The Structure and Signals of Neutron Stars, from Birth to Death
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Properties of localized protons in neutron star matter at finite temperatures

Adam Szmagliński, Sebastian Kubis, Włodzimierz Wójcik

Institute of Physics, Cracow University of Technology

Realistic Nuclear Models:

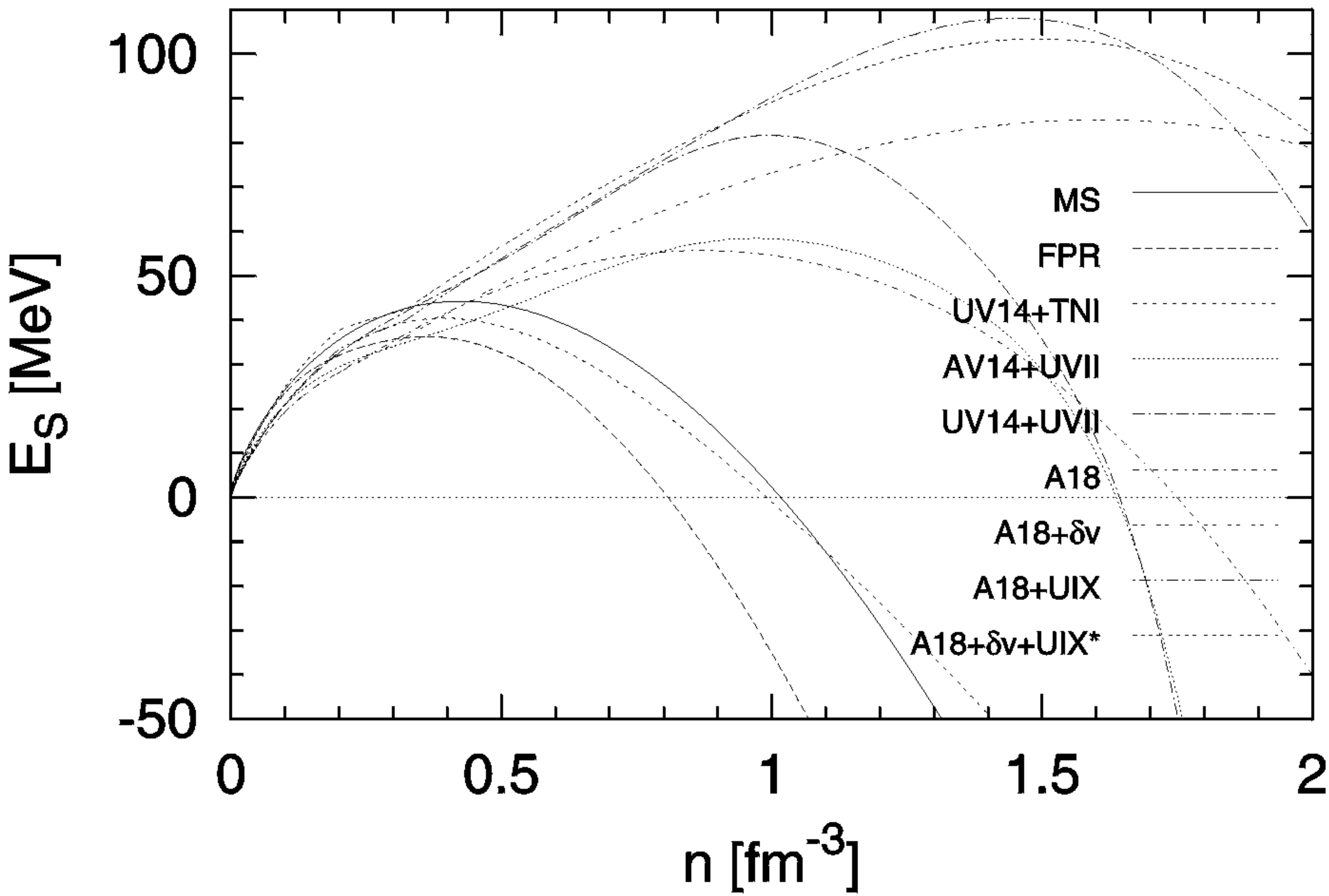
1. Skyrme (SI’, SII’, SIII’, SL, Ska, SKM, SGII, RATP, T6)
2. Myers-Świątecki (MS)
3. Friedman-Pandharipande-Ravenhall (FPR)
4. UV14+TNI (UV)
5. AV14+UVII (AV)
6. UV14+UVII (UVU)
7. A18
8. A18+δv
9. A18+UIX
10. A18+δv+UIX*

Symmetry Energy of Nuclear Matter

$$E(n,x)=E\left(n,\frac{1}{2}\right)+E_S(n)(2x-1)^2$$

$$x=n_P/n$$

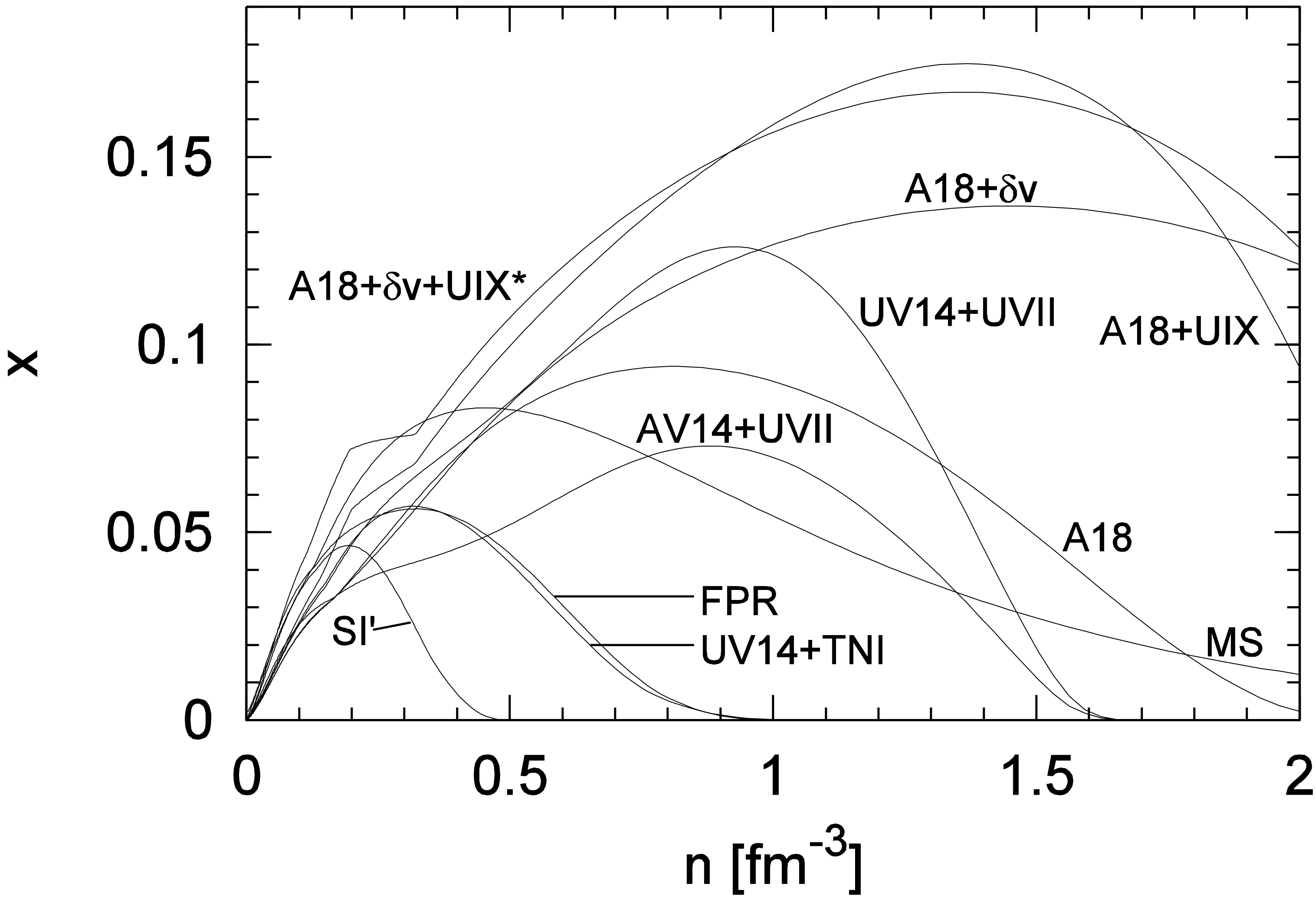
$$E_S(n)=\frac{1}{8}\left.\frac{\partial^2 E(n,x)}{\partial x^2}\right|_{x=\frac{1}{2}}$$



