

Micropattern Gas Detectors

Focus on Signals

RD51 electronics school Feb 3-5, 2014

Werner Riegler, CERN

The principle mechanisms and formulas for signal generation in particle detectors are reviewed.

Examples including micropattern detectors will be discussed.

Creation of the Signal

Charged particles leave a trail of ions (and excited atoms) along their path:
Electron-Ion pairs in gases and liquids, electron hole pairs in solids.

Photons from de-excitation are usually converted to electrons for detection.

The produced charges can be registered → Position measurement → Time measurement → Tracking Detectors

Cloud Chamber: Charges create drops → photography.

Bubble Chamber: Charges create bubbles → photography.

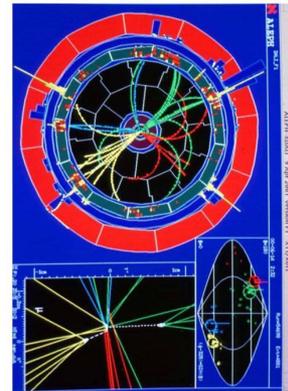
Emulsion: Charges 'blacked' the film.

Spark Chamber: Charges produce a conductive channel that create a discharge → photography

Gas and Solid State Detectors: Moving Charges (electric fields) induce electronic signals on metallic electrodes that can be read by dedicated electronics.

→ In solid state detectors the charge created by the incoming particle is sufficient (not exactly correct, in Avalanche Photo Diodes one produces avalanches in a solid state detector)

→ In gas detectors (e.g. wire chamber) the charges are internally multiplied in order to provide a measurable signal.



Cloud Chamber, C.T.R. Wilson 1910

Charges act as condensation nuclei in supersaturated water vapor

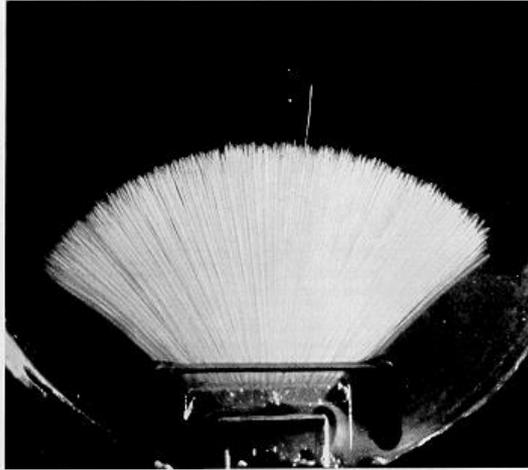


Fig. 13. K. PHILIPP, Naturwiss. 14, 1203 (1926).

Alphas, Philipp 1926

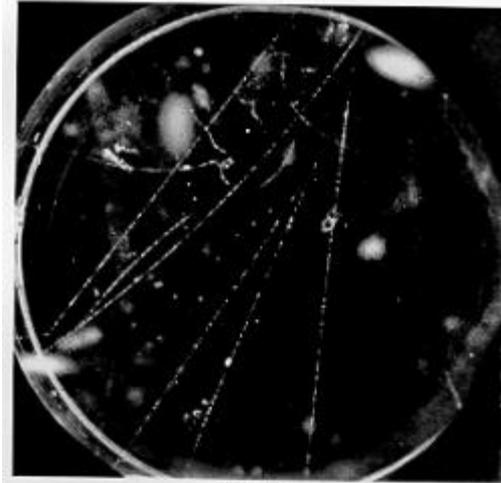


Plate 115

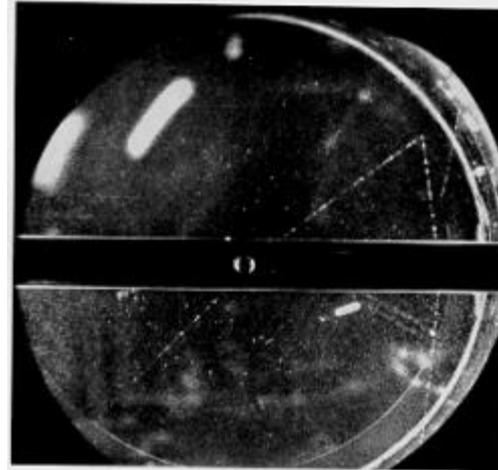
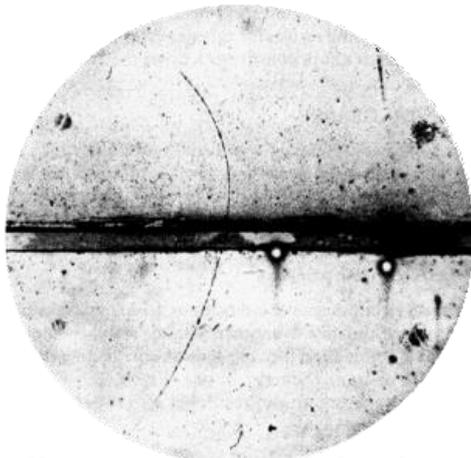


Plate 110

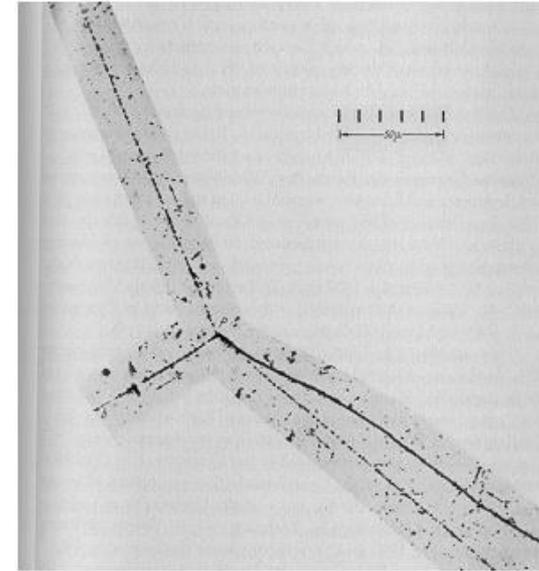
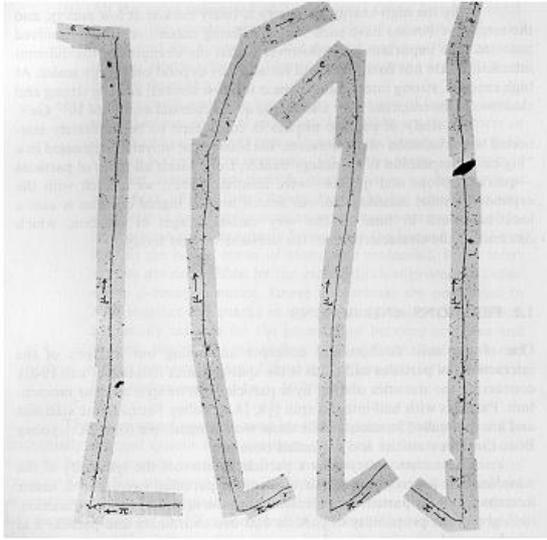


Positron discovery, Carl Andersen 1933

V- particles, Rochester and Wilson, 1940ies

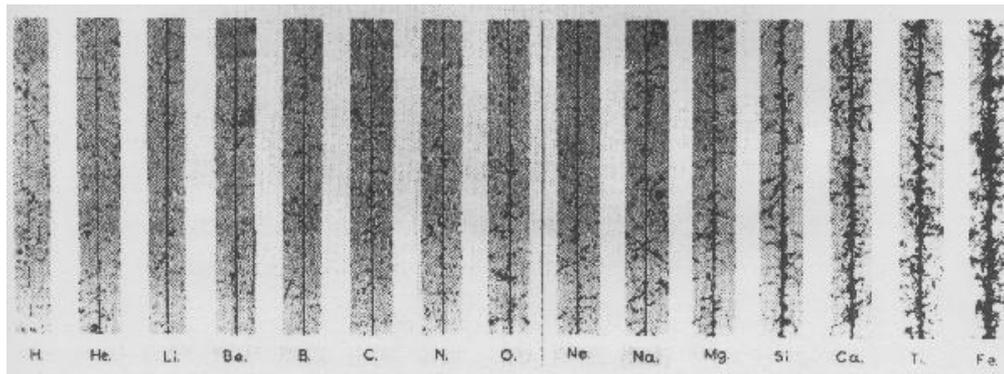
Nuclear Emulsion, M. Blau 1930ies

Charges initiate a chemical reaction that blackens the emulsion (film)



C. Powell, Discovery of muon and pion, 1947

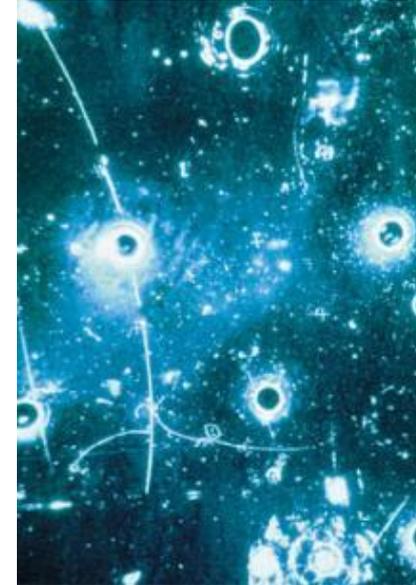
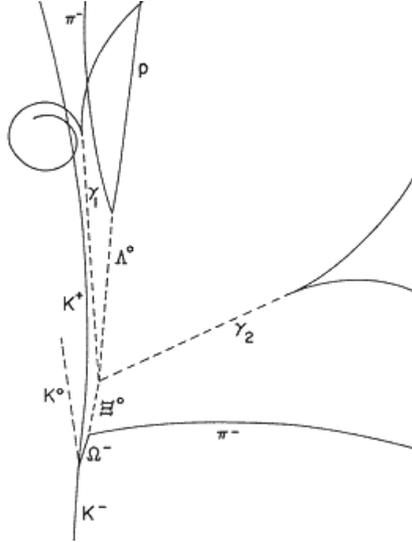
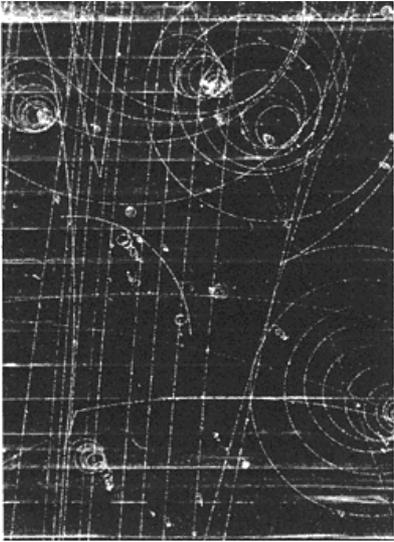
Kaon Decay into 3 pions, 1949



Cosmic Ray Composition

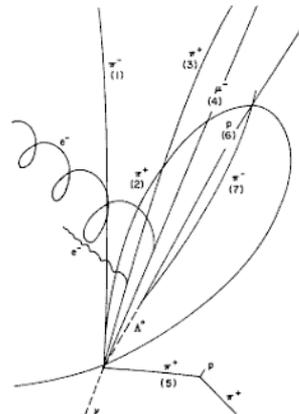
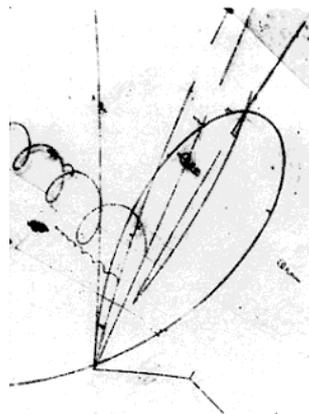
Bubble Chamber, D. Glaser 1952

Charges create bubbles in superheated liquid, e.g. propane or Hydrogen (Alvarez)



Discovery of the ϕ^- in 1964

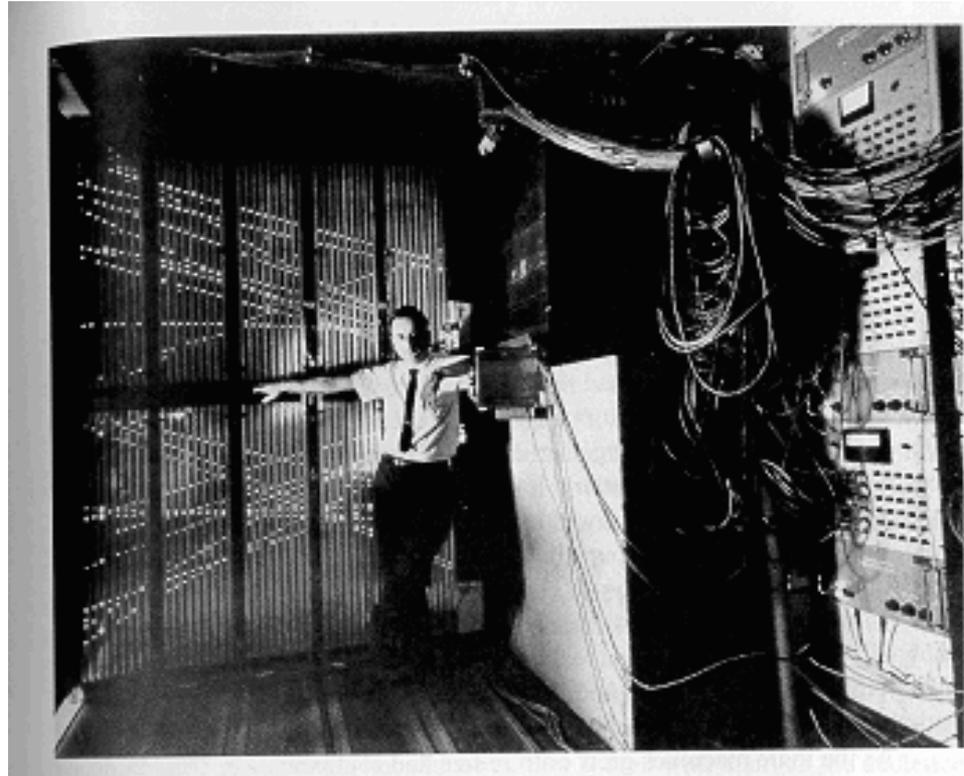
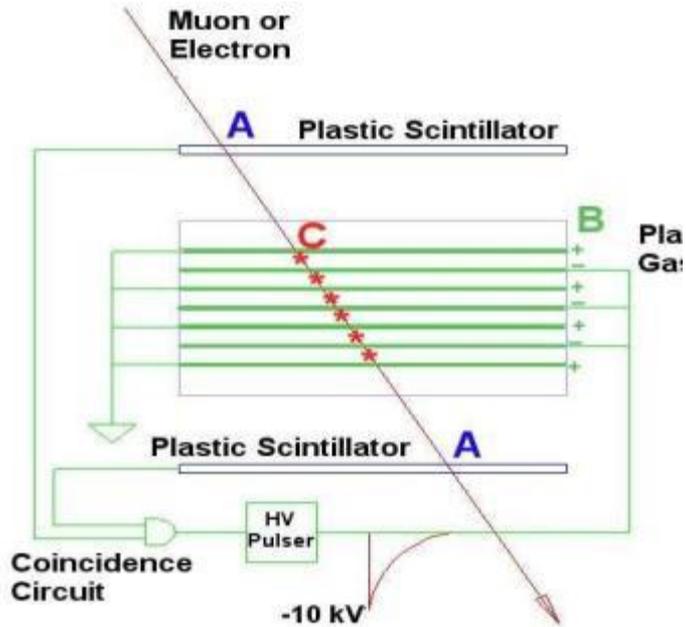
Neutral Currents 1973



Charmed Baryon, 1975

Spark Chamber, 1960ies

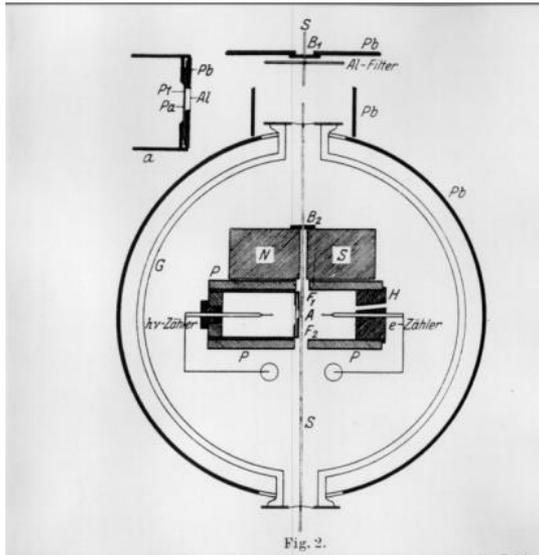
Charges create 'conductive channel' which initiates a spark in case HV is applied.



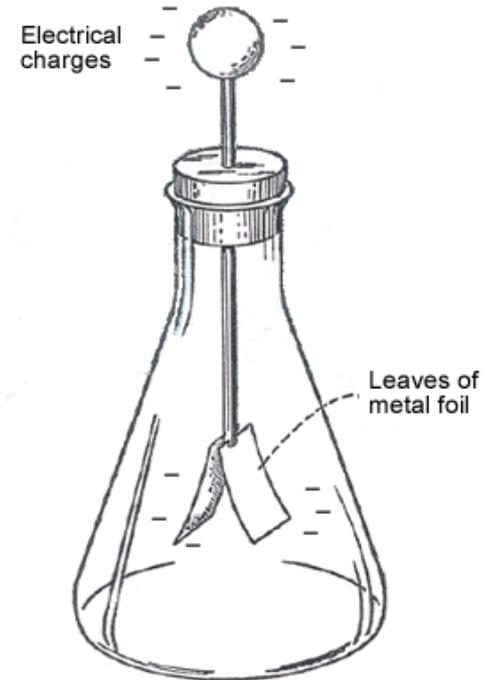
Discovery of the Muon Neutrino 1960ies

Tip Counter, Geiger 1914

Charges create a discharge of a needle which is at HV with respect to a cylinder.

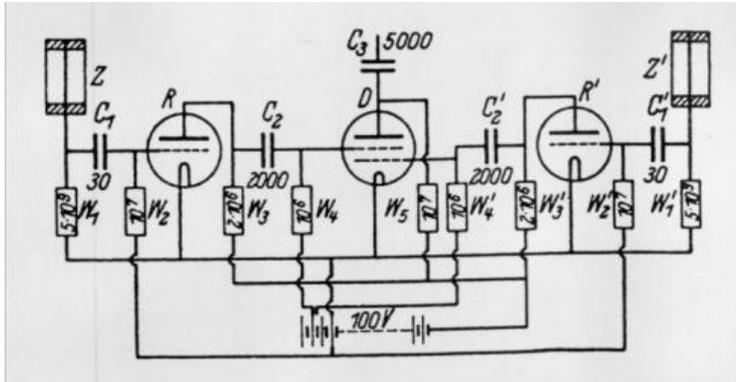


The needle is connected to an electroscope that can detect the produced charge.

