

Three-loop corrections to the lightest Higgs Boson mass in the MSSM

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Higgs boson mass in the MSSM

- MSSM: 5 Higgs bosons: h, H, A, H^\pm
- prediction of M_h
- tree-level:

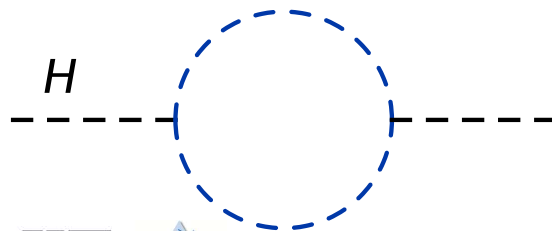
$$\mathcal{M}_{H,tree}^2 = \frac{\sin 2\beta}{2} \begin{pmatrix} M_Z^2 \cot \beta + M_A^2 \tan \beta & -M_Z^2 - M_A^2 \\ -M_Z^2 - M_A^2 & M_Z^2 \tan \beta + M_A^2 \cot \beta \end{pmatrix}$$

$$\Leftrightarrow M_h \leq M_Z$$

- quantum corrections:

$$\mathcal{M}_H^2 = \mathcal{M}_{H,tree}^2 + \begin{pmatrix} \hat{\Sigma}_{\phi_1} & \hat{\Sigma}_{\phi_1\phi_2} \\ \hat{\Sigma}_{\phi_1\phi_2} & \hat{\Sigma}_{\phi_2} \end{pmatrix}$$

squark



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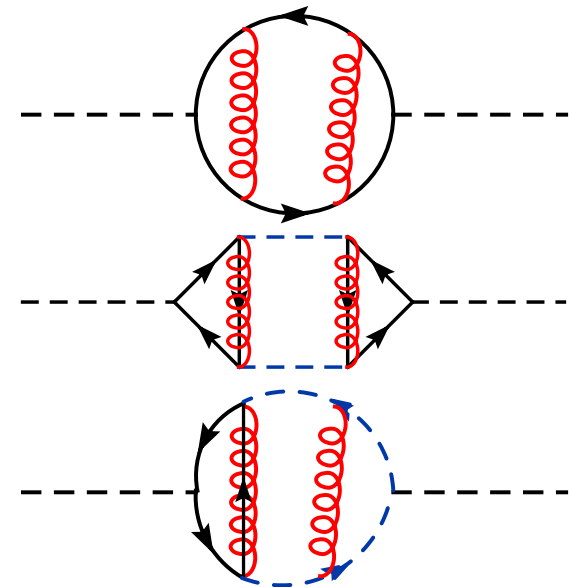
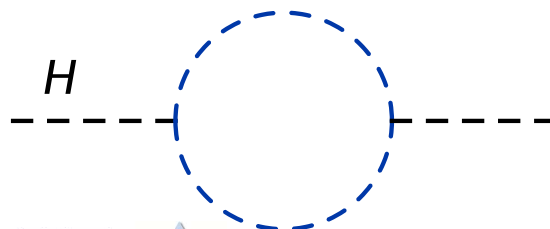
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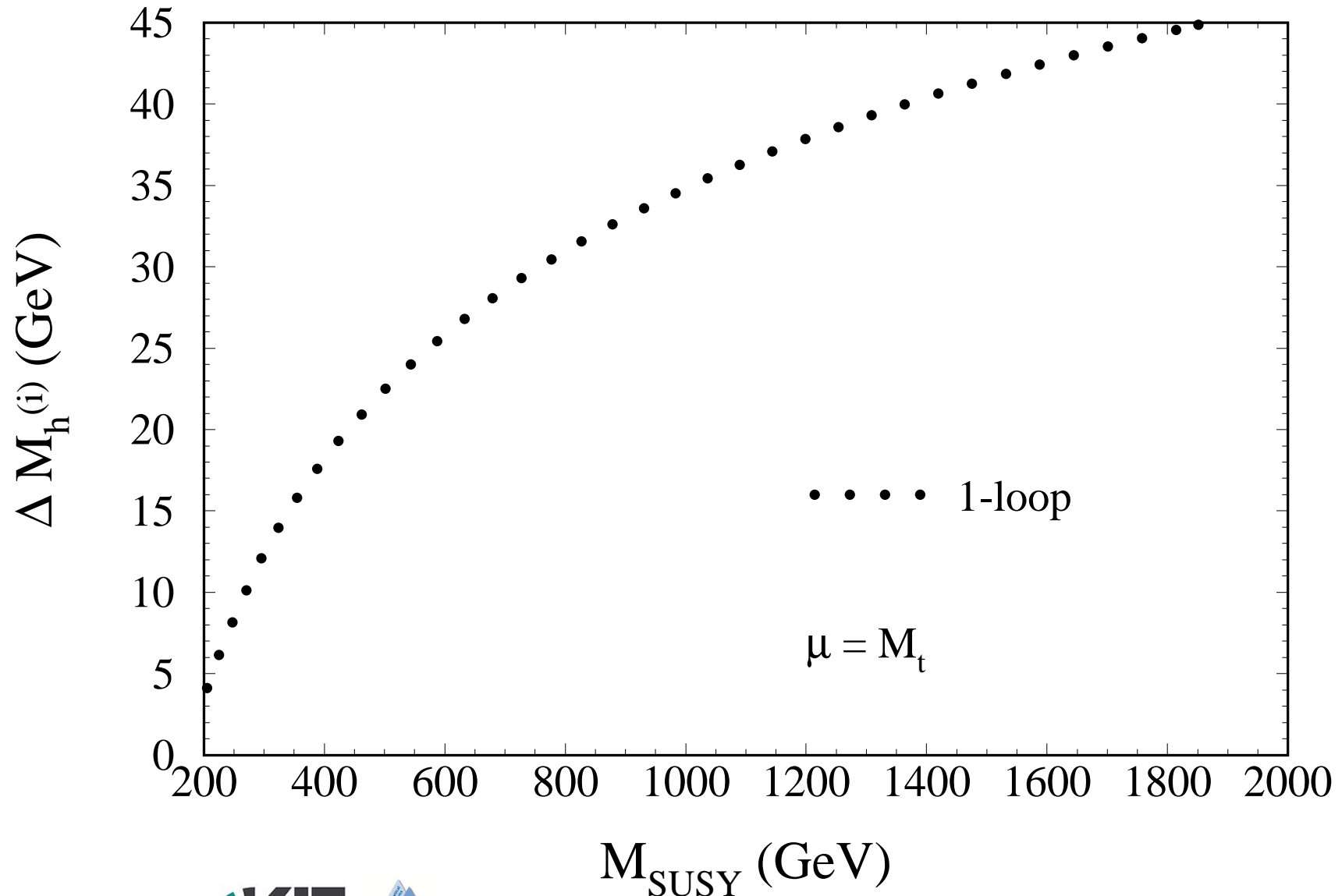
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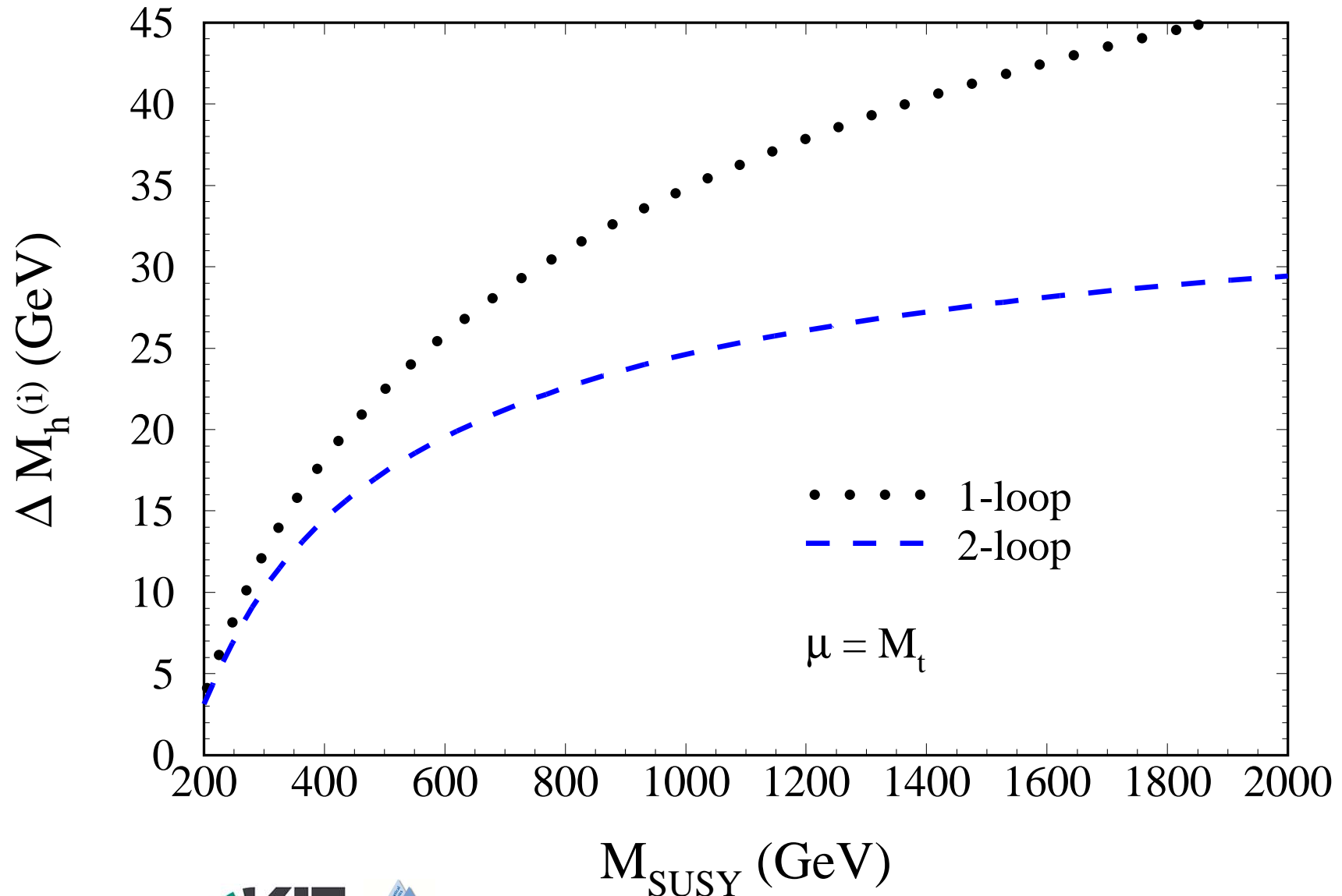
1- and 2-loop results