100
50
0
-50
E_S [MeV]
0
0.5
1
1.5
2
n [fm^-3]
MS
FPR
UV14+TNI
AV14+UVII
UV14+UVII
A18
A18+δv
A18+UIX
A18+δv+UIX*

Small values of the symmetry energy:

1. Low proton concentration
2. Charge separation instability
   - realized eg. through proton localization



A18+δv+UIX*
A18+δv
UV14+UVII
A18+UIX
AV14+UVII
A18
FPR
UV14+TNI
SI'
MS
x
0.15
0.1
0.05
0
0.5
1
1.5
2
n [fm^-3]

Model of Proton Impurities in Neutron Star Matter (M. Kutschera, W. Wójcik)

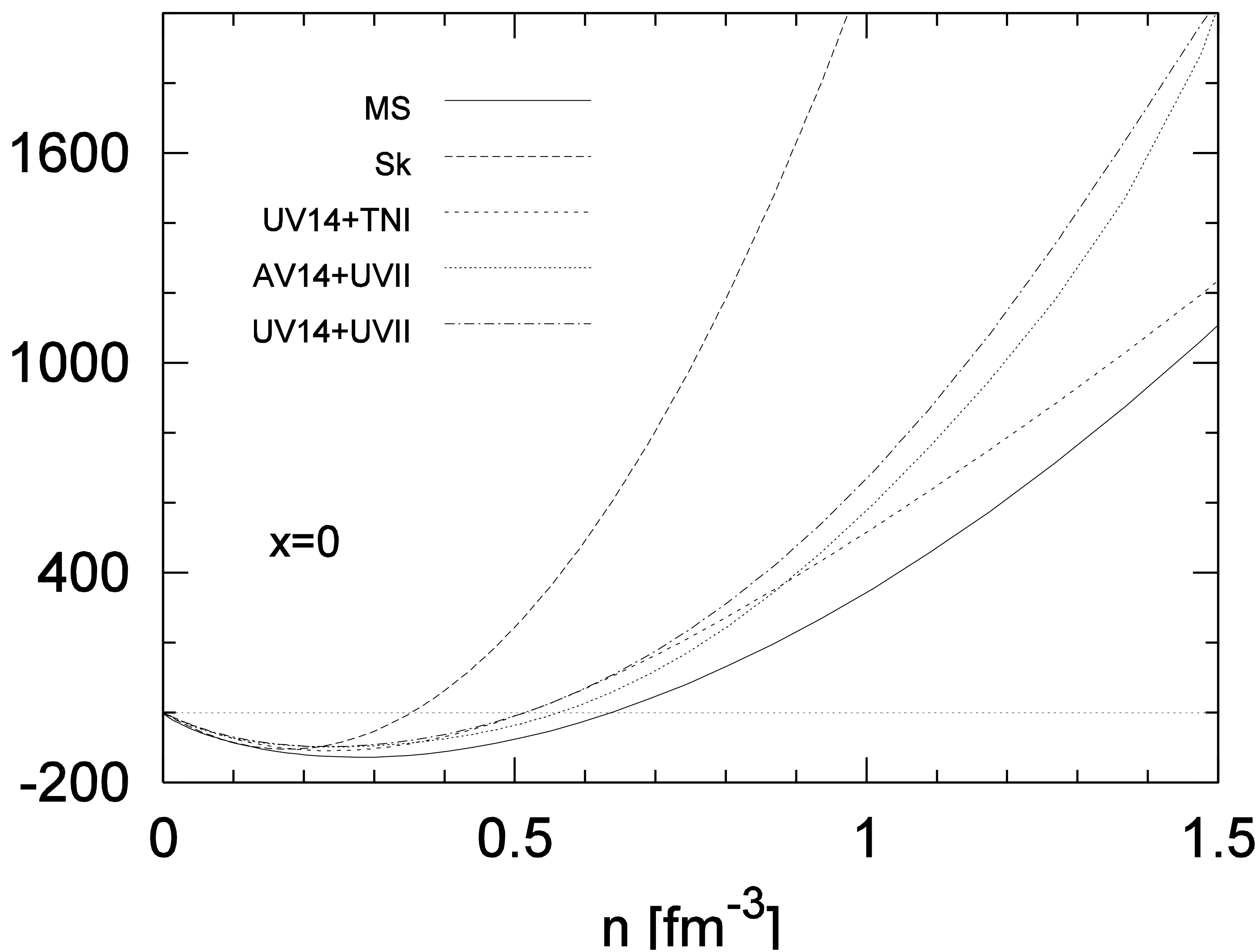
We divide the system into spherical Wigner-Seitz cells, each of them enclosing a single proton:

The volume of the cell: $V = \dfrac{1}{n_P}$

The energy of the cell of uniform phase: $E_0 = V\varepsilon(n_N, n_P)$

$$\varepsilon(n_N, n_P) \approx \varepsilon(n_N) + \mu_P(n_N) n_P$$

$$E_0 = \mu_P(n_N) + V\varepsilon(n_N)$$


MS
Sk
UV14+TNI
AV14+UVII
UV14+UVII
x=0
1600
1000
400
-200
0
0.5
1
1.5
μ_P [MeV]
n [fm^{-3}]

$$\Psi_P(r) = \left(\frac{2}{3}\pi R_P^2\right)^{-\frac{3}{4}} \exp\left(-\frac{3}{4}\frac{r^2}{R_P^2}\right)$$

$$E_L = \int_V \left\{ \Psi_P^*(r)\left[-\frac{\nabla^2}{2m_P} + \mu_P(n(r))\right]\Psi_P(r) + \varepsilon(n(r)) + B_N\left(\vec{\nabla} n(r)\right)^2 \right\} d^3r$$

$$\Delta E = E_L - E_0 = \frac{9}{8m_P R_P^2} +$$

$$+ 4\pi \int_0^\infty r^2 \left\{ p(r)\left(\mu_P(n(r)) - \mu_P(n_N)\right) + \varepsilon(n(r)) - \varepsilon(n_N) + B_N\left(\frac{dn(r)}{dr}\right)^2 \right\} dr$$

where $p(r) \equiv \Psi_P^*(r)\Psi_P(r)$

The self-consistent variational method

We look for such functions $\Psi(r)$ and $n(r)$, that minimize functional:

$$f\left[n(r),\Psi_P(r)\right]=\Delta E-\lambda\int_V\left[n(r)-n_N\right]d^3r-E\left[\int_V\Psi_P^*(r)\Psi_P(r)d^3r-1\right]$$

