
N^3 LO chiral predictions for spin observables
in nd elastic scattering and the deuteron breakup
at low energies.

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Outline

Introduction:

- Formalism for few nucleons

3NF from χ EFT

- 3NF at N²LO and N³LO
- Automatized partial wave decomposition (aPWD)
- Fixing free parameters (LECs) at N³LO

Observables

- Nd elastic scattering cross section
- Deuteron breakup – SST cross section
- Polarization observables for Nd elastic scattering

Outlook

- Another ways to fix LECs
- New regularization scheme for nuclear forces

Introduction – 2N and 3N systems

- Nonrelativistic formalism

- 2N:

Schrödinger equation,

Lippmann-Schwinger equation for the t-matrix

(interaction + free propagation)

$$t(E) = V + VG_0(E)V + VG_0VG_0(E)V + \dots$$

$$G_0(E) \equiv \lim_{\varepsilon \rightarrow 0^+} \frac{1}{E - H_0 + i\varepsilon}$$

- 3N: Faddeev equation

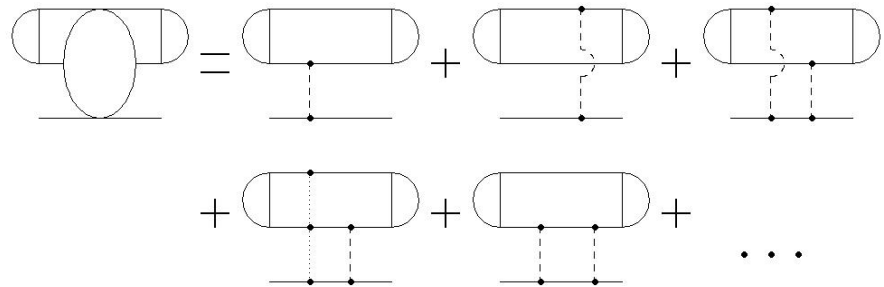
$$T = tP\phi + (1 + tG_0)V_{123}^{(1)}(1 + P)\phi + tPG_0T + (1 + tG_0)V_{123}^{(1)}(1 + P)G_0T$$

Transition amplitudes

$$U = PG_0^{-1} + V_{123}^{(1)}(1 + P)\phi +$$

$$+ PT + V_{123}^{(1)}(1 + P)G_0T$$

$$U_0 = (1 + P)T$$



3NF at N²LO

- N²LO (E.Epelbaum, Prog.Part.Nucl.Phys. 57, 654(2006)):

$$V_{123} = V_{2\pi}^{(3)} + V_{1\pi,cont}^{(3)} + V_{cont}^{(3)}$$

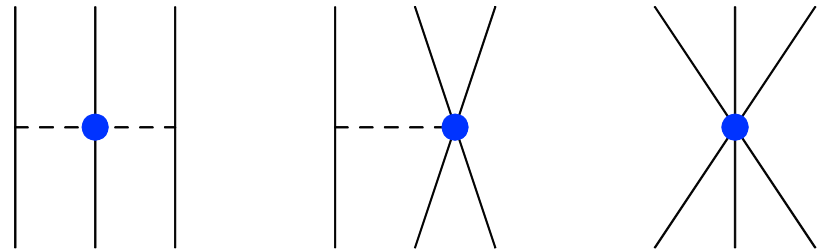
$$V_{2\pi}^{(3)} = \sum_{i \neq j \neq k} \frac{1}{2} \left(\frac{g_A}{2F_\pi} \right)^2 \frac{(\vec{\sigma}_i \circ \vec{q}_i)(\vec{\sigma}_j \circ \vec{q}_j)}{(\vec{q}_i^2 + M_\pi^2)(\vec{q}_j^2 + M_\pi^2)} F_{ijk}^{\alpha\beta} \tau_i^\alpha \tau_j^\beta$$

$$\vec{q}_i \equiv \vec{p}_i' - \vec{p}_i$$

$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta} \left[-\frac{4c_1 M_\pi^2}{F_\pi^2} + \frac{2c_3}{F_\pi^2} \vec{q}_i \circ \vec{q}_j \right] + \sum_\gamma \frac{c_4}{F_\pi^2} \varepsilon^{\alpha\beta\gamma} \tau_k^\gamma \vec{\sigma}_k \circ [\vec{q}_i \times \vec{q}_j]$$

$$V_{1\pi,cont}^{(3)} = - \sum_{i \neq j \neq k} \frac{g_A}{8F_\pi^2} D \frac{\vec{\sigma}_j \circ \vec{q}_j}{\vec{q}_j^2 + M_\pi^2} (\vec{\tau}_i \circ \vec{\tau}_j) (\vec{\sigma}_i \circ \vec{q}_j)$$

$$V_{cont}^{(3)} = \frac{1}{2} \sum_{j \neq k} E (\vec{\tau}_j \circ \vec{\tau}_k)$$



Two free parameters: D and E

3NF at N³LO long range part

N³LO V. Bernard, E. Epelbaum, H. Krebs, U-G. Meißner,
Phys Rev C77 (2008) 064004.