[Brignole,Carena,Casas,Dedes,Degrassi,Ellis,Espinosa,Haber,Hempfling,Heinemeyer,Hoang,Hollik,Martin,Quirós,Ridolfi,
Riotto,Rzehak,Slavich,Wagner,Weiglein,Zhang,Zwirner,... '94-'07]



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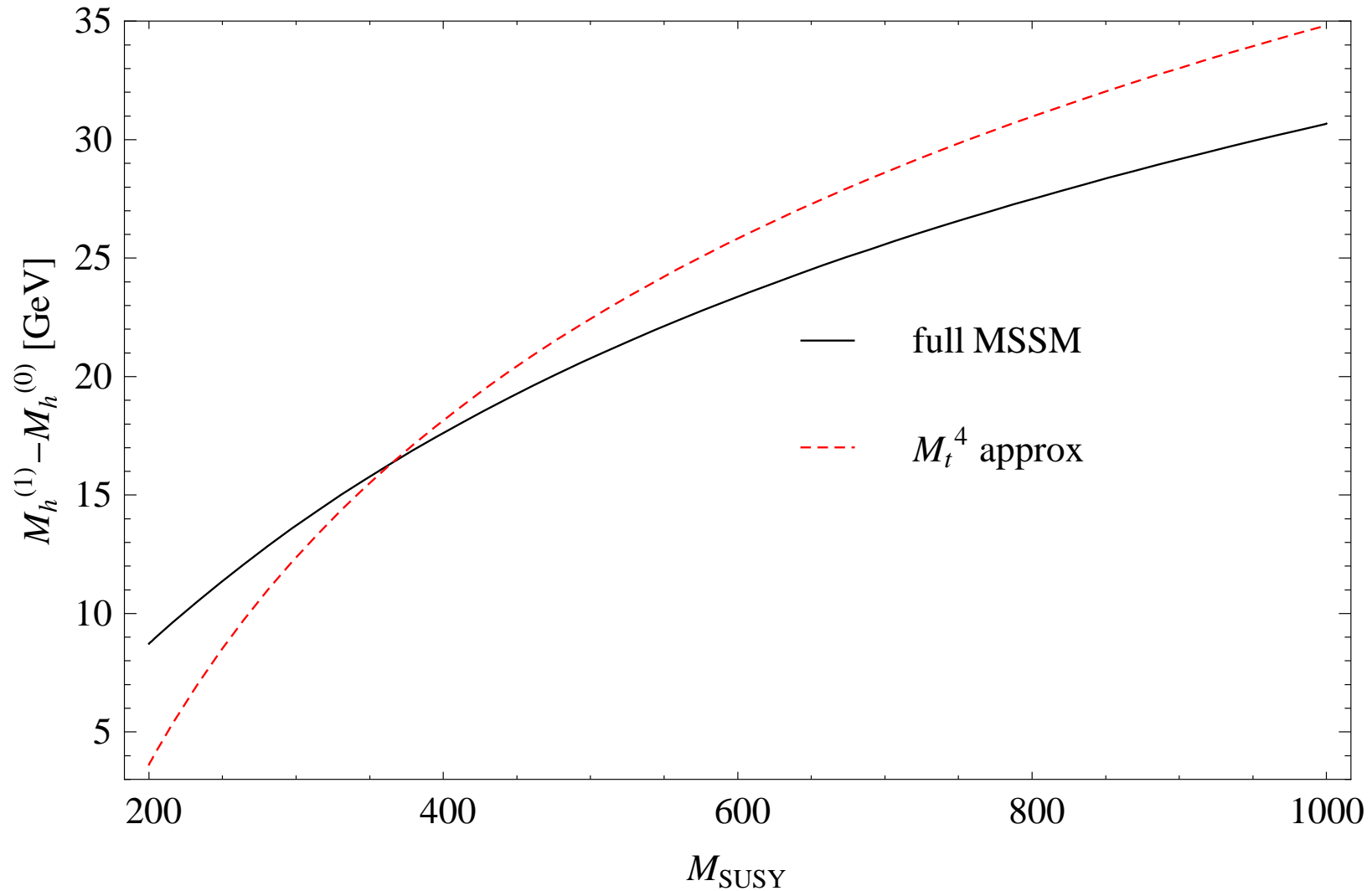


3-loop corrections to M_h

- remaining uncertainty: 3-5 GeV
- LHC: $\delta M_h \sim 100 - 200$ MeV
ILC: $\delta M_h \sim 50$ MeV
- Framework:
 - “ $q = 0$ ” $\Leftrightarrow \hat{\Sigma}(q = 0)$
 - SUSY-QCD: quarks, gluons \oplus squarks, gluinos
 - Leading contribution: $\sim G_F M_t^4 \alpha_s^2$

