NLO QCD Corrections to $t\bar{t}Z$ Production

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The top quark

- Discovered in 1995 by CDF and D0
- Heaviest known elementary particle
  - $m_t \approx v/\sqrt{2}$: top may play a special role in EWSB.
- $t\bar{t}$ production at Tevatron depends only on top mass and color charge.
- EW properties largely unknown (e.g. $tt\gamma$, ttZ, ttH couplings)
  - sensitive to new physics (e.g. Z', T quark, $\ldots$)
  - ttZ couplings:
    - inaccessible at Tevatron
    - can be studied at ILC via $e^+e^- \longrightarrow \gamma^*/Z^* \longrightarrow t\bar{t}$
    - or at LHC via $pp \longrightarrow t\bar{t}Z$

Top quark electroweak couplings

- General ttZ coupling:

$$\Gamma_\mu^{ttZ}(k) = -ie\Big[\gamma_\mu F_{1V}^Z(k^2) + \gamma_\mu\gamma_5 F_{1A}^Z(k^2) + \frac{i}{2m_t}\sigma_{\mu\nu}k^\nu F_{2V}^Z(k^2) + \frac{1}{2m_t}\sigma_{\mu\nu}\gamma_5 k^\nu F_{2A}^Z(k^2)\Big]$$

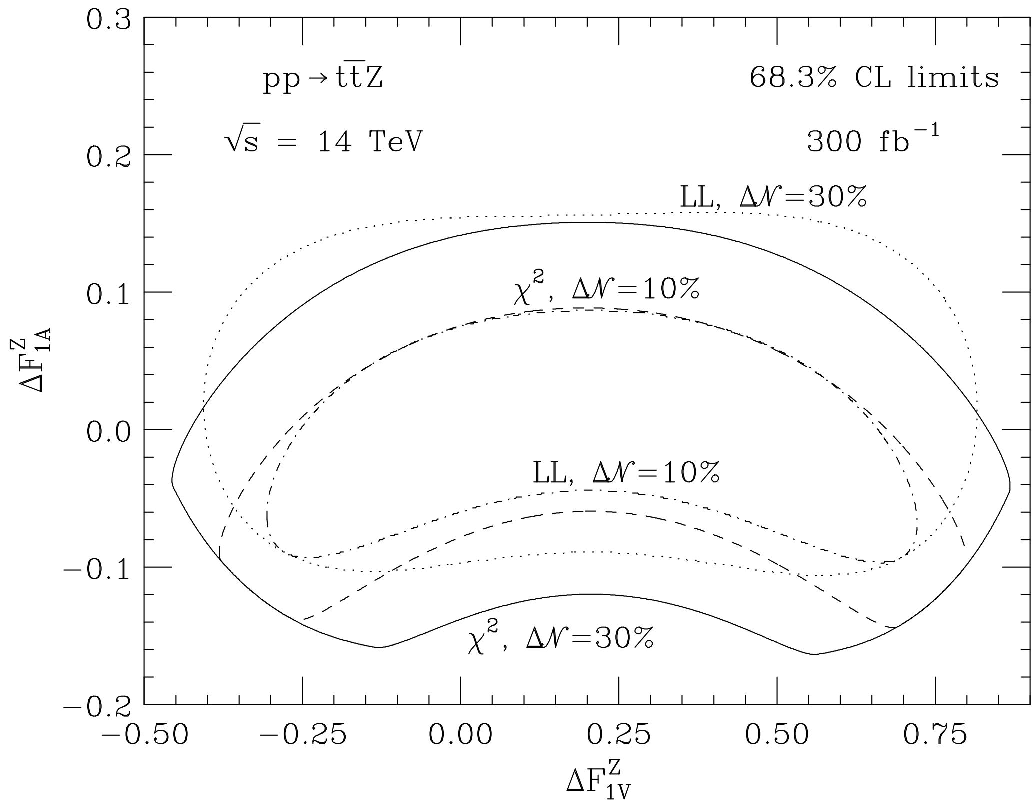
- No direct measurements of form factors
- F_{1V}^Z and F_{1A}^Z tightly but indirectly constrained by LEP:

$$-0.044 \leqslant -\Delta F_{1A}^Z(0)\left[1 + 0.842\Delta F_{1A}^Z(0)\right] \ln\left(\frac{\Lambda^2}{m_t^2}\right) \leqslant 0.065,$$

$$-0.029 \leqslant -\left[\Delta F_{1A}^Z(0) - \frac{3}{5}\Delta F_{1V}^Z(0)\right] \ln\left(\frac{\Lambda^2}{m_t^2}\right) \leqslant 0.143$$