λ, E - Lagrange multipliers

boundary conditions:

$$\int_V\left[n(r)-n_N\right]d^3r=0 \qquad \int_V\Psi_P^*(r)\Psi_P(r)d^3r-1=0$$

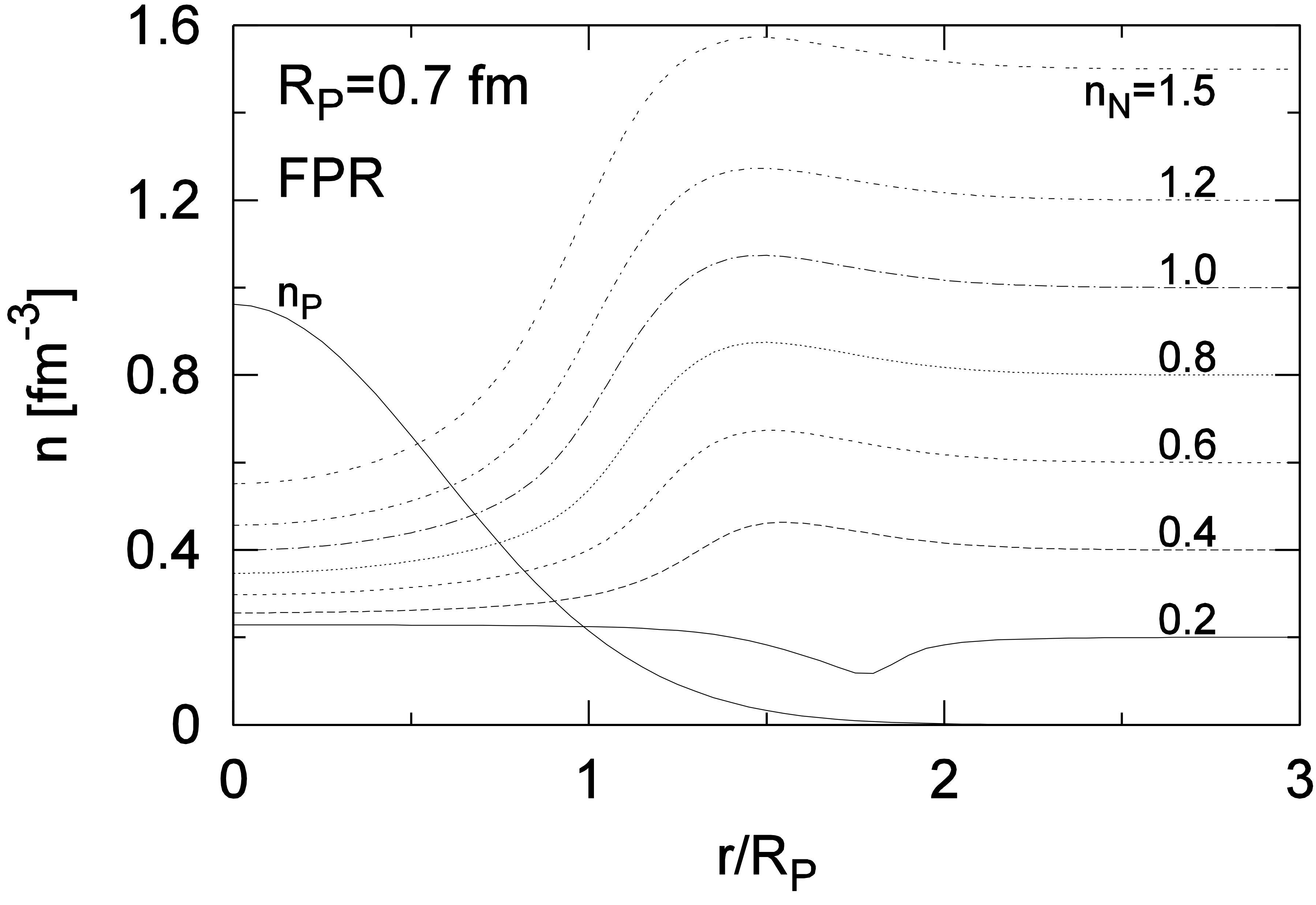
By differentiation with respect to $\Psi_P^*(r)$ and $n(r)$ we obtain

following Euler-Lagrange equations:

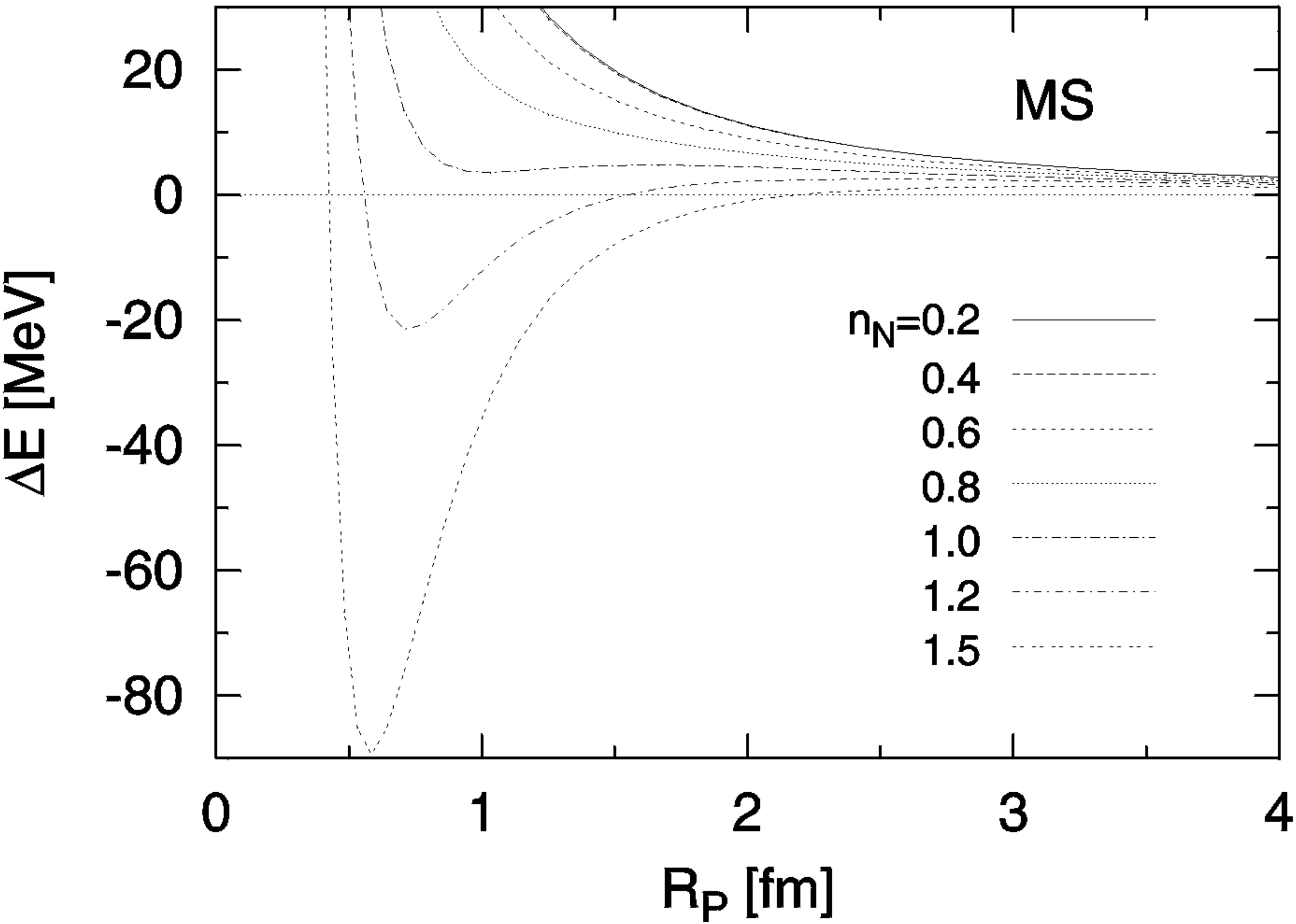
$$-\frac{1}{2m_P}\nabla^2\Psi_P(r)+\left[\mu_P(n(r))-\mu_P(n_N)\right]\Psi_P(r)=E_P\Psi_P(r)$$

