NNLO QCD effects in $b \to X_c + l + \bar{\nu}_l$

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Introduction

- Study of CP violation in the b-quark sector is one of the most successful experimental endevours of the last decade.
- Two excellent experiments, BABAR at SLAC and BELLE at KEK.
- The most important outcome of these studies is that Cabbibo-Kobayashi-Maskawa (CKM) picture of CP violation is confirmed, at least as the “leading” approximation.
- By now, the paradigm has shifted – currently, the goal is to look for small $\sim 10\%$ deviations from the CKM picture and/or search for New Physics signals in rare decays of B-mesons.
- This means (once again!) precision B-physics and we know how hard it can be from precision EW. Most likely, the precision B-physics program will really take off with the advent of SuperB factory at KEK (start in 2012 ?).

Introduction

- If we want to learn something about New Physics from B-decays, we need to understand the Standard Model B-physics quite well.
- The Standard Model physics in B-decays is difficult since we are dealing with properties of a QCD bound state where one particle is heavy and others are light and relativistic.
- Nevertheless, it was realized early on that one can make use of the fact that the mass of the b-quark is large compared to $\Lambda_{\rm QCD}$, to separate perturbative and non-perturbative physics.
- This is done by employing effective field theories such as HQET and SCET and/or operator product expansion (OPE).

$B \to X_c + l + \bar{\nu}_l$

- The inclusive decay $B \to X_c + l + \bar{\nu}_l$ is one of the simplest processes in B-physics.
- Since the decay is inclusive, the OPE is applicable and a relatively simple expansion in $\Lambda_{\rm QCD}/m_b$ emerges. The number of non-perturbative operators that contribute is small.
- The rate for $B \to X_c + l + \bar{\nu}_l$ reads

$$\Gamma_{\rm sl} = \frac{G_F^2 m_b^5 |V_{\rm cb}|^2}{192\pi^3} \left[f_{\rm pert} \left(\frac{m_c}{m_b} \right) + c_1 \frac{\mu_\pi^2}{m_b^2} + c_2 \frac{\mu_G^2}{m_b^2} + ..\mathcal{O}(m_b^{-3}) \right] ,$$

  where the first term is perturbative and the other two are non-perturbative.
- $f_{\rm pert}$ can be computed as an expansion in α_s whereas the non-perturbative terms are small thanks to the large b-quark mass

$$\frac{\mu_\pi^2}{m_b^2} \sim \frac{\mu_G^2}{m_b^2} \sim \frac{\Lambda_{\rm QCD}^2}{m_b^2} \ll 1.$$

- Similar results can be derived for the decay rates with cuts, provided that cuts are not too stringent.

$B \to X_c + l + \bar{\nu}_l$

- An important caveat is the issue of the quark mass. Since

  $$\Gamma_{\rm sl} \sim m_b^5,$$

  the decay rate depends strongly on the value of m_b.
- It was realized that there is a strong correlation between the size of perturbative corrections to $\Gamma_{\rm sl}$ and the choice of scheme to define m_b. In particular, the pole mass – a choice that seems natural a'priori – leads to large perturbative corrections, that grow from order to order in the perturbative expansion.
- A commonly accepted solution is to change the definition of the mass parameter in such a way that large (universal) infra-red effects are absorbed into $m_b(\mu)$; this stabilizes perturbative expansion and allows for accurate predictions.
- Similar problems arise also for non-perturbative operators/matrix elements μ_π, μ_G; they also require self-consistent definitions.
- Many schemes exist to define m_b and other non-perturbative parameters; using one or the other is a matter of taste. Uraltsev,Beneke,Hoang

