The six-photon amplitudes

BERNICOT Christophe
in collaboration with GUILLET Jean-Phillipe

based on JHEP **01** (2008) 059

16/05/2008 - University at Buffalo Loopfest VII

Motivations.

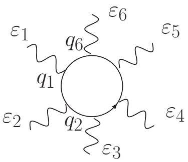
THE LHC

The Six-Photon Amplitudes.

NLO calculation: two difficulties

The Six-Photon Amplitudes.

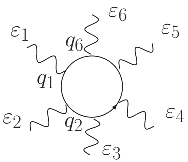
NLO calculation: two difficulties



$$tr\left(\varepsilon_1\gamma_{\mu_1}...\varepsilon_6\gamma_{\mu_6}\right) \quad \int d^n q \frac{q_1^{\mu_1}...q_6^{\mu_6}}{(q_1^2+i\lambda)...(q_6^2+i\lambda)}$$

The Six-Photon Amplitudes.

NLO calculation: two difficulties



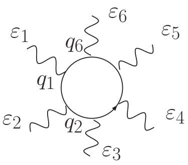
$$tr\left(\varepsilon_1\gamma_{\mu_1}...\varepsilon_6\gamma_{\mu_6}\right) \quad \int d^n q \frac{q_1^{\mu_1}...q_6^{\mu_6}}{(q_1^2+i\lambda)...(q_6^2+i\lambda)}$$

HOW TO SIMPLIFY ???

HOW TO REDUCE ???

THE SIX-PHOTON AMPLITUDES.

NLO calculation: two difficulties



$tr\,(\varepsilon_1\gamma_{\mu_1}...\varepsilon_6\gamma_{\mu_6})$ $\quad \int d^n q \dfrac{q_1^{\mu_1}...q_6^{\mu_6}}{(q_1^2+i\lambda)\,...(q_6^2+i\lambda)}$

HOW TO SIMPLIFY ???

HOW TO REDUCE ???

COMPACT NOTATION ⇒ HELICITY AMPLITUDES METHODS

REDUCE INTEGRALS ⇒ UNITARITY-CUTS

see talk : Bern, Britto, Forde, Giele, Kilgore, Zanderighi.

Computation of the Six-photon Amplitudes.

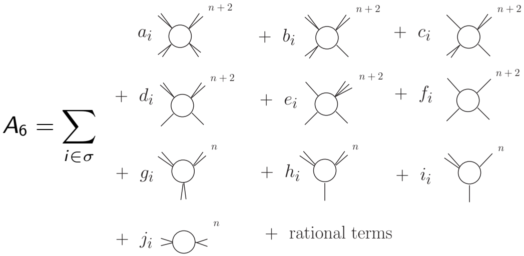
G.Mahlon [arXiv:hep-ph/9311213]
Z.Nagy,D.E.Soper [arXiv:hep-ph/0610028]
T.Binoth,T.Gehrmann, G.Heinrich,P.Mastrolia [arXiv:hep-ph/0703311]
G.Ossola, C.G.Papadopoulos,R.Pittau [arXiv:0704.1271[hep-ph]]

$$\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_6 \rightarrow \mathbf{0}$$

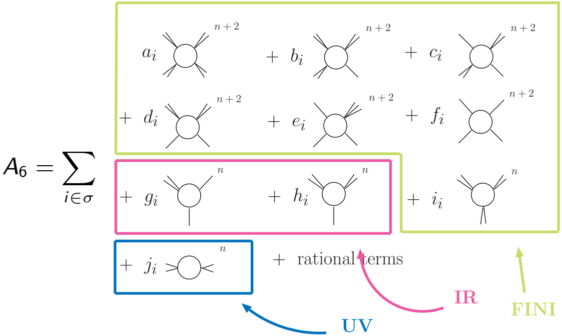
- C.Bernicot, J.Ph.Guillet [arXiv:hep-ph/0711.4713]
- Les Houches 2007: The NLO multileg working group: summary report [arXiv:hep-ph/0803.0494]

$$3\ \text{QED} : \begin{cases} \text{QED} \\ \text{scalar QED} \\ \text{QED}^{\mathcal{N}=1} \end{cases}$$

Reduction of the amplitude...



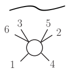
REDUCTION OF THE AMPLITUDE...

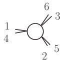


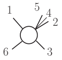
- Separate the UV/IF/finite structures
- Eliminate Gram determinant problems.