$$\frac{\partial\mu_P(n(r))}{\partial n(r)}\Psi_P^*(r)\Psi_P(r)+\mu_N(n(r))-2B_N r\left(\frac{d^2n(r)}{dr^2}r+2\frac{dn(r)}{dr}\right)-\lambda=0$$

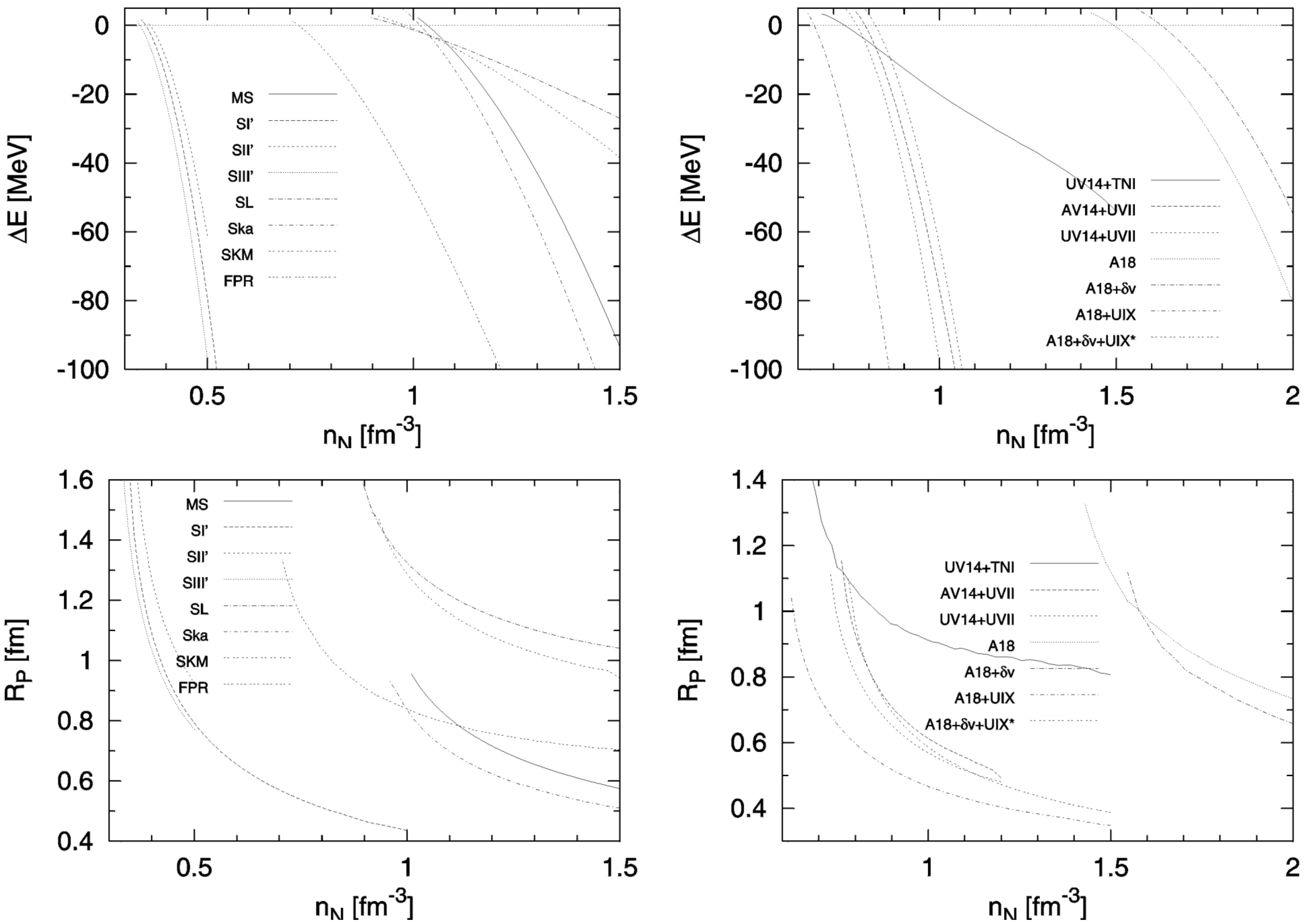
$r\to\infty\Rightarrow\lambda=\mu_N(n_N)$ $\qquad$ R_P - variational parameter



R_P=0.7 fm
FPR
n_N=1.5
1.2
1.0
0.8
0.6
0.4
0.2
n_P
n [fm^{-3}]
r/R_P
1.6
1.2
0.8
0.4
0
0
1
2
3



MS
n_N=0.2
0.4
0.6
0.8
1.0
1.2
1.5
ΔE [MeV]
R_P [fm]
20
0
-20
-40
-60
-80
0
1
2
3
4



ΔE [MeV]
n_N [fm^{-3}]
MS
SI'
SII'
SIII'
SL
Ska
SKM
FPR
UV14+TNI
AV14+UVII
UV14+UVII
A18
A18+δv
A18+UIX
A18+δv+UIX*
R_P [fm]

Potential	n_{loc}	R_P^{loc}
MS	1.033	0.906
SI’	0.351	1.570
SII’	0.361	1.688
SIII’	0.337	1.552
SL	0.964	1.384
Ska	1.016	0.804
SKM	0.979	1.330
FPR	0.721	1.262
UV14+TNI	0.731	1.209
AV14+UVII	0.789	0.971
UV14+UVII	0.766	0.913
A18	1.493	1.136
A18+δv	1.627	0.915
A18+UIX	0.645	0.911
A18+δv+UIX*	0.819	0.878

Properties of nuclear matter at T>0

We minimize the free energy difference

$$F = E - T\left(\left(1 - x\right)S_N + xS_P\right)$$

Kinetic energy density

$$\tau_{N,P} = \frac{2}{\left(2\pi\right)^2}\left(2m^*_{N,P}T\right)^{5/2} J_{3/2}\left(\eta_{N,P}\right)$$

Where the entropy per baryon

$$S_{N,P} = \frac{5}{3}\frac{1}{n_{N,P}}\frac{1}{\left(2\pi\right)^2}\left(2m^*_{N,P}T\right)^{3/2} J_{3/2}\left(\eta_{N,P}\right) - \frac{1}{2}\eta_{N,P}$$

