Overview of pQCD calculations (some aspects)

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Today:

- NLO: some “successes and failures”
- Resummation
- Additional aspects

NLO: “successes and failures”

Hard-scattering reactions play central role in QCD:

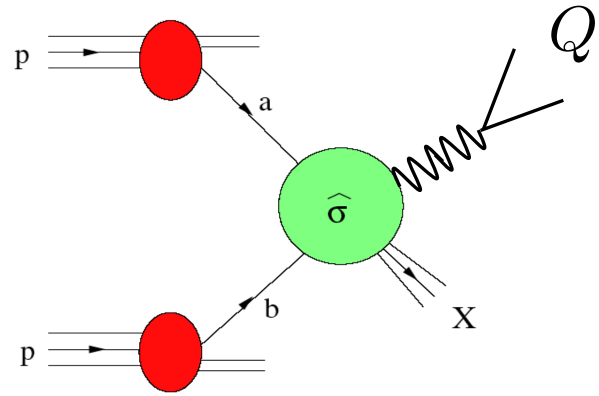
- Probes of nucleon structure

- Involved in most of today's Hadron Collider physics ("New Physics", Heavy Ions,…)

- Test our understanding of QCD at high energies, and our ability to do "first-principles" computations

Cornerstones: factorization & asymptotic freedom

Factorized cross section:
e.g. Drell-Yan



$$Q^4 \frac{d\sigma}{dQ^2} = \sum_{ab} \int dx_a dx_b \, f_a(x_a,\mu) \, f_b(x_b,\mu) \, \omega_{ab}\left(z = \frac{Q^2}{\hat{s}}, \alpha_s(\mu), \frac{Q}{\mu} \right) + \ldots$$

- $f_{a,b}$ parton distributions: non-pert., but universal
- ω_{ab} partonic cross sections: process-dep., but pQCD

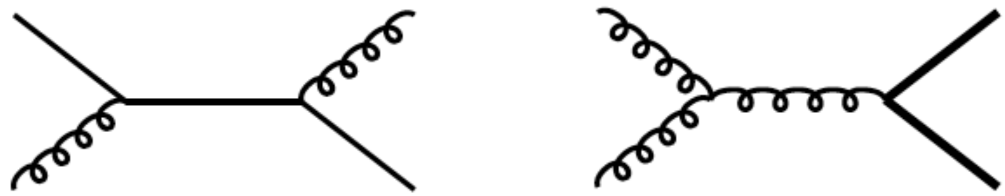
$$\omega_{ab} = \omega_{ab}^{(\mathrm{LO})} + \frac{\alpha_s}{2\pi} \omega_{ab}^{(\mathrm{NLO})} + \ldots$$

- $\mu \sim Q$ factorization / renormalization scale
- corrections power-suppressed in Q

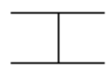
Efforts on perturbative calculations:

- fixed-order perturbation theory:
  LO, NLO, NNLO

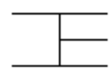
LO:



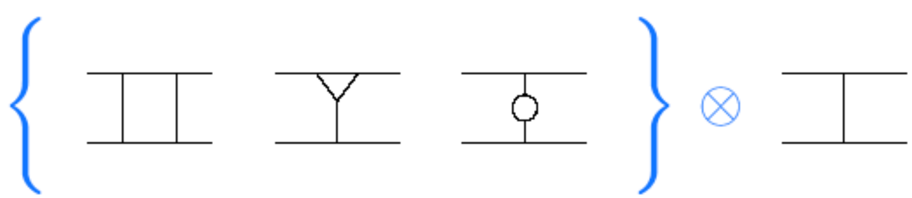
. . .

all $2 \to 2$  parton-parton scattering processes

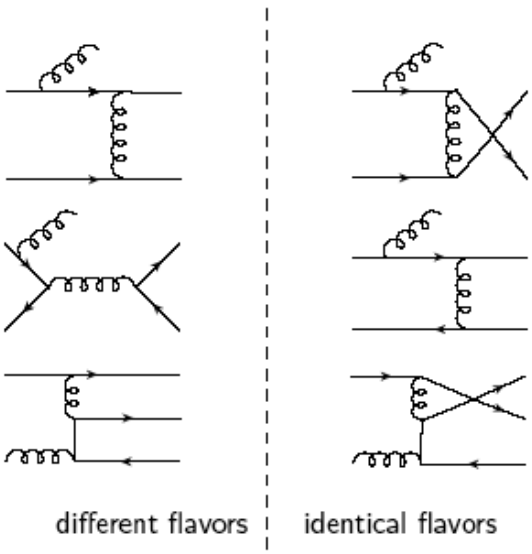
NLO:

(1) all $2 \to 3$  parton-parton scattering processes

(2) 1-loop (virtual) corrections to all LO processes,
interfering with Born process



some typical NLO $2 \to 3$ Feynman diagrams:

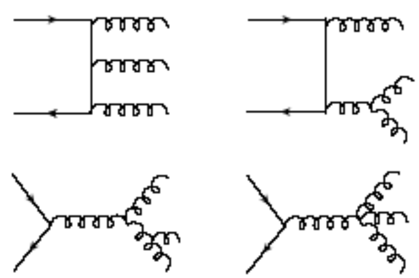


$q \neq q'$ $\quad$ $q = q'$

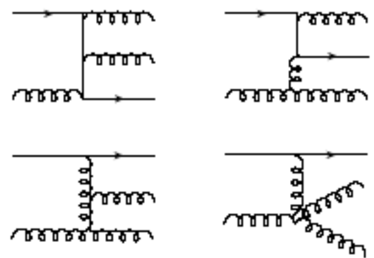
$qq' \to qq'g$

$q\bar{q} \to q'\bar{q}'g$

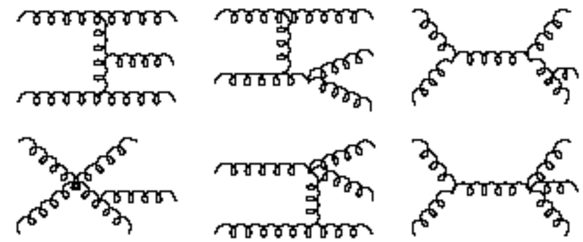
$qg \to qq'\bar{q}'$



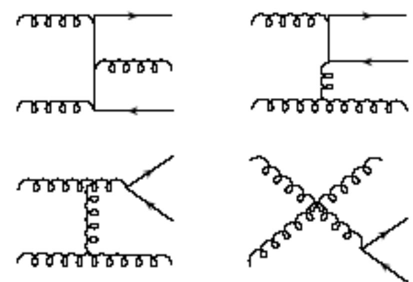
$q\bar{q} \to ggg$



$qg \to qgg$



$gg \to ggg$



$gg \to q\bar{q}g$

- all contributions individually singular $\Rightarrow$ choose $d = 4 - 2\varepsilon$ dimensions
- singularities then occur as poles in $1/\varepsilon$

UV $1/\varepsilon$*-singularities*

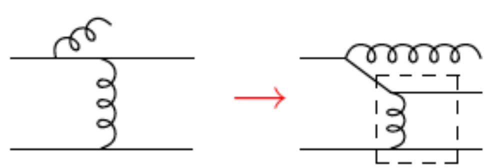
removed by renormalization of $\alpha_s(\mu)$

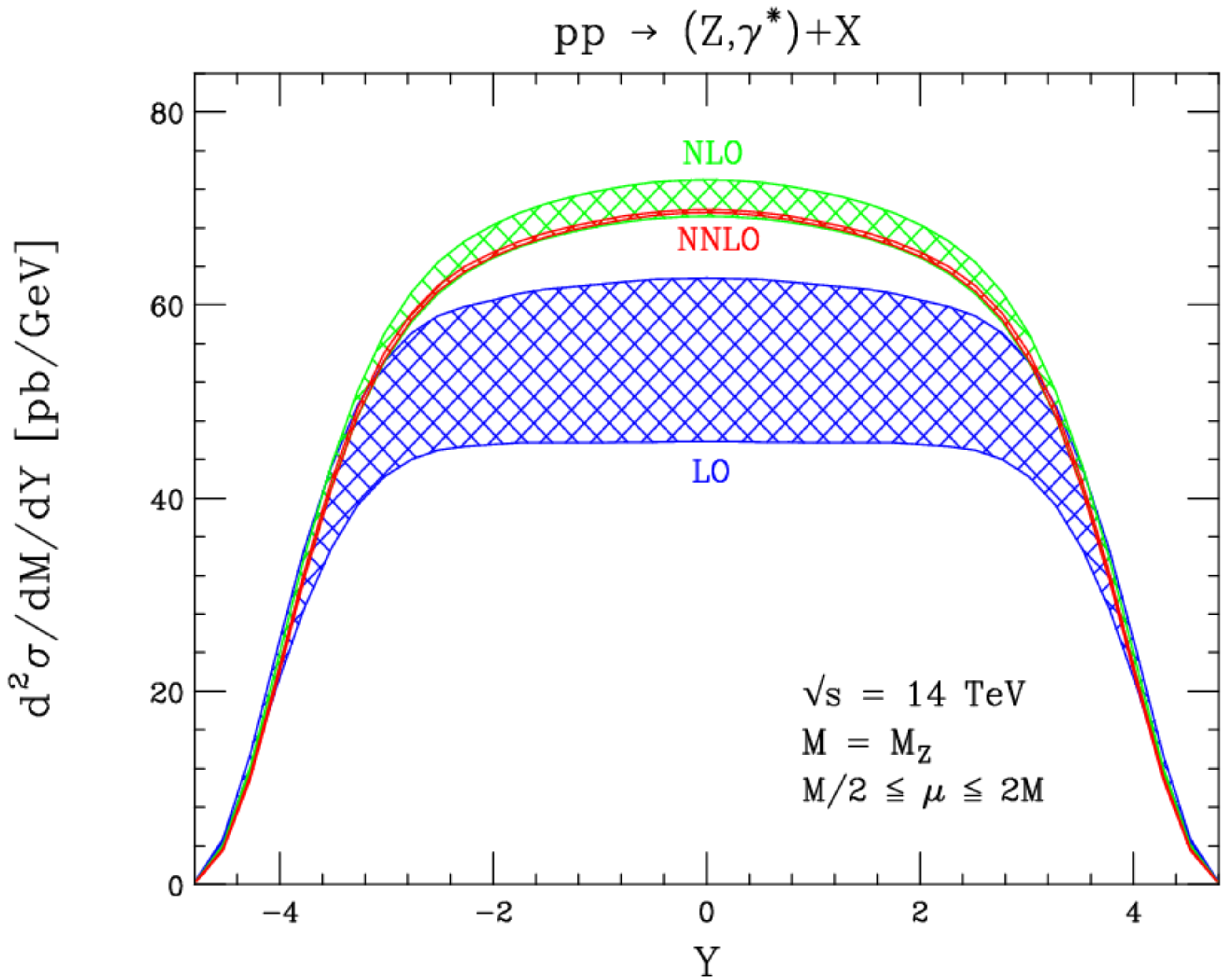
IR singularities ($1/\varepsilon^2$, $1/\varepsilon$)

cancel in sum of 1-loop and $2 \to 3$ contributions

collinear $1/\varepsilon$*-singularities*

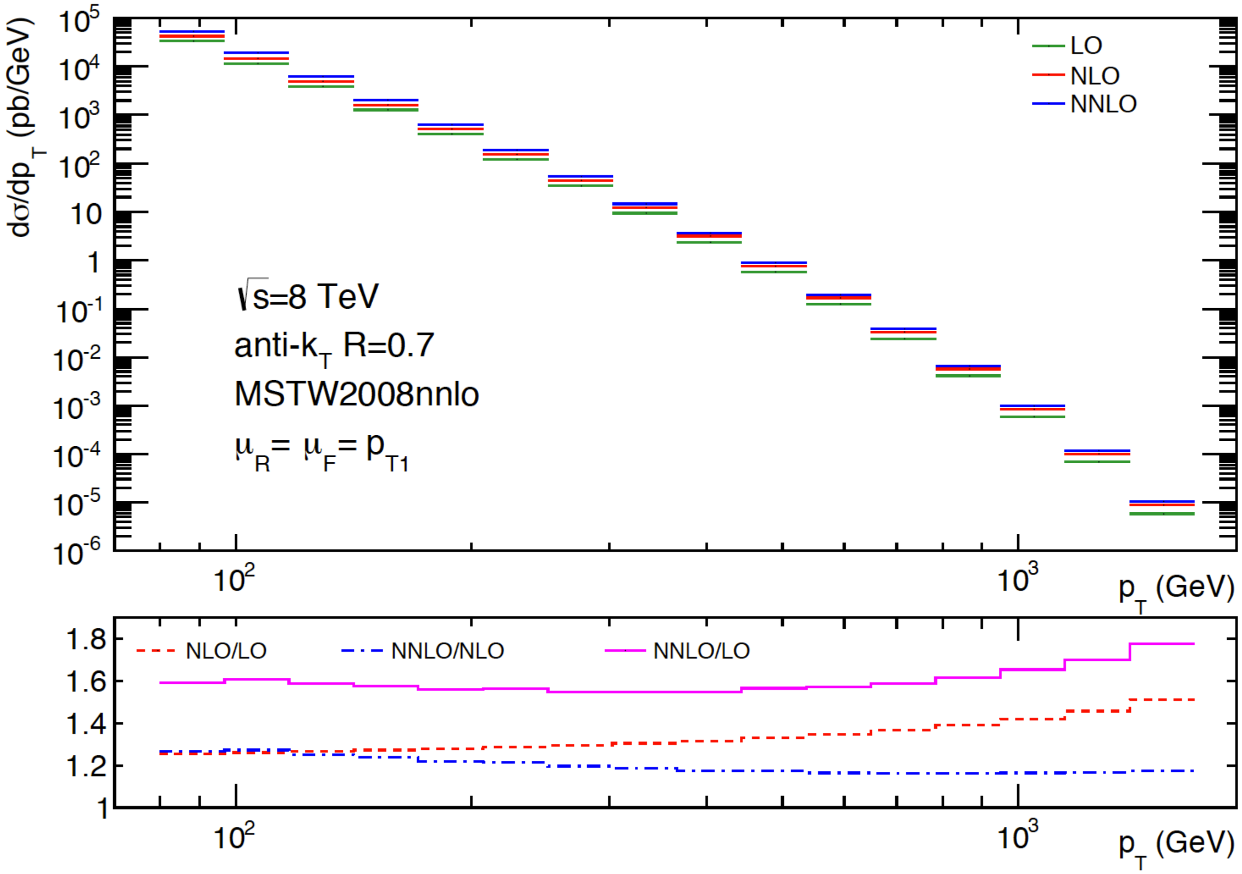
removed by factorization $\Rightarrow$ factorization scale μ

e.g.:  $\sim \frac{1}{\varepsilon} \int dx \Delta P_{qq}(x) \Delta\hat{\sigma}_{qq\to qq}$