[Baur, Juste, Orr, Rainwater, 2005]

Top quark electroweak couplings

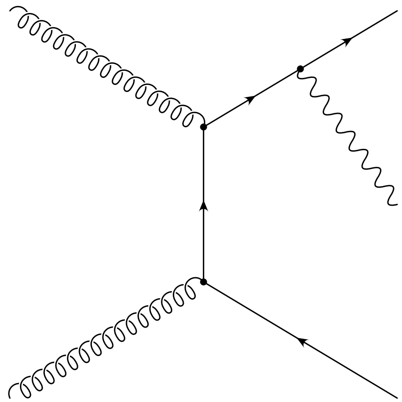


[Baur, Juste, Orr, Rainwater, 2005]

- Dominant uncertainty is signal normalization $\mathcal{N}$.
- Reducing $\Delta\mathcal{N}$ from 30% to 10% improves sensitivity by factor of 2.

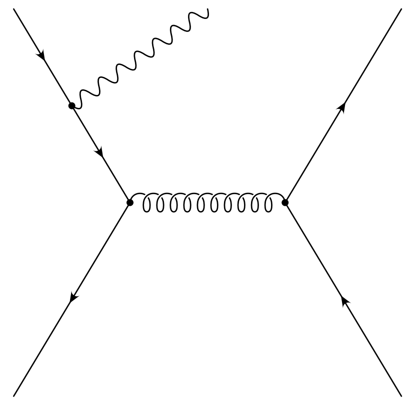
$pp \longrightarrow t\bar{t}Z$: LO

$gg \longrightarrow t\bar{t}Z$



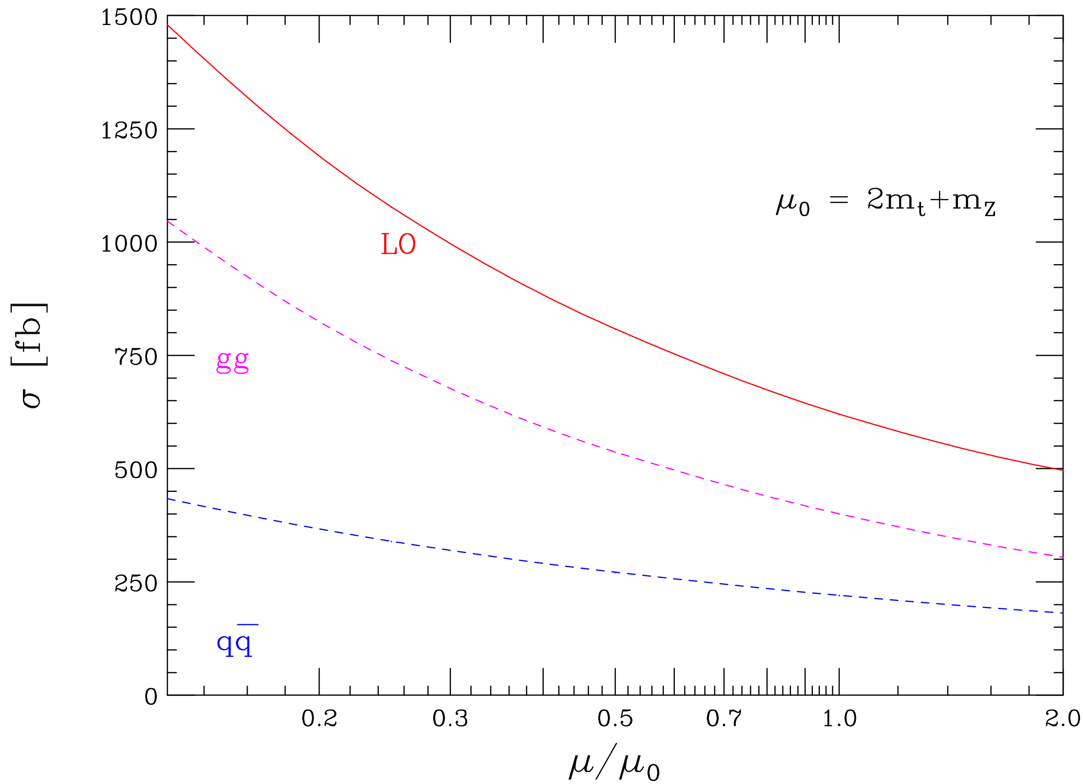
8 diagrams

$q\bar{q} \longrightarrow t\bar{t}Z$



4 diagrams

$pp \to t\bar{t}Z$: LO



- Significant scale dependence contributes to $\Delta\mathcal{N}$.
