Uniwersytet Wrocławski

QCD in dense matter - prospects for and beyond NJL-kind effective models

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2013/09/B/ST2/01560

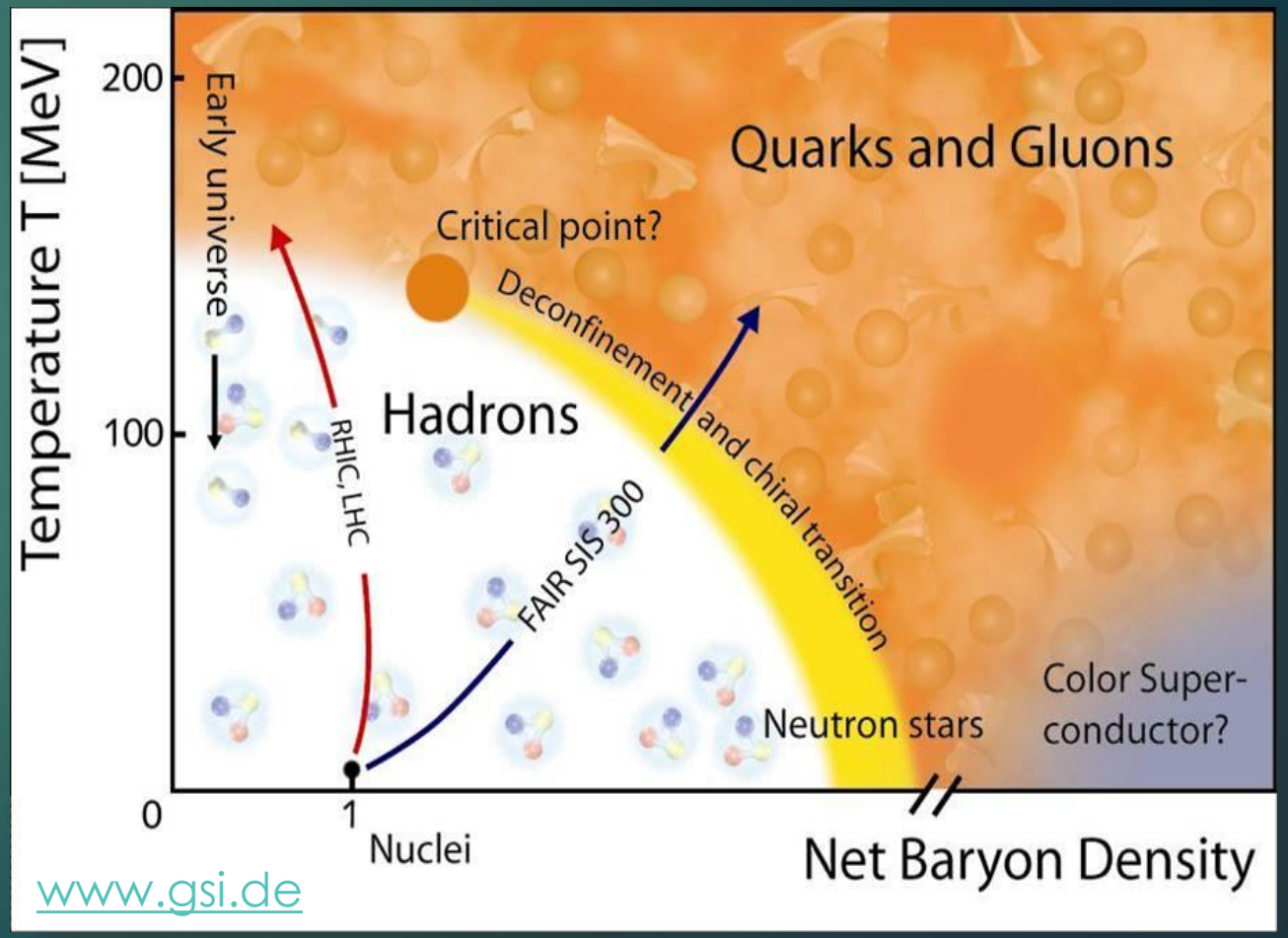
QCD Phase Diagram

- **dense baryonic matter**

<u>HIC in collider experiments</u>
Won't cover the whole diagram
Hot and 'rather' symmetric

<u>NS as a 2nd accessible option</u>
Cold and 'rather' asymmetric

Problem is more complex than
It looks at first gaze



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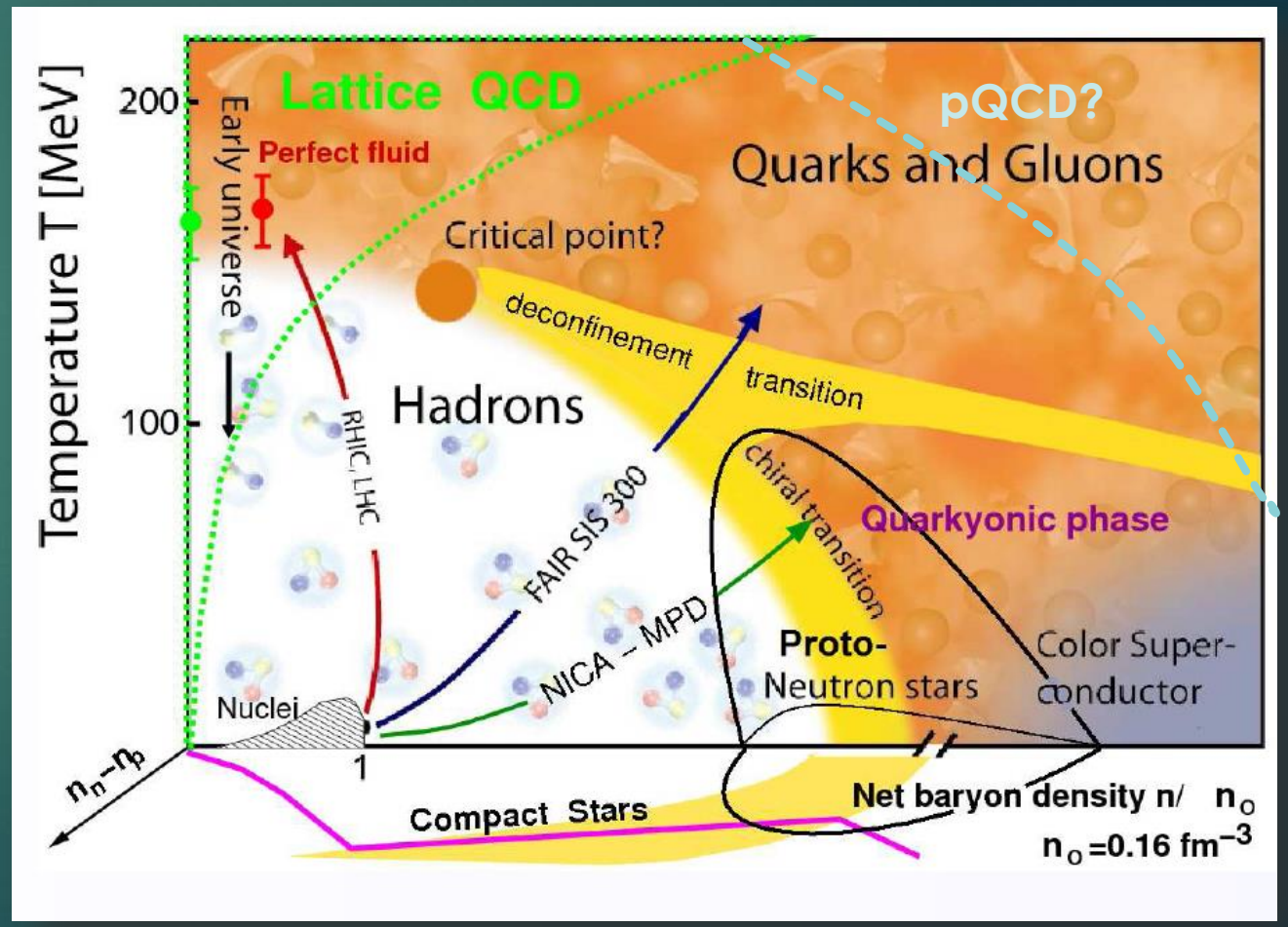
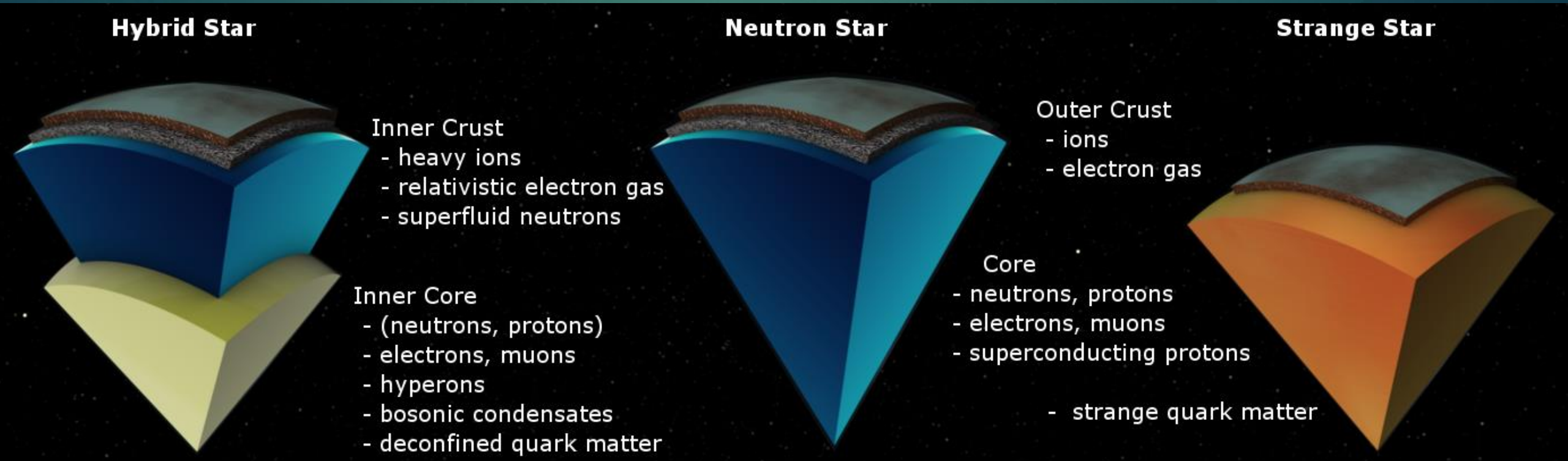
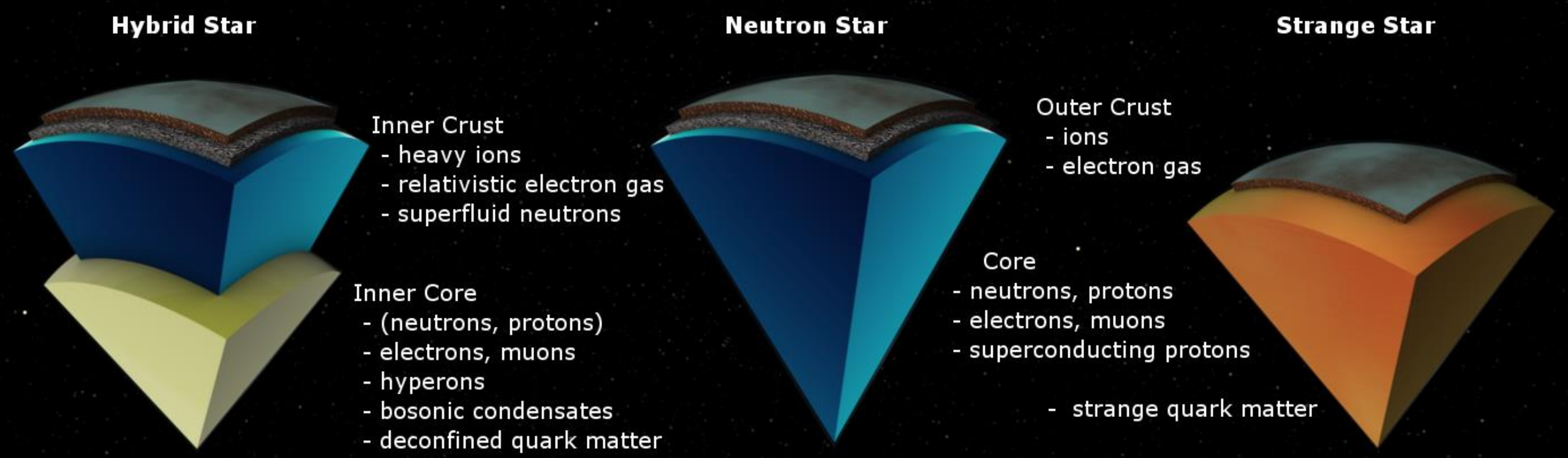