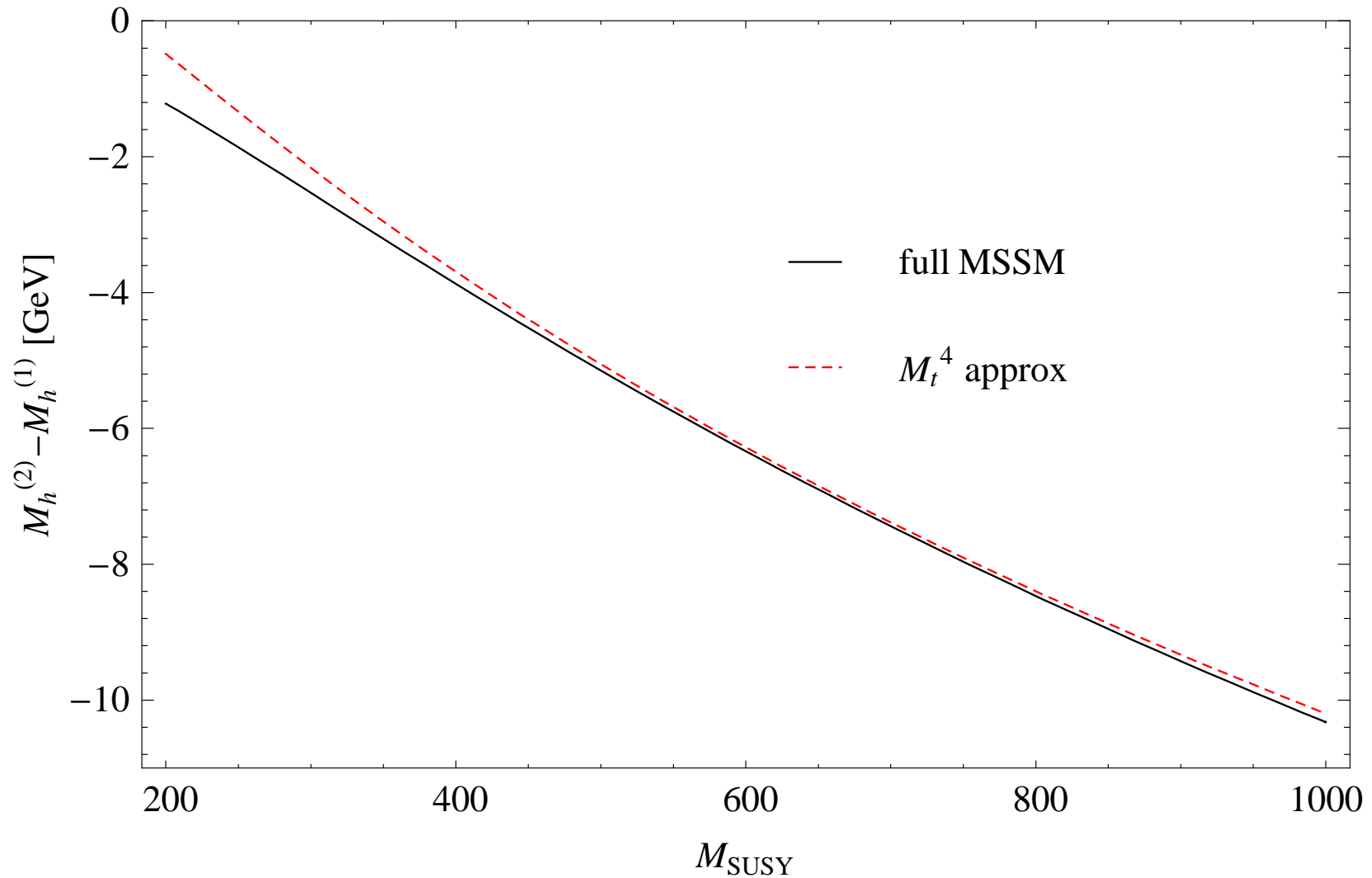
[Degrassi et al.'02; Allanach et al.'04]

FeynHiggs vs $G_F M_t^4$



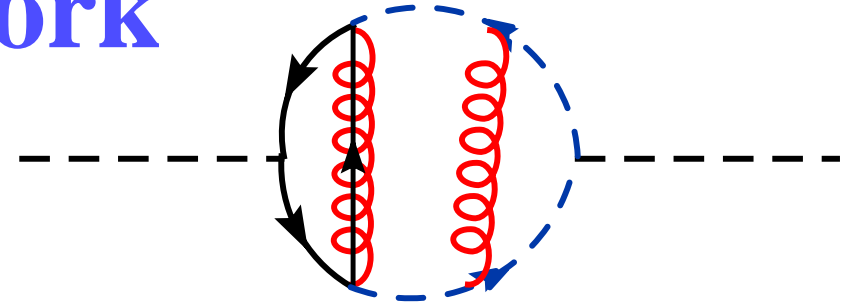
FeynHiggs: [Heinemeyer,Hollik,Weiglein,et al:'00...]

FeynHiggs vs $G_F M_t^4 \alpha_s$



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Framework



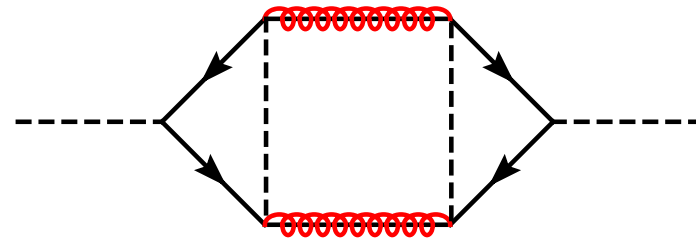
- 16,000 Feynman diagrams
- Dimensional Reduction (DRED) $\Leftrightarrow \epsilon$ scalars
- automated set-up

[Siegel'79]

(generation, asymptotic expansion, vacuum integrals):

qgraf, q2e/exp, MATAD

[Nogueira'91, Harlander, Seidelsticker, MS'97'99; MS'96-'00]



- Majorana character of gluino:
 \Leftrightarrow PERL program which implements prescription of

[Denner, Eck, Hahn, Kublbeck'92]

Renormalization

Renormalization (all CTs recomputed in our limit)

● α_s : 1-loop $\overline{\text{DR}}$

● $M_{\tilde{g}}$: 1-loop OS

[Pierce et al.'96]

● M_ϵ : 1-loop OS: $M_\epsilon = 0$

● $M_t, M_{\tilde{t}}$: 2-loop OS

[Bednyakov et al.'02; Martin'03'05]

● $\Theta_{\tilde{t}}$:

● use: $2M_t A_t = (M_{\tilde{t},1}^2 - M_{\tilde{t},2}^2) \sin 2\theta_t + 2M_t \mu_{\text{SUSY}} \cot \beta$

● $\delta\Theta_{\tilde{t}}^{1\text{-loop}} \sim 1/(M_{\tilde{t},1}^2 - M_{\tilde{t},2}^2)$