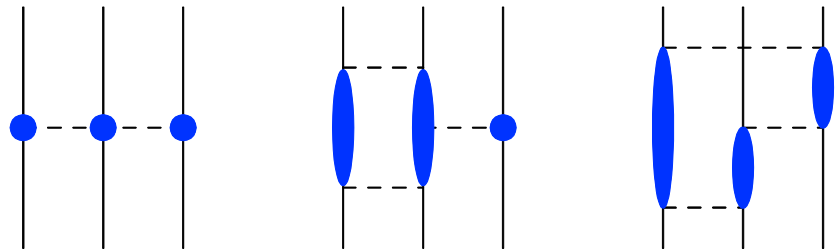
- $V_{2\pi}$ – already at N²LO, at N³LO the same operator structure but new values of C_1, C_3, C_4 and momentum dependence in formfactors

Two new topologies:

- Two pion – one pion exchange $V_{2\pi-1\pi}$
- The ring term V_{ring}

No new free parameters

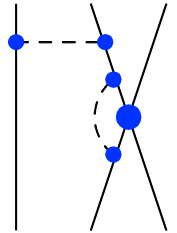
More operator structures
and more complicated
momentum dependence



3NF at N³LO short range part

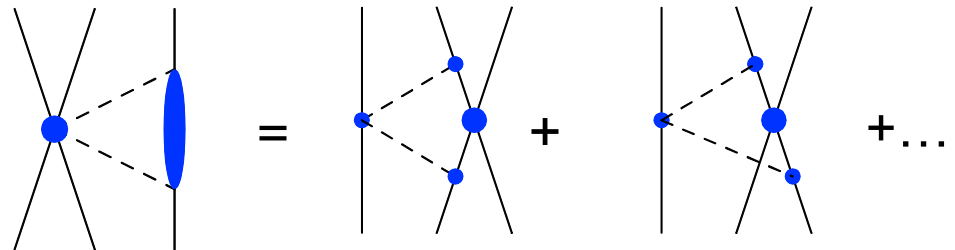
N³LO: V. Bernard, E. Epelbaum, H. Krebs, U-G. Meißner,
Phys Rev C84 (2011) 054001.

- 1 π -contact – already at N²LO (one free parameter D)
at N³LO all terms cancel thus no new contributions at this order

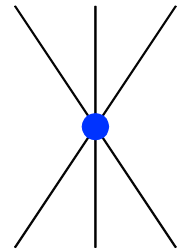


- 2 π -contact

No new free parameters



- Three nucleon contact term – already at N²LO
One free parameter E

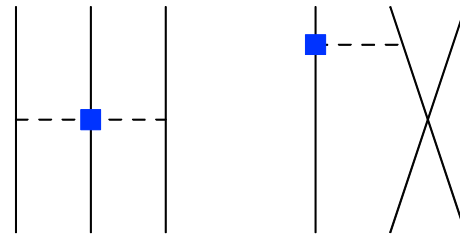


3NF at N³LO short range part

N³LO: V.Bernard, E.Epelbaum, H.Krebs, U-G.Meißner,
Phys Rev C84 (2011) 054001.

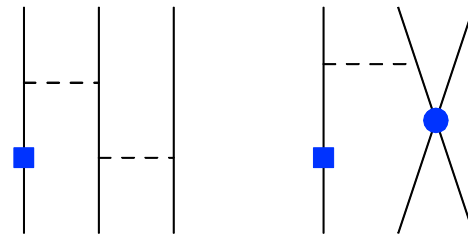
- Relativistic 1/m corrections to 2 π and 1 π -contact terms coming from:

corrections to π NN
and $\pi\pi$ NN vertices



retardation effects

No new free parameters



In total: two free parameters at N³LO.

The chiral 3NF depends also on additional regularization parameter.

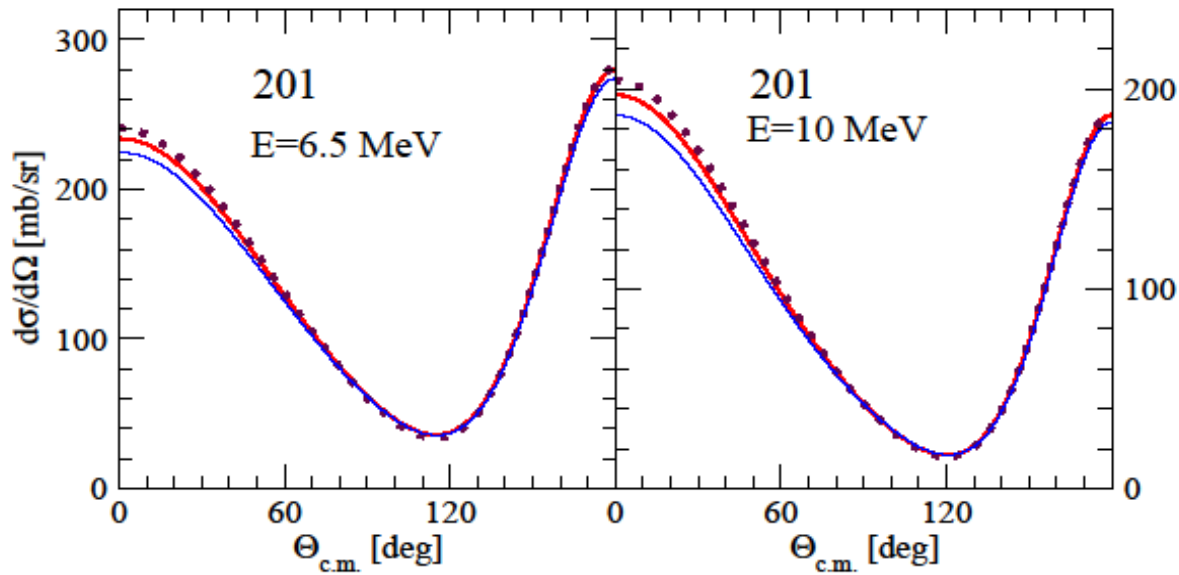
aPWD (automatized partial wave decomposition)

