NEUTRINOS

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CERN Accademic Training , December 2013

Plan of Lectures

I. Standard Neutrino Properties and Mass Terms (Beyond Standard)

II. Effects of ν Mass. Neutrino Oscillations in Vacuum and Matter

III. The Data: The Emerging Picture and Some Implications

Summary I

- In the SM:
  - Accidental global symmetry: $B \times L_e \times L_\mu \times L_\tau \leftrightarrow m_\nu \equiv 0$
  - neutrinos are left and $m_\nu = 0 \Rightarrow$ chirality $\equiv$ helicity $\Rightarrow$ spinors u_- or v_+
  - No distinction between Majorana or Dirac Neutrinos
- If $m_\nu \neq 0 \longrightarrow$ Need to extend SM

  $\longrightarrow$ different ways of adding m_ν to the SM
  - *breaking* total lepton number ($L = L_e + L_\mu + L_\tau$) $\longrightarrow$ Majorana ν: $\nu = \nu^C$
  - *conserving* total lepton number $\longrightarrow$ Dirac ν: $\nu \neq \nu^C$

  $\longrightarrow$ Lepton Mixing$\equiv$ breaking of $L_e \times L_\mu \times L_\tau$
- From direct searches of ν-mass: Tritium β decay: $\sqrt{\sum m_i^2 |U_{ei}|^2} \leq 2.2 eV$

Plan of Lecture II

Other Direct Probes of Neutrino Mass Scale

Neutrino Oscillations in Vacuum

Matter Effects: MSW

ν-less Double-β Decay

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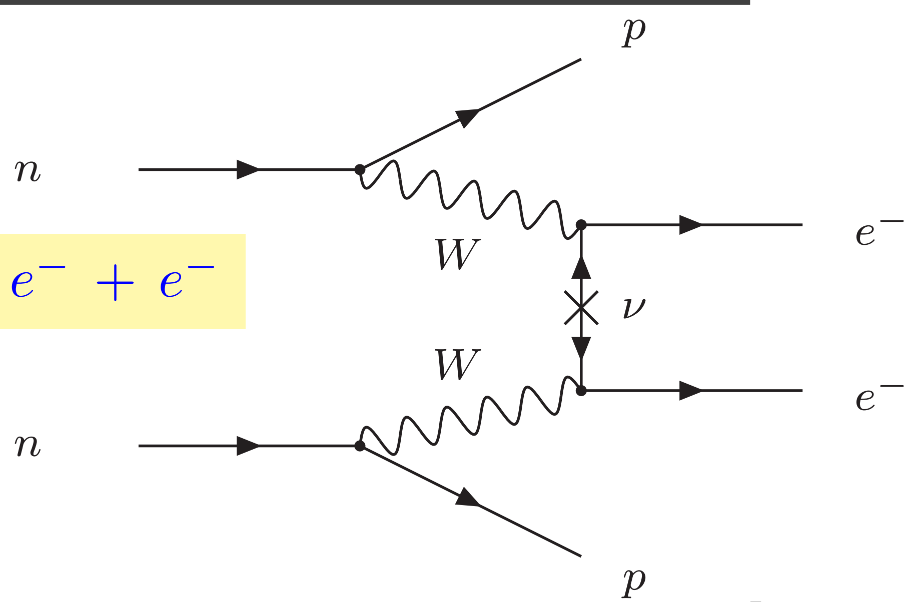
$$(A, Z) \rightarrow (A, Z+2) + e^- + e^-$$



- Amplitude involves the product of two leptonic currents: $\left[\overline{e}\gamma^\mu L\nu\right]\left[\overline{e}\gamma^\mu L\nu\right]$

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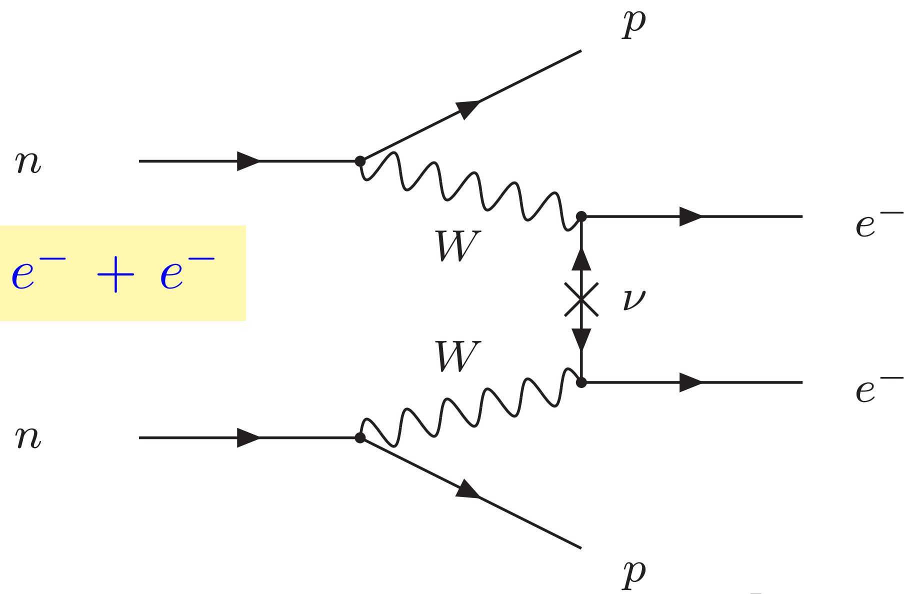

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  - If ν Dirac $\Rightarrow$ ν *annihilates* a *neutrino* and *creates* an *antineutrino*

    $\Rightarrow$ no same state $\Rightarrow$ Amplitude = 0
  - If ν Majorana $\Rightarrow$ $\nu = \nu^c$ *annihilates and creates* a *neutrino=antineutrino*

    $\Rightarrow$ same state $\Rightarrow$ Amplitude $\propto \overbrace{\nu\,(\nu^c)}^{}{}^T \neq 0$
- Amplitude of ν-less-$\beta\beta$ decay is proportional to $|\langle m_{\beta\beta}\rangle| = |\sum U_{ej}^2 m_j|$

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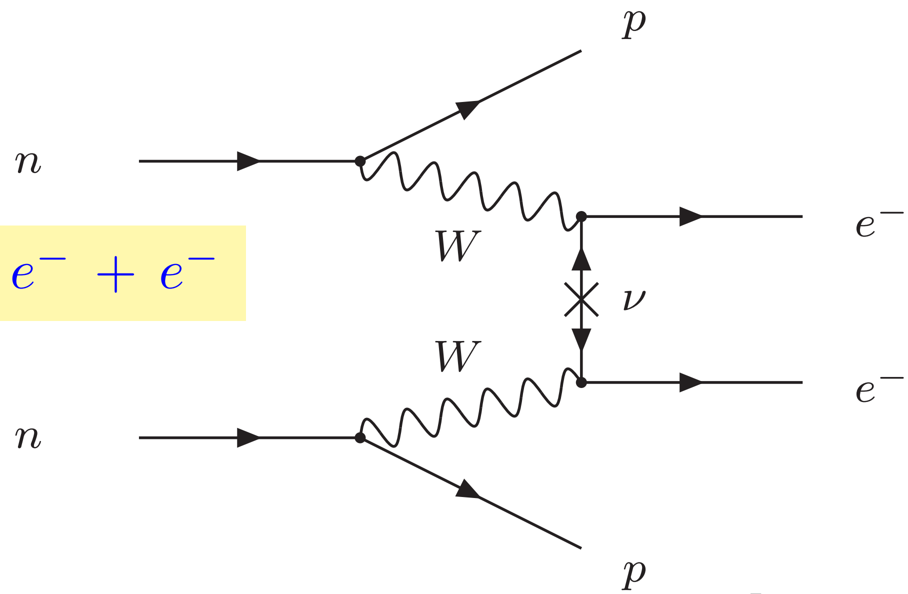

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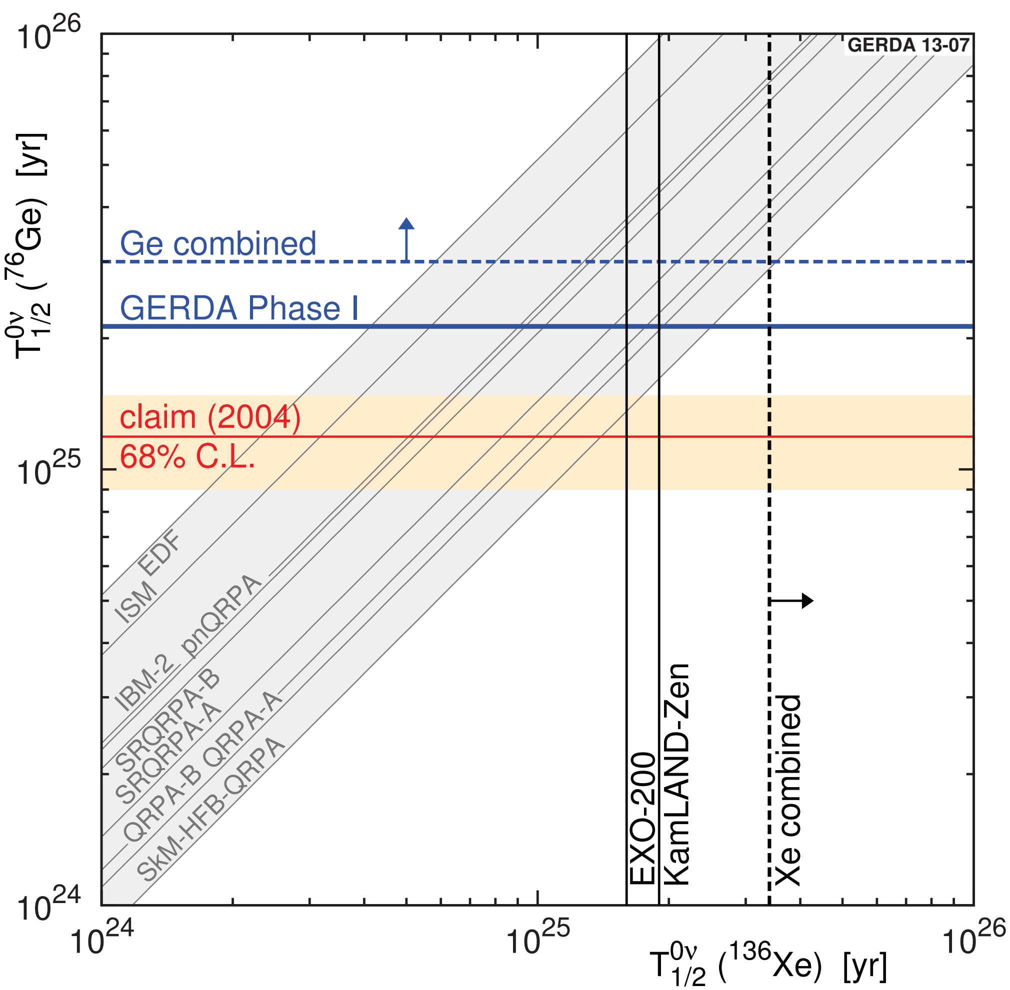
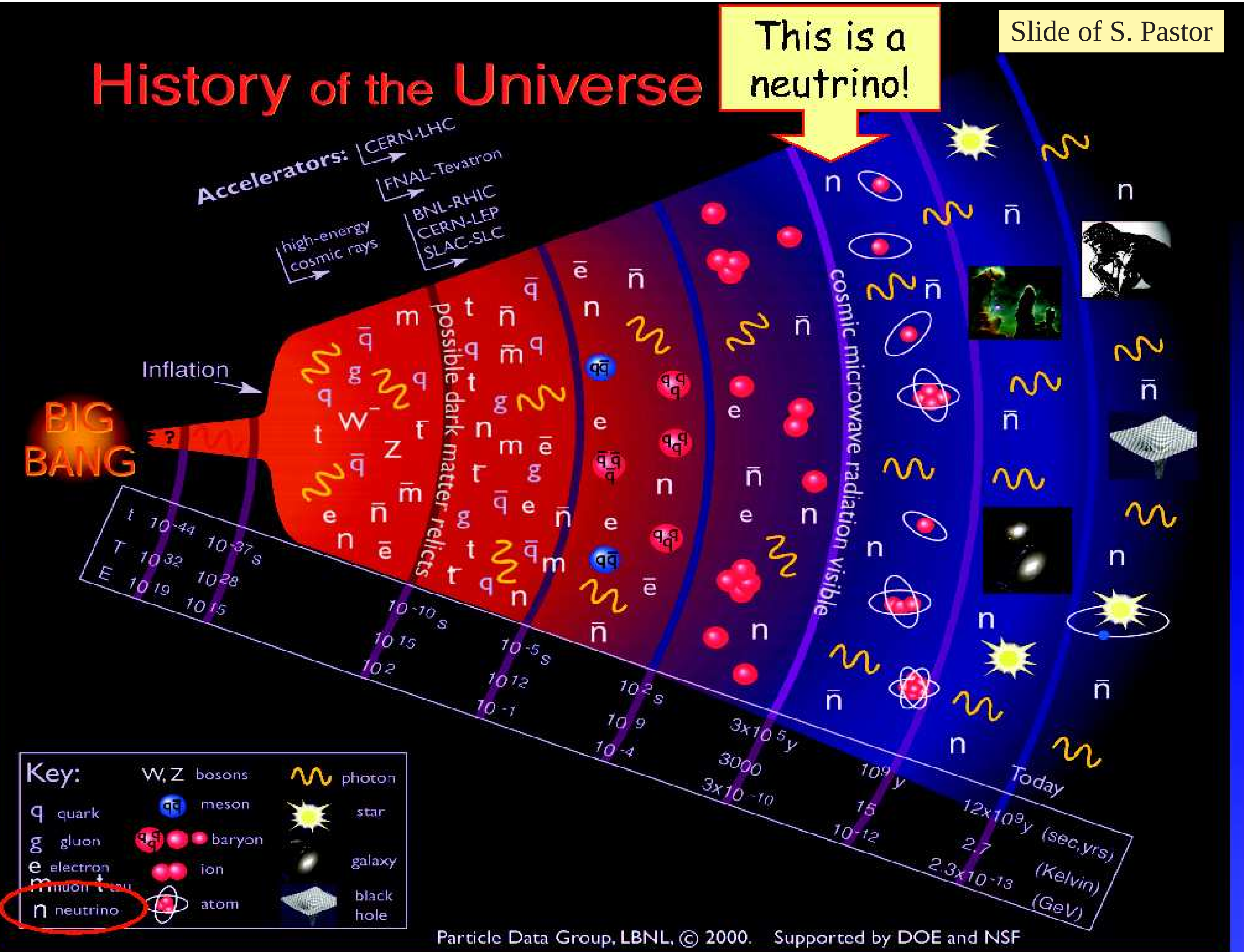
    $\Rightarrow$ same state $\Rightarrow$ Amplitude $\propto$ $\overbrace{\nu\ (\nu^c})^T \neq 0$
- Amplitude of ν-less-$\beta\beta$ decay is proportional to $|\langle m_{\beta\beta}\rangle| = |\sum U_{ej}^2 m_j|$
- Problem is uncertainty in the nuclear matter elements

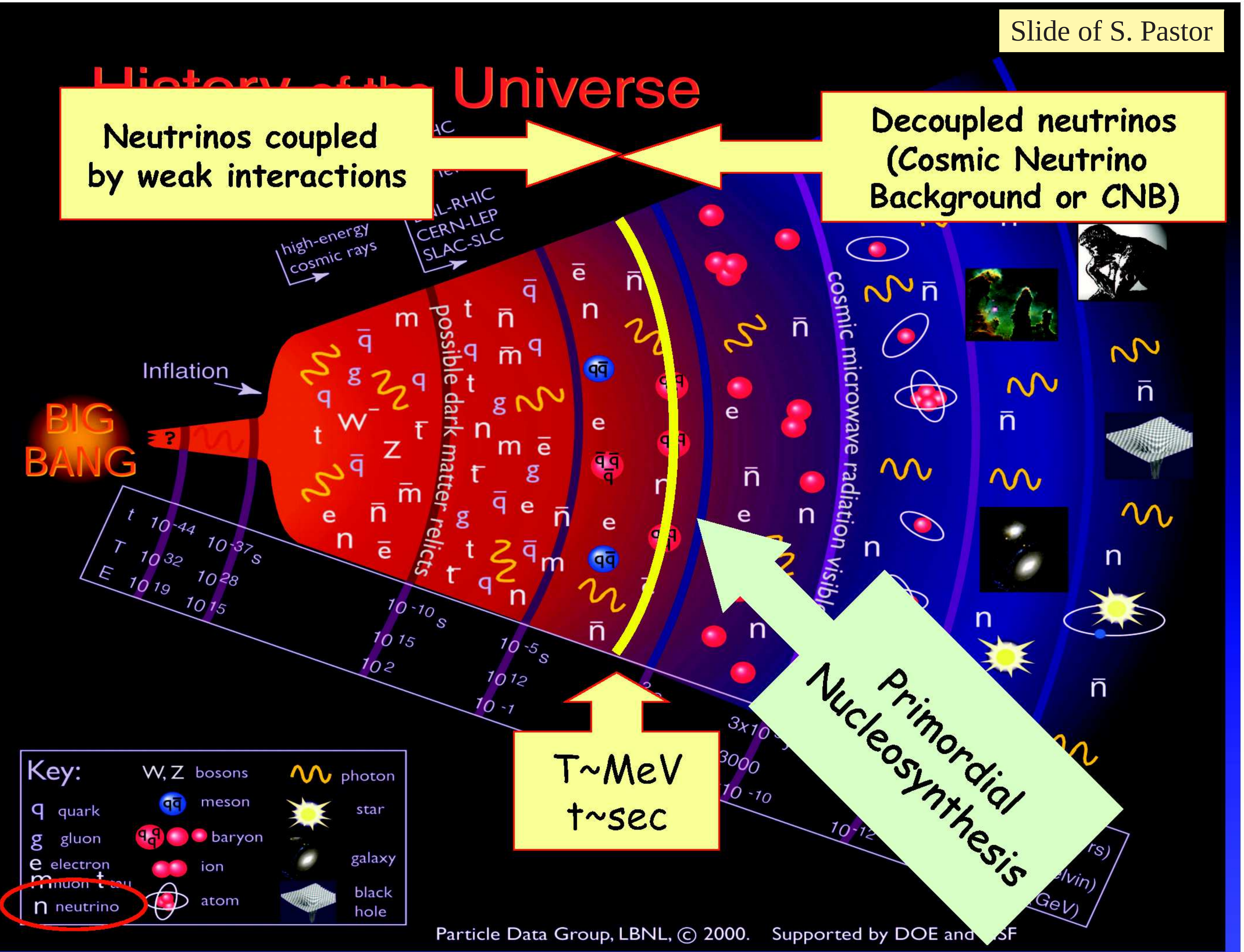
$0\nu\beta\beta$ Decay: Circa 2013

- Bounds from ^{136}Xe exp EXO and KamLAND-ZEN, and from ^{76}Ge exp Gerda



$m_{\beta\beta} \lesssim 0.15 - 0.5$ eV


Slide of S. Pastor
History of the Universe
This is a neutrino!
Accelerators:
CERN-LHC
FNAL-Tevatron
BNL-RHIC
CERN-LEP
SLAC-SLC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relicts
cosmic microwave radiation visible
t 10-44 10-37 s
T 1032 1028
E 1019 1015
10-10 s
1015
102
10-5 s
1012
10-1
102 s
109
10-4
3x105 y
3000
3x10-10
109 y
15
10-12
Today
12x109y (sec,yrs)
2.7 (Kelvin)
2.3x10-13 (GeV)
Key:
W,Z bosons
photon
q quark
meson
star
g gluon
baryon
e electron
ion
galaxy
m muon t tau
n neutrino
atom
black hole
Particle Data Group, LBNL, © 2000. Supported by DOE and NSF



