Status of calculations for top-antitop production at threshold

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A. Hoang, C. Reisser, PRF arXiv:1002.3223 [hep-ph]
M. Beneke, B. Jantzen, PRF arXiv:1004.2188 [hep-ph]
B. Jantzen, PRF arXiv:1307.4337 [hep-ph]
PRF arXiv:1402.1123 [hep-ph]

CSIC

CPAN Ingenio 2010

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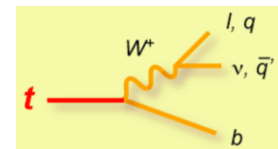
I. Top-pair production near threshold

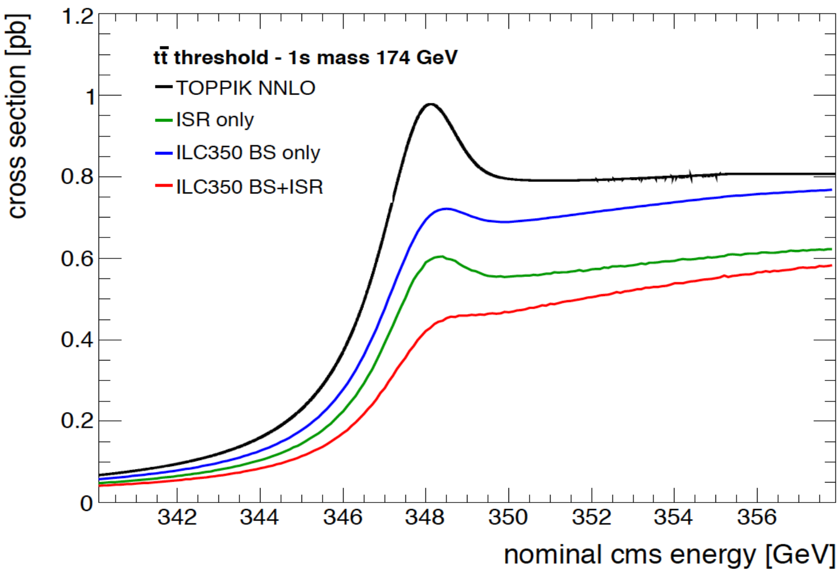
- Heaviest known quark (plays an important role in EWSB in many models)
- Important for quantum effects affecting precision observables
- Very unstable, decays "before hadronization" $(\Gamma_t \approx 1.5\ \text{GeV} \gg \Lambda_{\text{QCD}})$

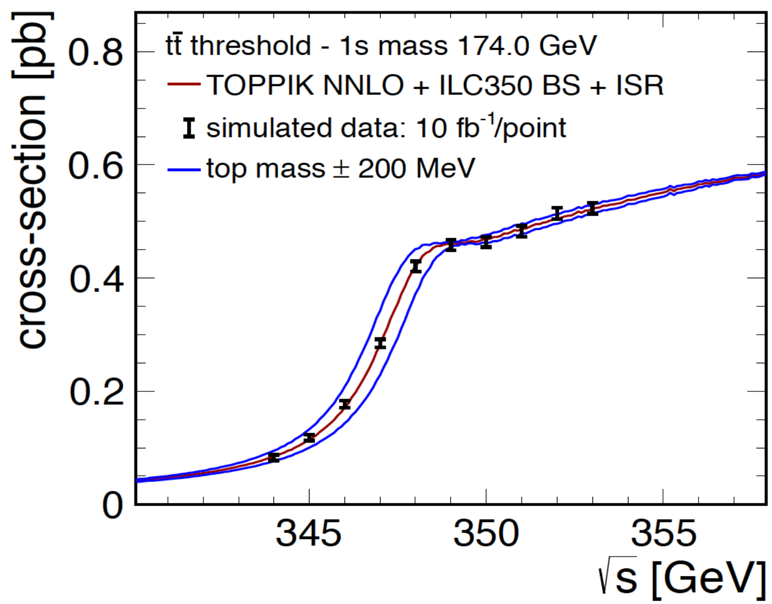


Threshold scan $\sqrt{s} \simeq 2m_t$

Precise determination of the top mass, the width and the Yukawa coupling in a well-defined theoretical scheme

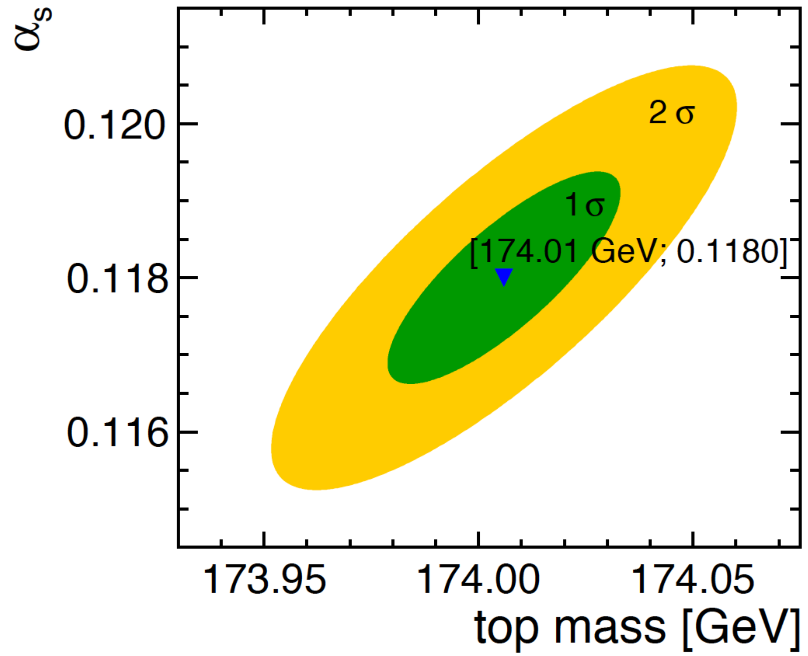
Seidel, Simon, Tesar (2012)





Prospects for the top mass measurement at the ILC

Seidel, Simon, Tesar (2012)



overall normalization uncertainty in the theory cross section assumed

ILC 2D 1S top mass and α_s combined fit

measurement	m_t stat. error	m_t th. syst. (1%/3%)	α_s stat. error	α_s th. syst. (1%/3%)
six point 2D	31 MeV	2 MeV / 1 MeV	0.0011	0.0006 / 0.0018
ten point 2D	27 MeV	5 MeV / 9 MeV	0.0008	0.0007 / 0.0022

Systematic uncertainties in background subtraction and knowledge of the luminosity spectrum dominate top mass uncertainty

$\rightarrow \quad \delta m_t \lesssim 100$ MeV

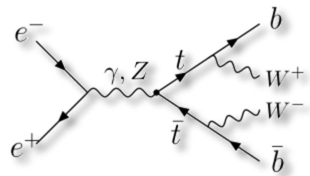
$t \to bW^+$ with $\Gamma_t \approx 1.5$ GeV $\gg \Lambda_{QCD} \Rightarrow t\bar{t}$ is perturbative at threshold

Note: once EW effects are turned on, the physical final state is $W^+W^-b\bar{b}$

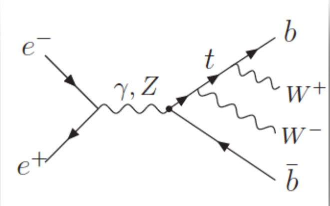
$\Rightarrow \quad \sigma(e^+e^- \to W^+W^-b\bar{b})$ in the $t\bar{t}$ resonance region

- power counting for EW effects: $\frac{\Gamma_t}{m_t} \sim \alpha_{\text{EW}} \sim \alpha_s^2 \sim v^2 \ll 1$

- **Resonant contributions** to $e^+e^- \to W^+W^-b\bar{b}$
  top and antitop close to the mass-shell

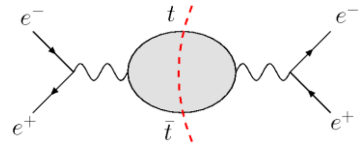


- **Non-resonant corrections**: account for the production of the bW pairs by highly virtual tops or diagrams with only one or no top



Theory signal: **resonant** + **non-resonant** corrections
Note that theory contributions **do not** always involve the production of a top-antitop pair. Such contributions **should not** be clasified as **background** (even non-resonant production of $W^+W^-b\bar{b}$ without tops!, starts at NNNLO)