- The efficient method of partial wave decomposition is required to include many operator structures present in 3NF into calculations.
- aPWD reduces partial wave decomposition to 1-dim integration for NN force and 5-dim integration for 3NF. However this integration has to be repeated for different momenta and three-body partial waves.
- The integrands can be relatively easy prepared using e.g. Mathematica[®].
- This can be further reduced for local 3NF
- Details are given in:
 - J.Golak et al., Eur. Phys. J. A43, (2010) 241
 - R.Skibiński et al., Eur. Phys J. A47 (2011) 48.

Fixing LECs

- At this moment we use the ^3H binding energy and the $^2a_{\text{nd}}$ scattering length to fix D and E LECs, which arise from short range terms.
- Such a procedure requires a solution of the Faddeev and Schrödinger equations but only for $J=1/2+$ states.
- Neutron-deuteron system – there is no Coulomb force.
- In principle other observables can also be used to determine D and E, for example μ -capture on ^3He ($J=1/2+$, first step in this direction in J.Golak et al., arXiv:1406.6781)
- For cut-offs $\Lambda=1$ and 4 (regularization parameter $\Lambda=450$ MeV) we are able to fix D and E, obtaining $D=13.78$ and $E=0.372$ and $D=9.095$ and $E=-0.0845$, respectively. For other cut-offs (with bigger Λ) one have to go to too big (absolute values) of D and E to reproduce $^2a_{\text{nd}}$. There are also problems with the convergence of the Faddeev equation in some non-physical range of values of D and E.
- More details in: R.Skibiński et al., Phys. Rev. C84 (2011) 054005
H.Wiłała et al., J. Phys. G: Nucl. Part. Phys. 41 (2014) 094011
J.Golak et al., arXiv:1410.0756

Nd elastic scattering at low energies

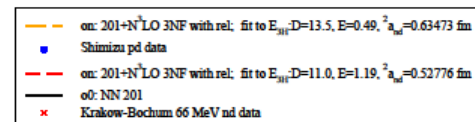
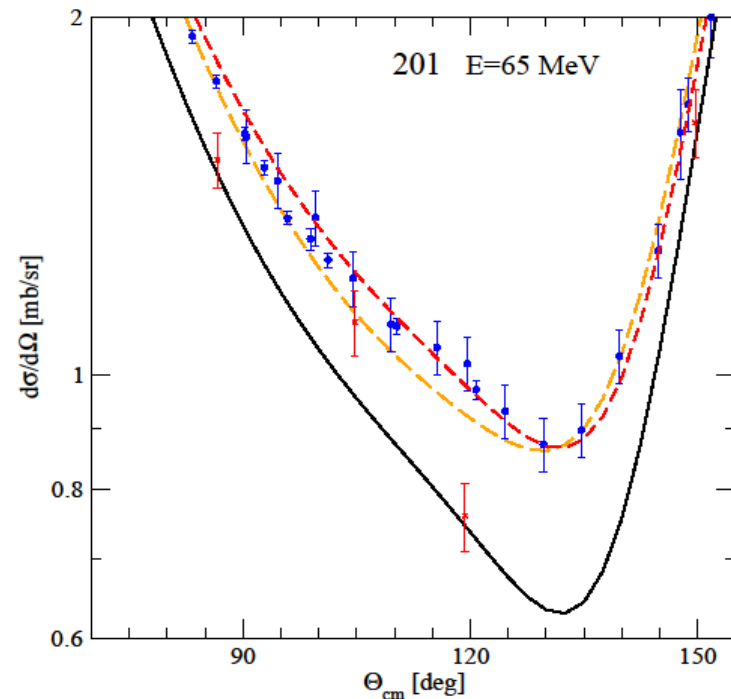
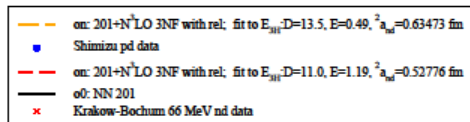
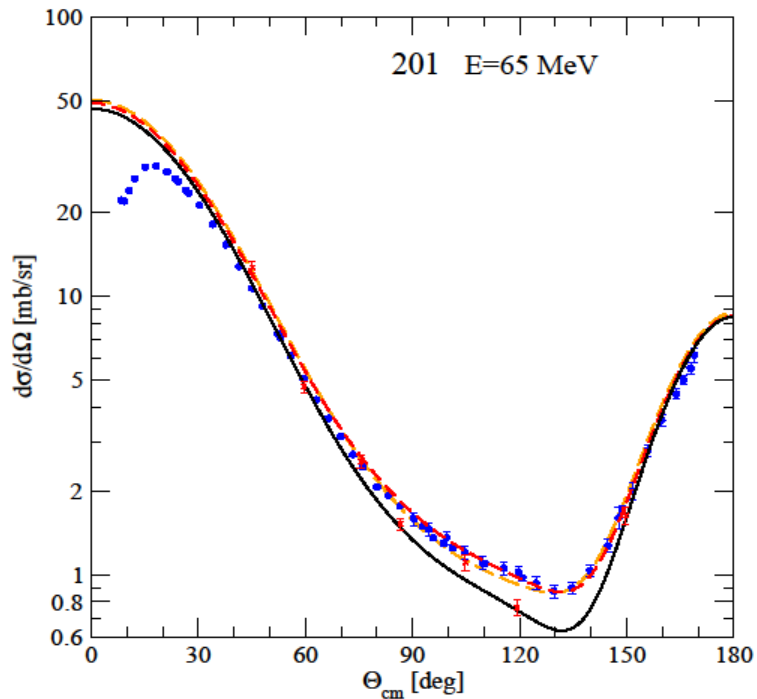