● $\Theta_{\tilde{t}} \rightarrow 0$

Checks

- exact 2-loop result: agreement with [Degrassi, Slavich, Zwirner'01]
- independence of ξ_S $\left[D_g = \frac{i}{q^2} \left(g^{\mu\nu} + \xi_S \frac{q^\mu q^\nu}{q^2} \right) \right]$
- SUSY limit: $M_t = M_{\tilde{t}}; M_{\tilde{g}} = 0 \Leftrightarrow \Delta M_h = 0$
- simple limit: $M_t \neq M_{\tilde{t}}; M_{\tilde{g}} = 0 \Leftrightarrow$

$$\Delta M_h = \frac{3G_F M_t^4}{\sqrt{2}\pi^2} \left\{ L_{tS} + \frac{\alpha_s}{\pi} [-1 - 4L_{tS} + 2L_{tS}^2] + \left(\frac{\alpha_s}{\pi} \right)^2 \left[-\frac{593}{27} - \frac{3}{4}L_{\mu t} \right. \right. \\ \left. \left. + \frac{23}{81}\pi^2 + \frac{401}{18}\zeta_3 + \left(-\frac{47}{4} - 3L_{\mu t} + \frac{4}{9}\pi^2 - \frac{4}{9}\pi^2 \ln 2 \right) L_{tS} \right. \right. \\ \left. \left. + \left(-\frac{1}{12} + \frac{3}{2}L_{\mu t} \right) L_{tS}^2 + \frac{5}{2}L_{tS}^3 \right] \right\}$$

$$L_{\mu t} = \ln(\mu^2/M_t^2), \quad L_{tS} = \ln(M_t^2/M_{\text{SUSY}}^2)$$

3-loop result: $M_t \ll M_{\text{SUSY}} \ll M_{\tilde{q}}$

$$M_{\tilde{t},1} = M_{\tilde{t},2} = M_{\tilde{g}} = M_{\text{SUSY}}$$

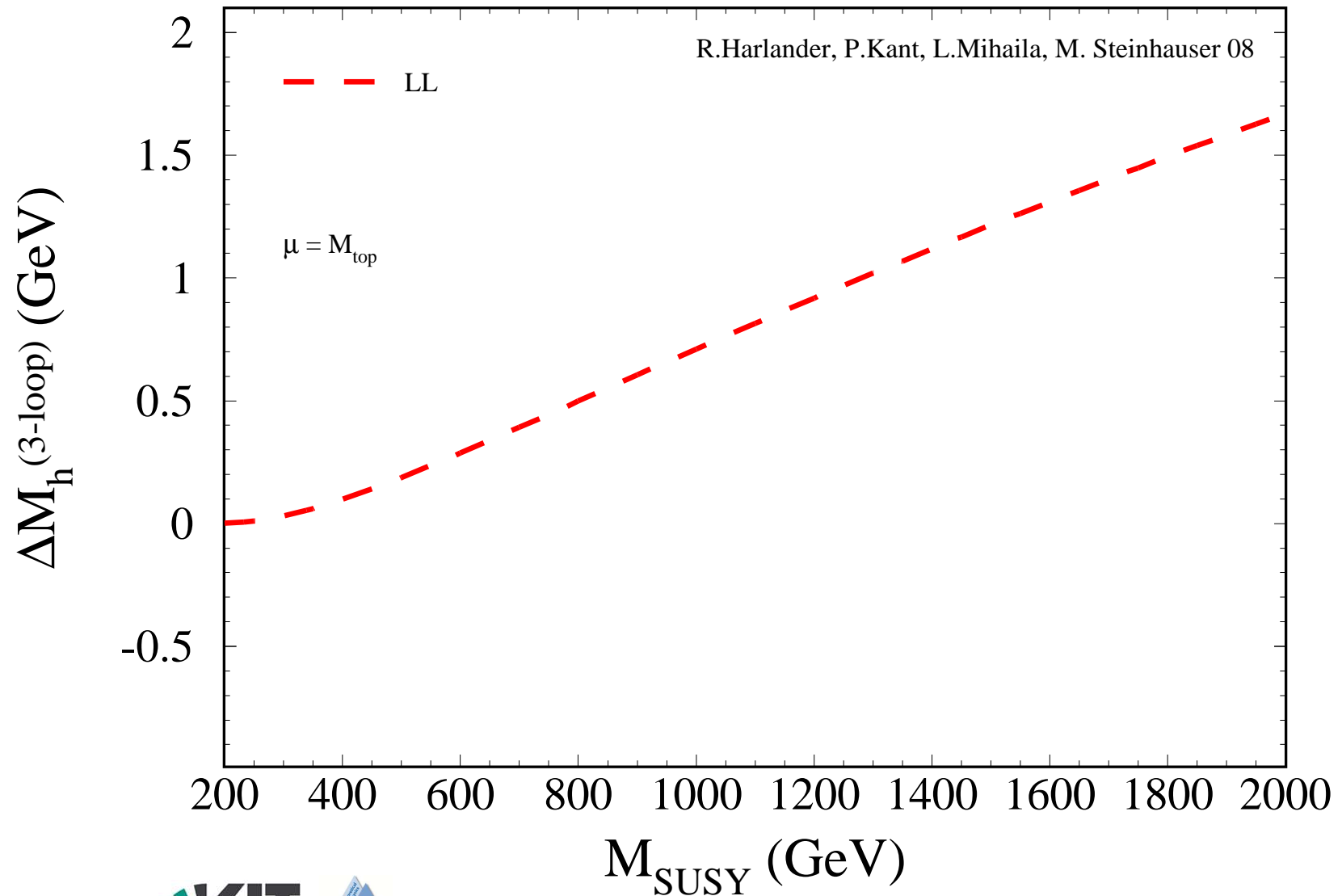
$$\begin{aligned} \Delta M_h = & \frac{3G_F M_t^4}{\sqrt{2}\pi^2} \left\{ L_{tS} + \frac{\alpha_s}{\pi} [-4L_{tS} + 2L_{tS}^2] \right. \\ & + \left(\frac{\alpha_s}{\pi} \right)^2 \left[\frac{671}{324} + \frac{1}{27}\pi^2 + \frac{1}{9}\zeta_3 + \left(-\frac{1591}{108} \right. \right. \\ & + \left. \left. \frac{1}{3}\pi^2 - \frac{4}{9}\pi^2 \ln 2 + \frac{55}{18}L_{t\tilde{q}} + \frac{5}{6}L_{t\tilde{q}}^2 \right) L_{tS} \right. \\ & + \left(\frac{13}{18} - \frac{5}{3}L_{t\tilde{q}} \right) L_{tS}^2 + \frac{53}{18}L_{tS}^3 \\ & \left. + \left(\frac{475}{108} - \frac{5}{9}\pi^2 \right) L_{t\tilde{q}} - \frac{25}{36}L_{t\tilde{q}}^2 - \frac{5}{18}L_{t\tilde{q}}^3 + \dots \right] \left. \right\} \end{aligned}$$

$$L_{tS} = \ln \frac{M_t^2}{M_{\text{SUSY}}^2}, \quad L_{t\tilde{q}} = \ln \frac{M_t^2}{M_{\tilde{q}}^2},$$

[Harlander,Kant,Mihaila,MS'08]