Neutron Stars

- Variety of scenarios regarding inner structure: with or without QM
- Question whether/how QCD phase transition occurs is not settled
- Most honest approach: take both (and more) scenarios into account and compare to available data



Neutron Stars = Quark Cores?



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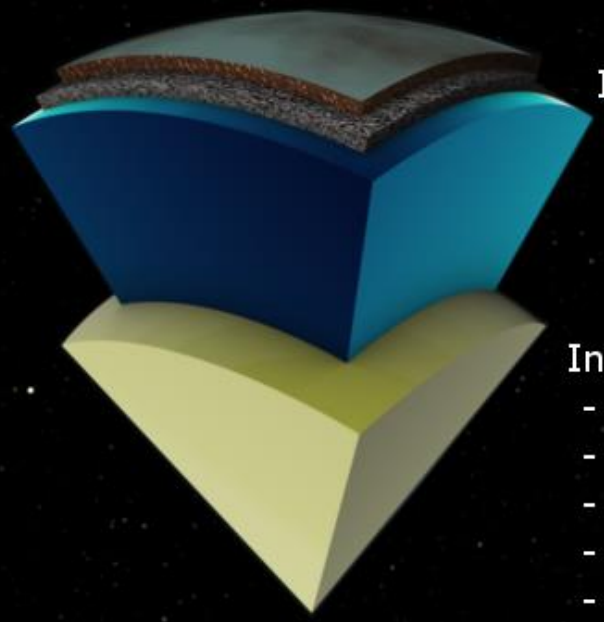
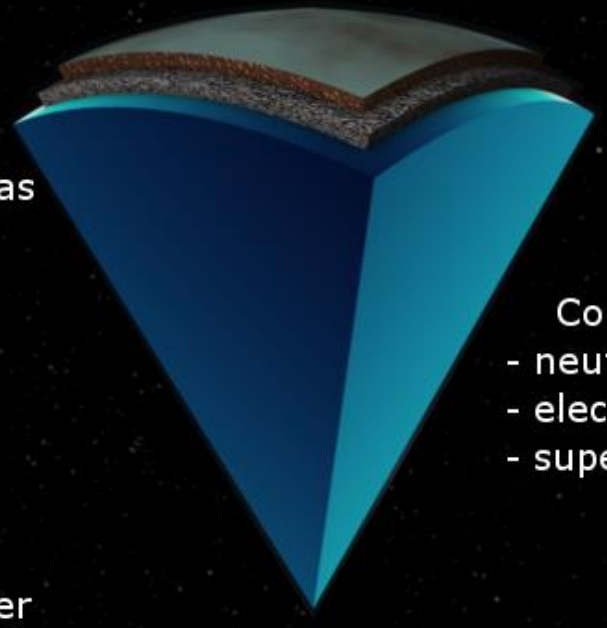
Hybrid Star



Inner Crust
- heavy ions
- relativistic electron gas
- superfluid neutrons

Inner Core
- (neutrons, protons)
- electrons, muons
- hyperons
- bosonic condensates
- deconfined quark matter

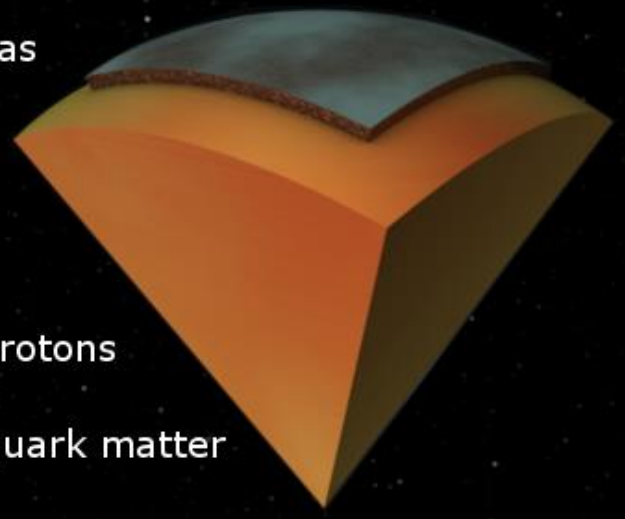
Neutron Star



Outer Crust
- ions
- electron gas

Core
- neutrons, protons
- electrons, muons
- superconducting protons

Strange Star



- strange quark matter

Neutron Stars = Quark Cores?

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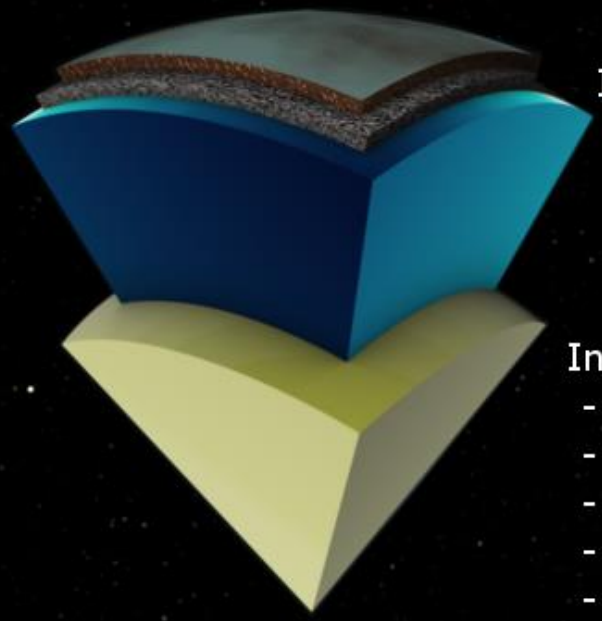

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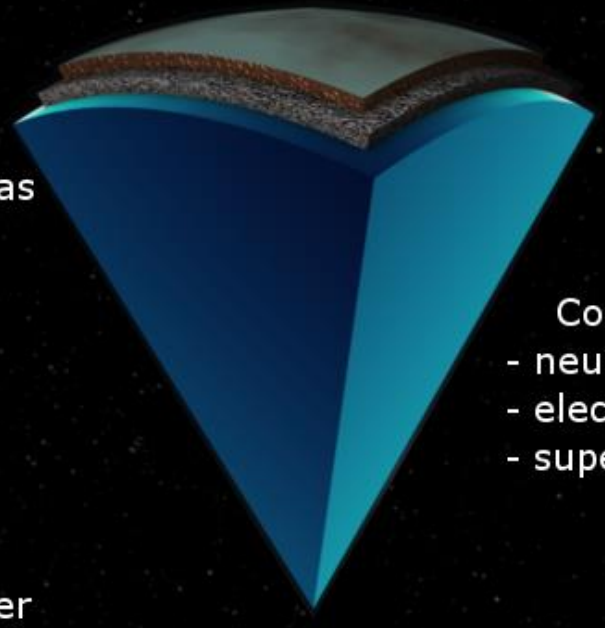

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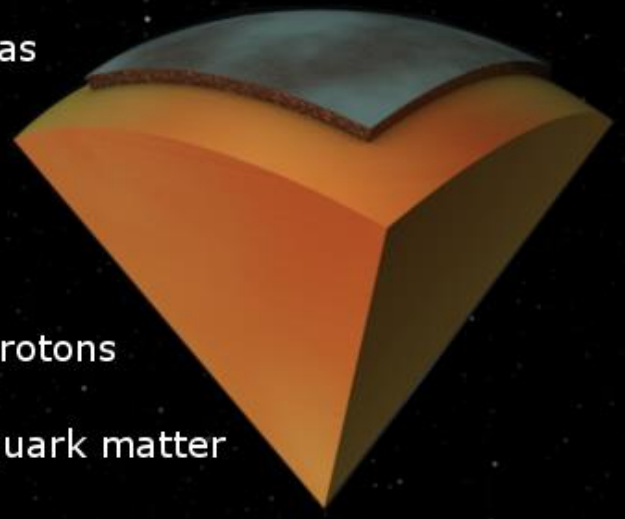

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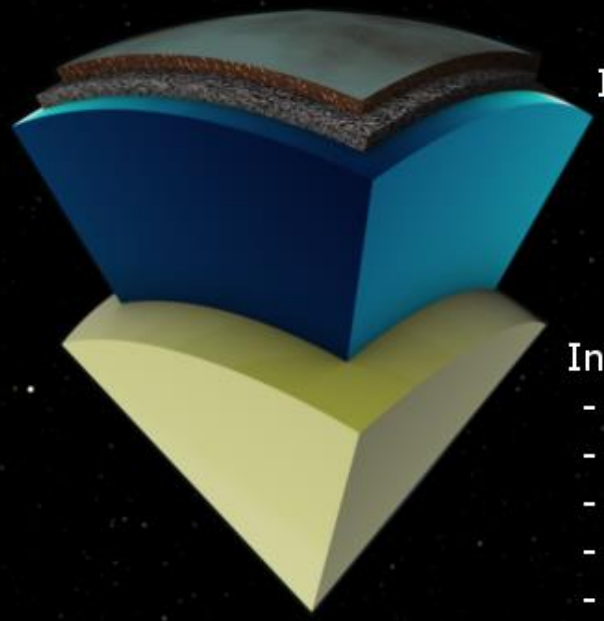
Neutron Stars = Quark Cores?