.....
N³LO NN+3NF

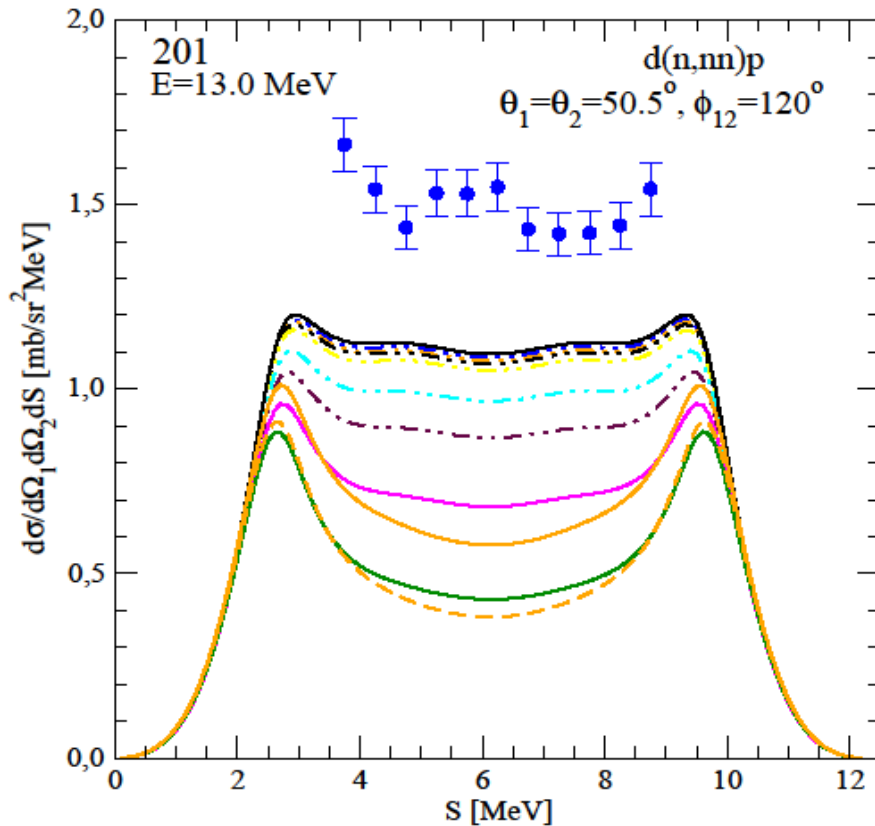
—————
N³LO NN

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CDBonn

Nd elastic scattering at E=65 MeV, cut-off=1

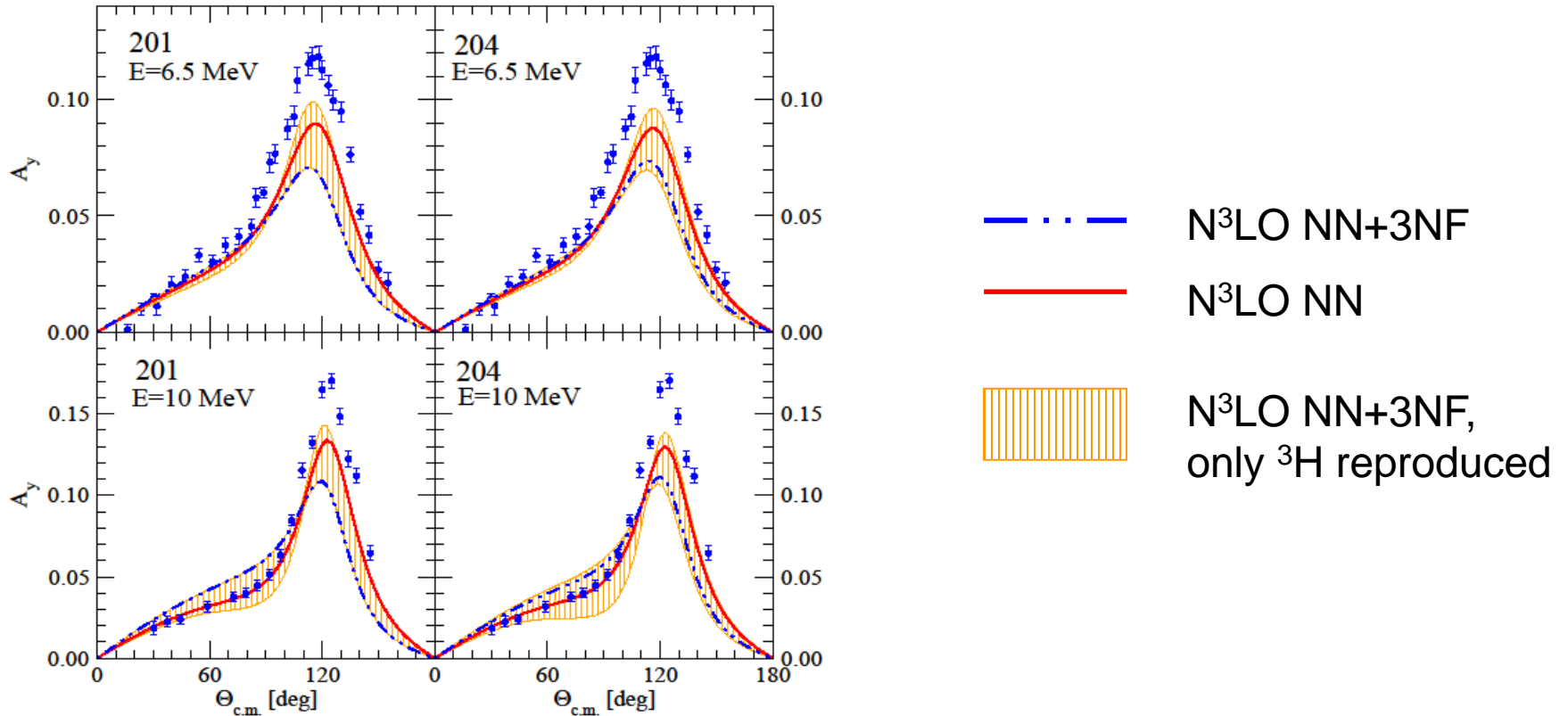


SST cross section at E=13 MeV, cut-off=1

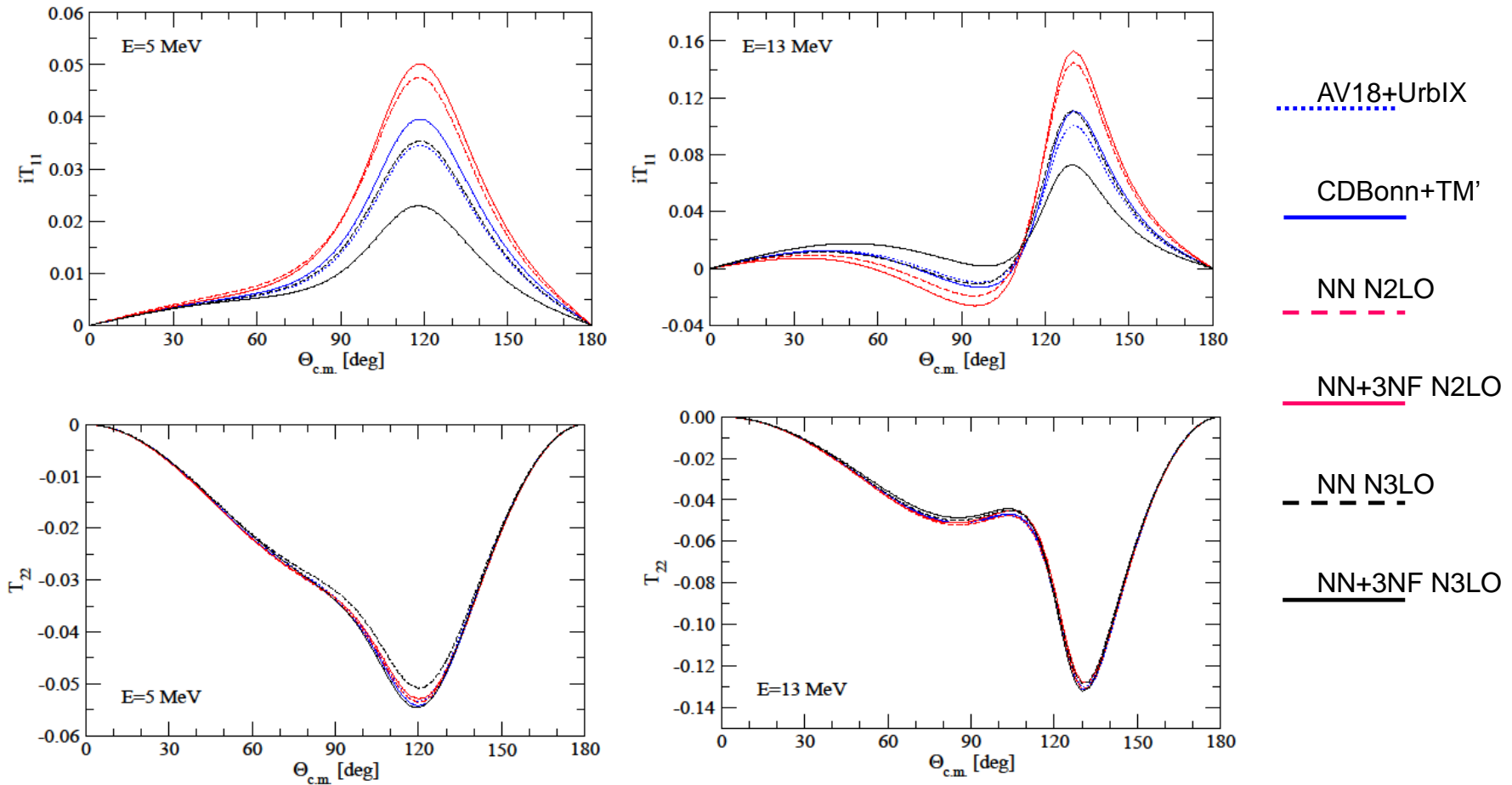


- - on: 201+N³LO 3NF with rel; fit to E_{3H1}-D=13.78, E=0.372, ²a_{nd}=0.644 fm⁹⁷
- TUNL nd data
- - on: 201+N³LO 3NF with rel; fit to E_{3H1}-D=12.0, E=0.974, ²a_{nd}=0.5748 fm
- - on: 201+N³LO 3NF with rel; fit to E_{3H1}-D=11.0, E=1.19, ²a_{nd}=0.52776 fm
- - on: 201+N³LO 3NF with rel; fit to E_{3H1}-D=9.0, E=1.425, ²a_{nd}=0.41699 fm
- - on: 201+N³LO 3NF with rel; fit to E_{3H1}-D=5.0, E=1.395, ²a_{nd}=0.08008 fm
- - on: 201+N³LO 3NF with rel; fit to E_{3H1}-D=3.0, E=1.219, ²a_{nd}=-0.18916 fm
- o0: NN 201
- - on: 201+N³LO 3NF with rel; fit to E_{3H1}-D=1.0, E=0.971, ²a_{nd}=-0.58864 fm
- - on: 201+N³LO 3NF with rel; fit to E_{3H1}-D=-1.0, E=0.6655, ²a_{nd}=-1.2228 fm
- - on: 201+N³LO 3NF with rel; fit to E_{3H1}-D=-3.0, E=0.3155, ²a_{nd}=-2.3853 fm
- - on: 201+N³LO 3NF with rel; fit to E_{3H1}-D=-5.0, E=-0.071, ²a_{nd}=-5.1286 fm