Top and antitop close to mass shell, use **Non-Relat. EFT**



Top quarks move slowly near threshold: $v = \sqrt{1 - \frac{4m_t^2}{s}} \sim \alpha_s \ll 1$

$\hookrightarrow$ sum $\left(\frac{\alpha_s}{v}\right)^n$ from "Coulomb gluons" to all orders

$$\frac{\alpha_s}{v} \sim 1$$

$$\text{LO} \sim \left(\frac{\alpha_s}{v}\right)^n$$
$$\text{NLO} \sim \{\alpha_s, v\} \times \left(\frac{\alpha_s}{v}\right)^n$$
$$\text{NNLO} \sim \{\alpha_s^2, \alpha_s v, v^2\} \times \left(\frac{\alpha_s}{v}\right)^n$$

✔ **fixed-order approach:** all **N³LO** pieces known (compilation of all contributions to check convergence of perturbative series still pending) **Beneke, Kiyo, Schuller '05-08**

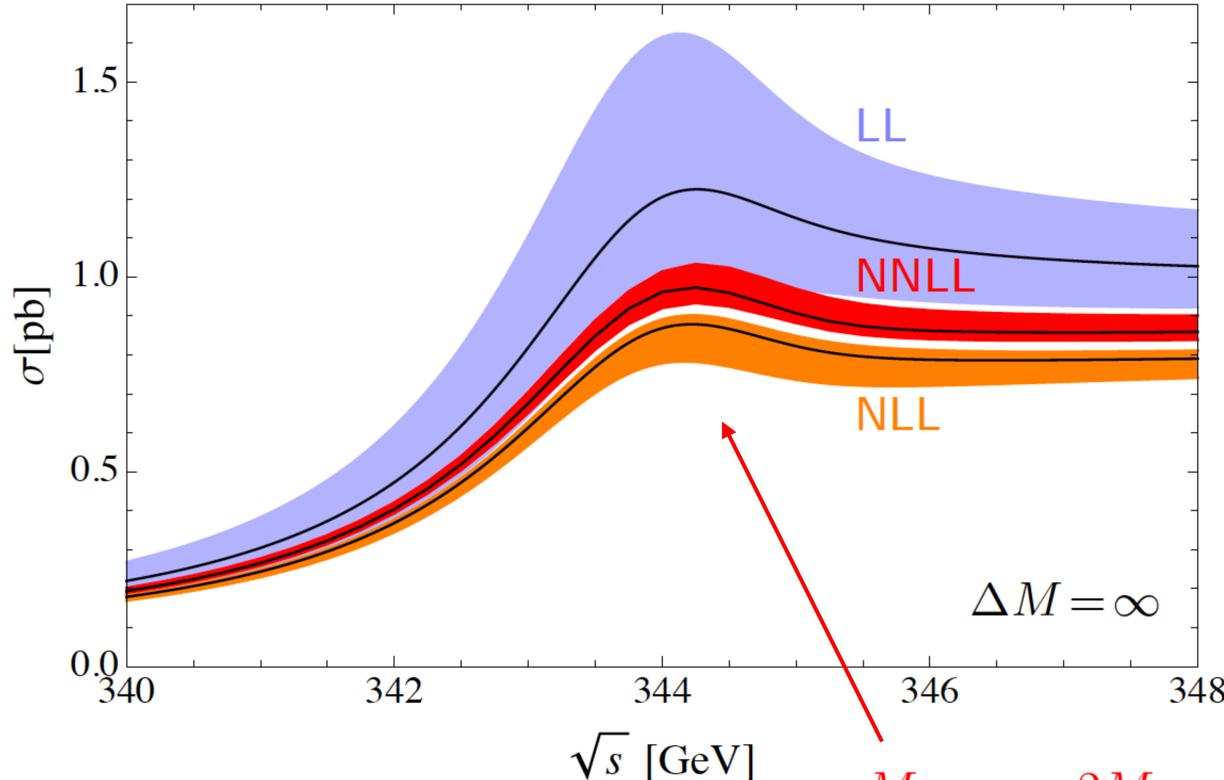
Further RG improvement by summing also $(\alpha_s \ln v)^m$: LL, NLL, ...

$$\frac{\alpha_s}{v} \sim 1 \qquad \alpha_s \ln v \sim 1$$

$$\text{LL} \sim \left(\frac{\alpha_s}{v}\right)^n \sum_m (\alpha_s \ln v)^m$$
$$\text{NLL} \sim \left(\frac{\alpha_s}{v}\right)^n \sum_m (\alpha_s \ln v)^m \times \{\alpha_s, v\}$$

✔ **RG improved calculation: NNLL** almost complete (missing NNLL piece small)

Hoang, Manohar, Stewart, Teubner '00-01;
Hoang ´03; Pineda, Signer '06;
Hoang, Stahlhofen '06-13



Hoang, Stahlhofen (2013)

$\delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim \pm 5\%$

$M_{\rm peak} \simeq 2M_{1S} = 2m_t^{\rm pole} - 0.22\,\alpha_s^2 m_t^{\rm pole}$

"threshold masses"

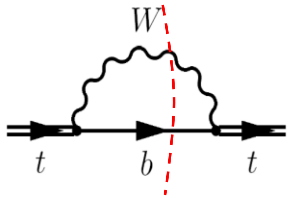
- missing QCD soft NNLL contributions small
- EW effects beyond LO and specially non-resonant effects give contributions at the level of the QCD uncertainty

$(\text{at LO:} \quad E = \sqrt{s} - 2m_t \to E + i\Gamma_t)$

Electroweak effects at LO Fadin, Khoze (1987)

- Replacement rule: $E = \sqrt{s} - 2m_t \to E + i\Gamma_t$

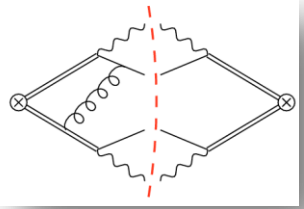
$\Rightarrow$ unstable top propagator $\frac{i}{p^0 - \mathbf{p}^2/(2m) + i\Gamma_t/2}$



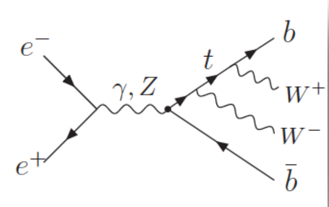
Electroweak effects at NLO

- Exchange of "Coulomb photon": trivially extension of QCD corrections

- Gluon exchange involving the bottom quarks in the final state $\Rightarrow$ these contributions vanish at NLO for the total cross section, Fadin, Khoze, Martin; Melnikov, Yakovlev (1994)
also negligible if loose top invariant-mass cuts are applied; remains true at **NNLO** Hoang, Reisser (2005); Beneke, Jantzen, RF (2010)

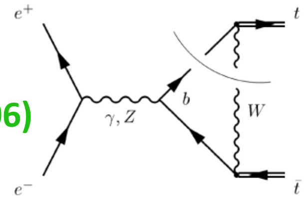


- Non-resonant corrections to $e^+e^- \to W^+W^-b\bar{b}$ which account for the production of the Wb pairs by highly virtual tops or with only one or no top
$\hookrightarrow$ fully known at this order Beneke, Jantzen, RF (2010)

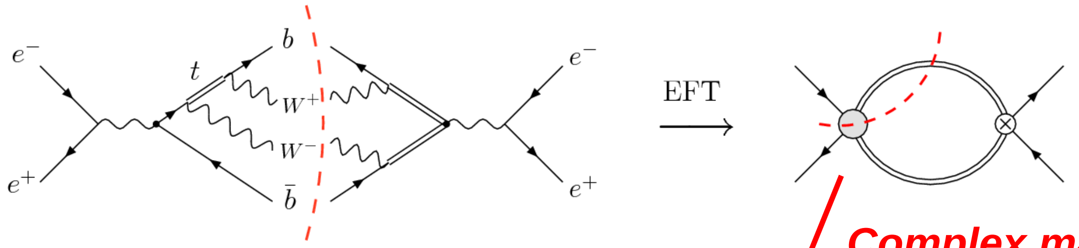


Electroweak (non-trivial) effects at NNLO

- **absorptive parts** in the 1-loop matching coeffs. of the production operators (arising from bW cuts) Hoang, Reisser (2006)