$\Rightarrow$ **IMPROVE UNTARITY METHODS TO THIS BASE**

Results for $A^{scalar/fermion}_{---+++}$.

$$A^{scalar/fermion}_{---+++} = \sum_{\sigma(1,2,3)} \sum_{\sigma(4,5,6)} \Bigg(d^{scalar/fermion} \Bigg($$


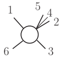
$$\Bigg)^{n+2} + \frac{g^{scalar/fermion}}{12} \Bigg($$


$$\Bigg)^{n} + \frac{e^{scalar/fermion}}{4} \Bigg($$


$$\Bigg)^{n+2} + h.c. \Bigg)$$

$$d^{scalar} = -\frac{i\ e^6}{\pi^2} \frac{\langle 24\rangle[16]}{\langle 45\rangle[31]} \frac{[1P_{425}2][6P_{425}4]}{[1P_{425}5][3P_{425}4]} \frac{\langle 1P_{425}4\rangle}{[1P_{425}4]}$$

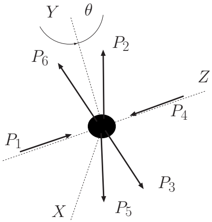
$$\langle ab\rangle = \langle a-|b+\rangle \qquad \langle abcd\rangle = \langle a-|b+\rangle\langle b+|c-\rangle\langle c-|d+\rangle$$



$$^{n+2} = \frac{1}{i\pi^{-n/2}} \int d^{n+2}Q \frac{1}{(Q)^2 (Q+p_1)^2 (Q+p_1+p_6)^2 (Q+p_1+p_6+p_3)^2}$$

A Plot of those Amplitudes ...

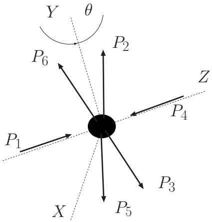
Nagy-Soper kinematical configuration (*Phys.Rev.* D74 (2006) 093006)



$$\left\{\begin{array}{l} \overrightarrow{p_2} = (-33.5, -15.9, -25.0) \\ \overrightarrow{p_3} = (11.0, 13.2, 22.0) \\ \overrightarrow{p_5} = (12.5, -15.3, -0.3) \\ \overrightarrow{p_6} = (10.0, 18.0, 3.3) \end{array}\right.$$

A Plot of those Amplitudes ...

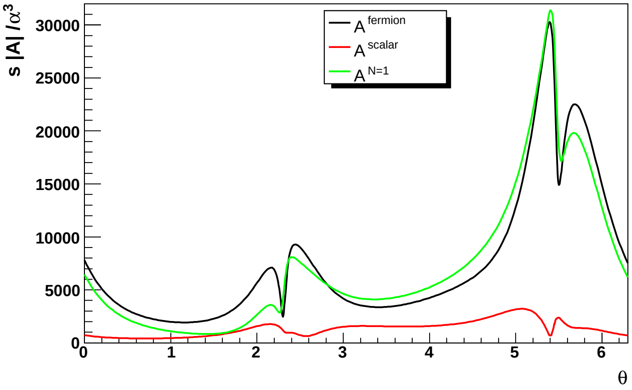
Nagy-Soper kinematical configuration (*Phys.Rev.* *D*74 (2006) 093006)



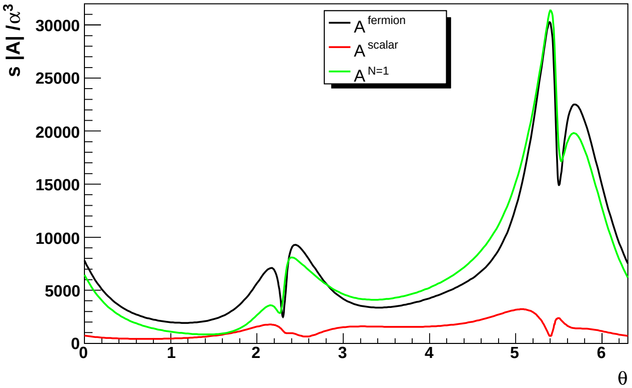
$$\left\{ \begin{array}{l} \overrightarrow{p_2} = (-33.5, -15.9, -25.0) \\ \overrightarrow{p_3} = (11.0, 13.2, 22.0) \\ \overrightarrow{p_5} = (12.5, -15.3, -0.3) \\ \overrightarrow{p_6} = (10.0, 18.0, 3.3) \end{array} \right.$$

rotation of the final state around the Y-axis

The $A^{scalar/fermion}_{---+++}$ amplitudes

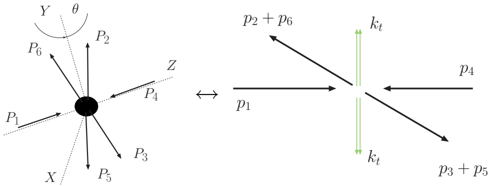


The $A^{scalar/fermion}_{---+++}$ amplitudes



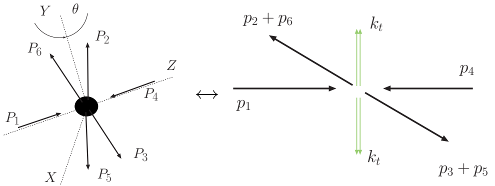
WHAT ARE THOSE DIPS ???