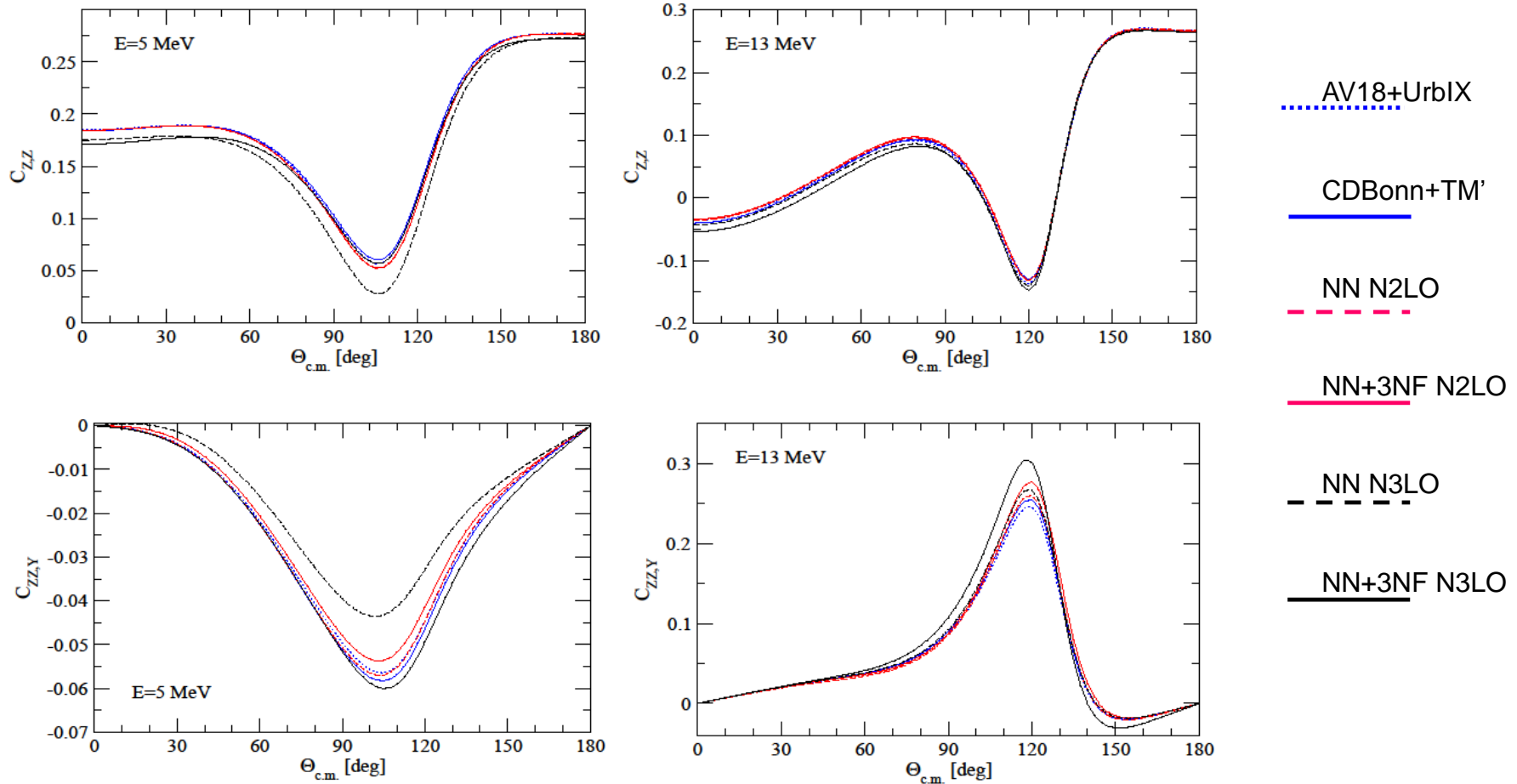
The nucleon analyzing power A_y