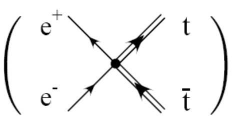


$\Rightarrow$ reproduce interferences between double and single resonant amplitudes



Complex matching conditions

$$J_{\mathbf{p}} = \left[C_{\rm LL}^{\rm born} + C_{\rm NLL}^{\rm QCD} + C_{\rm NNLL}^{\rm QCD} + i C_{\rm NNLL}^{bW,{\rm abs}} + C_{\rm NNLL}^{\rm EW} + \dots \right]$$



- **real part** of hard one-loop EW corrections Kuhn, Guth (1992); Hoang, Reisser (2006)

- **NNLO non-resonant** contributions (gluon corrections to NLO ones)
  Exact computation is hard, but dominant terms known for moderate invariant mass cuts. Hoang, Reisser, RF (2010); Jantzen, RF (2013)
  An approximation for the total cross section recently obtained RF (2014)

II. Non-resonant (electroweak) NLO contributions

II. Non-resonant NLO contributions

Beneke, Jantzen , RF (2010)

$\Rightarrow$ cuts through $bW^+\bar{t}$ (see diagrams) and $\bar{b}W^-t$ (not shown) in the 2-loop forward scattering amplitude

- treat loop-momenta as hard:

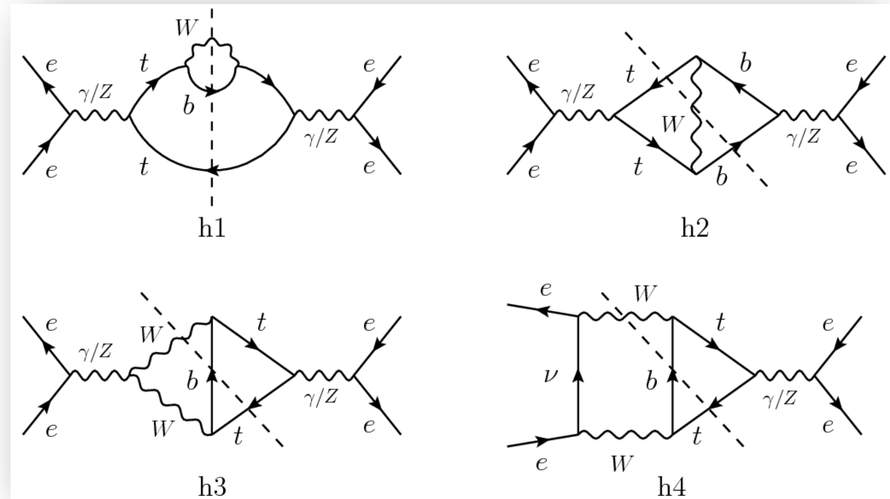
  $p_t^2 - m_t^2 \sim \mathcal{O}(m_t^2) \gg \Sigma(p_t^2) \sim m_t^2 \alpha_{\rm EW}$

  $\rightarrow \Gamma_t = 0$

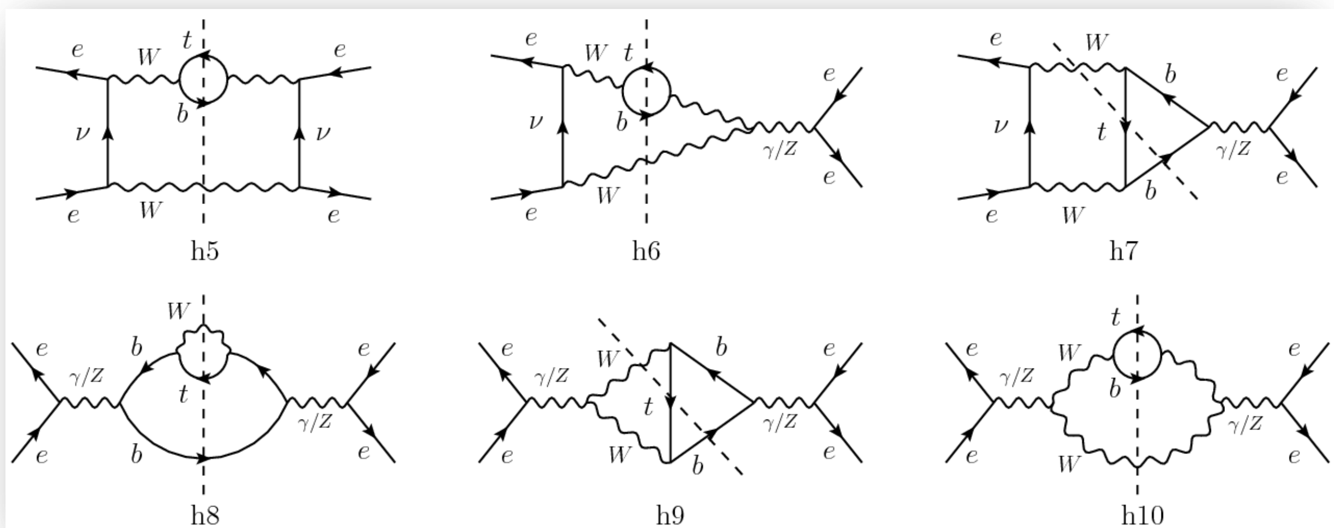
- suppressed w.r.t. LO ($\sim v$) by

  $\alpha_{\rm EW}/v \sim \alpha_s$

bW^+ from highly virtual top



bW^+ without intermediate top



In terms of the invariant mass of the bW^+ system, $p_t^2 = (p_b + p_{W^+})^2$, ($p_t \to$ also momentum of the top line for h1-h4) diagrams h1-h10 read:

$$\int_{\Delta^2}^{m_t^2} dp_t^2 \, (m_t^2 - p_t^2)^{1/2-\epsilon} \, H_i\left(\frac{p_t^2}{m_t^2}, \frac{M_W^2}{m_t^2}\right)$$

with $\Delta^2 = M_W^2$ for the total cross section

Applying top invariant-mass cuts

Restrict invariant masses of the reconstructed $t, \bar{t}$: $|\sqrt{p_{t,\bar{t}}^2} - m_t| \leq \Delta M_t$

$\hookrightarrow$ lower integration limit $\Delta^2 = m_t^2 - \Lambda^2$ where $\Lambda^2 = (2m_t - \Delta M_t)\Delta M_t$

