

# Hadronic Vacuum Polarisation in g-2 and $\alpha_{\text{QED}}$



Thomas Teubner

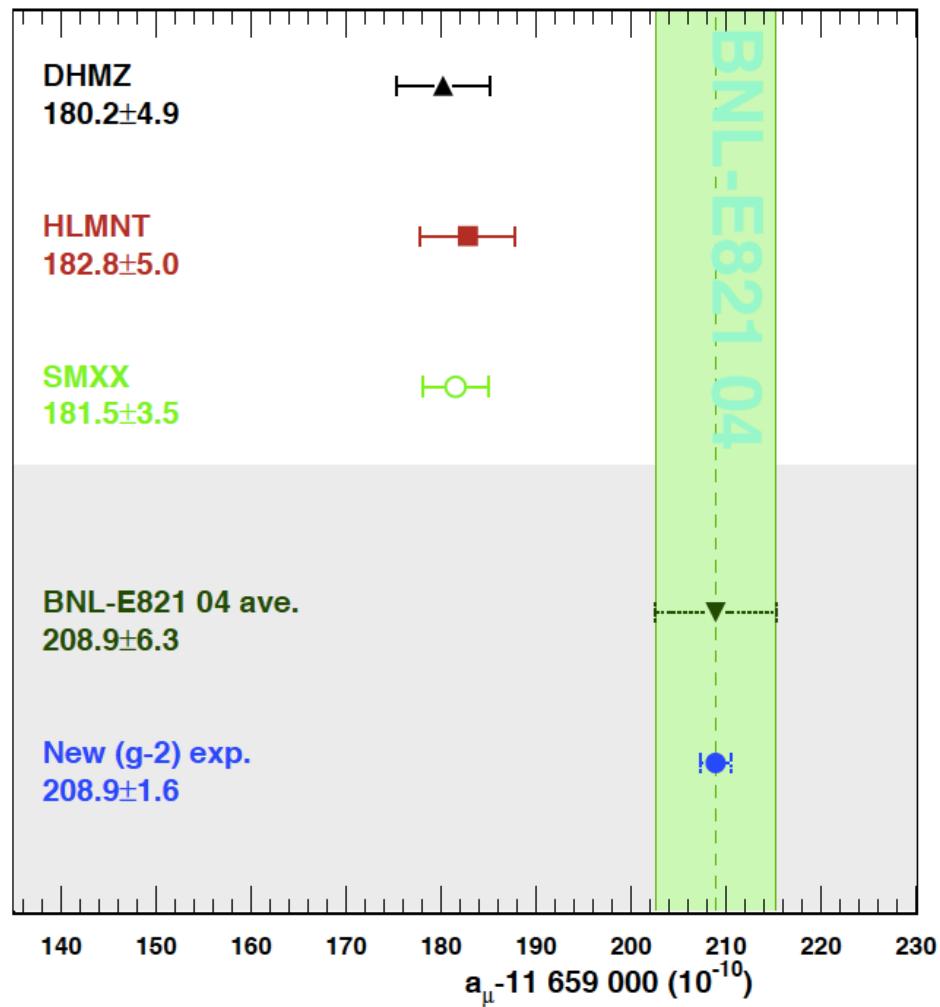


- The charge
- $a_{\mu}^{\text{HVP,LO}}$ : Status. Details/puzzles/troubles
- higher order HVP
- next steps
- $\Delta\alpha_{\text{had}}$  at low  $q^2$  and at the Z

# Motivation. Our charge

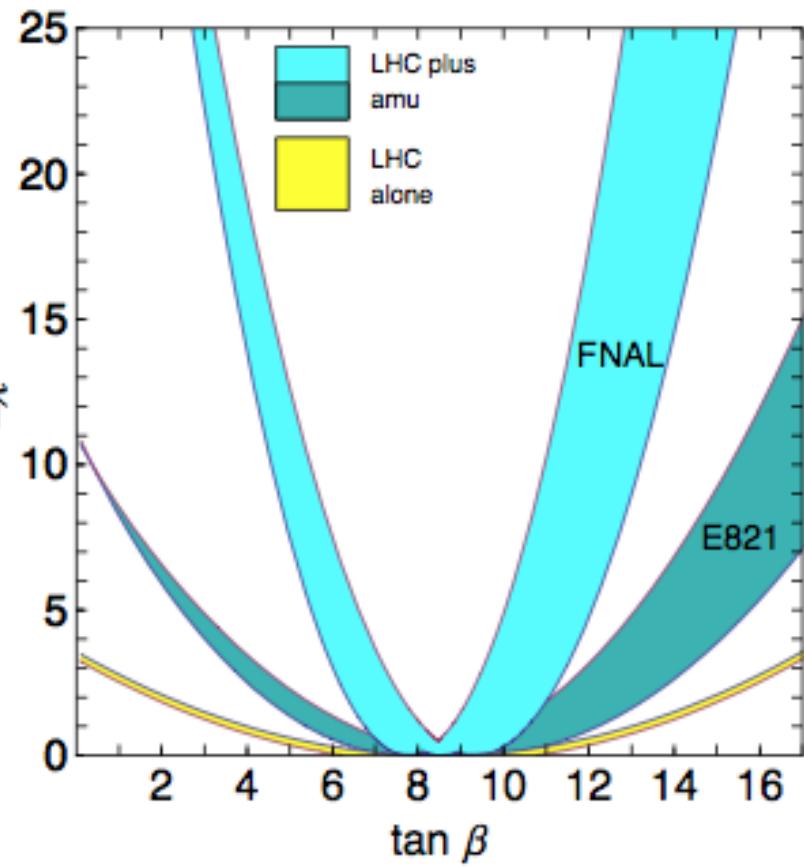
Future picture may look like:

- if mean values stay and with no  $a_{\mu}^{\text{SM}}$  improvement:  $5\sigma$  discrepancy
- if also EXP+TH can improve  $a_{\mu}^{\text{SM}}$  ‘as expected’ (consolidation of LbyL on level of Glasgow consensus, about factor 2 for HVP): NP at  $7\text{-}8\sigma$
- or, if mean values get closer, very strong exclusion limits on many NP models (extra dims, new dark sector, xxxSSSM)...



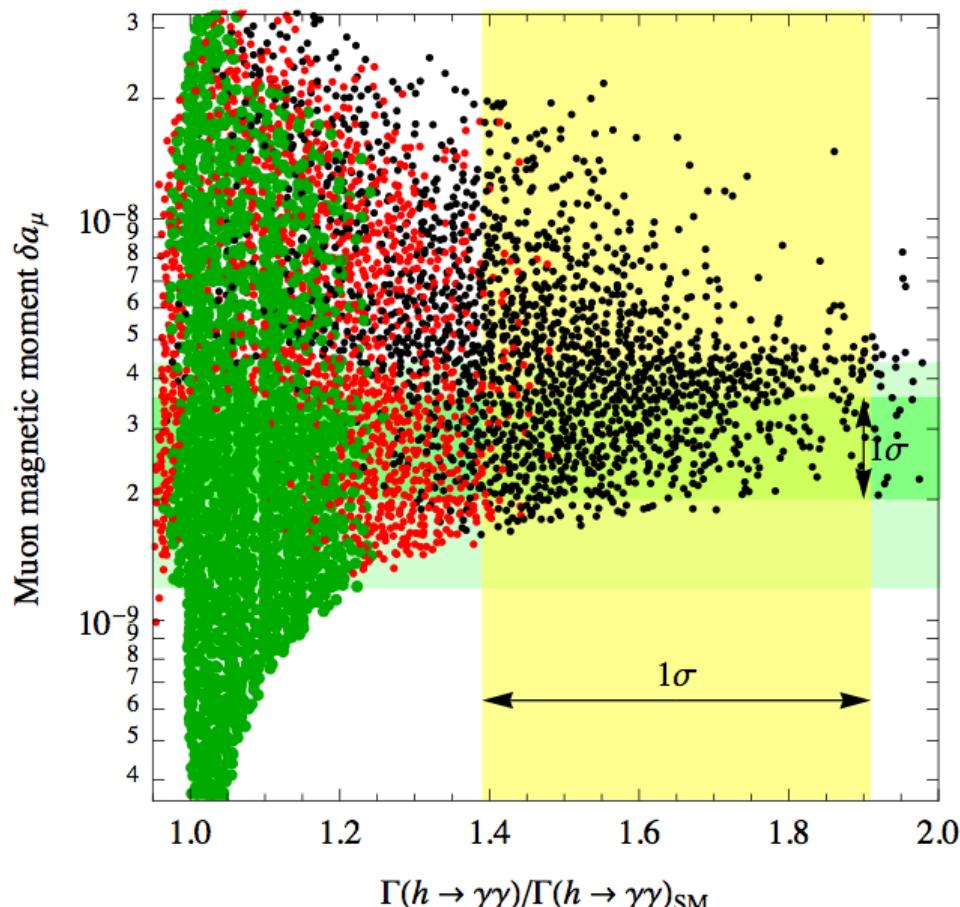
# Motivation: Precision g-2 constrains SUSY

LHC with ( $100 \text{ fb}^{-1}$ ) can determine  $\tan(\beta)$  to 50%, with g-2 to 10%



Miller, de Rafael, Roberts and Stöckinger,  
Ann. Rev. Nucl. Part. Sci. 62 (2012) 237

g-2 complements LHC data selecting in the vast SUSY (param/model) space



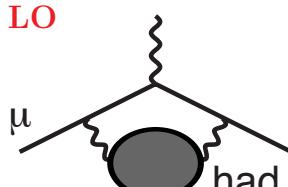
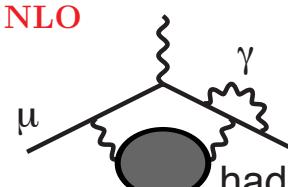
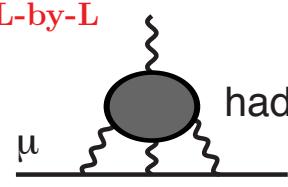
Guidice, Paradisi, Strumia JHEP 1210, 186

# The charge: half $\Delta a_\mu^{\text{SM}}$

$$a_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{hadronic}} + a_\mu^{\text{NP?}}$$

- QED: Kinoshita et al. 2012: 5-loop completed (12672 diags) [some 4-l checks] ✓
- EW: 2-loop (and Higgs mass now known) ✓
- Hadronic: the limiting factor of the SM prediction ✗

$$a_\mu^{\text{had}} = a_\mu^{\text{had, VP LO}} + a_\mu^{\text{had, VP NLO}} + a_\mu^{\text{had, Light-by-Light}}$$

**LO**  **NLO**  **L-by-L** 

**L-by-L:** - so far use of model calculations, form-factor data will help improving, also  
- lattice QCD. Detailed discussion in other talks.