Slide of S. Pastor
History of the Universe
Neutrinos coupled
by weak interactions
Decoupled neutrinos
(Cosmic Neutrino
Background or CNB)
high-energy
cosmic rays
BNL-RHIC
CERN-LEP
SLAC-SLC
Inflation
BIG
BANG
possible dark matter relicts
cosmic microwave radiation visible
t 10-44 10-37 s
T 1032 1028
E 1019 1015
10-10 s
1015
102
10-5 s
1012
10-1
T~MeV
t~sec
Primordial
Nucleosynthesis
Key:
W, Z bosons
photon
q quark
meson
star
g gluon
baryon
e electron
ion
galaxy
m muon t tau
n neutrino
atom
black
hole
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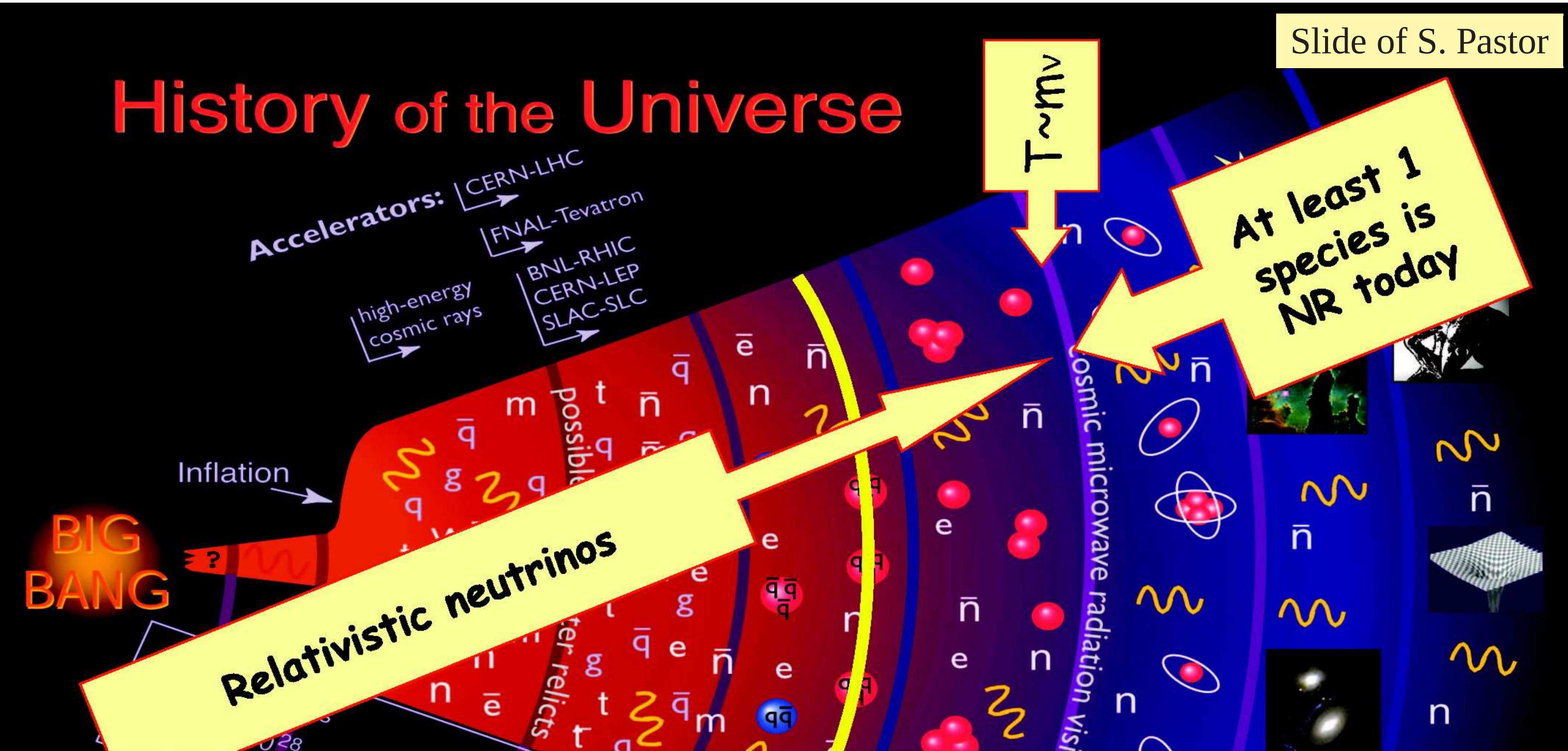

Neutrino cosmology is interesting because Relic neutrinos are very abundant:

• The CNB contributes to radiation at early times and to matter at late times (info on the number of neutrinos and their masses)

• Cosmological observables can be used to test standard or non-standard neutrino properties

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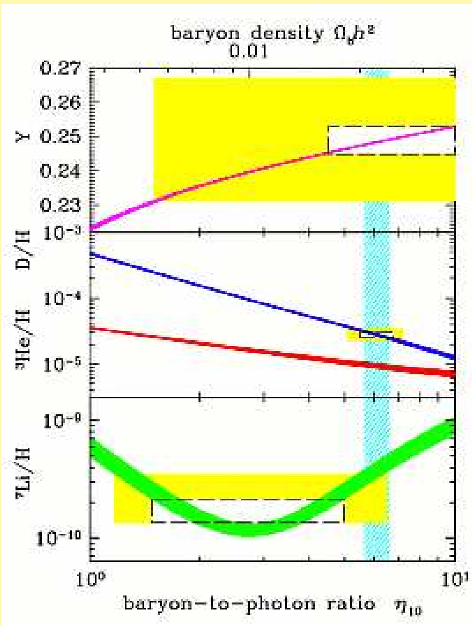
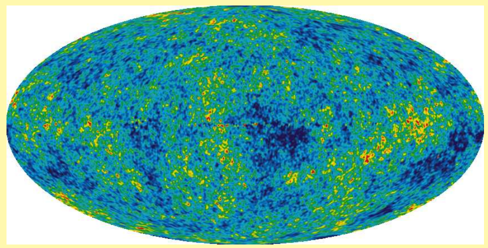
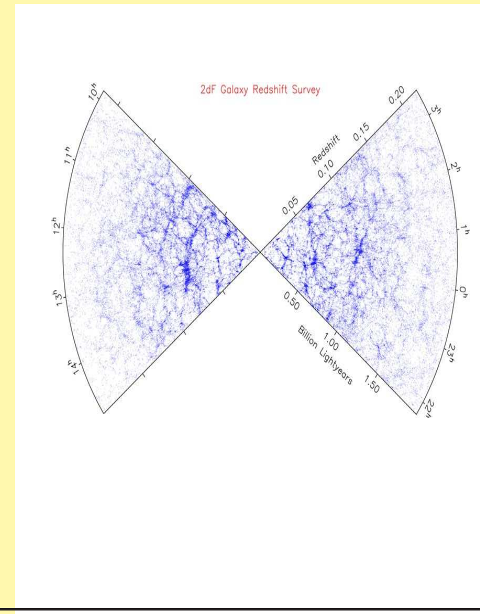
Massive ν in Cosmology

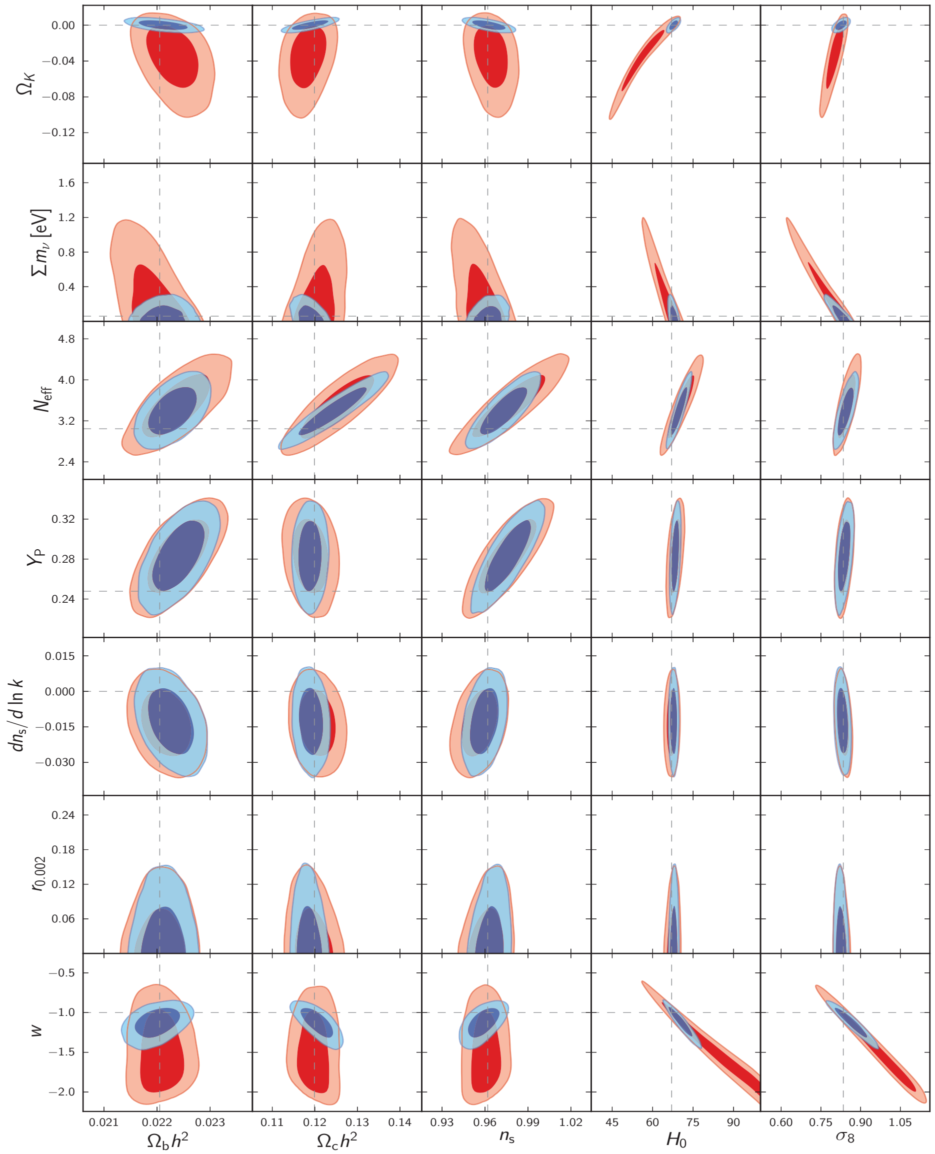
Relic $\nu's$: Effects in several cosmological observations at several epochs

		
Primordial Nucleosynthesis BBN	Cosmic Microwave Background CMB	Large Scale Structure Formation LSS
$T \sim$ MeV	$T \lesssim$ eV	
Number of $\nu's$ ($N_{\rm rel}$)	$N_{\rm rel}$ and $\sum m_\nu$	

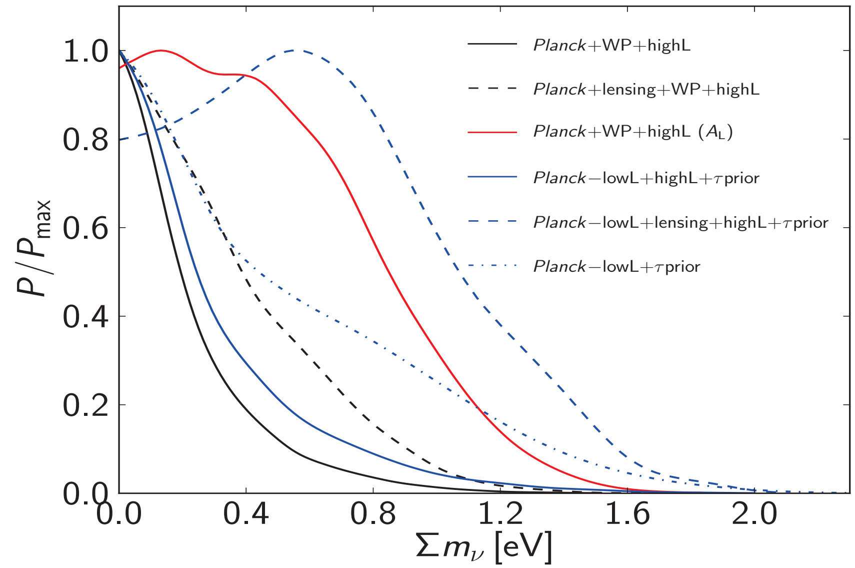
Observables also depend on all other cosmological parameters

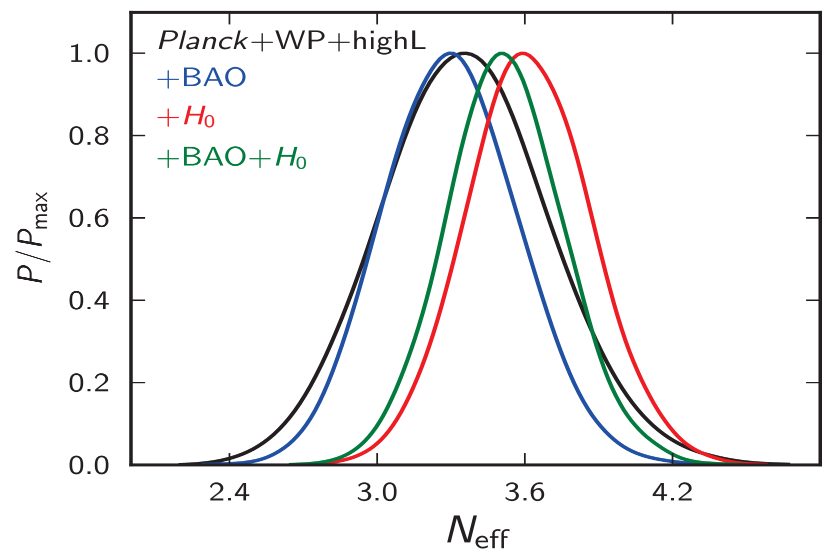
Cosmological Analysis by Planck

Correlations



Range of Bounds





Example of cosmological bounds on m_ν

Dependence on Data Samples and Cosmological Model

Model	Observables	Σm_ν (eV) 95%
$\Lambda\mathrm{CDM} + m_\nu$	Planck-lowL+τprior	≤ 1.31
$\Lambda\mathrm{CDM} + m_\nu$	Planck+WP+highL(A_L)	≤ 1.08
$\Lambda\mathrm{CDM} + m_\nu$	Planck+Lens+WP+highL(A_L)	≤ 0.85
$\Lambda\mathrm{CDM} + m_\nu$	Planck+WP+highL	≤ 0.66
$o\Lambda\mathrm{CDM} + m_\nu$	Planck+WP+highL	≤ 0.98
$\Lambda\mathrm{CDM} + m_\nu$	Planck+Lens+WP+highL+BAO	≤ 0.25
$o\Lambda\mathrm{CDM} + m_\nu$	Planck+Lens+WP+highL+BAO	≤ 0.36

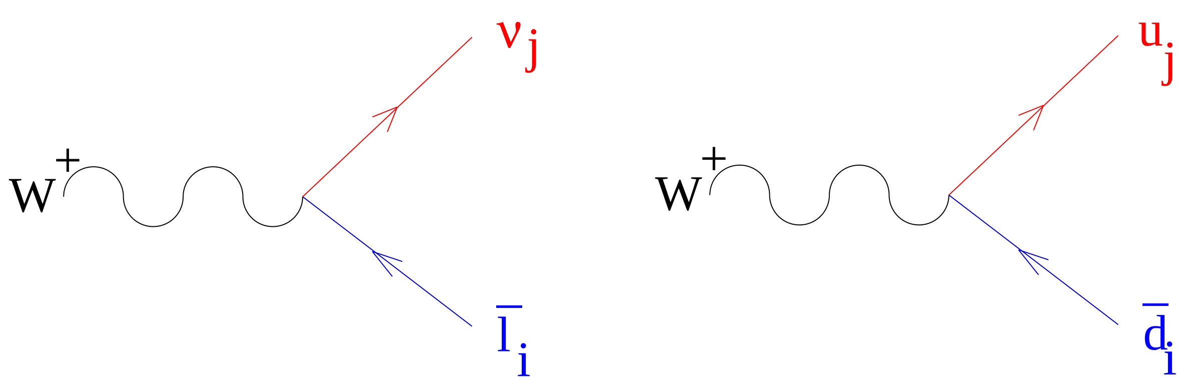
Lesson for Particle Physicists:

Careful with what you call *Cosmological bound on* m_ν

Effects of ν Mass

- Neutrino masses can have kinematic effects
- Also if neutrinos have a mass the charged current interactions of leptons are not diagonal (same as quarks)

$$\frac{g}{\sqrt{2}} W^+_\mu \sum_{ij} \left(U^{ij}_{LEP}\, \overline{\ell^i}\, \gamma^\mu\, L\, \nu^j + U^{ij}_{CKM}\, \overline{U^i}\, \gamma^\mu\, L\, D^j \right) + h.c.$$



Effects of ν Mass: Flavour Transitions

- Flavour ($\equiv$ Interaction) basis (production and detection): ν_e, ν_μ and ν_τ
- Mass basis (free propagation in space-time): ν_1, ν_2 and ν_3 $\ldots$

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- In general interaction eigenstates $\neq$ propagation eigenstates

$$U_{\rm LEP}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\\ \ldots\end{pmatrix} = \begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

$\Rightarrow$ Flavour is not conserved during propagation

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$\Rightarrow$ Flavour is not conserved during propagation

$\Rightarrow$ ν can be detected with different (or same) flavour than produced

- The probability $P_{\alpha\beta}$ of producing neutrino with flavour α and detecting with flavour β has to depend on:
  - Misalignment between interaction and propagation states ($\equiv U$)
  - Difference between propagation eigenvalues
  - Propagation distance

Vacuum Mass Oscillations

Vacuum Mass Oscillations

- If neutrinos have mass, a weak eigenstate $|\nu_\alpha\rangle$ produced in $l_\alpha + N \to \nu_\alpha + N'$ is a linear combination of the mass eigenstates ($|\nu_i\rangle$)

$$|\nu_\alpha\rangle = \sum_{i=1}^{n} U_{\alpha i}\, |\nu_i\rangle$$

U is the leptonic mixing matrix.