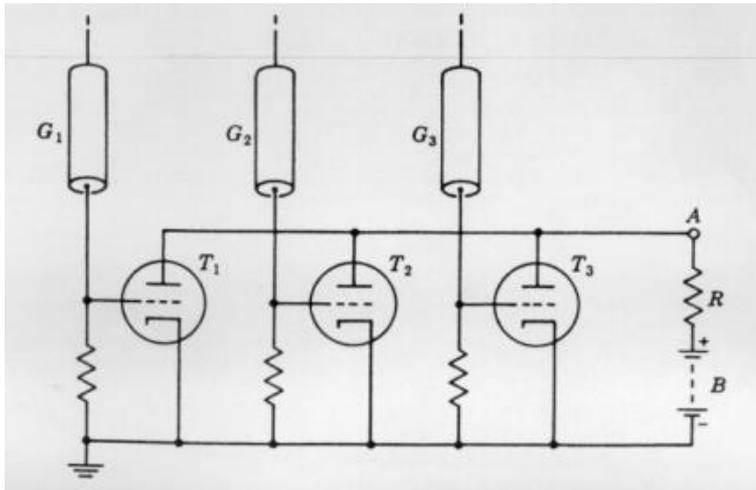
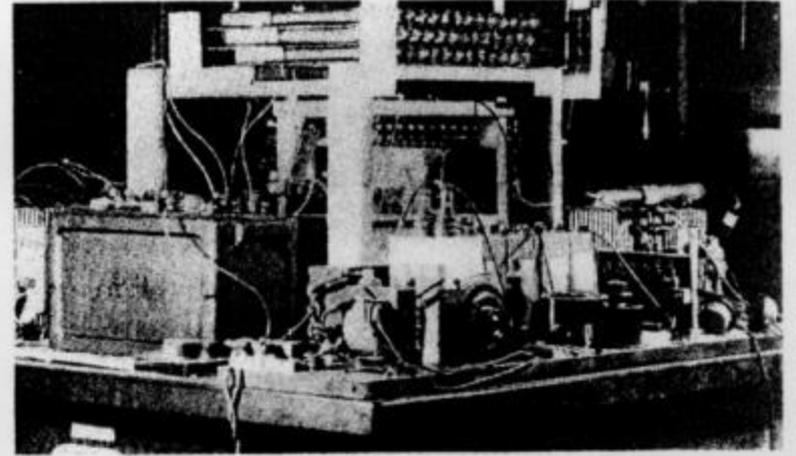


Electric Registration of Geiger Müller Tube Signals

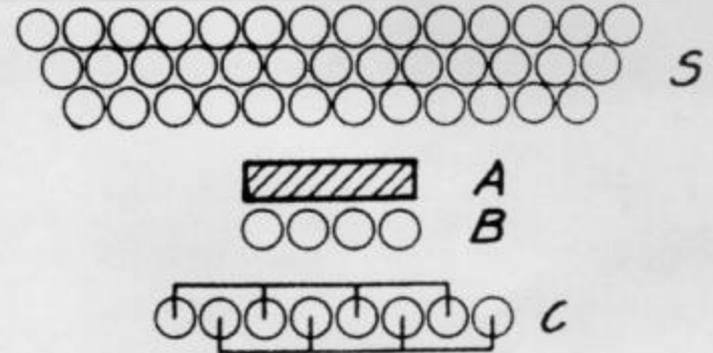
Charges create a discharge in a cylinder with a thin wire set to HV. The charge is measured with a electronics circuit consisting of tubes \rightarrow electronic signal.



W. Bothe, 1928



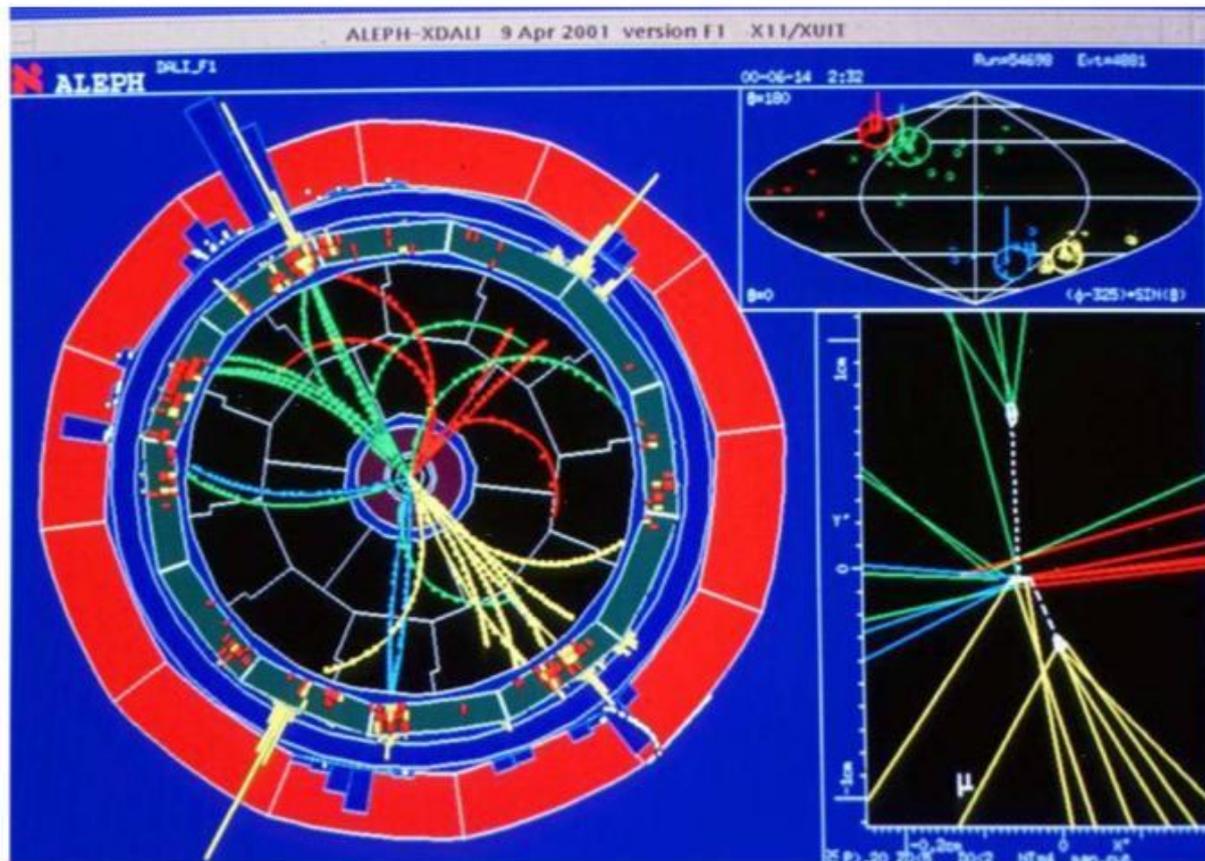
B. Rossi, 1932



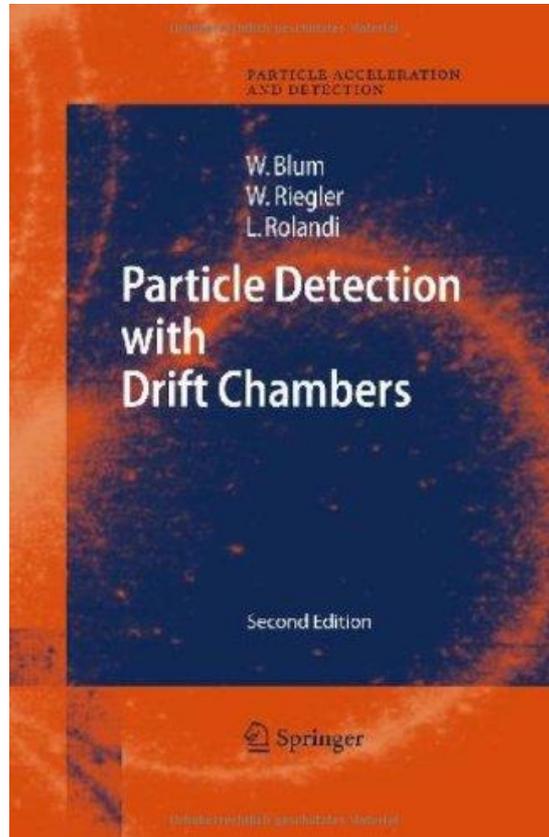
Cosmic Ray Telescope 1930ies

Ionization Chambers, Wire Chambers, Solid State Detectors

! The movement of charges in electric fields induces signal on readout electrodes (No discharge, there is no charge flowing from cathode to Anode) !



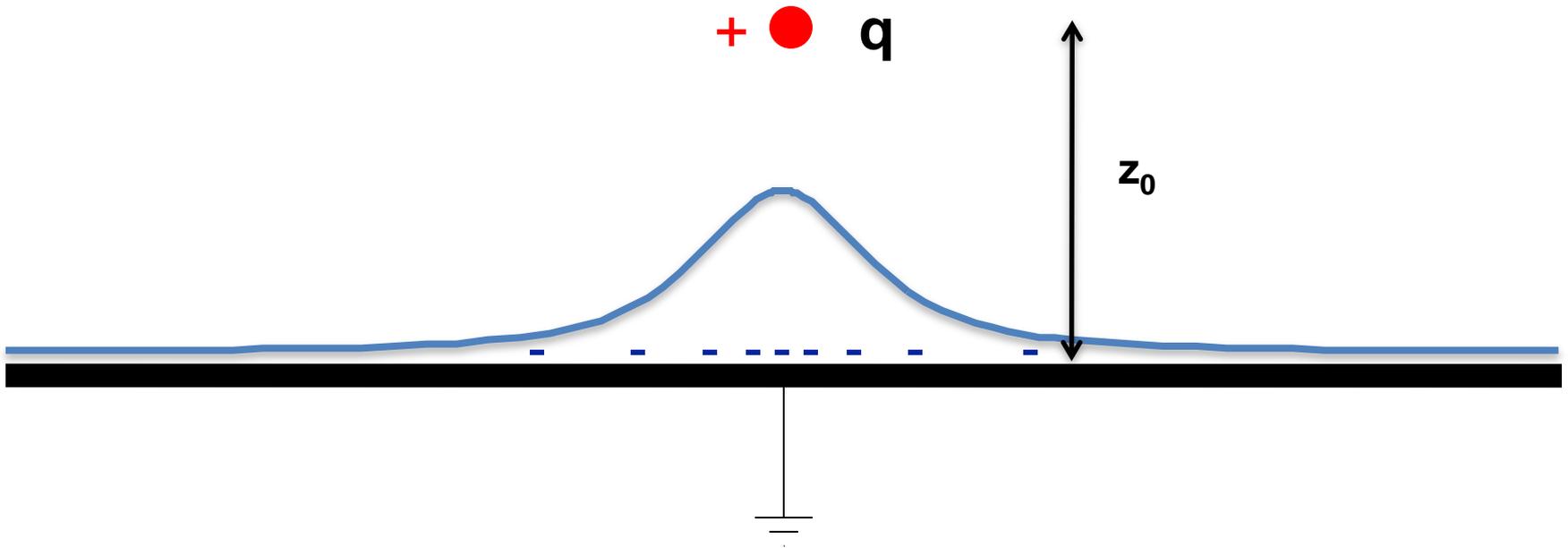
The Principle of Signal Induction on Metal Electrodes by Moving Charges



Many details on signal induction principles, electronics noise and signal processing can be found in this book.

Induced Charges

A point charge q at a distance z_0 above a grounded metal plate 'induces' a surface charge.



Electrostatics, Things we Know

Poisson Equation:

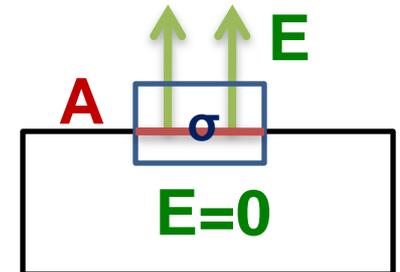
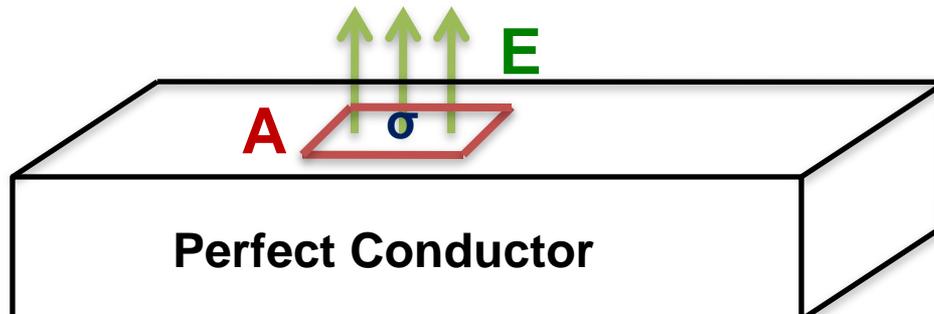
$$\Delta\varphi = -\frac{\rho}{\epsilon_0} \quad \vec{E} = -\vec{\nabla}\varphi$$

Gauss Law:

$$\oint \vec{E} d\vec{A} = \frac{1}{\epsilon_0} \int \rho dV$$

→ **Metal Surface: Electric Field is perpendicular to the surface. Charges are only on the surface. Surface Charge Density σ and electric E field on the surface are related by**

$$E A = \frac{1}{\epsilon_0} \sigma A \quad \rightarrow \quad \sigma = \epsilon_0 E$$



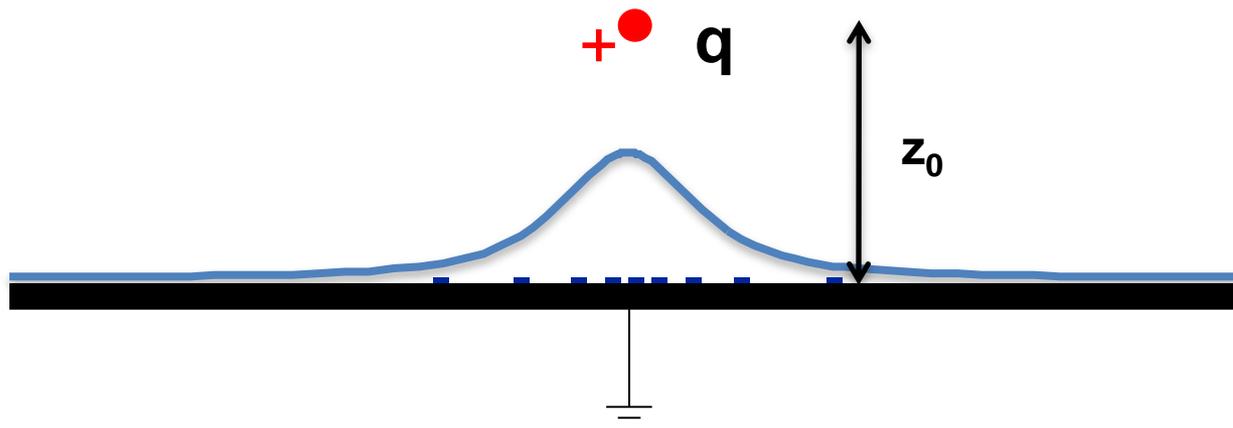
Induced Charges

In order to find the charge induced on an electrode we therefore have to

a) Solve the Poisson equation with boundary condition that $\varphi=0$ on the conductor surface.

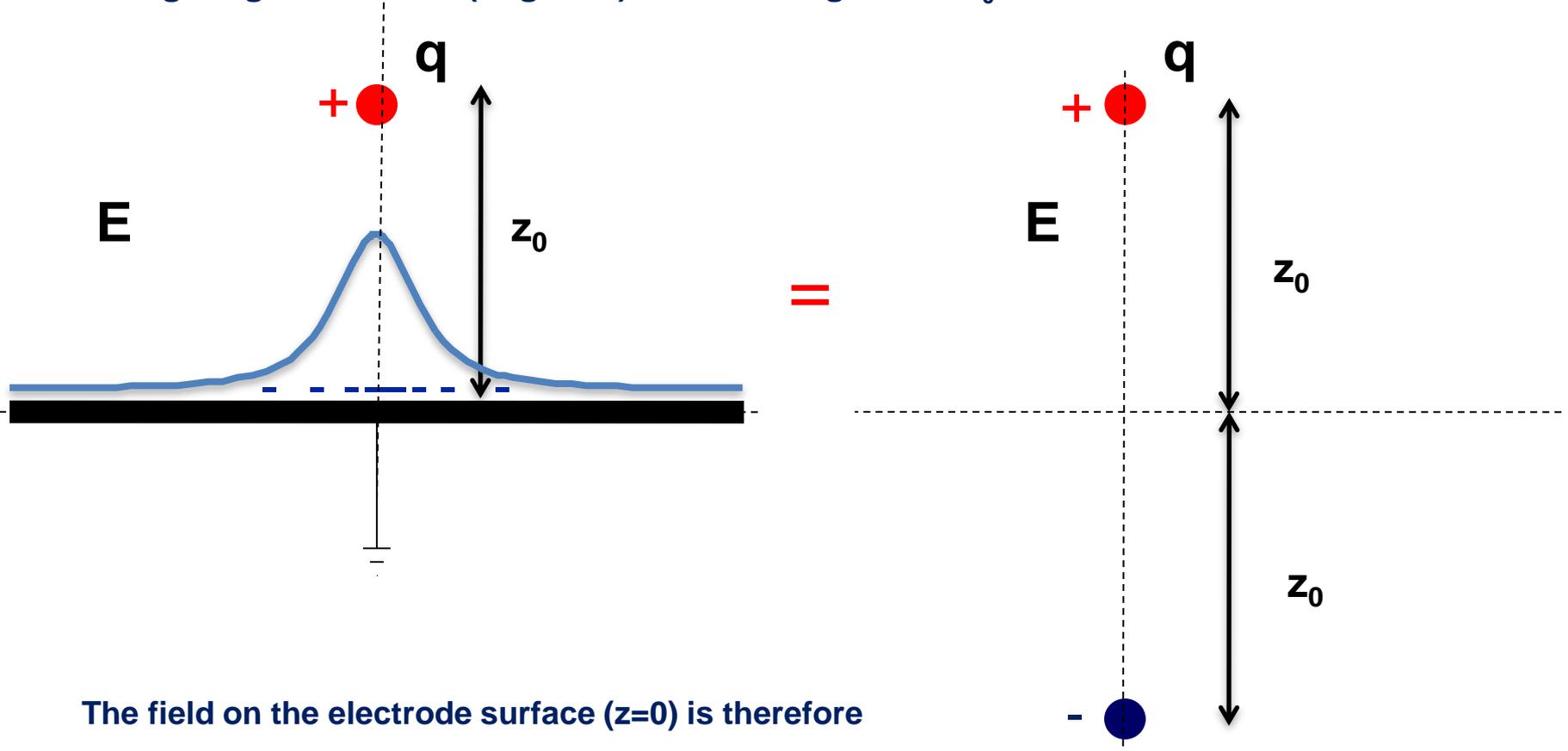
b) Calculate the electric field E on the surface of the conductor

c) Integrate $e_0 E$ over the electrode surface.



Induced Charges

The solution for the field of a point charge in front of a metal plate is equal to the solution of the charge together with a (negative) mirror charge at $z=-z_0$.