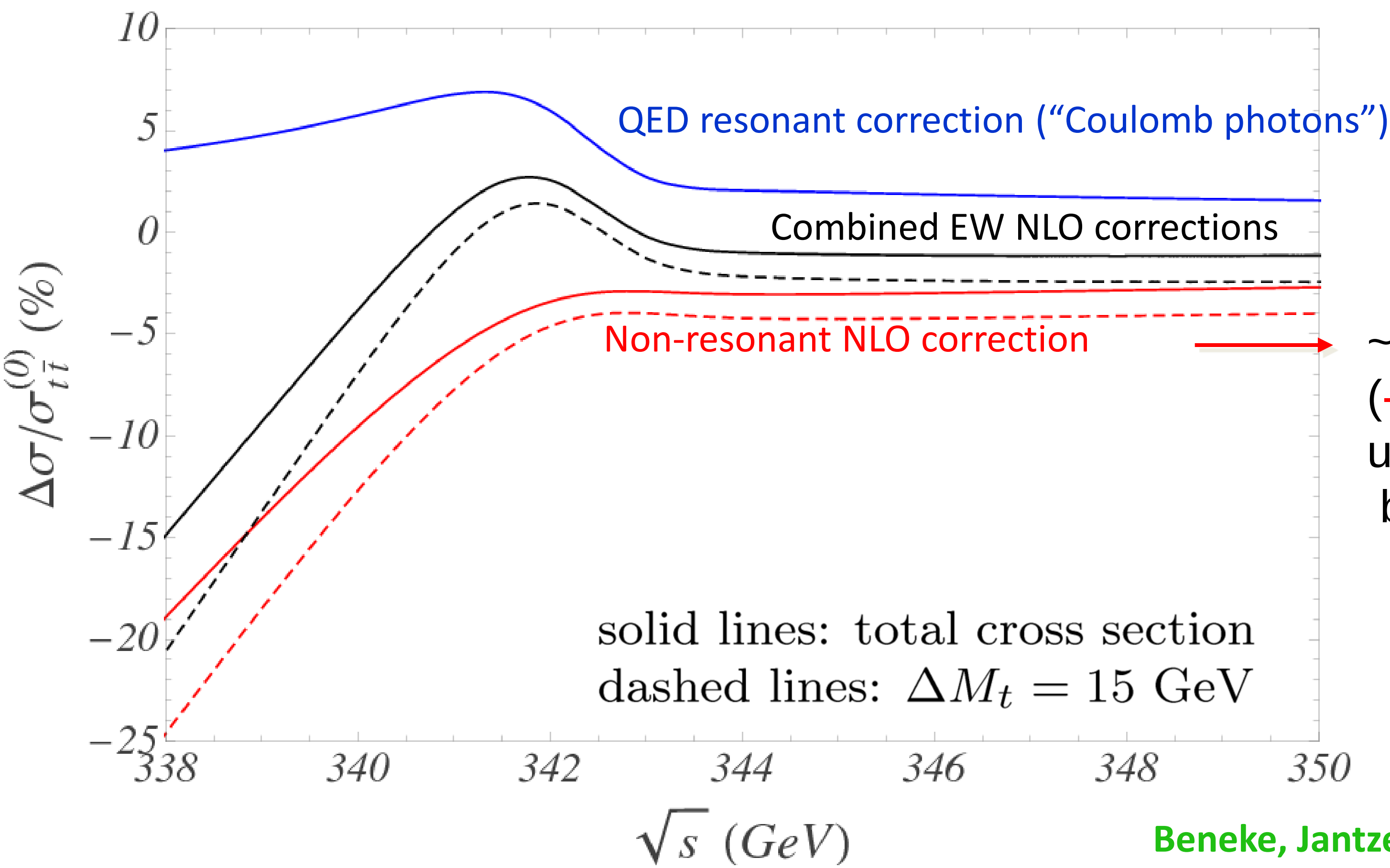
We focus on loose cuts with $\Lambda^2 \gg m_t\Gamma_t$ (corresponding to $\Delta M_t \gg \Gamma_t$)

$\rightsquigarrow$ cut has no effect in the resonant contributions

[In contrast: for tight cuts with $\Lambda^2 \sim m_t\Gamma_t$ ($\Delta M_t \sim \Gamma_t$), non-resonant contributions vanish and cuts only affect the resonant contributions]

Relative sizes of EW NLO corrections with respect LO

LO includes resummation of Coulomb gluons $\propto (\alpha_s/v)^n$ $[\alpha_s^{\overline{\rm MS}}(30\,{\rm GeV}) = 0.142]$



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(-3% above and
up to -20%
below threshold)

Beneke, Jantzen , RF (2010)

Phase space matching

Alternative approach to compute non-resonant contributions

Hoang, Reisser, RF (2010)

- Non-resonant contributions obtained for moderate invariant-mass cuts, $m_t\Gamma_t \ll \Lambda^2 \ll m_t^2$, as a series:

$$\frac{\Gamma_t}{\Lambda}\sum_{n,\ell,k}\left[\left(\frac{m_t\Gamma_t}{\Lambda^2}\right)^n \times \left(\frac{\Lambda^2}{m_t^2}\right)^\ell\right] \times \left(\alpha_s\frac{m_t}{\Lambda}\right)^k \qquad n,\ell,k = 0,1,\ldots$$

- NLO, NNLO and (partial) N^3LO contributions obtained (counting $\Lambda \sim m_t$) ✓

- Beyond NLO, phase space matching approach cannot be applied to larger cuts up to the total cross section ×

- Expansion of full NLO non-resonant contributions in $(\Lambda/m_t)^n$ agrees with first two terms in series above
  [higher powers receive contributions from diagrams h5-h10 with no top, not taken into account in the psm approach → remainder contributions, small at NLO ✓]

III. Non-resonant NNLO contributions

Finite-width divergences in the resonant contributions

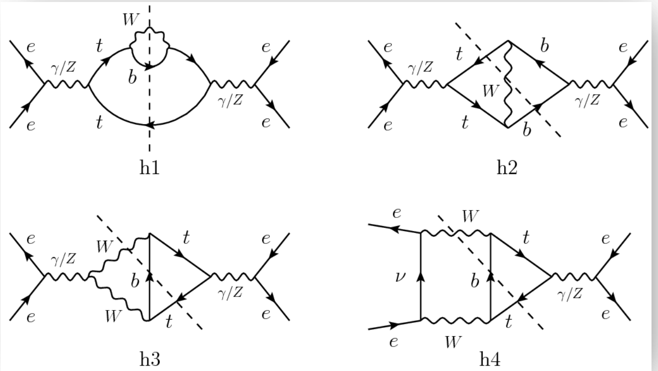
Resonant contributions obtained by assuming the top quarks are nearly on-shell (potential), but integrated over all momenta

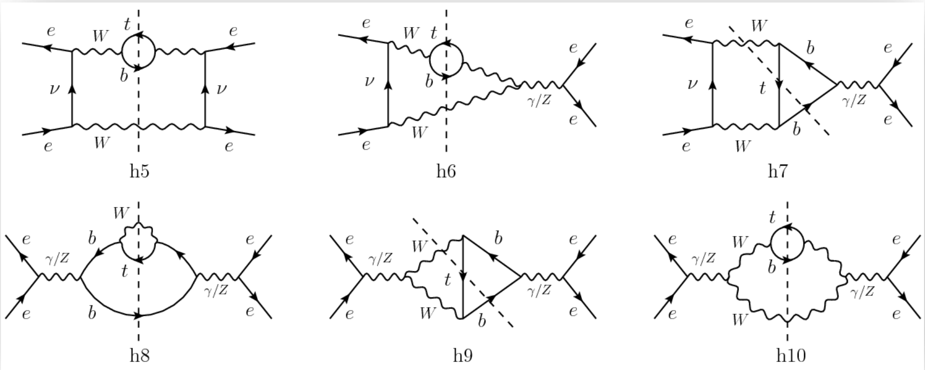
→ uncancelled **UV-singularity** from hard momenta: $\Delta\sigma^{\text{NNLO}} \sim m_t^2 \frac{\alpha_s \Gamma_t}{\epsilon}$

Related to finite top width in EFT cut propagator

- for stable top $\to \pi\, \delta\!\left(p^0 - \frac{\vec{p}^{\,2}}{2m_t}\right)$,
- for unstable top $\to \frac{\Gamma_t/2}{(p^0 - \vec{p}^{\,2}/2m_t)^2 + (\Gamma_t/2)^2}$ Breit–Wigner, UV-behaviour changed!