Hybrid Star | **Neutron Star** | **Strange Star**



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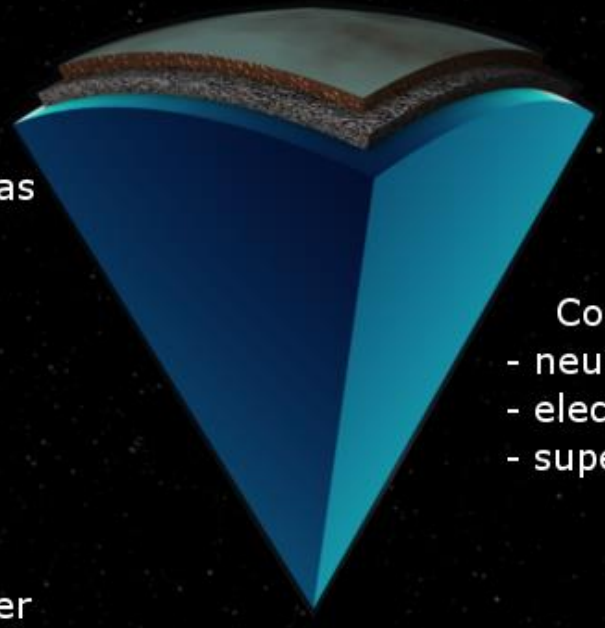

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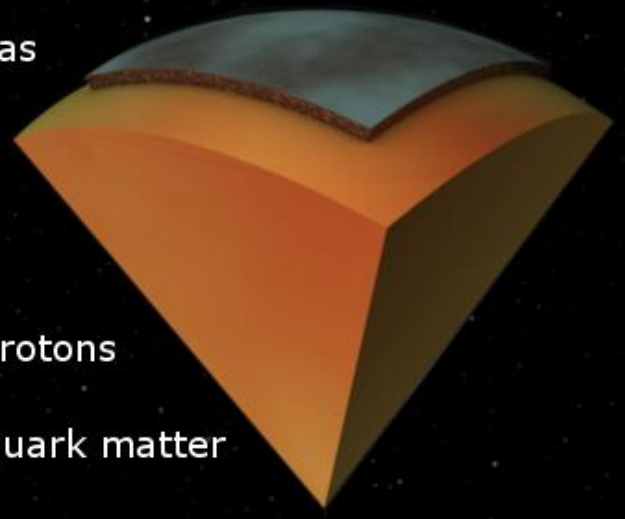
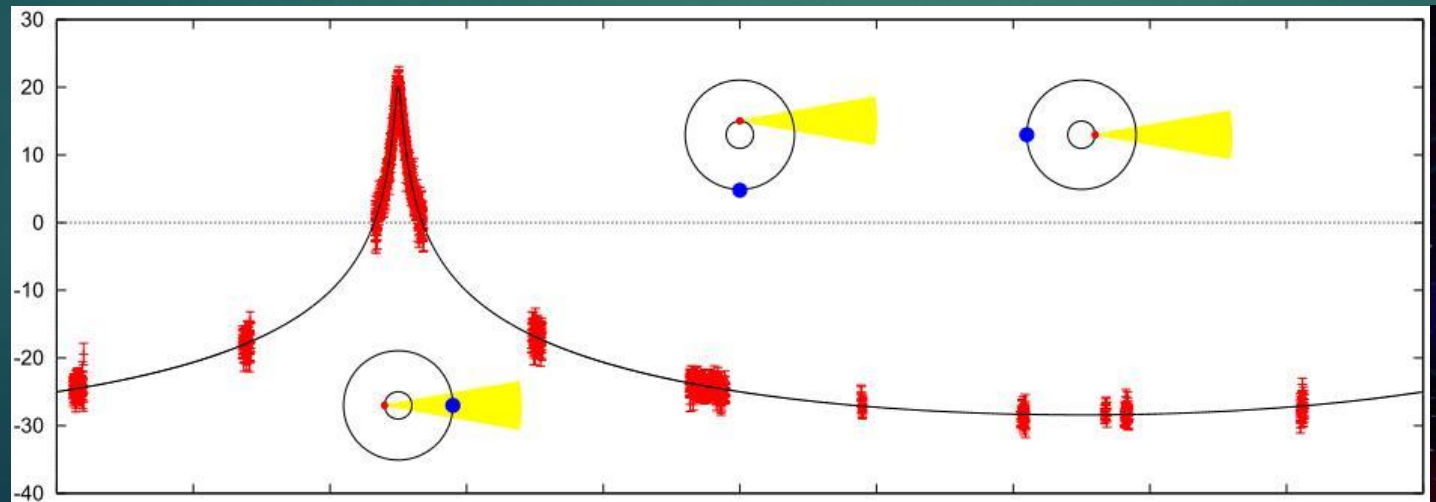
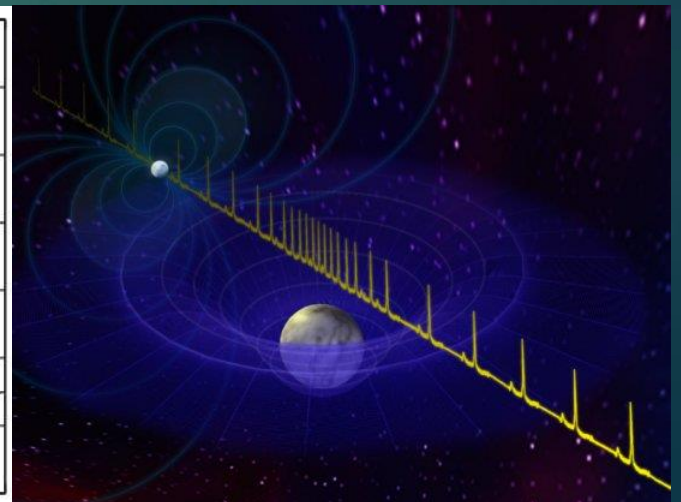


Neutron Star Data

- Data situation in general terms is good (masses, temperatures, ages, frequencies)
- Ability to explain the data with different models in general is good, too.

  … which sounds good, but becomes tiresome if everybody explains everything …
- For our purpose only a few observables are of real interest
- Most promising: High Massive NS with 2 solar masses (Demorest et al., Nature 467, 1081-1083 (2010))





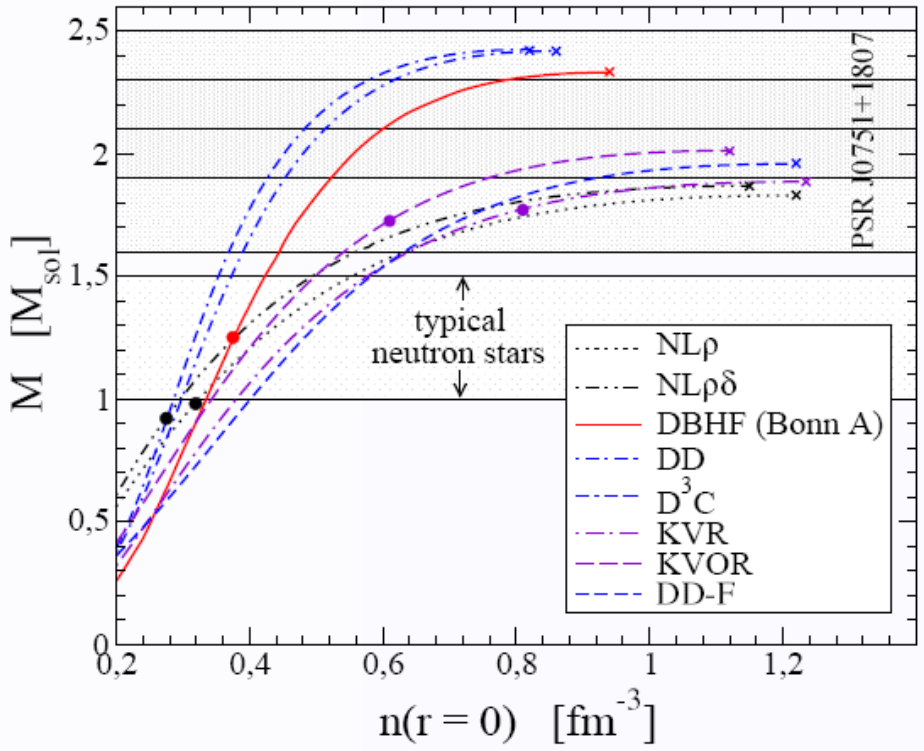
NS masses and the (QM) Equation of State

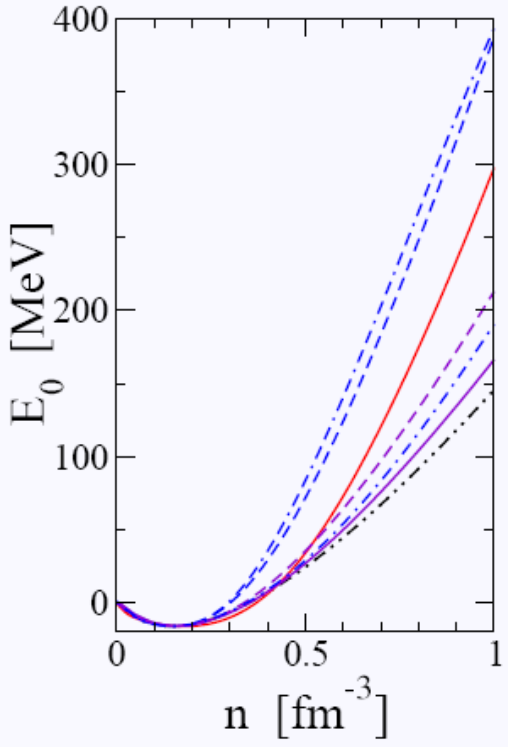
- **NS mass is sensitive mainly to the sym. EoS (In particular true for heavy NS)**

