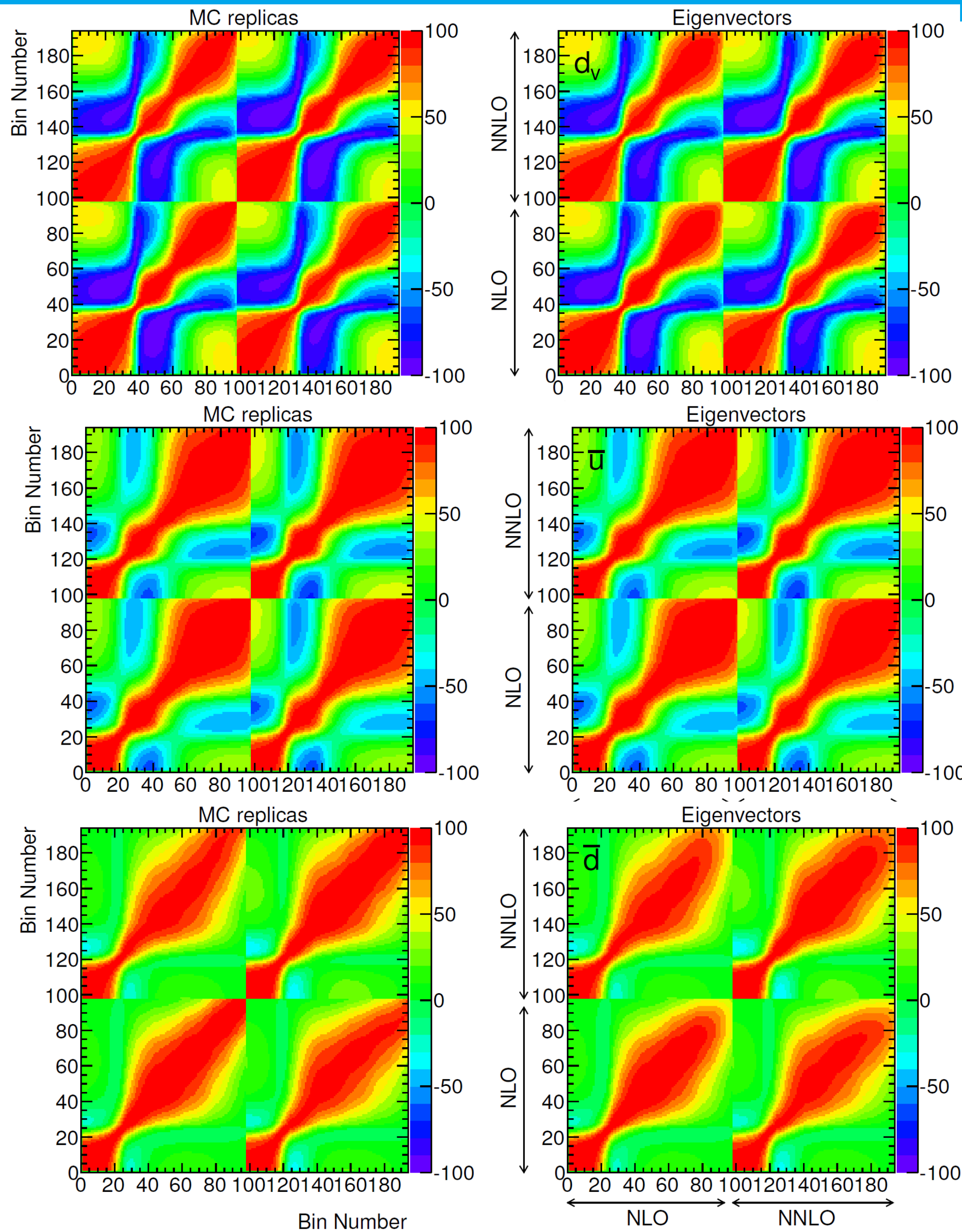
- Sets of LO, NLO and NNLO PDFs correlated at different orders were extracted using HERAFitter.
- The MC method was used to determine the experimental PDF uncertainties. The uncertainties were decomposed into the eigenvector representation.
- A high degree of correlation was observed for PDFs at different orders and similar x.
- The correlated NLO and NNLO PDFs were used to calculate W+/Z ratio. Significant correlation of the PDF uncertainties was observed.
- The total theoretical uncertainty is reduced for the mixed-order calculation by 30-40% due to reduced scale uncertainties.
- PDFs will be released in LHAPDF v6 soon: HF14cor*