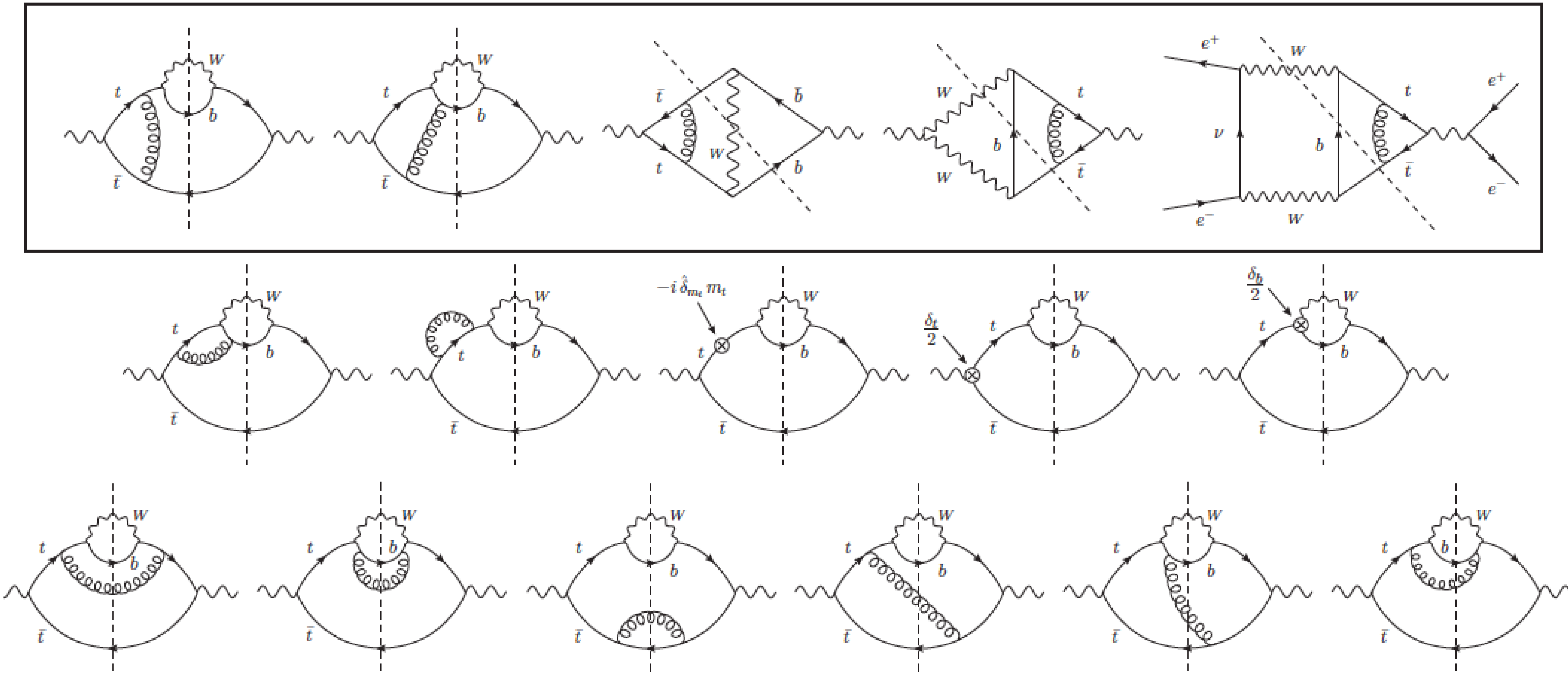
These divergences must cancel with non-resonant (hard) NNLO terms, which arise from **gluon corrections** to NLO non-resonant diagrams h1-h10





ENDPOINT-DIVERGENT NON-RESONANT NNLO DIAGRAMS

$\hookrightarrow$ expanded near endpoint $\rightsquigarrow$ potential top momentum $p_t = p_b + p_W\,(+p_g)$



boxed diagrams $\rightsquigarrow$ endpoint-singular $\frac{1}{\epsilon} - 2\ln\frac{\Lambda^2}{\mu^2}$ terms from potential gluons

$+$ "finite" endpoint-divergent $\frac{m_t}{\Lambda}$ & $\frac{m_t^2}{\Lambda^2}$ terms from hard & potential gluons

Endpoint-divergent non-resonant NNLO contribution

$\hookrightarrow$ dominant contribution for small Λ (or small ΔM_t)

$$\left[\begin{array}{c} x = M_W^2/m_t^2 \\ C_{\dots}(s) = \gamma/Z\text{-prop. \& } e^{\pm}\text{-coupl.} \end{array}\right]$$

$$\sigma^{(2),\mathrm{ep}}_{\mathrm{non-res}} = \frac{64\pi^2\alpha^2}{s}\,\frac{\Gamma_t^{\mathrm{Born}}}{m_t}$$

$$\times \Bigg\{ \Big[Q_t^2\,C_{\gamma\gamma}(s) - 2Q_t v_t\,C_{\gamma Z}(s) + v_t^2\,C_{ZZ}(s)\Big] \left\{4N_c C_F\,\frac{\alpha_s}{\pi}\,\frac{m_t^2}{\Lambda^2} + \frac{6\sqrt{2}}{\pi^2}\left(\delta\Gamma_t^{(1)} - \frac{4C_F}{\pi}\alpha_s\right)\frac{m_t}{\Lambda}\right\}$$

$$+ N_c C_F\,\frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon_{\mathrm{ep}}} + 2\ln\frac{\mu_s^2}{\Lambda^2}\right)\Bigg\{\Big[Q_t^2\,C_{\gamma\gamma}(s) - 2Q_t v_t\,C_{\gamma Z}(s) + v_t^2\,C_{ZZ}(s)\Big]\frac{7+7x+22x^2}{6(x-1)\,(1+2x)}$$

$$+ \frac{1}{3}a_t^2\,C_{ZZ}(s) + \frac{1}{2}Q_t a_t\,C_{\gamma Z}(s) - \frac{1}{2}v_t a_t\,C_{ZZ}(s)$$

$$+ \Big[Q_t Q_b\,C_{\gamma\gamma}(s) - (Q_t\,(v_b+a_b) + Q_b v_t)\,C_{\gamma Z}(s) + v_t\,(v_b+a_b)\,C_{ZZ}(s)\Big]\frac{1-5x-2x^2}{6(1+x)\,(1+2x)}$$

$$+ \left[Q_t\,C_{\gamma\gamma}(s) - \left(v_t + Q_t\,\frac{c_w}{s_w}\right)C_{\gamma Z}(s) + v_t\,\frac{c_w}{s_w}\,C_{ZZ}(s)\right]\frac{2+5x-2x^2}{6x\,(1+2x)}$$

$$- \Big[Q_t\,C_\gamma(s) + v_t\,C_Z(s)\Big]\left[\ln\left(\frac{2}{x}-1\right) + \frac{(1-x)\,(1-2x-23x^2)}{12x^2}\right]\frac{x}{4(1-x)^3\,(1+2x)}\Bigg\}\Bigg\}$$

$$+ \text{finite } \Lambda\text{-independent terms} + \mathcal{O}(\Lambda/m_t)$$

Jantzen, RF (2013)

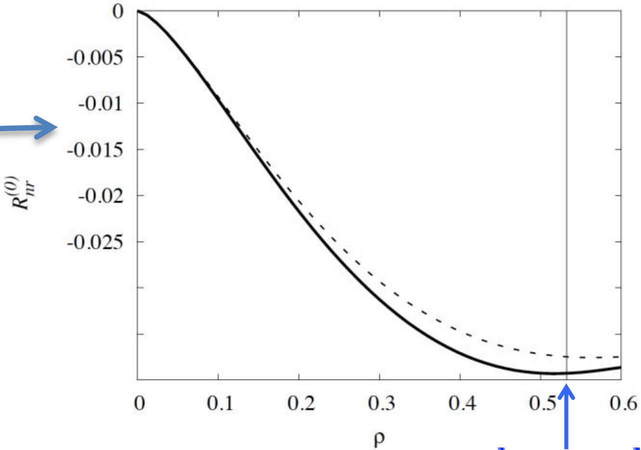
- UV and IR singularities cancelled between diagrams ✓
- $1/\epsilon$ endpoint singularities & finite-width divergences cancel each other ✓
- comparison to HRR result: m_t^2/Λ^2 ✓, m_t/Λ ✓, $\Lambda^0\ln(\Lambda^2)$ ✓

Non-resonant NNLO contribution for total cross section

Alternative framework [Penin, Piclum, 2012] **computes non-resonant contributions to the total cross section by expanding in**

$\rho = 1 - M_W/m_t \approx 0.5$

- at NLO, the first term in ρ deviates from the exact result by less than 5%



ρ phys. value

- at NNLO

$$R_{nr}^{(1)} = \frac{N_c C_F \alpha_s}{\pi^2 \rho} \frac{\Gamma_t}{m_t} \Bigg\{ \left[Q_e^2 Q_t^2 + \frac{2 Q_e Q_t v_e v_t}{1 - x_Z} + \frac{(a_e^2 + v_e^2) v_t^2}{(1 - x_Z)^2} \right] \times \left[\left(3L_E + \frac{3}{2} + 6 \ln 2 \right) \pi^2 + (18 + 24 \ln 2)\, \rho^{1/2} \right] + \frac{1}{s_w^4} \left[\frac{22}{3} + \frac{17\pi^2}{6} - \frac{17}{2} \ln 2 + (2 - 3\pi^2 + 9 \ln 2) \frac{3\sqrt{2}}{4} \ln\left(1 + \sqrt{2}\right) - \frac{27\sqrt{2}}{8} \left(\ln^2\left(1 + \sqrt{2}\right) + \mathrm{Li}_2\left(2\sqrt{2} - 2\right) \right) \right] \rho^{1/2} + \mathcal{O}(\rho) \Bigg\}.$$