$B \to X_c + l + \bar{\nu}_l$

- Because accurate theoretical predictions are available, one can use $B \to X_c + l + \bar{\nu}_l$ in a variety of ways.
- Experimentally, one measures the total rate and various moments of the lepton energy and hadronic invariant mass. A host of accurate measurements of these observables exists thanks to CLEO, BELLE, BABAR and DELPHI.
- When all this information is combined, one can determine all the unknown parameters that enter $\Gamma_{\rm sl}$, such as V_{cb}, m_b, μ_π^2, μ_G^2 and α_s. This means that quite a lot can be learned from this observable!
- The precision with what these parameters are currently derived from the fits is very high. For example Bauer, Ligeti, Manohar, Trott

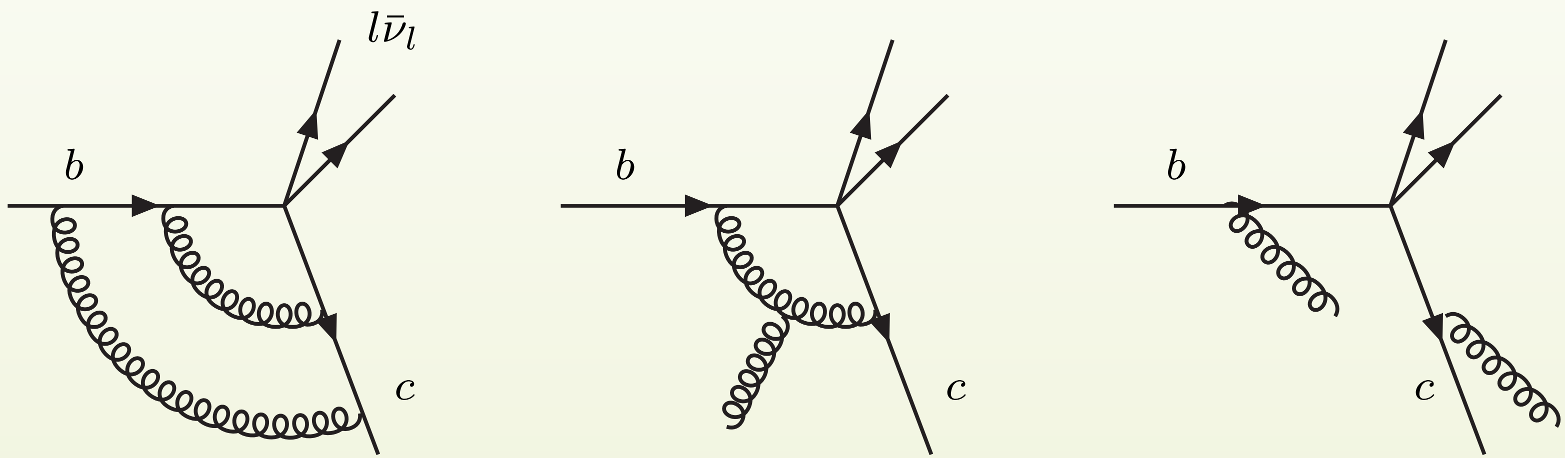
$$|V_{cb}| = (41.4 \pm 0.6 \pm 0.1_{\tau_B}) \times 10^{-3};$$
$$m_b^{1S} = 4.68 \pm 0.03 \text{ GeV};$$

$B \to X_c + l + \bar{\nu}_l$

- The natural question then is – do we indeed know everything we need to know about $\Gamma_{\rm sl}$ to analyze it with such a precision?
- The answer to this question is not very clear and, amazingly, the issue is related to perturbative corrections. Note that:
  - just a few years ago even $\mathcal{O}(\alpha_s)$ corrections to fully differential $\Gamma_{\rm sl}$ were not known;
    Trott; Gambino and Uraltsev
  - moreover, until very recently $\mathcal{O}(\alpha_s^2)$ corrections were not known beyond the BLM (large β_0) approximation;
  - in addition, $\mathcal{O}(\alpha_s)$ corrections to Wilson coefficient of the operator μ_π^2 were computed very recently and they are still not known for μ_G^2.
    Becher, Boos, Lunghi
- While all these effects may turn out to be small and not very important for the analysis, it is desirable to have them calculated and included into the fits.
- To be sufficiently realistic, the calculation of NNLO QCD corrections $b \to X_c + l + \nu_l$ should be fully differential. Luckily, the technique for performing such calculation exists.