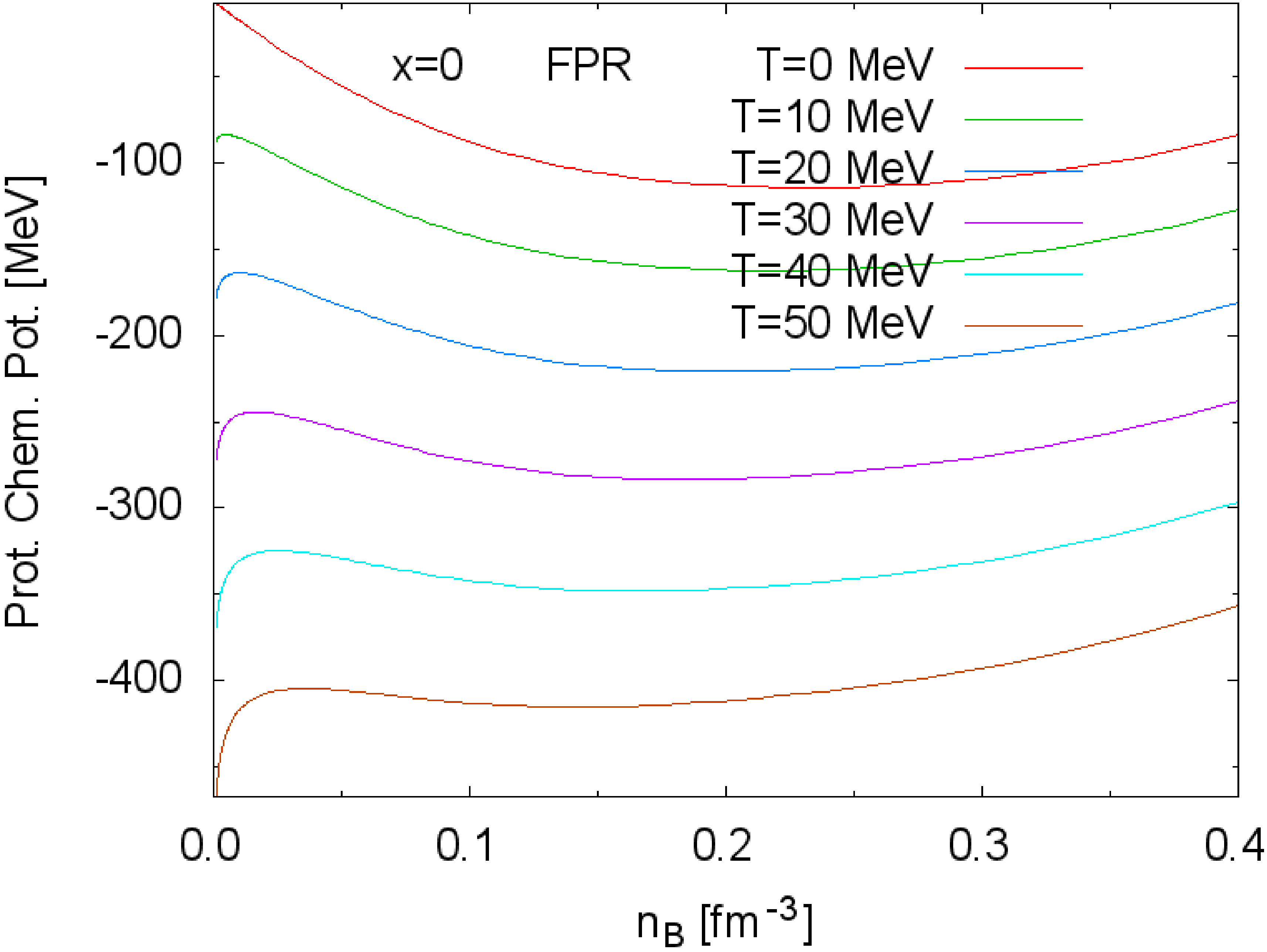
The unknown quantity $\eta_{N,P}$ comes from:

$$n_{N,P} = \frac{2}{(2\pi)^2}\left(2m^*_{N,P}\right)^{3/2} J_{1/2}\left(\eta_{N,P}\right)$$

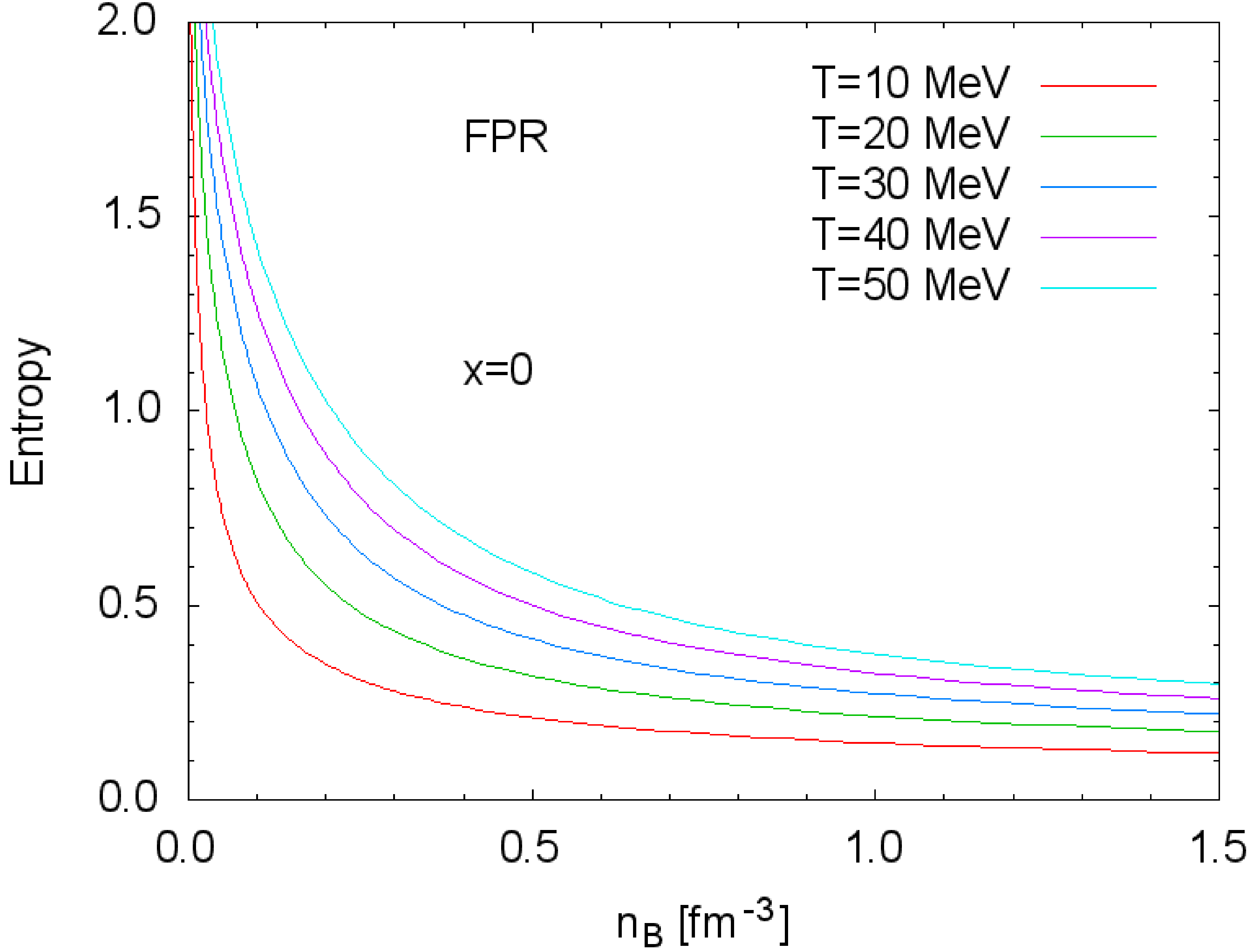
Fermi integrals are defined $$J_\nu(\eta) = \int_0^\infty dx \frac{x^\nu}{1+e^{x-\eta}}$$