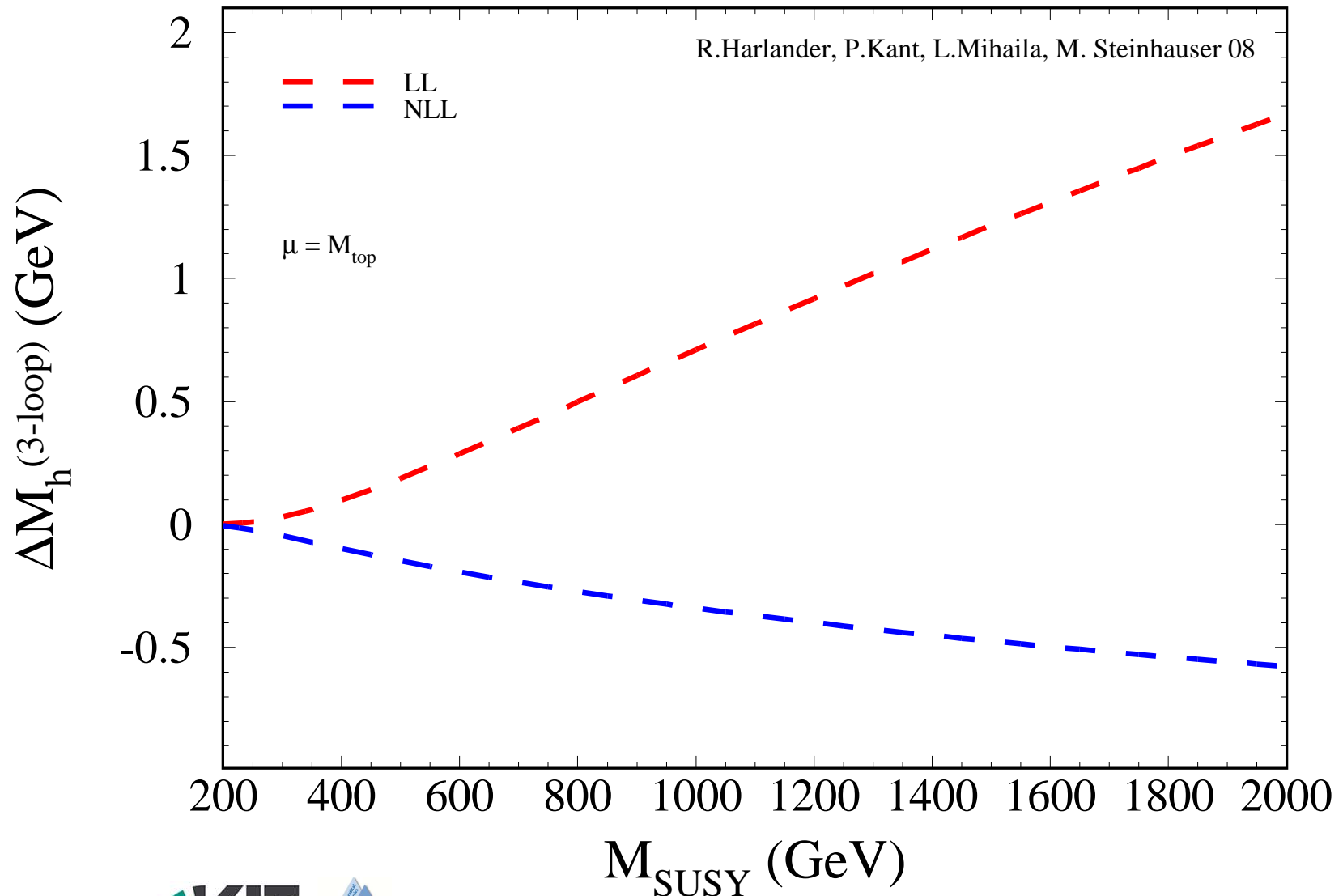
Logarithmic expansion

LL: agreement with [Martin'07]