The field on the electrode surface ($z=0$) is therefore

$$E_z(x, y) = -\frac{qz_0}{2\pi\epsilon_0(x^2 + y^2 + z_0^2)^{\frac{3}{2}}}$$

$$E_x = E_y = 0$$

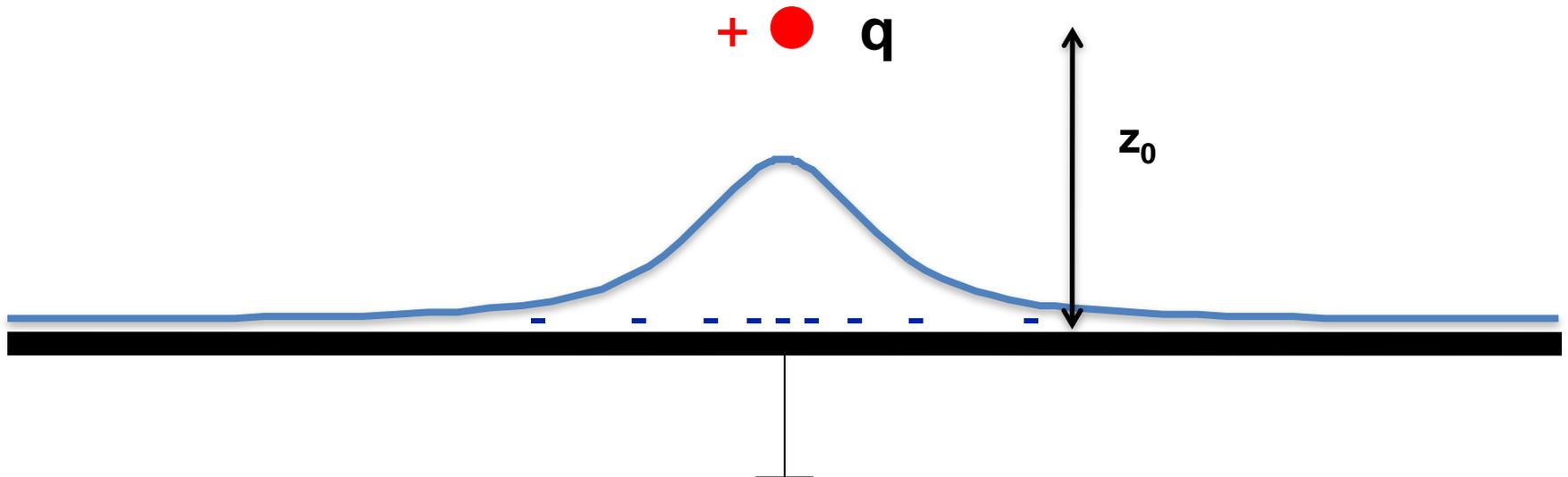
Induced Charges

We therefore find a surface charge density of

$$\sigma(x, y) = \varepsilon_0 E_z(x, y) = -\frac{qz_0}{2\pi(x^2 + y^2 + z_0^2)^{\frac{3}{2}}}$$

And therefore a total induced charge of

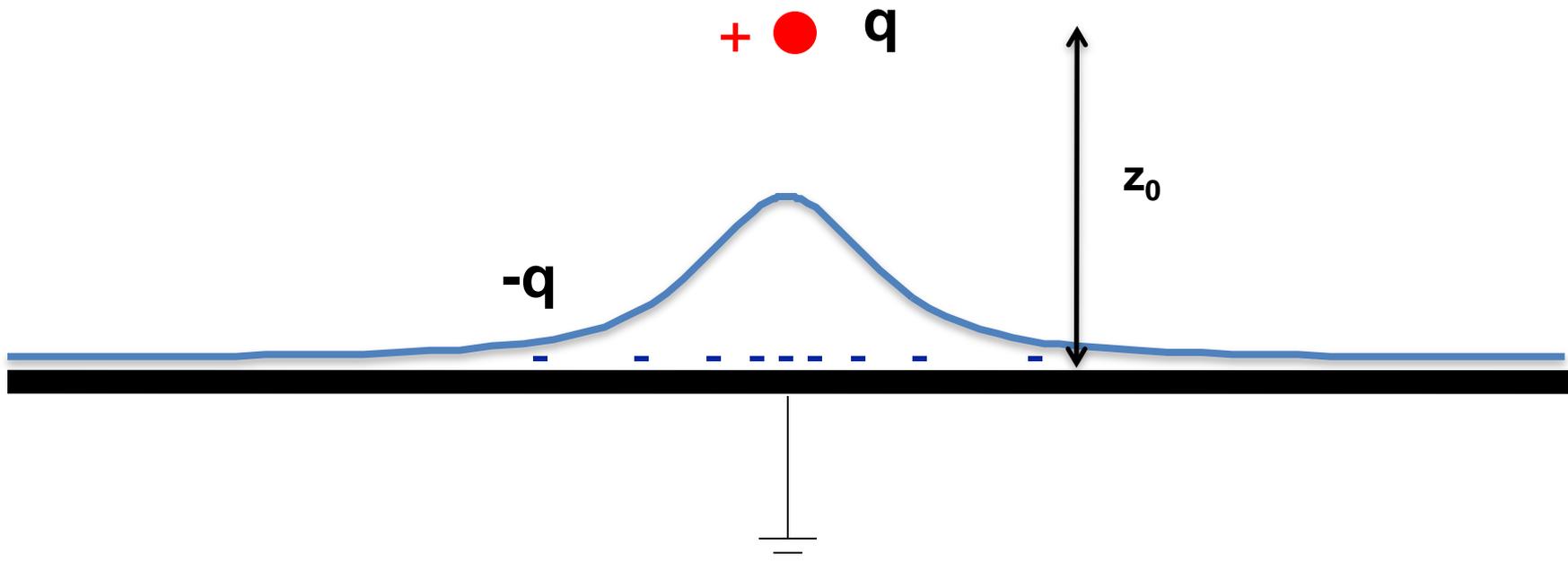
$$Q = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma(x, y) dx dy = -q$$



Induced Charges

The total charge induced by a point charge q on an infinitely large grounded metal plate is equal to $-q$, independent of the distance of the charge from the plate.

The surface charge distribution is however depending on the distance z_0 of the charge q .



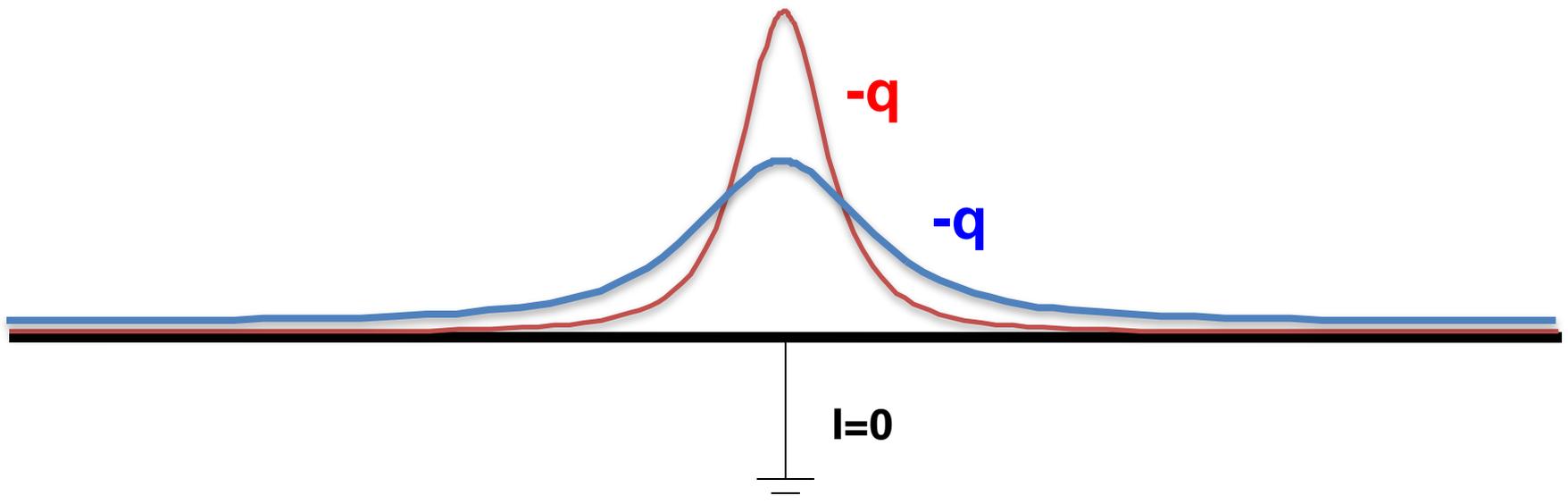
Induced Charges

Moving the point charge closer to the metal plate, the surface charge distribution becomes more peaked, the total induced charge is however always equal to $-q$.

● q

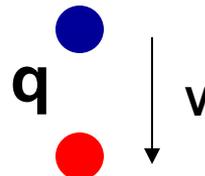
● q

$$\sigma(x, y) = -\frac{qz_0}{2\pi(x^2 + y^2 + z_0^2)^{\frac{3}{2}}}$$



Signal Induction by Moving Charges

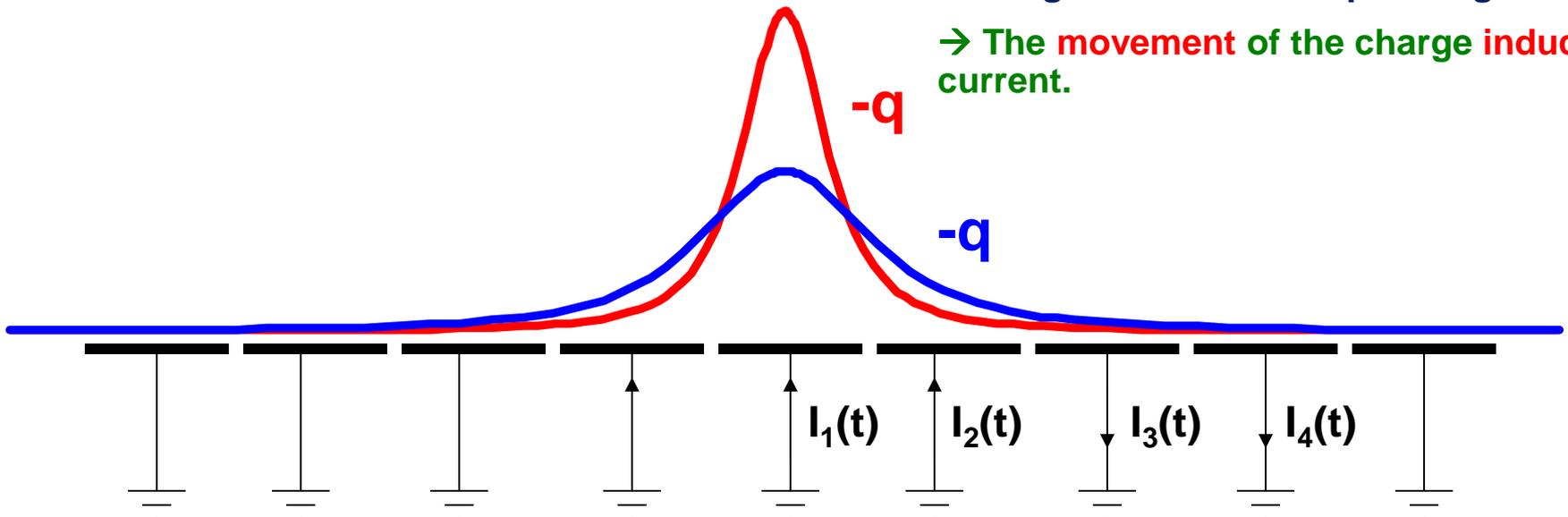
If we segment the grounded metal plate and if we ground the individual strips, the surface charge density doesn't change with respect to the continuous metal plate.



The charge induced on the individual strips is now depending on the position z_0 of the charge.

If the charge is moving there are currents flowing between the strips and ground.

→ The movement of the charge induces a current.



$$Q_1(z_0) = \int_{-\infty}^{\infty} \int_{-w/2}^{w/2} \sigma(x, y) dx dy = -\frac{2q}{\pi} \arctan\left(\frac{w}{2z_0}\right) \quad z_0(t) = z_0 - vt$$

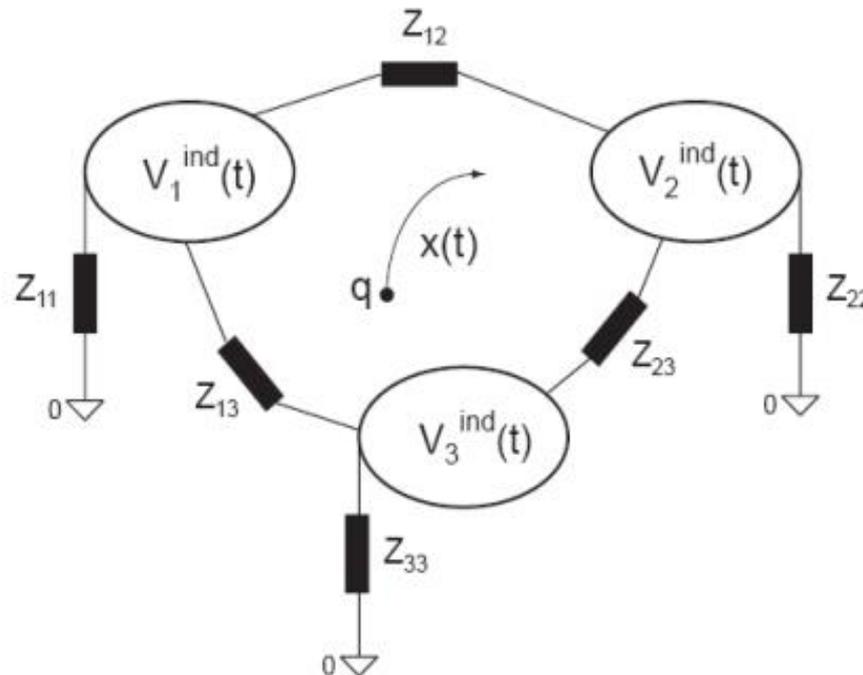
$$I_1^{ind}(t) = -\frac{d}{dt} Q_1[z_0(t)] = -\frac{\partial Q_1[z_0(t)]}{\partial z_0} \frac{dz_0(t)}{dt} = \frac{4qw}{\pi[4z_0(t)^2 + w^2]} v$$

Formulation of the Problem

In a real particle detector, the electrodes (wires, cathode strips, silicon strips, plate electrodes ...) are not grounded but they are connected to readout electronics and interconnected by other discrete elements.

We want to answer the question:

What are the voltages induced on metal electrodes by a charge q moving along a trajectory $x(t)$, in case these metal electrodes are connected by arbitrary linear impedance components ?

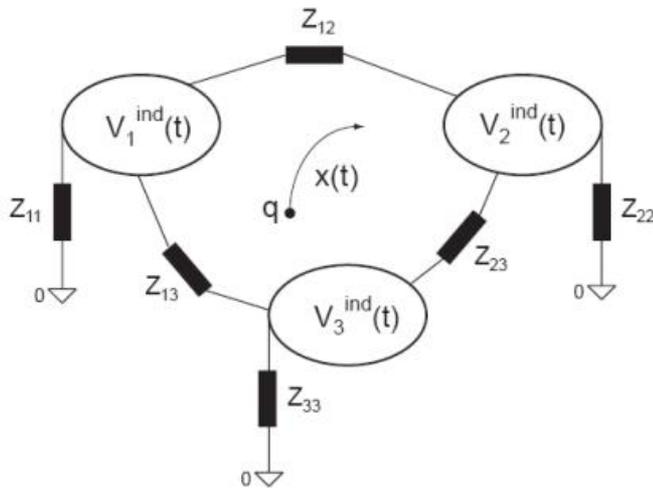


Formulation of the Problem

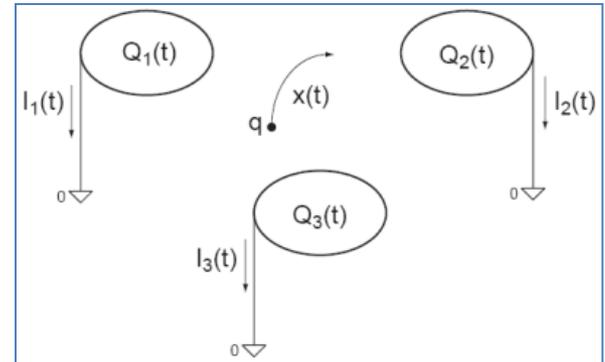
We will divide the problem into two parts:

We first calculate the currents induced on grounded electrodes.

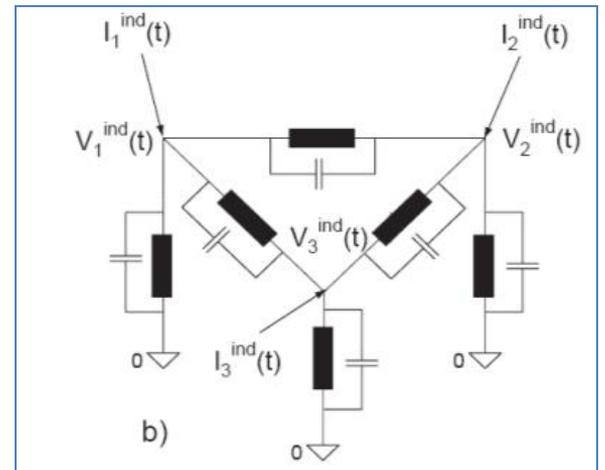
A theorem, that we will proof later, states that we then have to place these currents as ideal current sources on a circuit containing the discrete components and the mutual electrode capacitances



=



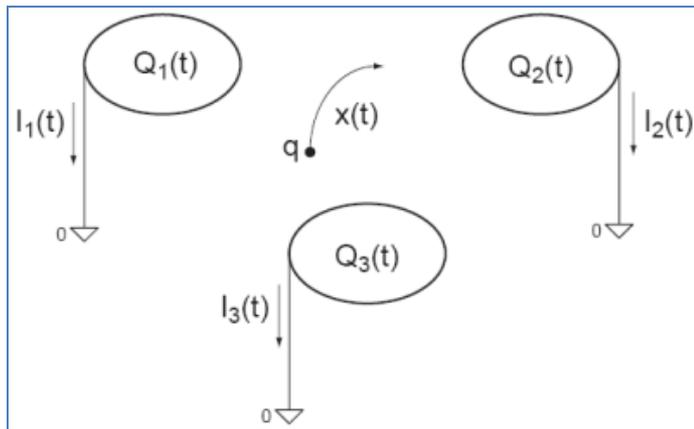
+



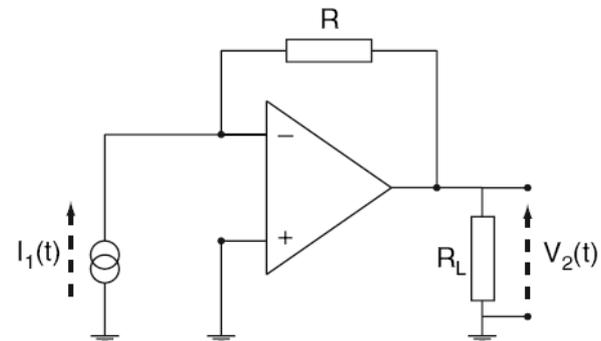
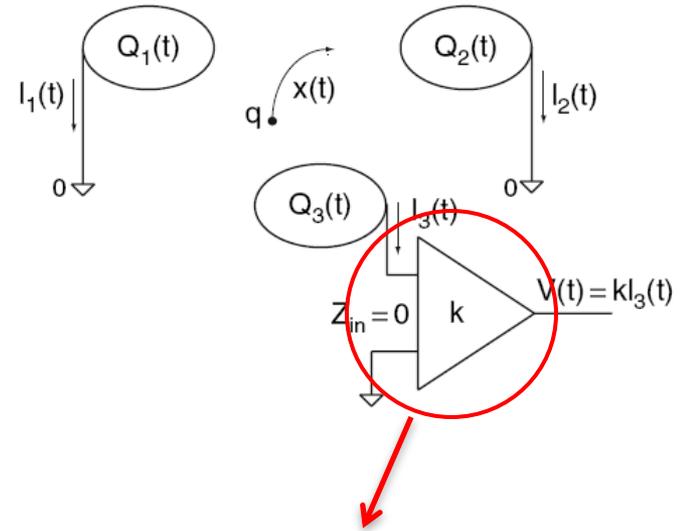
The second step is typically performed by using an analog circuit simulation program. We will first focus on the induced currents.