**HVP:** - most precise prediction by using  $e^+e^-$  hadronic cross section (+ tau) **data**  
- alternative: lattice QCD, but: need also QED corrections; systematics?  
- HVP at NLO and NNLO (Steinhauser et al, 1403.6400) done w. disp. integral:  
 $a_\mu^{\text{HVP, NNLO}} = + 1.24 \times 10^{-10}$

# The charge: half $\Delta a_\mu^{\text{SM}}$

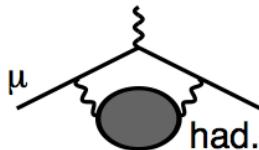
- Several groups have produced hadronic compilations over the years.
- Here: Hagiwara+Liao+Martin+Nomura+T
- Many more precise data in the meantime and more expected for near future
- At present **HVP still dominates** the SM error

<b>QED</b> contribution	$11\ 658\ 471.808\ (0.015) \times 10^{-10}$	Kinoshita & Nio, Aoyama et al
<b>EW</b> contribution	$15.4\ (0.2) \times 10^{-10}$	Czarnecki et al
<b>Hadronic</b> contribution		
<b>LO</b> hadronic	$694.9\ (4.3) \times 10^{-10}$	HLMNT11
<b>NLO</b> hadronic	$-9.8\ (0.1) \times 10^{-10}$	HLMNT11
<b>light-by-light</b>	$10.5\ (2.6) \times 10^{-10}$	Prades, de Rafael & Vainshtein
<b>Theory TOTAL</b>	$11\ 659\ 182.8\ (4.9) \times 10^{-10}$	
<b>Experiment</b>	$11\ 659\ 208.9\ (6.3) \times 10^{-10}$	world avg
<b>Exp – Theory</b>	$26.1\ (8.0) \times 10^{-10}$	$3.3\ \sigma$ discrepancy

(Numbers taken from HLMNT11, arXiv:1105.3149)

# Hadronic Vacuum Polarisation, essentials:

Use of data compilation for HVP:



pQCD not useful. Use the dispersion relation and the optical theorem.

$$\text{had.} = \int \frac{ds}{\pi(s-q^2)} \text{Im } \text{had.}$$

$$2 \text{Im } \text{had.} = \sum_{\text{had.}} \int d\Phi \left| \text{had.} \right|^2$$

$$a_\mu^{\text{had,LO}} = \frac{m_\mu^2}{12\pi^3} \int_{s_{\text{th}}}^\infty ds \frac{1}{s} \hat{K}(s) \sigma_{\text{had}}(s)$$

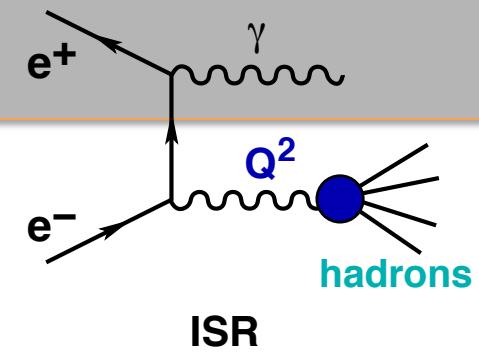
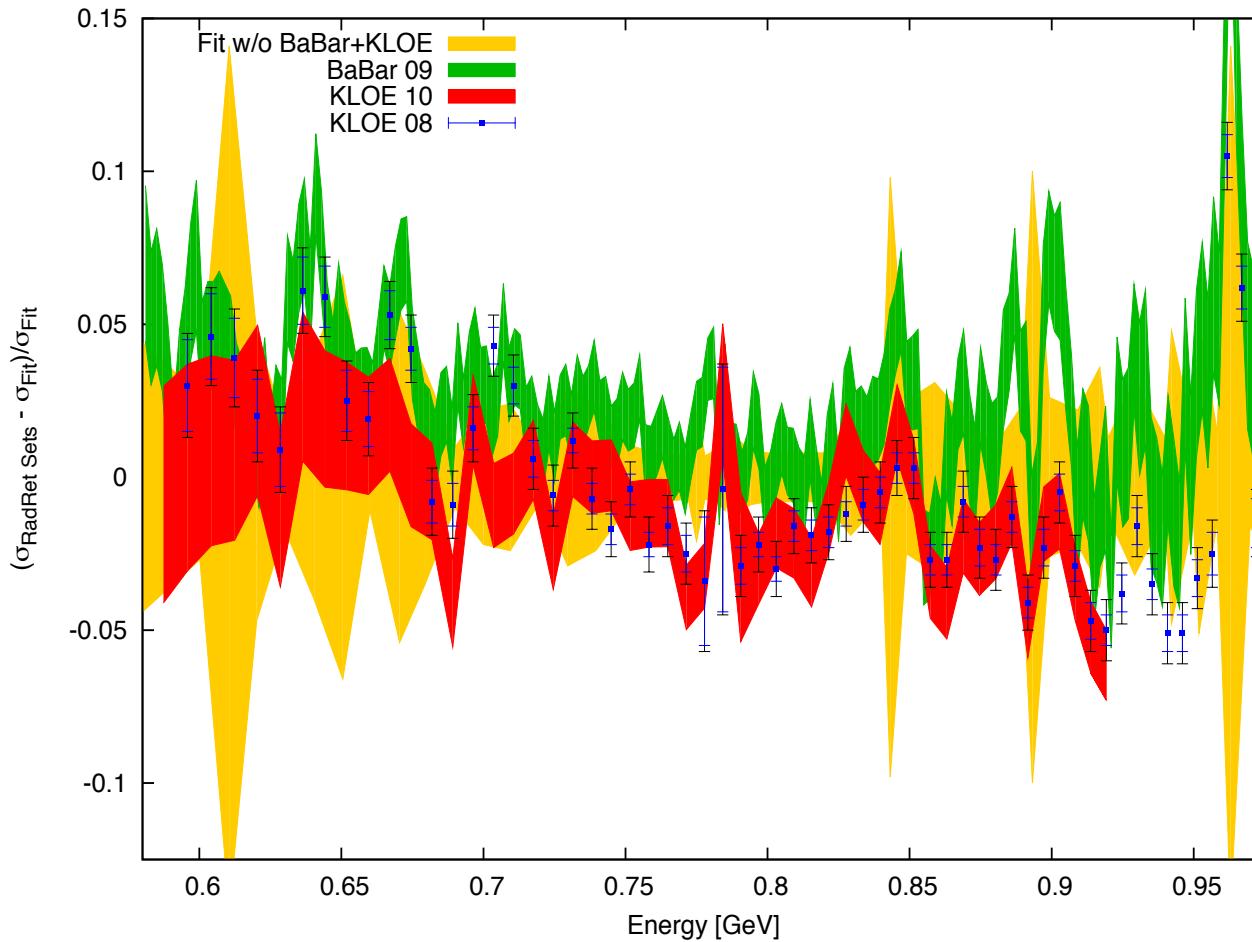
- Weight function  $\hat{K}(s)/s = \mathcal{O}(1)/s$   
 $\Rightarrow$  Lower energies more important  
 $\Rightarrow \pi^+\pi^-$  channel: 73% of total  $a_\mu^{\text{had,LO}}$

How to get the most precise  $\sigma_{\text{had}}^0$ ?  $e^+e^-$  data:

- Low energies: sum  $\sim 25$  exclusive channels
- $\sqrt{s} = 1.4 - 2$  GeV: sum exclusive channels, use iso-spin relations for missing channels, old inclusive data disfavoured
- Above  $\sim 1.8$  GeV: can start to use pQCD (away from flavour thresholds), supplemented by narrow resonances ( $J/\Psi, Y$ )
- Challenge of data combination (locally in  $\sqrt{s}$ ): from many experiments, in different energy bins, errors from different sources, correlations; sometimes inconsistencies/bias
- $\sigma_{\text{had}}^0$  means 'bare'  $\sigma$ , but WITH FSR: RadCors [ HLMNT:  $\delta a_\mu^{\text{had, RadCor VP+FSR}} = 2 \times 10^{-10}$  ! ]
- traditional direct 'scan' (tunable  $e^+e^-$  beams) vs 'Radiative Return' [+  $\tau$  spectral functions]

# Data ‘puzzle’ in the $\pi^+\pi^-$ channel:

Radiative Return (ISR) data compared to  $2\pi$  fit w/out them

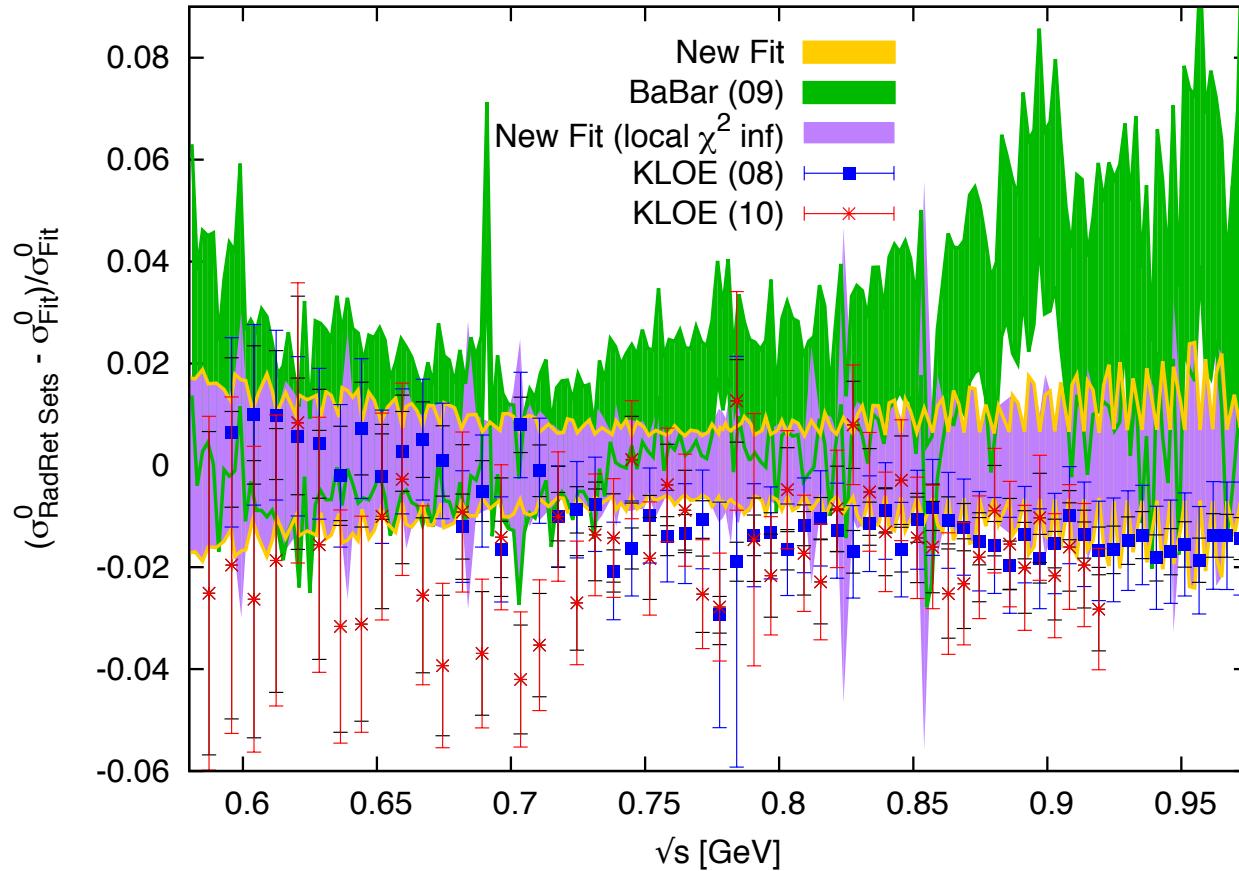


New KLOE12 data  
confirm this tension

Note:  $a_\mu^{\pi\pi, \text{w/out Rad Ret}} = 498.7 \pm 3.3$  BUT  $a_\mu^{\pi\pi, \text{with Rad Ret}} = 504.2 \pm 3.0$