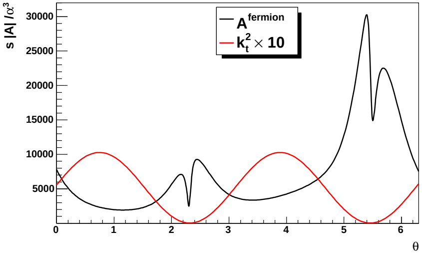
... What is it ???



Y
θ
P_2
P_6
Z
P_4
P_1
P_3
X
P_5
$\leftrightarrow$
$p_2 + p_6$
k_t
p_4
p_1
k_t
$p_3 + p_5$

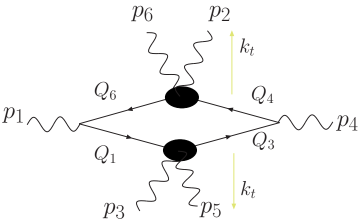
... WHAT IS IT ???





⇒ **DOUBLE PARTON SCATTERING**

… Double Parton Scattering …



DOUBLE PARTON SCATTERING:

$$\text{Configuration such as } \begin{cases} Q_1 \propto Q_6 \propto p_1 \\ Q_3 \propto Q_4 \propto p_4 \end{cases}$$

$$\Leftrightarrow k_t = 0$$

$\Rightarrow$ A LANDAU SINGULARITY !

The Landau Singularities.

- Loop amplitude:

$$A = \int d^n Q \frac{\mathrm{Num}\,(Q)}{D_1^2 \ldots D_N^2} = \Gamma(N) \int d^n Q dz_i \frac{\mathrm{Num}\,(Q)}{\left(\sum_{i=1}^{N} z_i D_i^2\right)^N}$$

The Landau Singularities.

- Loop amplitude:

$$A = \int d^n Q \frac{\mathrm{Num}\,(Q)}{D_1^2 \ldots D_N^2} = \Gamma(N) \int d^n Q dz_i \frac{\mathrm{Num}\,(Q)}{\left(\sum_{i=1}^{N} z_i D_i^2\right)^N}$$

- Definition of a Landau Singularity (1957):
  $=$ Kinematical configuration such as $\frac{\mathrm{Num}(Q)}{\left(\sum_{i=1}^{N} z_i D_i^2\right)^N}$ not analytic.

The Landau Singularities.

- Loop amplitude:

$$A = \int d^n Q \frac{\mathrm{Num}\,(Q)}{D_1^2 \ldots D_N^2} = \Gamma(N) \int d^n Q dz_i \frac{\mathrm{Num}\,(Q)}{\left(\sum_{i=1}^{N} z_i D_i^2\right)^N}$$

- Definition of a Landau Singularity (1957):
  = Kinematical configuration such as $\frac{\mathrm{Num}(Q)}{\left(\sum_{i=1}^{N} z_i D_i^2\right)^N}$ not analytic.
- Definition of a Divergence:
  = Kinematical configuration such as $A \rightarrow +\infty$.

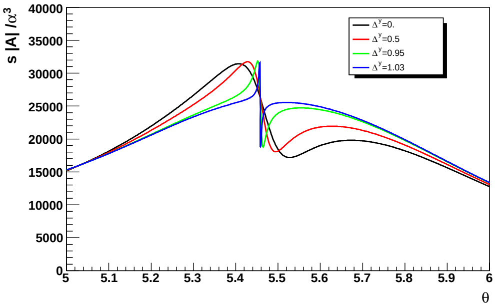
$\Rightarrow$ **In the case of 6-photon : DIVERGENCE OR NOT ?**

Divergence or Not ???