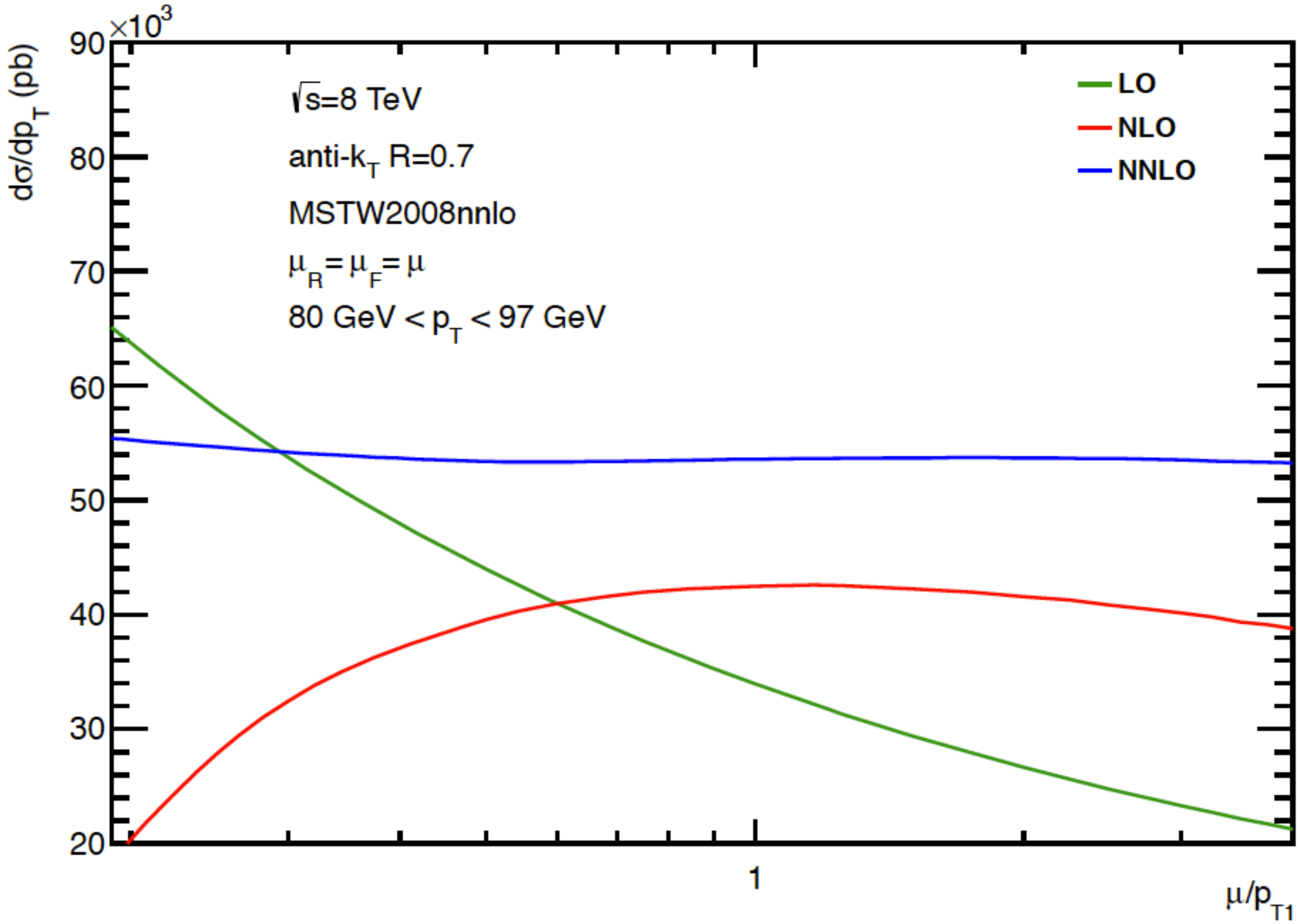


Anastasiou et al.

- NNLO corrections in all-gluon channel:

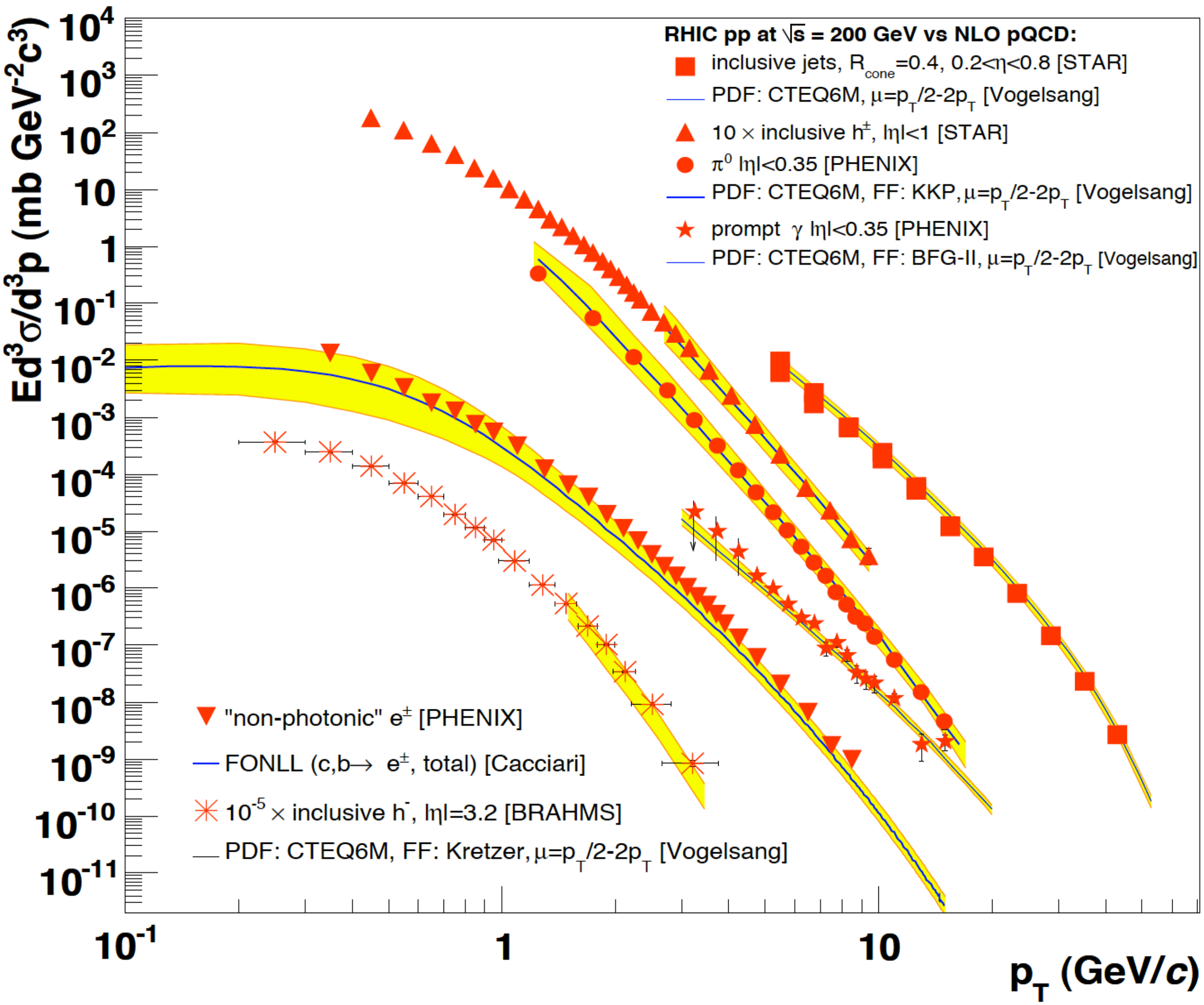


Currie, Gehrmann-De Ridder, Glover, Pires, arXiv:1310.3993

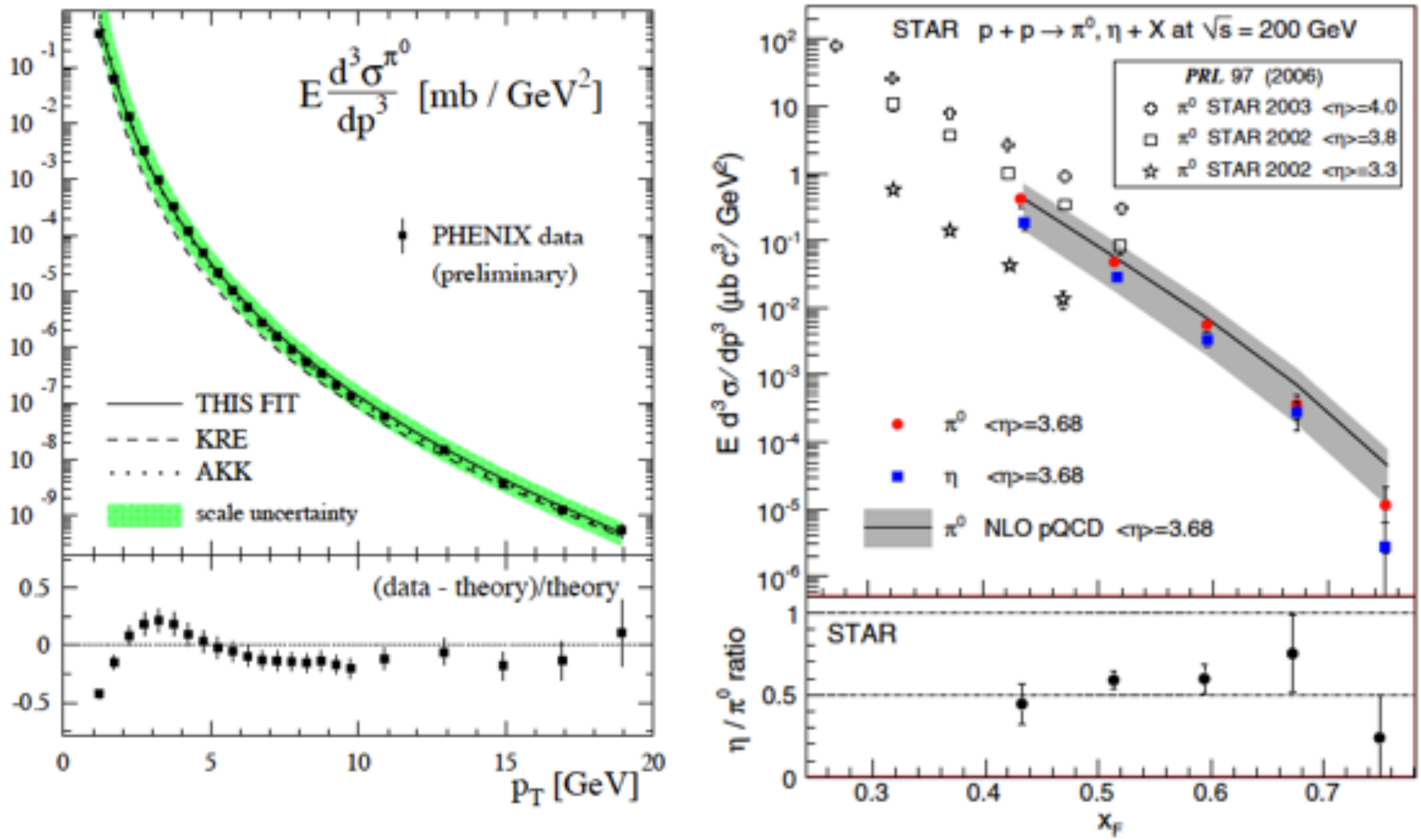


Gehrmann-De Ridder, Gehrmann, Glover, Pires, arXiv:1301.7310

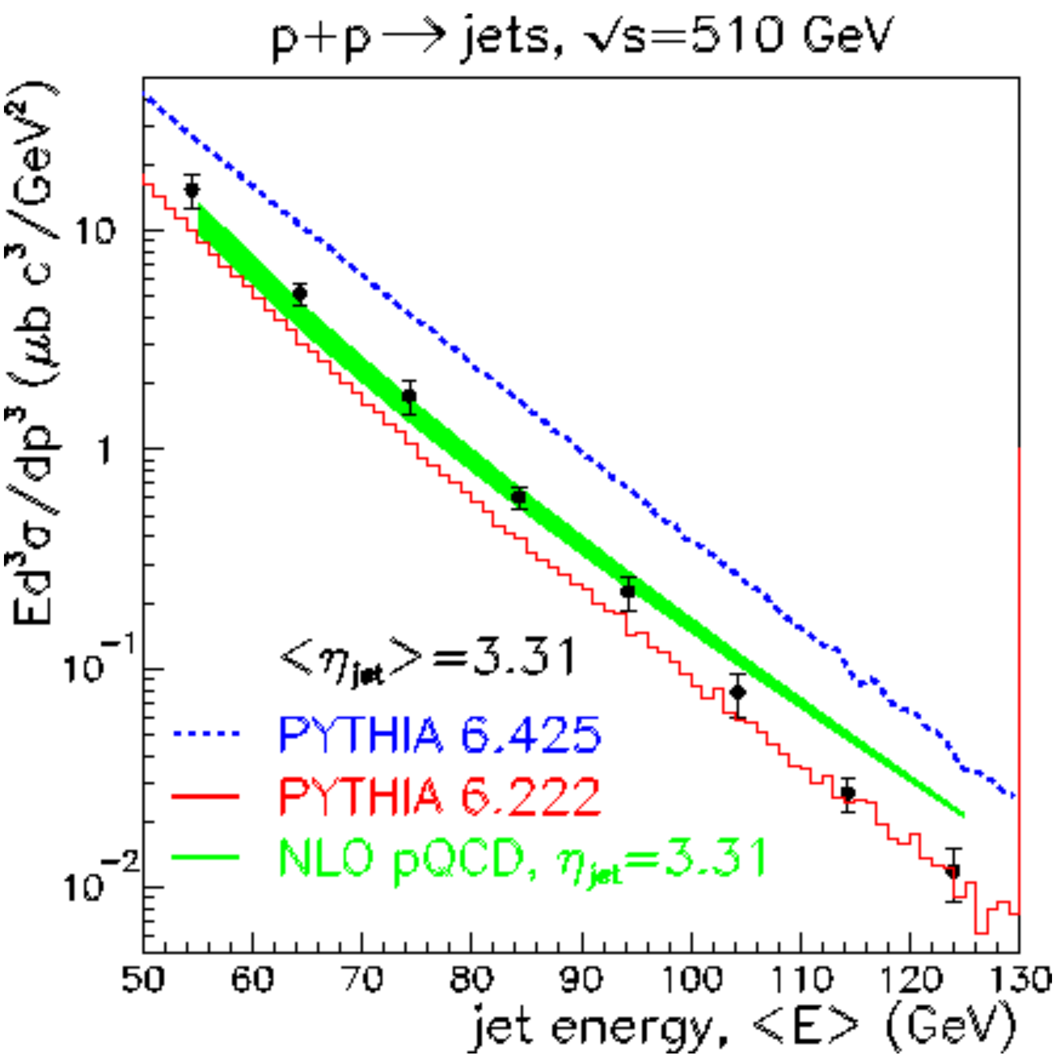
- Some successes:



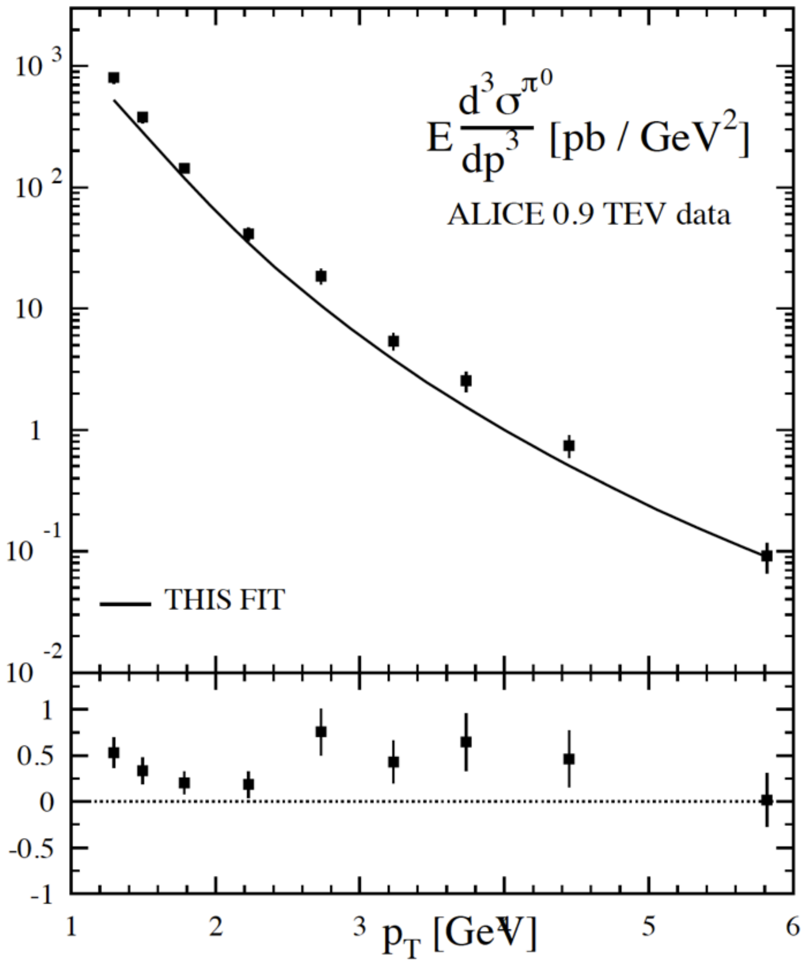
- More recently: (courtesy M. Stratmann → DSS)

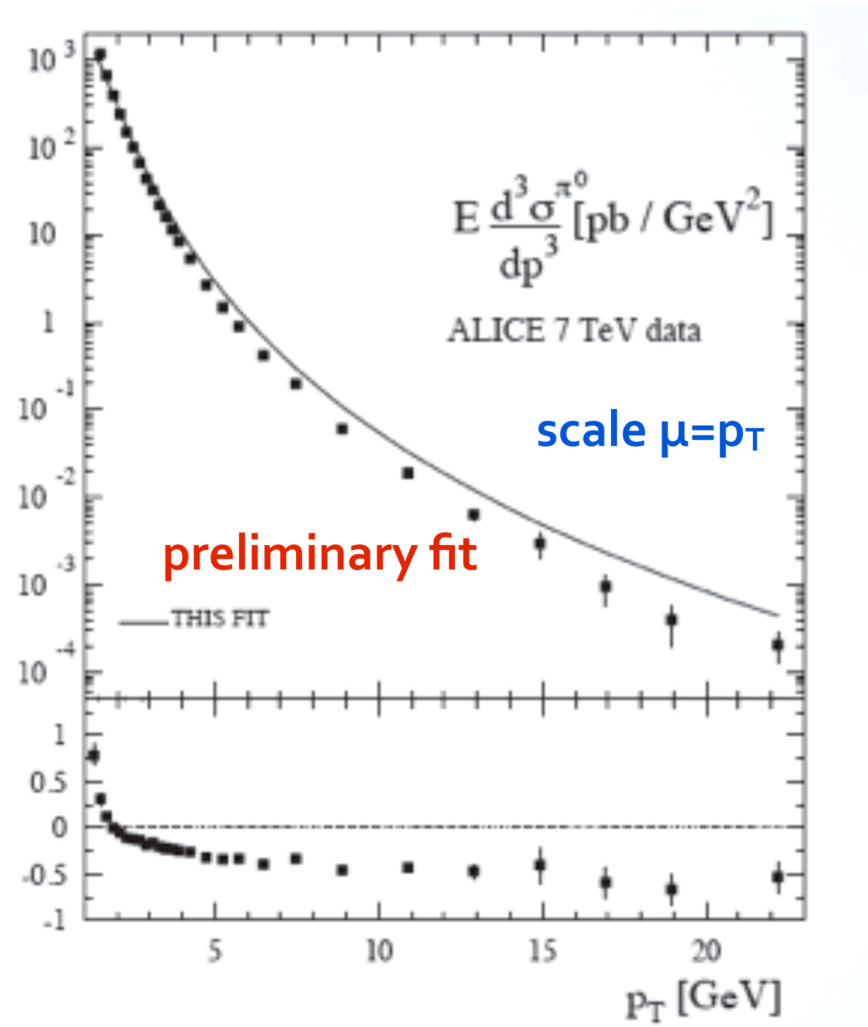


- AnDY experiment (BNL), L. Bland et al.



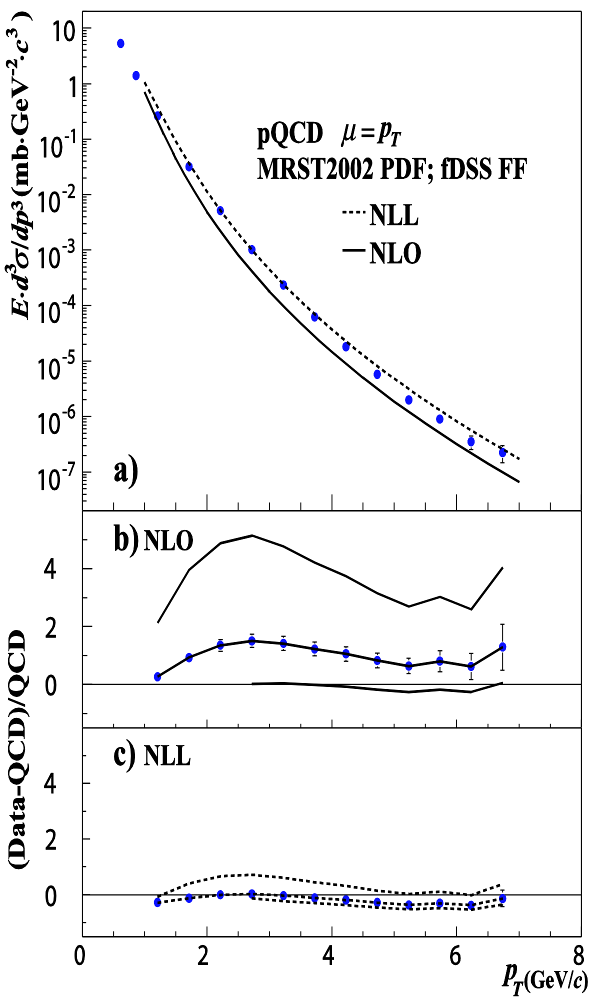
- **7 TeV data very low** - hard (impossible?) to fit with *all* other pion data



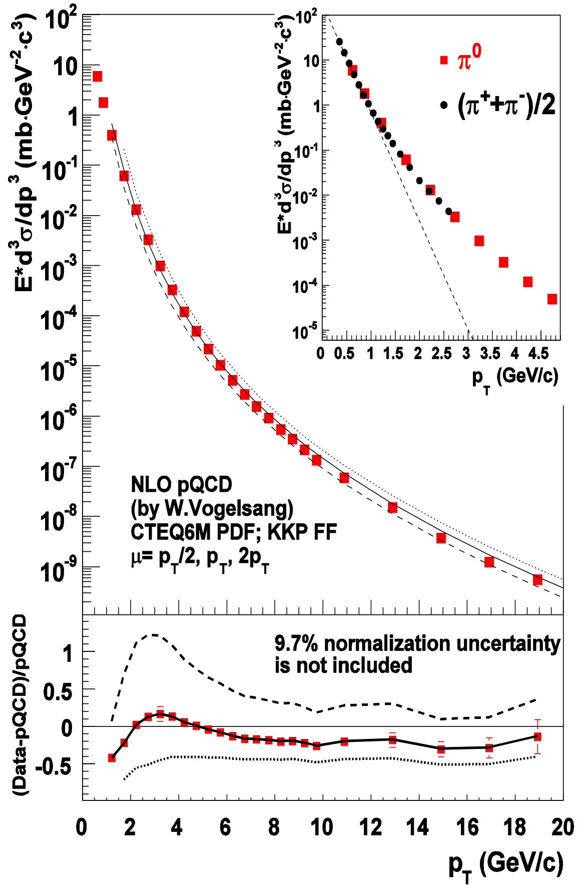


de Florian, Stratmann, Sassot (DSS)