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- it can be detected with flavour β with probability

$$P_{\alpha\beta} = |\langle \nu_\beta(t) | \nu_\alpha(0)\rangle|^2 = |\sum_{i=1}^{n} U_{\alpha i} U^*_{\beta i} \langle \nu_i(t) | \nu_i(0)\rangle|^2$$

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$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{j\neq i}^{n} \mathrm{Re}[U^\star_{\alpha i} U_{\beta i} U_{\alpha j} U^\star_{\beta j}] \sin^2\left(\frac{\Delta_{ij}}{2}\right) + 2\sum_{j\neq i} \mathrm{Im}[U^\star_{\alpha i} U_{\beta i} U_{\alpha j} U^\star_{\beta j}] \sin\left(\Delta_{ij}\right)$$

with $\Delta_{ij} = (E_i - E_j)t$

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  $(\mathbf{2})$ *relativistic* ν

$$E_i = \sqrt{p_i^2 + m_i^2} \simeq p_i + \frac{m_i^2}{2E_i}$$

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(**2**) *relativistic* ν

$$E_i = \sqrt{p_i^2 + m_i^2} \simeq p_i + \frac{m_i^2}{2E_i}$$

(**3**) Lowest order in mass $p_i \simeq p_j = p \simeq E$

$$\frac{\Delta_{ij}}{2} = 1.27 \frac{m_i^2 - m_j^2}{\mathrm{eV}^2} \frac{L/E}{\mathrm{Km/GeV}}$$

Vacuum Oscillations

- The oscillation probability:

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{j\neq i}^{n} \mathrm{Re}[U^{\star}_{\alpha i}U_{\beta i}U_{\alpha j}U^{\star}_{\beta j}]\sin^2\left(\frac{\Delta_{ij}}{2}\right) + 2\sum_{j\neq i} \mathrm{Im}[U^{\star}_{\alpha i}U_{\beta i}U_{\alpha j}U^{\star}_{\beta j}]\sin\left(\Delta_{ij}\right)$$

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$\to$ conserves CP

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– The last piece $2\sum_{j\neq i} \mathrm{Im}[U^{\star}_{\alpha i} U_{\beta i} U_{\alpha j} U^{\star}_{\beta j}] \sin\left(\Delta_{ij}\right)$ opposite sign for $\overline{\nu}$

$\longrightarrow$ violates CP

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- $P_{\alpha\beta}$ depends on Theoretical Parameters
  - $\Delta m_{ij}^2 = m_i^2 - m_j^2$ The mass differences
  - $U_{\alpha j}$ The mixing angles (and Dirac phases)

and on Two *Experimental* Parameters:
  - E The neutrino energy
  - L Distance ν source to detector

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and on Two *Experimental* Parameters:
  - E The neutrino energy
  - L Distance ν source to detector

- No information on mass scale nor Majorana phases

2-ν Oscillations

- For 2-ν: $U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

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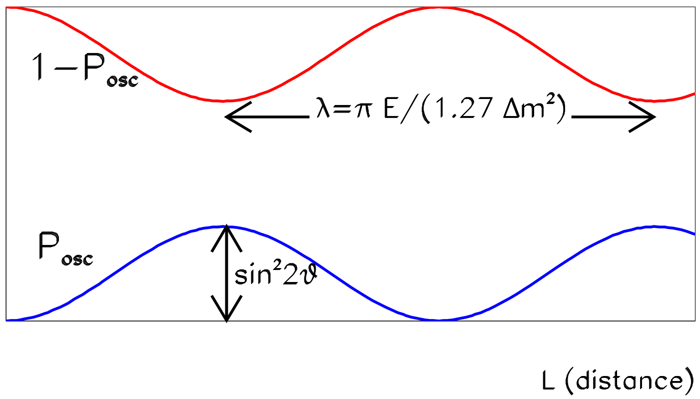
$P_{osc} = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$ Appear

$P_{\alpha\alpha} = 1 - P_{osc}$ Disappear



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- A state mixture of 2 neutrino species $|\nu_e\rangle$ and $|\nu_X\rangle$ or equivalently of $|\nu_1\rangle$ and $|\nu_2\rangle$

$$\Phi(x) = \Phi_e(x)|\nu_e\rangle + \Phi_X(x)|\nu_X\rangle = \Phi_1(x)|\nu_1\rangle + \Phi_2(x)|\nu_2\rangle$$

Equations of Motion for Weak Eigenstates

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- Using (1) in Dirac Eq. we can factorize ϕ_i and α_x and get:

$$-i\frac{\partial\nu_1(x)}{\partial x} = \left\{E_1^2 - m_1^2\right\}^{1/2}\nu_1(x)$$
$$-i\frac{\partial\nu_2(x)}{\partial x} = \left\{E_2^2 - m_2^2\right\}^{1/2}\nu_2(x)$$

- In the relativistic limit and first order in mass $\sqrt{E^2 - m_i^2} \simeq E - \frac{m_i^2}{2E}$

$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \begin{pmatrix} E - \frac{m_1^2}{2E} & 0 \\ 0 & \frac{E - m_2^2}{2E}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

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- And the flavour transition probability

$$P(\nu_\alpha \to \nu_\beta) = |\nu_\beta(L)|^2 = B_1^2 + B_2^2 + 2B_1B_2\cos(2\omega L) = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

ν Oscillations: Experimental Probes

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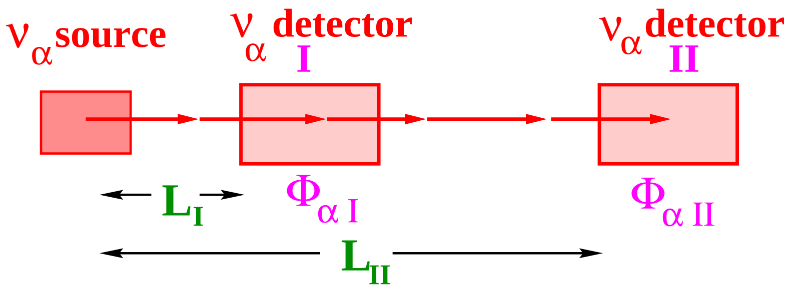
Disappearance Experiment



Compares $\Phi_{\alpha\ I}$ and $\Phi_{\alpha\ II}$ to look for loss

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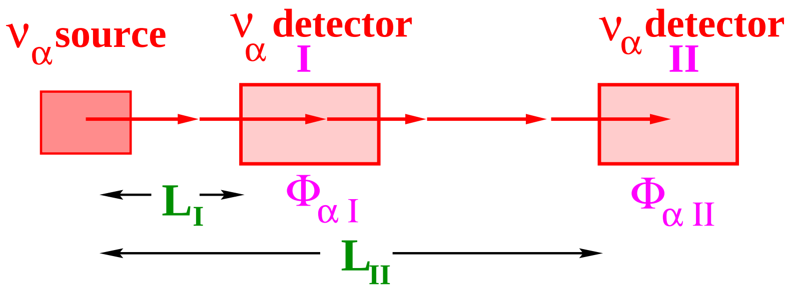

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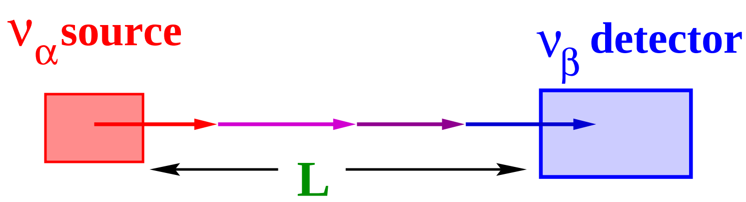

Searches for β diff α

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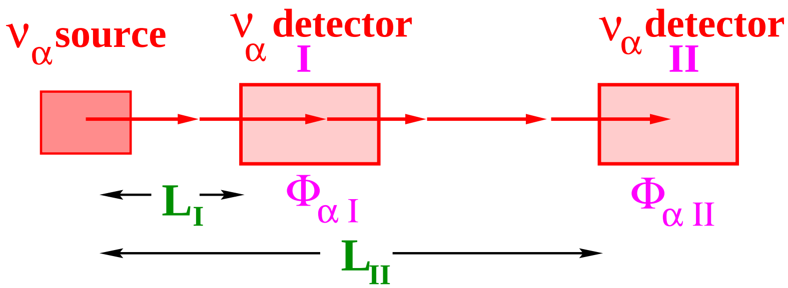

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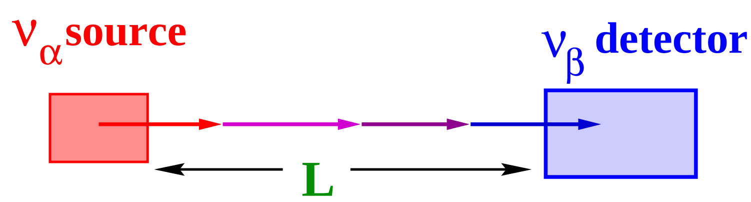

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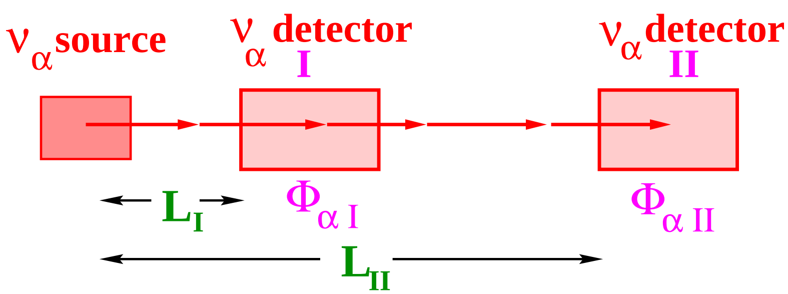

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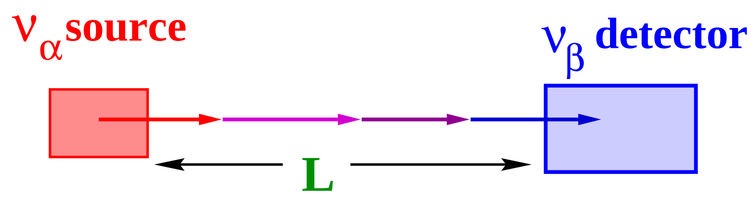
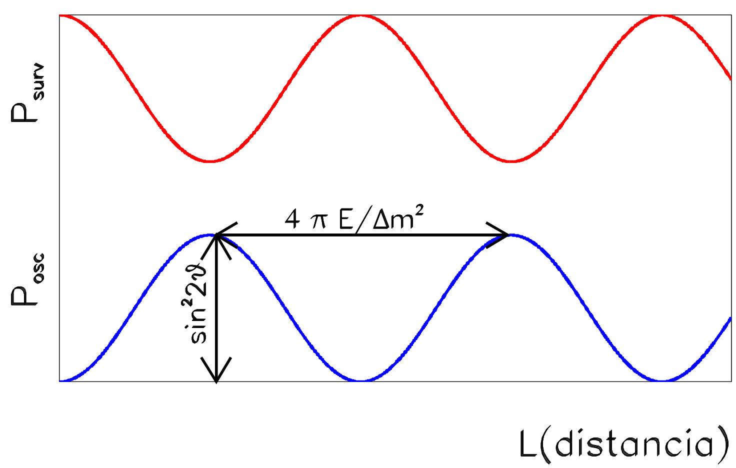


Searches for β diff α

- To detect oscillations we can study the neutrino flavour as function of the Distance to the source



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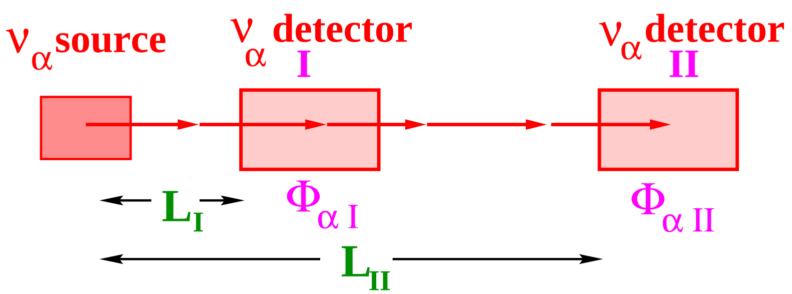

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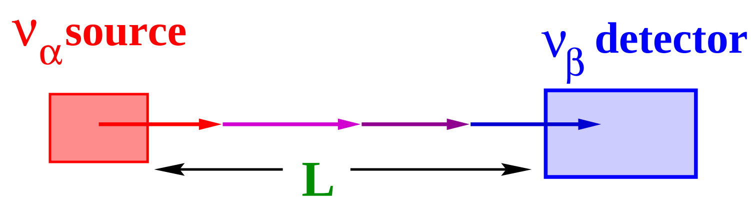

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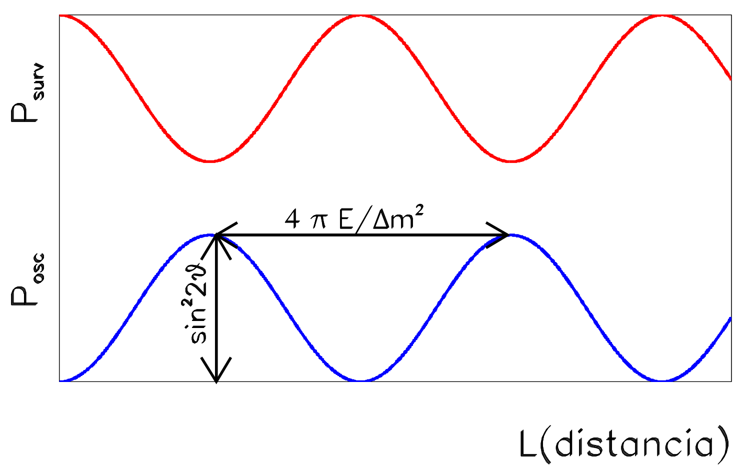
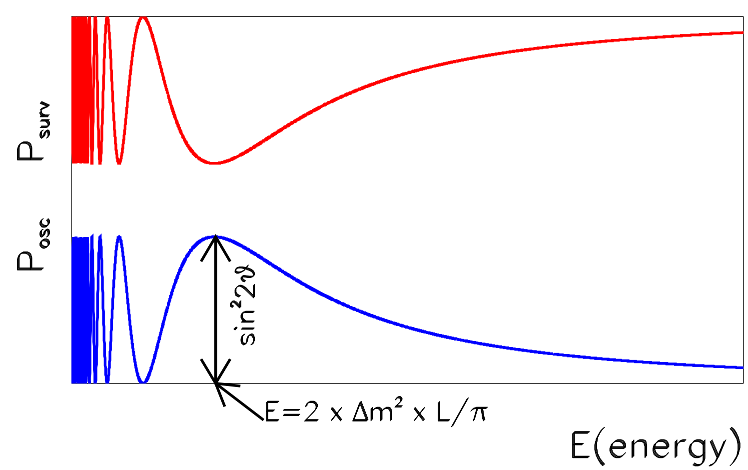


As function of the neutrino Energy



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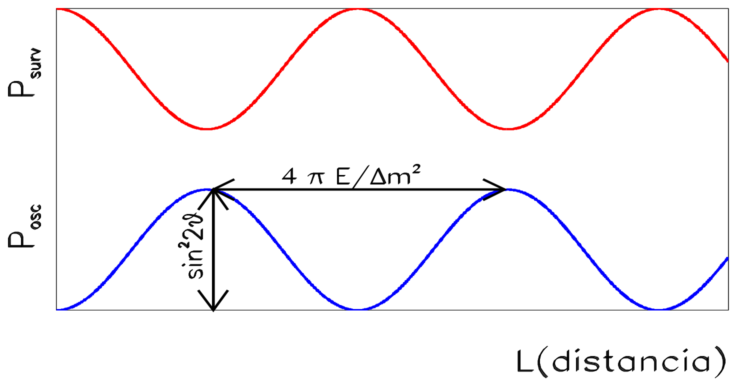