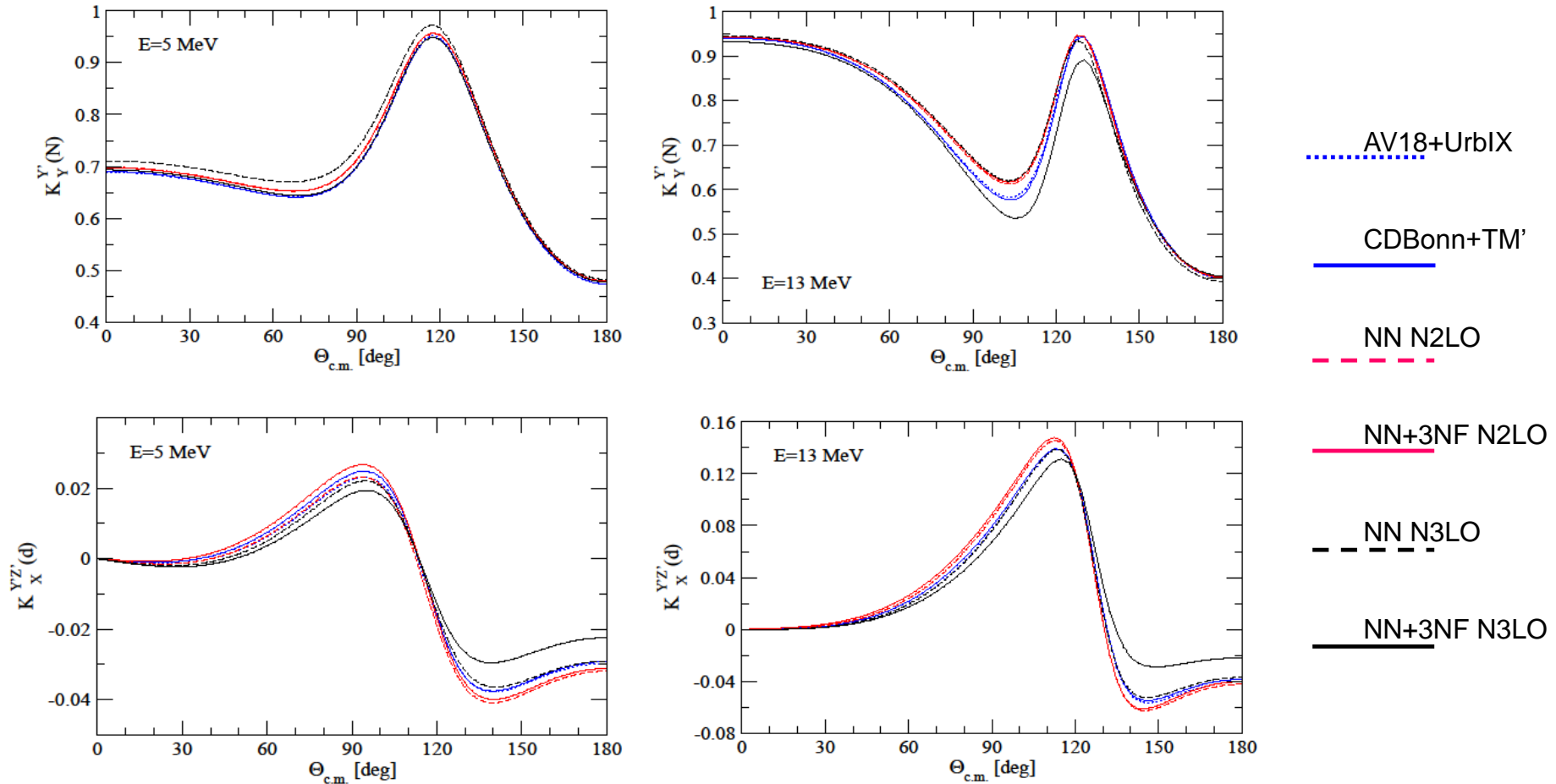
The deuteron tensor analyzing powers iT_{11} and T_{22}