Currents on Grounded Electrodes

We can imagine this case by reading the signal with an ideal current amplifier of zero input impedance

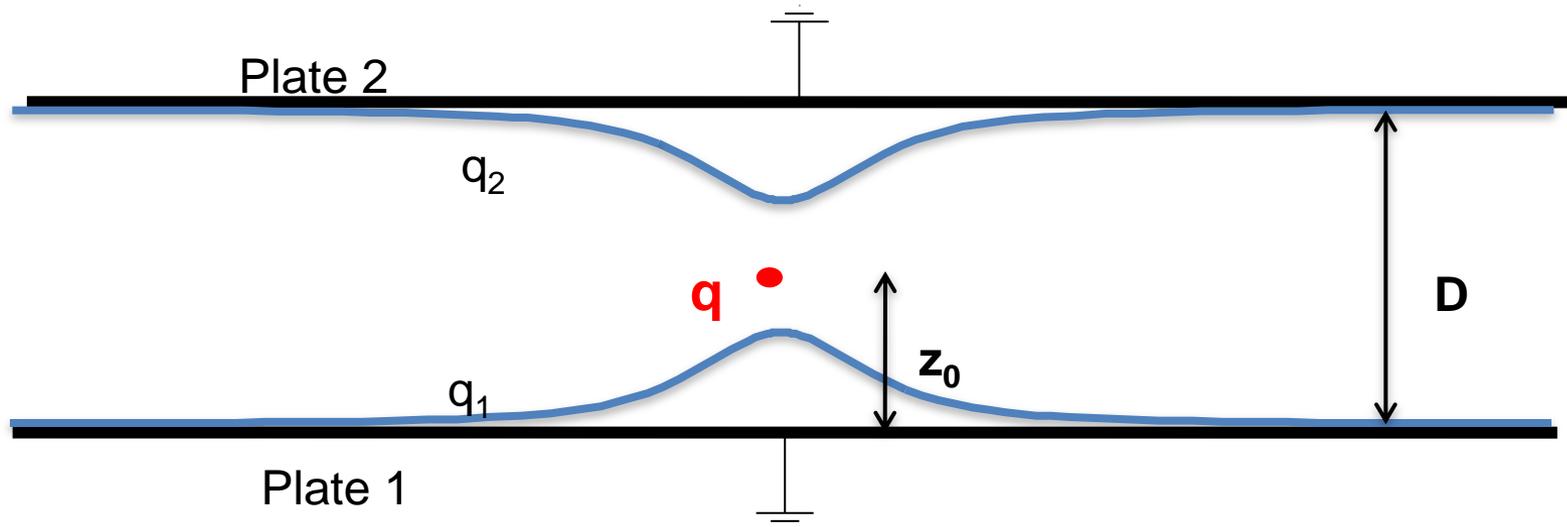


=



$$V_2(t) = -R I_1(t)$$

Parallel Plate Chamber

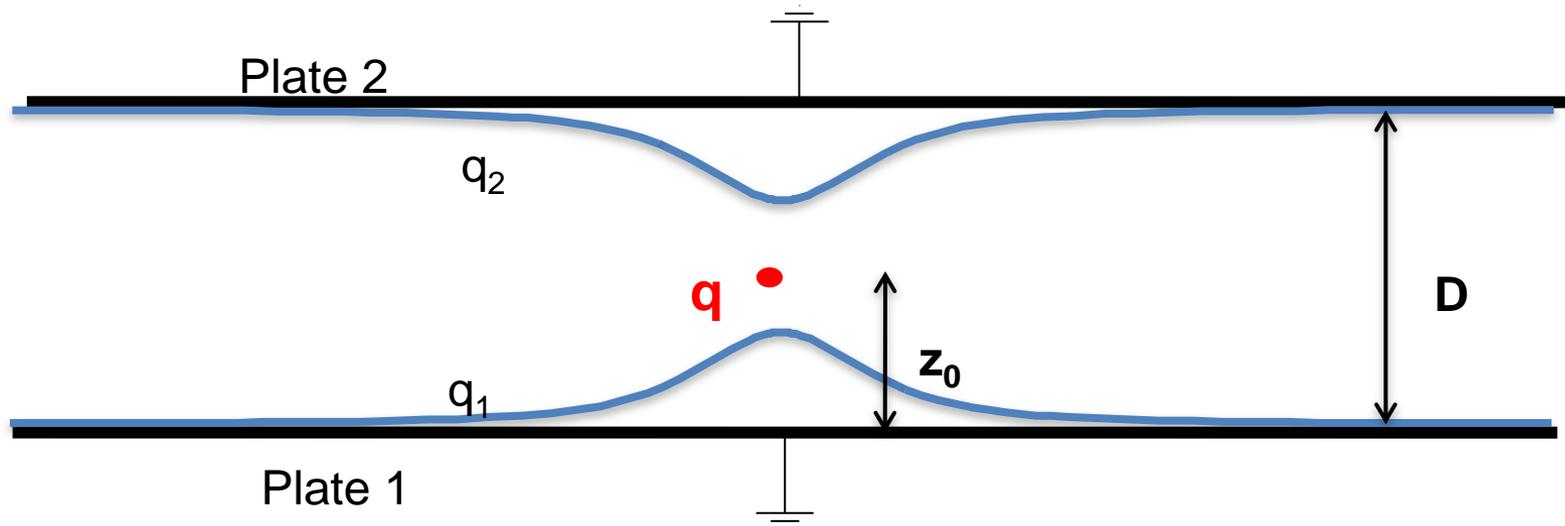


$$\phi(r, z) = \frac{q}{\varepsilon_0 \pi D} \sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{D} z\right) \sin\left(\frac{n\pi}{D} z_0\right) K_0\left(\frac{n\pi}{D} r\right) \quad [5]$$

$$E(r, z) = \frac{q}{\varepsilon_0 \pi D} \sum_{n=1}^{\infty} \frac{n\pi}{D} \cos\left(\frac{n\pi}{D} z\right) \sin\left(\frac{n\pi}{D} z_0\right) K_0\left(\frac{n\pi}{D} r\right)$$

$$\sigma_1(r) = \varepsilon_0 E(r, z = 0) = \frac{q}{\pi D} \sum_{n=1}^{\infty} \frac{n\pi}{D} \sin\left(\frac{n\pi}{D} z_0\right) K_0\left(\frac{n\pi}{D} r\right)$$

Parallel Plate Chamber



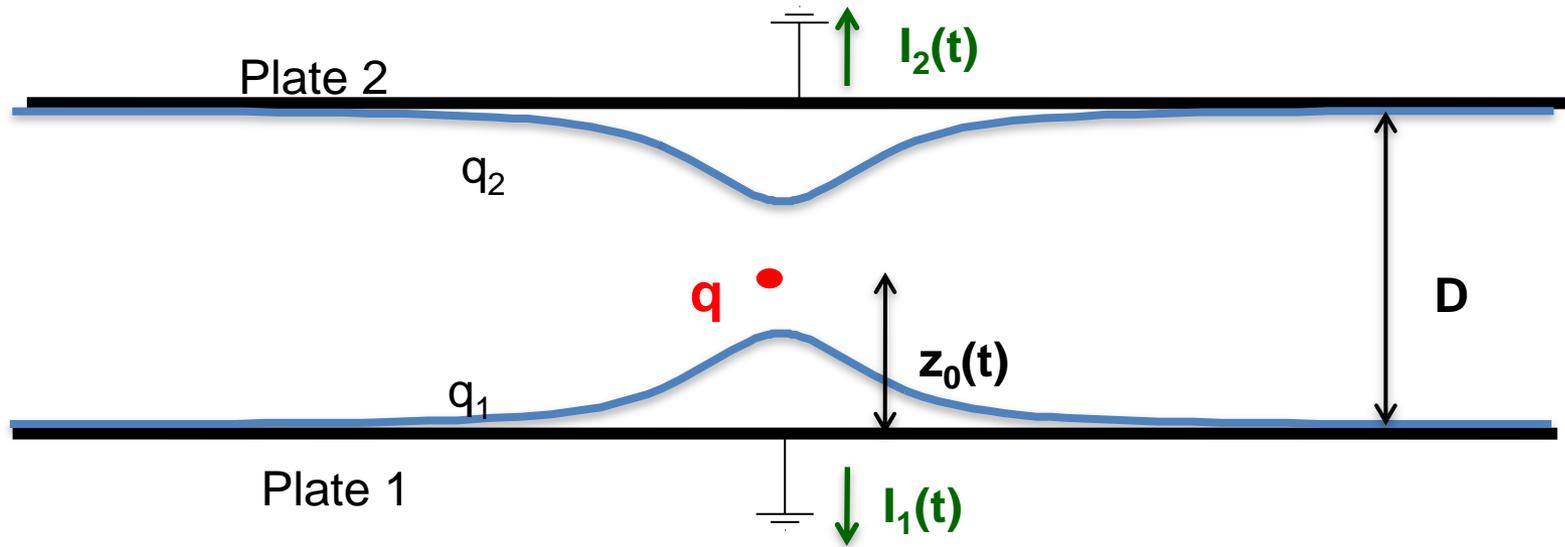
$$q_1 = \int_0^\infty 2r\pi\sigma(r)dr = \frac{q}{\pi D} \sum_{n=1}^{\infty} \frac{n\pi}{D} \sin\left(\frac{n\pi}{D} z_0\right) \int_0^\infty 2r\pi K_0\left(\frac{n\pi}{D} r\right) dr = \frac{2q}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi}{D} z_0\right) =$$

$$= -q \left(1 - \frac{z_0}{D}\right)$$

$$q_2 = \dots = -q \frac{z_0}{D}$$

$$q_1 + q_2 = -q$$

Parallel Plate Chamber



$$q_1 = -q \left(1 - \frac{z_0}{D}\right)$$

$$q_2 = -q \frac{z_0}{D}$$

$$z_0(t) = vt$$

$$q_1(t) = -q \left(1 - \frac{vt}{D}\right)$$

$$q_2(t) = -q \frac{vt}{D}$$

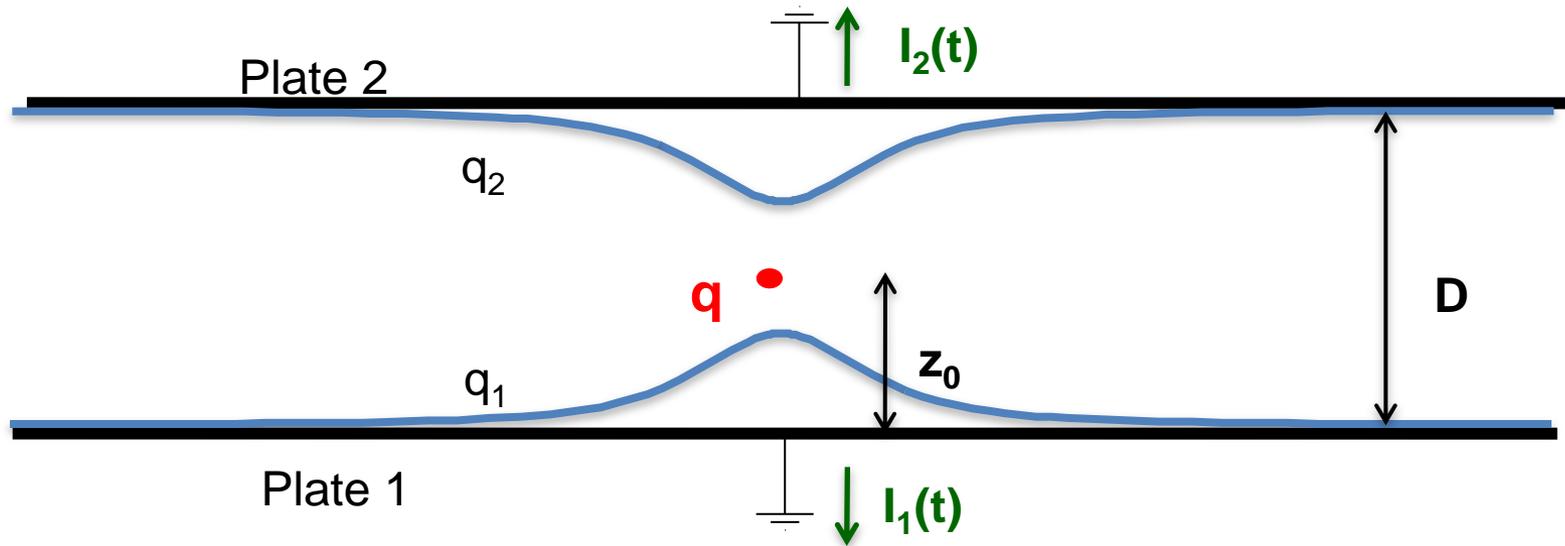
$$q_1(t) + q_2(t) = q$$

$$I_1(t) = -\frac{dq_1(t)}{dt} = -\frac{qv}{D}$$

$$I_2(t) = -\frac{dq_2(t)}{dt} = +\frac{qv}{D}$$

$$I_1(t) + I_2(t) = 0$$

Parallel Plate Chamber



$$q_1(t) = -q \left(1 - \frac{vt}{D} \right) \quad q_2(t) = -q \frac{vt}{D} \quad I_1(t) = -\frac{dq_1(t)}{dt} = -\frac{qv}{D} \quad I_2(t) = -\frac{dq_2(t)}{dt} = +\frac{qv}{D}$$

The sum of all induced charges is equal to the moving charge at any time.

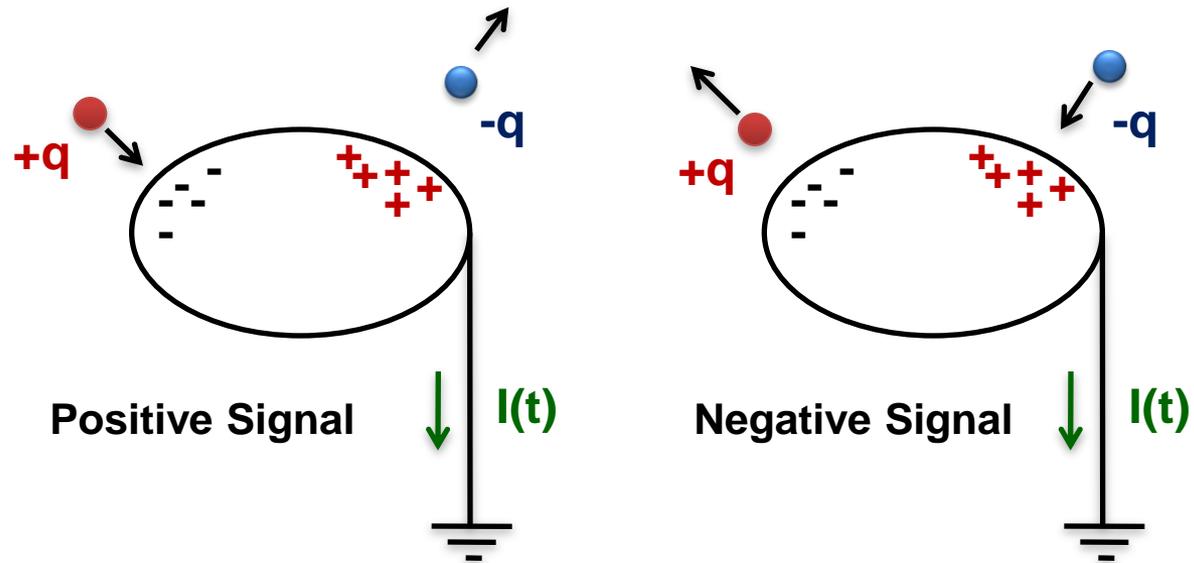
The sum of the induced currents is zero at any time.

The field calculation is complicated, the formula for the induced signal is however very simple – there might be an easier way to calculate the signals ?

→ Ramo-Shockley theorem !

Signal Polarity Definition

$$I(t) = -\frac{dQ(t)}{dt}$$



The definition of $I = -dQ/dt$ states that the positive current is pointing away from the electrode.

The signal is positive if:

Positive charge is moving from electrode to ground or

Negative charge is moving from ground to the electrode

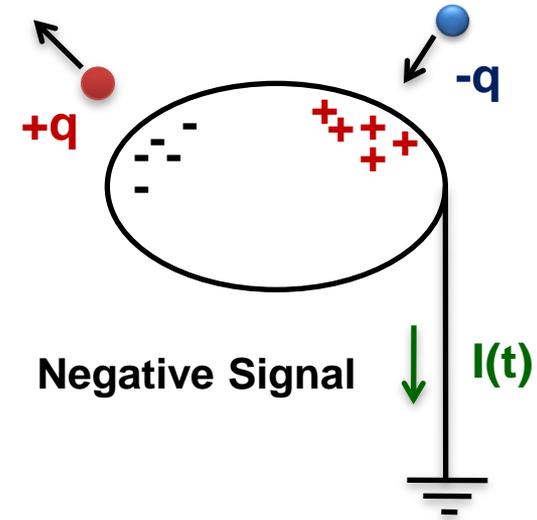
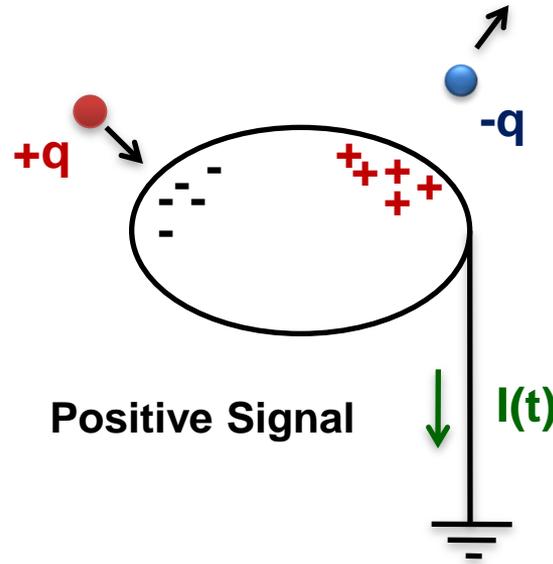
The signal is negative if:

Negative charge is moving from electrode to ground or

Positive charge is moving from ground to the electrode

Signal Polarity Definition

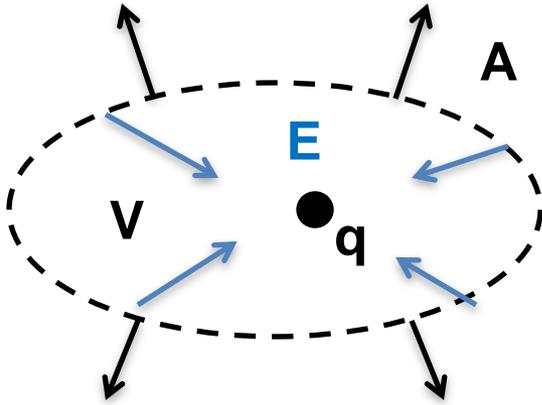
$$I(t) = -\frac{dQ(t)}{dt}$$



By this we can guess the signal polarities:

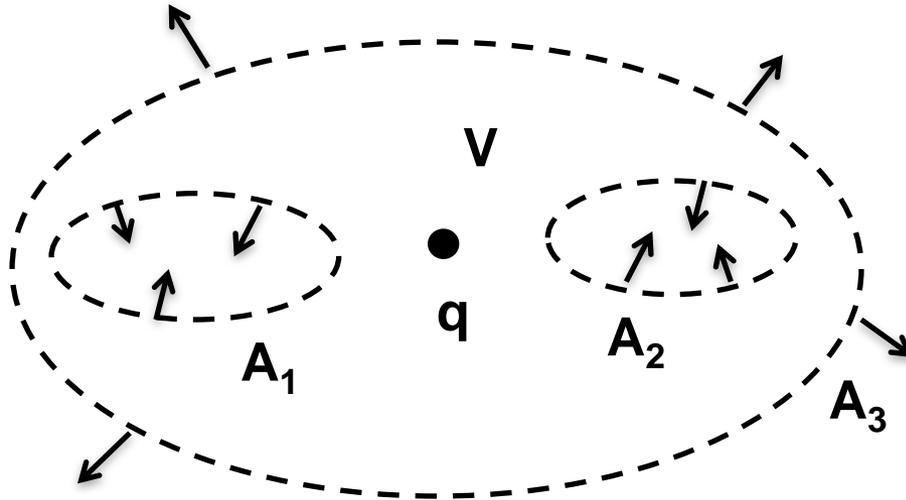
In a wire chamber, the electrons are moving towards the wire, which means that they attract positive charges that are moving from ground to the electrode. The signal of a wire that collects electrons is therefore negative.