$B \to X_c + l + \bar{\nu}_l$ at NNLO QCD

- The second order effects in $b \to X_c + l + \bar{\nu}_l$ naturally fall into three classes: two-loop virtual corrections, one-loop corrections to single gluon emission and double real emission corrections.



- The inclusive rate is a “one-scale” problem (m_c/m_b), but hard enough for analytic calculations. For fully-differential rate, the number of scales is infinite and analytic calculations are out of question.
- We perform the calculation numerically, employing the sector decomposition technique.

$B \to X_c + l + \bar{\nu}_l$ at NNLO QCD

- It is often assumed that the larger number of massive particles we have to deal with, the more complicated the problem is. While true for analytic calculations, it is not necessarily true for numerical ones.
- Indeed, for $b \to c$, the proximity of m_b and m_c means that the there are no large/small ratios of kinematic variables – this is great news for numerics.
- Moreover, $m_b \neq 0$, $m_c \neq 0$ is a very welcome fact for the NNLO computation since finite quark masses cut off collinear singularities.
- As a matter of fact, it is collinear singularities rather than soft that are difficult to disentangle when sector decomposition is used; the (near) absence of collinear singularities makes the calculation relatively straightforward.

Sector decomposition

- Sector decomposition is an easy-to-automate procedure that allows algorithmic extraction of infrared/collinear singularities from loop integrals and real emission processes. Binoth, Heinrich; Anastasiou, K.M., Petriello
- Consider the following integral

$$I = \int_0^1 \mathrm{d}x\mathrm{d}y \frac{x^{-\epsilon}}{(x+y)^2}.$$

Split the integration region into $x < y$ and $y < x$. In the first region, do $x \to xy$; in the second region $y \to xy$. Then,

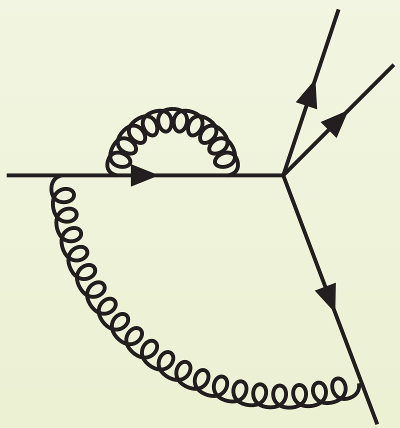
$$I = \int_0^1 \mathrm{d}x\mathrm{d}y \; \frac{x^{-\epsilon}y^{-\epsilon-1}}{(1+x)^2} + \int_0^1 \mathrm{d}x\mathrm{d}y \; \frac{x^{-\epsilon-1}}{(1+y)^2}.$$

- Extract singularities using the plus-distribution prescription

$$x^{n\epsilon-1} = \frac{1}{n\epsilon}\delta(x) + \sum_{i=0}^{\infty} \frac{(n\epsilon)^i}{i!} \left[\frac{\ln^i(x)}{x}\right]_+ .$$

Integrands for virtual corrections

- Two-loop virtual corrections – how to construct an integrand?
- When tensor integrals are present, I found it useful to construct an integrand over Feynman parameters by doing loop-after-loop integration over the virtual momenta.
- Such a procedure requires more analytic work but the advantage is that tensor integrals do not generate unphysical (power-like) singularities in Feynman parameters.
- However, self-energy insertions do generate unphysical singularities; to deal with those we have to use an integral representation for mass counter-terms to cancel power-like singularities point-by-point in the Feynman parameter space.



$$\Leftrightarrow \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\delta m}{(2bk)^2 k^2 (2ck)}$$

Imaginary parts

- Sector decomposition takes care of soft and collinear singularities but it doesn't regulate "threshold" singularities, responsible for the appearance of the imaginary parts of Feynman diagrams.
- In principle, the technique for dealing with those singularities is known; it requires to deform the integration contour into the complex plane.