Change Nagy-Soper configuration

$$\begin{cases} \overrightarrow{p_2} = (-33.5, -15.9 - \Delta^y, -25.0) \\ \overrightarrow{p_3} = (11.0, 13.2 + \Delta^y, 22.0) \\ \overrightarrow{p_5} = (12.5, -15.3 + \Delta^y, -0.3) \\ \overrightarrow{p_6} = (10.0, 18.0 - \Delta^y, 3.3) \end{cases}$$

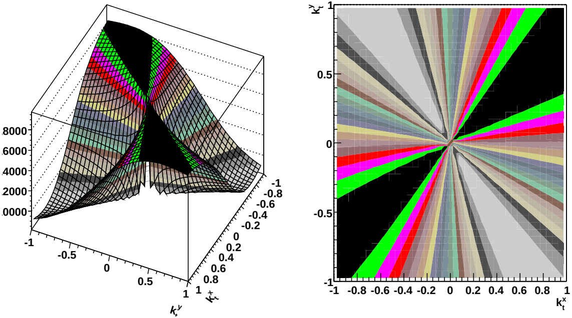
Singularity reached for $\Delta^y = 1,05$ GeV



⇒ **NO DIVERGENCE !**

Around the singularity ...

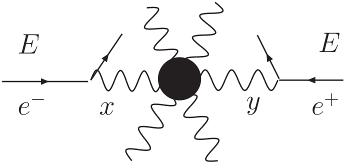
Plot the QED amplitude around the singularity



⇒ SADDLE POINT !

Cross Section... 1

Cross section of the physical process



$$E = 100 GeV$$

Structure fonction is a Weisacker-Williams.

CROSS SECTION... 2

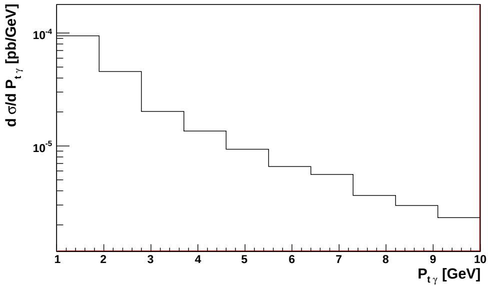
Integration with BASES and GOLEM.

$$\sigma \sim 4.1\ 10^{-7} pBarn \pm 0.2\% \Rightarrow \text{insignificant}$$

very stable result, no integration problem !!!

Differential Cross Section... 1

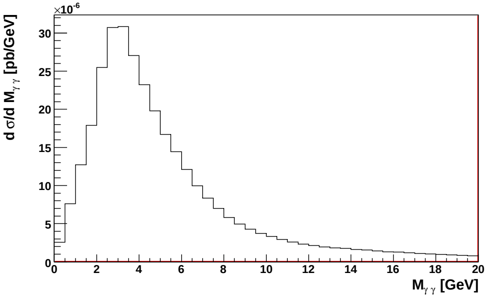
Differential cross section versus the p_t of one photon.



Divergence for $p_t \to 0$ correspond to Coulomb singularity.

Differential Cross Section... 2

Differential cross section versus the invariant mass of two photons.



$M_{\gamma\gamma} \to 0 \equiv$ 5-photon amplitudes.

Conclusion.

- $$\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_6 \longrightarrow 0$$
  $$A^{scalar}, A^{fermion}, A^{\mathcal{N}=1}$$

Conclusion.

- $$\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_6 \rightarrow 0$$
  $$A^{scalar}, A^{fermion}, A^{\mathcal{N}=1}$$

- Powerful method to reduce tensor integrals : unitarity-cuts.

Conclusion.

- $$\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_6 \to 0$$
  $$A^{scalar}, A^{fermion}, A^{\mathcal{N}=1}$$
- Powerful method to reduce tensor integrals : unitarity-cuts.
- No Infrared divergence
  Landau singularities don't create divergences in special case of the 6-photon amplitudes.

Conclusion.

- $$\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_6 \to 0$$
  $$A^{scalar}, A^{fermion}, A^{\mathcal{N}=1}$$

- Powerful method to reduce tensor integrals : unitarity-cuts.

- No Infrared divergence
  Landau singularities don't create divergences in special case of the 6-photon amplitudes.

- Calculation of differential cross sections.

Conclusion.

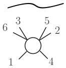
- $$\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_6 \longrightarrow 0$$
  $$A^{scalar}, A^{fermion}, A^{\mathcal{N}=1}$$

- Powerful method to reduce tensor integrals : unitarity-cuts.

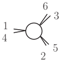
- No Infrared divergence
  Landau singularities don't create divergences in special case of the 6-photon amplitudes.

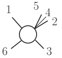
- Calculation of differential cross sections.