- NLO QCD calculation is necessary!

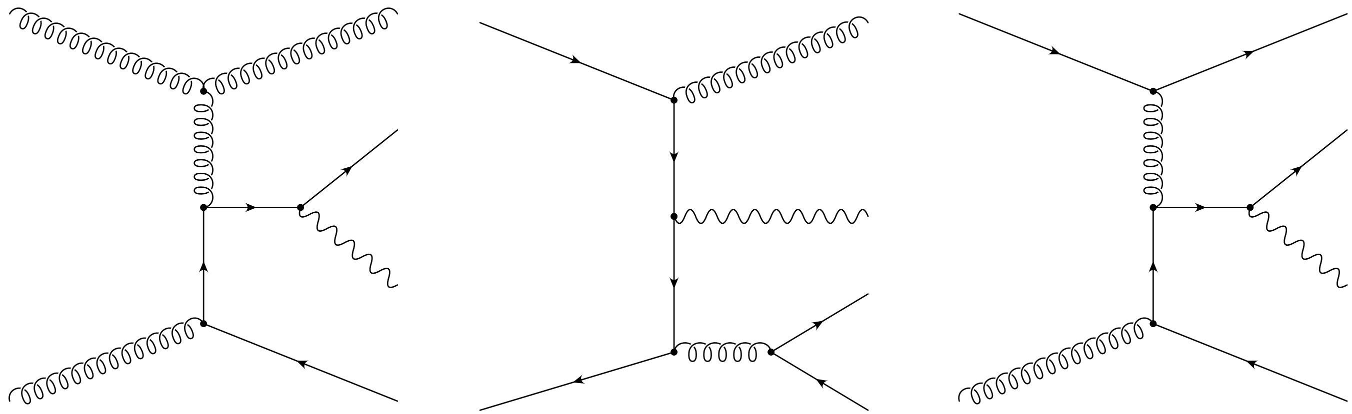
$pp \to t\bar{t}Z$: NLO

Full set of complexities that appear in multi-leg one-loop calculations:

- Multiple mass scales
- Soft and collinear singularities on boundaries of phase space and Feynman-parameter space
- Threshold singularities in interior of Feynman-parameter space
- Pentagon diagrams with 1–4 massive propagators

NLO: Real radiative corrections

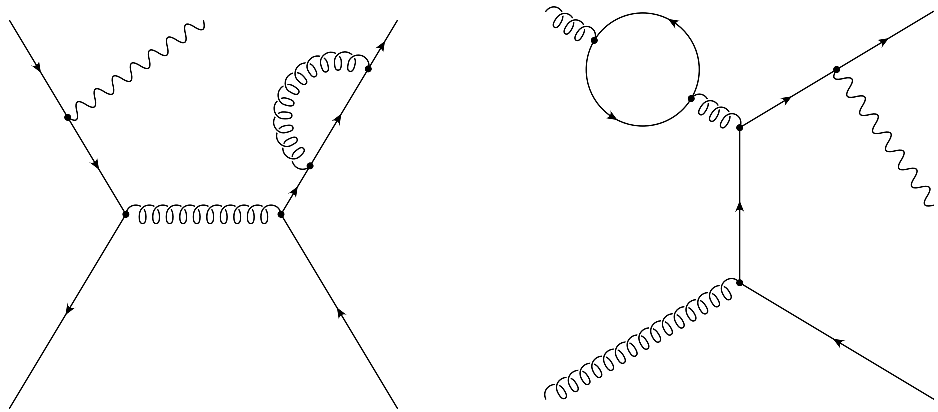
$gg \to t\bar{t}Zg$ | $q\bar{q} \to t\bar{t}Zg$ | $qg \to t\bar{t}Zq$