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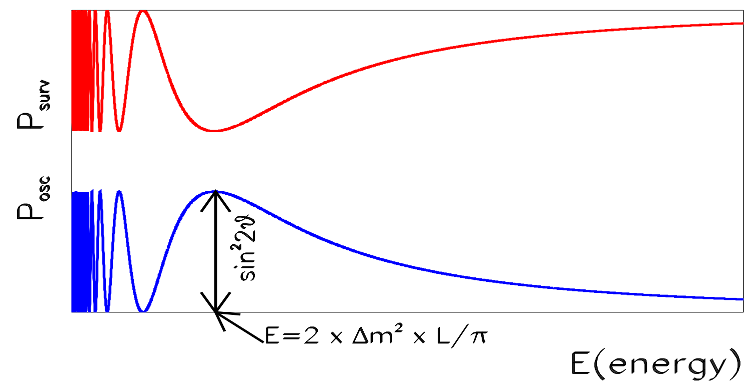

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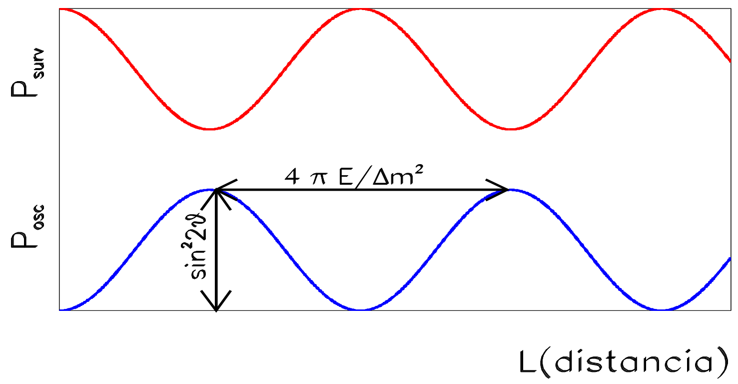

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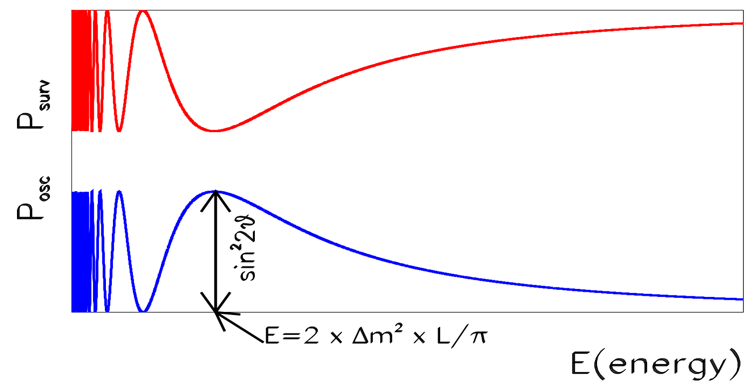
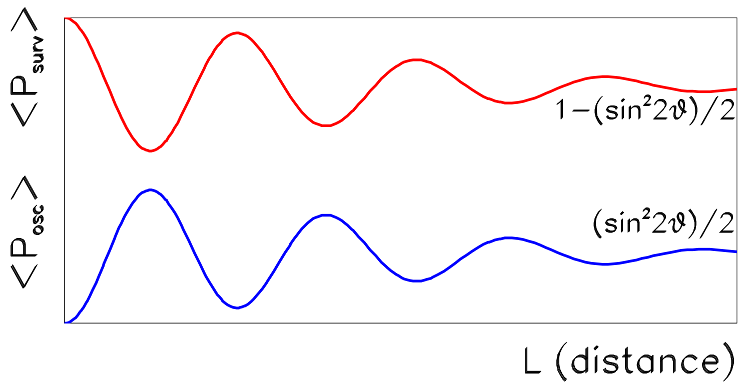
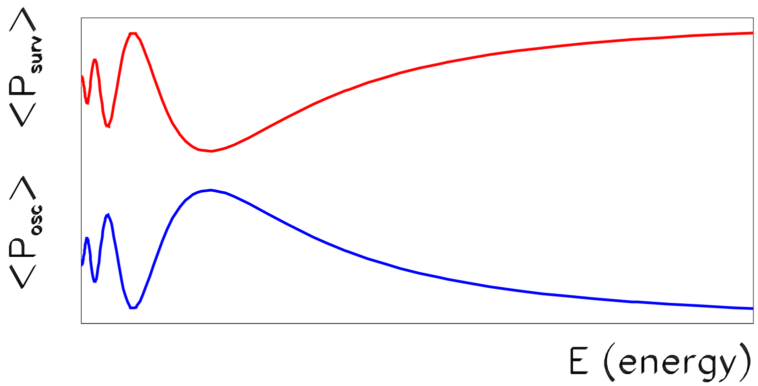


- In real experiments $\Rightarrow$ $\langle P_{\alpha\beta}\rangle = \int dE_\nu \frac{d\Phi}{dE_\nu} \sigma_{CC}(E_\nu) P_{\alpha\beta}(E_\nu)$





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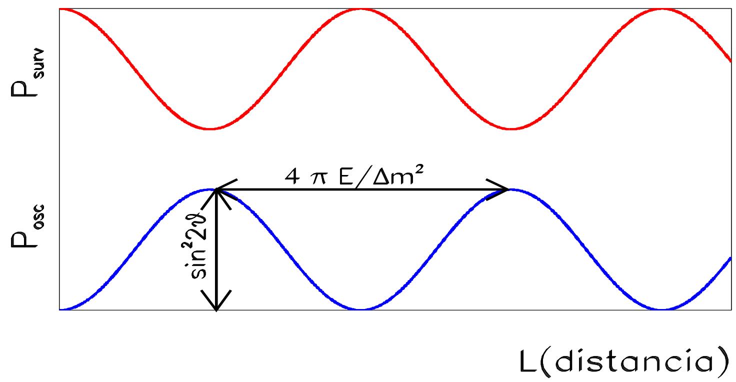
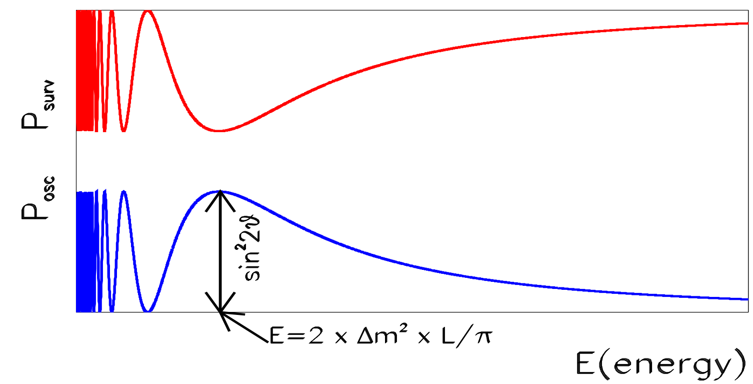


As function of the neutrino Energy



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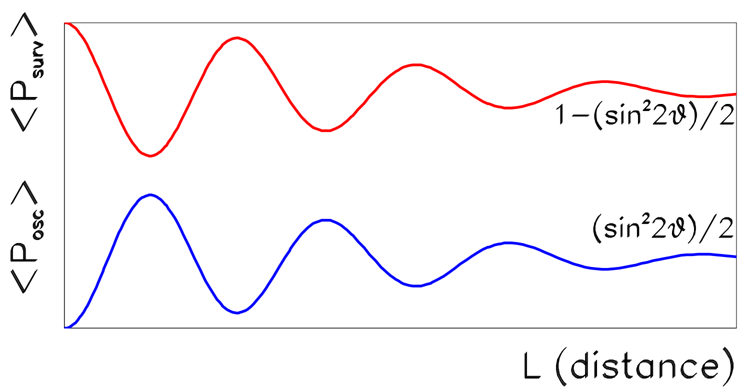
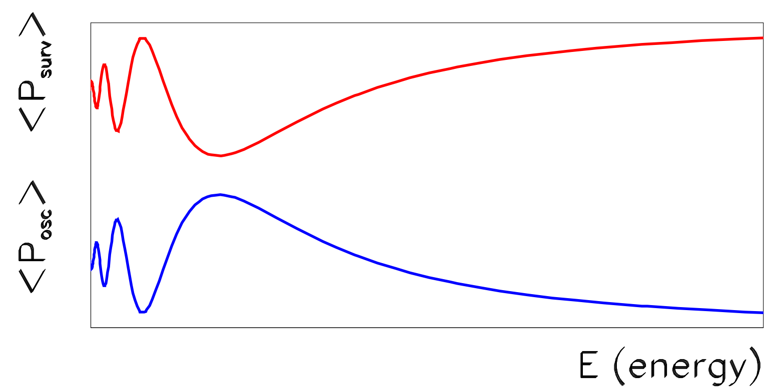


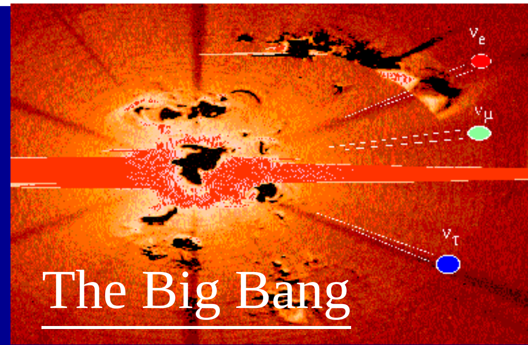


- Maximal sensitivity for $\Delta m^2 \sim E/L$

– $\Delta m^2 \ll E/L \;\Rightarrow \langle \sin^2(1.27\Delta m^2 L/E)\rangle \simeq 0 \rightarrow \langle P_{osc}\rangle \simeq 0$

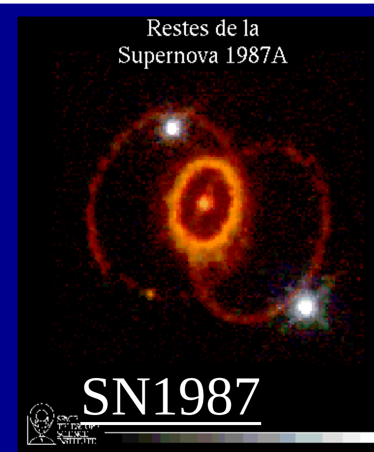
– $\Delta m^2 \gg E/L \;\Rightarrow \langle \sin^2(1.27\Delta m^2 L/E)\rangle \simeq \frac{1}{2} \rightarrow \langle P_{osc}\rangle \simeq \frac{1}{2}\sin^2(2\theta)$

Sources of ν's



$\rho_\nu = 330/\text{cm}^3$

E_ν =0.0004 eV



$E_\nu \sim$ MeV

The Sun

ν_e

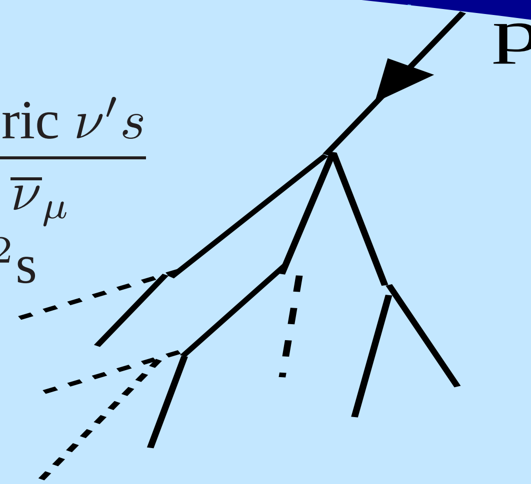
$\Phi_\nu^{Earth} = 6 \times 10^{10} \nu/\text{cm}^2\text{s}$

$E_\nu \sim$ 0.1–20 MeV

Atmospheric $\nu's$

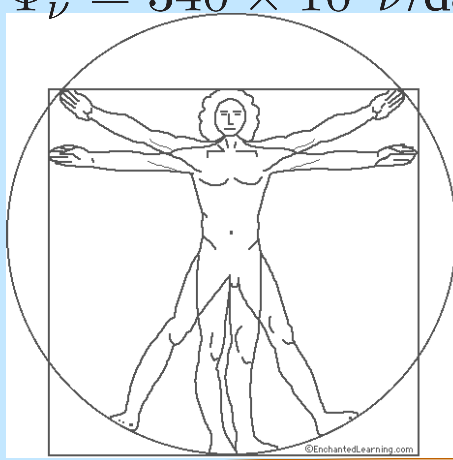
$\nu_e, \nu_\mu, \overline{\nu}_e, \overline{\nu}_\mu$

$\Phi_\nu \sim 1\nu/\text{cm}^2\text{s}$



Human Body

$\Phi_\nu = 340 \times 10^6 \nu$/day



Nuclear Reactors

$E_\nu \sim$ few MeV

$\overline{\nu}_e$

Earth's radioactivity

$\Phi_\nu \sim 6 \times 10^6 \nu/\text{cm}^2\text{s}$

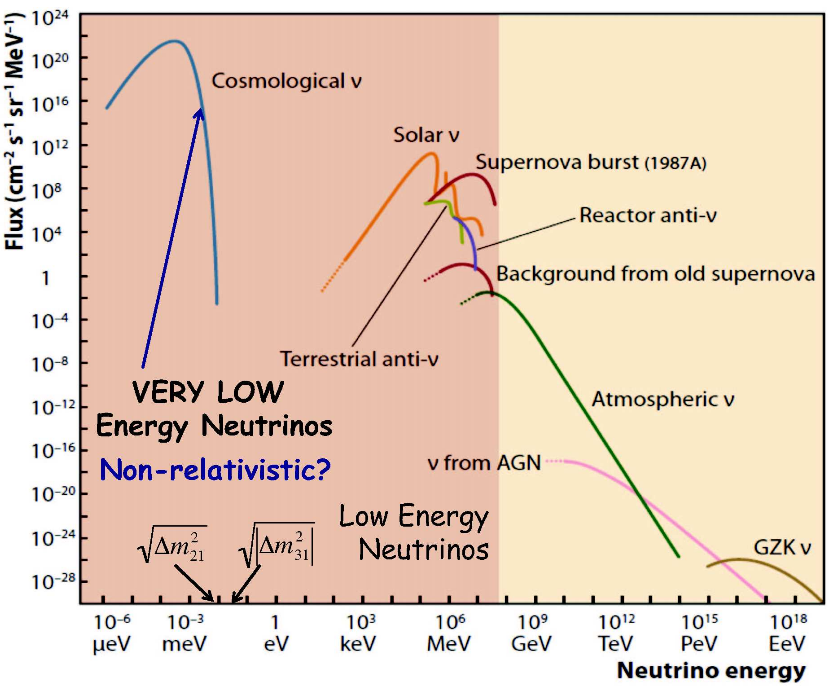
Accelerators

$E_\nu \simeq$ 0.3–30 GeV

To allow observation of neutrino oscillations:

– Nature has to be good: $\theta \not\ll 0$

– Need the right set up ($\equiv$right $\langle \frac{L}{E} \rangle$) for Δm^2



Source	E (GeV)	L (Km)	Δm^2 (eV^2)
Solar	10^{-3}	10^{7}	10^{-10}
Atmos	0.1–10^2	10–10^3	10^{-1}–10^{-4}
Reactor	10^{-3}	SBL: 0.1–1	10^{-2}–10^{-3}
		LBL: 10–10^2	10^{-4}–10^{-5}
Accel	10	SBL: 0.1	$\gtrsim 0.01$
		LBL: 10^2– 10^3	10^{-2}–10^{-3}

ν Interactions

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- Due to SM Weak Interactions

$$\sigma^{\nu p} \sim 10^{-38}\mathrm{cm}^2 \frac{E_\nu}{\mathrm{GeV}}$$

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- Let's consider for example atmospheric $\nu' s$?