Nucleon chemical potentials are the derivatives of the free energy density

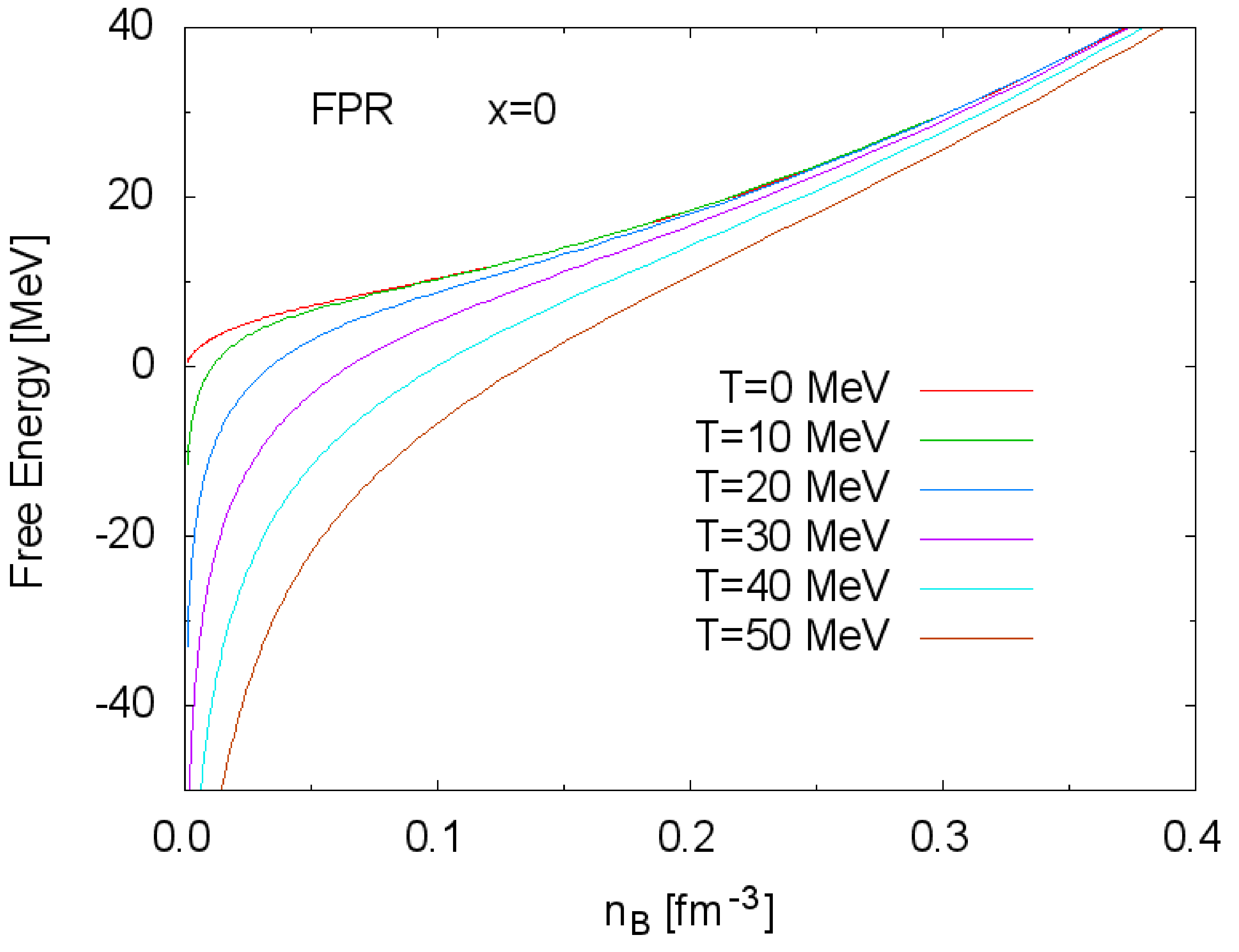
$$\mu_{N,P} = \frac{\partial f}{\partial n_{N,P}}$$



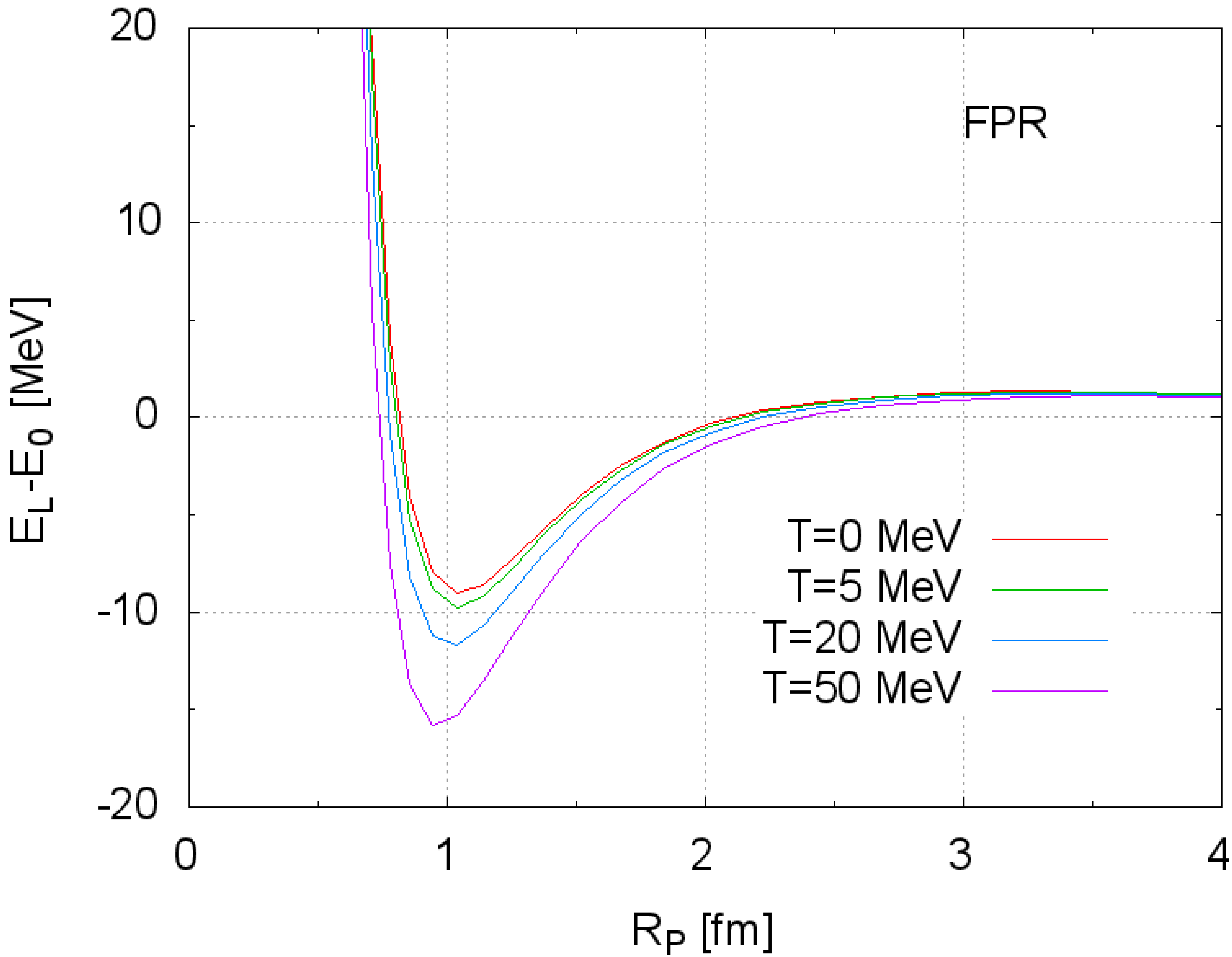
x=0
FPR
T=0 MeV
T=10 MeV
T=20 MeV
T=30 MeV
T=40 MeV
T=50 MeV
-100
-200
-300
-400
0.0
0.1
0.2
0.3
0.4
Prot. Chem. Pot. [MeV]
n_B [fm^-3]