Sum of Induced Charges and Currents



$$\oint_{\vec{A}} \vec{E} d\vec{A} = \frac{1}{\epsilon_0} \oint_V \rho dV = \frac{q}{\epsilon_0}$$

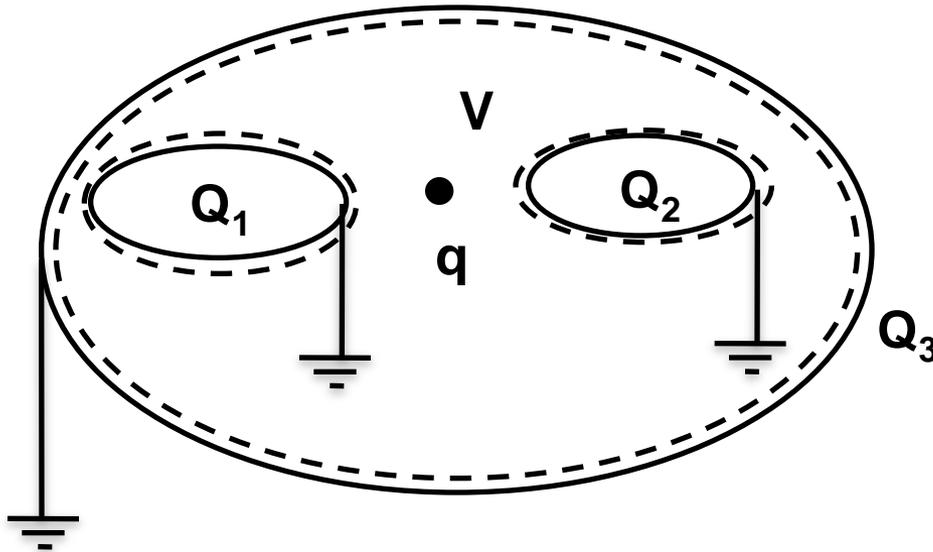
The surface A must be oriented towards the outside of the volume V.



$$\mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2 + \mathbf{A}_3$$

$$\oint_{\vec{A}} \vec{E} d\vec{A} = \oint_{\vec{A}_1} \vec{E} d\vec{A} + \oint_{\vec{A}_2} \vec{E} d\vec{A} + \oint_{\vec{A}_3} \vec{E} d\vec{A} = \frac{q}{\epsilon_0}$$

Sum of Induced Charges and Currents



$$\oint_{\vec{A}} \vec{E} d\vec{A} = \oint_{\vec{A}_1} \vec{E} d\vec{A} + \oint_{\vec{A}_2} \vec{E} d\vec{A} + \oint_{\vec{A}_3} \vec{E} d\vec{A} = \frac{q}{\epsilon_0}$$

In case the surfaces are metal electrodes we know that

$$Q_1 = - \oint_{\vec{A}_1} \epsilon_0 \vec{E} d\vec{A} \quad Q_2 = - \oint_{\vec{A}_2} \epsilon_0 \vec{E} d\vec{A} \quad Q_3 = - \oint_{\vec{A}_3} \epsilon_0 \vec{E} d\vec{A}$$

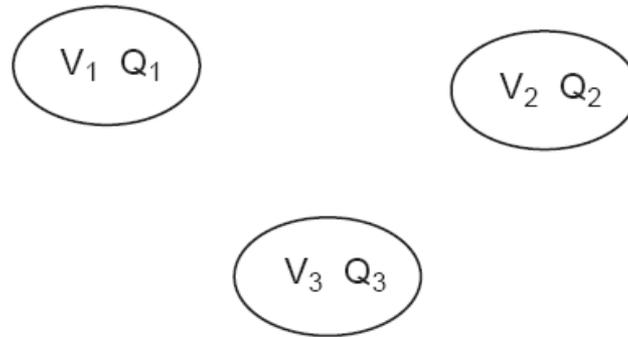
And we therefore have

$$Q_1 + Q_2 + Q_3 = -q$$

In case there is one electrode enclosing all the others, the sum of all induced charges is always equal to the point charge.

The sum of all induced currents is therefore zero at any time !

Charged Electrodes



Setting the three electrodes to potentials V_1 , V_2 , V_3 results in charges Q_1 , Q_2 , Q_3 . In order to find them we have to solve the Laplace equation

$$\Delta\varphi = 0$$

with boundary condition

$$\varphi|_{\vec{A}_1} = V_1 \quad \varphi|_{\vec{A}_2} = V_2 \quad \varphi|_{\vec{A}_3} = V_3$$

And then calculate

$$Q_1 = \oint_{\vec{A}_1} -\vec{\nabla}\varphi d\vec{A} \quad Q_2 = \oint_{\vec{A}_2} -\vec{\nabla}\varphi d\vec{A} \quad Q_3 = \oint_{\vec{A}_3} -\vec{\nabla}\varphi d\vec{A}$$

Green's Second Theorem

Gauss Law which is valid for Vector Field and Volume V surrounded by the Surface A:

$$\oint_{\vec{A}} \vec{E} d\vec{A} = \oint_V \vec{\nabla} \cdot \vec{E} dV$$

By setting

$$\vec{E} = \varphi \vec{\nabla} \psi \quad \oint_{\vec{A}} \varphi \vec{\nabla} \psi d\vec{A} = \oint_V \vec{\nabla} \varphi \cdot \vec{\nabla} \psi dV + \oint_V \varphi \Delta \psi dV$$

and setting

$$\vec{E} = \psi \vec{\nabla} \varphi \quad \oint_{\vec{A}} \psi \vec{\nabla} \varphi d\vec{A} = \oint_V \vec{\nabla} \psi \cdot \vec{\nabla} \varphi dV + \oint_V \psi \Delta \varphi dV$$

and subtracting the two expressions we get Green's second theorem:

$$\oint_{\vec{A}} (\varphi \vec{\nabla} \psi - \psi \vec{\nabla} \varphi) d\vec{A} = \oint_V (\varphi \Delta \psi - \psi \Delta \varphi) dV$$

Green's Theorem, Reciprocity

$$V_1 \quad Q_1$$

$$V_2 \quad Q_2$$

$$V_3 \quad Q_3$$

$$\Delta\varphi = 0$$

$$\bar{V}_1 \quad \bar{Q}_1$$

$$\bar{V}_2 \quad \bar{Q}_2$$

$$\bar{V}_3 \quad \bar{Q}_3$$

$$\Delta\psi = 0$$

$$\varphi|_{\vec{A}_1} = V_1 \quad \varphi|_{\vec{A}_2} = V_2 \quad \varphi|_{\vec{A}_3} = V_3$$

$$\psi|_{\vec{A}_1} = \bar{V}_1 \quad \psi|_{\vec{A}_2} = \bar{V}_2 \quad \psi|_{\vec{A}_3} = \bar{V}_3$$

$$Q_1 = \oint_{\vec{A}_1} -\vec{\nabla}\varphi d\vec{A} \quad Q_2 = \oint_{\vec{A}_2} -\vec{\nabla}\varphi d\vec{A} \quad Q_3 = \oint_{\vec{A}_3} -\vec{\nabla}\varphi d\vec{A}$$

$$\bar{Q}_1 = \oint_{\vec{A}_1} -\vec{\nabla}\psi d\vec{A} \quad \bar{Q}_2 = \oint_{\vec{A}_2} -\vec{\nabla}\psi d\vec{A} \quad \bar{Q}_3 = \oint_{\vec{A}_3} -\vec{\nabla}\psi d\vec{A}$$

$$\oint_{\vec{A}} (\varphi \vec{\nabla}\psi - \psi \vec{\nabla}\varphi) d\vec{A} = \int_V (\varphi \Delta\psi - \psi \Delta\varphi) dV$$



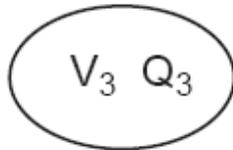
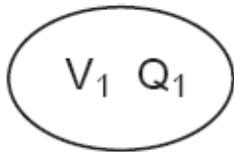
$$\sum_{n=1}^N Q_n \bar{V}_n = \sum_{n=1}^N \bar{Q}_n V_n$$

Reciprocity Theorem

It relates two electrostatic states, i.e. two sets of voltages and charges

Electrostatics, Capacitance Matrix

From the reciprocity theorem it follows that the voltages of the electrodes and the charges on the electrodes are related by a matrix



$$Q_n = \sum_{m=1}^N c_{nm} V_m$$

The matrix c_{nm} is called the capacitance matrix with the important properties

$$c_{nm} = c_{mn} \quad c_{nm} < 0 \quad \sum_{m=1}^N c_{nm} > 0$$

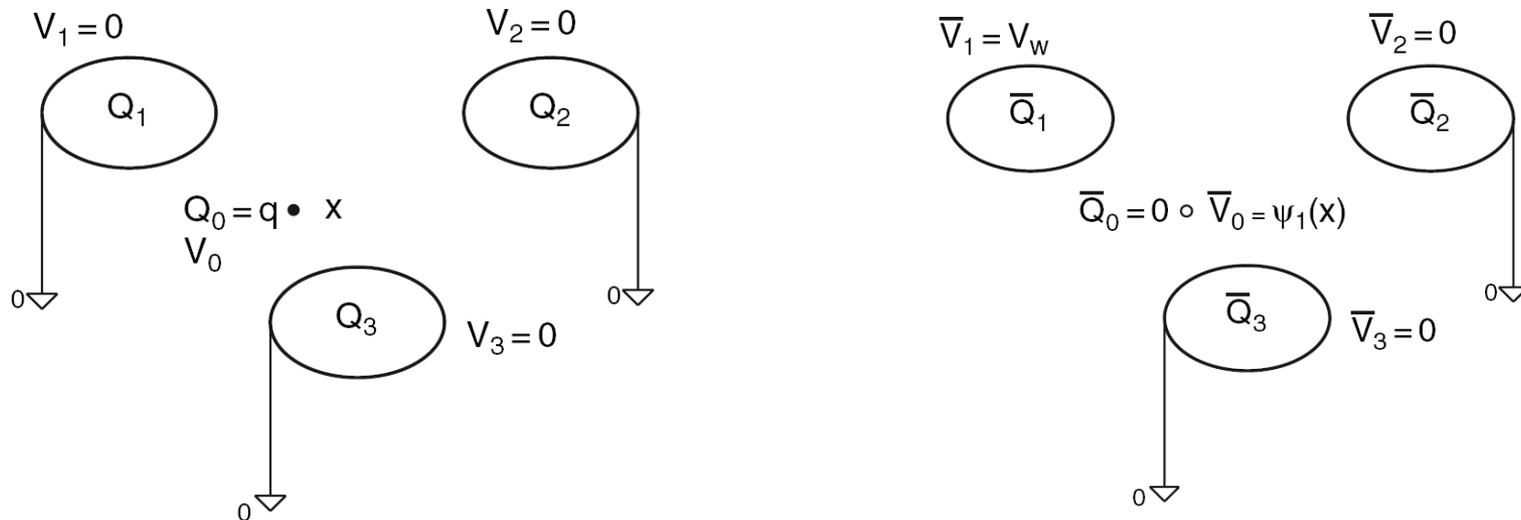
The capacitance matrix elements are not to be confused with the electrode capacitances of the equivalent circuit. They are related by

$$C_{nm} = -c_{nm} \quad n \neq m \quad C_{nn} = \sum_{m=1}^N c_{nm}$$

Induced Charge

We assume three grounded electrodes and a point charge in between. We want to know the charges induced on the grounded electrodes. We assume the point charge to be an very small metal electrode with charge q , so we have a system of 4 electrodes with $V_1=0, V_2=0, V_3=0, Q_0=q$.

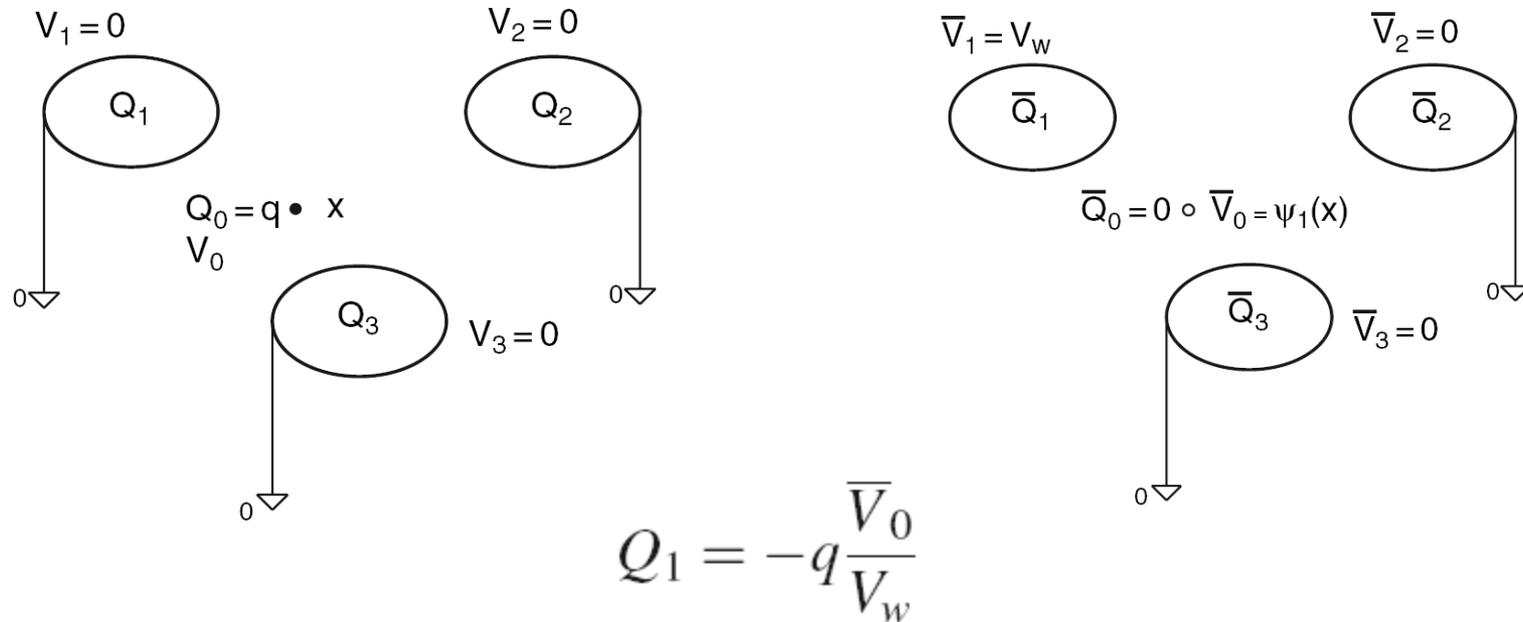
We can now assume another set of voltages and charges where we remove the charge from electrode zero, we put electrode 1 to voltage V_w and keep electrodes 2 and 3 grounded.



Using the reciprocity theorem $\sum_{n=1}^N Q_n \bar{V}_n = \sum_{n=1}^N \bar{Q}_n V_n$ we get

$$q\bar{V}_0 + Q_1 V_w = 0 \quad \rightarrow \quad Q_1 = -q \frac{\bar{V}_0}{V_w}.$$

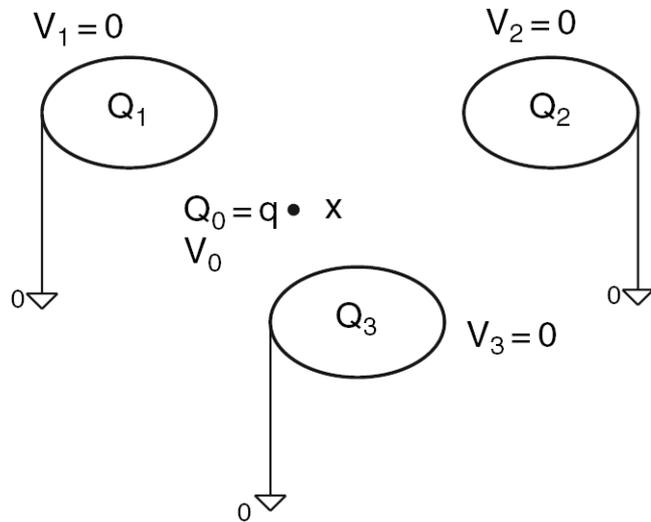
Induced Charge



The voltage \bar{V}_0 is the voltage of the small uncharged electrode for the second electrostatic state, and because a small uncharged electrode is equal to having no electrode, \bar{V}_0 is the voltage at the place x of the point charge in case the charge is removed, electrode 1 is put to voltage V_w and the other electrodes are grounded.

We call the potential $\psi(x)$ the weighting potential of electrode 1.