- Folcloric:
  QM is soft, hence no NS with QM core

- Fact:
  QM is softer, but able to support QM core in NS

- Problem:
  (transition from NM to) QM is barely understood





$M(n)$ correlated to $E_0(n)$

stiff: higher M_{max} at smaller densities

soft: smaller M_{max} at higher densities

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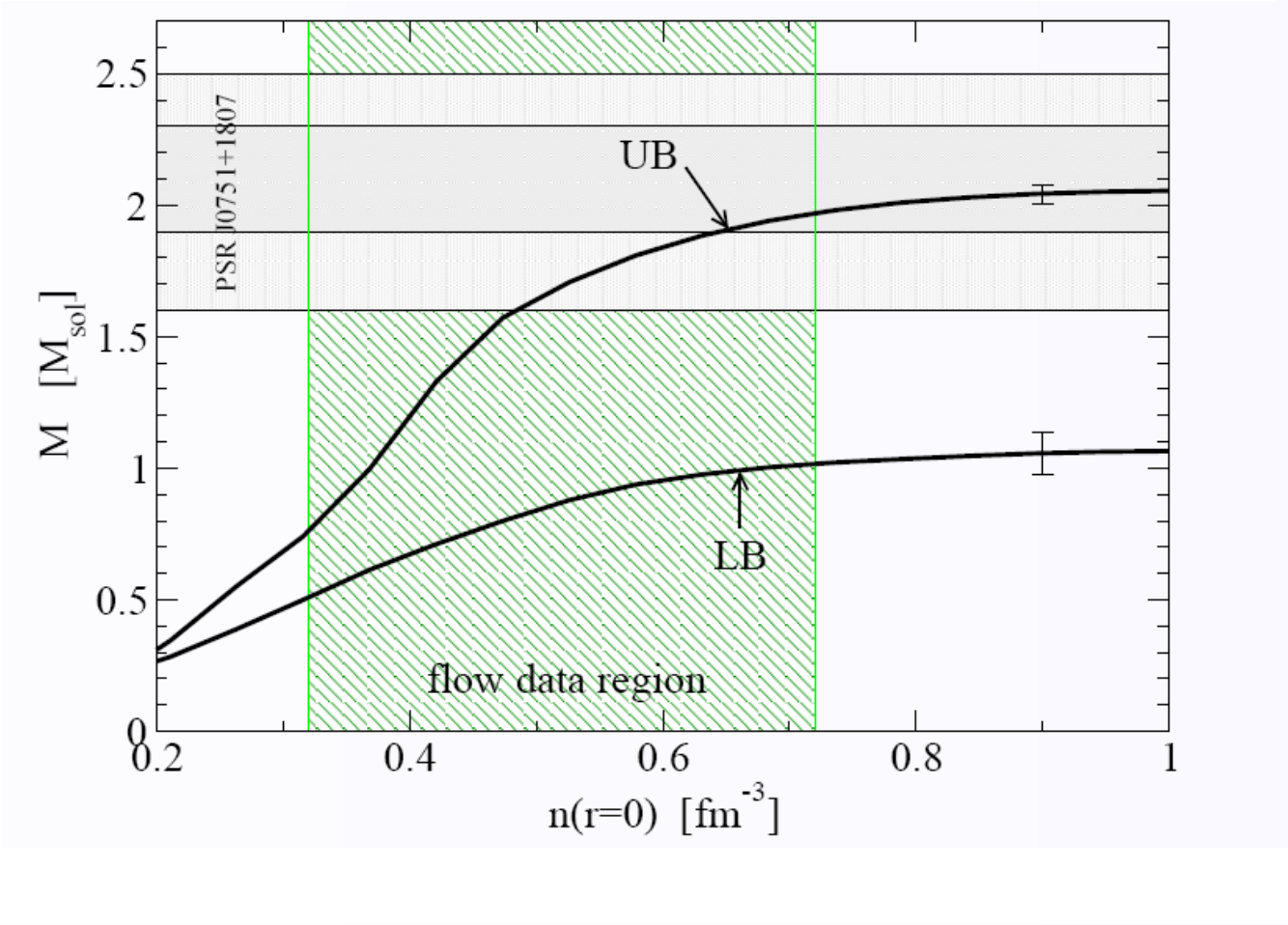

(applied "universal" $\beta^2 E_S$ (error bars!))

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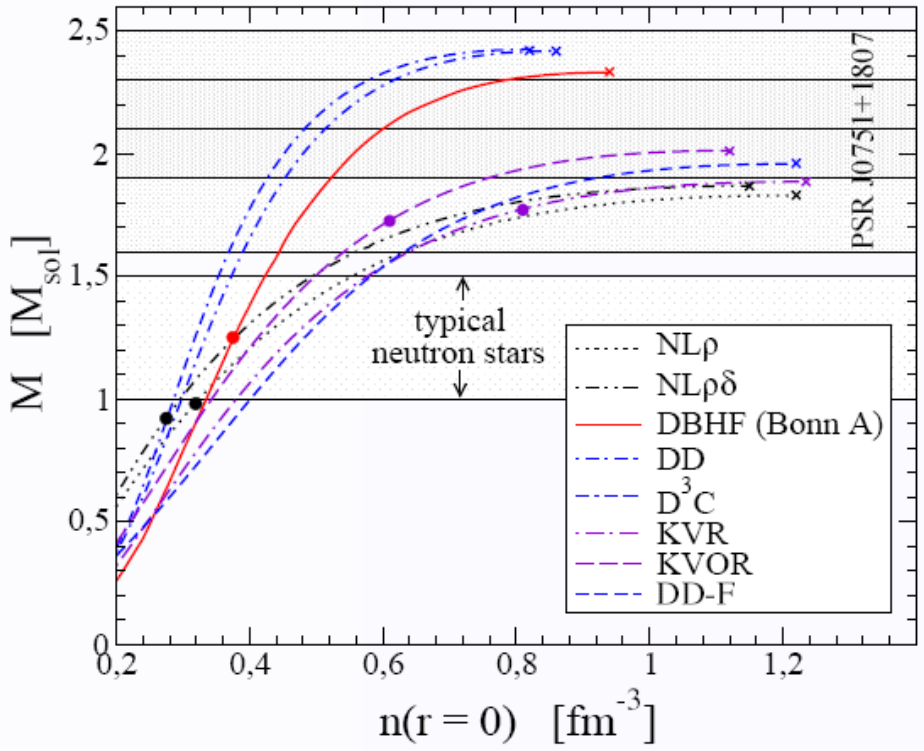

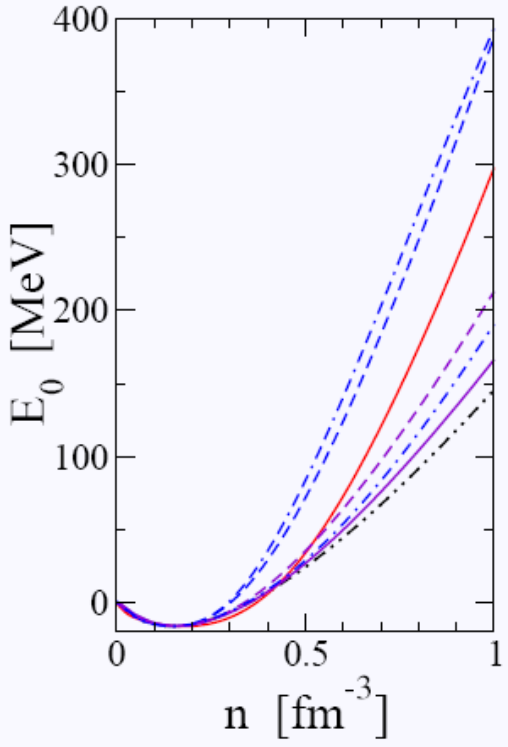

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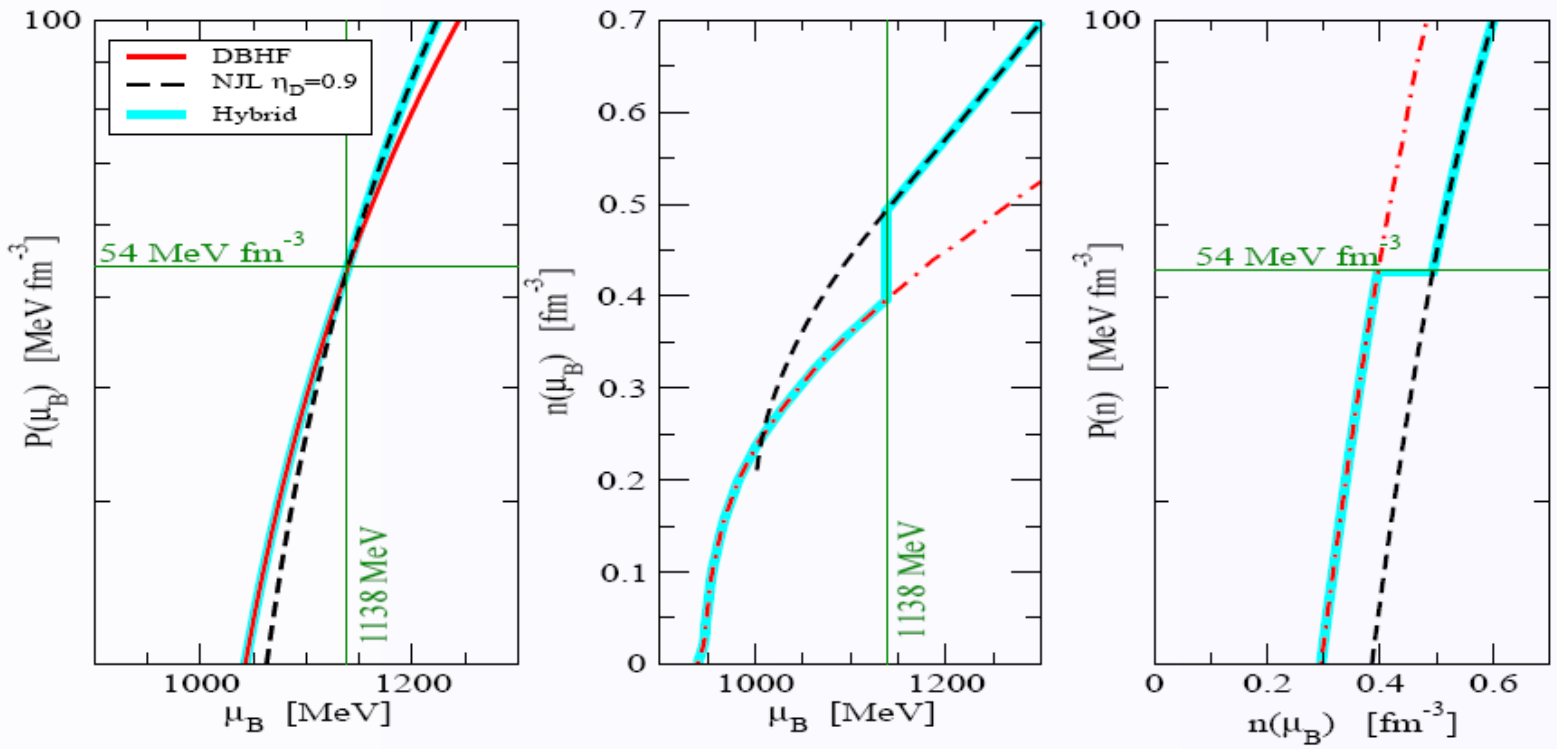
➛ traditional: two-phase construction



➛ "masquerade" problem: quark and hadron eos almost identical!