# Data combination in the $\pi^+\pi^-$ channel:

Radiative Return data in the combined fit of HLMNT 11



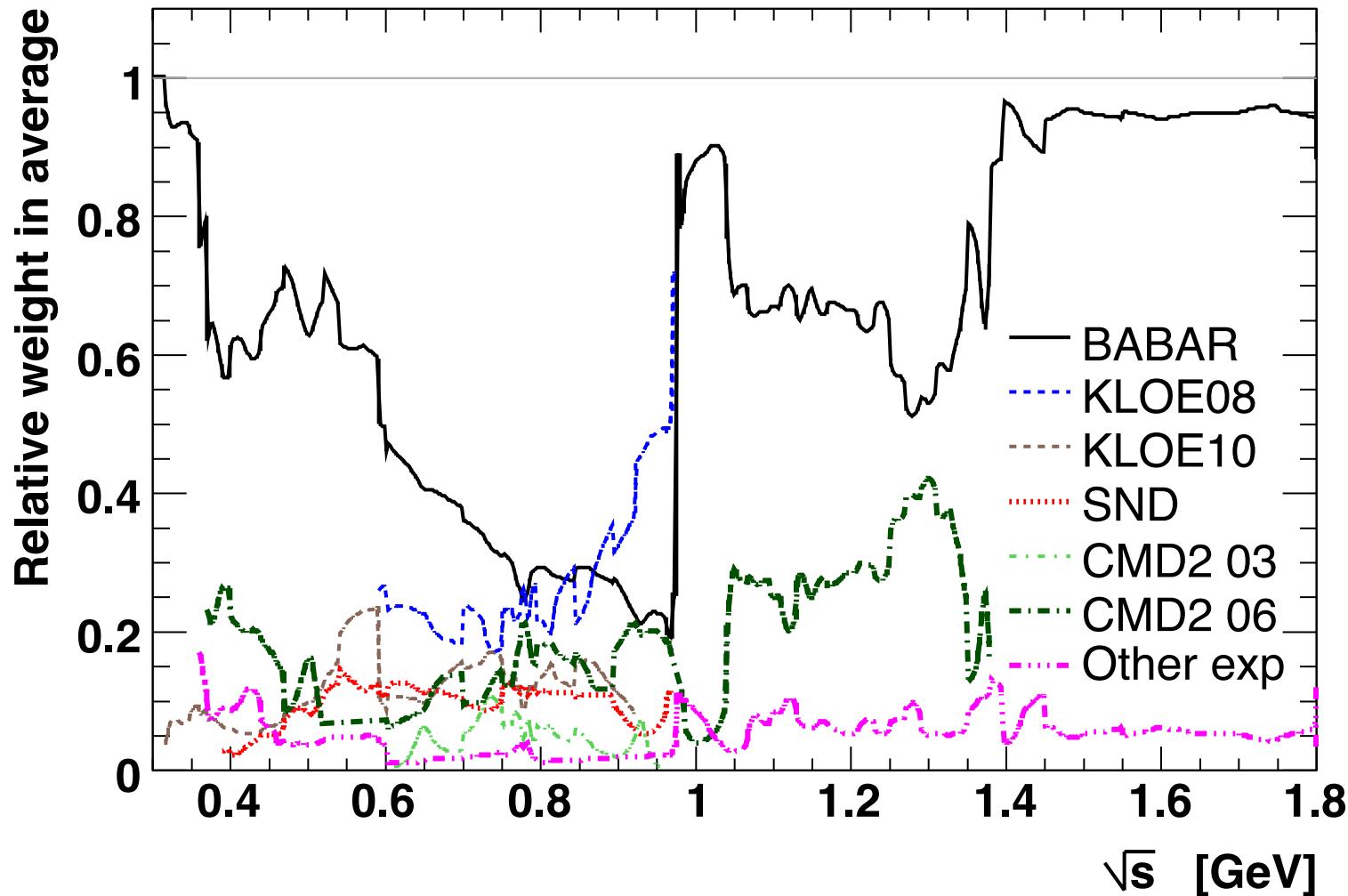
2 $\pi$  fit: overall  
 $\chi^2_{\text{min}}/\text{dof} \sim 1.5$   
needs error inflation,  
limited gain in error

Note:  $a_\mu^{\pi\pi, \text{w/out Rad Ret}} = 498.7 \pm 3.3$  BUT  $a_\mu^{\pi\pi, \text{with Rad Ret}} = 504.2 \pm 3.0$

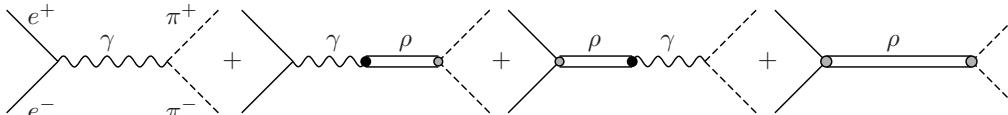
→ i.e. a shift of +5.5 in HLMNT [DHMZ:  $a_\mu^{\pi\pi}$  even higher by 2.1 units]

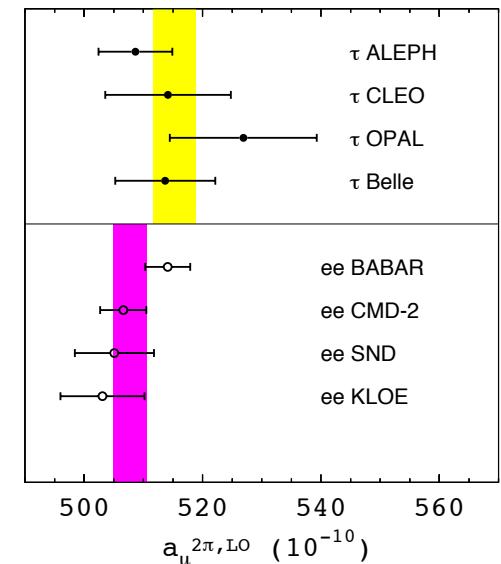
# Data combination in $\pi^+\pi^-$ : an elephant's weight

Weighting factors from Davier et al. [DHMZ EPJC71(2011)1515]

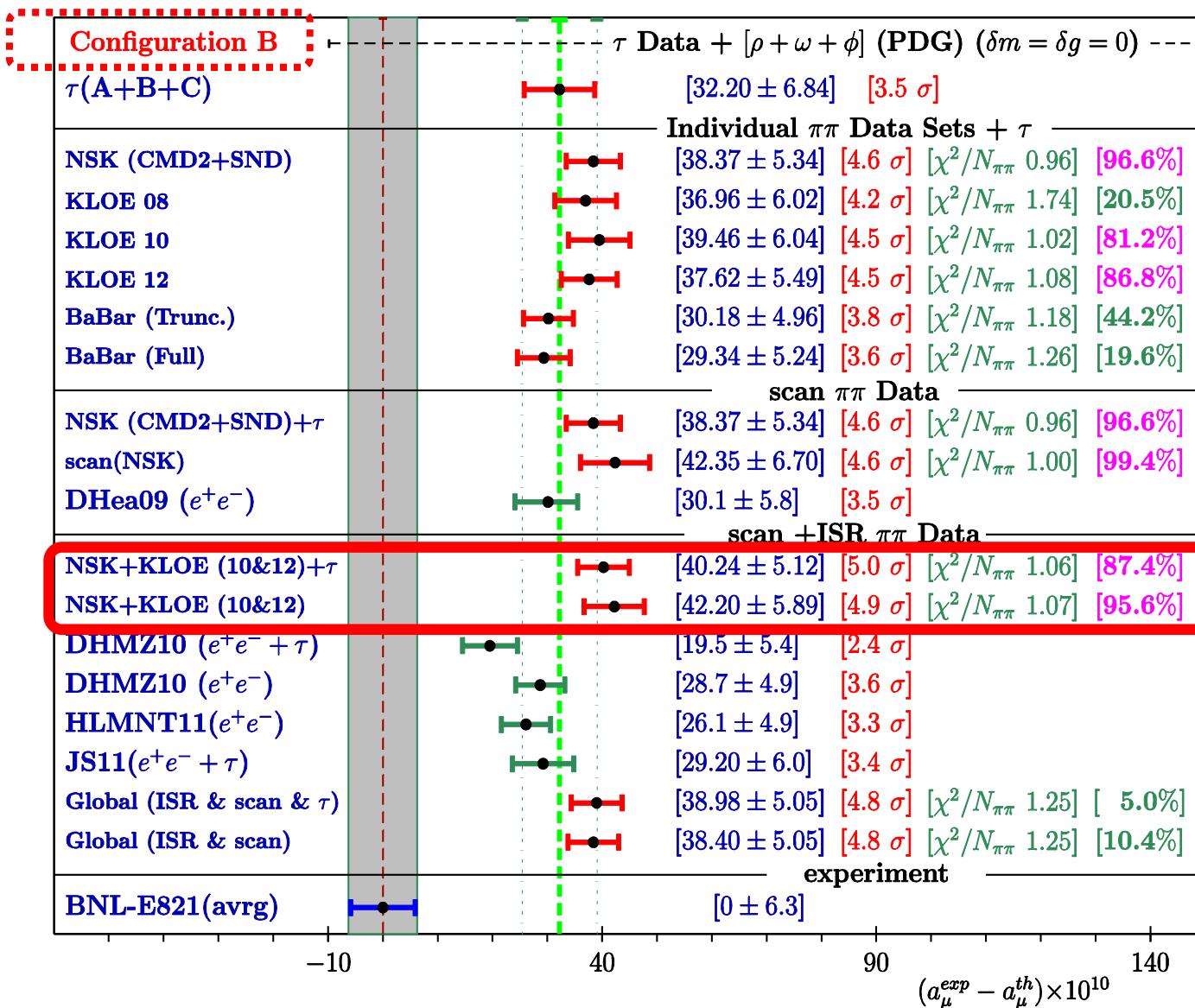


# Another ‘puzzle’: Use of tau spectral function data?

- Use CVC (iso-spin symmetry) to connect  $\tau^- \rightarrow \pi^0 \pi^- \nu_\tau$  spectral functions to  $e^+ e^- \rightarrow \omega, \rho \rightarrow \pi^+ \pi^-$  but have to apply **iso-spin corrections**
- Early calculations by [Alemany, Davier, Hoecker](#): use of  $\tau$  data complementing  $e^+ e^-$  data originally resulted in an improvement w.r.t. use of  $e^+ e^-$  data alone; discrepancy smaller with tau data; later increased tension between  $e^+ e^-$  and  $\tau$
- Recent compilation by [Davier et al](#) (Fig. from PRD86, 032013):
- [Jegerlehner+Szafron](#): crucial role of  $\gamma$ - $\rho$  mixing:  