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$\Longrightarrow$ Need huge detectors with Exposure $\sim$ KTon $\times$ year

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- In *coherent* interactions $\Rightarrow$ ν and medium remain unchanged

  Interference of scattered and unscattered ν waves

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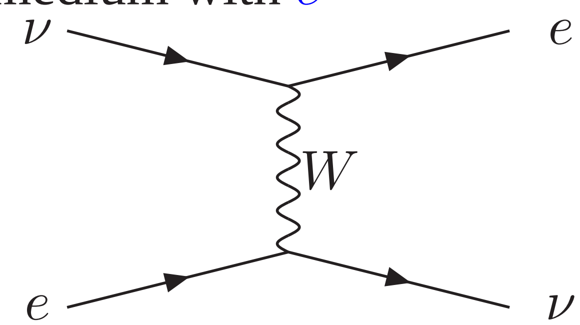

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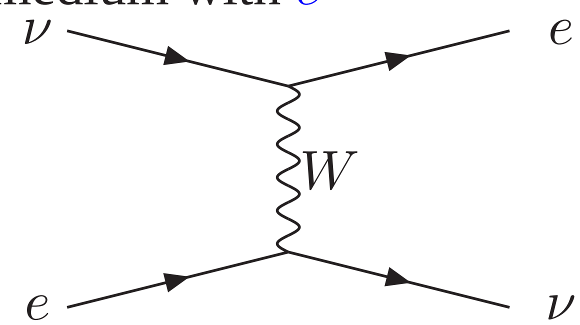

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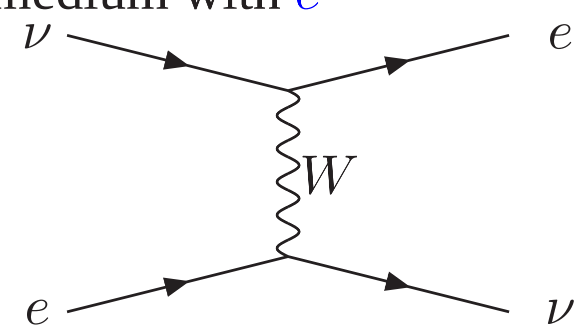

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- Other potentials for ν_e $(\overline{\nu}_e)$ due to different particles in medium

medium	V_C	V_N
e^+ and e^-	$\pm\sqrt{2}G_F(N_e - N_{\bar{e}})$	$\mp\frac{G_F}{\sqrt{2}}(N_e - N_{\bar{e}})(1 - 4\sin^2\theta_W)$
p and $\bar{p}$	0	$\pm\frac{G_F}{\sqrt{2}}(N_p - N_{\bar{p}})(1 - 4\sin^2\theta_W)$
n and $\bar{n}$	0	$\mp\frac{G_F}{\sqrt{2}}(N_n - N_{\bar{n}})$
Neutral $(N_e = N_p)$	$\pm\sqrt{2}G_F N_e$	$\mp\frac{G_F}{\sqrt{2}}N_n$

Neutrinos in Matter: Evolution Equation

Evolution Eq. for $|\nu\rangle = \nu_1|\nu_1\rangle + \nu_2|\nu_2\rangle \equiv \nu_e|\nu_e\rangle + \nu_X|\nu_X\rangle$ $(X = \mu, \tau, \text{sterile})$

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(a) In vacuum in the weak basis

$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_e\\ \nu_X\end{pmatrix} = E - \frac{m_1^2+m_2^2}{2E} - \begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_X\end{pmatrix}$$

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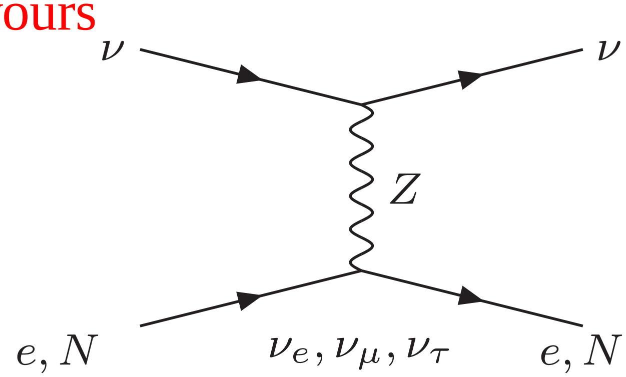
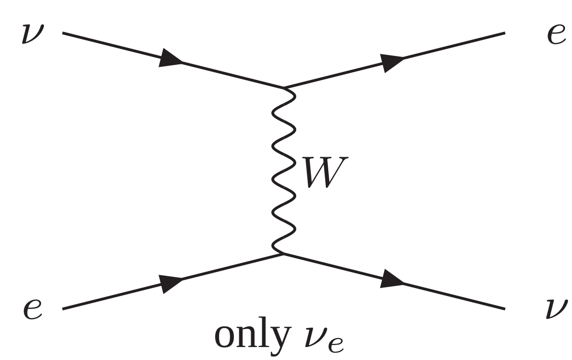
(a)$\neq$ (b) because different flavours have different interactions

For example $X = \mu, \tau$:

$$V_{CC} = V_e - V_X = \sqrt{2}G_F N_e$$

(opposite sign for $\overline{\nu}$)





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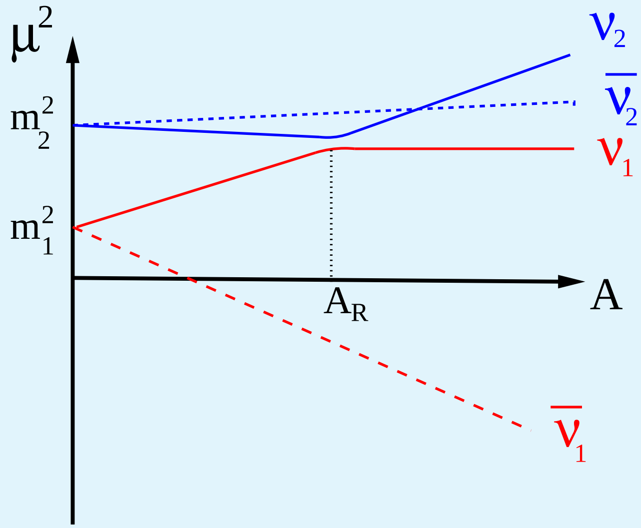
The effective masses: $(A = 2E(V_e - V_X))$

$$\mu_{1,2}(x) = \frac{m_1^2 + m_2^2}{2} + E(V_e + V_X)$$
$$\pm \frac{1}{2}\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}$$



At *resonant* potential: $A_R = \Delta m^2 \cos 2\theta$

Minimum $\Delta\mu^2 = \mu_2^2 - \mu_1^2$

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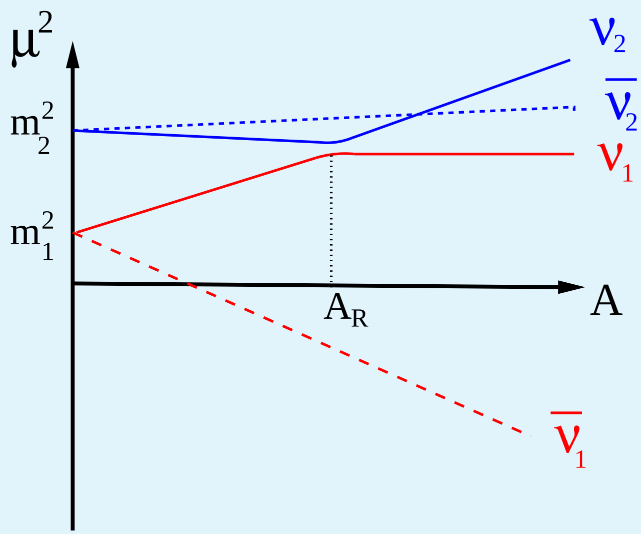

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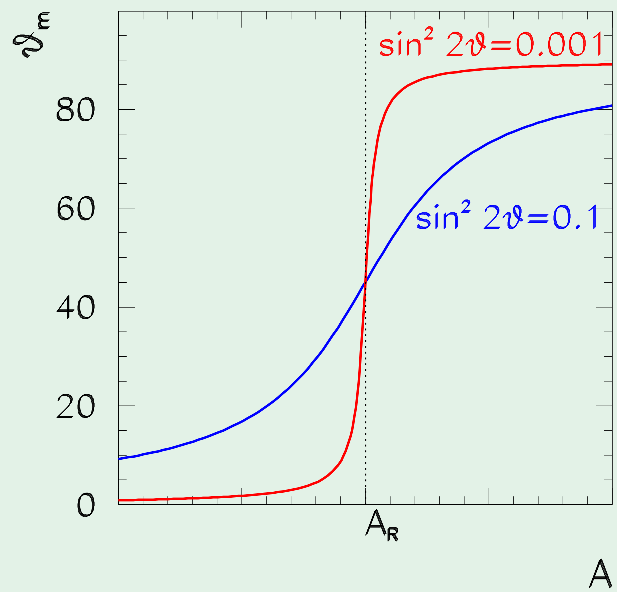
The mixing angle in matter

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - A}$$



* At $A = 0$ (vacuum) ⇒ $\theta_m = \theta$
* At $A = A_R$ ⇒ $\theta_m = \frac{\pi}{4}$
* At $A >> A_R$ ⇒ $\theta_m = \frac{\pi}{2}$

The oscillation length in vacuum

$$L_0^{osc} = \frac{4\pi E}{\Delta m^2}$$

The oscillation length in matter

$$L^{osc} = \frac{L_0^{osc}}{\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}} \equiv \frac{4\pi E}{\Delta \mu^2}$$

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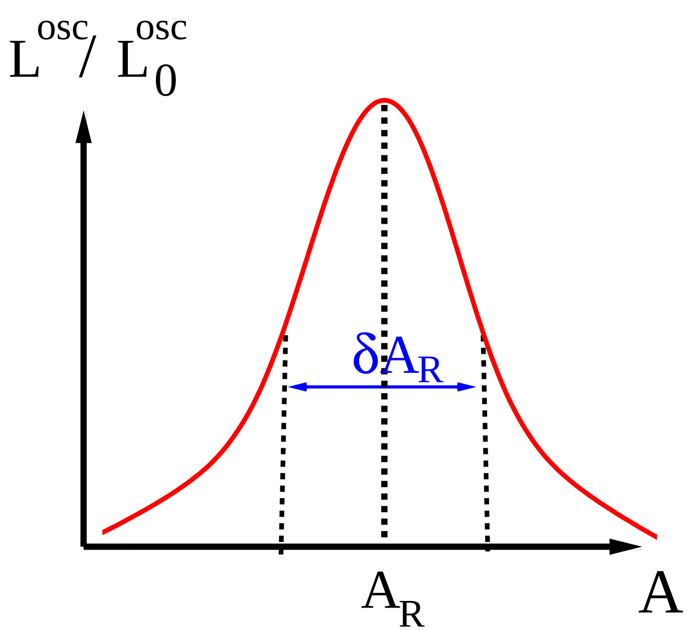
L^{osc} presents a resonant behaviour



At the resonant point

$$L_R^{osc} = \frac{L_0^{osc}}{\sin 2\theta}$$

The width of the resonance in potential:

$$\delta V_R = \frac{\Delta m^2 \sin 2\theta}{E}$$

The width of the resonance in distance:

$$\delta r_R = \frac{\delta V_R}{|\frac{dV}{dr}|_R}$$

- In terms of the mass eigenstates in matter: $$\begin{pmatrix} \nu_e \\ \nu_X \end{pmatrix} = U[\theta_m(x)] \begin{pmatrix} \nu_1^m(x) \\ \nu_2^m(x) \end{pmatrix}$$

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$\Rightarrow$ the evolution equation in flavour basis (removing diagonal part)

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• For $\Delta\mu^2(x) \gg 4\,E\,\dot\theta_m(x)$ $\left[\frac{1}{V}\frac{dV}{dx}\Big|_R \ll \frac{\Delta m^2}{2E}\frac{\sin^2 2\theta}{\cos 2\theta}\right]$ $\equiv$ Slowly varying matter potent

$\Rightarrow$ ν_i^m behave approximately as *evolution eigenstates*

$\Rightarrow$ ν_i^m do not mix in the evolution This is the *adiabatic* transition approximation

- The evolution equation in instantaneous mass basis

$$i\begin{pmatrix}\dot{\nu}_1^m \\ \dot{\nu}_2^m\end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\Delta\mu^2(x) & -4\,i\,E\,\dot{\theta}_m(x) \\ 4\,i\,E\,\dot{\theta}_m(x) & \Delta\mu^2(x)\end{pmatrix}\begin{pmatrix}\nu_1^m \\ \nu_2^m\end{pmatrix}$$

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The adiabaticity condition

$$\frac{1}{V}\frac{dV}{dx}\Big|_R \ll \frac{\Delta m^2}{2E}\frac{\sin^2 2\theta}{\cos 2\theta} \equiv 2\pi\,\delta r_R \gg L_R^{osc}$$

$\Rightarrow$ Many oscillations take place in the resonant region

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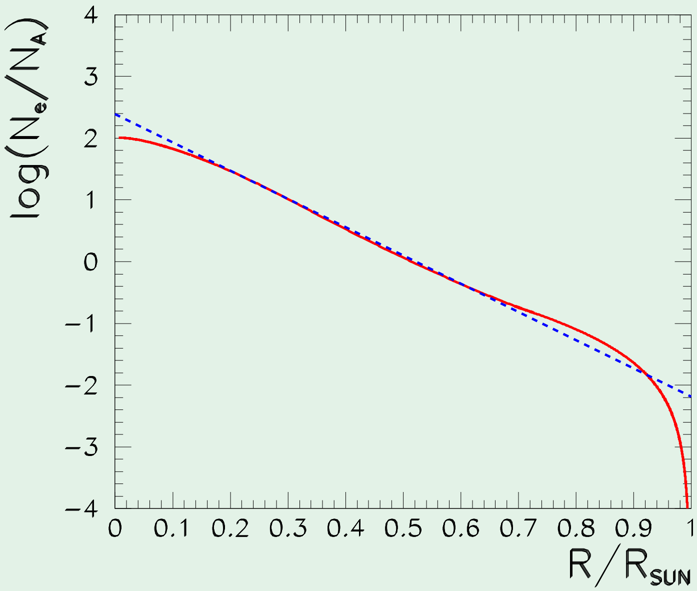

$$V_{CC} = \sqrt{2} G_F N_e \sim 10^{-14} \frac{N_e}{N_A} \text{ eV}$$

At core: $V_{CC,0} \sim 10^{-14}$–10^{-12} eV

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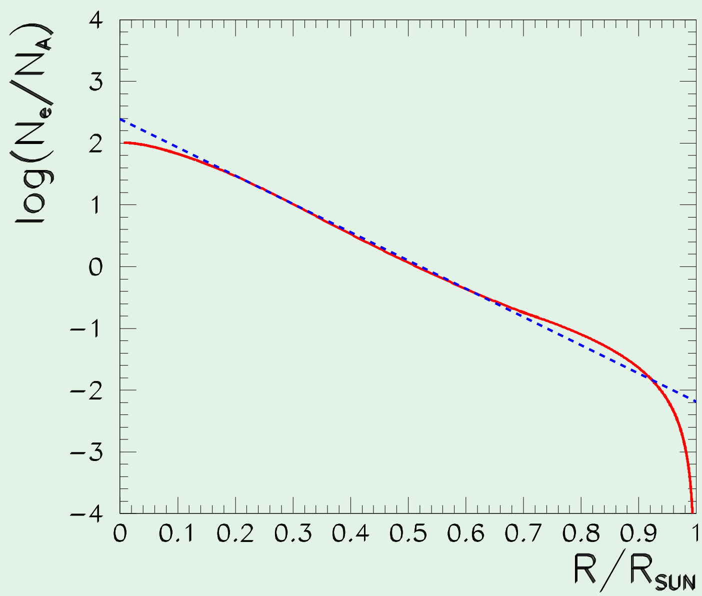

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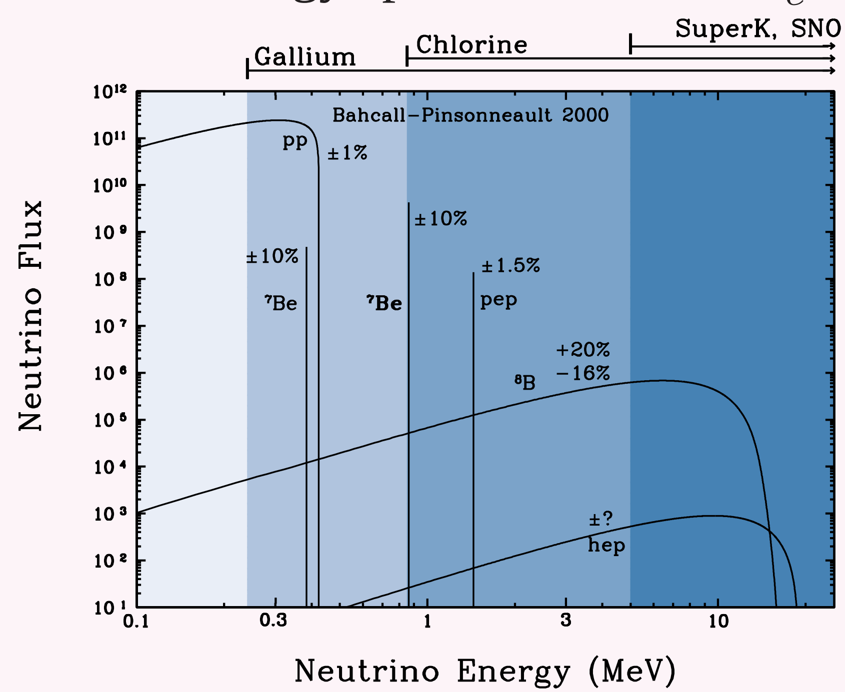

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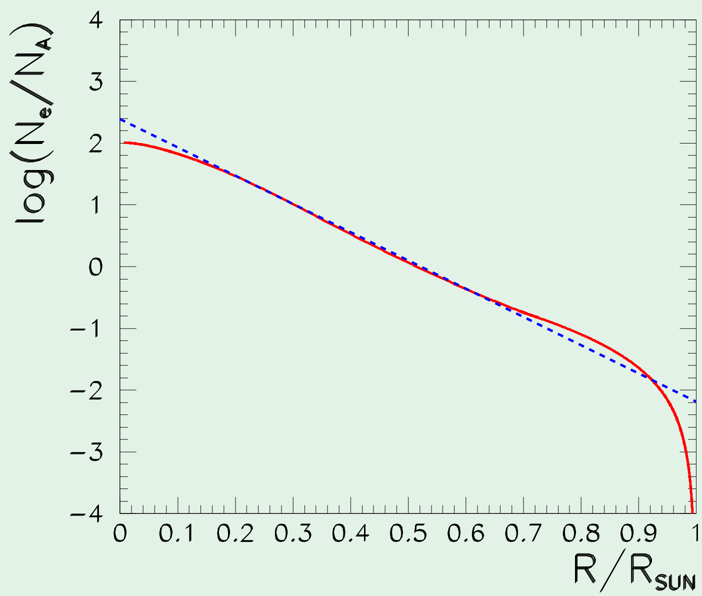

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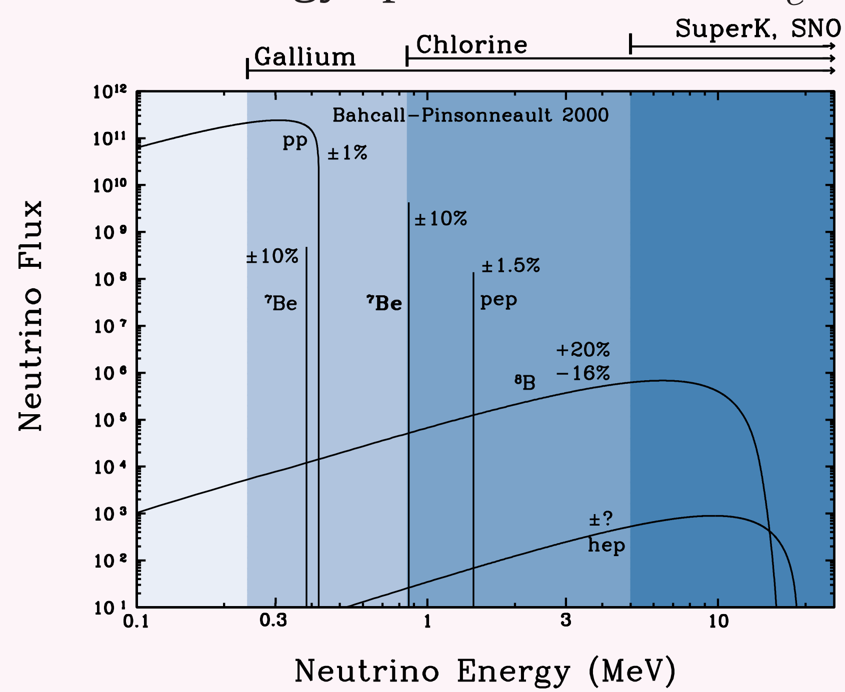