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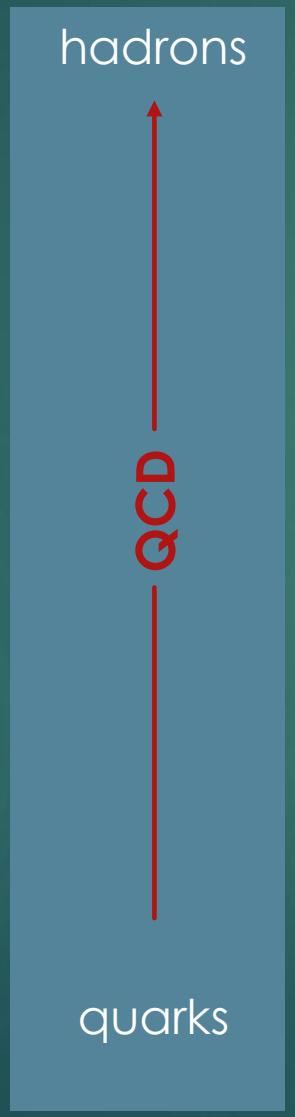
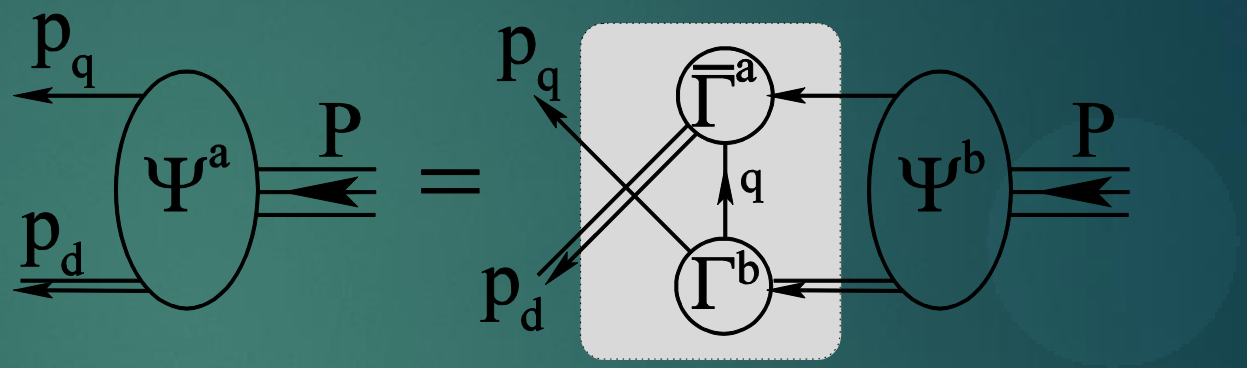


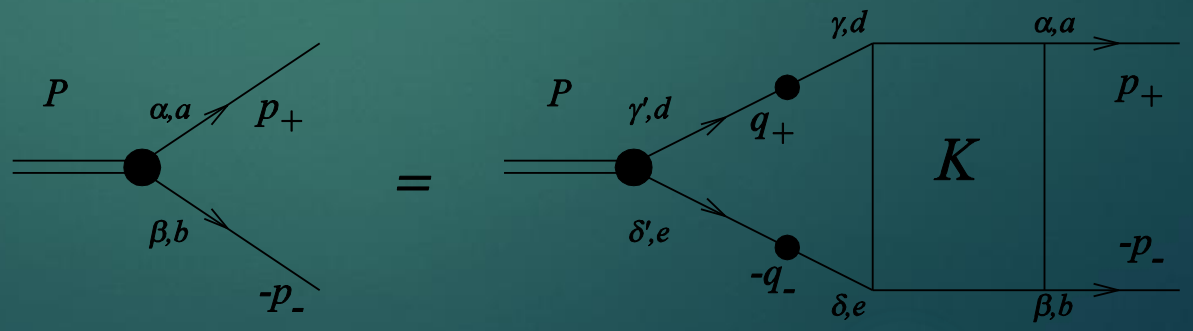
Dense Nuclear Matter in terms of Quark DoF is barely understood

Problem is attacked in vacuum **Faddeev Equations**



Baryons as composites of confined quarks and diquarks

Bethe Salpeter Equations



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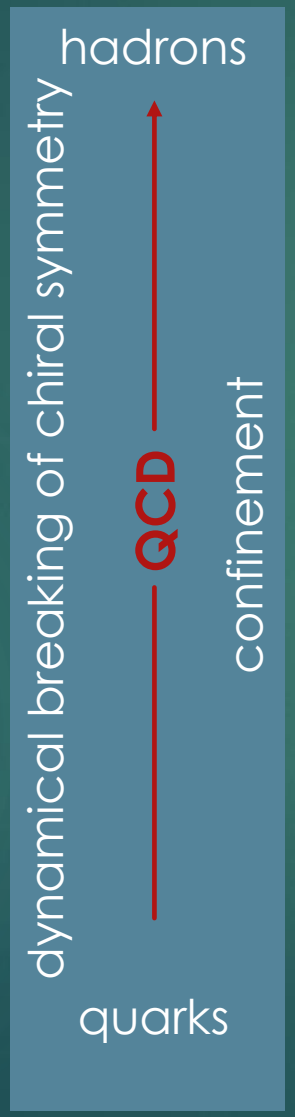

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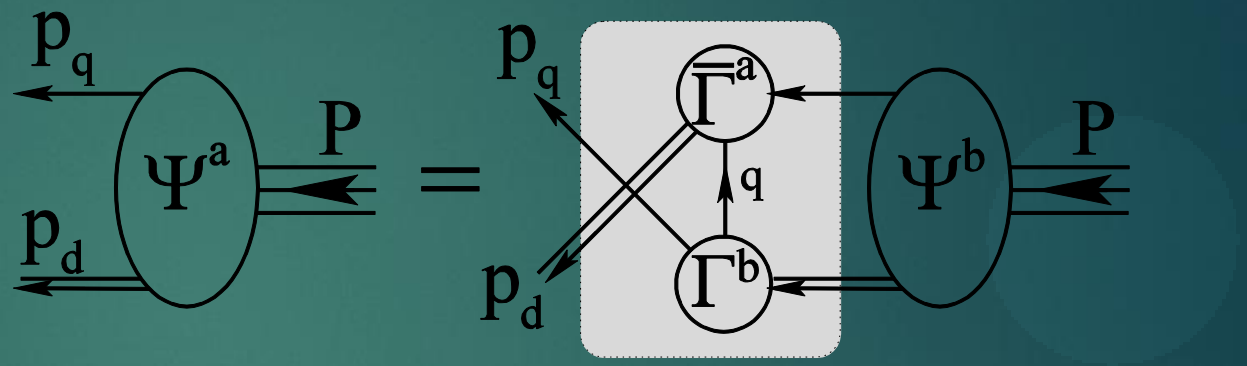

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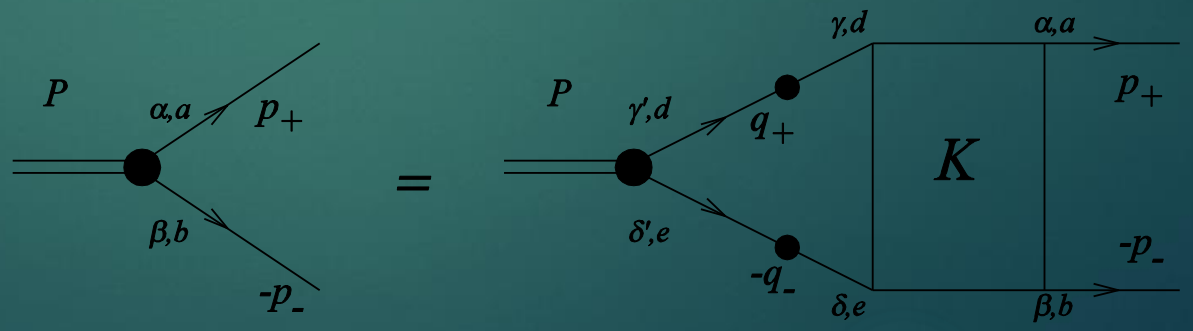


QCD in dense matter

- LQCD fails in dense (like DENSE) matter (Fermion-sign problem)
- Perturbative QCD fails in non-perturbative domain

  DCSB is explicitly not covered by perturbative approach:

$$B(p^2) = m\left(1 - \frac{\alpha_S}{\pi}\ln\left[\frac{p^2}{m^2}\right]\right) \qquad \lim_{m_0\to 0}\Sigma_S(p^2, m_0^2) = 0$$

- Solution: 'some' non-perturbative approach 'as close as possible' to QCD

  some = solvable; as close as possible = if possible DCSB, if possible confinement
- State of the art: Nambu-Jona-Lasinio model(s) (+bag models, +hybrids)

NJL type models

- S: DCSB
- V: renormalizes μ
- D: diquarks → 2SC, CFL
- TD Potential minimized in mean-field approximation
- Effective model by its nature; can be motivated (1g-exchange) doesn't have to though and can be extended (KMT, PNJL)
- possible to describe nucleons; not to be confused with confinement!