- They found discrepancy gone but  $\tau$  data improves  $e^+ e^-$  analysis only marginally
- Analyses by [Benayoun et al](#): combined fit of  $e^+ e^-$  and  $\tau$  based on **Hidden Local Symmetry (HLS)**: no big tension betw.  $e^+ e^-$  and  $\tau$ , but w. [BaBar](#), hence not used; increased  $\Delta a_\mu$  of  $>\approx 4.5\sigma$
- [Davier+Malaescu](#) refute criticism, claim fair agreement betw. BaBar and their  $\tau$  comp.
- [HLMNT](#): stick to  $e^+ e^-$  (and do not use  $\tau$  data). With  $e^+ e^-$  (incl. BaBar) discrepancy of  $3-3.5\sigma$



# HLS fit based results from Maurice Benayoun et al.



Results for g-2  
and the discrepancy  
as presented last  
week by Benayoun

MB: Preferred fits  
discard BaBar 2pi  
data (red framed)

## HVP Results with scan & $\tau$ data

- (Updated) Central value shifted by  $\approx 3 \cdot 10^{-10}$

Channel	Solution B	Direct Estimate
$\pi^+\pi^-$	$495.40 \pm 1.92$	$498.53 \pm 3.73$ $(497.72 \pm 2.12)$
$\pi^0\gamma$	$4.61 \pm 0.04$	$3.35 \pm 0.11$
$\eta\gamma$	$0.64 \pm 0.01$	$0.48 \pm 0.01$
$\eta'\gamma$	$0.01 \pm 0.00$	---
$\pi^+\pi^-\pi^0$	$41.16 \pm 0.59$	$43.24 \pm 1.47$
$K_L K_S$	$11.90 \pm 0.08$	$12.31 \pm 0.33$
$K^+K^-$	$17.59 \pm 0.21$	$17.88 \pm 0.54$
Total up to 1.05 GeV	$571.30 \pm 2.02$	$575.79 \pm 4.06$

Diff=3.1 units

# Data comb. in the $\pi^+\pi^-$ channel: Benayoun et al

$\downarrow$  another shift by -4.3

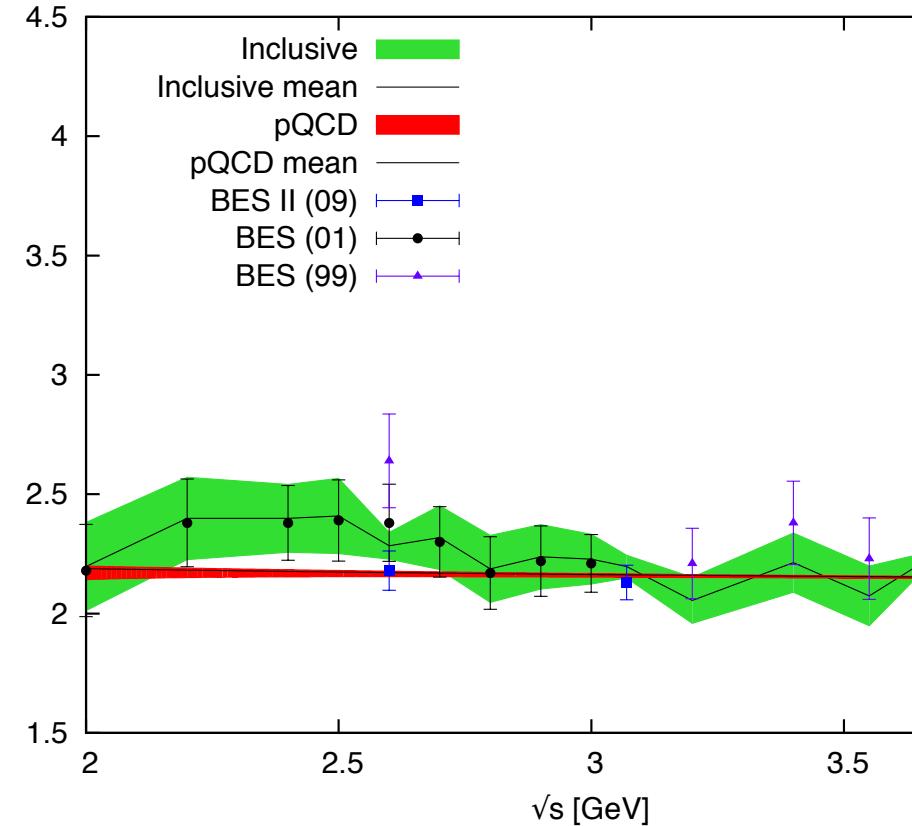
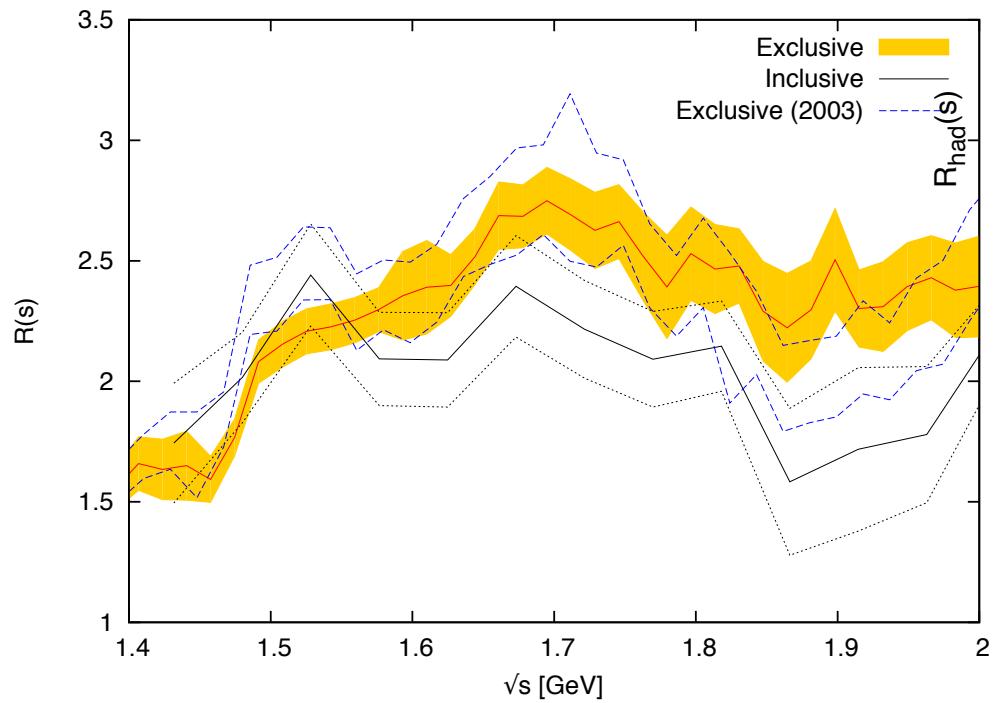
Channel	NSK +KLOE 10&12 + $\tau$ ( $ABC < 1$ GeV)	scan only(NSK) + $\tau$ ( $ABC < 1$ GeV)	Direct Estimate
$\pi^+\pi^-$	$491.12 \pm 1.35$	$495.40 \pm 1.92$	$498.53 \pm 3.73$ $(497.72 \pm 2.12)$
$\pi^0\gamma$	$4.63 \pm 0.04$	$4.61 \pm 0.04$	$3.35 \pm 0.11$
$\eta\gamma$	$0.64 \pm 0.01$	$0.64 \pm 0.01$	$0.48 \pm 0.01$
$\eta'\gamma$	$0.003 \pm 0.000$	$0.003 \pm 0.000$	---
$\pi^+\pi^-\pi^0$	$40.78 \pm 0.64$	$41.16 \pm 0.59$	$43.24 \pm 1.47$
$K_L K_S$	$11.94 \pm 0.08$	$11.90 \pm 0.08$	$12.31 \pm 0.33$
$K^+K^-$	$17.48 \pm 0.21$	$17.59 \pm 0.21$	$17.88 \pm 0.54$
<b>Total up to 1.05 GeV</b>	$566.58 \pm 1.50$	$571.30 \pm 2.02$	$575.79 \pm 4.06$

# Data combination in the $\pi^+\pi^-$ channel:

- Taking only direct scan as baseline:
- **Benayoun et al**: -3.1 from HLS-based fit, -4.3 from KLOE10+12
- **HLMNT**: +5.5 from KLOE and BaBar (compared to scan only)
- So the big difference ( $\sim 13 \times 10^{-10}$ ,  $3.3 \rightarrow 5\sigma$ ) comes to a big part from the data input, i.e. if BaBar's  $2\pi$  is used or not.  
(If used: error relatively poor despite stats due to inflation)
- Future SND, CMD-3, BELLE and BESIII  $2\pi$  data may dilute the strong significance of BaBar  
[also more data from BaBar to be analysed!]
- Ideally find out why the different data sets are not consistent.  
**If this could be achieved the  $2\pi$  channel would be great!**

# $\sigma_{\text{had}}$ at higher energies; old 2011 status to be improved soon

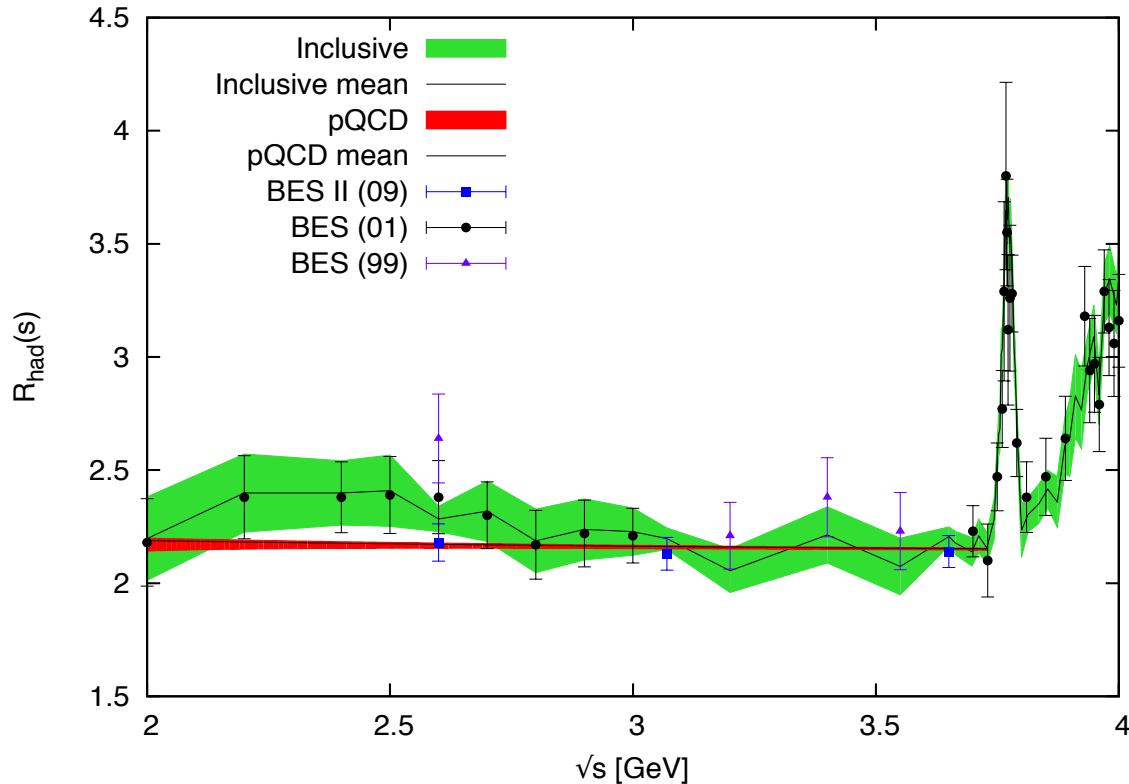
Inclusive vs. sum of exclusive,  
match to inclusive/pQCD:



→ small step at transition from sum of exclusive to incl. (or pQCD), similar accuracy

# New data from BESIII eagerly awaited:

- ▶ Perturbative QCD vs. inclusive data above 2 GeV (below the charm threshold)



- Latest BES data (blue markers) in perfect agreement with perturbative QCD;  
data slightly higher than pQCD for  $\sqrt{s} > 2.6$  GeV
- HLMNT use pQCD for  $2.6 < \sqrt{s} < 3.7$  GeV and with (larger) BES errors
  - would have small shift downwards ( $\sim -1.4 \cdot 10^{-10}$  for  $a_\mu$ ) if used from 2 GeV
  - Davier et al. use pQCD from 1.8 GeV

# Channels with biggest errors. PQCD at 2 GeV?

## Importance of various ‘channels’

[Numbers from HLMNT, ‘local error infl.’,  $\cdot 10^{-10}$ ]

- Errors contributions to  $a_\mu$  from leading and subleading channels (ordered) up to 2 GeV

Purely from data:

channel	error
$\pi^+ \pi^-$	3.09
$\pi^+ \pi^- \pi^0 \pi^0$	1.26
$3\pi$	0.99
$2\pi^+ 2\pi^-$	0.47
$K^+ K^-$	0.46
$2\pi^+ 2\pi^- 2\pi^0$	0.24
$K_S^0 K_L^0$	0.16

‘Higher multiplicity’ region from 1.4 to 2 GeV  
with use of isospin relations for some channels:  
[Use of old inclusive data disfavoured.]

Channel	contr.	$\pm$ error
$K \bar{K} 2\pi$	3.31	$\pm 0.58$
$\pi^+ \pi^- 4\pi^0$	0.28	$\pm 0.28$
$\eta \pi^+ \pi^-$	0.98	$\pm 0.24$
$K \bar{K} \pi$	2.77	$\pm 0.15$
$2\pi^+ 2\pi^- \pi^0$	1.20	$\pm 0.10$

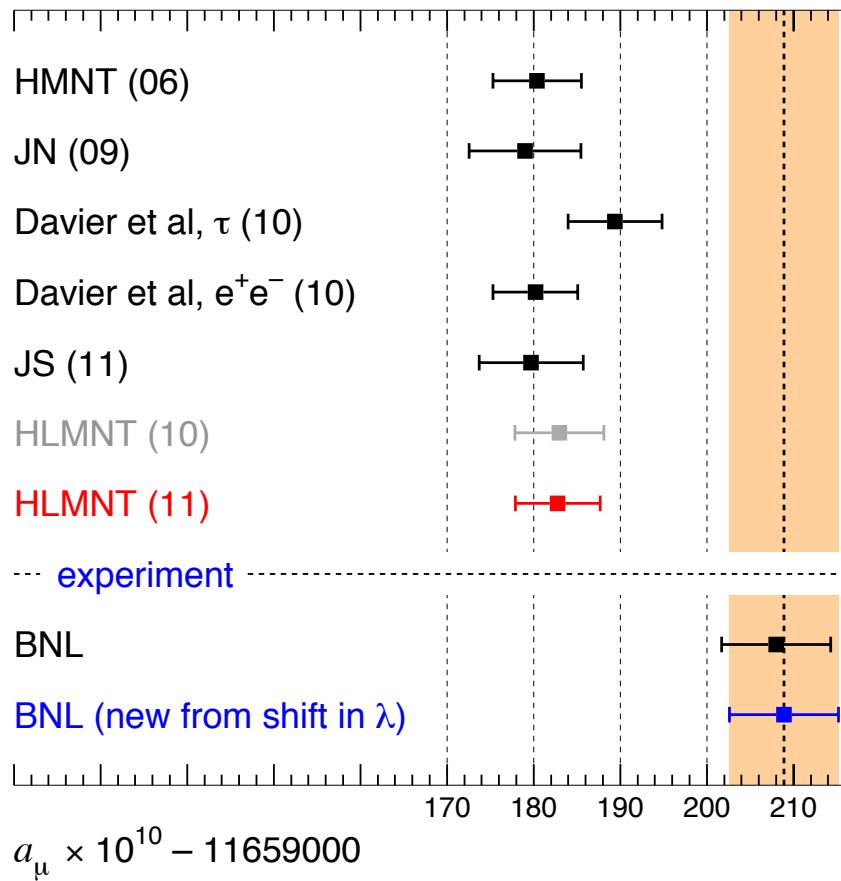
- ‘Inclusive’ region from 2 to  $\sim 11$  GeV:  $41.19 \pm 0.82$

Can be ‘squeezed’ by using pQCD (done by DHMZ from 1.8 GeV);

region from 2 to 2.6 GeV:  $15.69 \pm 0.63 \rightarrow 14.49 \pm 0.13$ , only small changes for higher energies.

# SM status: Recent ‘history’ plot

$a_\mu^{\text{HVP, LO}} (10^{-10})$



- Fair agreement between different  $e^+e^-$  analyses, including recent updates:

HLMNT (11):  $694.9 \pm 3.7$  (exp)  $\pm 2.1$  (rad)

Jegerlehner (11):  $690.8 \pm 4.7$

Davier et al (11):  $692.3 \pm 4.2$

- The ‘extremes’ (both with  $\tau$  data):

Davier et al (11):  $701.5 \pm 4.7$  (+  $\sim 1.5$  shift from their 2013  $\tau$  re-analysis 1312.1501)

Benayoun et al (12):  $681.2 \pm 4.5$

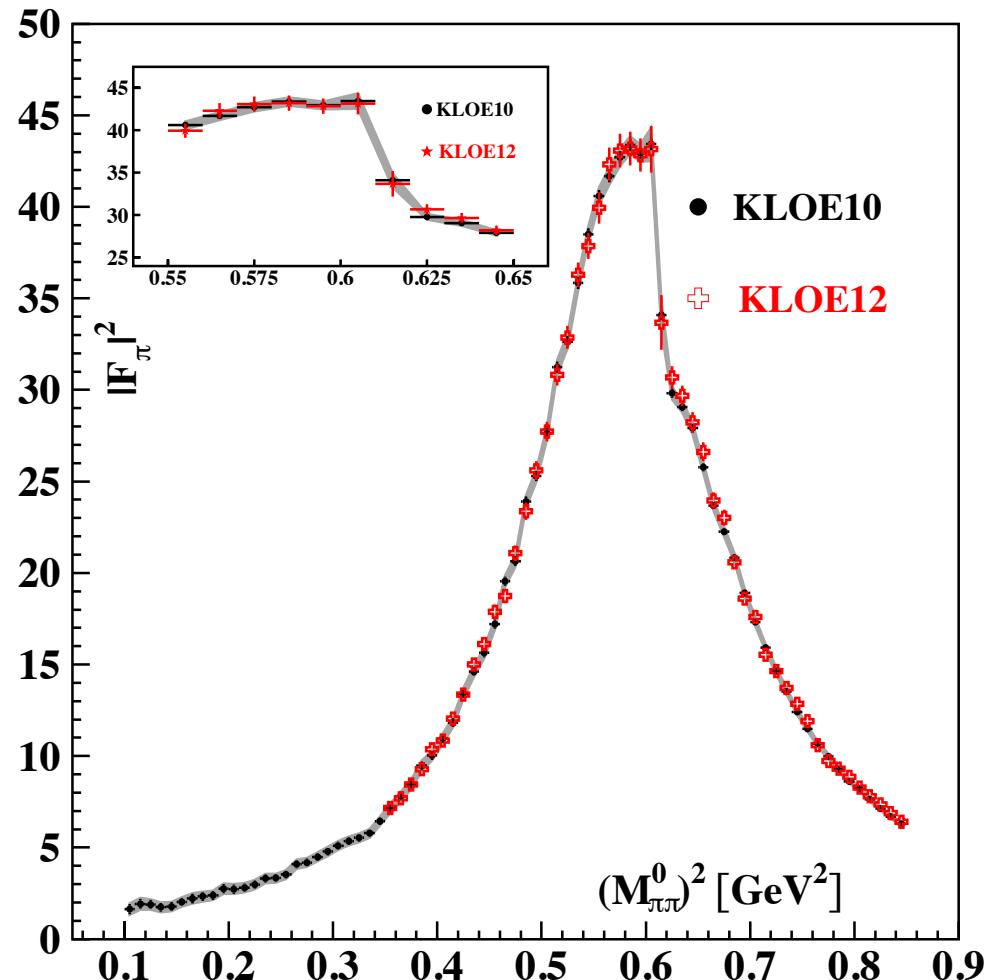
- New data available already do not shift the mean value strongly, but are incrementally improving the determination of  $a_\mu^{\text{HVP}}$ :

# $\sigma_{\text{had}}$ : recent new data

## KLOE

$\pi^+\pi^-$  data with  $\sigma_{\mu\mu}$   
normalisation:

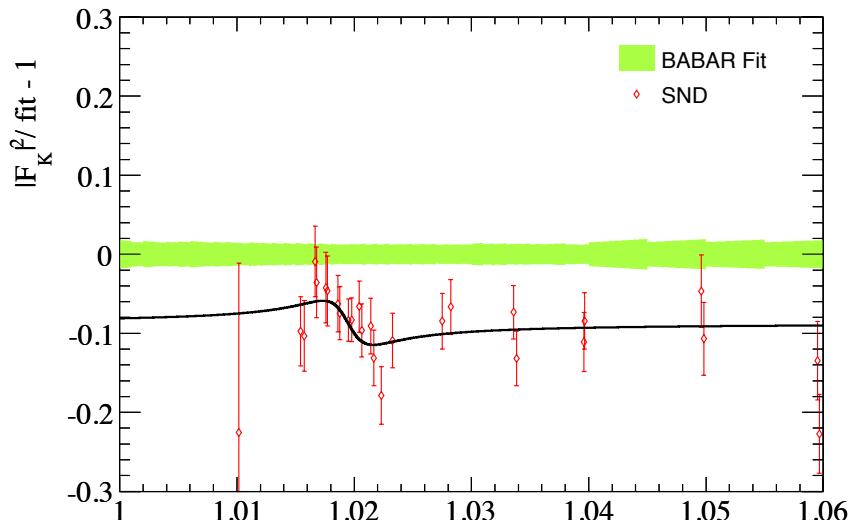
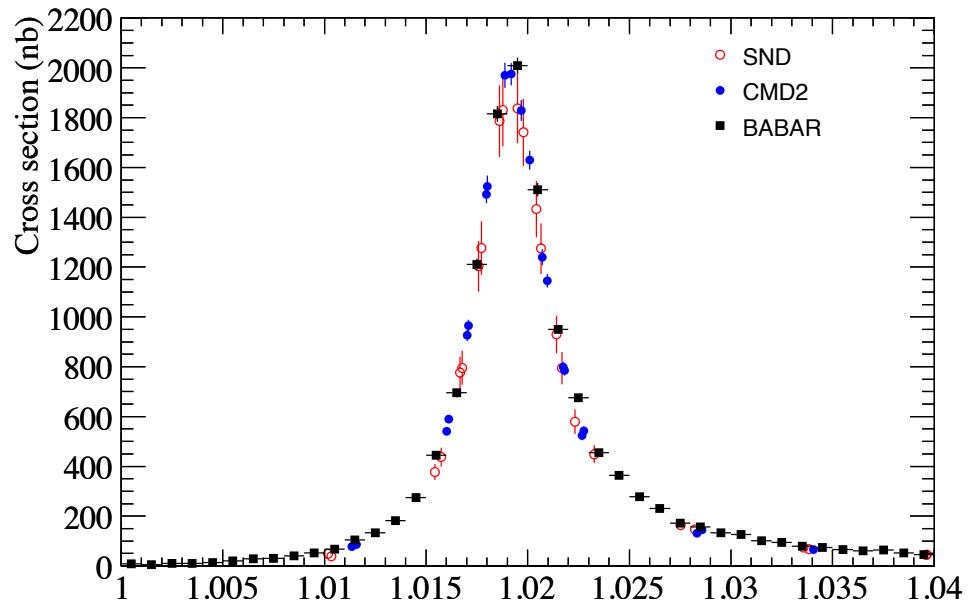
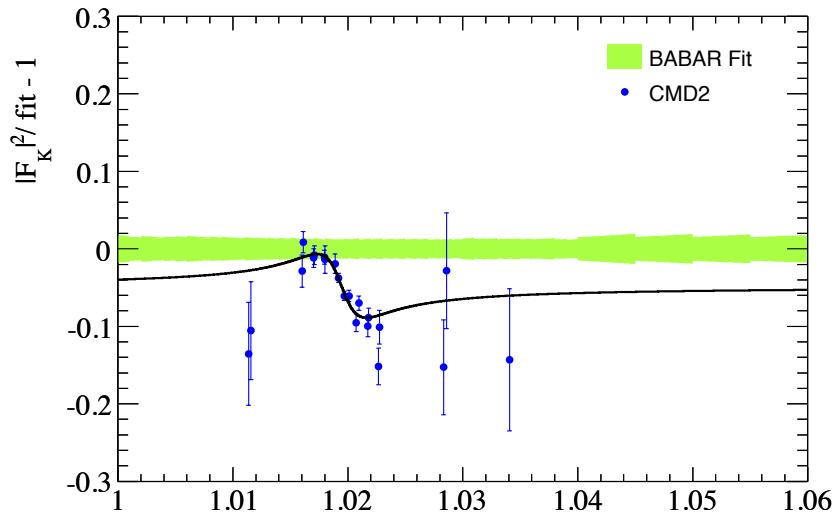
- confirm previous KLOE measurements
- will not decrease tension with BaBar once included in next round of ‘global’  $\sigma_{\text{had}}$  compilations, but slightly increase significance of KLOE
- Open question: Why are BaBar’s data so different from KLOE’s?  
Are there any issues with the MCs or analysis techniques used?



# $\sigma_{\text{had}}$ : recent new data: $K^+K^-(\gamma)$ from BaBar

arXiv:1306.3600

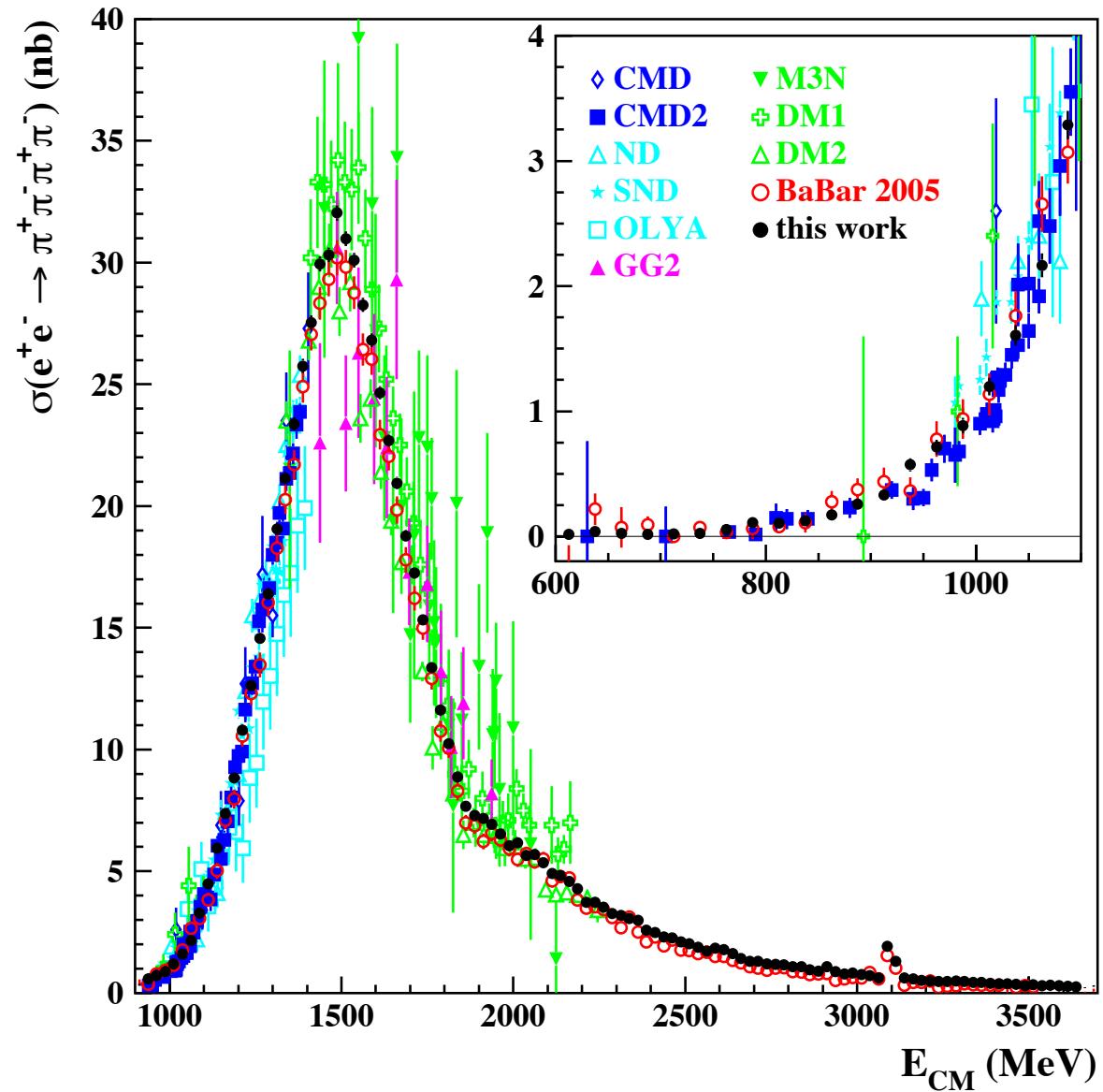
- $a_\mu = 22.94 \pm 0.18 \pm 0.22$  up to 1.8 GeV vs.  $21.63 \pm 0.27 \pm 0.68$  for combined previous data
- significant shift up, error down?!
- may need to use mass shifts or  $\Delta E$  for best combination;  $m_\phi = ?$
- Comp. plots BaBar vs Novosibirsk:



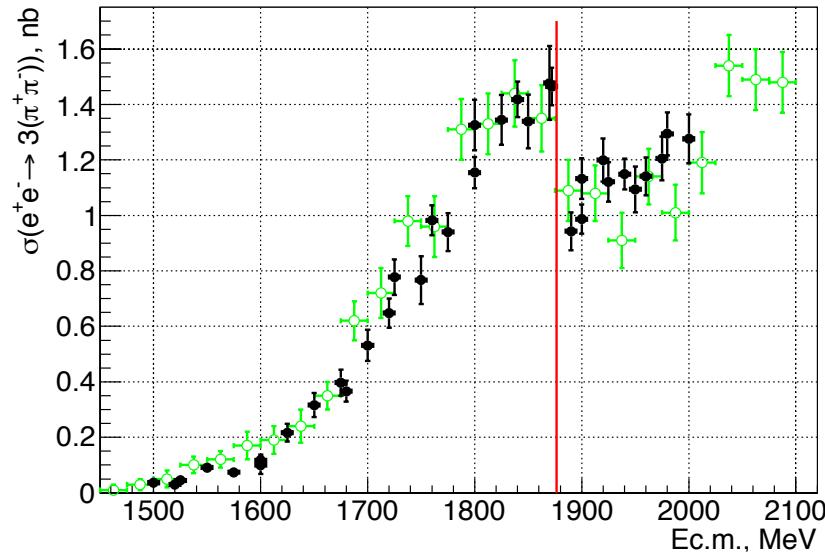
# $\sigma_{\text{had}}$ : recent new data: $2\pi^+2\pi^-(\gamma)$ from BaBar

PRD85(2012)112009

- shift of  $+0.3 \times 10^{-10}$  for  $a_\mu$
- error down to a third
- combination?

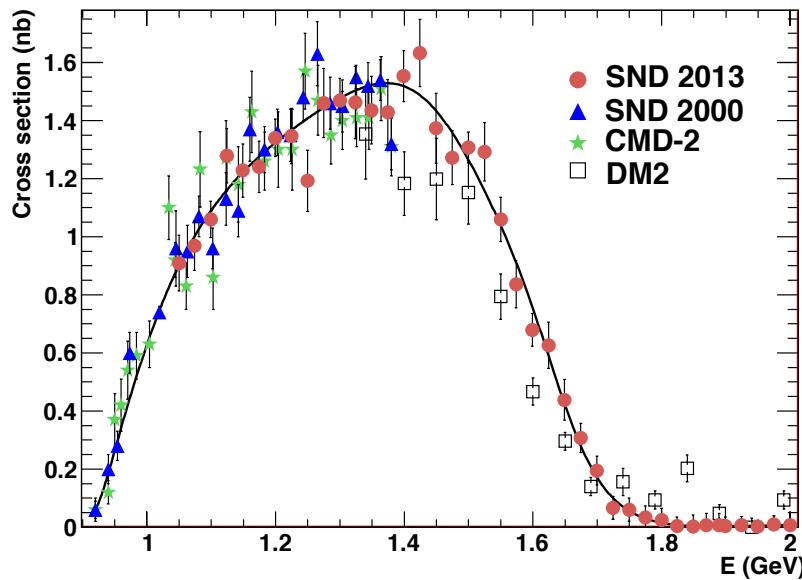


# $\sigma_{\text{had}}$ : recent new data from Novosibirsk



CMD-3 6 $\pi$  charged,  
PLB723(2013)82

- solid black: CMD-3, open green: BaBar
- full analysis will include  $2(\pi^+\pi^-\pi^0)$



SND  $\omega\pi^0$ , PRD88(2013)054013

- many more analyses reported with preliminary results, incl.  $3\pi$ ,  $4\pi(2n)$
- looking forward to rich harvest from SND and CMD-3