Induced Charge

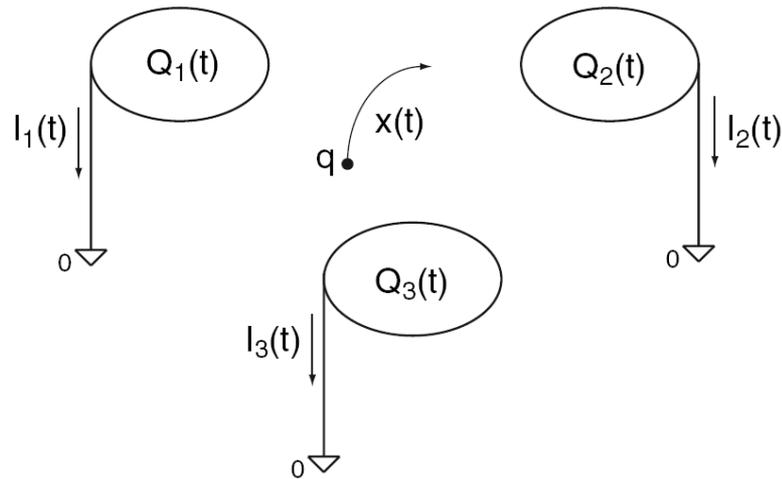


$$Q = -\frac{q}{V_w} \psi(\vec{x})$$

The charge induced by a point charge q at position x on a grounded electrode can be calculated the following way: One removes the point charge, puts the electrode in question to potential V_w while keeping the other electrodes grounded.

This defines the potential 'weighting potential' $\psi(x)$ from which the induced charge can be calculated by the above formula.

Induced Current, Ramo Schottky Theorem



$$Q = -\frac{q}{V_w} \psi(\vec{x})$$

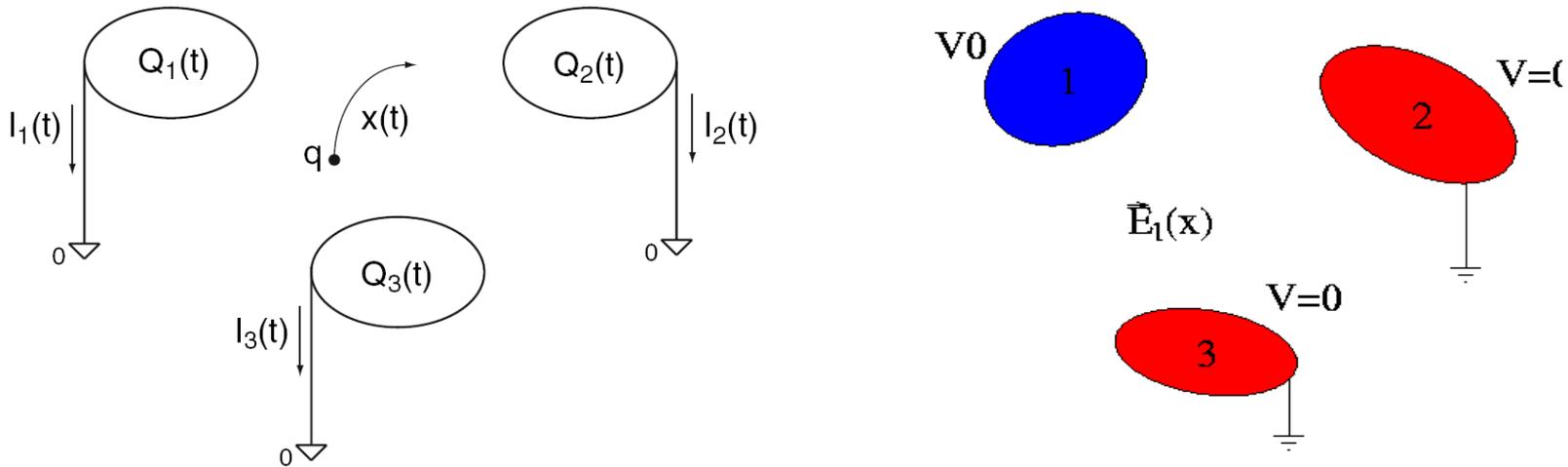
In case the charge is moving along a trajectory $x(t)$, the time dependent induced charge is

$$Q(t) = -\frac{q}{V_w} \psi(\vec{x}(t))$$

And the induced current is

$$I(t) = -\frac{dQ}{dt} = \frac{q}{V_w} \vec{\nabla} \psi(\vec{x}(t)) \frac{d\vec{x}(t)}{dt} = -\frac{q}{V_w} \vec{E}(\vec{x}(t)) \vec{v}(t)$$

Induced Current



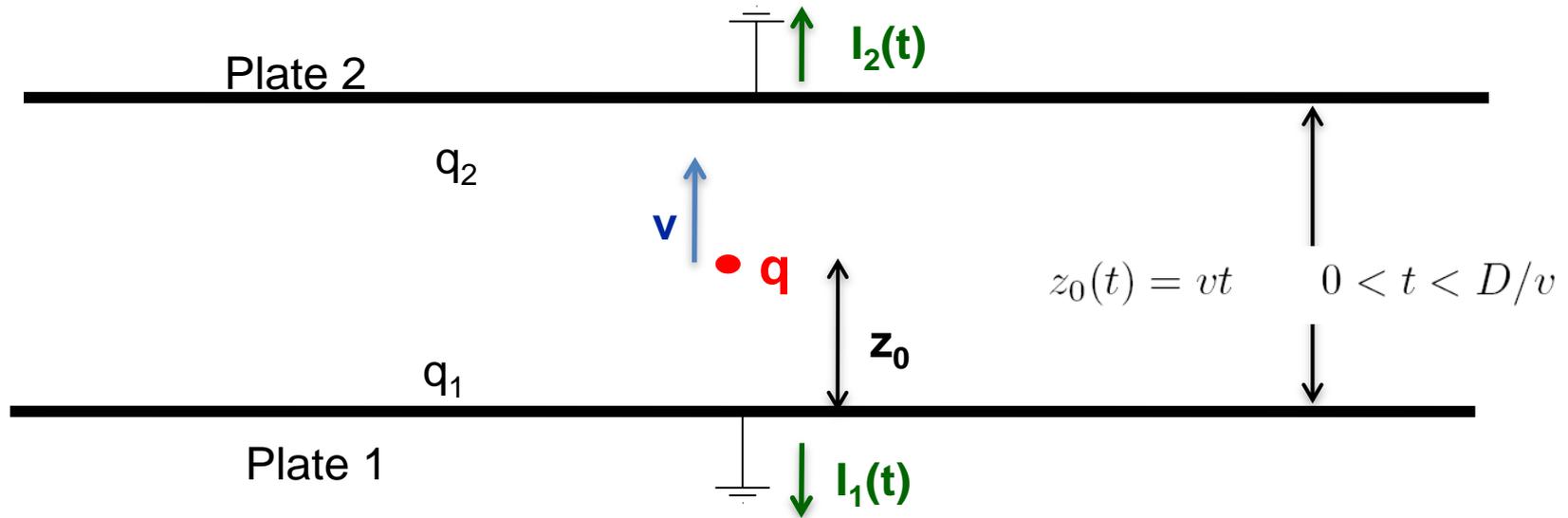
The current induced on a grounded electrode n by a moving point charge q is given by

$$I_n(t) = -\frac{q}{V_w} \vec{E}_n(\vec{x}(t)) \cdot \vec{v}(t)$$

Where the weighting field \vec{E}_n is defined by removing the point charge, setting the electrode in question to potential V_w and keeping the other electrodes grounded.

Removing the charge means that we just have to solve the Laplace equation and not the Poisson equation !

Parallel Plate Chamber



Weighting field E_1 of plate 1: Remove charge, set plate1 to V_w and keep plate 2 grounded

$$E_1 = \frac{V_w}{D}$$

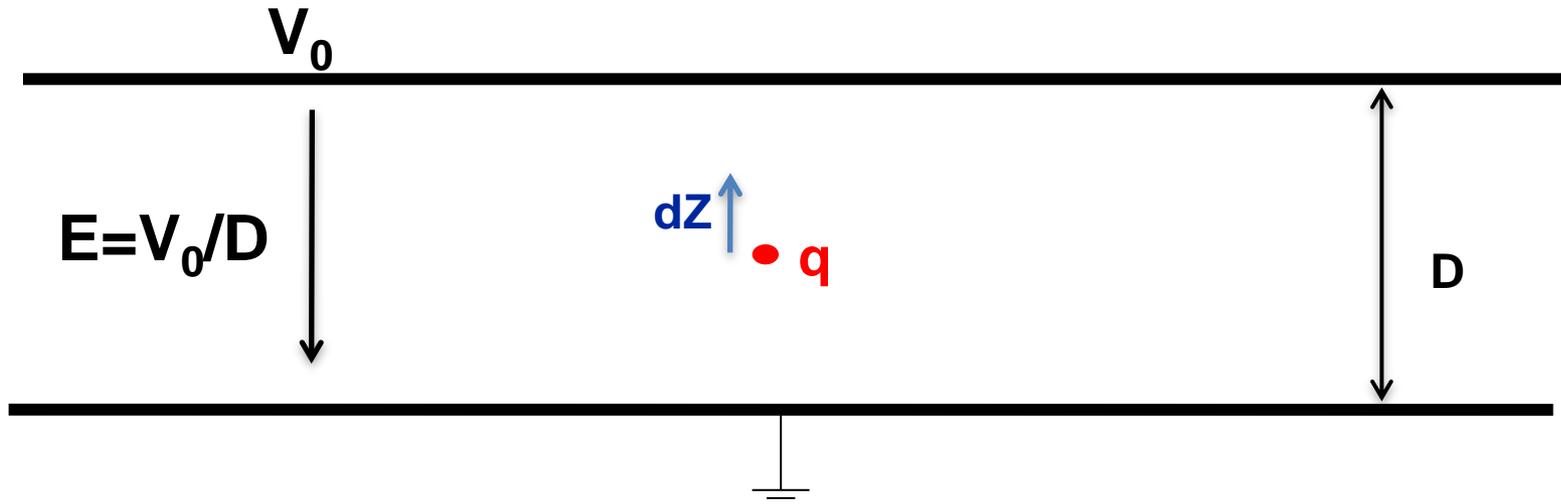
Weighting field E_2 of plate 2: Remove charge, set plate2 to V_w and keep plate1 grounded

$$E_2 = -\frac{V_w}{D}$$

So we have the induced currents

$$I_1 = -\frac{q}{V_w} \frac{V_w}{D} E_1 v = -\frac{qv}{D} \quad I_2 = -\frac{q}{V_w} \frac{V_w}{D} E_2 v = \frac{qv}{D}$$

Arguing with Energy ? Not a good Idea !



$$Energy = \frac{1}{2} C V_0^2 = \frac{1}{2} \frac{Q^2}{C}$$

$$dW = q E dz = \frac{q V_0}{D} dz$$

$$d(Energy) = dW$$

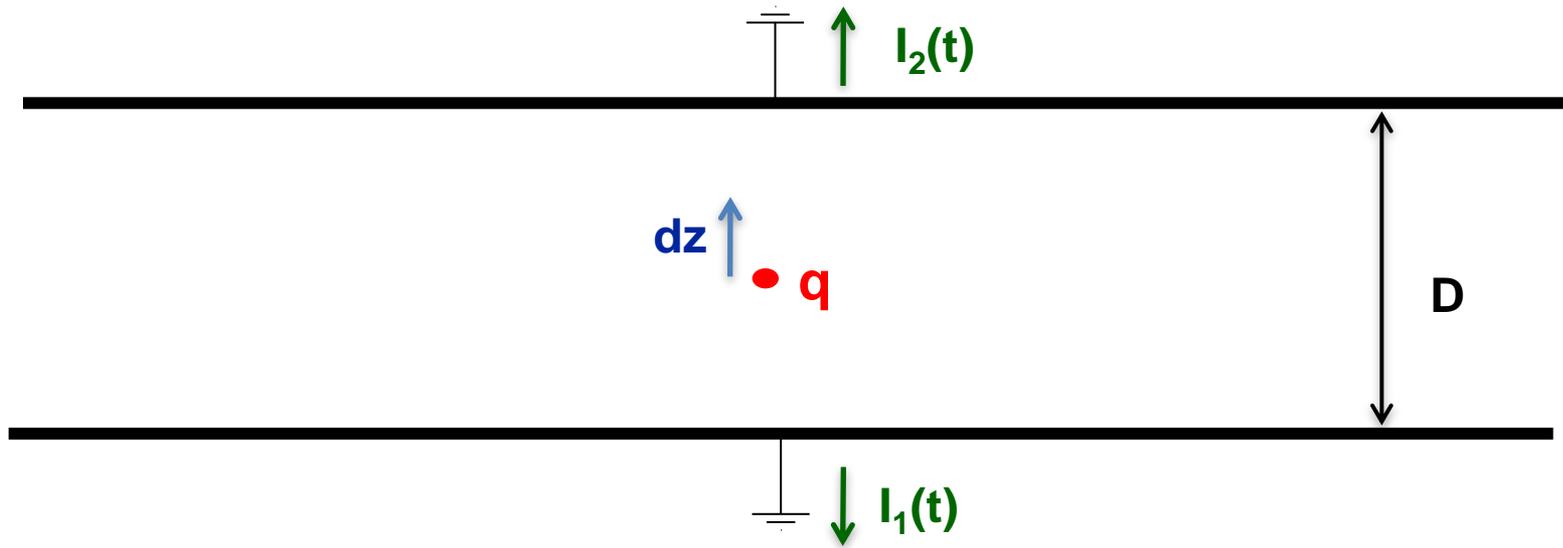
$$d \frac{1}{2} \frac{Q^2}{C} = \frac{q V_0}{D} dz$$

$$\frac{1}{2} \frac{1}{C} Q dQ = \frac{q V_0}{D} dz$$

$$dQ = \frac{q}{D} dz$$

This argument gives the correct result, it is however only correct for a 2 electrode system because there the weighting field and the real field are equal. In addition the argument is very misleading.

Arguing with Energy ? Not a good Idea !



An induced current signal has nothing to do with Energy. In a gas detector the electrons are moving at constant speed in a constant electric field, so the energy gained by the electron in the electric field is lost into collisions with the gas, i.e. heating of the gas.

In absence of an electric field, the charge can be moved across the gap without using any force and currents are flowing.

The electric signals are due to induction !

Total Induced Charge

If a charge is moving from point \mathbf{x}_0 to point \mathbf{x}_1 , the induced charge is

$$Q_n^{ind} = \int_{t_0}^{t_1} I_n^{ind}(t) dt = -\frac{q}{V_w} \int_{t_0}^{t_1} \mathbf{E}_n[\mathbf{x}(t)] \dot{\mathbf{x}}(t) dt = \frac{q}{V_w} [\psi_n(\mathbf{x}_1) - \psi_n(\mathbf{x}_0)]$$

If a pair of charges $+q$ and $-q$ is produced at point \mathbf{x}_0 and q moves to \mathbf{x}_1 while $-q$ moves to \mathbf{x}_2 , the charge induced on electrode n is given by

$$Q_n^{ind} = \int_{t_0}^{t_1} I_n^{ind}(t) dt = \frac{q}{V_w} [\psi_n(\mathbf{x}_1) - \psi_n(\mathbf{x}_2)]$$

If the charge q moves to electrode n while the charge $-q$ moves to another electrode, the total induced charge on electrode n is q , because the ψ_n is equal to V_w on electrode n and equal to zero on all other electrodes.

In case both charges go to different electrodes the total induced charge is zero.

After ALL charges have arrived at the electrodes, the total induced charge on a given electrode is equal to the charge that has ARRIVED at this electrode.

Current signals on electrodes that don't receive a charge are therefore strictly bipolar.

Induced Charge, 'Collected' Charge

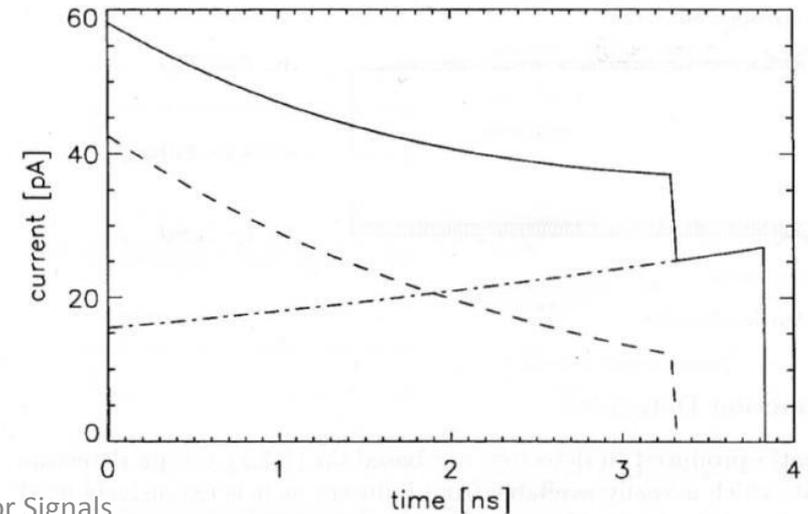
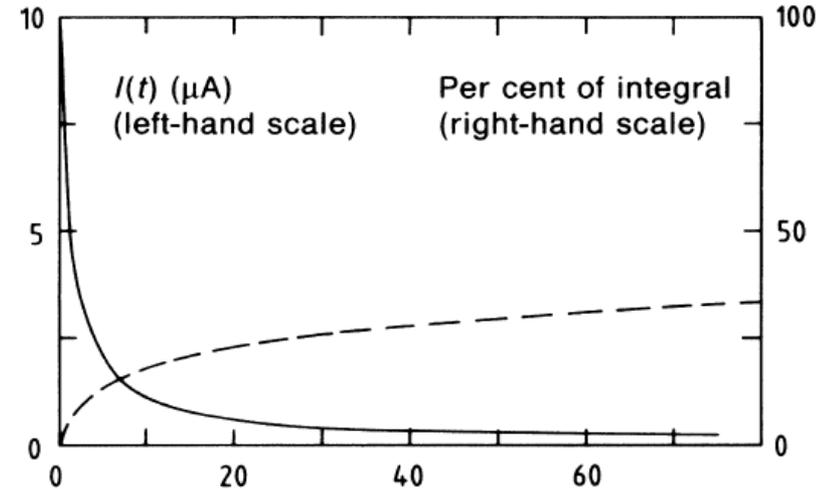
The fact that the total induced charge on an electrode, once ALL charges have arrived at the electrodes, is equal to the actual charge that has ARRIVED at the electrode, leads to very different 'vocabulary for detectors in different detectors.

In wire chambers the ions take hundreds of microseconds to arrive at the cathodes. Because the electronics 'integration time' is typically much shorter than this time, the reality that the signal is 'induced' is very well known for wire chambers, and the signal shape is dominated by the movement of the ions.

The longer the amplifier integration time, the more charge is integrated, which is sometimes called 'collected', but it has nothing to do with collecting charge from the detector volume ...

In Silicon Detectors, the electrons and holes take only a few ns to arrive at their electrodes, so e.g. for typical 'integration times' of amplifiers of 25ns, the shape is dominated by the amplifier response. The peak of the amplifier output is the proportional to the primary charge, and all the charge is 'collected'

Still, the signal is not due to charges entering the amplifier from the detector, it is due to induction by the moving charge. Once the charge has actually arrived at the electrode, the signal is over!