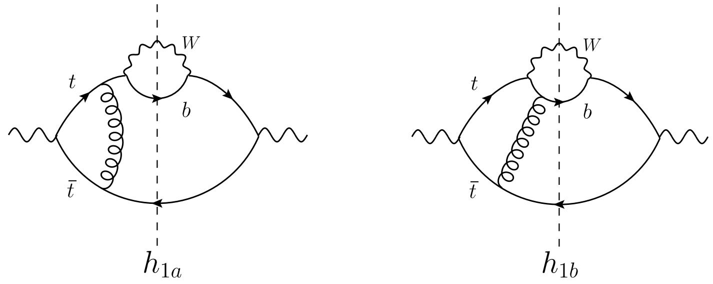
$L_E = \ln\left(\frac{\sqrt{E^2 + \Gamma_t^2}}{\rho m_t}\right)$ → infrared regularization dependent term. But dim. reg. used for the resonant contributions, not clear how to combine both

Moreover: infrared structure does not match with divs in resonant side

→ diagram missing in this computation

Non-resonant NNLO contribution: rho-expansion

Dominant term in rho reevaluated [RF 2014] **:**

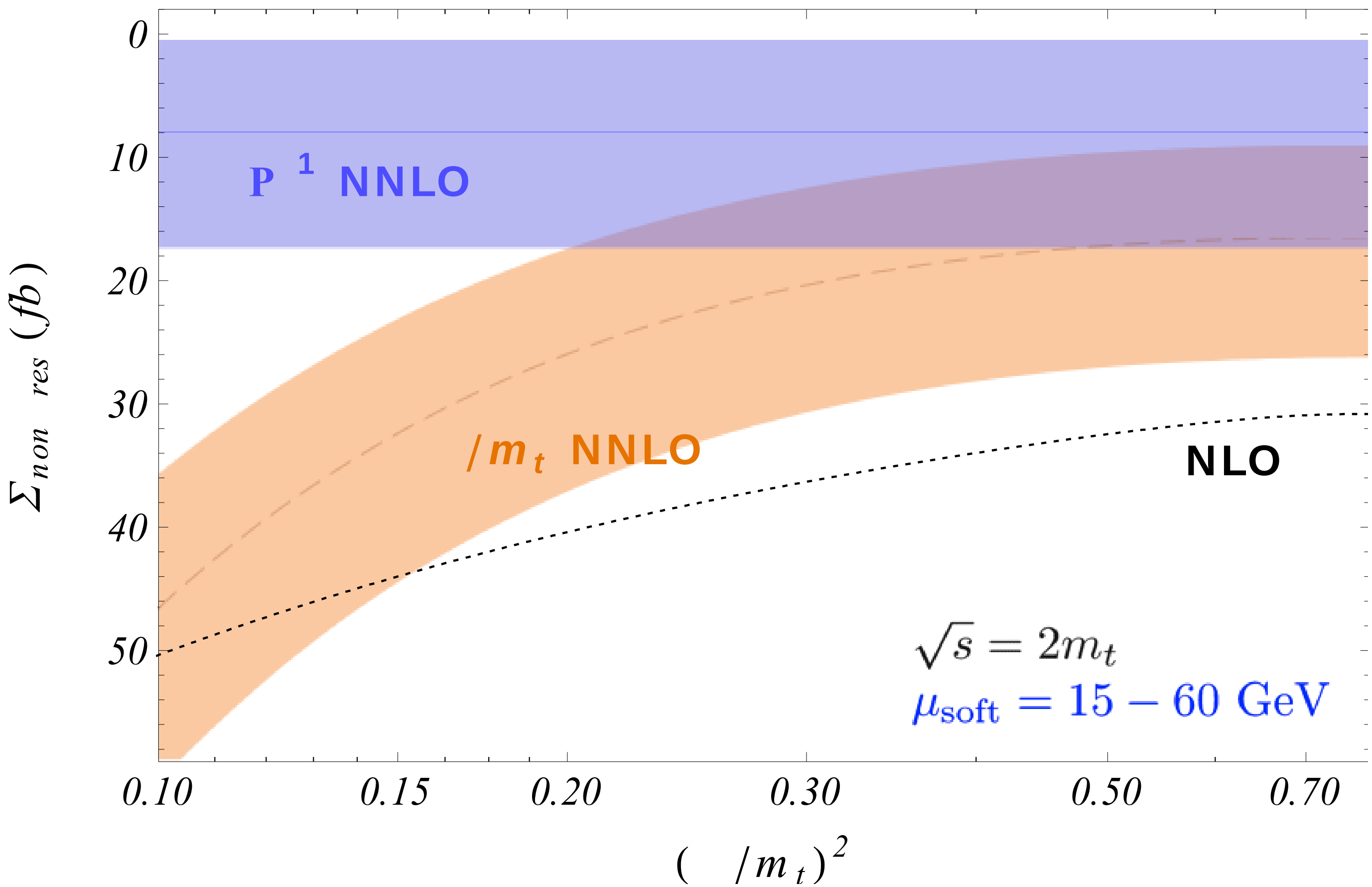


$$\sigma^{(2),\rho}_{\text{non-res}} = \frac{8\pi\alpha^2}{s} m_t \Gamma_t^{\text{Born}} N_c C_F \alpha_s \left[\frac{Q_t^2}{s} - \frac{2\,Q_t v_t v_e}{(s-M_Z^2)} + \frac{v_t^2(v_e^2+a_e^2)\,s}{(s-M_Z^2)^2} \right] \times \frac{1}{\rho}\left(\frac{1}{2\epsilon} + \frac{2}{3} - \ln\frac{\rho}{2} + \ln\frac{\mu_{\text{soft}}^2}{m_t^2} + \mathcal{O}(\rho^{1/2}) \right),$$

- $1/\epsilon$ cancel the $\alpha_s \Gamma_t/\epsilon$ divergences in the resonant contributions
- subleading term of $\mathcal{O}(\rho^{-1/2})$; should be computed to test ρ-expansion

IV. Results & comparisons

Results for the non-resonant NLO & NNLO contributions



$1/\rho$ NNLO $\lesssim 1\%$

NLO $\gtrsim 3\%$

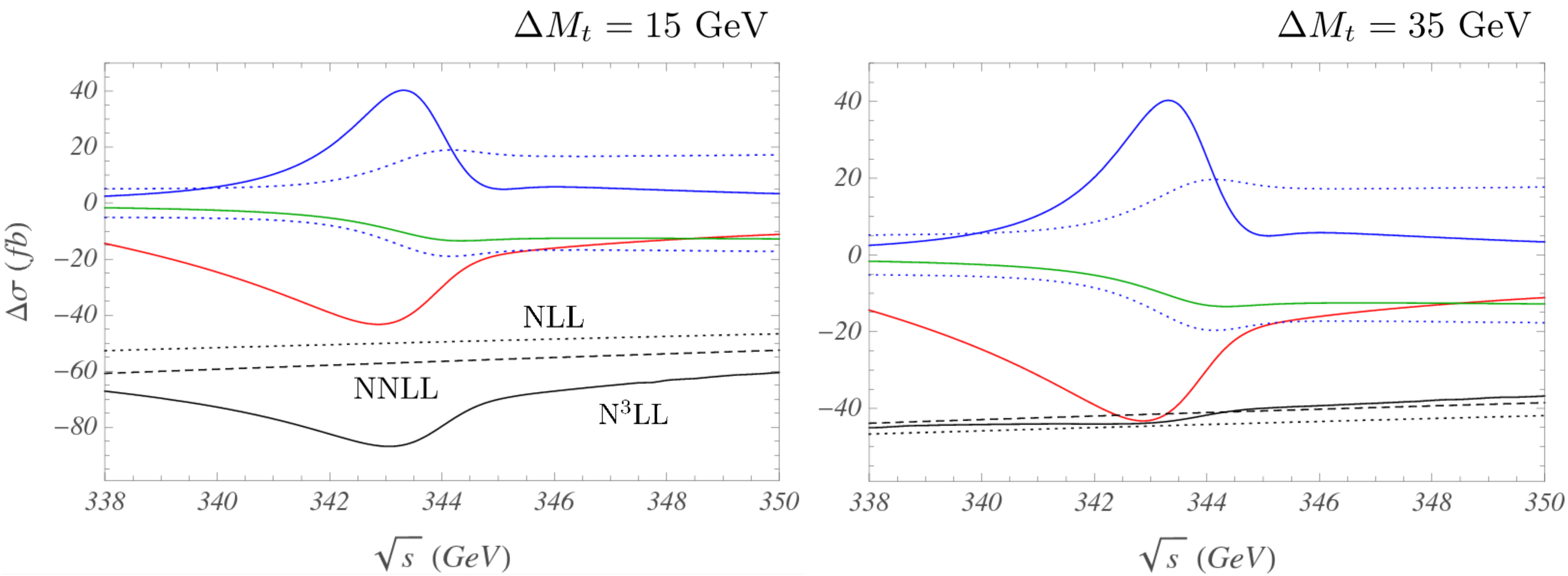
total cross section reached for $\Lambda^2/m_t^2 = 0.78$