50 diagrams | 44 diagrams | 24 diagrams

- Use QGRAF [Nogueira, 1993], FORM [Vermaseren, 2000], and MAPLE to calculate amplitudes and generate FORTRAN or C++ code; use CUBA [Hahn, 2004] for numerical integration.
- Two-cutoff phase space slicing to isolate soft and collinear singularities [Harris, Owens, 2002]

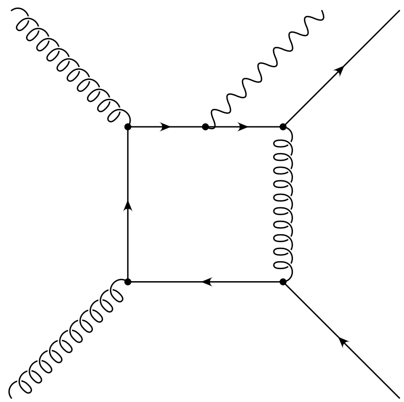
NLO: Renormalization



- Top mass and wavefunction renormalized on shell
- α_s renormalized using $\overline{\text{MS}}$ for 5 light quarks, zero-momentum subtraction for top

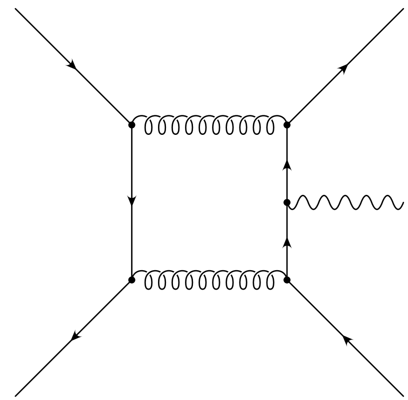
NLO: Virtual corrections

$gg \longrightarrow t\bar{t}Z$



160 diagrams

$q\bar{q} \longrightarrow t\bar{t}Z$



62 diagrams

- Soft and collinear singularities extracted using sector decomposition [Binoth, Heinrich, 2000]
- Threshold singularities avoided using contour deformation [Nagy, Soper, 2006; Lazopoulos, Melnikov, Petriello, 2007]

Sector decomposition

- Use $\delta(x_1 + \cdots + x_n - 1)$ to form n primary sectors.
- Singularities appear when some set $\{x_i, x_j, \ldots\}$ of Feynman parameters vanish.
- e.g. $\int_0^1 dx_1 \int_0^1 dx_2 \, (x_1 + x_2)^{-2-\epsilon}$
- Split integration region into sectors: $x_1 > x_2$ and $x_1 < x_2$
- Change variables in each sector: $x_2 \to x_1 x_2$, $x_1 \to x_1 x_2$
- Integral becomes $\int_0^1 dx_1 \int_0^1 dx_2 \left[x_1^{-1-\epsilon}(1 + x_2)^{-2-\epsilon} + x_2^{-1-\epsilon}(1 + x_1)^{-2-\epsilon} \right]$
- Extract singularities using plus distributions:

$$x^{-1-\epsilon} = -\frac{1}{\epsilon}\delta(x) + \sum_{n=0}^{\infty} \frac{(-\epsilon)^n}{n!} \left(\frac{\ln^n x}{x} \right)_+$$

Contour deformation

- After sector decomposition, we have a denominator $(\Delta - i0)^{-a-b\epsilon}$, where

$$\Delta = Z + Y_i x_i + \frac{1}{2} X_{ij} x_i x_j + \frac{1}{3} W_{ijk} x_i x_j x_k + \cdots$$

- Avoid threshold singularities by deforming the integration contour.
- Set $x_i = y_i - i\tau_i$, where