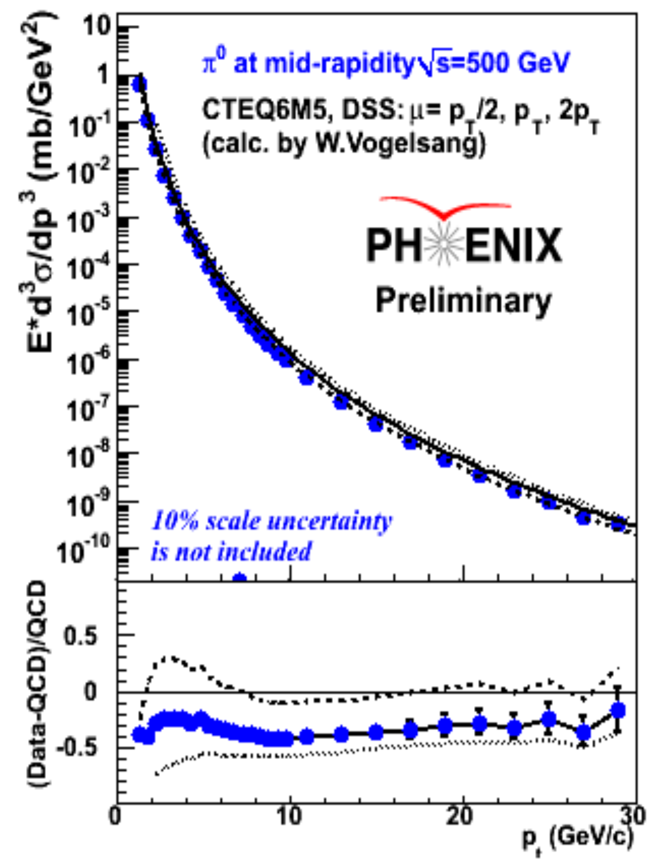
√s=62 GeV



√s=200 GeV



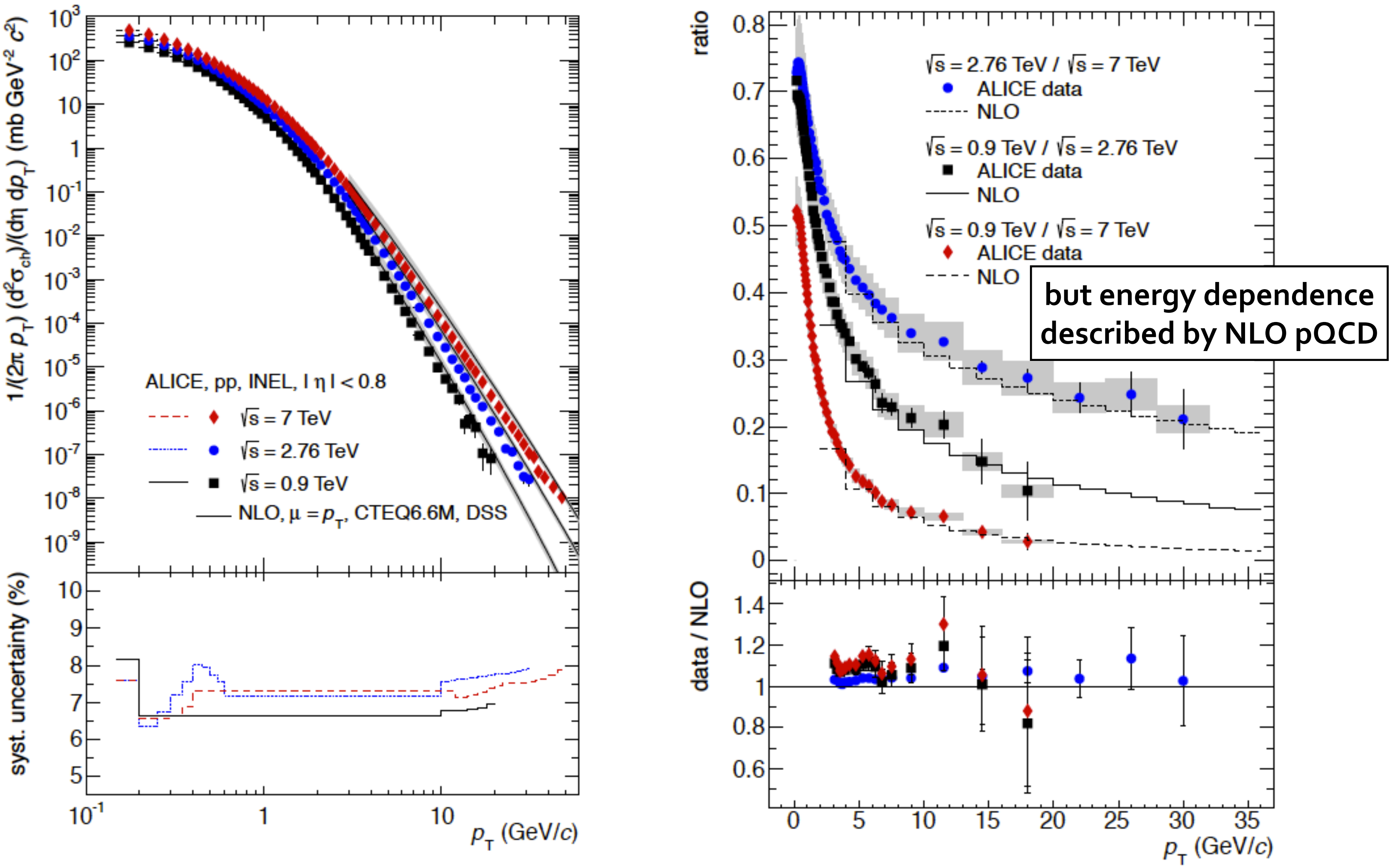
√s=500 GeV



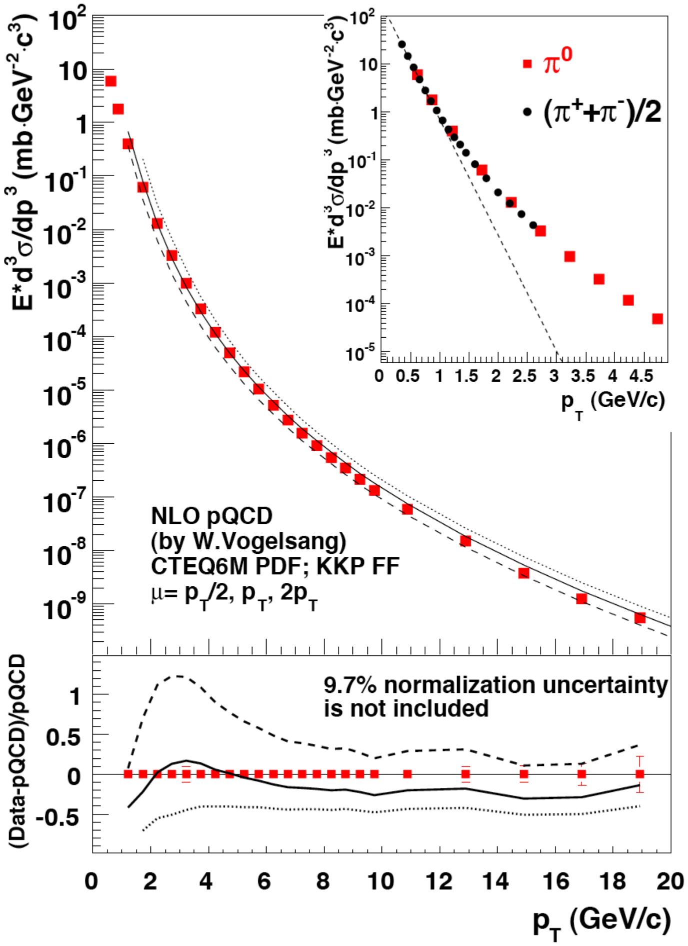
A. Bazilevsky (PHENIX)

puzzling new results on *unidentified* charged hadrons arXiv:1307.1093

(courtesy M. Stratmann)

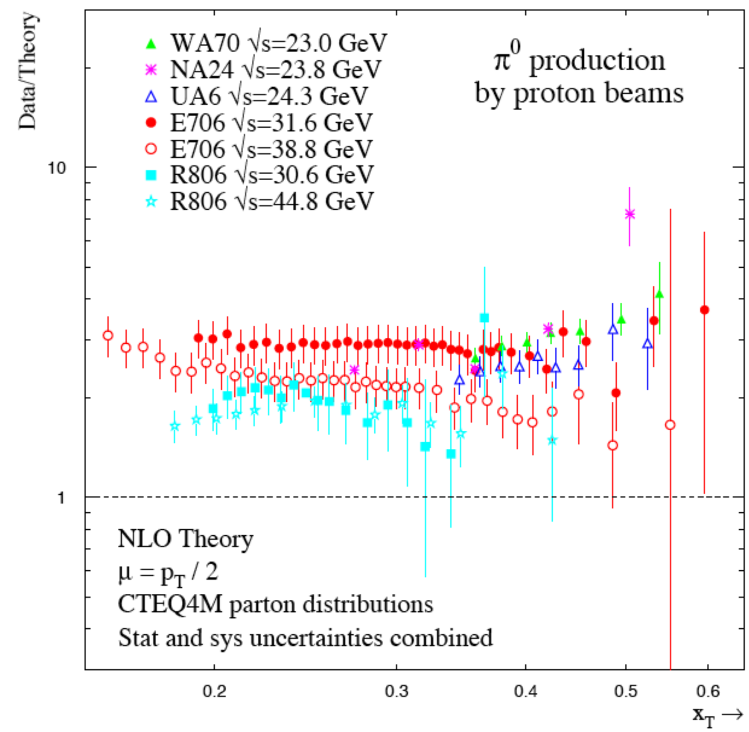


- a more long-standing problem : π^0 at lower energies



…well described by NLO at RHIC

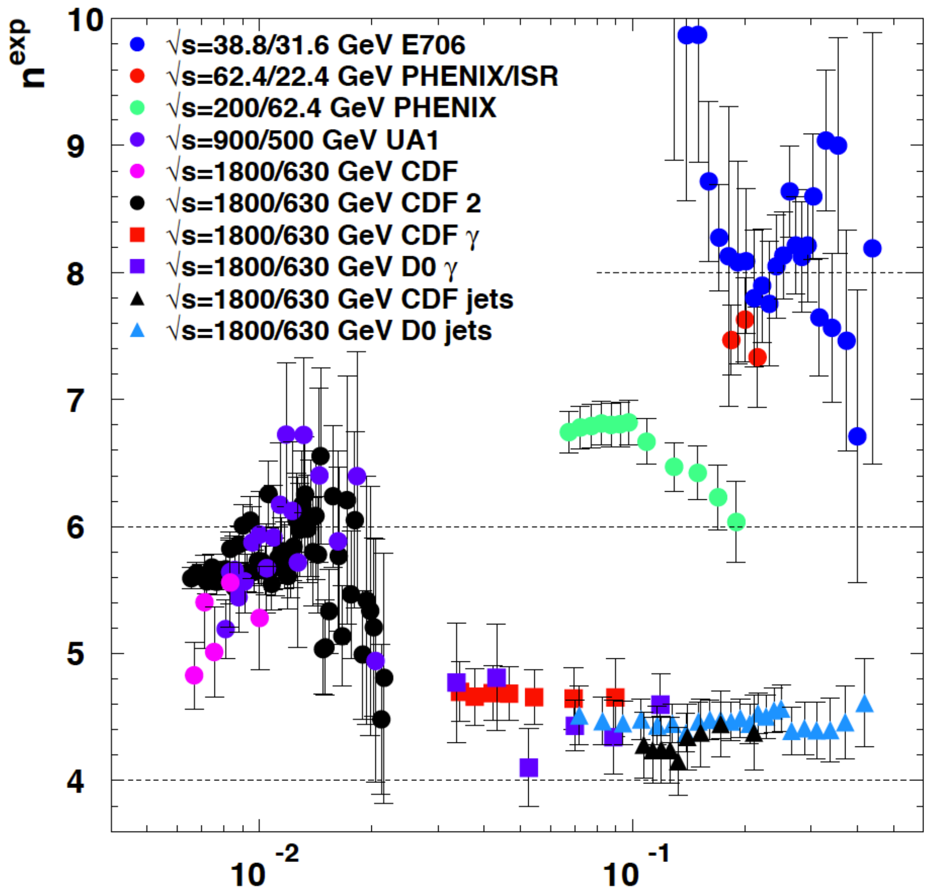
Aurenche et al.; Bourrely, Soffer

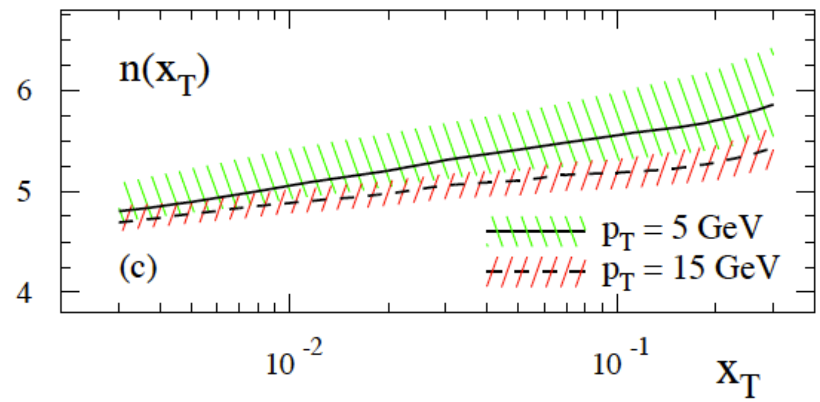


…but data much higher than NLO
at fixed-target energies !