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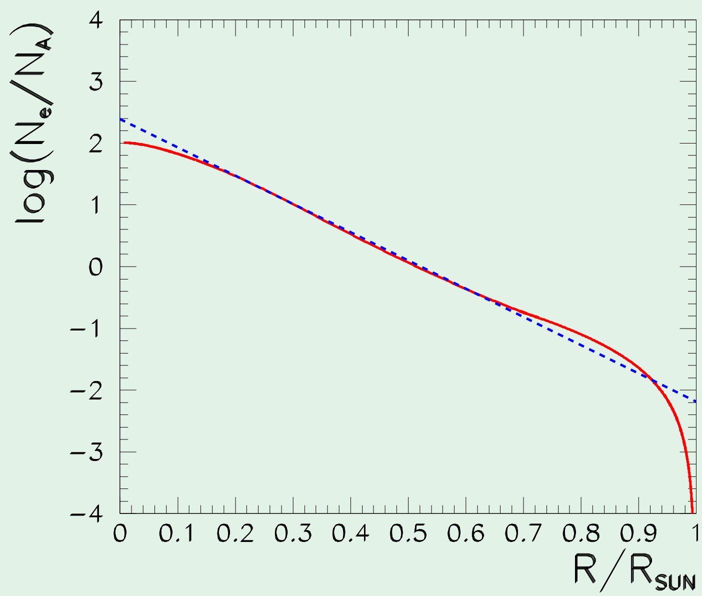

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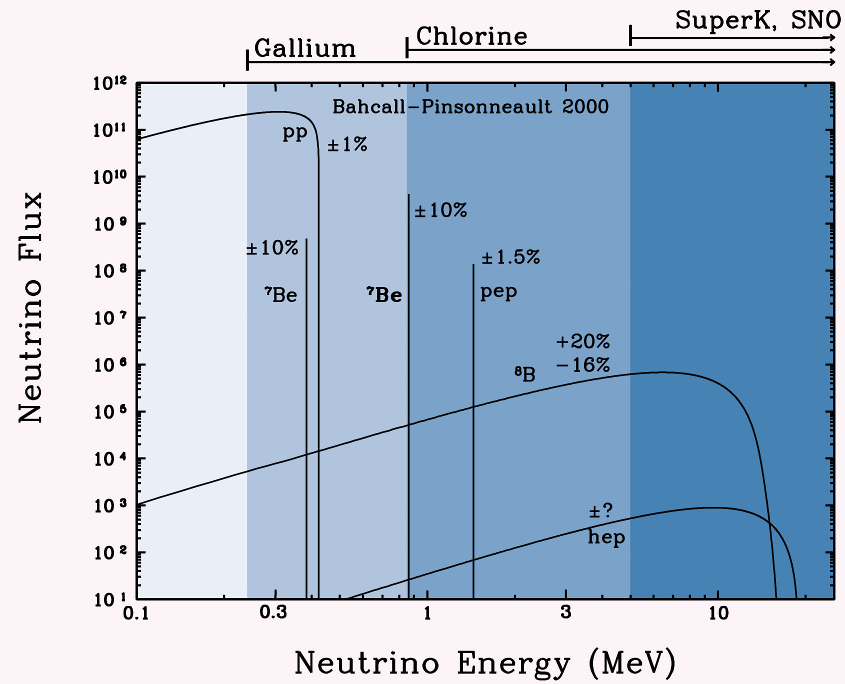

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$\Rightarrow$ ν can cross resonance condition in its way out of the Sun

For $\theta \ll \frac{\pi}{4}$: In vacuum $\nu_e = \cos\theta\,\nu_1 + \sin\theta\,\nu_2$ is mostly ν_1

In Sun core $\nu_e = \cos\theta_{m,0}\,\nu_1 + \sin\theta_{m,0}\,\nu_2$ is mostly ν_2

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$\Rightarrow$ Adiabatic transition

$*$ ν is mostly ν_2 before and after resonance

$*$ $\theta_m \downarrow$ *dramatically* at resonance

$\Rightarrow \nu_e$ component $\downarrow \Rightarrow P_{ee} \downarrow$

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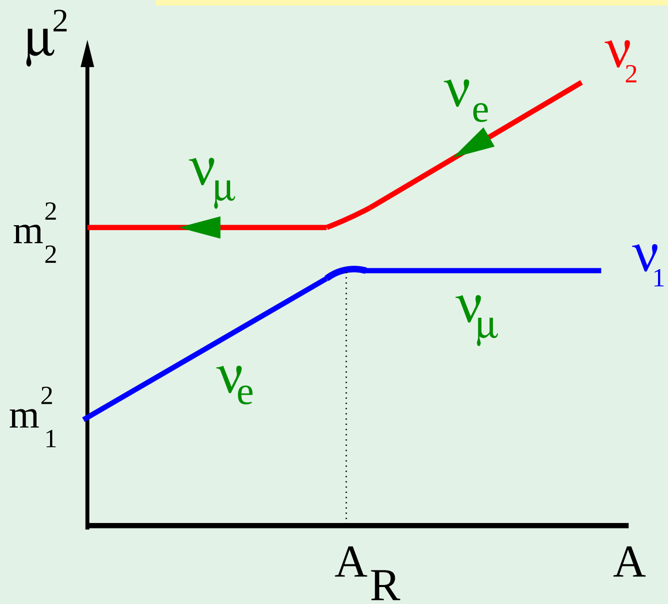

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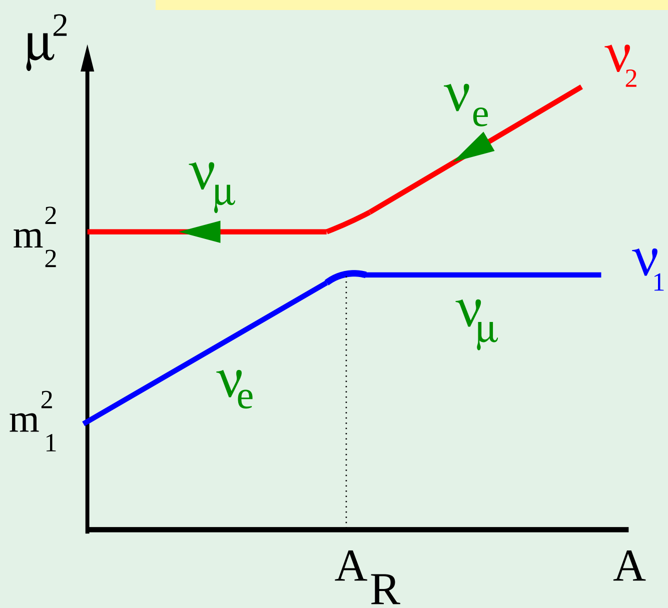

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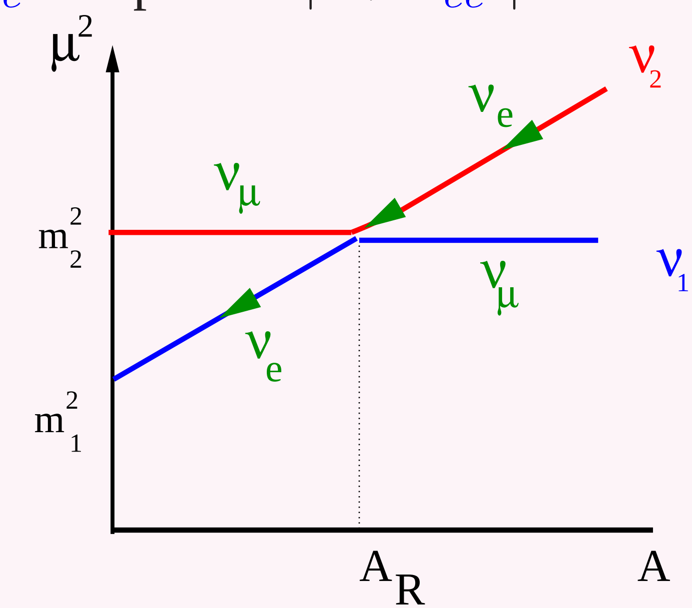
$\Rightarrow$ Non-Adiabatic transition

$*$ ν is mostly ν_2 till the resonance

$*$ At resonance the state can jump into ν_1 (with probability P_{LZ})

$\Rightarrow \nu_e$ component $\uparrow \Rightarrow P_{ee} \uparrow$



$$P_{ee} = \frac{1}{2}[1 + (1-2P_{LZ})\cos 2\theta_{m,0}\cos 2\theta]$$

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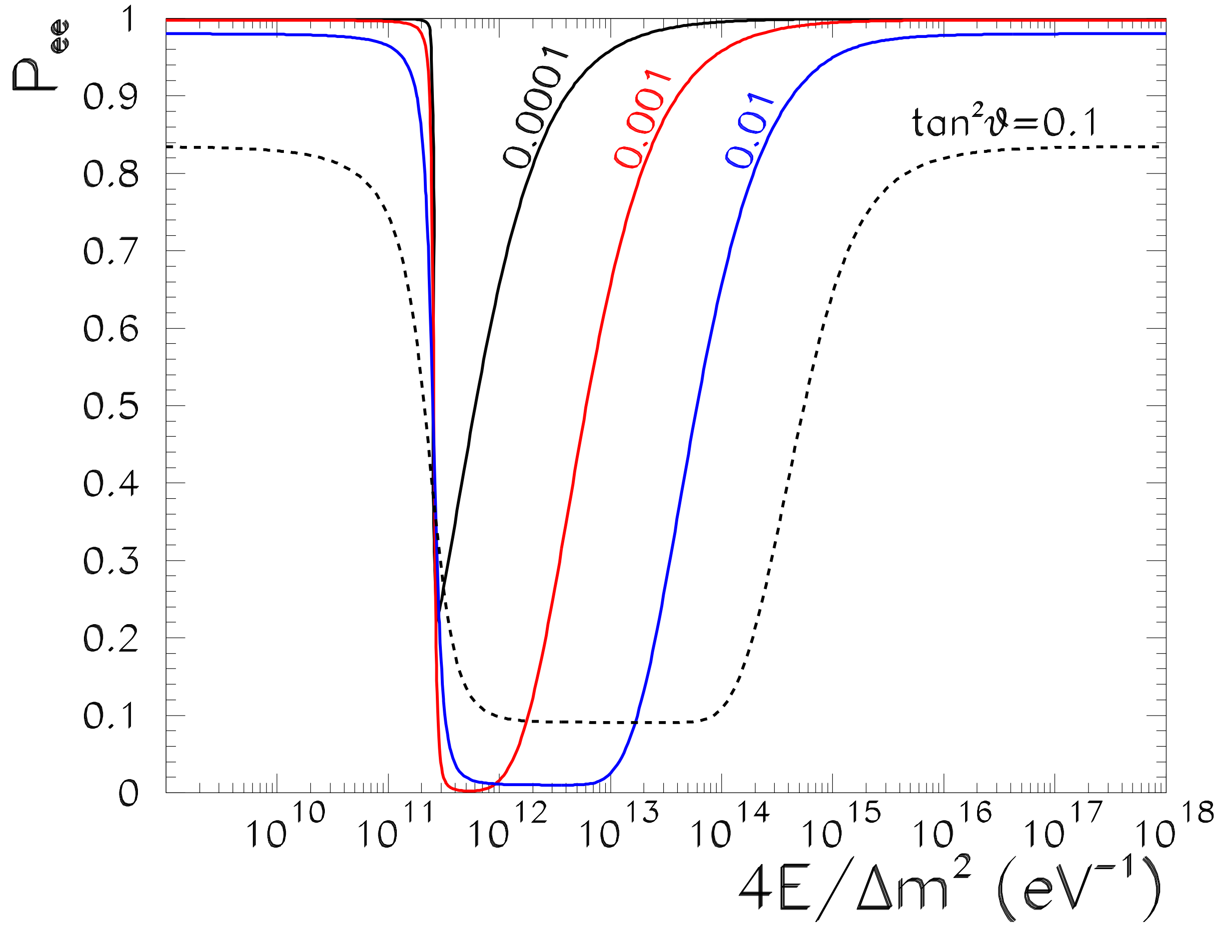

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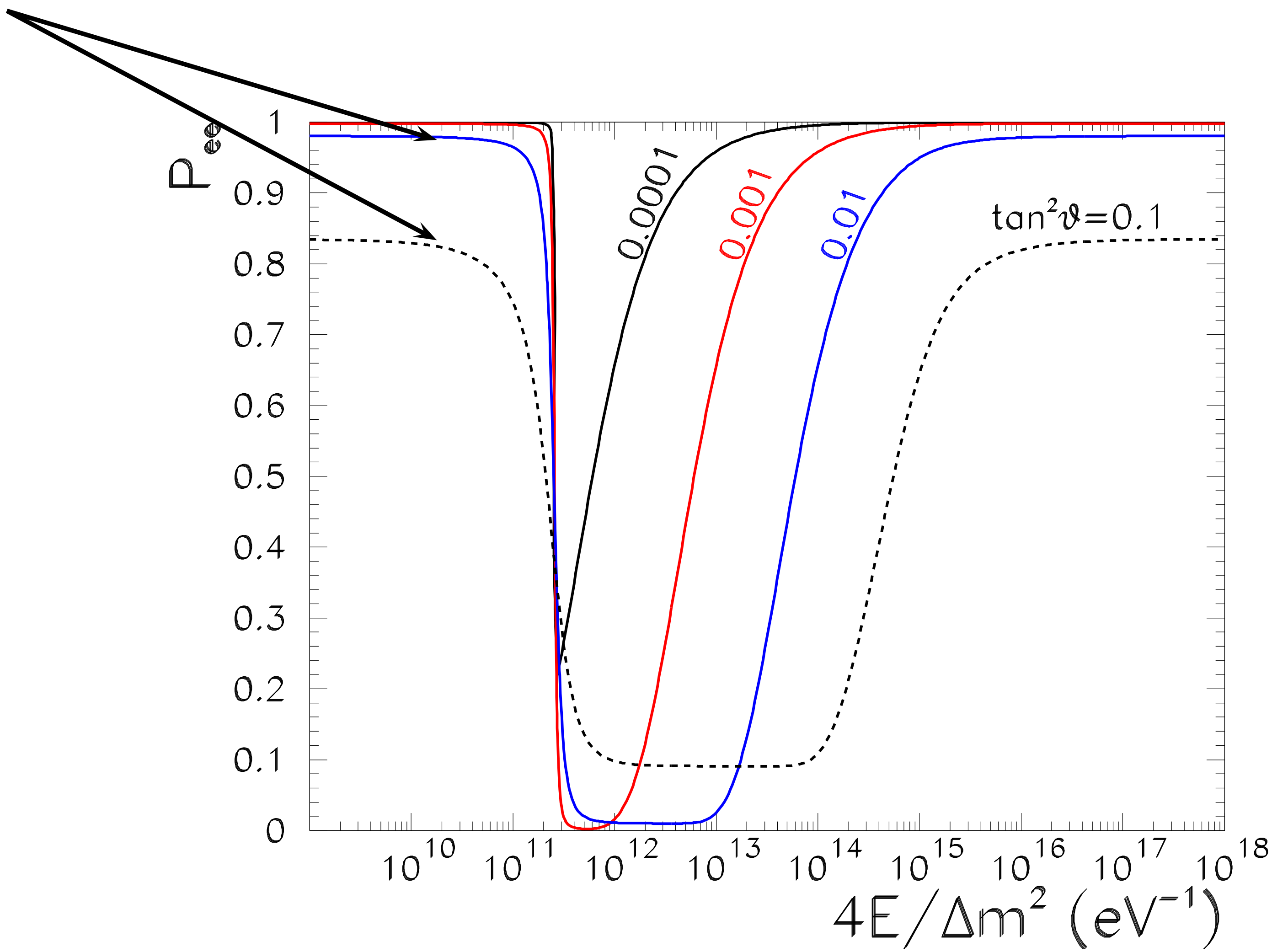
ν does not cross resonance: $$P_{ee} = 1 - \frac{1}{2}\sin^2 2\theta > \frac{1}{2}$$