Backup

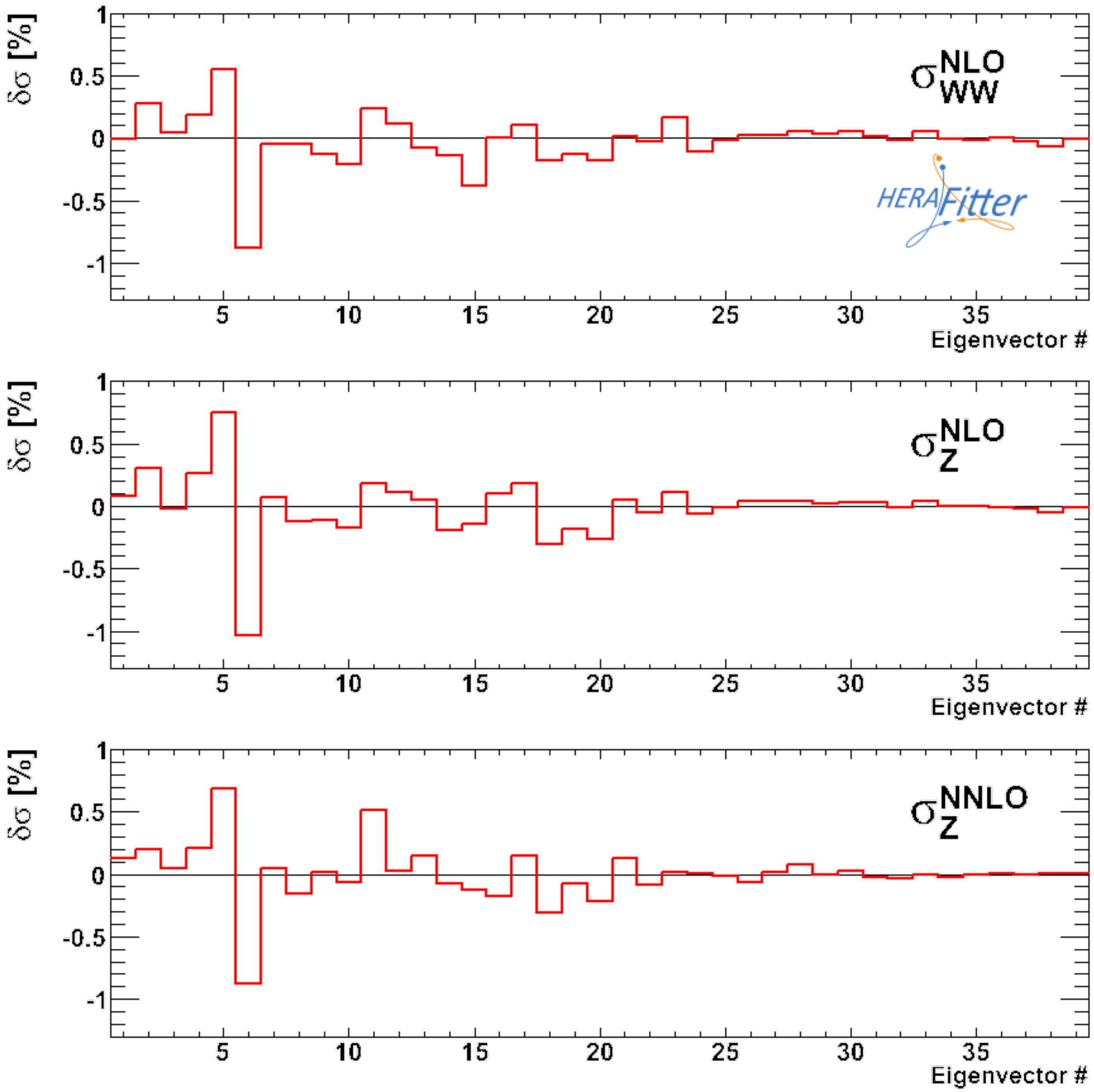
References for theory uncertainties.

- gg->H (MH=125 GeV, √s=8 TeV)
- NLO: Anastasiou et al. 1202.3638 + Ball et al. 1211.5142.
- NNLO+NNLL: Higgs XS working group
- ttbar
- NLO: Cacciari et al. 1111.6859
- NNLO+NNLL: Czakan et al. 1303.6254

NLO-NNLO correlations (cont.)



WW/Z : correlation of experimental uncertainties



DESY