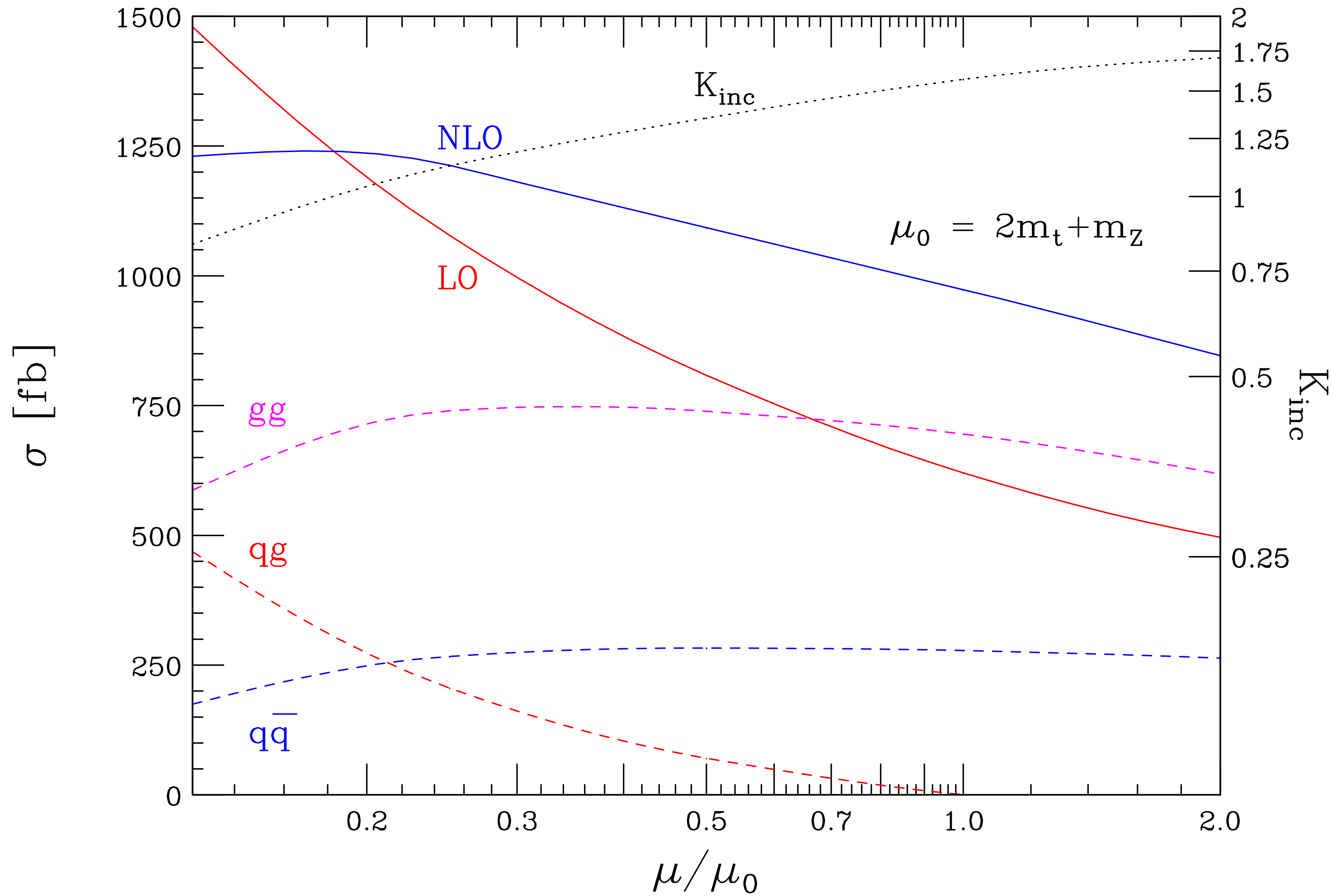
$$\tau_i = \lambda y_i (1 - y_i) \left[Y_i + X_{ij} y_j + W_{ijk} y_j y_k + \cdots \right]$$

- $\mathrm{Im}\,\Delta < 0$ for small λ; we can check that we cross no poles as we increase λ.