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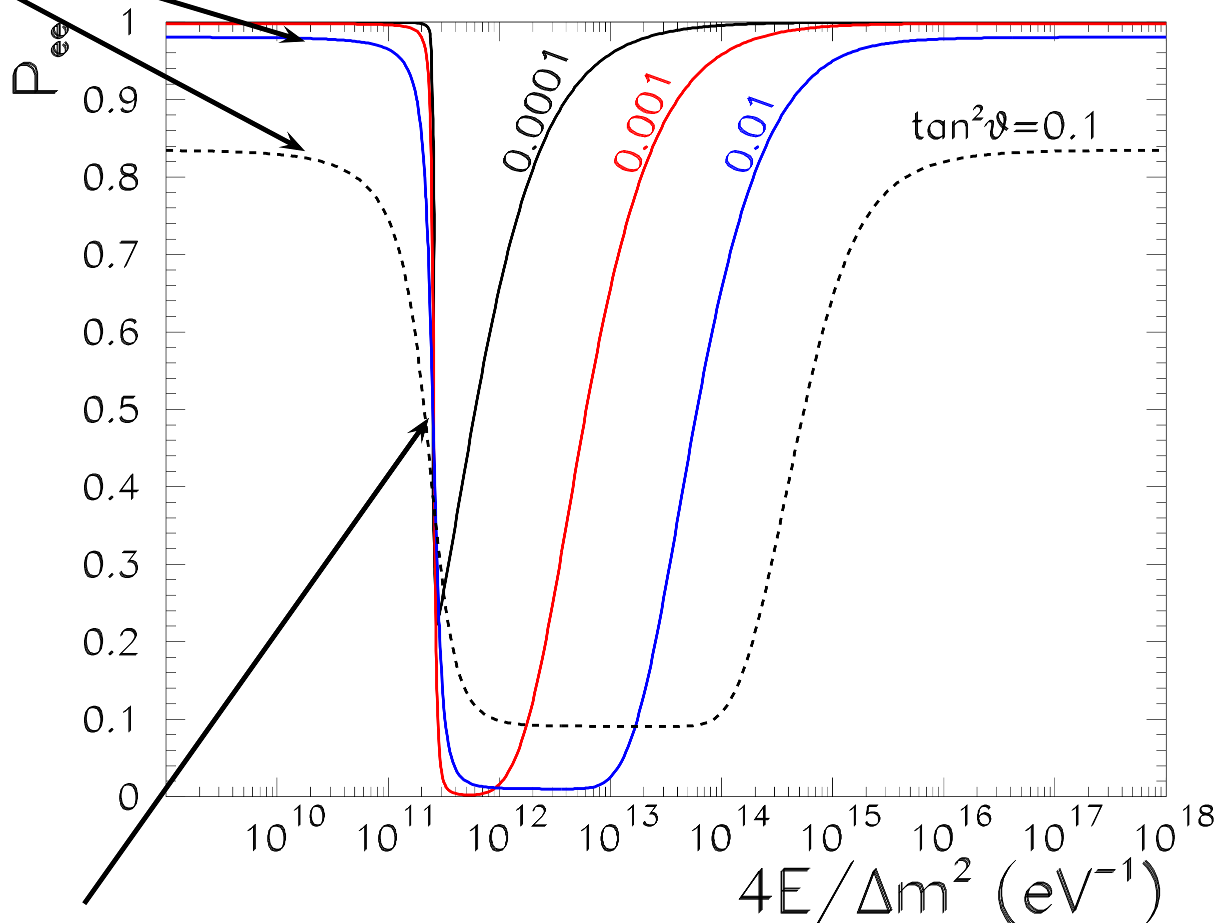

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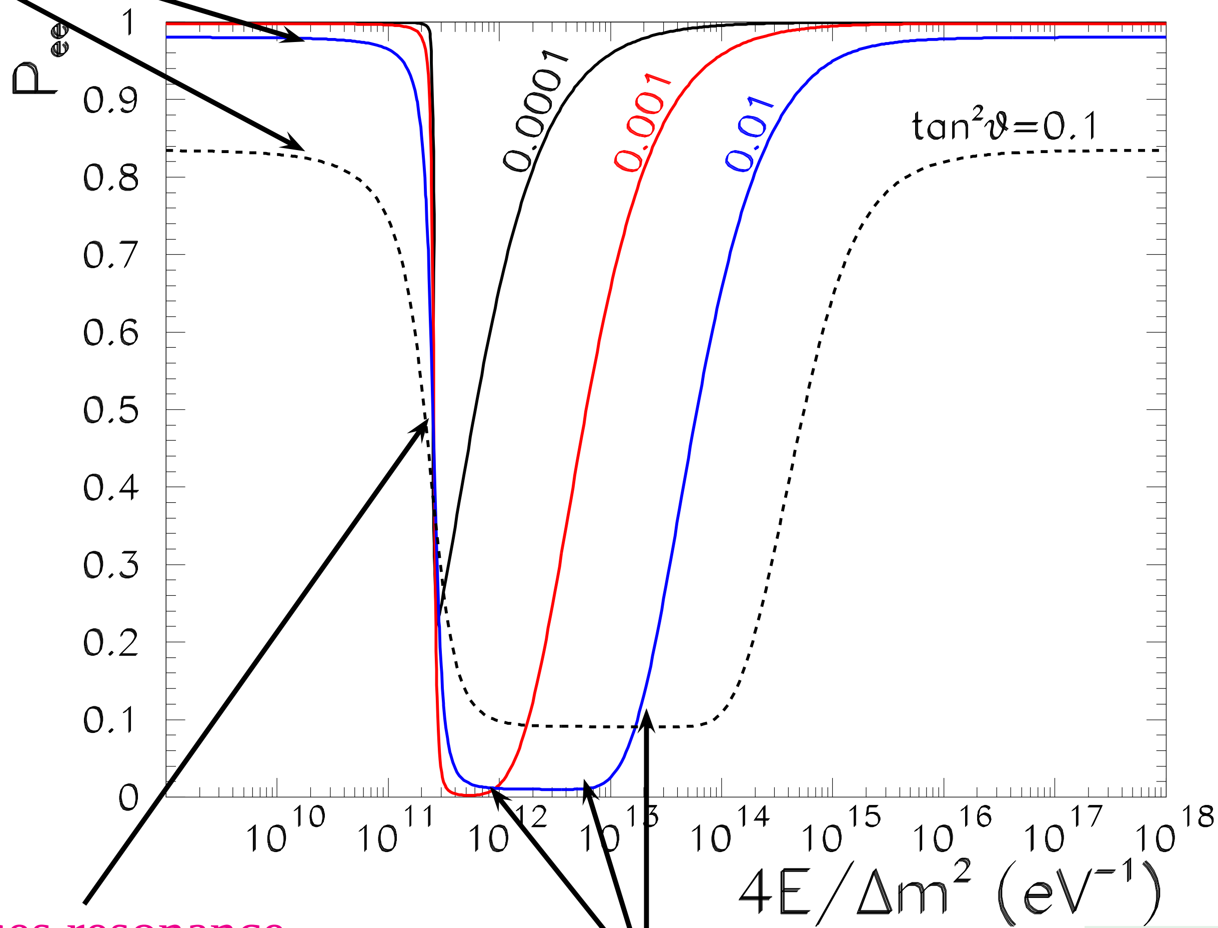

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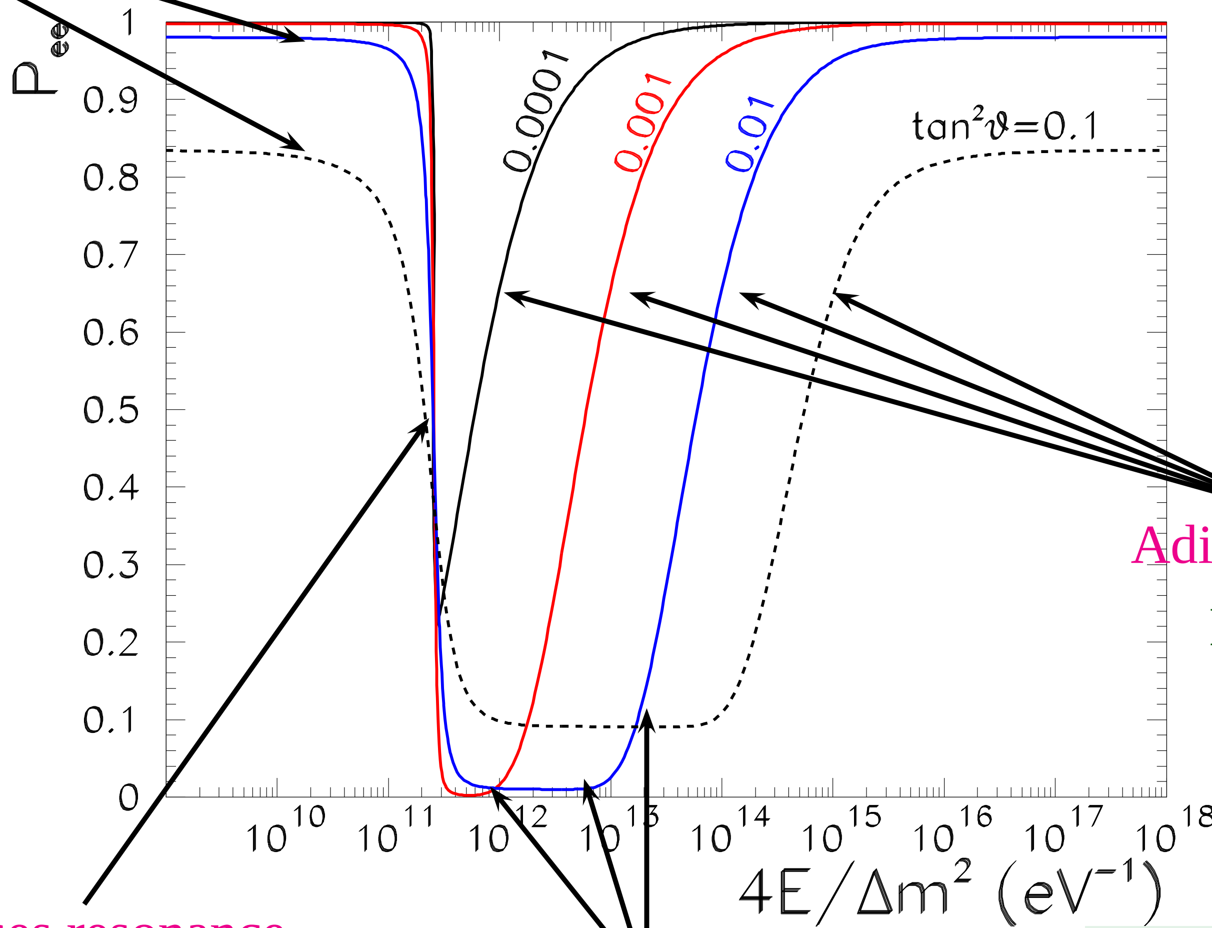

Adiabacity breaking

Effect of P_{LZ}

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- ν traveling through matter $\Rightarrow$ Modification of oscillation pattern
- Matter effect is crucial to interpretation of solar data

$0\nu\beta\beta$ Decay: Future

JJ Gomez-Cadenas, sl etal ArXiv:1109.5515

Experiment	$M_{\beta\beta}$ (kg$_{\beta\beta}$)	ε	ΔE (keV)	c (10^{-3} counts/(keV $\cdot$ kg$_{\beta\beta}$ $\cdot$ year))	Bgr/ROI (cts/yr)
EXO-200	141	0.34	100	0.78–5	11–71
GERDA-1	15.2	0.95	4.2	12–70	0.77–4.5
GERDA-2	30.4	0.84	2	1.2–7	0.07–0.43
CUORE-0	10.9	0.83	5	180–390	9.8–21.3
CUORE	206	0.83	5	36–130	37.1–134
KamLAND-Zen	357	0.61	250	0.22–1.8	19.6–161
MAJORANA Demonstrator	17.2	0.85	2	1.2–12	0.04–0.41
SNO+	44	0.50	220	9–70	87–680
NEXT	89.2	0.33	18	0.2–1	0.32–1.6
SuperNEMO Demonstrator	7	0.28	130	0.6–6	0.55–5.5

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- Energy Density

$$\rho_\nu = g_\nu \int \frac{d^3p}{(2\pi)^3} \sqrt{E^2 + m_\nu^2} f_\nu = \begin{cases} \frac{7\pi^2}{120} \left(\frac{4}{11}\right)^{\frac{4}{3}} T_{\text{CMB}}^4 & \text{m}_\nu = 0 \\ m_\nu n_\nu & m_\nu \gg T \end{cases}$$

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Contribution to present Ω = $\begin{cases} \Omega_\nu h^2 \simeq 1.7 \times 10^{-5} & \text{m}_\nu = 0 \\ \Omega_\nu h^2 = \frac{\sum_i m_{\nu_i}}{94\text{eV}} & m_\nu \gg T \end{cases}$

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$\Rightarrow$ a conservative limit $\Omega_\nu h^2 < 0.1 \Rightarrow \sum_i m_{\nu_i} < 9\ \text{eV}$

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- In general at $T < m_e$ we can always write

$$\rho_r = \left[1 + \frac{7}{8} \times \left(\frac{4}{11} \right)^{\frac{4}{3}} N_{\rm eff} \right] \rho_\gamma$$

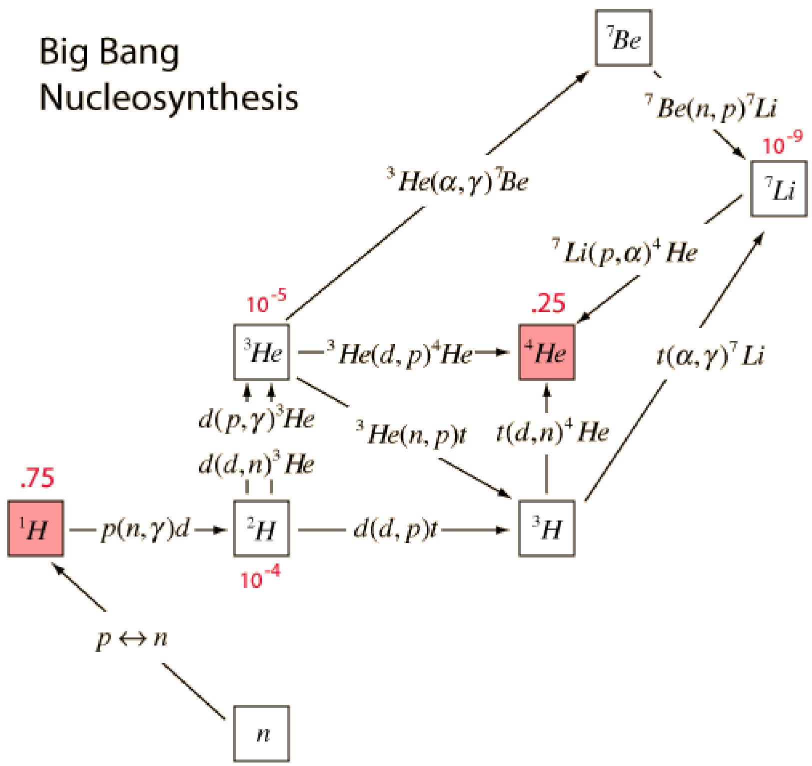
$\Delta N_{\rm eff} = N_{\rm eff} - 3$ (exactly -3.04) parametrizes;

– Any new relativistic states (accounting for their decoupling temperature

– Possible new ν-interactions (which change the relation between T_γ and T_ν)

Big Bang Nucleosynthesis

- BBN: Nuclear reactions producing D, ^{3}He, ^{4}He, ^{7}Li from protons p and neutrons n



- The produced abundances depend on:
  - The neutron life time: (well-known)

    $$\tau_n = 881.5 \pm 1.5 \text{ s}$$
  - G: (well-known)
  - $\eta_B = \frac{n_B}{n_\gamma}$, baryon to photon # density: (independently measured in CMB)
  - Nuclear reaction rates (larger uncertainties)
  - New physics which affects the expansion of Universe …

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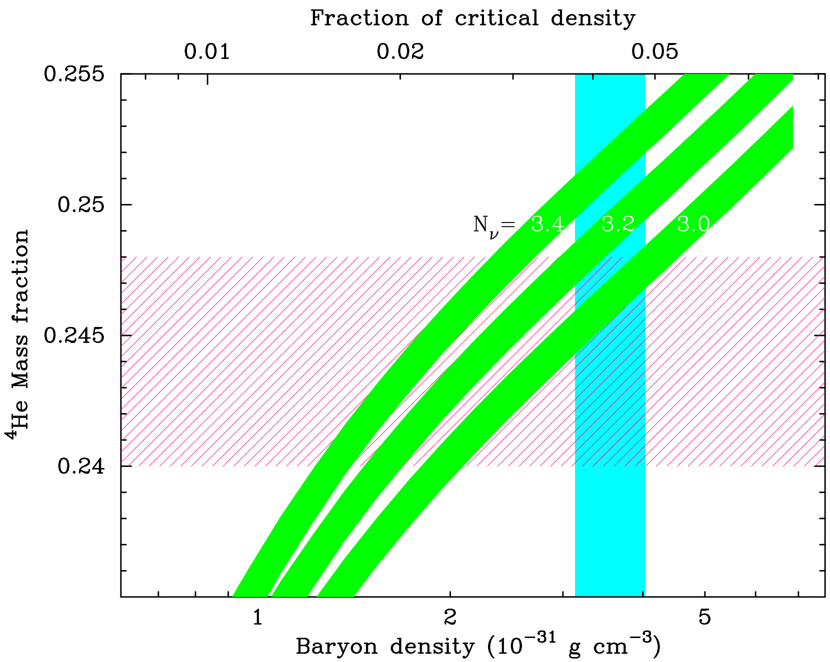

Burles, Nollet, Turner (1999)

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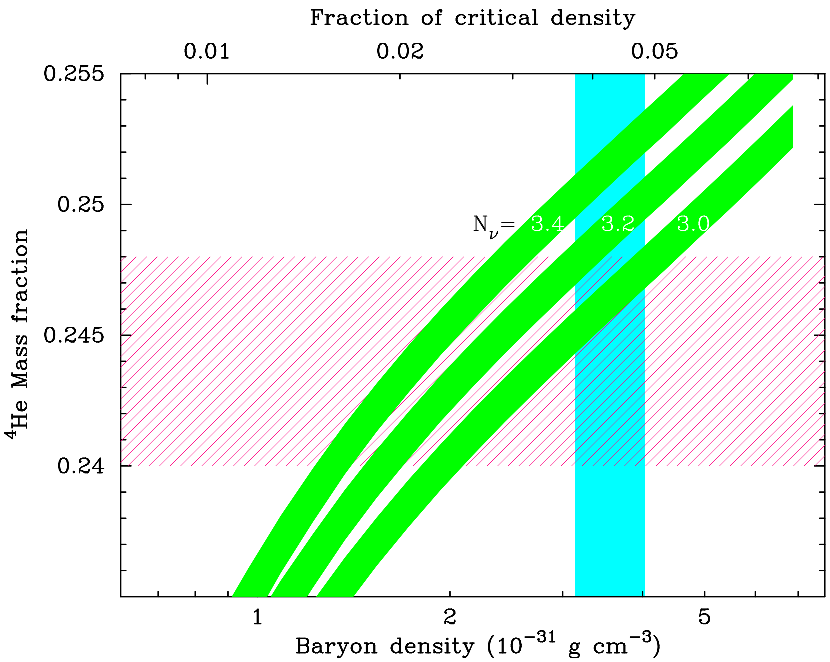