Total Induced Charge

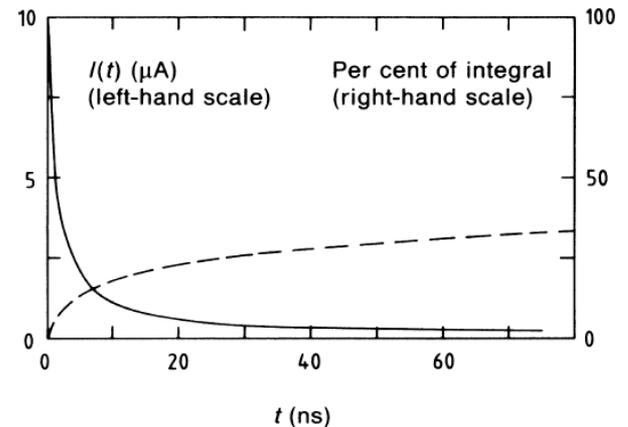
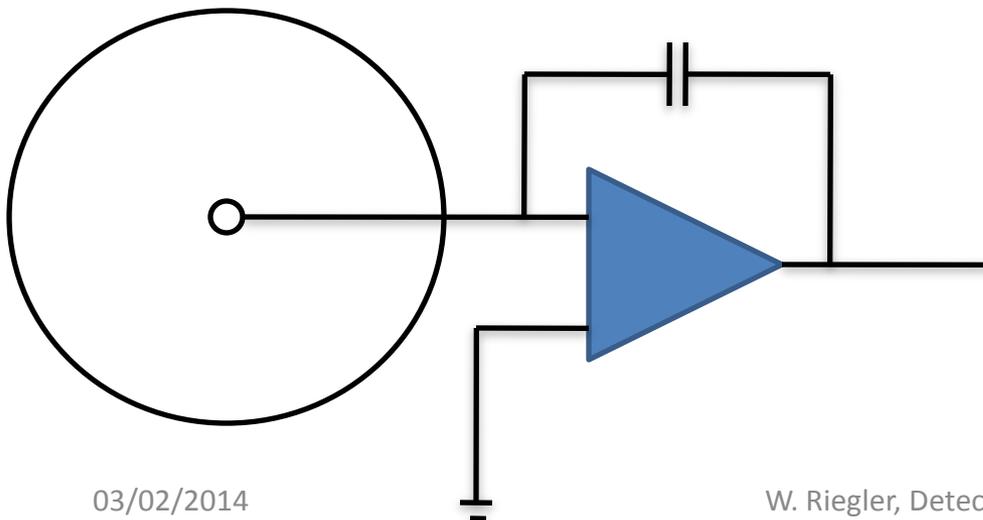
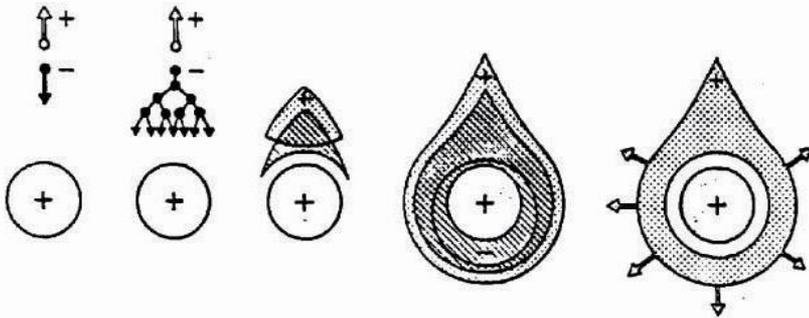
Imagine avalanche in a drift tube, caused by a single electron.

Let's assume that the gas gain is 10^4 .

We read out the wire signal with an ideal integrator

The 10^4 electrons arrive at the wire within $<1\text{ns}$, so the integrator should instantly see the full charge of $-10^4 e_0$ electrons ?

No ! The ions close to the wire induce the opposite charge on the wire, so in the very beginning there is zero charge on the integrator and only once the ions have moved away from the wire the integrator measures the full $-10^4 e_0$



Signal Calculation in 3 Steps

What are the signals induced by a moving charge on electrodes that are connected with arbitrary linear impedance elements ?

1) Calculate the particle trajectory in the ‘real’ electric field.

2) Remove all the impedance elements, connect the electrodes to ground and calculate the currents induced by the moving charge on the grounded electrodes.

The current induced on a grounded electrode by a charge q moving along a trajectory $x(t)$ is calculated the following way (Ramo Theorem):

One removes the charge q from the setup, puts the electrode to voltage V_0 while keeping all other electrodes grounded. This results in an electric field $E_n(x)$, the Weighting Field, in the volume between the electrodes, from which the current is calculated by

$$I_n(t) = -\frac{q}{V_0} \vec{E}_n[\vec{x}(t)] \frac{d\vec{x}(t)}{dt} = -\frac{q}{V_0} \vec{E}_n[\vec{x}(t)] \vec{v}(t)$$

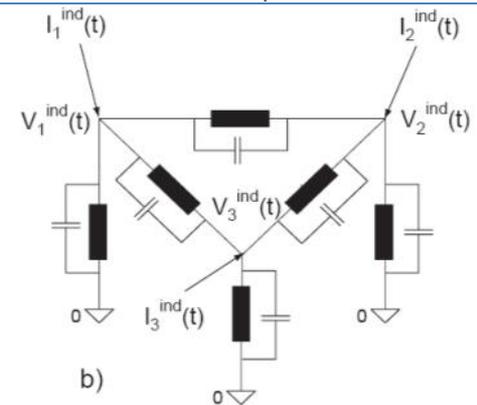
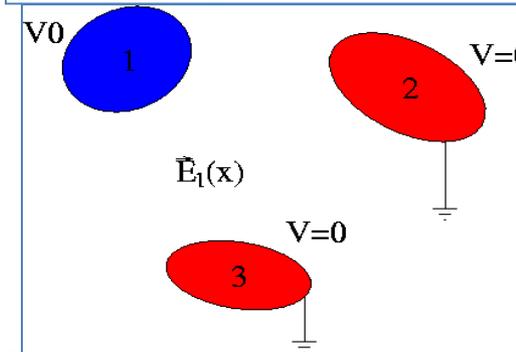
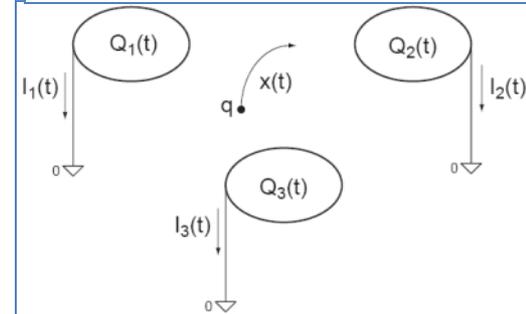
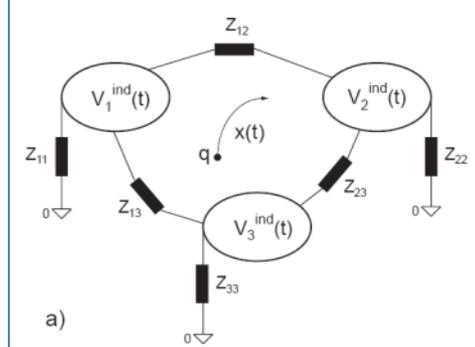
3) These currents are then placed as ideal current sources on a circuit where the electrodes are ‘shrunk’ to simple nodes and the mutual electrode capacitances are added between the nodes. These capacitances are calculated from the weighting fields by

$$c_{nm} = \frac{\epsilon_0}{V_w} \oint_{A_n} \vec{E}_m(x) dA$$

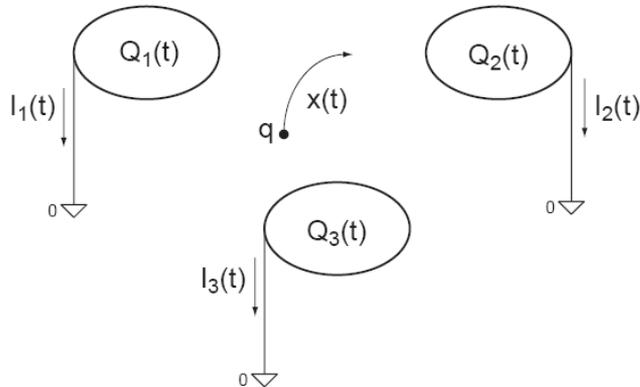
$$C_{nn} = \sum_m c_{nm} \quad C_{nm} = -c_{nm} \quad n \neq m$$

03/02/2014

W. Riegler, Detector Signals



General Signal Theorems

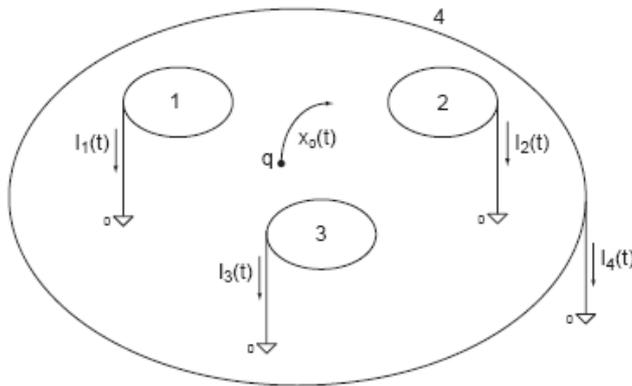


The following relations hold for the induced currents:

1) The charge induced on an electrode in case a charge in between the electrode has moved from a point \mathbf{x}_0 to a point \mathbf{x}_1 is

$$Q_n^{ind} = \int_{t_0}^{t_1} I_n^{ind}(t) dt = -\frac{q}{V_w} \int_{t_0}^{t_1} \mathbf{E}_n[\mathbf{x}(t)] \dot{\mathbf{x}}(t) dt = \frac{q}{V_w} [\psi_n(\mathbf{x}_1) - \psi_n(\mathbf{x}_0)]$$

and is independent on the actual path.



2) Once ALL charges have arrived at the electrodes, the total induced charge in the electrodes is equal to the charge that has ARRIVED at this electrode.

3) In case there is one electrode enclosing all the others, the sum of all induced currents is zero at any time.

Signals in a Parallel Plate Geometry

E.g.: Electron-ion pair in gas
 or Electron-ion pair in a liquid
 or Electron-hole pair in a solid

$$E_1 = V_0/D$$

$$E_2 = -V_0/D$$

$$I_1 = -(-q)/V_0 * (V_0/D) * v_e - q/V_0 (V_0/D) (-v_i)$$

$$= q/D * v_e + q/D * v_i$$

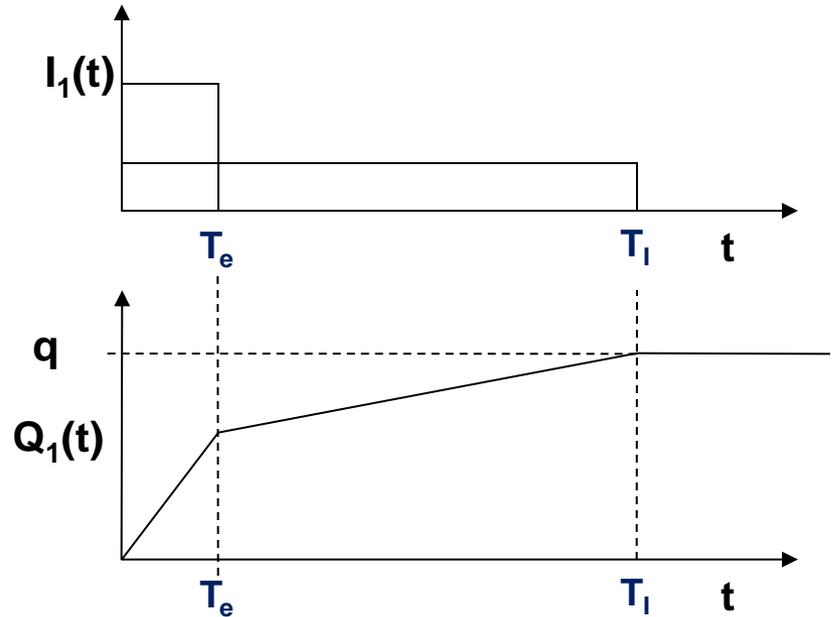
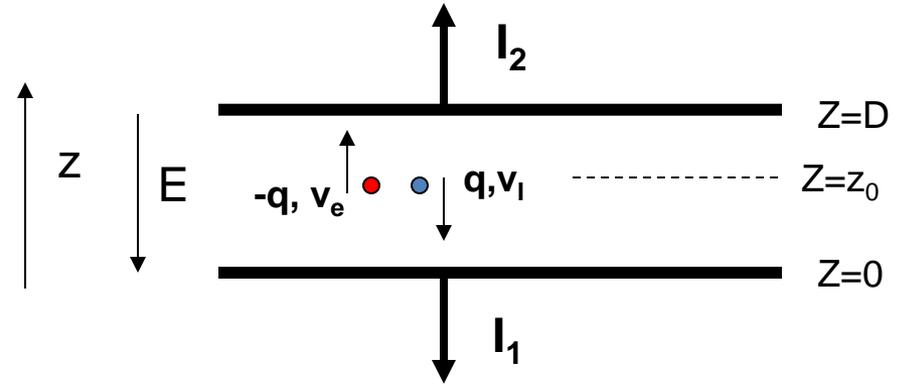
$$I_2 = -I_1$$

$$Q_1^{\text{tot}} = \int I_1 dt = q/D * v_e T_e + q/D * v_i T_i$$

$$= q/D * v_e * (D - z_0)/v_e + q/D * v_i * z_0/v_i$$

$$= q(D - z_0)/D + qz_0/D =$$

$$q_e + q_i = q$$



The total induced charge on a specific electrode, once all the charges have arrived at the electrodes, is equal to the charge that has arrived at this specific electrode.

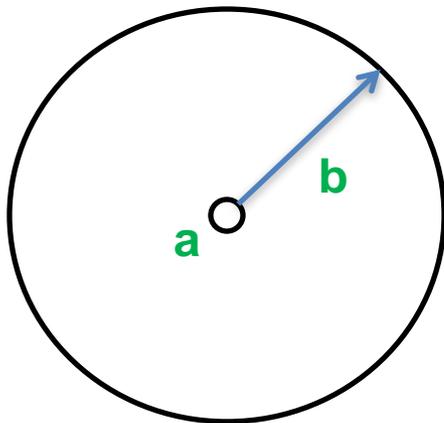
Wire Chamber Signals

Wire with radius (10-25 μm) in a tube of radius b (1-3cm):

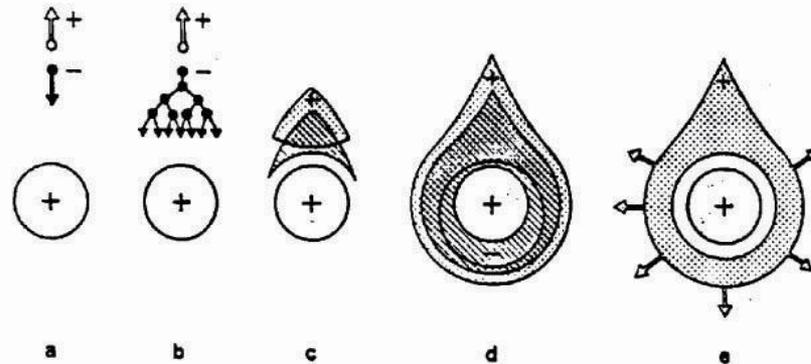
$$E(r) = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{V_0}{\ln \frac{b}{a}} \frac{1}{r}, \quad V(r) = \frac{V_0}{\ln \frac{b}{a}} \ln \frac{r}{a},$$

Electric field close to a thin wire (100-300kV/cm). E.g. $V_0=1000\text{V}$, $a=10\mu\text{m}$, $b=10\text{mm}$, $E(a)=150\text{kV/cm}$

Electric field is sufficient to accelerate electrons to energies which are sufficient to produce secondary ionization \rightarrow electron avalanche \rightarrow signal.



Wire



Wire Chamber Signals

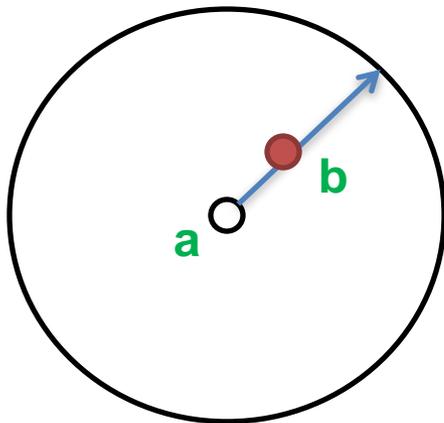
The electrons are produced very close to the wire, so for now we assume that N_{tot} ions are moving from the wire surface to the tube wall

$$E(r) = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{U}{\ln \frac{b}{a} r}, \quad V(r) = \frac{U}{\ln \frac{b}{a}} \ln \frac{r}{a}$$

Ions move with a velocity proportional to the electric field.

$$v = \mu E$$

$$\frac{dr(t)}{dt} = \mu \frac{U}{r(t) \ln(b/a)} \rightarrow r(t) = a \sqrt{1 + \frac{t}{t_0}} \quad t_0 = \frac{a^2 \ln(b/a)}{2\mu U}$$



Weighting Field of the wire: Remove charge and set wire to V_w while grounding the tube wall.

$$E_1(r) = \frac{V_w}{r \ln(b/a)}$$

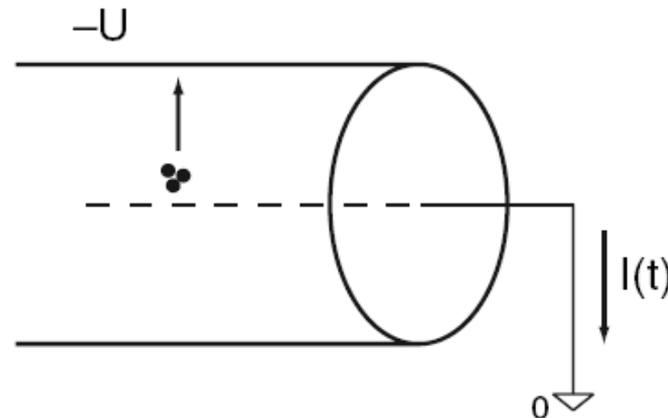
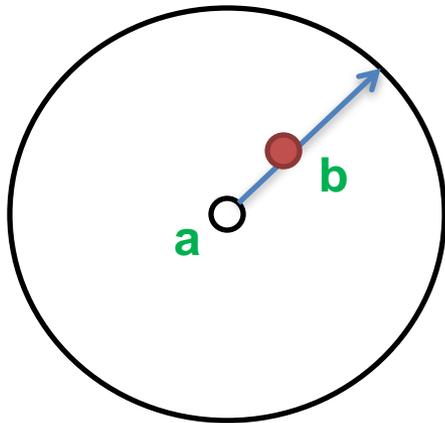
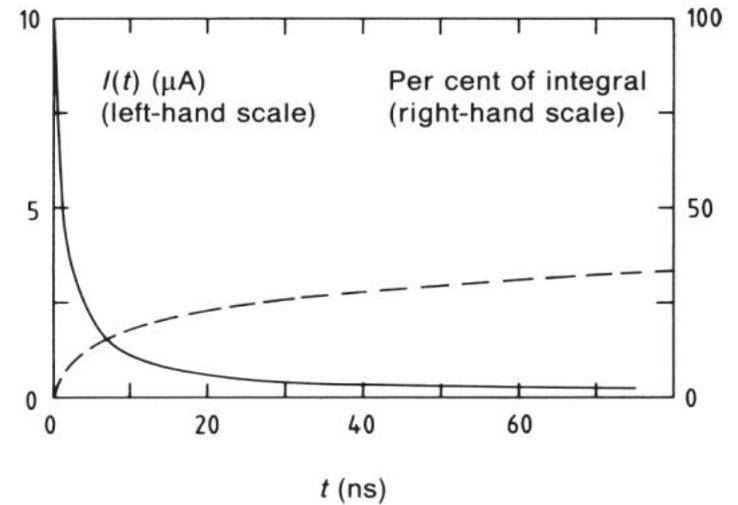
The induced current is therefore

$$I_1^{\text{ind}}(t) = -\frac{N_{\text{tot}} e_0}{V_w} E_1[r(t)] \dot{r}(t) = -\frac{N_{\text{tot}} e_0}{2 \ln(b/a)} \frac{1}{t + t_0}.$$

Wire Chamber Signals

$$I_1^{ind}(t) = -\frac{N_{tot}e_0}{V_w} E_1[r(t)] \dot{r}(t) = -\frac{N_{tot}e_0}{2 \ln(b/a)} \frac{1}{t + t_0}$$

$$Q_1^{ind}(t) = \int_0^t I_1^{ind}(t') dt' = -\frac{N_{tot}e_0}{2 \ln(b/a)} \ln \left(1 + \frac{t}{t_0} \right)$$



Conclusion

This principle of signal generation is identical for Solid State Detectors, Gas Detectors and Liquid Detectors.