Checks

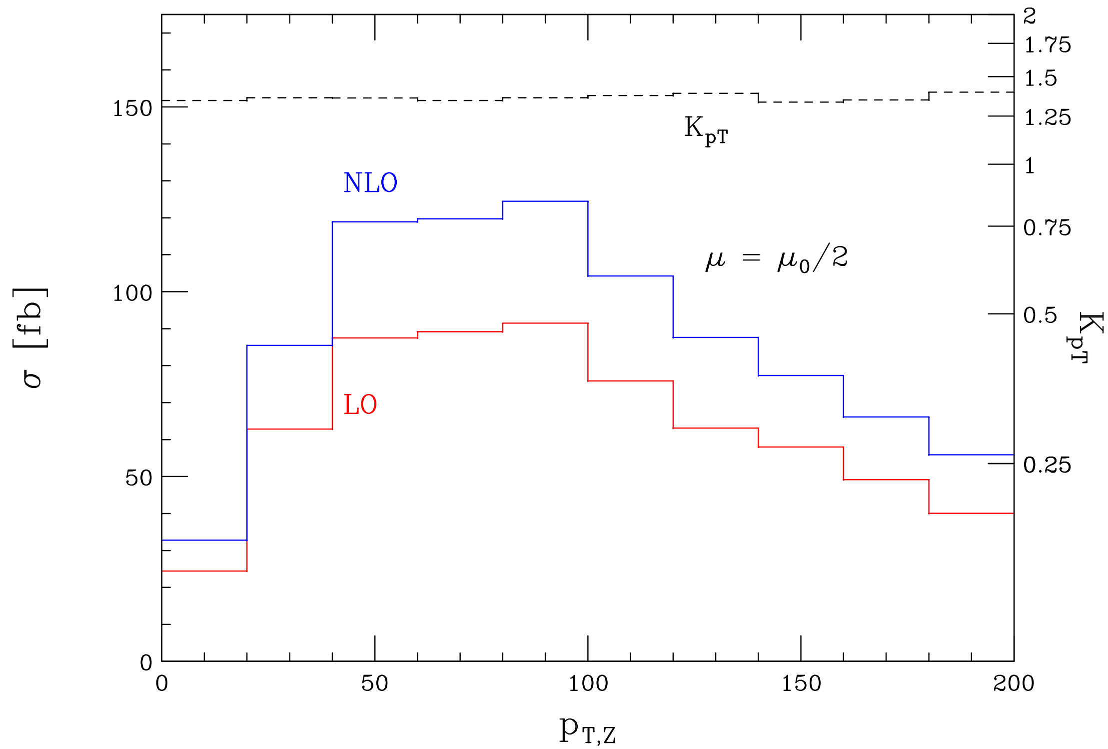
- LO result agrees with MADEVENT [Maltoni, Stelzer, 2003; Alwall *et al.*, 2007].
- Soft parts of virtual corrections agree with eikonal approximation.
- All poles in ϵ cancel.
- Virtual corrections independent of λ (magnitude of contour deformation)
- Real corrections independent of δ_s and δ_c (phase-space slicing cutoffs)
- Agreement between multiple independent codes

Results



- Vary scale from $\mu_0/4$ to μ_0: uncertainty is $\pm 11\%$.
- $K_{\text{inc}} = 1.35$ for $\mu = \mu_0/2$.

Results



- NLO corrections do not change the shape of the $p_T(Z)$ distribution.
- We expect the same for other kinematic distributions.

Summary

- We have computed the NLO QCD corrections to $pp \longrightarrow t\bar{t}Z$.
- Automated, fully numerical approach
  - $\Rightarrow$ arbitrary kinematic distributions
- $K = 1.35$ for $\mu = (2m_t + m_Z)/2$, independent of $p_T(Z)$
- Theoretical uncertainty reduced from $\sim$30% to $\pm$11%
  - $\Rightarrow$ improvement by factor of 1.5–2 in measurement of ttZ couplings
- Straightforward to extend to $t\bar{t}\gamma$, $t\bar{t}W$