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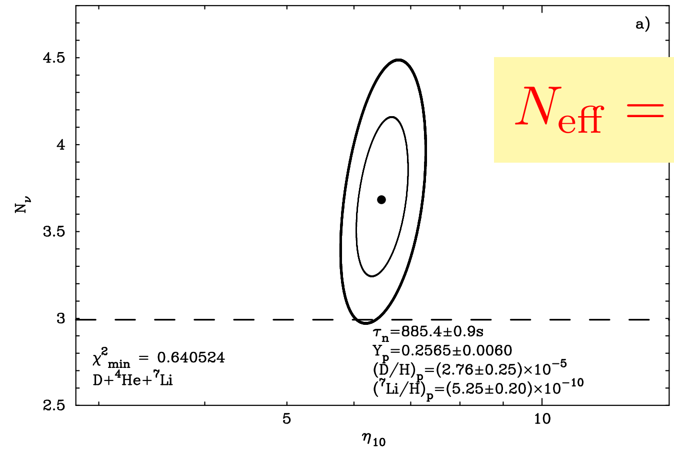
With latest Y_P determination



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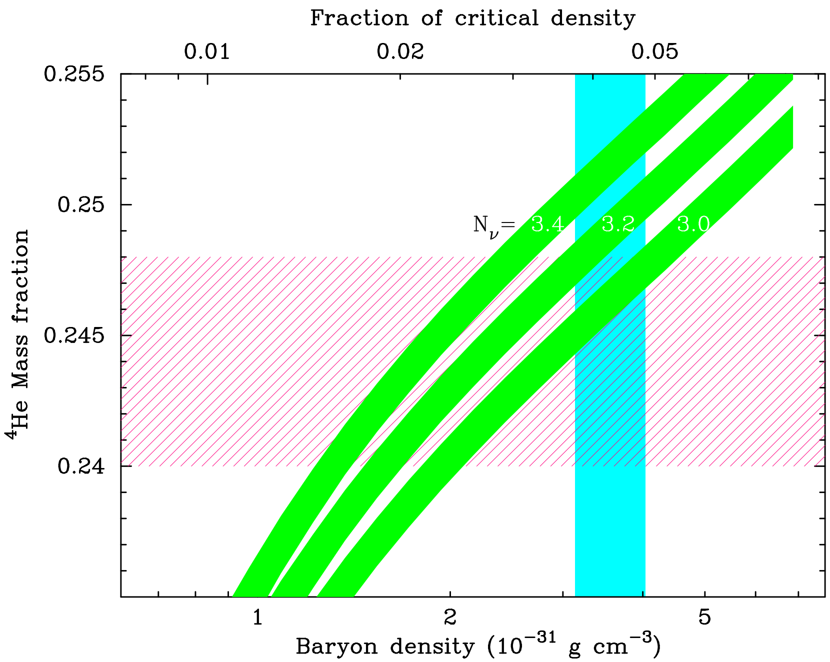

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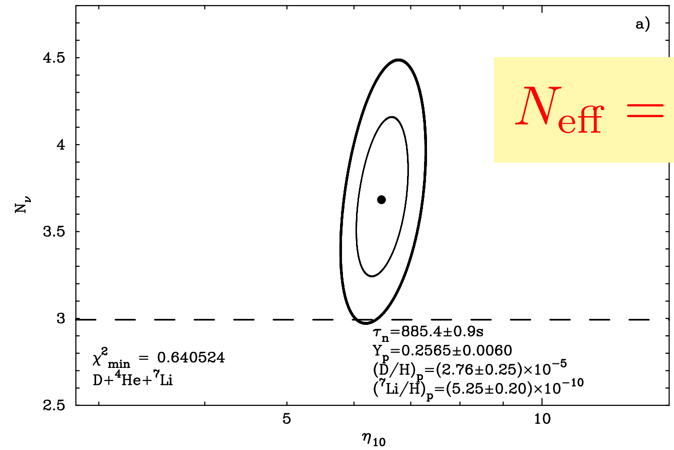

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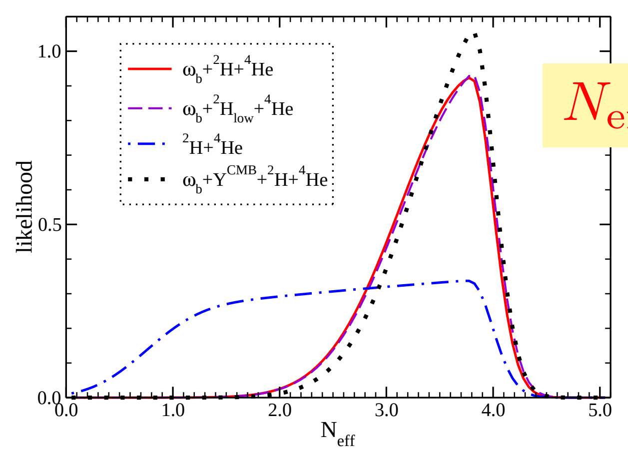
More conservatively



$N_{\rm eff} < 1$ at 95%

Mangano, Serpico (2011)

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So ν's direct effect of CMB if $\sum m_{\nu_i} > 1.7$ eV

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At recombination $T_\gamma^{rec} \simeq 3000$ K$\simeq 0.26$ eV $\Rightarrow$ $T_\nu^{rec} = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma^{rec} \simeq 0.18$ eV

The mean momenta of the neutrino $\langle p_\nu \rangle_{rec} = \frac{7\pi^4}{180\xi(3)} T_\nu^{rec} = 0.58$ eV

So ν's direct effect of CMB if $\sum m_{\nu_i} > 1.7$ eV

- For lighter $\nu's$ effect is indirect (and small): they change background evolution (time of matter-radiation equality)

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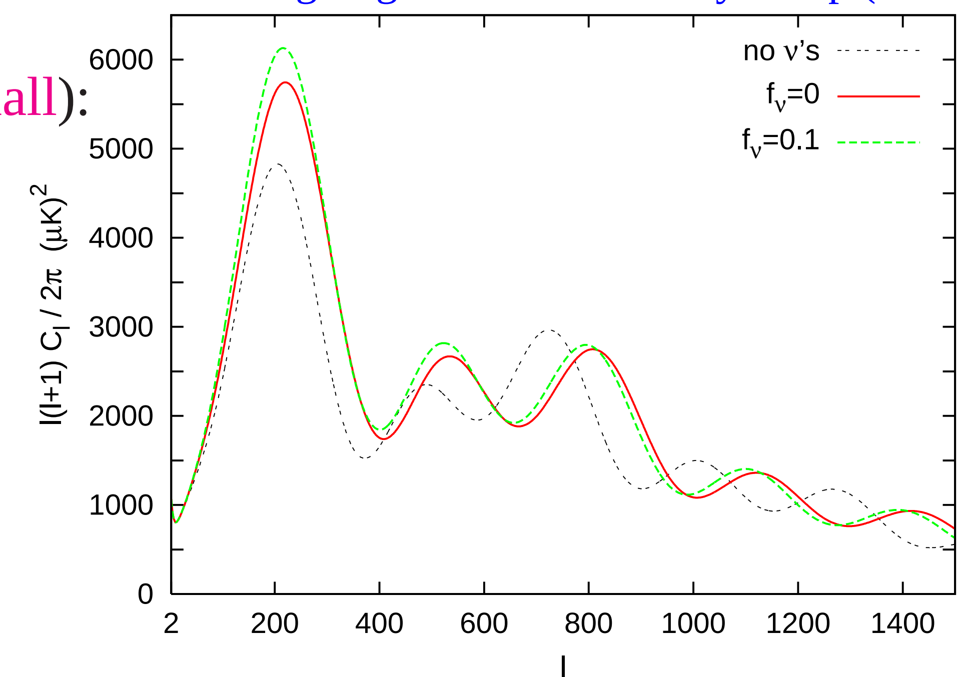

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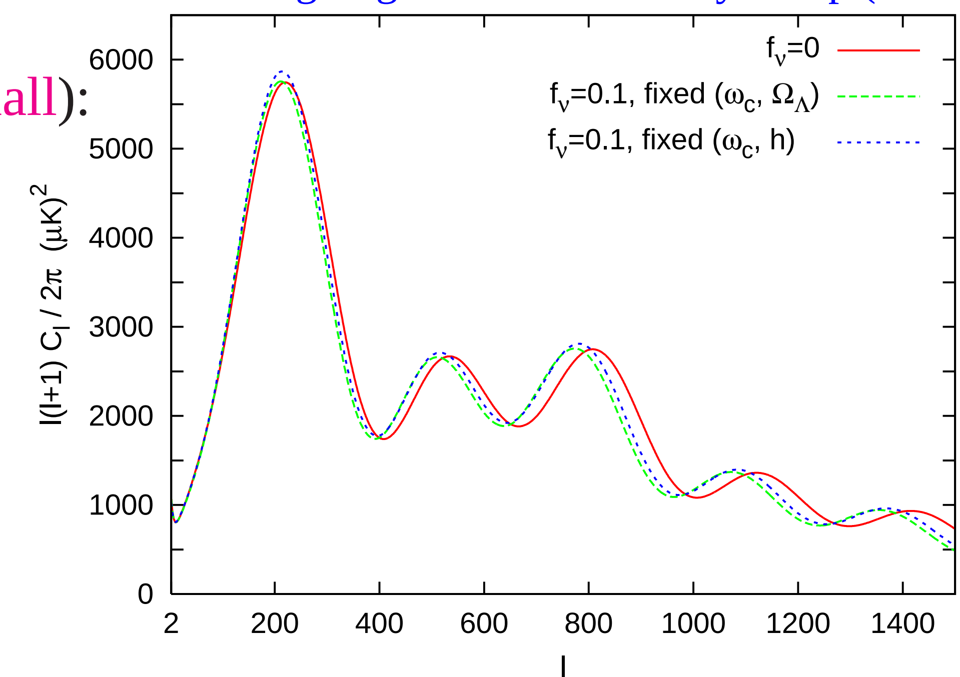

- But parameter degeneracies:
  Same effect by change of
  other cosmological parameters

CMB: Effect $N_{\rm eff}$

- In general at $T < m_e$ we can always write

$$\rho_r = \left[1 + \frac{7}{8} \times \left(\frac{4}{11}\right)^{\frac{4}{3}} N_{\rm eff}\right] \rho_\gamma$$

$\Delta N_{\rm eff} = N_{\rm eff} - 3$ (exactly -3.04) parametrizes;

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The conclusions is:

Combined analysis of Several Observables to break Degeneracies

Formation of Structures: Effects of ν's

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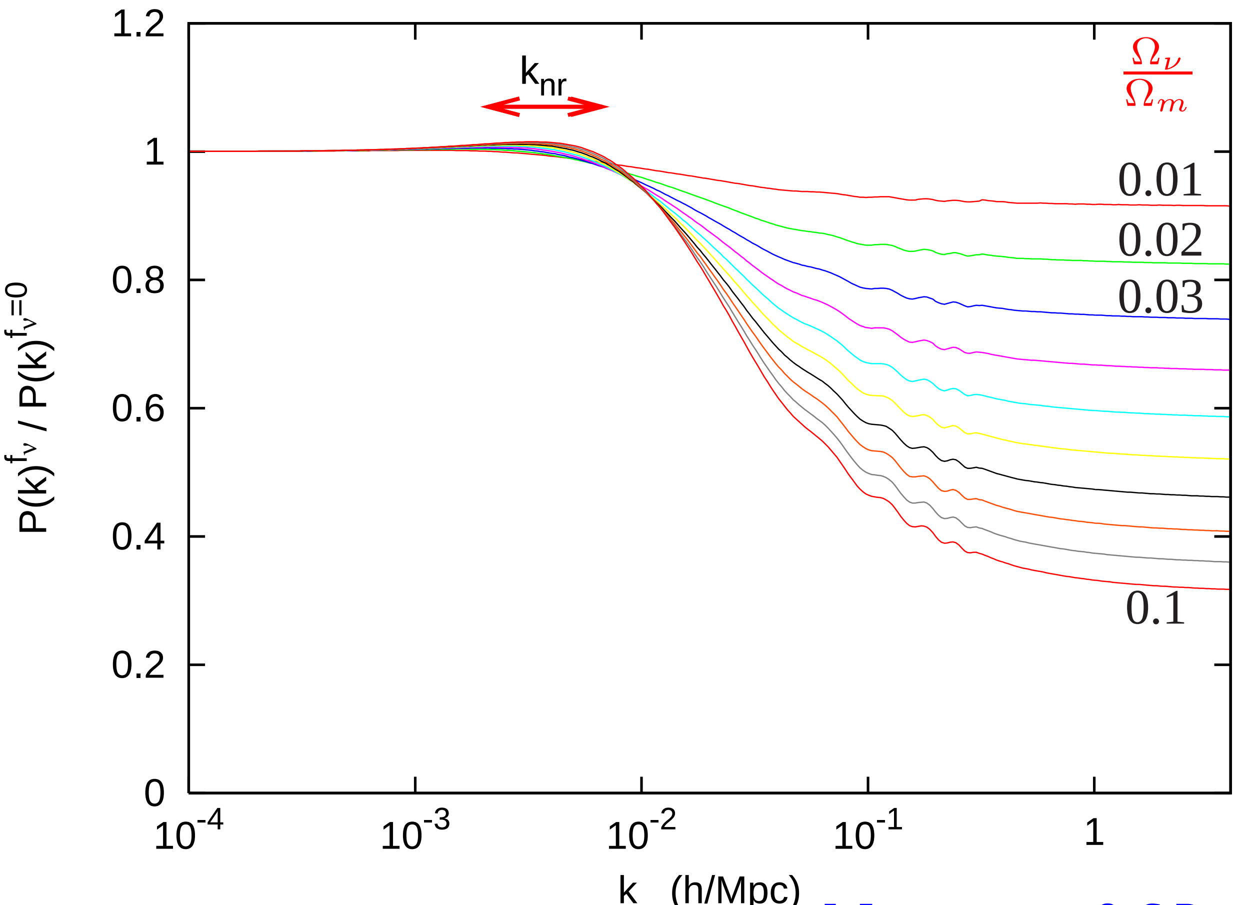
  $\Rightarrow$ Subdominant contribution of $\nu' s$ to DM Constrained by Observations

Formation of Structures: Effects of ν's

- The matter power spectra $\langle\delta_m(\vec{x}_1)\delta_m(\vec{x}_2)\rangle = \int \frac{d3k}{(2\pi)^3} \exp^{i\vec{k}(\vec{x}_1-\vec{x}_2)} P(k)$ is modified

$$\frac{\Delta P(k)}{P(k)} \simeq -8\frac{\Omega_\nu}{\Omega_m} \simeq -0.09\frac{\sum m_{\nu_i}}{1\,\mathrm{eV}}\frac{1}{\Omega_m h^2} \quad \text{for } k \gg k_{nr}$$



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Neutrinos in Matter:Effective Potentials

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- Lets consider ν_e in a medium with e, p, and n. The effective low-energy Hamiltonian:

$$H_W = \frac{G_F}{\sqrt{2}}[J^{(+)\alpha}(x)J^{(-)}_\alpha(x) + \frac{1}{4}J^{(N)\alpha}(x)J^{(N)}_\alpha(x)]$$

CC Int $J^{(+)}_\alpha(x) = \quad \overline{\nu_e}(x)\gamma_\alpha(1-\gamma_5)e(x) \qquad J^{(-)}_\alpha(x) = \overline{e}(x)\gamma_\alpha(1-\gamma_5)\nu_e(x)$

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- Example: The effect of CC with the e medium. The effective CC Hamiltonian:

$$H^{(e)}_C = \frac{G_F}{\sqrt{2}}\int d^3p_e f(E_e,T)\left\langle \langle e(s,p_e)|\overline{e}\gamma^\alpha(1-\gamma_5)\nu_e\overline{\nu_e}\gamma_\alpha(1-\gamma_5)|e(s,p_e)\rangle \right\rangle$$

Fierz rearrange

$$= \frac{G_F}{\sqrt{2}}\overline{\nu_e}\gamma_\alpha(1-\gamma_5)\nu_e\int d^3p_e f(E_e,T)\left\langle \langle e(s,p_e)|\overline{e}\gamma_\alpha(1-\gamma_5)e|e(s,p_e)\rangle \right\rangle$$

$f(E_e,T)$ statistical energy distribution of e in *homogeneous and isotropic* medium.

$$\int d^3p_e f(E_e,T) = 1$$

$\left\langle \ldots \right\rangle$ $\equiv$ averaging over electron spinors and summing over all e.

coherence $\Rightarrow$ s, p_e same for initial and final e

• Expanding the electron fields e in plane waves

$$\langle e(s,p_e)|\overline{e}\gamma_\alpha(1-\gamma_5)e|e(s,p_e)\rangle = \langle e(s,p_e)|\overline{u_s}(p_e)a_s^\dagger(p_e)\gamma_\alpha(1-\gamma_5)a_s(p_e)u_s(p_e)|e(s,p_e)\rangle$$

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$$\Big\langle \langle e(s,p_e)|a_s^\dagger(p_e)a_s(p_e)|e(s,p_e)\rangle \Big\rangle \equiv N_e(p_e)\frac{1}{2}\sum_s$$

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$$= \frac{N_e(p_e)}{2}\sum_s Tr\left[\overline{u_s}(p_e)\gamma_\alpha(1-\gamma_5)u_s(p_e)\right] = \frac{N_e(p_e)}{2}\sum_s Tr\left[u_s(p_e)\overline{u_s}(p_e)\gamma_\alpha(1-\gamma_5)\right]$$

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• Isotropy $\Rightarrow \int d^3p_e \vec{p_e} f(E_e,T) = 0$

• Also $\int d^3p_e f(E_e,T)N_e(p_e) = N_e$ electron number density

• The effective charged current Hamiltonian due to electrons in matter is then:

$$H_C^{(e)} = \frac{G_F N_e}{\sqrt{2}} \overline{\nu_e}(x)\gamma_0(1-\gamma_5)\nu_e(x) = \sqrt{2} G_F N_e \overline{\nu_{eL}}(x)\gamma_0 \nu_{eL}(x)$$

which contributes a potential term to the Dirac Eq of the neutrinos

$$V_C = \sqrt{2} G_F N_e$$