- Λ/m_t expansion for the NNLO non-resonant contributions valid for moderate invariant-mass cuts, $m_t\Gamma_t \ll \Lambda^2 \ll m_t^2$
  Extrapolation to $\Lambda^2_{\rm max} = m_t^2 - M_W^2$ approaches $1/\rho$ estimate for the total cross section

Non-QCD corrections beyond NNLO

Sizes of NNLL EW and non-resonant corrections

Hoang, Reisser, RF (2010)



NNLL QED effects

NNLL hard one-loop EW effects

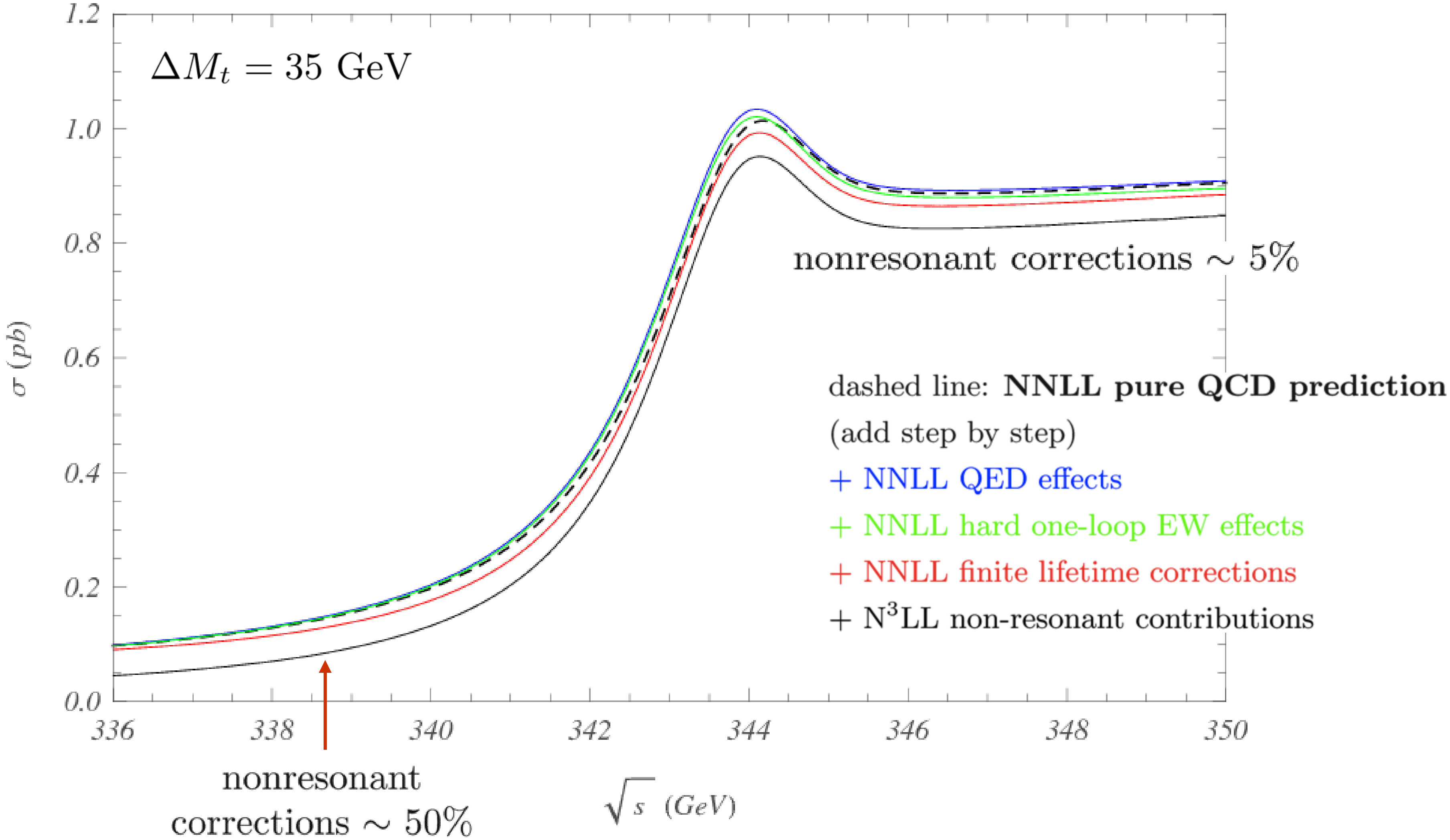
NNLL finite lifetime corrections

Non-resonant corrections
(NLL, NNLL, N^{3}LL phase space matching contributions)

- psm contributions are the largest of the 4 classes of EW effects
- almost constant (small linear $\sqrt{s}$-dependence from γ, Z propagators)
- convergence of the psm procedure particularly good for larger ΔM_t

Inclusive top-pair production cross section

Cut on bW invariant masses of the form $|\sqrt{p^2_{t,\bar{t}}} - m_t| \leq \Delta M_t$



IV. Summary

Resonant corrections (top and antitop close to mass shell)

- QCD contributions:
  - ✔ **fixed-order approach:** most of **N^3LO** pieces known (compilation of all contributions shall appear soon...)
  - ✔ **RG improved calculation: NNLL** almost complete
- Electroweak contributions known to **NNLL** accuracy

Theoretical uncertainties ~ 5% at **NNLL**, at **N^3LO ?** ...

3% theoretical uncertainty on the total cross section here may be possible...

Non-resonant corrections (bW pairs from virtual tops or with only one or no top)

✔ **computed at NLO** for the total cross section and with top invariant-mass cuts

✔ Beyond: **dominant NNLO and NNNLO terms known** when top invariant mass cuts are included. **NNLO estimate** for the total cross section: **~1% effect**

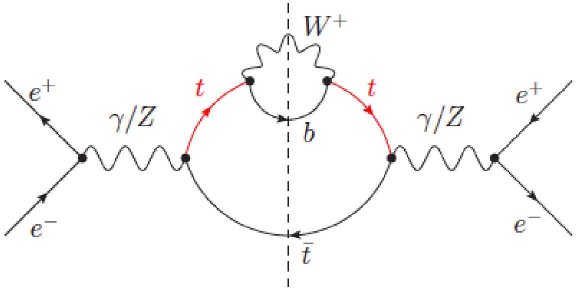
8 **In progress:** **full** NNLO corrections to total cross section (few percent at most), but **can become very important below the peak region**

→ **include non-resonant corrections in future ILC top-quark mass measurement study (analyse background to avoid double-counting!)**

Endpoint divergences in the non-resonant contributions

Endpoint divergences of the phase-space integration at $p_t^2 \to m_t^2$ (because $\Gamma_t = 0$ here):

NLO:

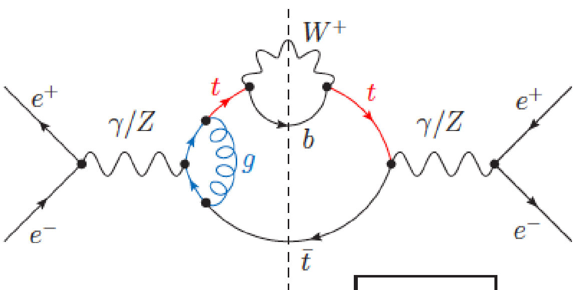


$$\sim \int^{m_t^2} \frac{\mathrm{d}p_t^2}{(m_t^2 - p_t^2)^{n+\epsilon}} \quad \text{with } n = \frac{3}{2}, \frac{1}{2}, \ldots$$