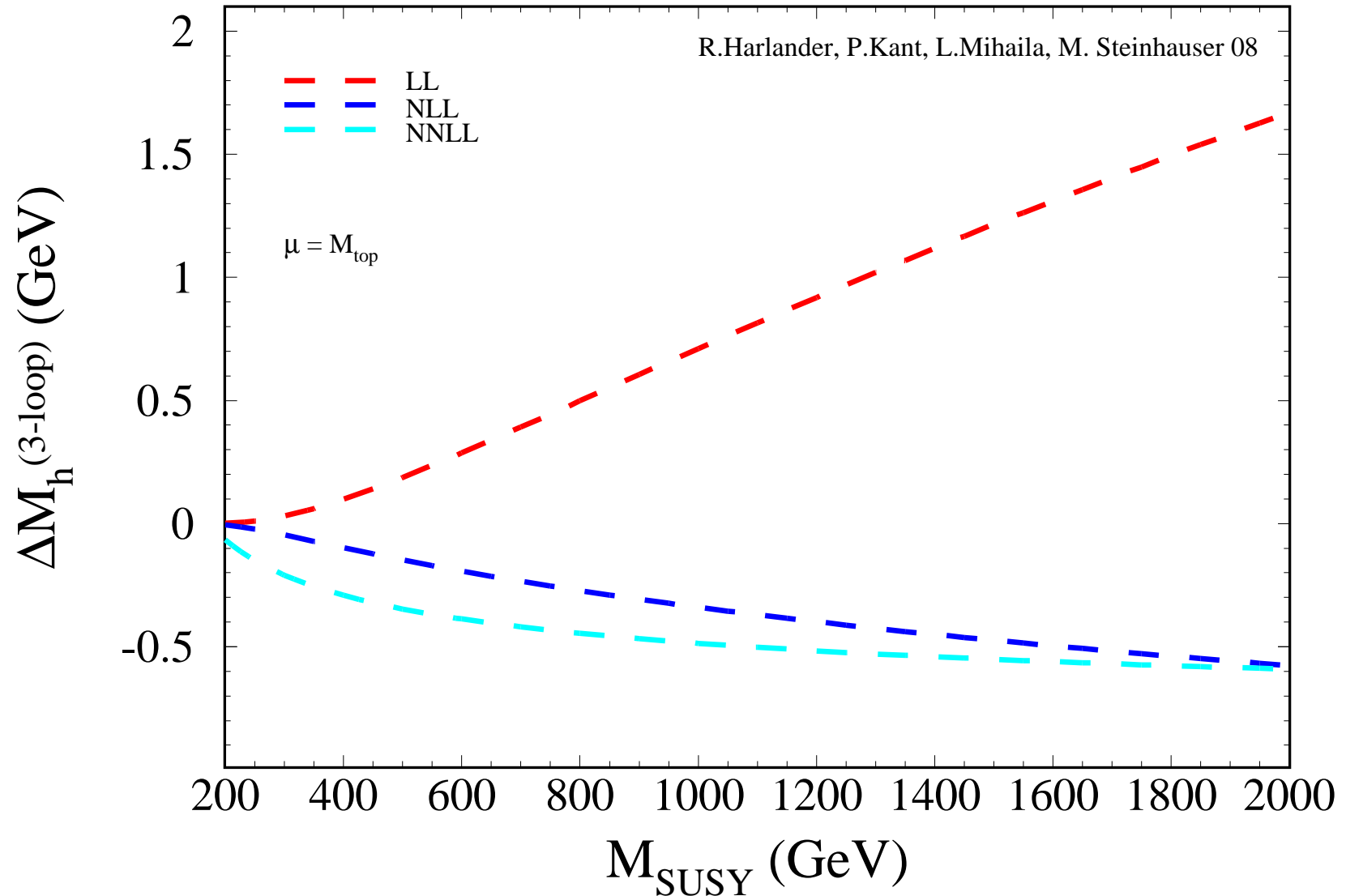


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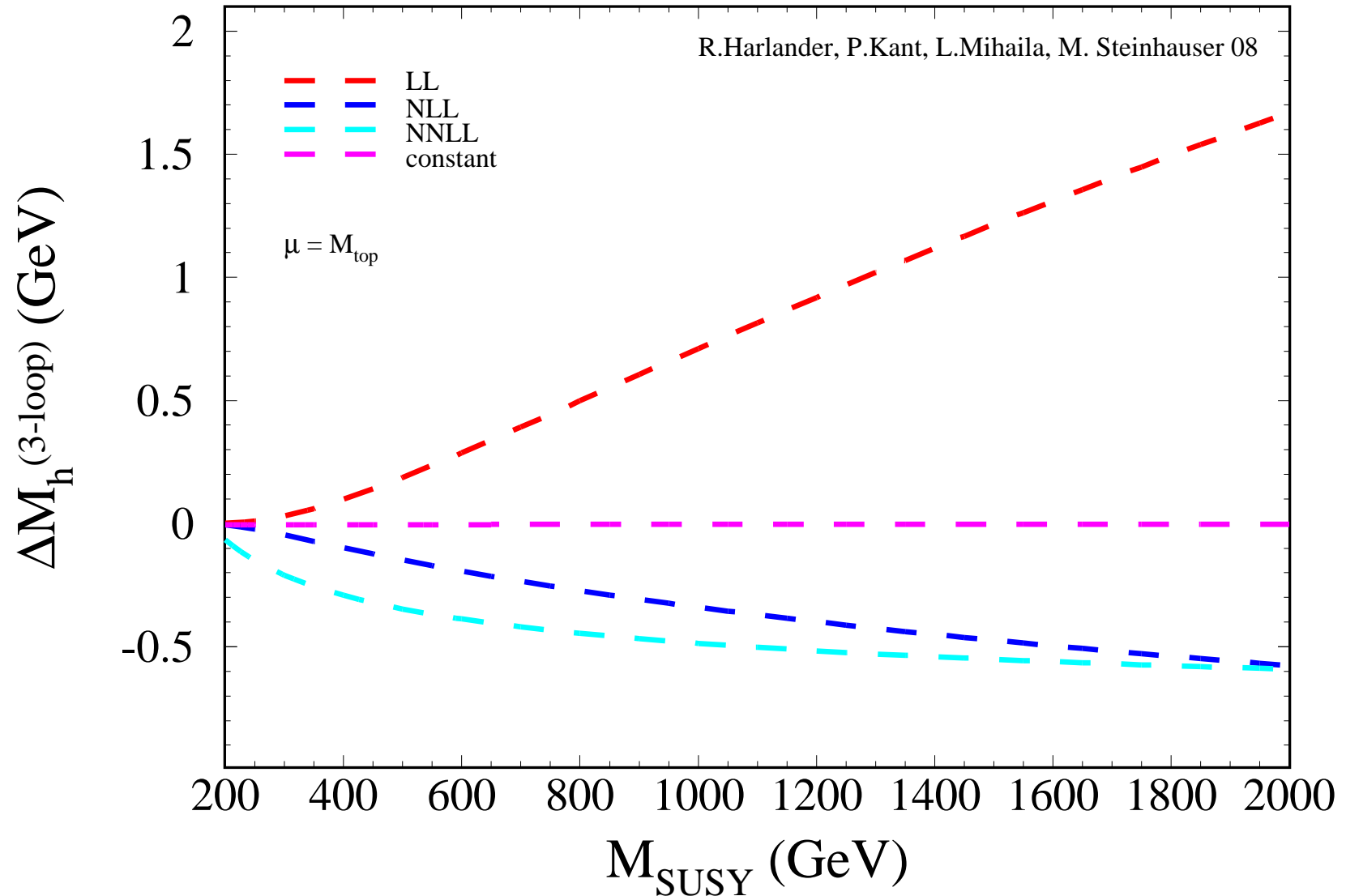
NLL: no agreement with [Martin'07]