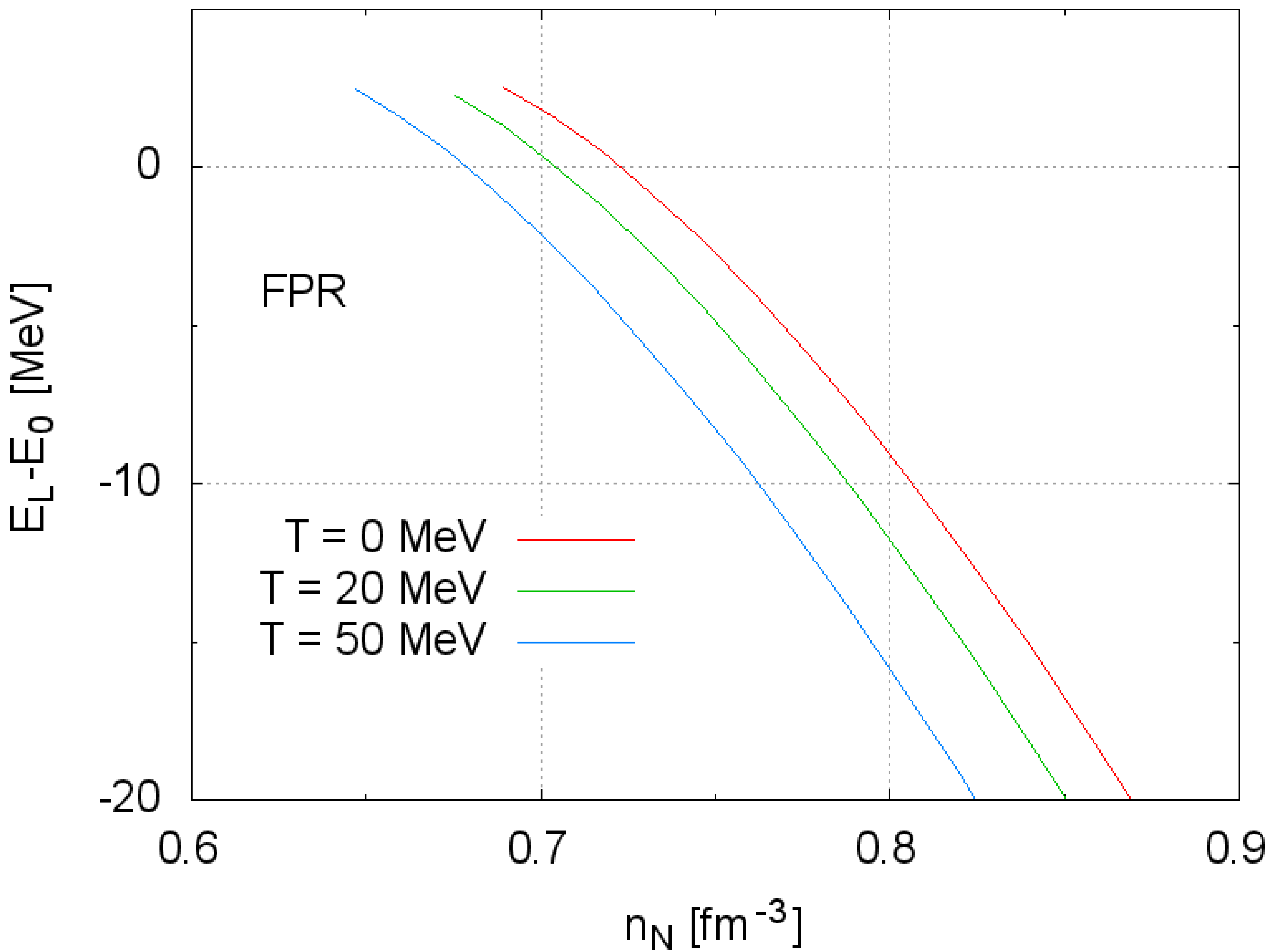


2.0
1.5
1.0
0.5
0.0
Entropy
0.0
0.5
1.0
1.5
n_B [fm^{-3}]
FPR
x=0
T=10 MeV
T=20 MeV
T=30 MeV
T=40 MeV
T=50 MeV