  Soper; Lazopoulos, K.M., Petriello; Anastasiou, Bierle, Daleo
- In $b \to X_c l \nu_l$, the imaginary part issue appears only for one-loop corrections to a single gluon emission; in this case there is a somewhat simpler way to deal with this problem. The reason is that for all one-loop integrals, one can find a particular Feynman parameter (x_1) such that the integrand depends linearly on it

$$I(0,1) = \int_0^1 \frac{\mathrm{d}x_1 x_1^{-\epsilon+n}}{(-a+bx_1+i0)^{1+\epsilon}}, \qquad n \geq -1, b > a > 0.$$

  We rewrite

$$I(0,1) = I(0,\infty) - I(1,\infty).$$

  The first integral is calculable analytically while the denominator of the integrand in the second one is sign-definite.

Checks on the result

- Numerical cancellation of singularities after including UV renormalization and considering IR-safe observables.
- The BLM corrections are known for the rate and (almost) arbitrary distribution; we find complete agreement. Gambino, Uraltsev
- We can check our result against known NNLO corrections to $b \to c$ at zero recoil. Czarnecki; K.M., Czarnecki; Tausk, Franzkowski
- By considering $m_c \ll m_b$, we can compare our result with NNLO QCD corrections to $B \to X_u l \bar{\nu}_l$. van Ritbergen; Seidensticker, Steinhauser
- A direct check on our calculation is provided by the analytic result for the full rate $B \to X_c l \bar{\nu}_l$ and the lepton moments with no cut on the lepton energy (expansion in m_c/m_b).

  Czarnecki, Pak

Results

- Let us now discuss some results. We define moments of leptonic (hadronic) energy, as a function of the cut on the lepton energy

$$M_n(E_{\rm cut}) = \frac{\langle (E_M/m_b)^n \; \theta(E_l - E_{\rm cut}) \; {\rm d}\Gamma \rangle}{\langle {\rm d}\Gamma_0 \rangle}, \quad M = L, H.$$

- Moments can be written as series in the QCD coupling constant

$$M_n = M_n^{(0)} + \left(\frac{\alpha_s(m_b)}{\pi} \right) M_n^{(1)} + \left(\frac{\alpha_s(m_b)}{\pi} \right)^2 \left(\beta_0 M_n^{(2,{\rm BLM})} + M_n^{(2)} \right) + \dots$$

- I have considered a few moments and the values of $E_{\rm cut} = 0 \text{ and } 1 \text{ GeV}$. I typically find that

$$\frac{\beta_0 M_n^{(2,{\rm BLM})} + M_n^{(2)}}{\beta_0 M_n^{(2,{\rm BLM})}} \approx 0.8, \qquad M = L, H,$$

independent of n and $E_{\rm cut}$.

Results

- I can summarize the calculation by saying that the non-BLM corrections change the rate by $1.5 - 2\%$ and do not affect shapes of distributions significantly.
- The latter should not be considered as a rock-solid statement; it derives from my study of limited number of leptonic and hadronic energy moments.
- If we take the computed corrections at face value, the 2% correction to the decay rate implies a 1% change in V_{cb} – a shift, comparable to the stated uncertainty.
- However, most likely, this is incorrect. There are two reasons for that. For example, estimates of non-BLM $\mathcal{O}(\alpha_s^2)$ are already included in some fits. In addition, there might be cancellations between non-BLM corrections in low-scale b-quark mass and the non-BLM corrections to the rate discussed here.
- To get a clear picture, one needs to include the second order QCD corrections to the fits and re-fit the experimental data consistently.
- Further possibilities include – additional refinements of QCD perturbative corrections in top decays, fully differential calculations for $b \to X_u l \bar{\nu}_l$ and $b \to X_c \tau \bar{\nu}_\tau$.