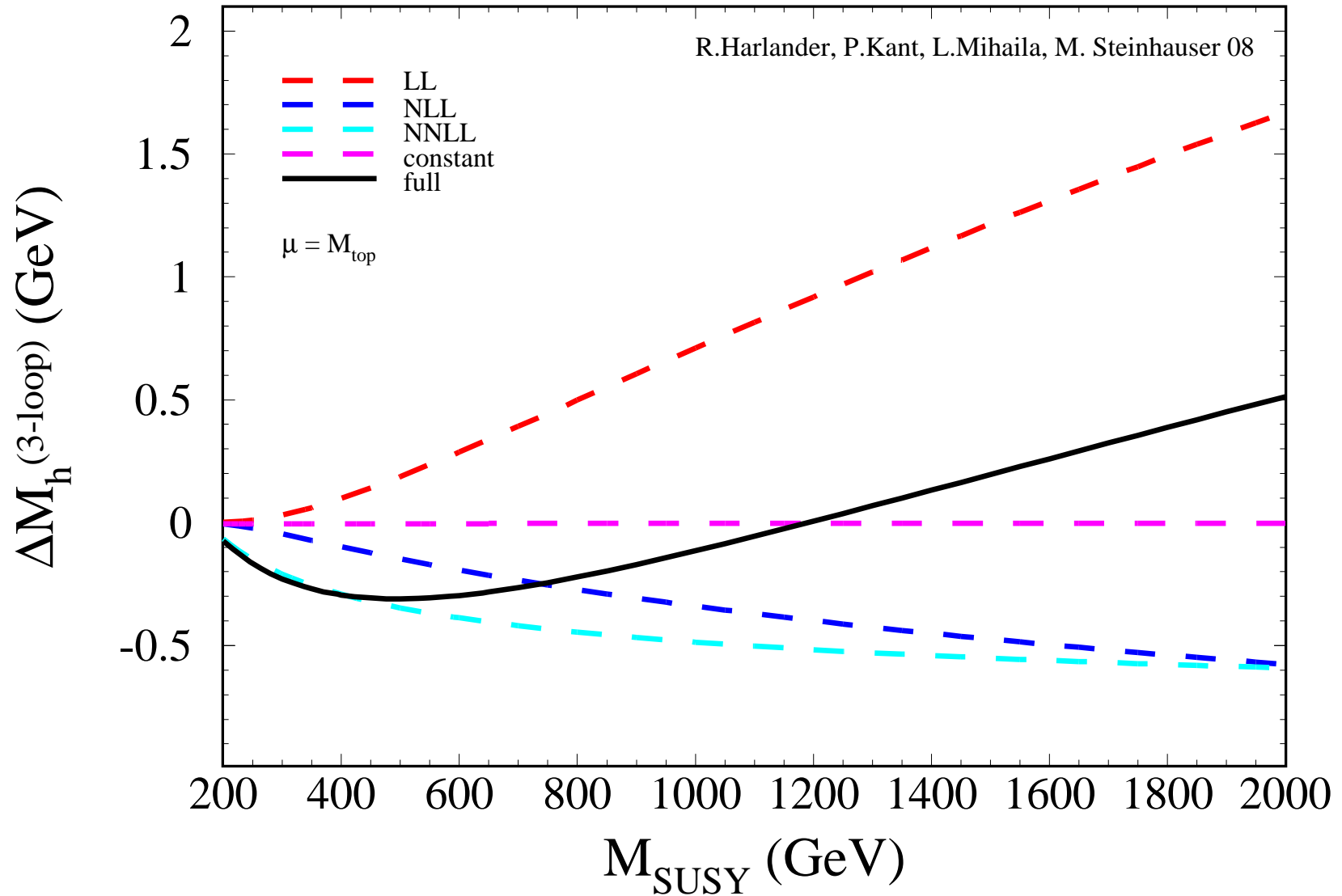
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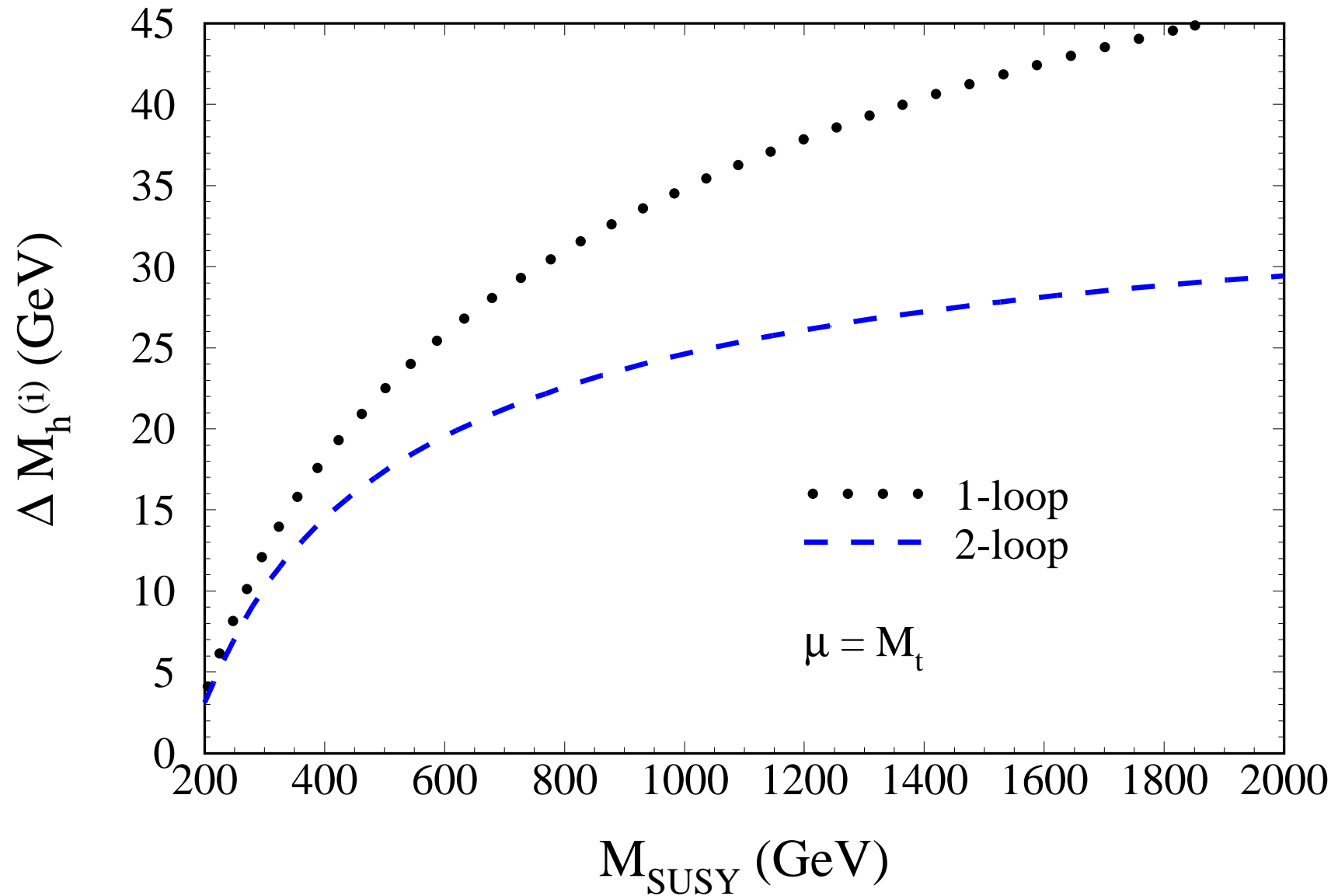
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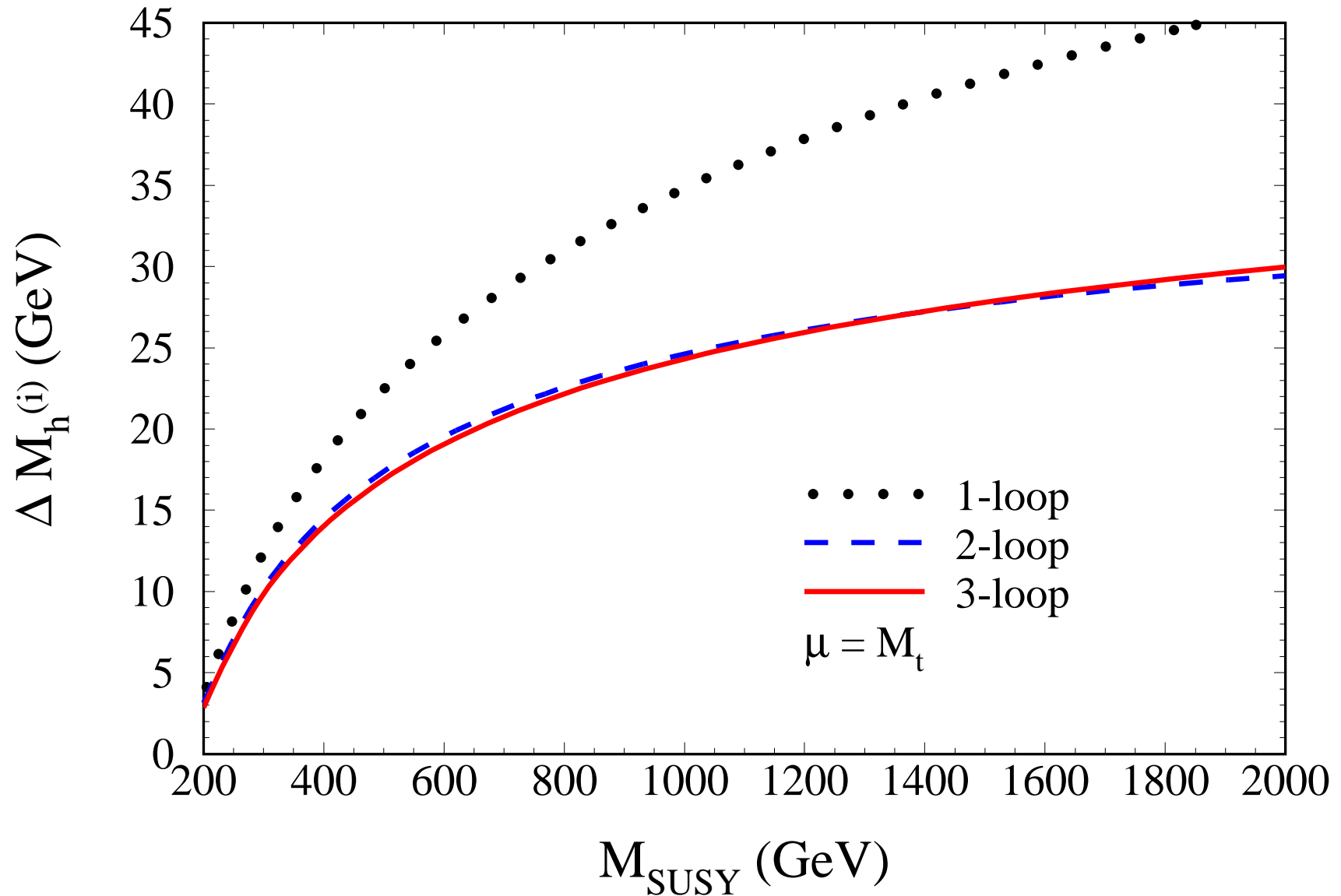