Parton Model: $$E\frac{d^3\sigma}{dp^3} = \frac{1}{p_\perp^4} f(x_\perp) = \frac{1}{\sqrt{s}^4}\tilde{f}(x_\perp)$$

$$n^{\exp}(x_\perp) \equiv -\frac{\ln\left(\sigma^{\rm inv}(x_\perp,\sqrt{s_1})/\sigma^{\rm inv}(x_\perp,\sqrt{s_2})\right)}{\ln\left(\sqrt{s_1}/\sqrt{s_2}\right)}$$



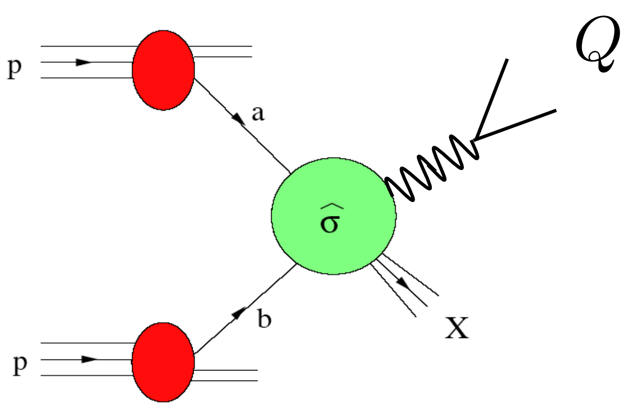


Sassot, Stratmann, Zurita

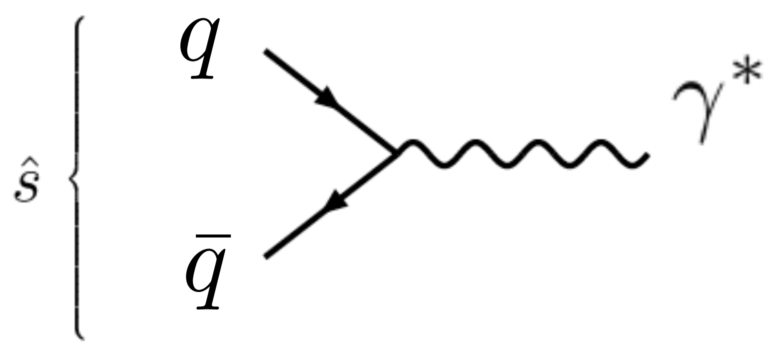
F. Arleo et al.

$$x_\perp = \frac{2p_\perp}{\sqrt{s}}$$

Resummation



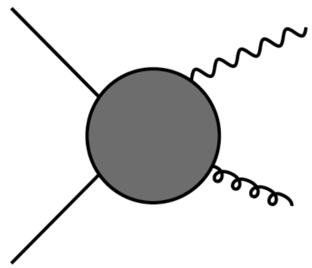
LO :



$$z = \frac{Q^2}{\hat{s}}$$

$$\omega_{q\bar{q}}^{(\mathrm{LO})} \propto \delta(1-z)$$

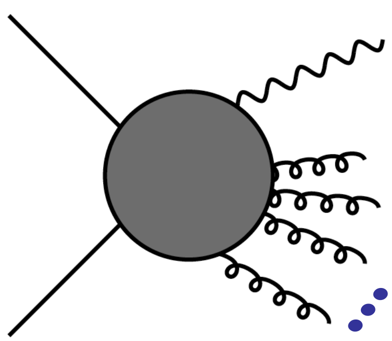
- NLO correction:



$$z \to 1:$$

$$\omega_{q\bar{q}}^{(\mathrm{NLO})} \propto \alpha_s \left(\frac{\log(1-z)}{1-z} \right)_+ + \dots$$

- higher orders:



$$\omega_{q\bar{q}}^{(\mathrm{N^kLO})} \propto \alpha_s^k \left(\frac{\log^{2k-1}(1-z)}{1-z} \right)_+ + \dots$$

"threshold logarithms"

- for z->1 real radiation inhibited

- logs emphasized by parton distributions :

$$d\sigma \sim \int_\tau^1 \frac{dz}{z} \mathcal{L}_{q\bar{q}}\left(\frac{\tau}{z}\right) \omega_{q\bar{q}}(z) \qquad \tau = \frac{Q^2}{S}$$

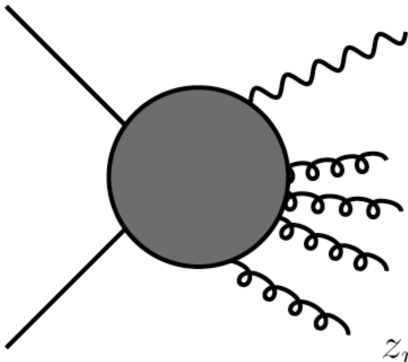
z = 1 relevant,
in particular as $\tau \to 1$

- logs more relevant at lower hadronic energies

Large logs can be resummed to all orders

Catani, Trentadue; Sterman; ...

- factorization of matrix elements in soft limit
- and of phase space when integral transform is taken:



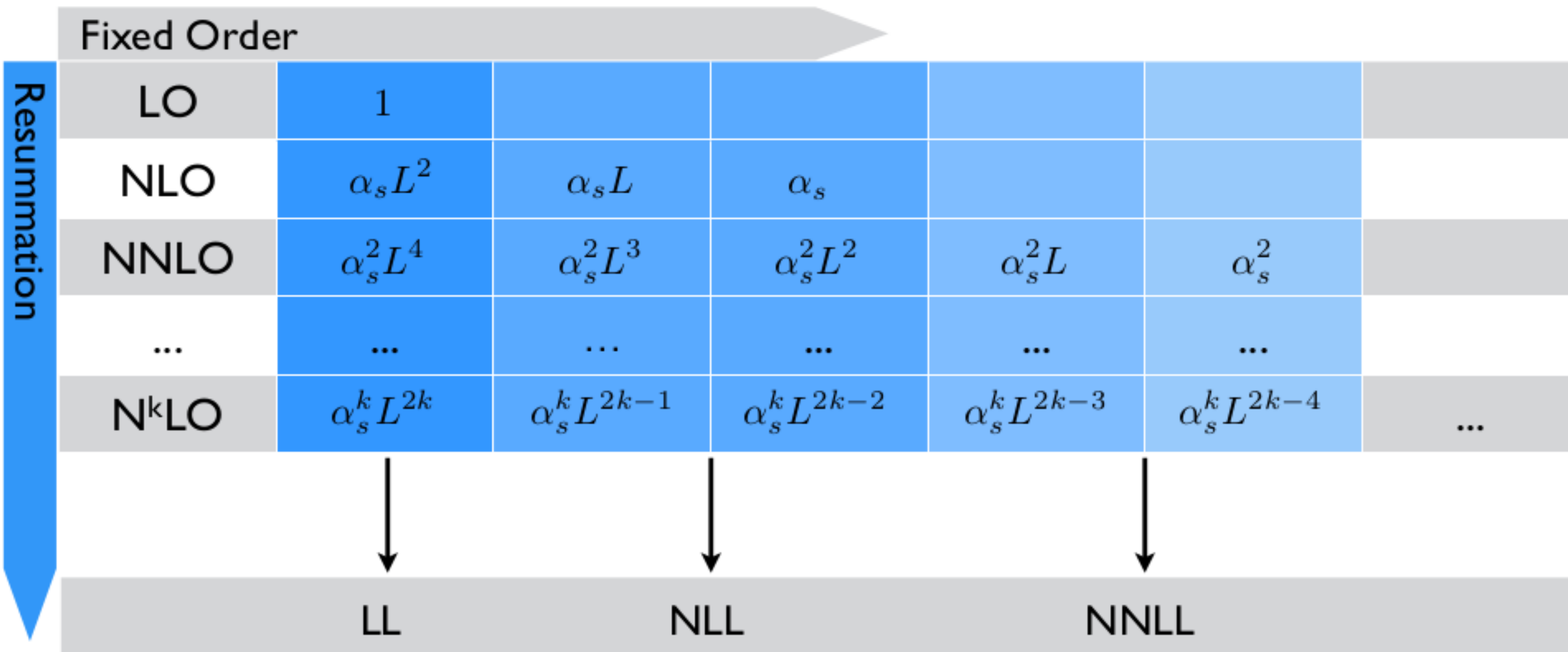
$$\delta\left(1 - z - \sum_{i=1}^{n} z_i\right) = \frac{1}{2\pi i}\int_C dN\, e^{N\left(1-z-\sum_{i=1}^{n} z_i\right)}$$

$$z_i = \frac{2E_i}{\sqrt{\hat{s}}}$$

$\overline{\text{MS}}$ scheme

$$\tilde{\omega}_{q\bar{q}}^{(\text{res})}(N) \propto \exp\left[2\int_0^1 dy\, \frac{y^N - 1}{1-y}\int_{\mu^2}^{Q^2(1-y)^2}\frac{dk_\perp^2}{k_\perp^2} A_q\left(\alpha_s(k_\perp^2)\right) + \ldots\right]$$

$$A_q(\alpha_s) = C_F\left\{\frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{C_A}{2}\left(\frac{67}{18} - \zeta(2)\right) - \frac{5}{9}T_R n_f\right] + \ldots\right\}$$


Fixed Order
Resummation
LO
1
NLO
$\alpha_s L^2$
$\alpha_s L$
α_s
NNLO
$\alpha_s^2 L^4$
$\alpha_s^2 L^3$
$\alpha_s^2 L^2$
$\alpha_s^2 L$
α_s^2
...
N^kLO
$\alpha_s^k L^{2k}$
$\alpha_s^k L^{2k-1}$
$\alpha_s^k L^{2k-2}$
$\alpha_s^k L^{2k-3}$
$\alpha_s^k L^{2k-4}$
LL
NLL
NNLL

- logs enhance cross section:

LL :

$$\tilde{\omega}_{q\bar{q}}^{(\mathrm{res})}(N) \;\propto\; \exp\left[+\frac{2C_F}{\pi}\,\alpha_s \ln^2 N + \ldots\right]$$

to NLL :

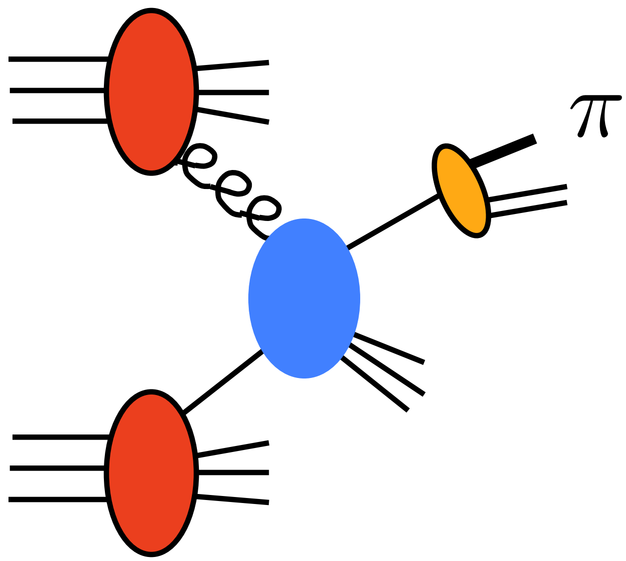
Catani,Mangano,Nason,Trentadue

$$\tilde{\omega}_{q\bar{q}}^{(\mathrm{res})}(N) \;\propto\; \exp\left\{2\ln\bar{N}\; h^{(1)}(\lambda) + 2h^{(2)}\left(\lambda, \frac{Q^2}{\mu^2}\right)\right\}$$

LL NLL

$$\lambda = \alpha_s(\mu^2)\, b_0\, \log(N e^{\gamma_E})$$

$$h^{(1)}(\lambda) \;=\; \frac{A_q^{(1)}}{2\pi b_0 \lambda}\left[2\lambda + (1-2\lambda)\ln(1-2\lambda)\right] \qquad h^{(2)} \;=\; \ldots$$

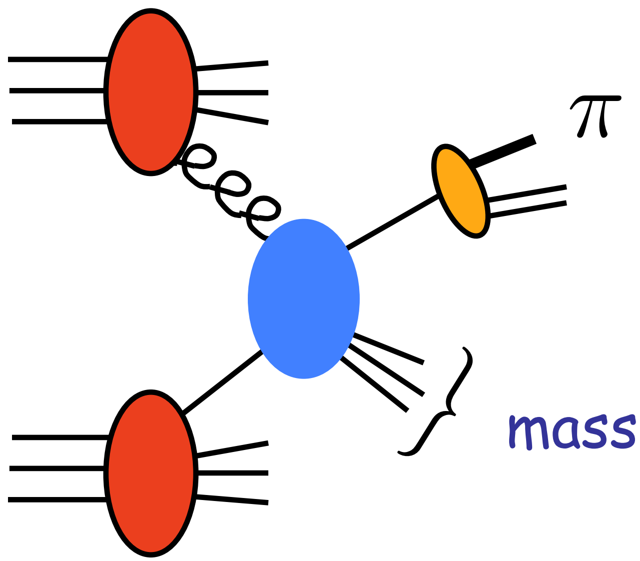


$$\Big\} \quad p_T, \eta \qquad x_T = \frac{2p_T}{\sqrt{S}}$$

$$\frac{d\sigma}{dp_T d\eta} = \int_{x_a^0}^{1} dx_a \int_{x_b^0}^{1} dx_b \int_{z_c^0}^{1} dz_c \; f_a(x_a)\, f_b(x_b)\, \frac{d\hat{\sigma}_{ab\to c}}{dp_T d\eta}\, D_c(z_c)$$