$\hookrightarrow$ endpoint divergence finite in dim. reg.:

$$\int_{m_t^2-\Lambda^2}^{m_t^2} \frac{\mathrm{d}p_t^2}{(m_t^2 - p_t^2)^{\frac{3}{2}+\epsilon}} = -\frac{2}{\Lambda} + \mathcal{O}(\epsilon)$$

NNLO:



$$\sim \int^{m_t^2} \frac{\mathrm{d}p_t^2}{(m_t^2 - p_t^2)^{n+a\epsilon}} \quad \text{with } n = 2, \frac{3}{2}, 1, \frac{1}{2}, \ldots$$

$\hookrightarrow$ endpoint divergence $\propto$ $\boxed{\alpha_s \dfrac{\Gamma_t}{\epsilon}}$ from $n = 1$:

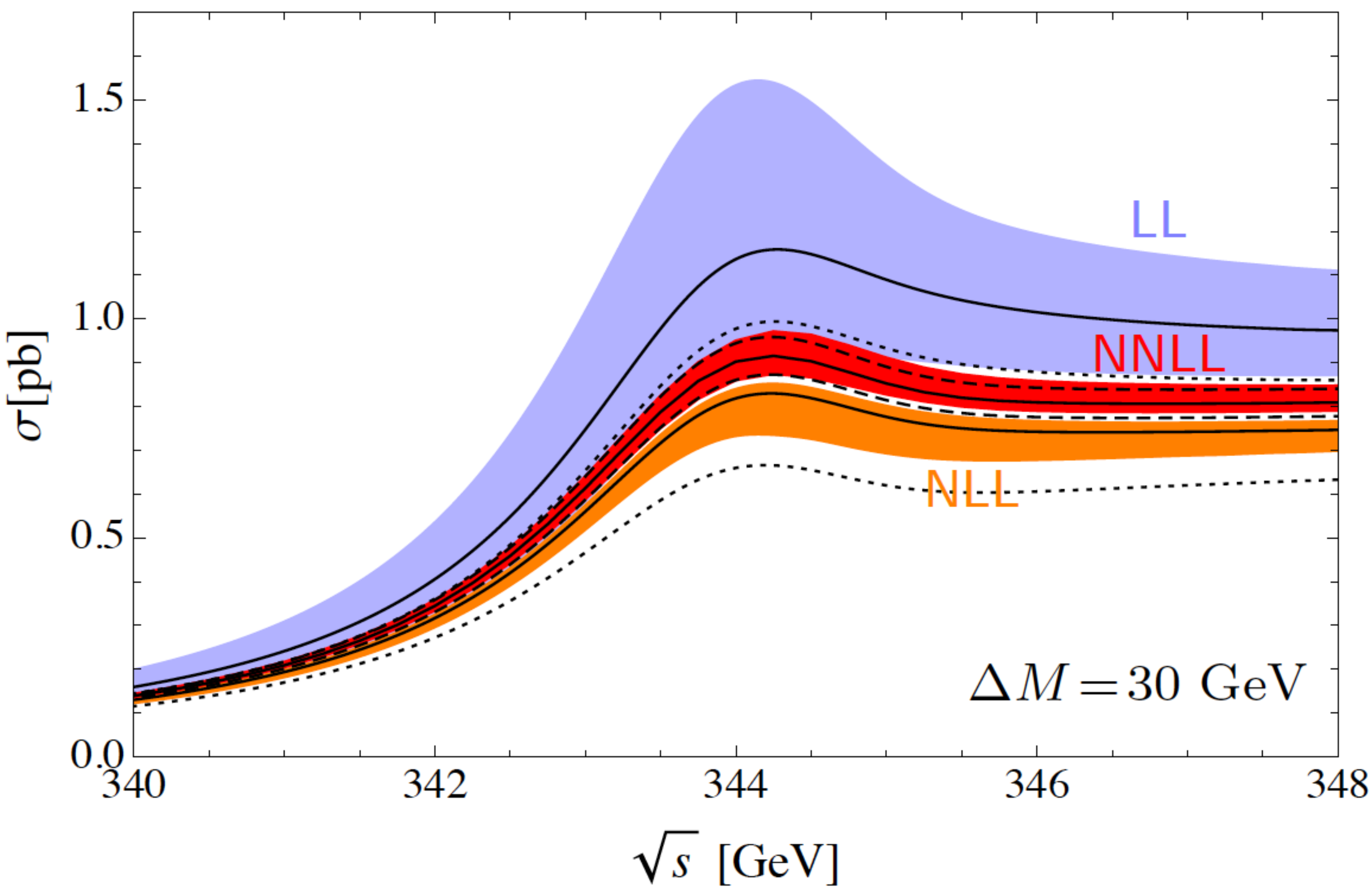
$$\mu^{4\epsilon} \int_{m_t^2-\Lambda^2}^{m_t^2} \frac{\mathrm{d}p_t^2}{(m_t^2 - p_t^2)^{1+2\epsilon}} = -\frac{1}{2\epsilon} + \ln\frac{\Lambda^2}{\mu^2} + \mathcal{O}(\epsilon)$$

Expand integrand in $(m_t^2 - p_t^2)/m_t^2 \iff$ asymptotic expansion of result in Λ/m_t

Inclusive top-pair production cross section

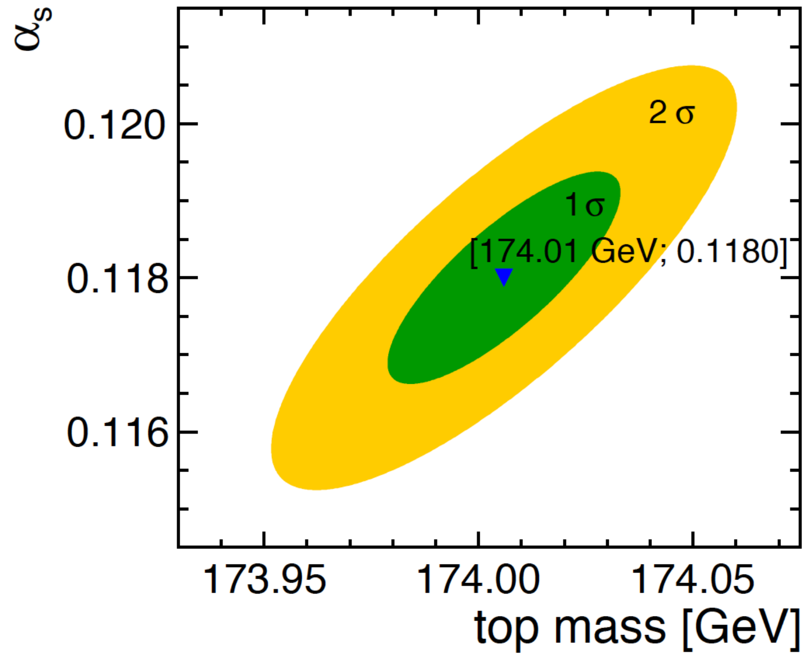
NNLL QCD + N^3LO non-resonant corrections

Hoang, Stahlhofen (2013)



- resonant EW & QED corrections not included

Prospects for the top mass measurement at the ILC



[Seidel, Simon, Tesar (2012)]

includes up to NNLO QCD corrections (no EW)

ILC 1D 1S top mass fit

measurement	m_t stat. error	m_t th. syst. (1%/3%)	α_s syst.
six point scan 1D fit	18 MeV	15 MeV / 47 MeV	18 MeV
ten point scan 1D fit	18 MeV	13 MeV / 39 MeV	17 MeV

ILC 2D 1S top mass and α_s combined fit

measurement	m_t stat. error	m_t th. syst. (1%/3%)	α_s stat. error	α_s th. syst. (1%/3%)
six point 2D	31 MeV	2 MeV / 1 MeV	0.0011	0.0006 / 0.0018
ten point 2D	27 MeV	5 MeV / 9 MeV	0.0008	0.0007 / 0.0022