Effective Lagrangian

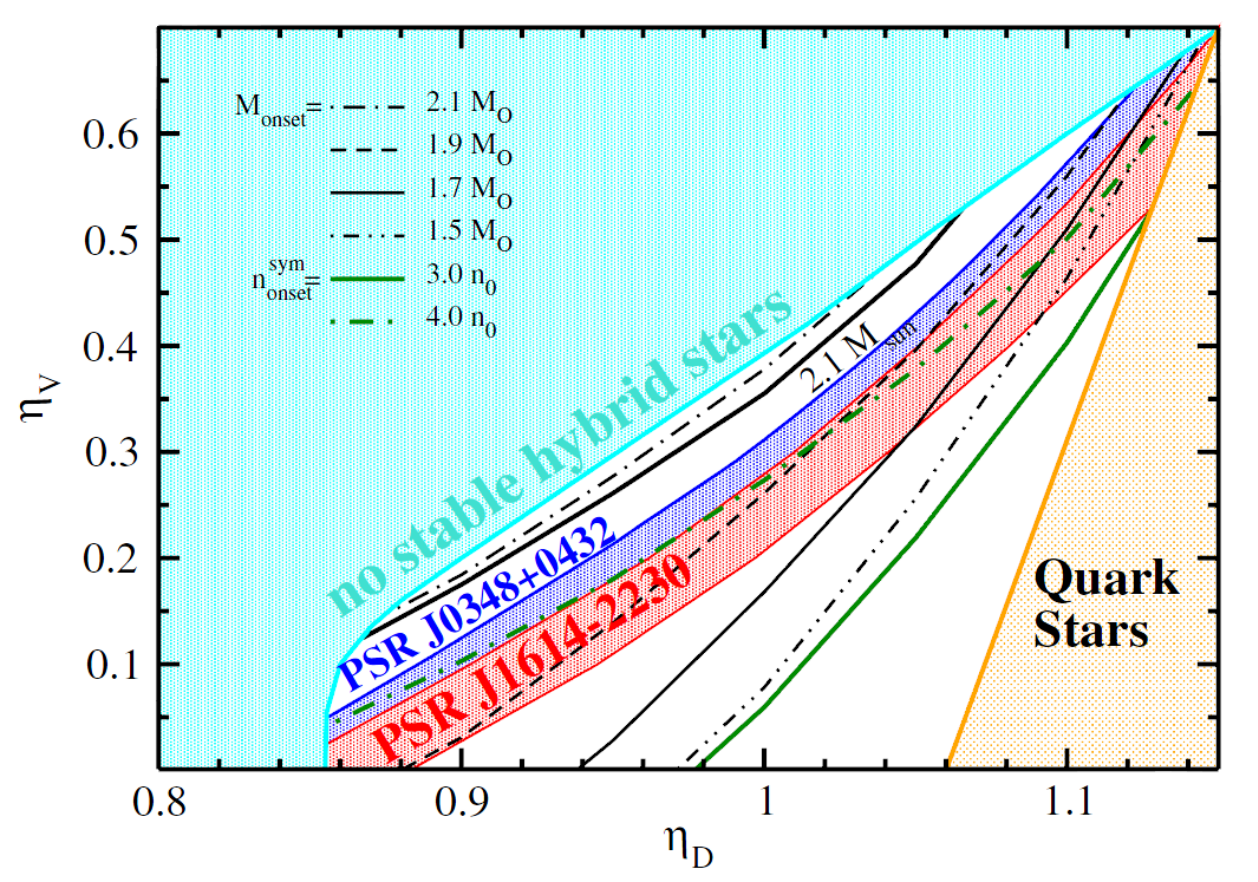
$$\mathcal{L}_{int} = G_S \eta_D \sum_{a,b=2,5,7} (\bar{q} i\gamma_5 \tau_a \lambda_b C \bar{q}^T)(q^T C i\gamma_5 \tau_a \lambda_a q) + G_S \sum_{a=0}^{8} \left[(\bar{q}\tau_a q)^2 + \eta_V (\bar{q} i\gamma_0 q)^2\right]$$

Thermodynamical potential

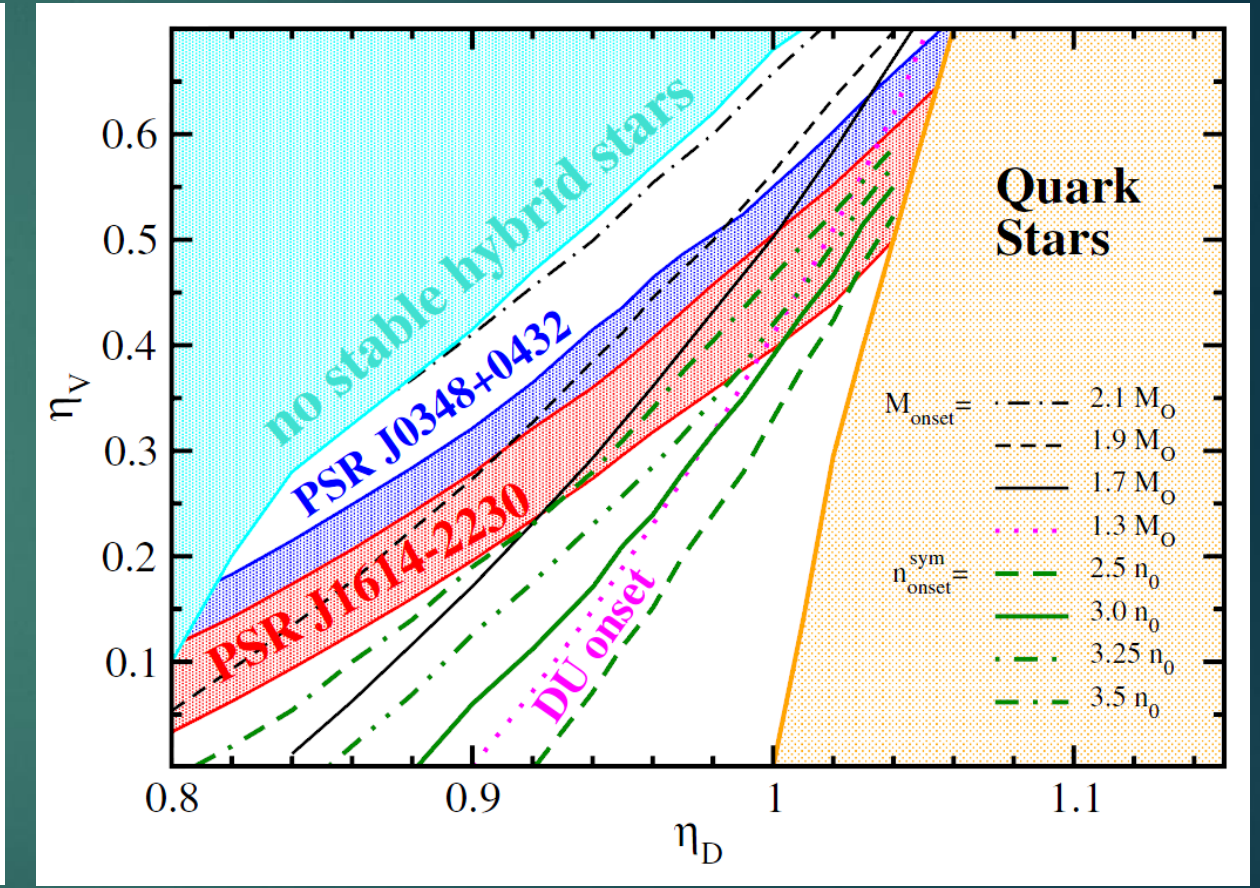
$$\Omega(T,\mu) = \frac{\phi_u^2 + \phi_d^2 + \phi_s^2}{8G_S} + \frac{\Delta_{ud}^2 + \Delta_{us}^2 + \Delta_{ds}^2}{4G_D} - \int \frac{d^3p}{(2\pi)^3} \sum_{n=1}^{18} \left[E_n + 2T ln\left(1 + e^{-E_n/T}\right)\right] + \Omega_{lep} - \Omega_0$$

NJL model study for NS

(TK, R.Łastowiecki, D.Blaschke, PRD **88**, 085001 (2013))



Set A



Set B

Conclusion: NS may or may not support a significant QM core.
Other interaction channels won't change this if their coupling strengths are not precisely known.

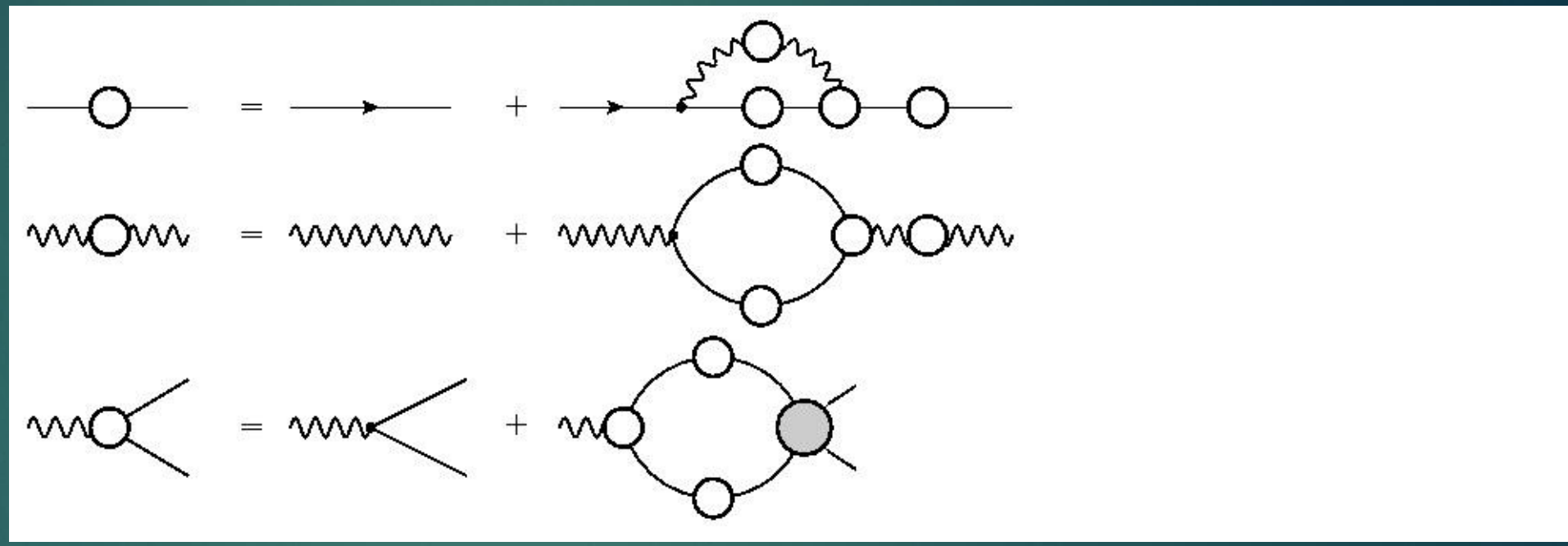
Beyond NJL

- NJL model can be understood as an approximate solution of Dyson-Schwinger equations