The signals are due to charges (currents) induced on metal electrodes by moving charges.

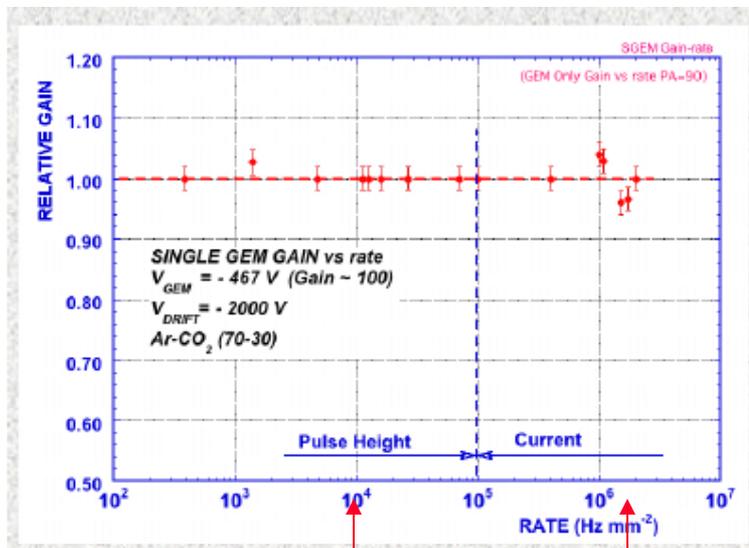
The easiest way to calculate signals induced by moving charges on metal electrodes is the use of Weighting fields (Ramo – Shockley Theorem) for calculation of currents induced on grounded electrodes.

These currents can then be placed as ideal current sources on an equivalent circuit diagram representing the detector.

Micro Pattern Gas Detector
Signal Characteristics,
Frontend Electronics Specification

GEM/Micromega Rate Limits

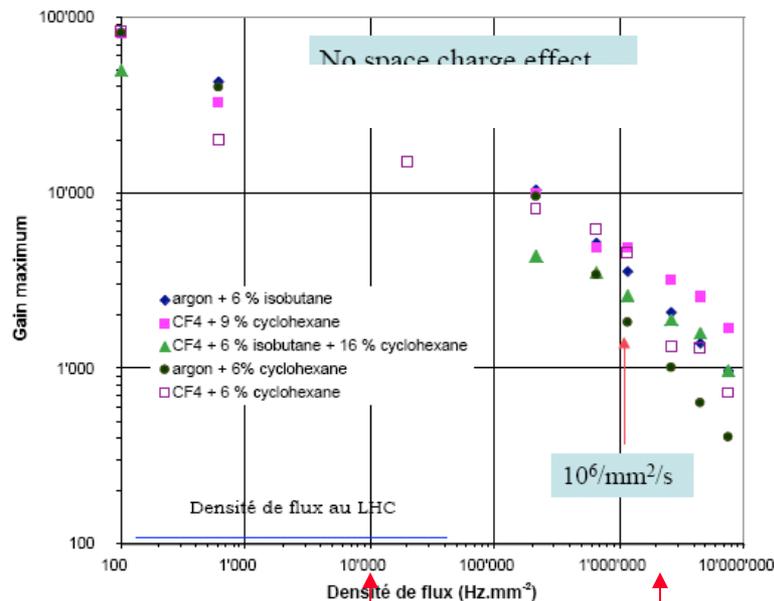
GEM



1MHz/cm²

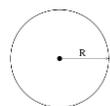
200MHz/cm²

MICROMEGA

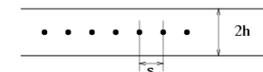


1MHz/cm²

200MHz/cm²



TRT straw tubes ($R=2\text{mm}$) and LHCb MWPCs ($s=1.5\text{mm}$, $h=2.5\text{mm}$) are limited at about 1MHz/cm² due to spacecharge.



In terms of rate capability due to space charge effects, MICROPATTERN detectors win over wire chambers by several orders of magnitude → BUT

Rate Limits

When talking about rate capability one also has to talk about **pulse-width** and **pad response function** and **dead-time** and **occupancy**

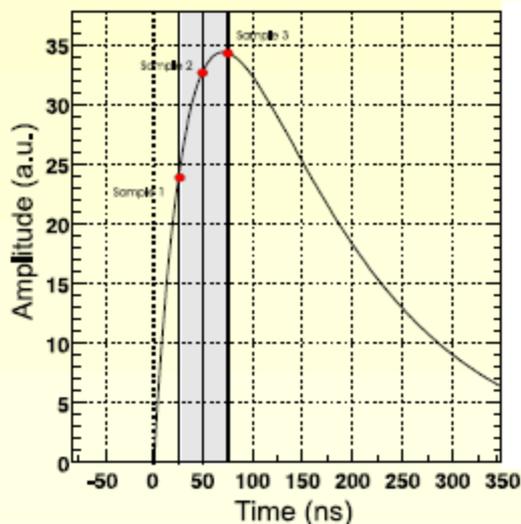
Compass Gems:

Rates of up to 2.5MHz/cm²
 Amplifier peaking time approx 50ns
 occupancy up to 25% (readout strips).

Compass MicroMegas:

Up to 0.5 MHz/cm²
 Rates of 200kHz/strip
 Amplifier peaking time approx 85ns

E.g. 1MHz on a readout channel & 100ns pulse-width → 10% inefficiency



Clearly in order to profit from 200MHz rate capability one has to

Increase the electronics bandwidth.
AND/OR Pixelize the readout electrodes

→ Pad response function, ballistic deficit, double track separation

GEM

'Only' fast electron signal, but

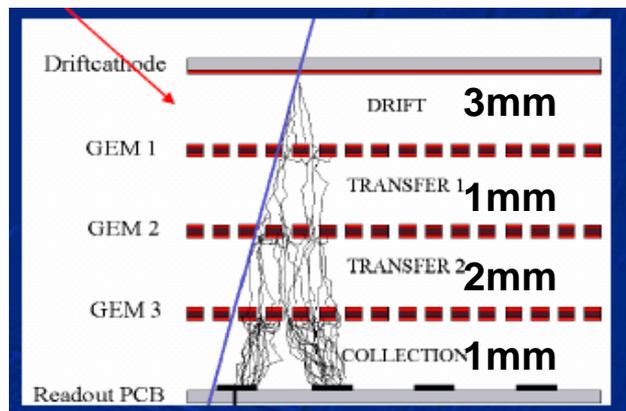
Single electron moving in the induction gap takes about $1\text{mm}/v_1=10\text{ns}$.

Collecting all electrons from the drift gap takes a maximum of $3\text{mm}/v_1=60\text{ns}$.

The GEM signal has a length of about **50 ns**.

Use fast electronics for GEM readout.

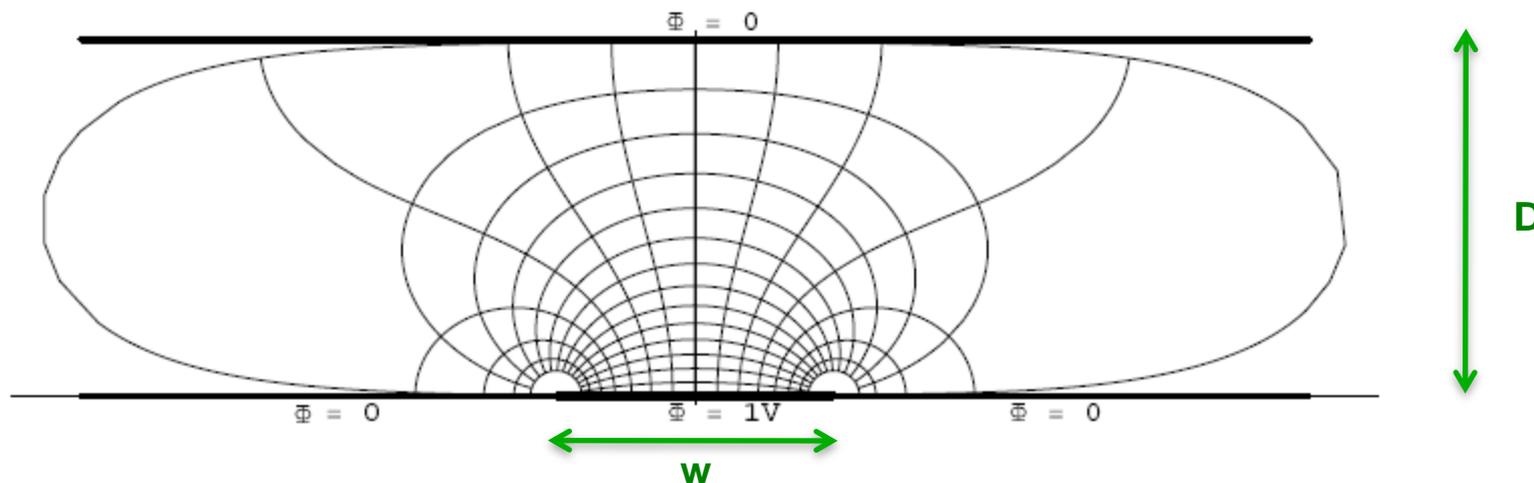
→ Increase of Pad Response Function
!



$V_1 \sim 50\mu\text{m}/\text{ns}$

$V_2 \sim 100\mu\text{m}/\text{ns}$

Pad response function of parallel plate geometry



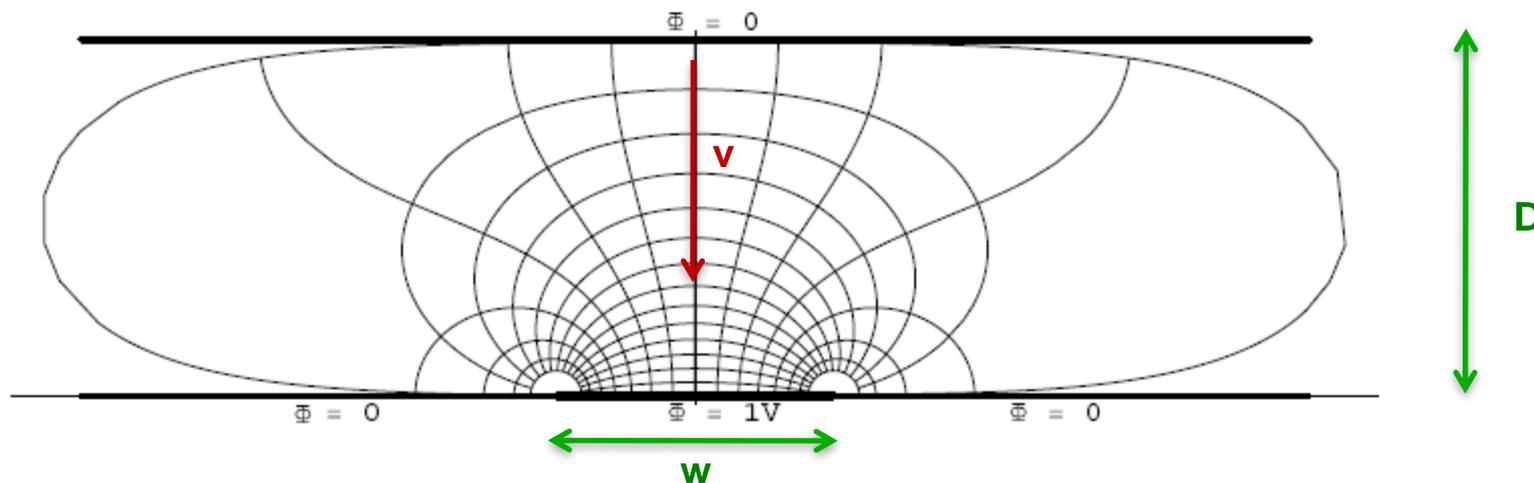
$$\Phi_1(x, z) = \frac{V_1}{\pi} \left[\arctan \left(\cot \left(\frac{z\pi}{2D} \right) \tanh \left(\pi \frac{x + w/2}{2D} \right) \right) - \arctan \left(\cot \left(\frac{z\pi}{2D} \right) \tanh \left(\pi \frac{x - w/2}{2D} \right) \right) \right]$$

$$E_{1x} = V_1 \frac{1}{2D} \left[\frac{\sin \left(\frac{z\pi}{D} \right)}{\cosh \left(\pi \frac{x-w/2}{D} \right) - \cos \left(\frac{z\pi}{D} \right)} - \frac{\sin \left(\frac{z\pi}{D} \right)}{\cosh \left(\pi \frac{x+w/2}{D} \right) - \cos \left(\frac{z\pi}{D} \right)} \right]$$

Weighting Field of a strip in an GEM or Micromega

$$E_{1z} = -V_1 \frac{1}{2D} \left[\frac{\sinh \left(\pi \frac{x-w/2}{D} \right)}{\cosh \left(\pi \frac{x-w/2}{D} \right) - \cos \left(\frac{z\pi}{D} \right)} - \frac{\sinh \left(\pi \frac{x+w/2}{D} \right)}{\cosh \left(\pi \frac{x+w/2}{D} \right) - \cos \left(\frac{z\pi}{D} \right)} \right]$$

Pad response function of parallel plate geometry



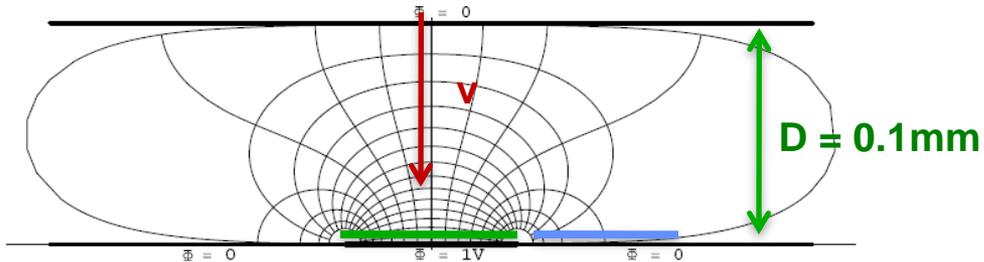
$$I(t,x) = -e_0 * E_z[x, D-v*t] * v$$

When **all** charges have arrived at the electrodes the induced charge is equal to the **total charge that has arrived** at the electrode.

If the electronics 'integration time' ~ 'peaking time' ~ 'shaping time' is larger than the time it takes the electron to pass the induction gap, the readout strips that don't receive any charge show zero signal.

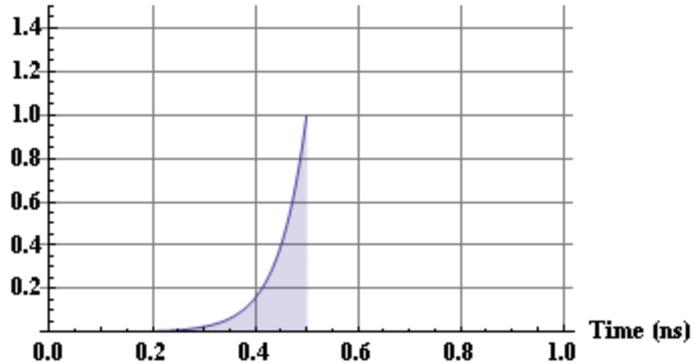
If the electronics 'integration time' ~ 'peaking time' ~ 'shaping time' is smaller than the time it takes the electron to pass the induction gap, the readout strips that don't receive any charge show a signal different from zero that is strictly bipolar.

Micromega Signal

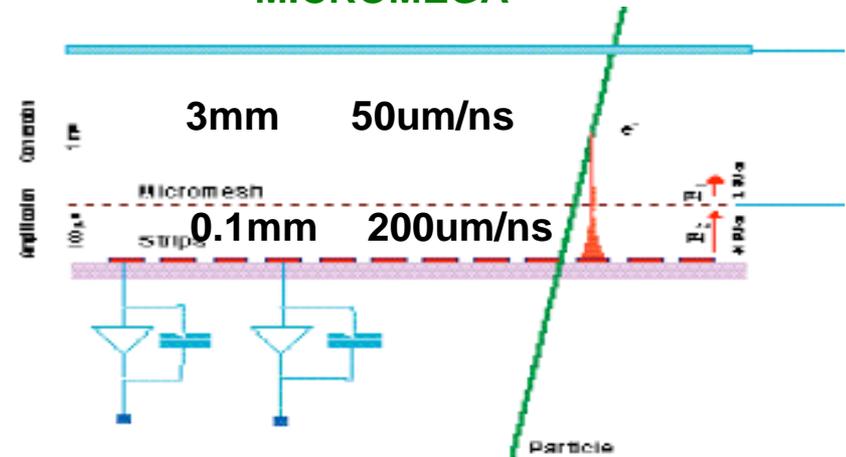


$$I(t,x) = -e_0 \cdot \text{Exp}[\alpha v t] \cdot E_z[x, D - v \cdot t] \cdot v \rightarrow \text{Electrons}$$

Induced Signal



MICROMEGA



Electrons movement in the induction gap takes about $0.1\text{mm}/v_1 = 0.5\text{ns}$.

Collecting all electrons from the drift gap takes a maximum of $3\text{mm}/v_1 = 60\text{ns}$.

The MICROMEGA electron signal has a length of about **50 ns**.

Typically $w \gg D$ – cluster size from electron component is dominated by diffusion and not by direct induction.

However, ion component has a length of about 100ns \rightarrow Ballistic Deficit for fast electrons (e.g. 10ns peaking time).

Conclusions

As long as the electronics integration time is significantly larger than the time of the charge movement in the induction gap (GEM and MICROMEGA) the signal just depends on the amount of charge that has arrived at the readout pad ('collection').

For the GEM detector, where only electrons move inside the induction gap, the transit time is about 10ns, so the above statement is true for electronics integration time of $\gg 10\text{ns}$.

For the Micromega, electrons are moving from the mesh to the readout plane in $< 0.5\text{ns}$, but the ions take up to 100ns to move back to the mesh, depending on the gas, geometry and operation point.

The signal shape and crosstalk will therefore strongly depends on the electronics integration time (ballistic deficit).