- Future prospect ...
  which we have begin, process with infrared divergences : $q\overline{q} + 4g \longrightarrow 0$

$$A^{scalar/spinor}_{---+++} = \frac{i\ e^6}{\pi^2} \sum_{\sigma(1,2,3)} \sum_{\sigma(4,5,6)} \Bigg(d^{scalar/spinor} \Bigg($$


$$\Bigg)^{n+2}$$

$$+ \frac{g^{scalar/spinor}}{12} \Bigg($$


$$\Bigg)^{n} + \frac{e^{scalar/spinor}}{4} \Bigg($$


$$\Bigg)^{n+2} + h.c. \Bigg)$$

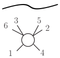
$$d^{scalar} = -\frac{\langle 24\rangle[16]}{\langle 45\rangle[31]} \frac{[1P_{425}2][6P_{425}4]}{[1P_{425}5][3P_{425}4]} \frac{\langle 1P_{425}4\rangle}{[1P_{425}4]}$$

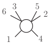
$$e^{scalar} = -\frac{\langle 2P_{425}1\rangle\langle 2P_{425}3\rangle[36][16]s_{425}}{\langle 4P_{425}1\rangle\langle 5P_{425}3\rangle\langle 5P_{425}1\rangle\langle 4P_{425}3\rangle} \frac{\langle 31\rangle}{[31]}$$

$$g^{scalar} = \frac{[4P_{25}1]}{[1P_{25}4]} \frac{[5P_{14}2]}{[2P_{14}5]} \frac{[6P_{25}3]}{[3P_{25}6]} \sum_{\gamma_\pm} \frac{[1K_2^b1]}{[4K_2^b4]} \frac{[2K_2^b2]}{[5K_2^b5]} \frac{[3K_2^b3]}{[6K_2^b6]}$$

$$K_2^{b\mu} = \gamma_\pm\,(-P_{25})^\mu - s_{25}\,(P_{14})^\mu$$

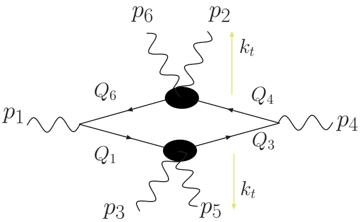
$$\gamma_\pm = -P_{25}.P_{14} \pm \sqrt{\Delta} \qquad \Delta = (P_{25}.P_{14})^2 - P_{14}^2 P_{25}^2$$



$$^{n+2} = \text{“Finite part”} \Bigg($$


$$\Bigg)^{n}$$

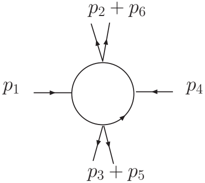
... Double Parton Scattering...



DOUBLE PARTON SCATTERING:

$$\text{Configuration such as } \begin{cases} Q_1 \propto Q_6 \propto p_1 \\ Q_3 \propto Q_4 \propto p_4 \end{cases} \Leftrightarrow \quad k_t = 0 \Leftrightarrow \det(S) = 0$$

$S_{ij} = (q_i - q_j)^2 - m_i^2 - m_j^2$ kinematical matrix of :



$$A = \sum_i a_i I^n_{4,i} + \sum_i b_i I^n_{3,i} + \sum_i c_i I^n_{2,i} + \text{rational terms}$$

But we have

$$I^n_4 = \sum_i \beta_i I^n_{3,i} - \frac{\det(G)}{\det(S)} I^{n+2}_4$$

So,

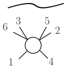
$$A = -\sum_i a_i \frac{\det(G)}{\det(S)} I^{n+2}_{4,i} + \sum_i (b_i + a_i\beta_i) I^n_{3,i} + \sum_i c_i I^n_{2,i} + \text{rational terms}$$

With the three cut technics it is very easy to calculate the coefficient $b_i + a_i\beta_i$.

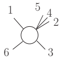
Results for $A^{\mathcal{N}=1}_{---+++}$

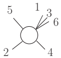
$$A^{\mathcal{N}=1}_{---+++} = \frac{i\ e^6}{\pi^2} \sum_{\sigma(1,2,3)} \sum_{\sigma(4,5,6)} d^{\mathcal{N}=1}$$

$$\Bigg(\langle 1P_{425}4P_{425}1\rangle \Big($$



$$\Big)^{n+2} + \frac{s_{13}\Big(\ \text{[diagram]}\ \Big)^{n+2} + s_{45}\Big(\ \text{[diagram]}\ \Big)^{n+2}}{2} \Bigg)$$





$$d^{\mathcal{N}=1} = \frac{[6P_{425}2]^2}{[31]\langle 45\rangle[1P_{425}5][3P_{425}4]}$$

NO TRIANGLE