Partonic variables: $\hat{x}_T = \dfrac{2p_T}{z_c\sqrt{\hat{s}}}$ $\qquad \hat{\eta} = \eta - \dfrac{1}{2}\ln\dfrac{x_a}{x_b}$

$$\frac{d\sigma}{dp_T d\eta} = \int_{x_a^0}^1 dx_a \int_{x_b^0}^1 dx_b \int_{z_c^0}^1 dz_c \; f_a(x_a)\, f_b(x_b)\, \frac{d\hat{\sigma}_{ab\to c}(\hat{x}_T, \hat{\eta})}{dp_T d\eta} D_c(z_c)$$



mass[2] $\equiv s_4$

$$s_4 = \hat{s}\,(\,1 - \hat{x}_T \cosh \hat{\eta}\,)$$

LO :

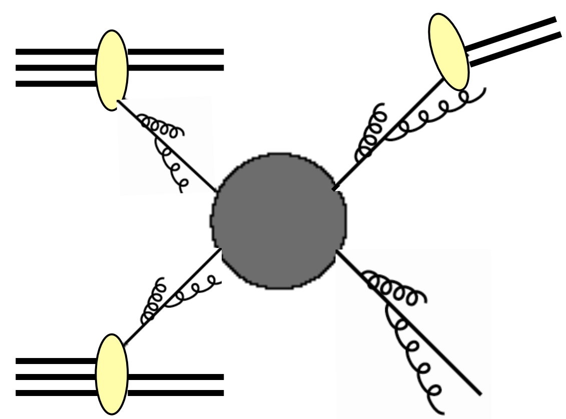
$$d\hat{\sigma}^{(\mathrm{LO})}_{ab\to c} \propto \delta\left(\frac{s_4}{\hat{s}}\right)$$

NLO :

$$d\hat{\sigma}^{(\mathrm{NLO})}_{ab\to c} \propto \alpha_s \left(\frac{\log(s_4/\hat{s})}{s_4/\hat{s}}\right)_+ + \ldots$$

yet higher orders:

$$d\hat{\sigma}^{(\mathrm{N^kLO})}_{ab\to c} \propto \alpha_s^k \left(\frac{\log^{2k-1}(s_4/\hat{s})}{s_4/\hat{s}}\right)_+ + \ldots$$



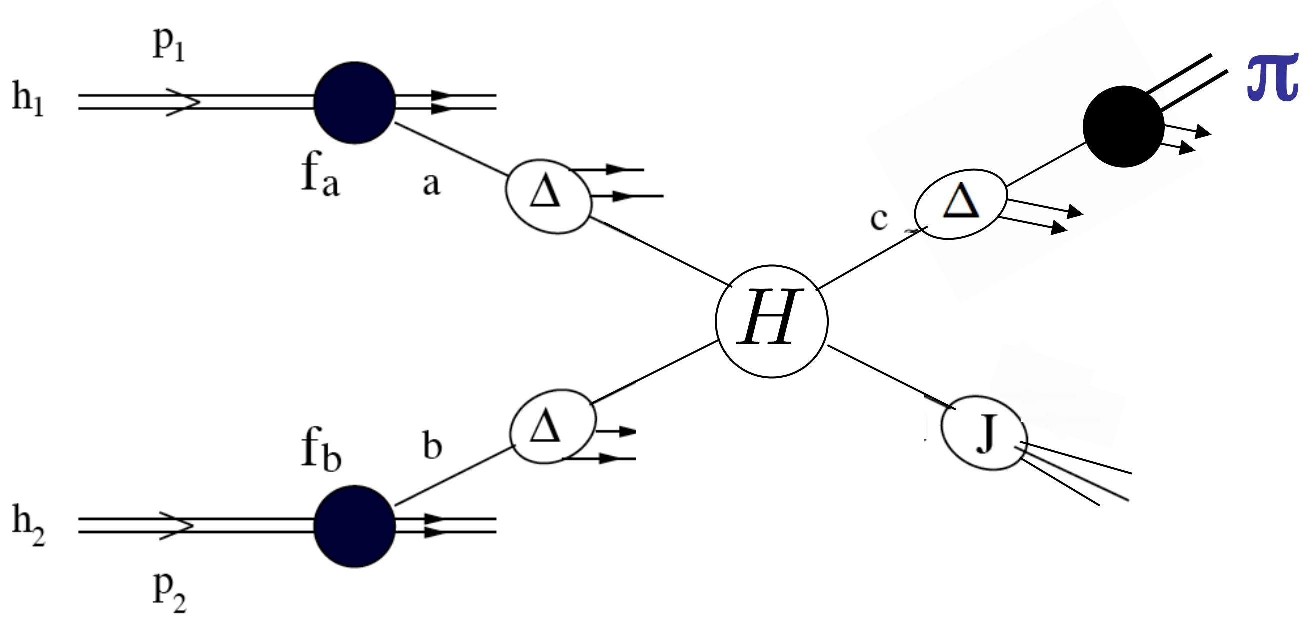
- color structure of hard scattering matters !

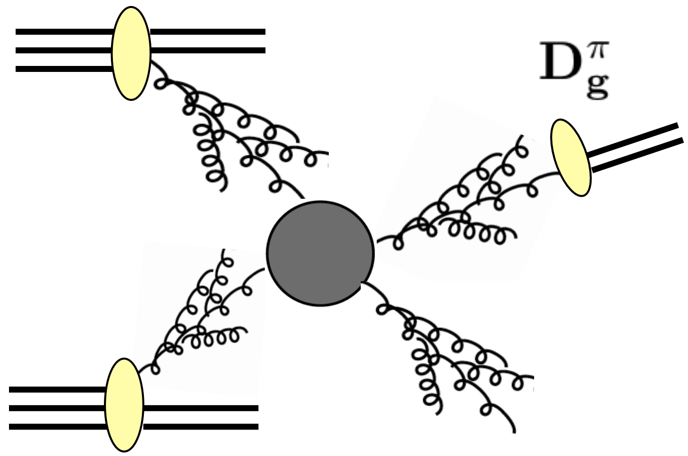
$$\int_0^1 \frac{ds_4}{\hat{s}} \left(1 - \frac{s_4}{\hat{s}}\right)^N \frac{d\hat{\sigma}^{\text{resum}}_{ab\to c}(\hat{x}_T, \hat{\eta})}{dp_T d\eta} = \Delta_a^N \, \Delta_b^N \, \Delta_c^N \; J^N_{\text{recoil}}$$

like DY

$$\times \sum_{IK} \left[H^{ab\to cd}_{IK} \, S^{ab\to cd}_{KI} \right] (\hat{\eta}, N)$$

Kidonakis,Oderda,Sterman
Bonciani,Catani,Mangano,Nason
Banfi,Salam,Zanderighi
Dokshitzer,Marchesini





Mellin moment in $\hat{x}_T^2$

Leading logarithms

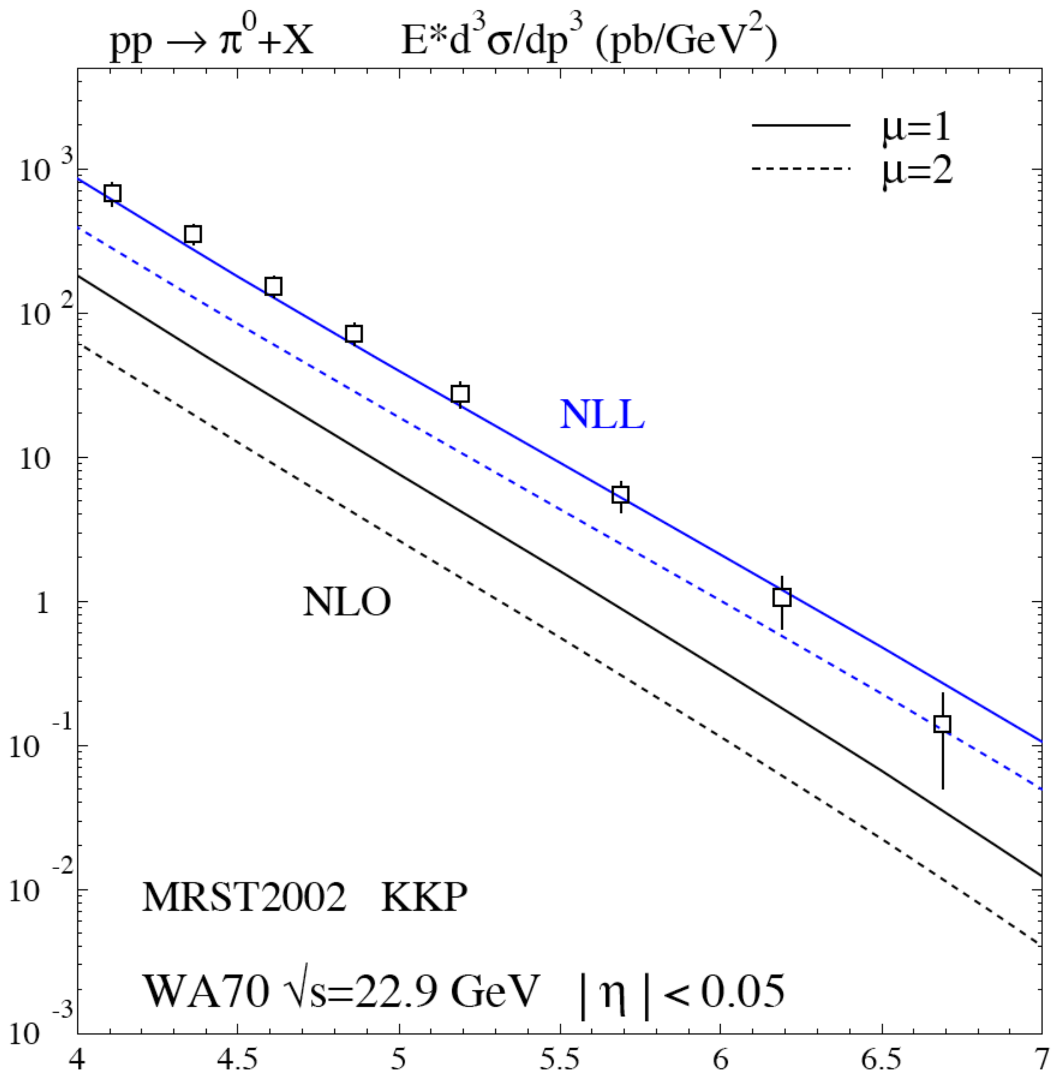
$gg \to gg$ $$\exp\left[\left(C_A + C_A + C_A - \frac{1}{2}C_A\right)\frac{\alpha_s}{\pi}\ln^2(N)\right]$$

- expect (large) enhancement !