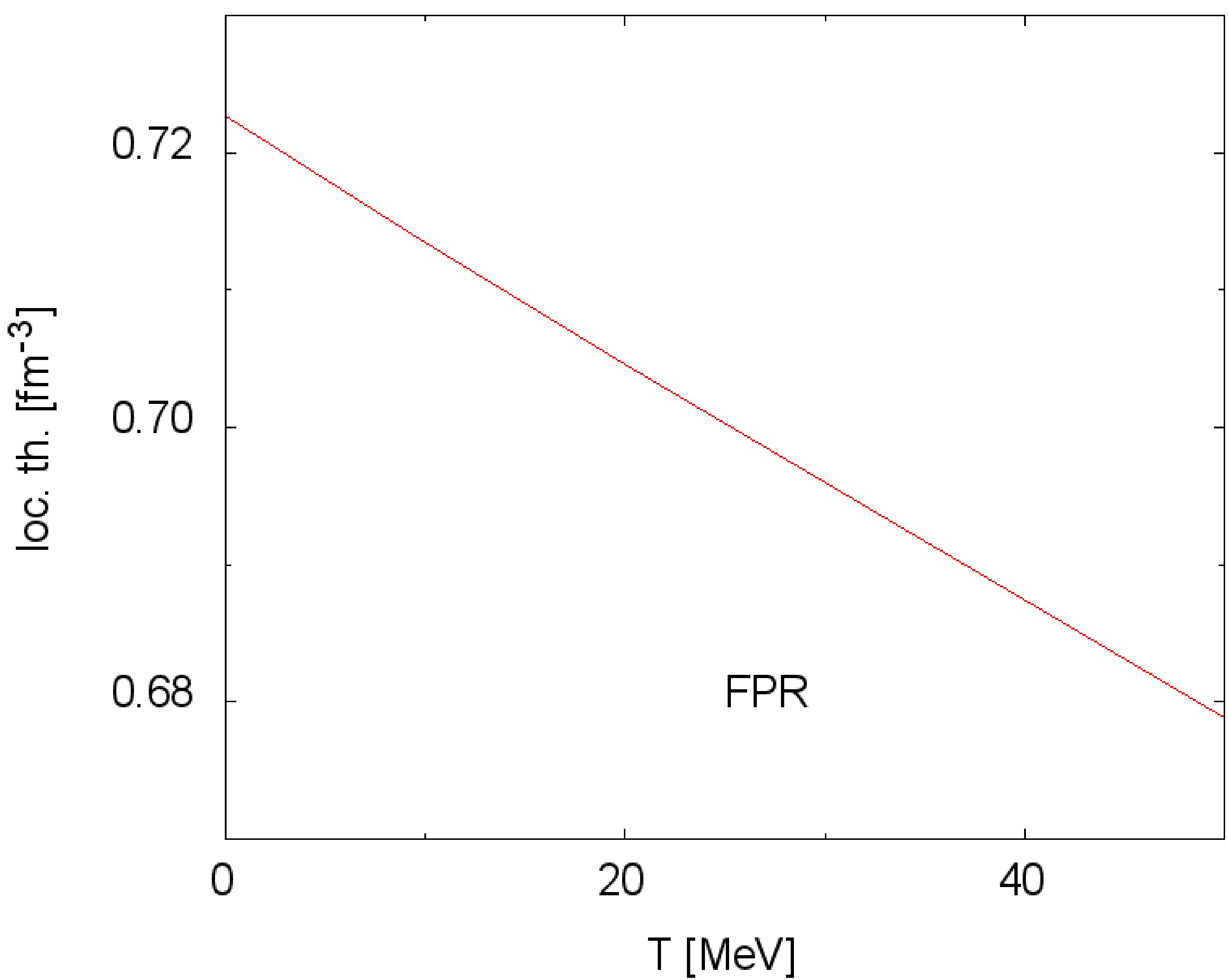


FPR x=0
Free Energy [MeV]
40
20
0
-20
-40
0.0
0.1
0.2
0.3
0.4
n_B [fm^{-3}]
T=0 MeV
T=10 MeV
T=20 MeV
T=30 MeV
T=40 MeV
T=50 MeV

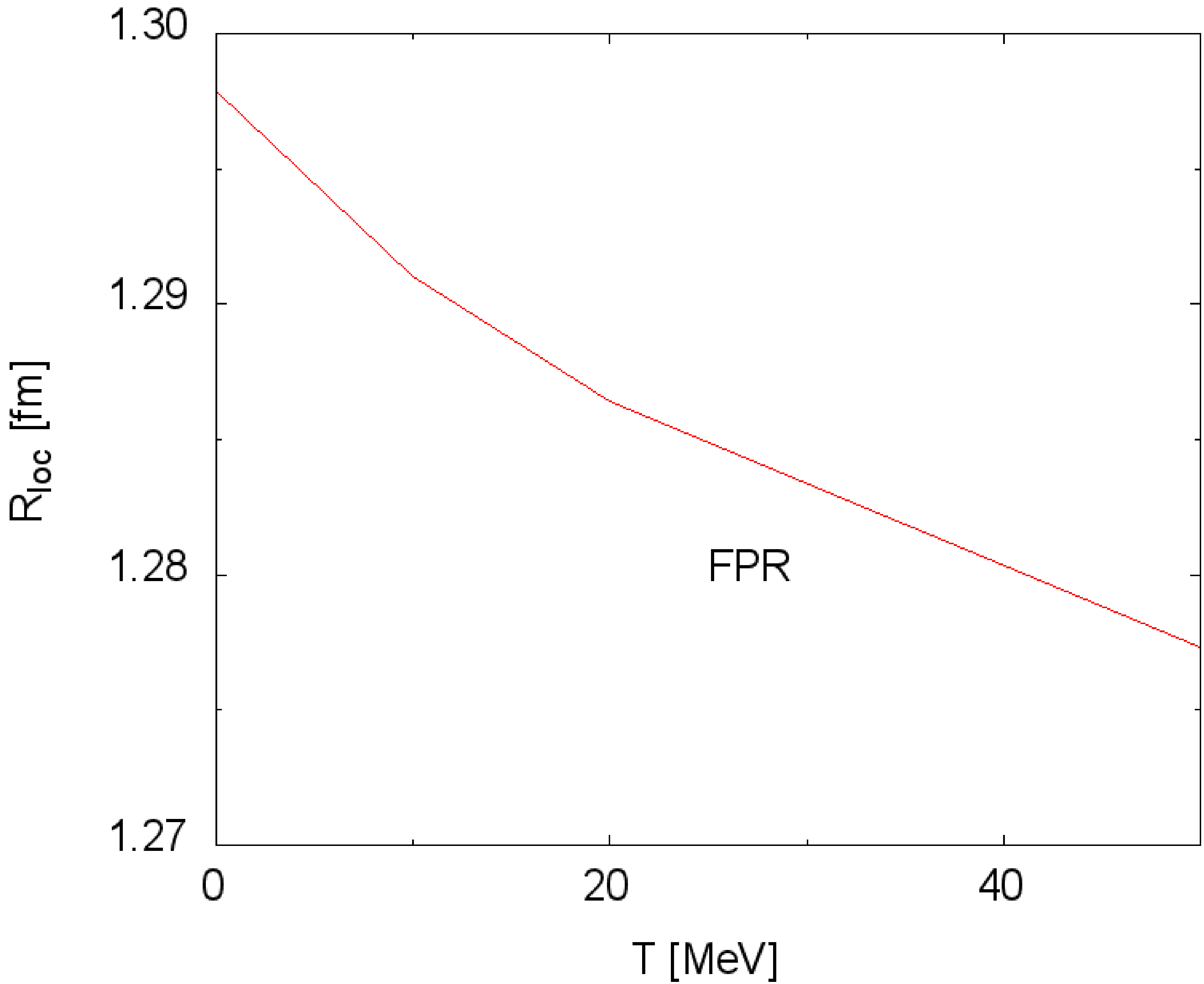

FPR
T=0 MeV
T=5 MeV
T=20 MeV
T=50 MeV
E_L-E_0 [MeV]
R_P [fm]
20
10
0
-10
-20
0
1
2
3
4



FPR
T = 0 MeV
T = 20 MeV
T = 50 MeV
0
-10
-20
0.6
0.7
0.8
0.9
E_L-E_0 [MeV]
n_N [fm^{-3}]



loc. th. [fm^{-3}]
0.72
0.70
0.68
FPR
0
20
40
T [MeV]



1.30
1.29
1.28
1.27
R_{loc} [fm]
0
20
40
T [MeV]
FPR

Conclusions

1. Symmetry energy implies the inhomogeneity of dense nuclear matter in neutron stars.
2. Proposal of self-consistent variational method – neutron background profile as a solution of variational equation with parameter (mean square radius of proton wave function).
3. Localization of protons as an universal state of dense nuclear matter in neutron stars.
4. Nonzero temperature lowers the localization threshold density and diminishing the size of the proton wave function.
5. Localization is still present at very high temperature.

References:

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[8] A. Szmagliński, S. Kubis, W. Wójcik, Acta Phys. Pol. **B45**, 249 (2014).