# Radiative corrections in data for $\sigma^0_{\text{had},\gamma}$

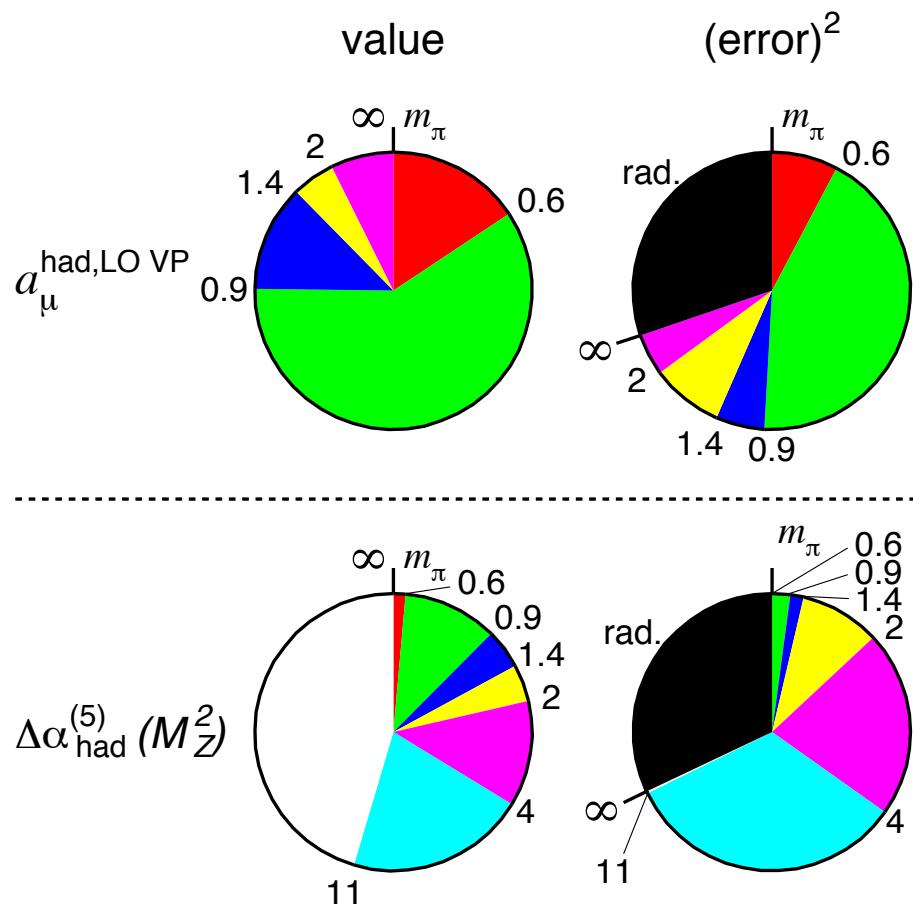
The black pieces are from

$$\delta a_\mu^{\text{had, RadCor VP+FSR}} = 2 \times 10^{-10}$$

an additional Radiative Correction error assigned due to uncertainties in the correct treatment of VP and FSR corrections

- VP: mostly relevant for older sets so will improve with time (care for  $\phi$ !?)
- FSR: most probably too conservative in HLMNT, re-visit
- Collaboration of EXP and TH!  
→ Work in WG Radio MonteCar Low

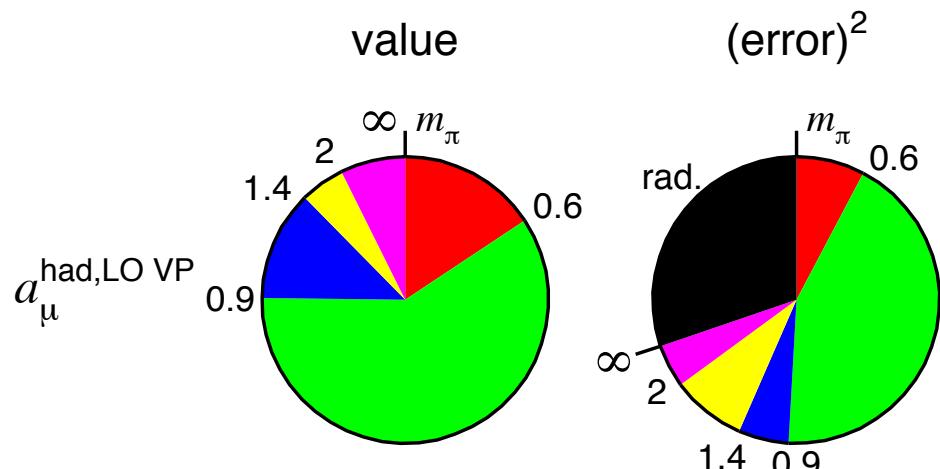
Pie diagrams from HLMNT 11:



# Future improvements (HVP):

- Most important  $2\pi$ :
  - close to threshold important; possible info also from spacelike
  - better and more data
  - understand discrepancy between sets, especially ‘BaBar puzzle’
  - possibility of direct scan & ISR in the same experiment(s)
- $\sqrt{s} > 1.4 \text{ GeV}$ :  
higher energies will improve with input from **SND**, **CMD-3**, **BESIII**, **BaBar**
- With channels more complete, test/replace iso-spin corrections
- Very good prospects to significantly squeeze the dominant HLO error!

Pie diagrams from HLMNT 11:



My personal ‘5 year prognosis’: [expected improvement in error]

- $2\pi$ : error down by about 30-50%
- subleading channels: by factor 2-3
- $\sqrt{s} > 2 \text{ GeV}$ : by about a factor 2

➔ I believe we can half the HVP error in time for the new  $g-2$

# Hadronic VP: $\Delta\alpha$

- Dyson summation of Real part of one-particle irreducible blobs  $\Pi$  into the effective, real running coupling  $\alpha_{\text{QED}}$ :

$$\Pi = \text{---} \quad \text{---} \quad \text{---}$$

Full photon propagator  $\sim 1 + \Pi + \Pi \cdot \Pi + \Pi \cdot \Pi \cdot \Pi + \dots$

$$\rightsquigarrow \alpha(q^2) = \frac{\alpha}{1 - \text{Re}\Pi(q^2)} = \alpha / (1 - \Delta\alpha_{\text{lep}}(q^2) - \Delta\alpha_{\text{had}}(q^2))$$

- The Real part of the VP,  $\text{Re}\Pi$ , is obtained from the Imaginary part, which via the *Optical Theorem* is directly related to the cross section,  $\text{Im}\Pi \sim \sigma(e^+e^- \rightarrow \text{hadrons})$ :

$$\Delta\alpha_{\text{had}}^{(5)}(q^2) = -\frac{q^2}{4\pi^2\alpha} P \int_{m_\pi^2}^\infty \frac{\sigma_{\text{had}}^0(s) ds}{s - q^2}, \quad \sigma_{\text{had}}(s) = \frac{\sigma_{\text{had}}^0(s)}{|1 - \Pi|^2}$$

[ $\rightarrow \sigma^0$  requires ‘undressing’, e.g. via  $\cdot(\alpha/\alpha(s))^2 \rightsquigarrow$  iteration needed]

- Observable cross sections  $\sigma_{\text{had}}$  contain the |full photon propagator|<sup>2</sup>, i.e. |infinite sum|<sup>2</sup>.  
 $\rightarrow$  To include the subleading Imaginary part, use dressing factor  $\frac{1}{|1-\Pi|^2}$ .

# Hadronic VP: $\Delta\alpha$

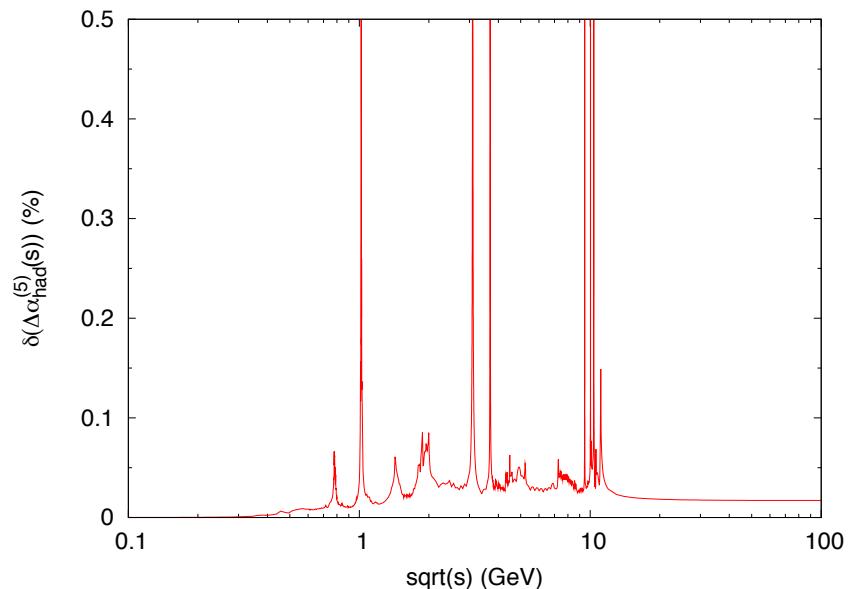
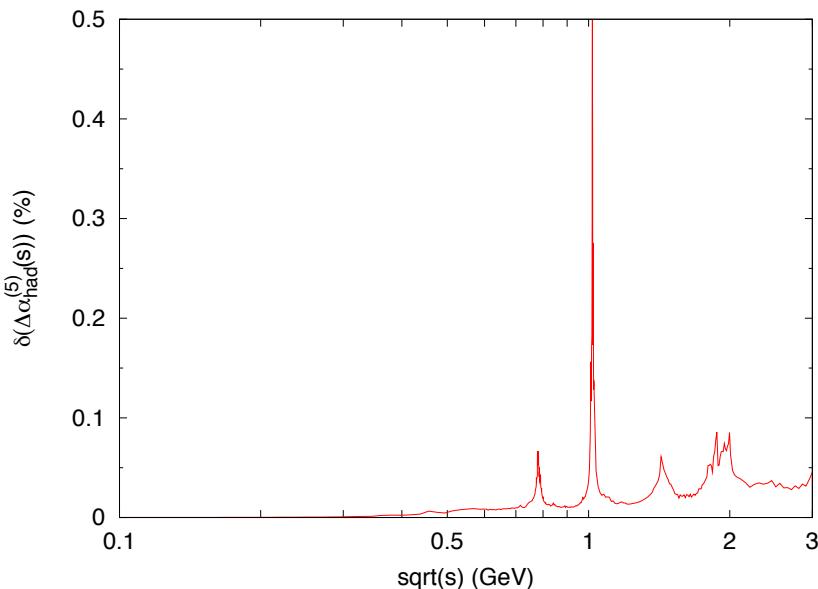
Parametrisations/routines based on ‘global’ data compilations available from a few groups:

- Novosibirsk: <http://cmd.inp.nsk.su/~ignatov/vpl/> tabulation with ROOT package
- Davier et al: [HVPTools](#) (status of distribution? still in preparation?)
- Fred Jegerlehner’s package: <http://www-com.physik.hu-berlin.de/~fjeger/software.html>
  - set of routines with analytic codes and tabulations
  - uses rhad from Harlander+Steinhauser for Im part
  - regular updates (last 5.4.2012)
- HLMNT routine
  - provided upon request by authors (Daisuke Nomura or TT)
  - standalone Fortran, partly analytic, partly tabulation
  - current version is VP\_HLMNT\_v2\_1 (from 27.1.2012), minor update imminent
  - flag to control if narrow resonances included or not, but  $\Phi$  and higher  $\Upsilon$  always included through direct data integration