3-loop corrections to M_h



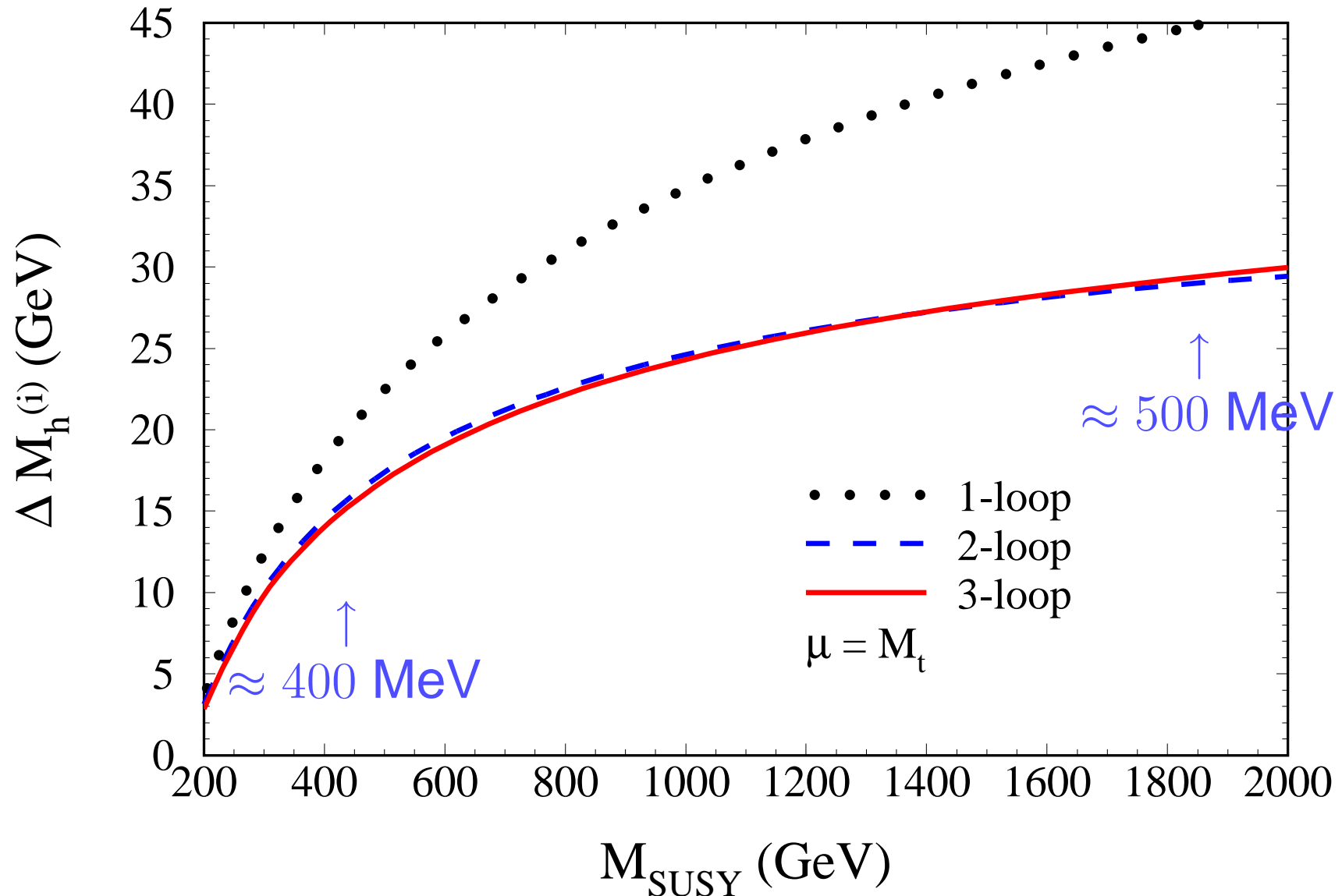
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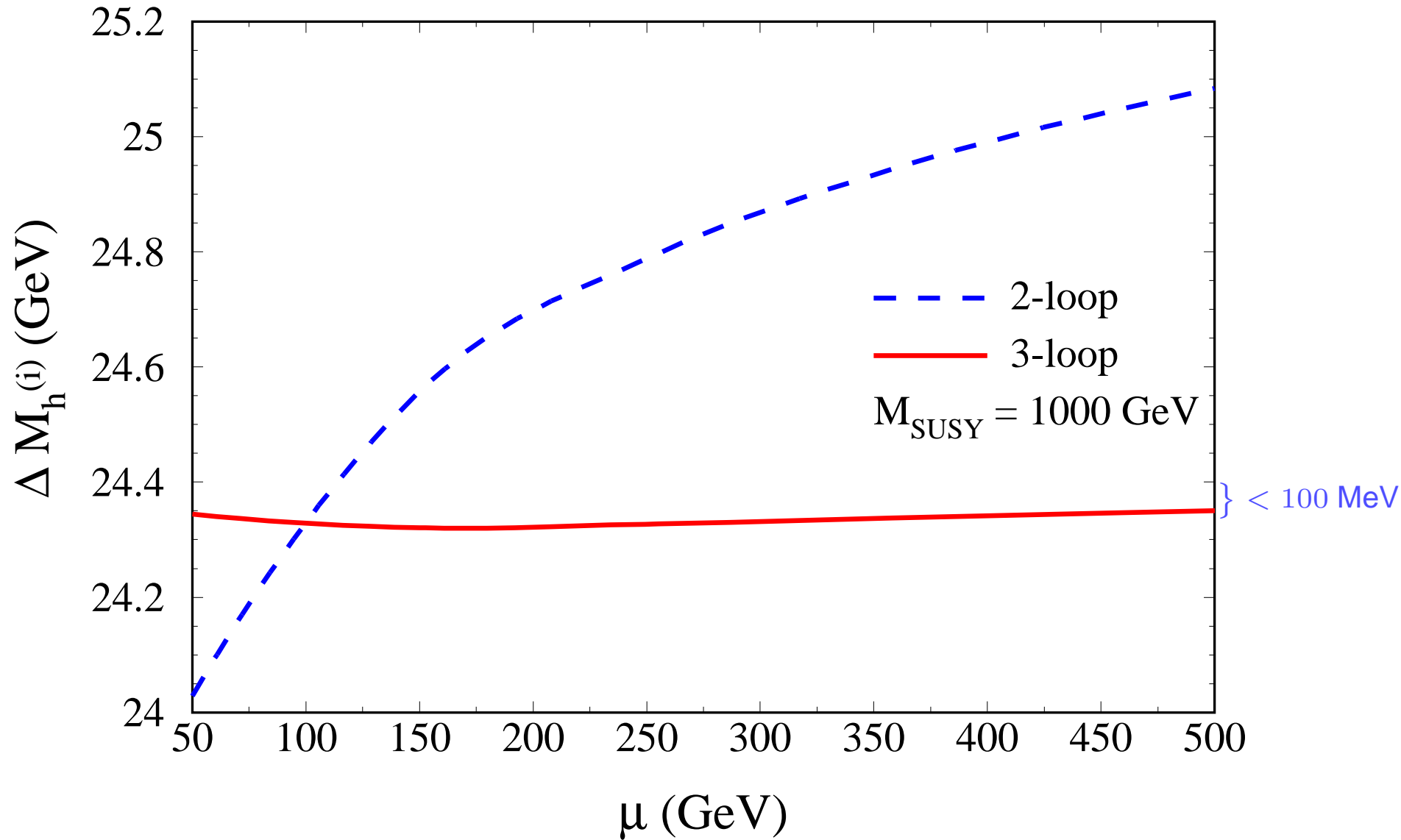
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Summary

- Higgs boson mass to 3 loops
- SUSY-QCD
- $\Delta M_h^{(3\text{loops})} > \Delta M_h^{(\text{LHC,ILC})}$
- significant reduction of scale uncertainty
- logarithmic expansion: LL + NLL \Rightarrow bad approximation