quark

gluon

q-g-vertex



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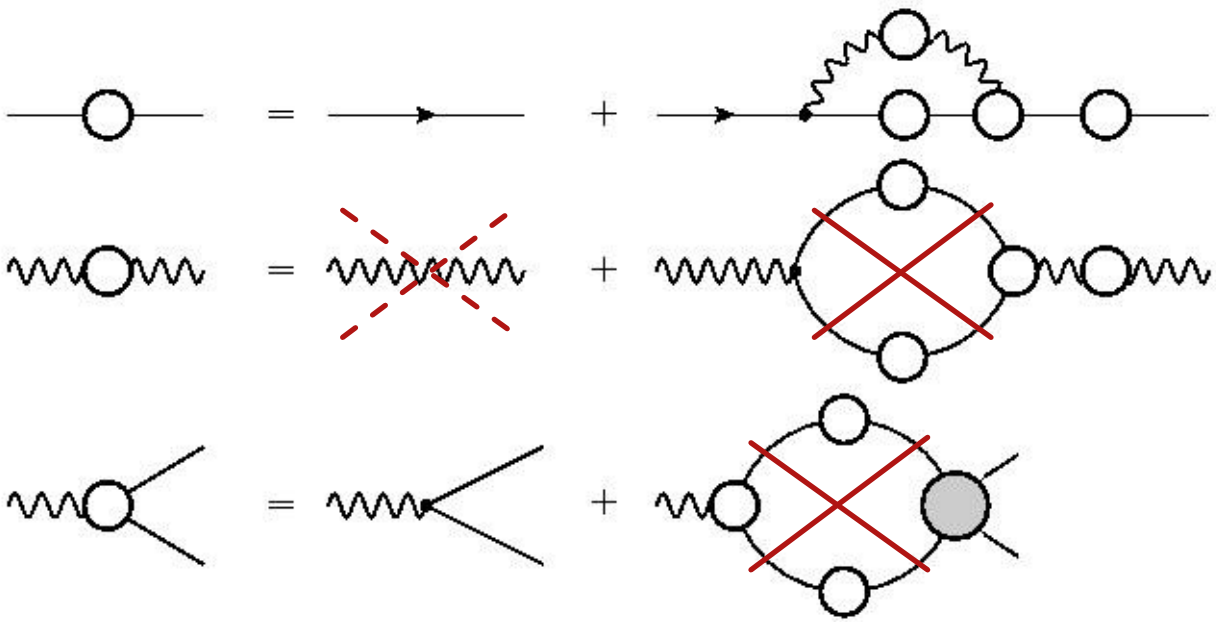

$$g^2 D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \frac{\mathcal{G}(k^2)}{k^2}$$

$$\Gamma^a_\mu(k,p)_{\text{bare}} = \gamma_\mu \frac{\lambda^a}{2}$$

single particle: quark self energy

Inverse Quark Propagator:

$$S(p;\mu)^{-1} = Z_2(\underbrace{i\,\vec{\gamma}\,\vec{p} + i\,\gamma_4(p_4 + i\mu)}_{=i\,\gamma\,p} + m_{\mathrm{bm}}) + \Sigma(p;\mu)$$

revokes Poincaré covariance

Renormalised Self Energy:

$$\Sigma(p;\mu) = Z_1 \int_q^{\Lambda} g^2(\mu) D_{\rho\sigma}(p-q;\mu) \frac{\lambda^a}{2} \gamma_\rho S(q;\mu) \Gamma^a_\sigma(q,p;\mu)$$

Loss of Poincaré covariance increases complexity

→ technically and numerically more challenging → no surprise, though

General Solution:

Vacuum: $\boxed{\mu = 0}$ $\quad S(p^2)^{-1} = i\,\gamma\,p\,A(p^2) + B(p^2)$

Medium: $\boxed{\mu \neq 0}$ $\quad S(p^2, p_4;\mu)^{-1} = i\,\vec{\gamma}\,\vec{p}\,A(p^2, p_4, \mu) + i\,\gamma_4(p_4 + i\mu)\,C(p^2, p_4, \mu) + B(p^2, p_4, \mu)$

Similar structured equations in vacuum and medium, but in medium:

1. one more gap
2. gaps are complex valued
3. gaps depend on (4-)momentum, energy and chemical potential

Effective gluon propagator

$$S(p;\mu)^{-1} = Z_2(i\,\vec{\gamma}\,\vec{p} + i\,\gamma_4(p_4 + i\mu) + m_{\mathrm{bm}}) + \Sigma(p;\mu)$$

$$\Sigma(p;\mu) = Z_1 \int_q^{\Lambda} g^2(\mu) D_{\rho\sigma}(p-q;\mu) \frac{\lambda^a}{2} \gamma_\rho S(q;\mu) \Gamma^a_\sigma(q,p;\mu)$$

Ansatz for self energy (rainbow approximation, effective gluon propagator(s))

$$Z_1 \int_q^{\Lambda} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma^a_\nu(q,p) \to \int_q^{\Lambda} \mathcal{G}((p-q)^2)\, D^{\mathrm{free}}_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \frac{\lambda^a}{2} \gamma_\nu$$

Specify behaviour $\mathcal{G}(k^2)$

$$\frac{\mathcal{G}(k^2)}{k^2} = 8\pi^4 D \delta^4(k) + \frac{4\pi^2}{\omega^6} D k^2 \mathrm{e}^{-k^2/\omega^2} + 4\pi \frac{\gamma_m \pi}{\frac{1}{2}\ln\left[\tau + \left(1 + k^2/\Lambda^2_{\mathrm{QCD}}\right)^2\right]} \mathcal{F}(k^2)$$

Infrared strength (zero width + finite width contribution)

running coupling for large k

Results at finite densities obtained for

1st term (Munczek/Nemirowsky (1983)) → Klähn et al. (2010)

2nd term → Chen et al.(2008,2011)

NJL model: $g^2 D_{\rho\sigma}(p-q) = \frac{1}{m_G^2} \delta_{\rho\sigma}$ delta function in configuration(!) space

NJL model within DS framework

$$
\begin{aligned}
B(p) &= m + \frac{16}{3m_G^2}\int \frac{d^4q}{(2\pi)^4}\frac{B(q)}{\vec{q}^2A^2(q)+\tilde{q}_4^2C^2(q)+B^2(q)},\\
\vec{p}^2A(p) &= \vec{p}^2 + \frac{8}{3m_G^2}\int \frac{d^4q}{(2\pi)^4}\frac{\vec{p}\vec{q}A(q)}{\vec{q}^2A^2(q)+\tilde{q}_4^2C^2(q)+B^2(q)},\\
\tilde{p}_4^2C(p) &= \tilde{p}_4^2 + \frac{8}{3m_G^2}\int \frac{d^4q}{(2\pi)^4}\frac{\tilde{p}_4\tilde{q}_4C(q)}{\vec{q}^2A^2(q)+\tilde{q}_4^2C^2(q)+B^2(q)}.
\end{aligned}
$$

To satisfy these equations all gap solutions have to be momentum independent. Simplest solution: A=1

$$
\frac{8}{3m_G^2}\int \frac{d^4q}{(2\pi)^4}\frac{\tilde{q}_4C(q)}{\vec{q}^2+\tilde{q}_4^2C^2(q)+B^2(q)} = iK
$$

$$
\begin{aligned}
\tilde{p}_4^2C &= \tilde{p}_4^2 + i\tilde{p}_4K\\
\Rightarrow \tilde{p}_4C &= p_4 + i(\mu+K)
\end{aligned}
$$

Renormalization of chem. pot. due to vector interaction

$$
B = m + \frac{16}{3m_G^2}\int \frac{d^4q}{(2\pi)^4}\frac{B}{\vec{q}^2+\hat{q}_4^2+B^2}
$$

mass gap equation

This is a 1 to 1 reproduction of the (basic) NJL model

NJL model within DS framework

$$\frac{8}{3m_G^2}\int \frac{d^4q}{(2\pi)^4}\frac{\widetilde{q}_4 C(q)}{\vec{q}^2+\widetilde{q}_4^2 C^2(q)+B^2(q)} = iK$$

$$\begin{aligned} \widetilde{p}_4^2 C &= \widetilde{p}_4^2 + i\widetilde{p}_4 K \\ \Rightarrow \widetilde{p}_4 C &= p_4 + i(\mu + K) \end{aligned}$$

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NJL model within DS framework

$$P[S] = \mathrm{Tr}\ln[S^{-1}] - \frac{1}{2}\mathrm{Tr}[\Sigma S]$$

Steepest descent approximation

$$P(\mu) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4}\,\mathrm{Tr}\ln S^{-1}(\vec{p}^2, \hat{p}_4) + \frac{3}{4}m_G^2 K^2 - \frac{3}{8}m_G^2 B^2$$

1 to 1 NJL (regularization issue ignored)

$$\frac{8}{3m_G^2}\int \frac{d^4q}{(2\pi)^4}\frac{\tilde{q}_4 C(q)}{\vec{q}^2 + \tilde{q}_4^2 C^2(q) + B^2(q)} = iK$$

$$\begin{aligned} \tilde{p}_4^2 C &= \tilde{p}_4^2 + i\tilde{p}_4 K \\ \Rightarrow \tilde{p}_4 C &= p_4 + i(\mu + K) \end{aligned}$$