de Florian, WV

WA70

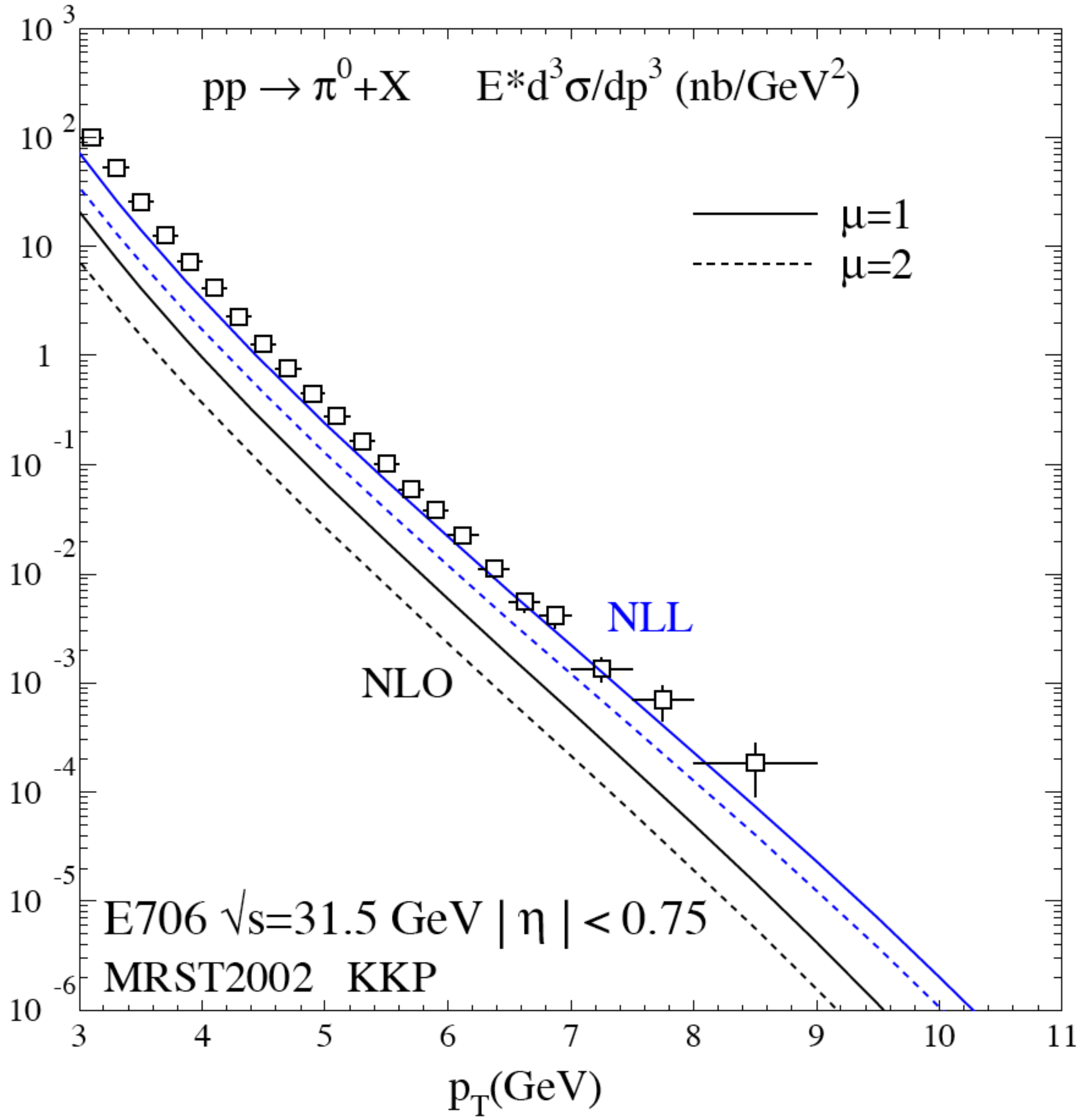
de Florian, WV



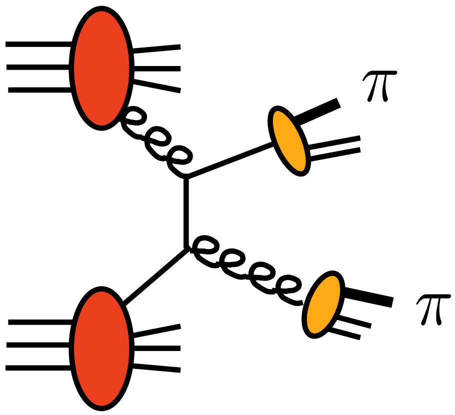
$pp \to \pi^0 X$

E706

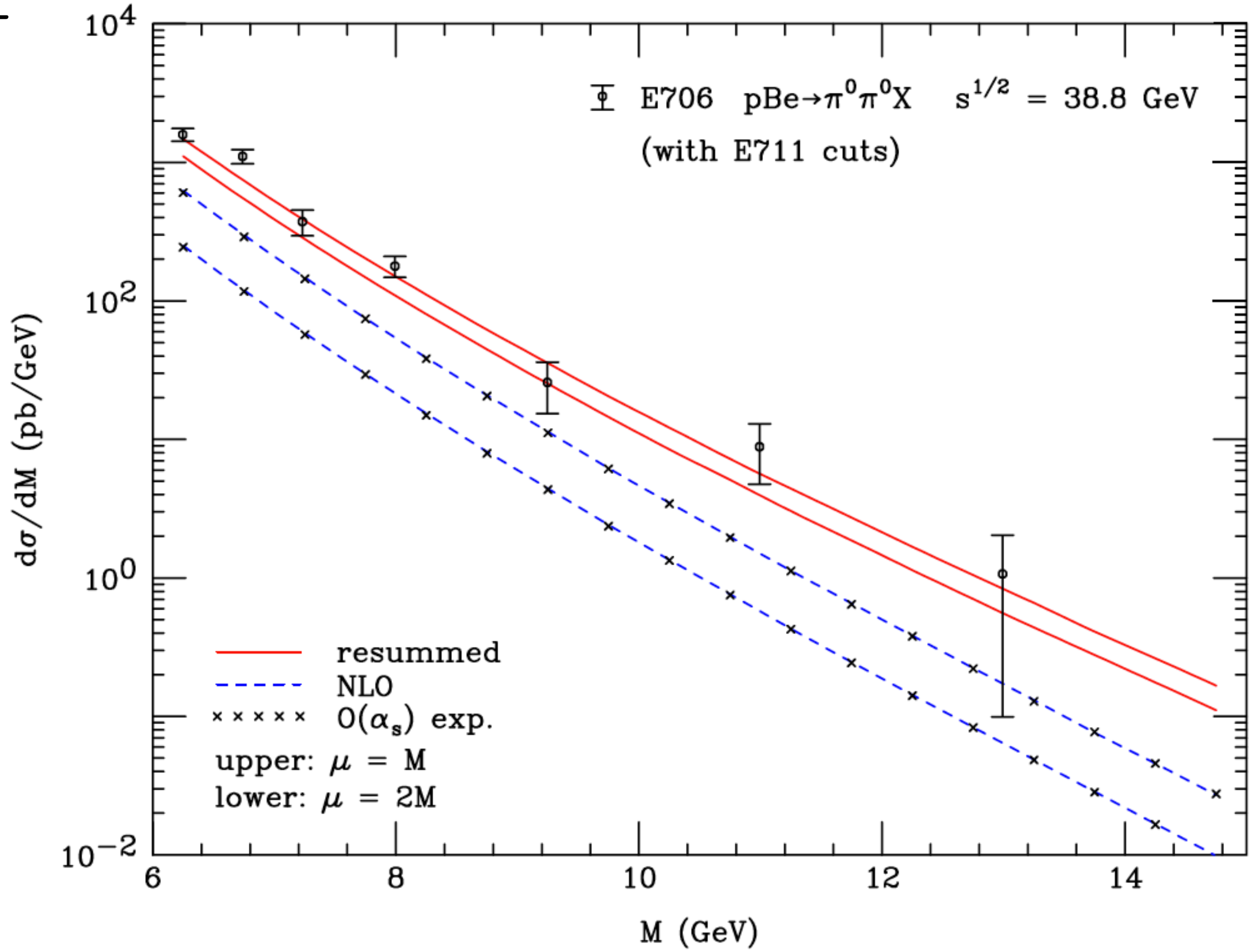
de Florian, WV



effects at RHIC, LHC more modest (except at forward rap.?)



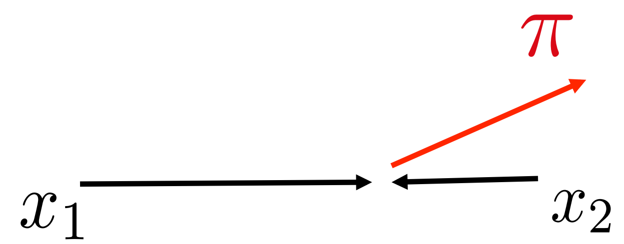
Almeida,Sterman,WV



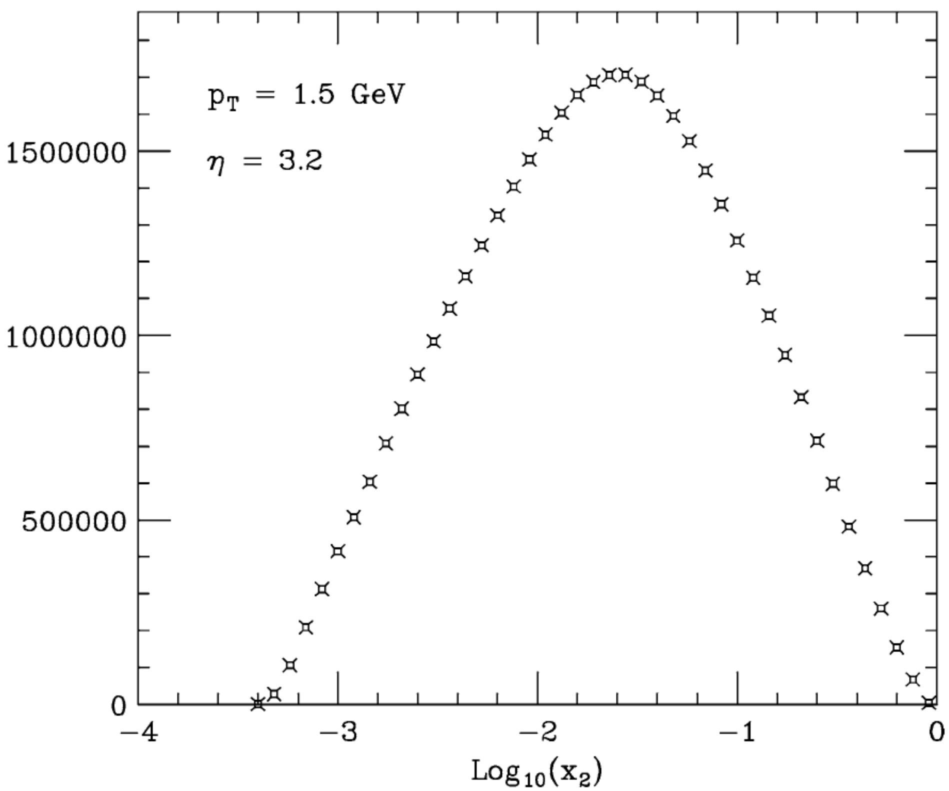
Additional aspects

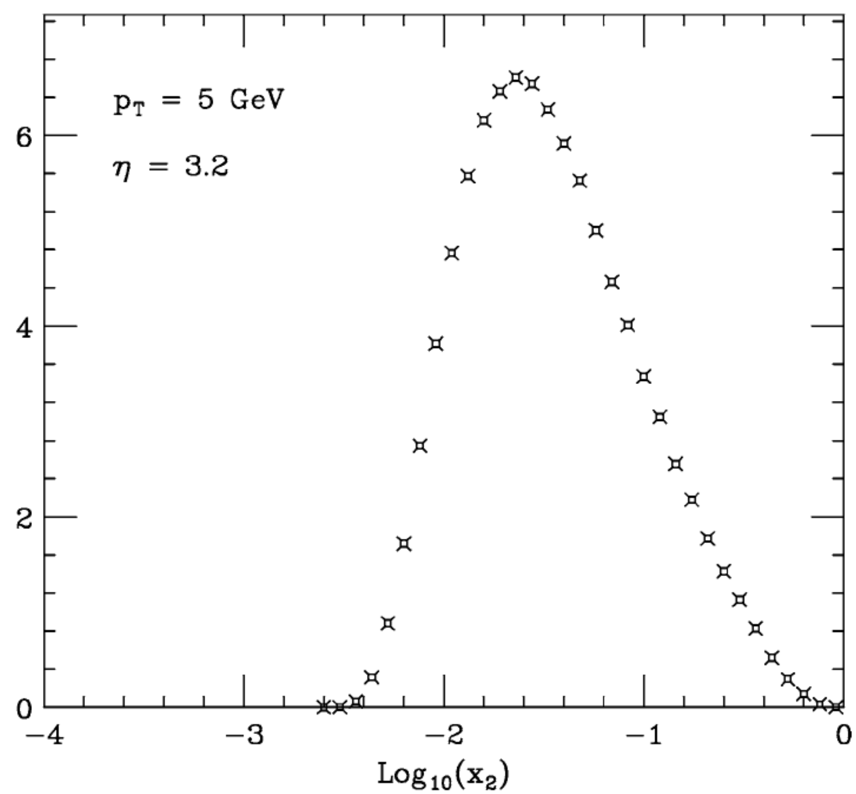
In regions where pQCD works well, can look more closely; for instance:

(1) Forward scattering RHIC / LHC



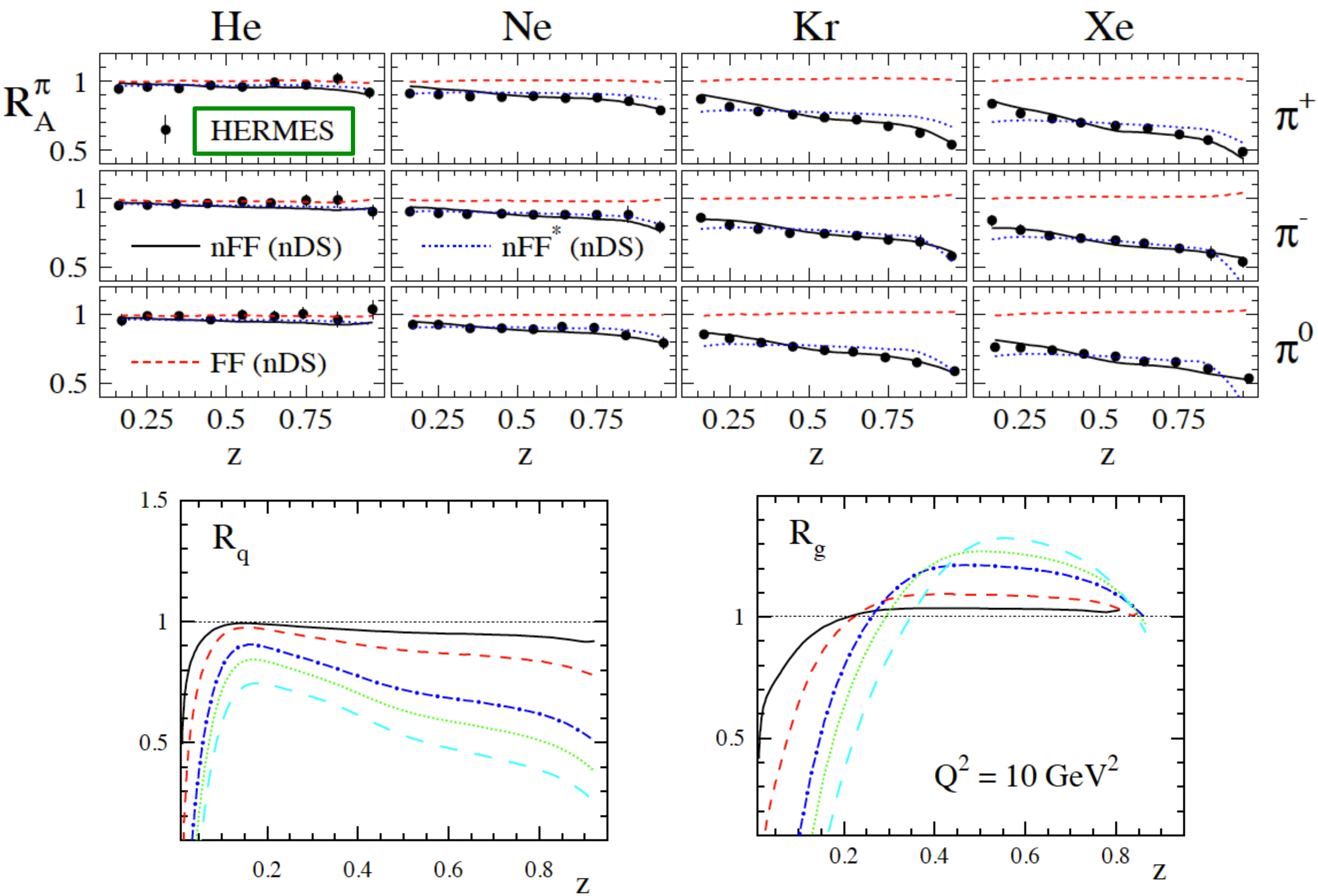
Guzey,Strikman,WV



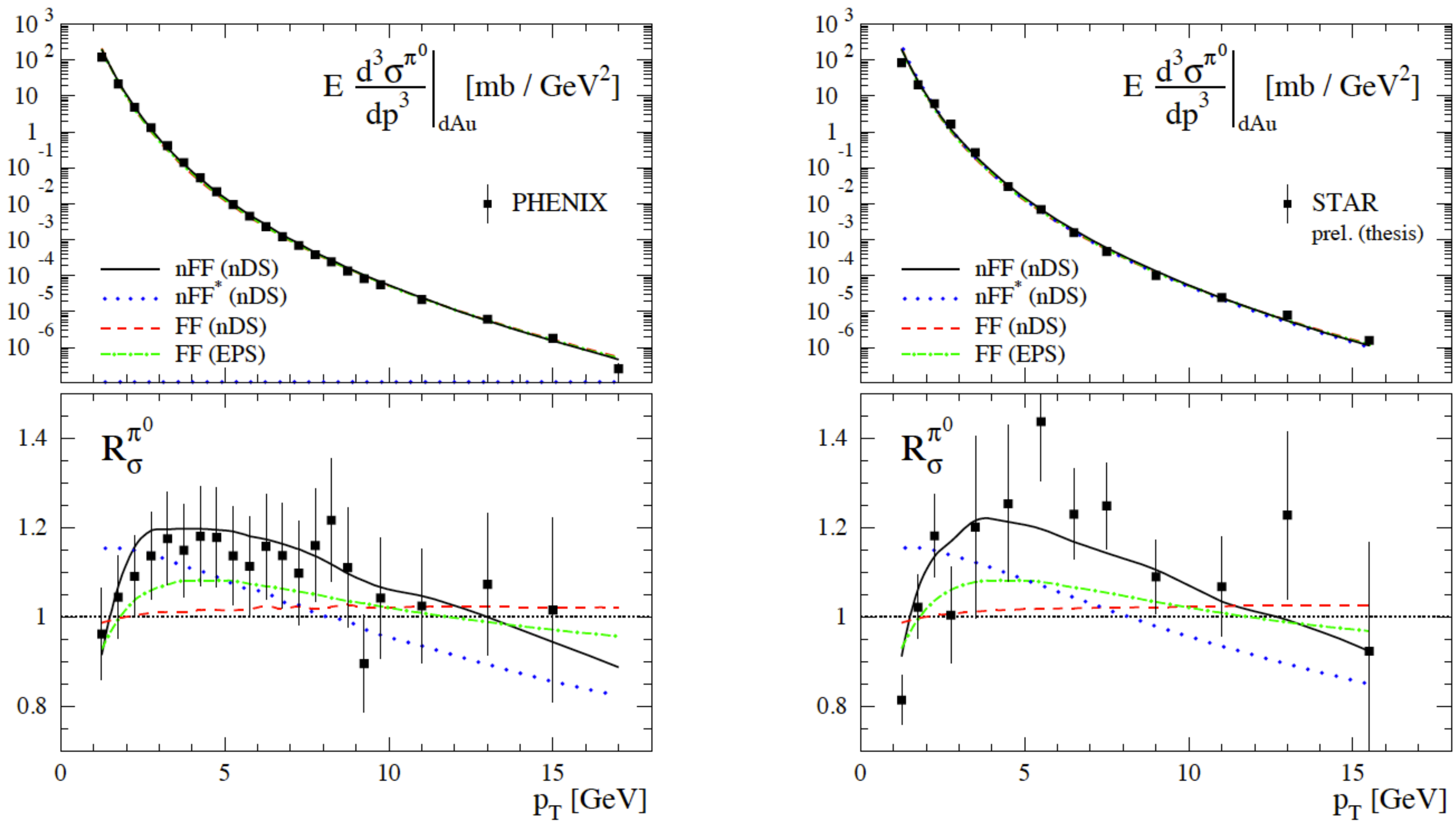


(2) In-medium fragmentation

Sassot,Stratmann,Zurita

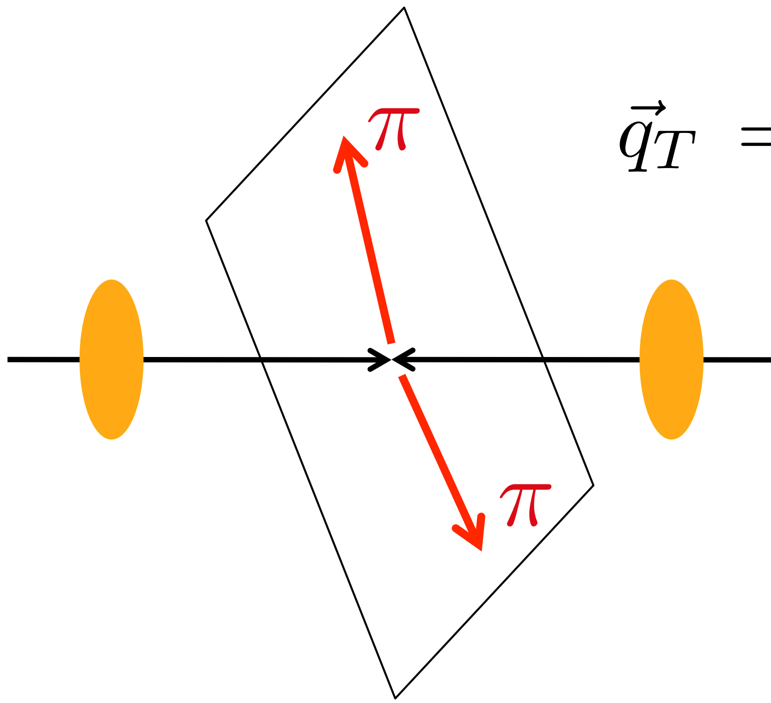


Sassot,Stratmann,Zurita



(3) Two-particle production:

large scale p_T, small (also measured) scale q_T

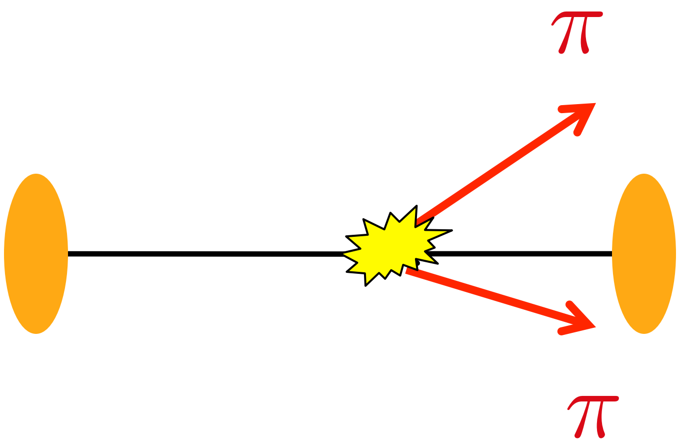


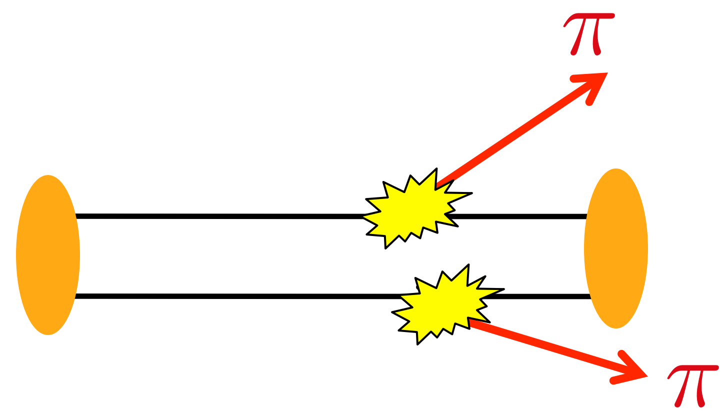
$$\vec{q}_T = \vec{p}_{T,1} + \vec{p}_{T,2}$$

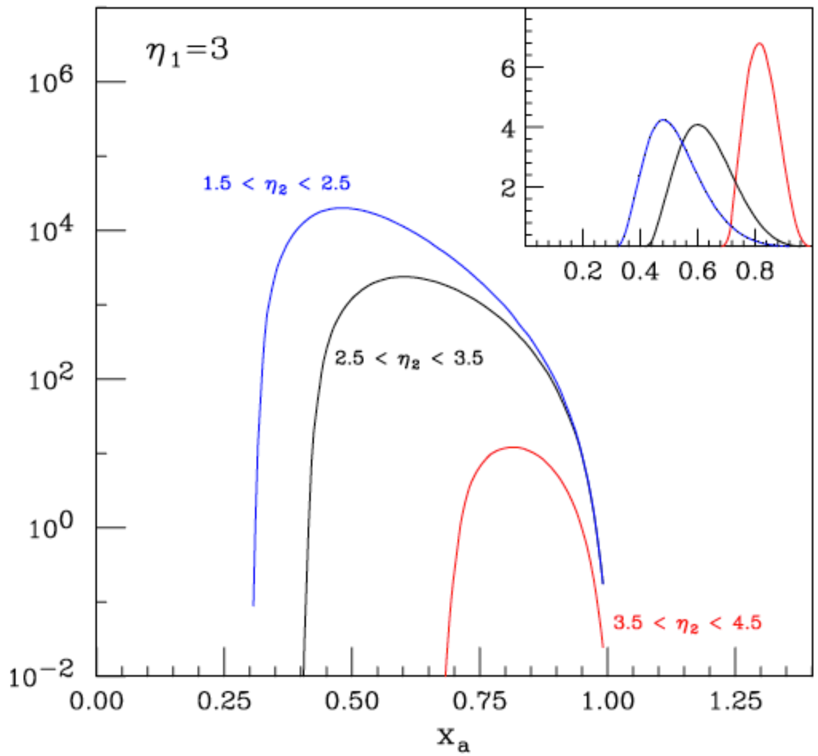
(Sudakov logs, TMD (non)factorization,…)

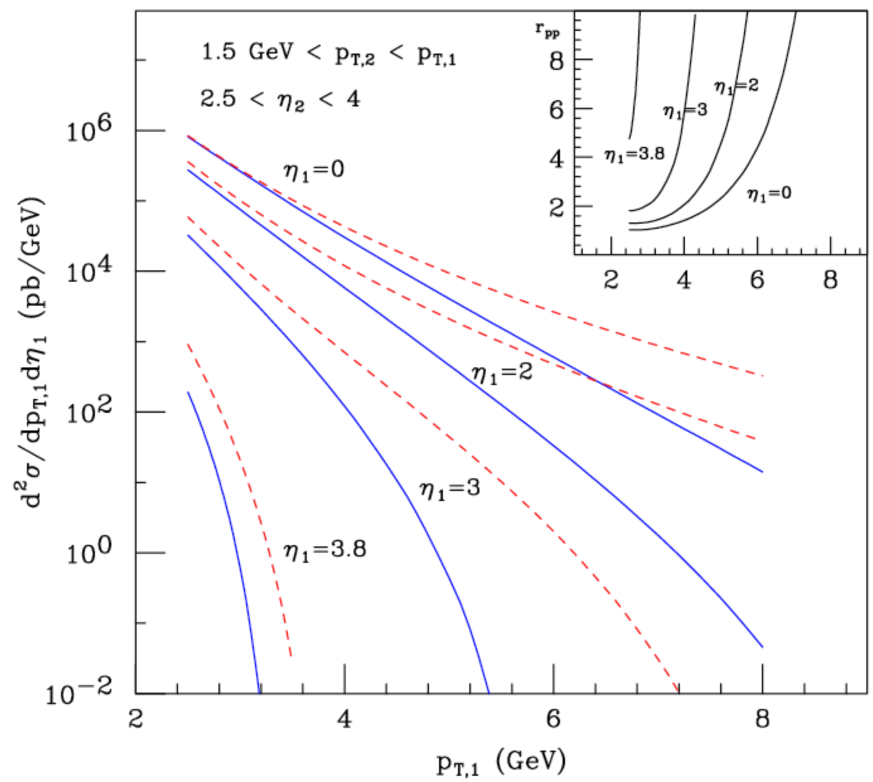
- double-scattering contributions

Strikman,WV









Conclusions:

- pQCD framework very advanced:

  fixed-order calculations

  all-order resummations
  (not a cure for everything, but often helpful)

  many applications relevant to pA