The spin correlation coefficients $C_{Z,Z}$ and $C_{ZZ,Y}$



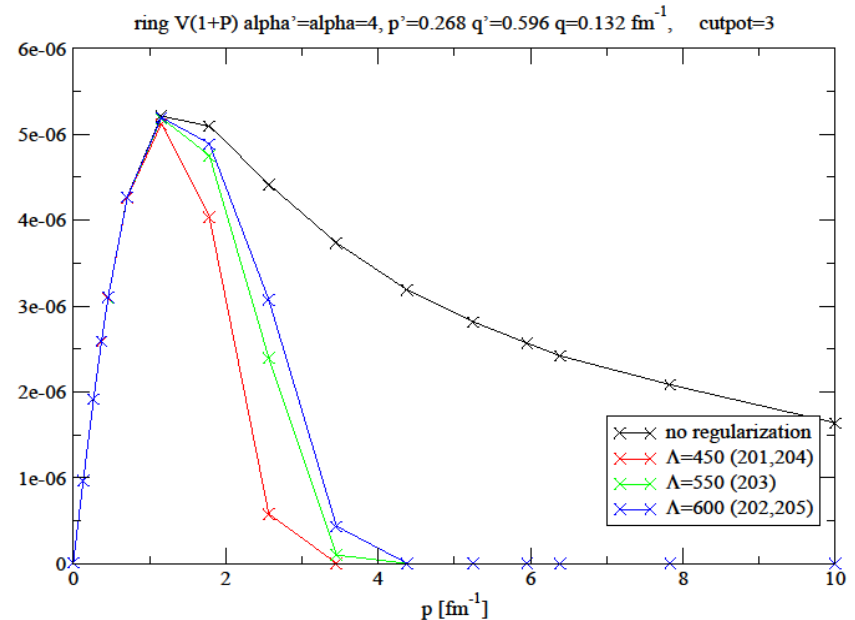
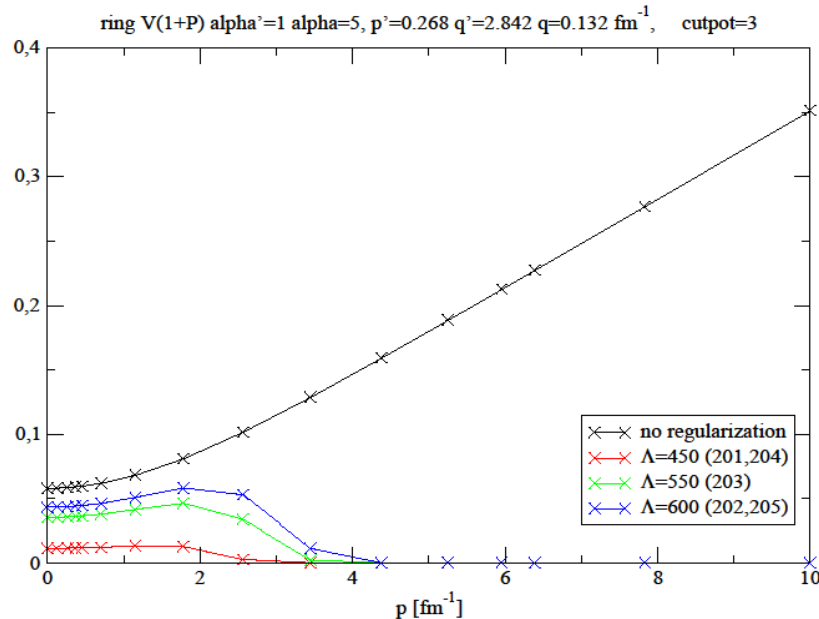
The spin transfer coefficients $K_Y^{Y'}$ (N) and $K_X^{Y'Z'}$



Importance of regularization

- Regularization $V^4 \rightarrow f(p',q',\Lambda) V^4(p',q',p',q) f(p,q,\Lambda)$ with

$$f(p, q, \Lambda) \equiv e^{-(p^2 + 0.75q^2)^3 / \Lambda^6}$$



- New, local regularization scheme for NN and 3N forces is under construction (E.Epelbaum). The preliminary results for NN and Nd scattering show much smaller dependence on cut-off parameters, then presented here.

Summary and Outlook

1. The full chiral N³LO potential with standard nonlocal regularization has been applied to the elastic Nd scattering and the deuteron breakup reaction. Unfortunately, there is no improvement in data description.
2. The strong dependence on cut-off parameters is seen and unnaturally large values of free parameters (not shown in my talk) have been obtained for some cut-offs.

The above calls for modifications of current forces:

1. The new, local regularization scheme for NN and 3N forces will be used .
2. LECs will be fixed using also weak processes.
3. New 3NFs, especially ones from chiral EFT with explicit Δ will be included.

Thank you for your attention