# Hadronic VP: $\Delta\alpha$

- Typical accuracy  $\delta(\Delta\alpha_{\text{had}}^{(5)}(s))$

Error of VP in the timelike regime at low and higher energies (HLMNT compilation):

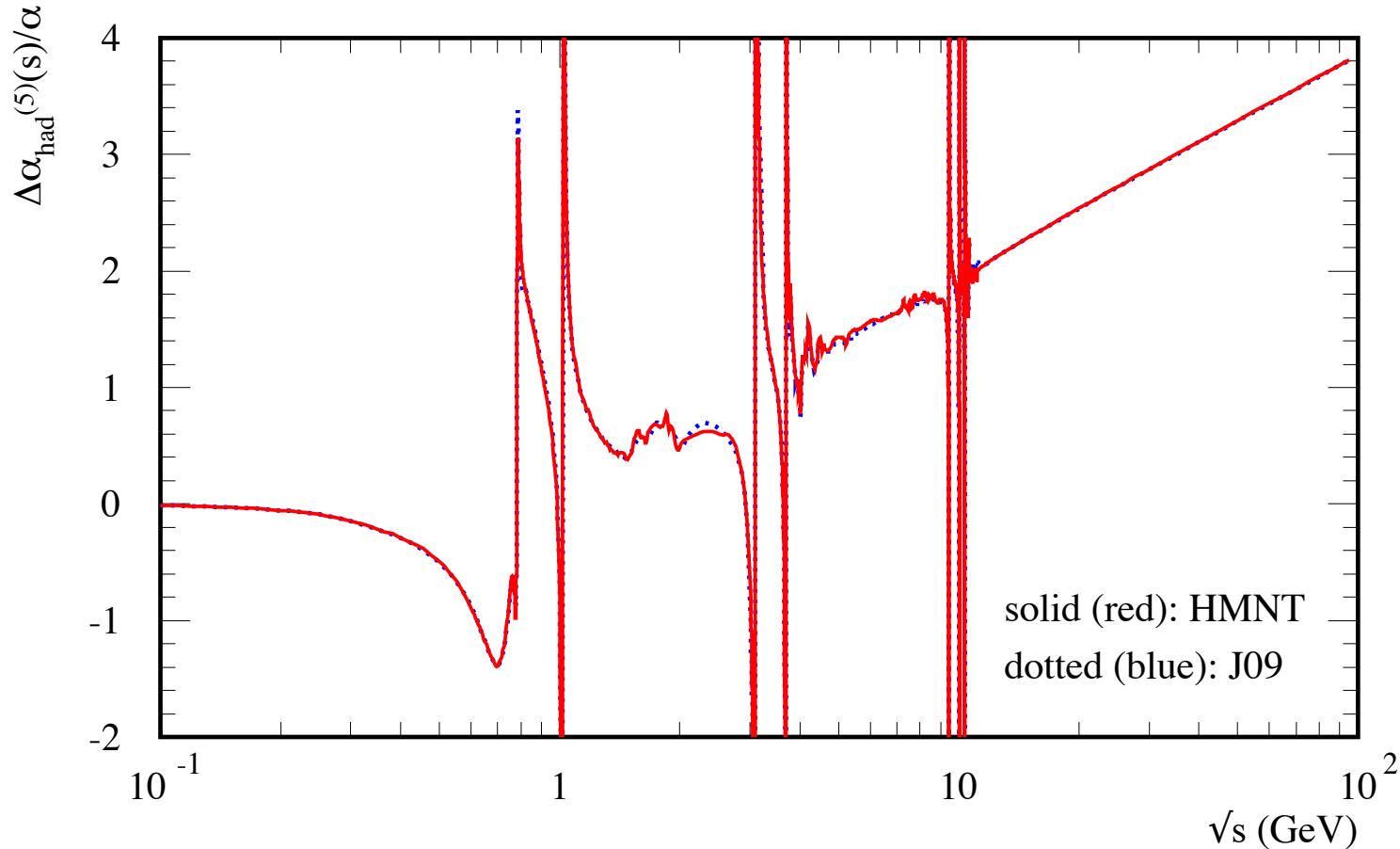


→ Below one per-mille (and typically  $\sim 5 \cdot 10^{-4}$ ), apart from Narrow Resonances where the bubble summation is not well justified.

Enough in the long term? Need for more work in resonance regions.

# Hadronic VP: $\Delta\alpha$

- $\Delta\alpha(q^2)$  in the time-like: HLMNT compared to Fred Jegerlehner's new routines



→ with new version big differences (with 2003 version) gone

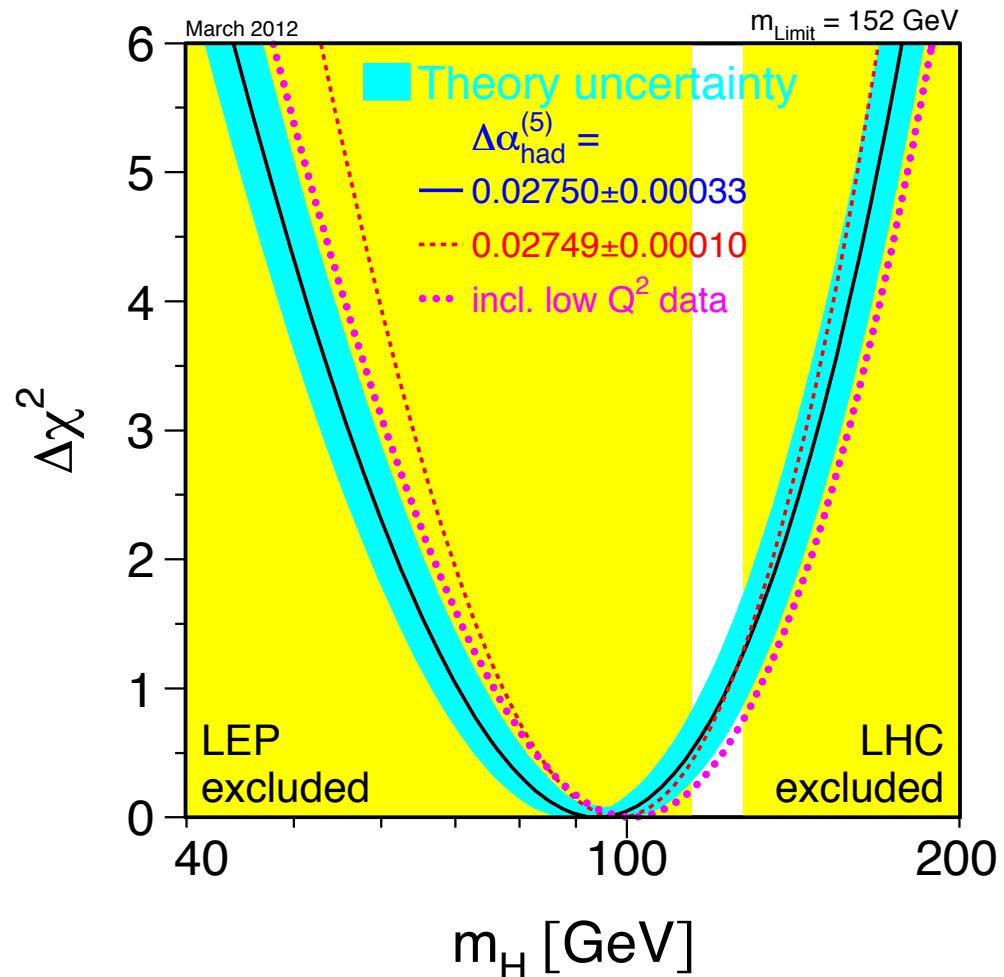
— smaller differences remain and reflect different choices, smoothing etc.

# Hadronic VP: $\Delta\alpha$

$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$  (units of  $10^{-4}$ )

- DHMZ:  $275.7 \pm 1.0$
- Jegerlehner:  $275.10 \pm 2.18$   
 $274.98 \pm 1.35$  (Adler)
- HLMNT:  $276.26 \pm 1.38$
- similar results by all groups now
- limited gain by using more pQCD

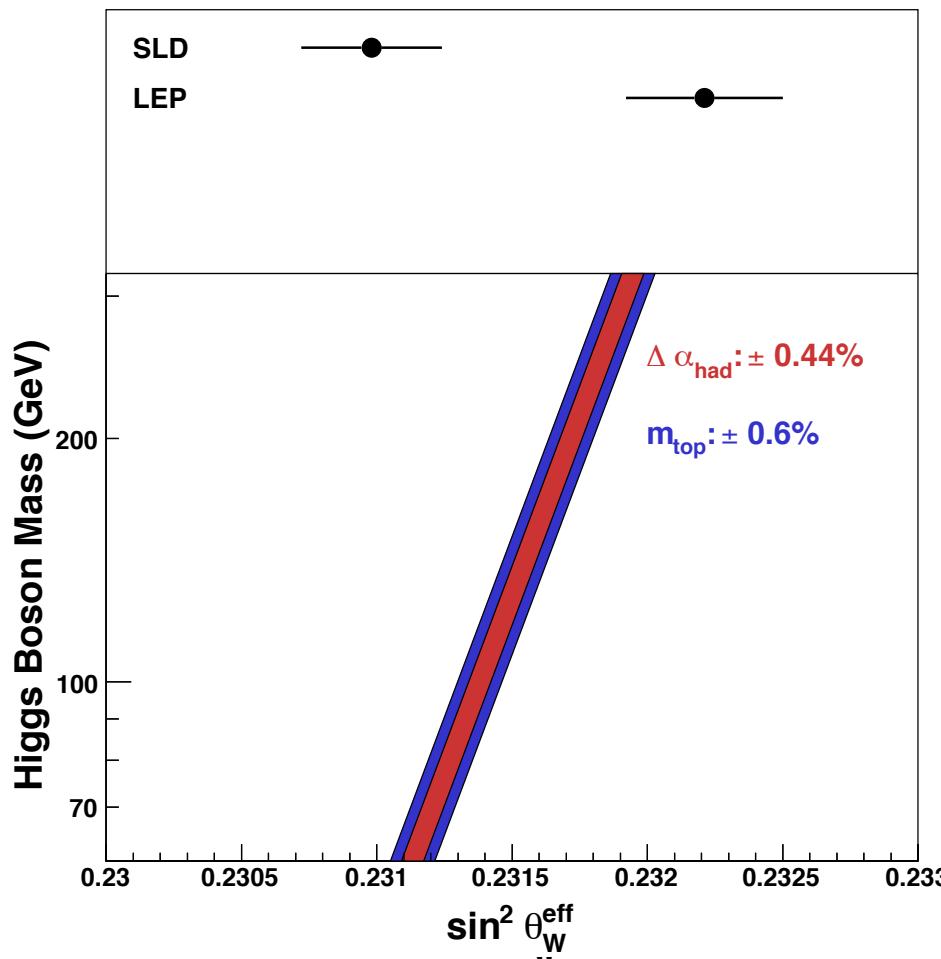
LEP EWWG: The last Blueband plot?



# $\Delta\alpha(M_Z^2)$ is limiting future electroweak precision fits

From Achim Denig

- Test overall consistency of the electroweak Standard Model
- Since the discovery of the Higgs boson more timely than ever



SM running  
 $\sin^2 \Theta_W (M_{\text{Higgs}})$   
limited by  $\alpha_{\text{em}}$