Renormalization of chem. pot. due to vector interaction

$$B = m + \frac{16}{3m_G^2}\int \frac{d^4q}{(2\pi)^4}\frac{B}{\vec{q}^2 + \hat{q}_4^2 + B^2}$$

mass gap equation

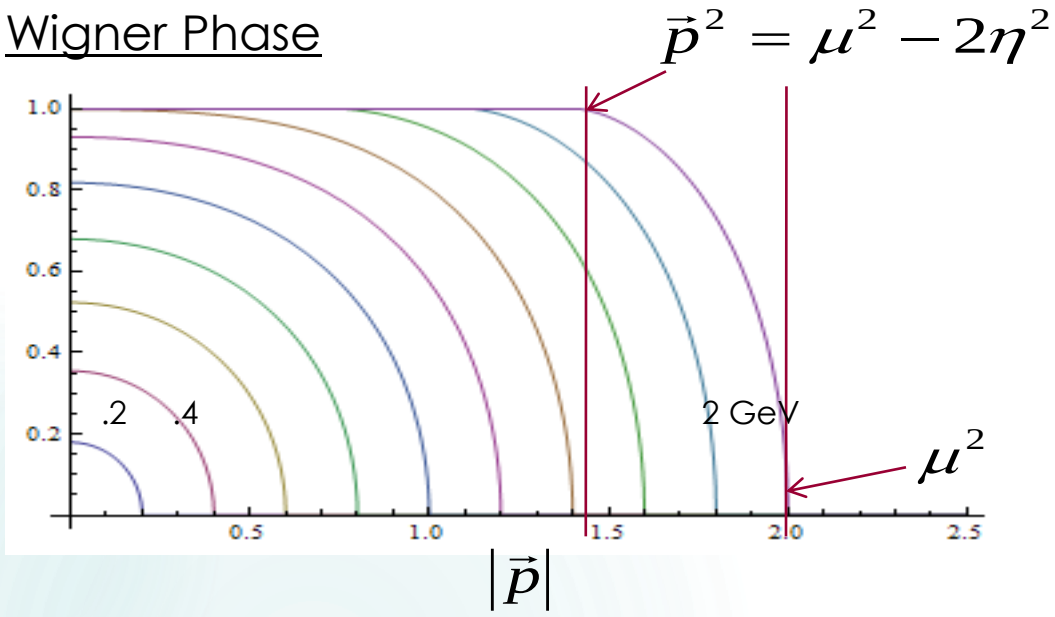
This is a 1 to 1 reproduction of the (basic) NJL model

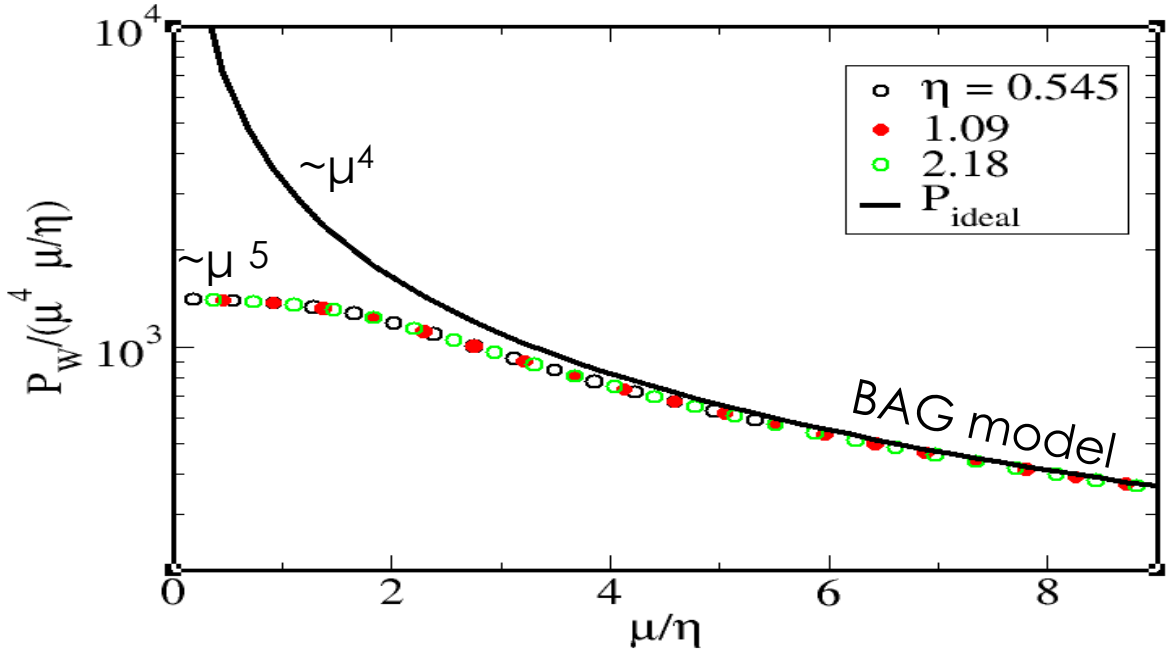
Model 1 (Munczek/Nemirowsky)

$$f_1(|\vec{p}|;\mu) = \frac{1}{4\pi}\int_{-\infty}^{\infty} dp_4\, \mathrm{tr_D}(-\gamma_4) S(p;\mu)$$

$$P(\mu<\eta) = P_0 + \int_0^{\mu} d\mu'\, n(\mu') \propto P_0 + const \times \mu^5$$

Wigner Phase





$\mu^2 \geq 2\eta^2$ to obtain $f_1(\vec{p}^2 = 0) = 1$ model is scale invariant regarding μ/η

$P(\mu) \propto \mu^5$ well satisfied up to $\mu/\eta \approx 1$

$(\eta = 1.09\,\mathrm{GeV})$

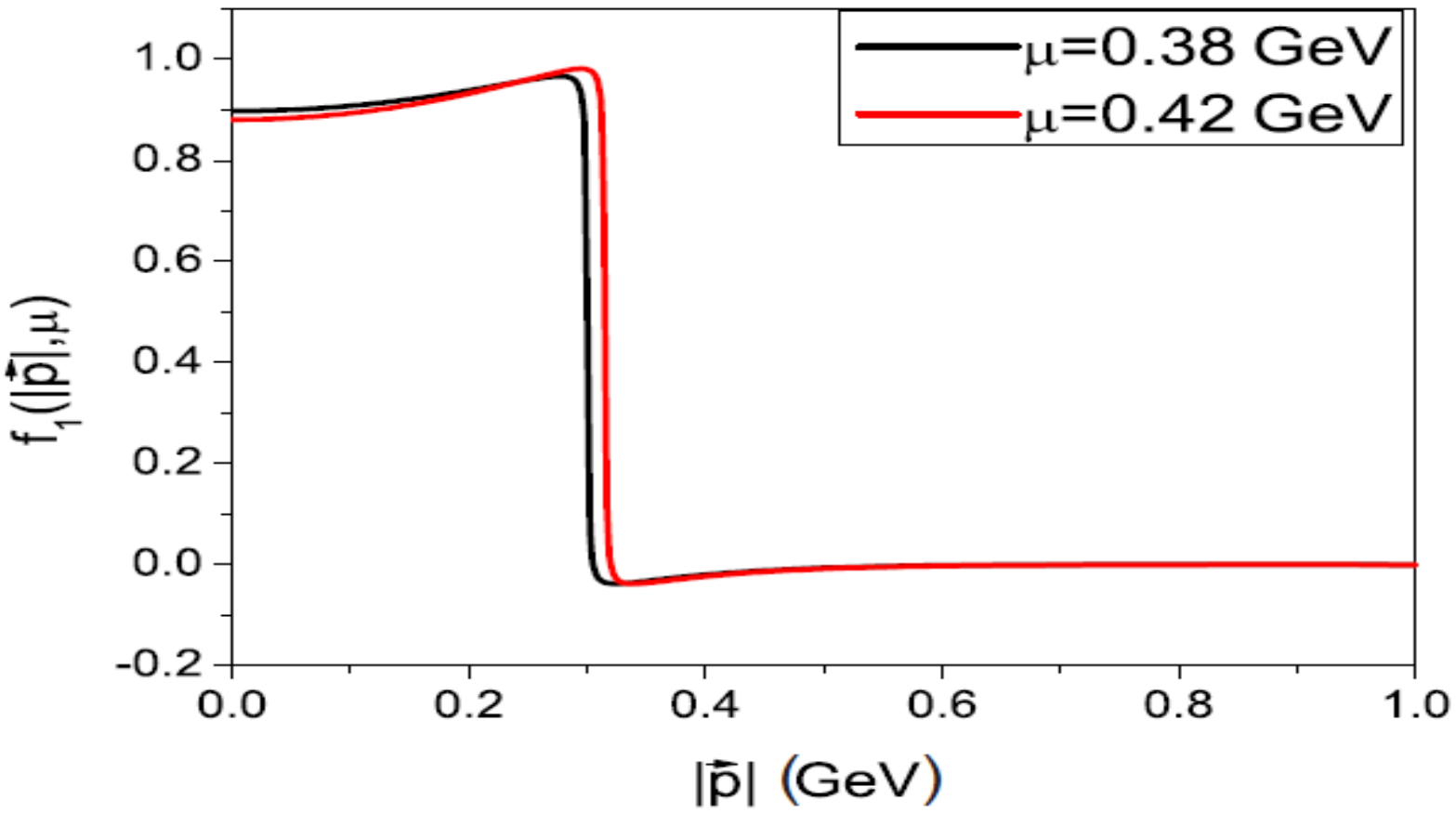
‚small' chem. Potential: $f_1(\vec{p}^2 = 0, \mu<\eta) \propto \mu \leftarrow$ $n(\mu<\eta) = \frac{2N_c N_f}{2\pi^2}\int d^3\vec{p}\; f_1(|\vec{p}|) \propto \mu^4$

T. Klahn, C.D. Roberts, L. Chang, H. Chen, Y.-X. Liu PRC 82, 035801 (2010)

Model 2

$$f_1(|\vec{p}|;\mu) = \frac{1}{4\pi}\int_{-\infty}^{\infty} dp_4\, \mathrm{tr}_D(-\gamma_4)S(p;\mu)$$

<u>Wigner Phase</u> Less extreme, but again, 1particle number density distribution different from free Fermi gas distribution



Chen et al. (TK) PRD 78 (2008)

Conclusions

NJL model is a powerful tool to explore possible features of dense QCD

It possibly might be a too powerful tool for unambiguous predictions

NJL mf approximation is a gluon mf approximation in DSE

NB: Momentum independent gap solutions in their very nature result in a quasi particle picture $\rightarrow$ no confinement

Accounting for momentum dependent gap solutions enriches the model space significantly

DS approach starts from QCD Lagrangian. Successful in vacuum.
Still plenty of open issues. -> Certainly a frontier area of in-medium physics.

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