Top-Mass from Jets at the ILC: Two loop results and more

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LoopFest VII

Based on:

A. Hoang, S. Fleming, S. Mantry, & I.S. (hep-ph/0703207)

A. Hoang & I.S. (arXiv:0709.3519)

A. Hoang, S. Fleming, S. Mantry, & I.S. (arXiv:0711.2079)

A. Jain, I. Scimemi, & I.S. (arXiv:0801.0743)

Outline

- Importance of Top Mass measurements. Which mass?

$$M_t^{\text{peak}} = m_t \; + \text{(nonperturbative effects)} + \text{(perturbative effects)}$$

want a short-distance Lagrangian mass parameter

- Factorization theorem for Jet Invariant Masses

$$e^+e^- \to t\bar{t} \qquad\qquad Q \gg m_t \gg \Gamma_t$$

  - Summation of Large Logs
  - Heavy-Quark Jet Function (perturbative shift, NNLO & NNLL)
  - Gluon Soft Function (nonperturbative shift)

- Cross Sections Results

Motivation

- The top mass is a fundamental parameter of the Standard Model

$$m_t = 172.6 \pm 1.4\,\mathrm{GeV}$$ (a 0.8% error) (theory error? what mass is it?)

- Important for precision e.w. constraints

eg. $m_H = 76^{+33}_{-24}\,\mathrm{GeV}$ $\quad m_H < 182\,\mathrm{GeV}$ (95% CL)

$76 \to 87$ $\qquad 182 \to 209$

A 2 GeV shift in m_t changes the central values by 15%

- Top Yukawa coupling is large. Top parameters are important for analyzing many new physics models. (eg. Higgs masses in MSSM)

- Top is very unstable, it decays before it has a chance to hadronize. This provides an intrinsic smearing for jet observables, making them more amenable to perturbative analysis.

$$\Gamma_t = 1.4\,\mathrm{GeV} \quad \text{from} \quad t \to bW$$

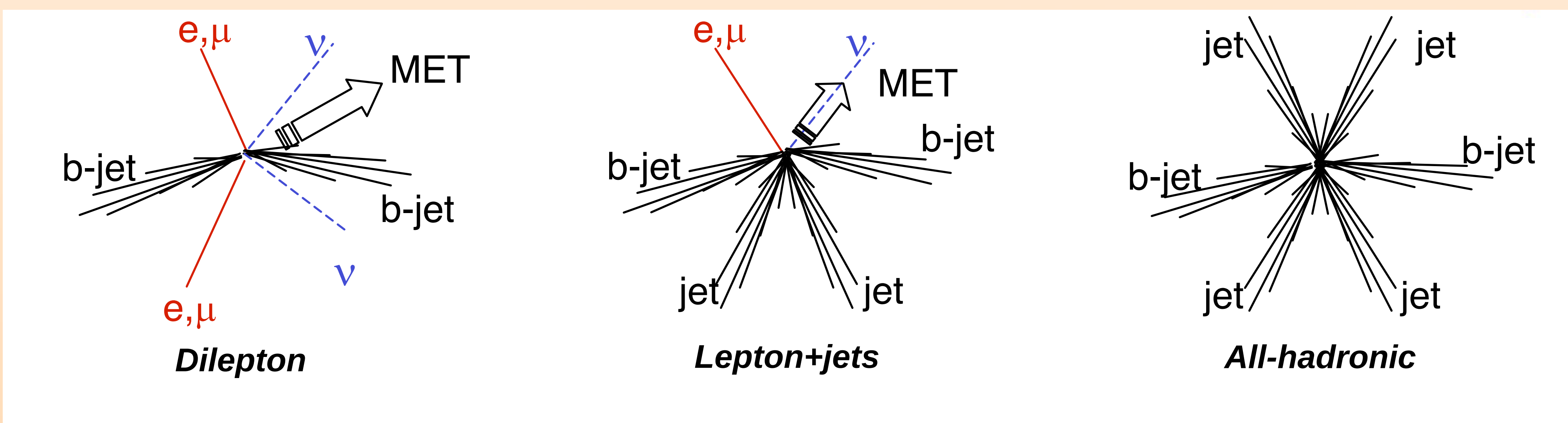
$Q \simeq 1\,\text{TeV}$ Production scale

$m_t = 172.6 \pm 1.4\,\text{GeV}$ Mass scale

$\Gamma_t \simeq 1.4\,\text{GeV}$ Short Lifetime

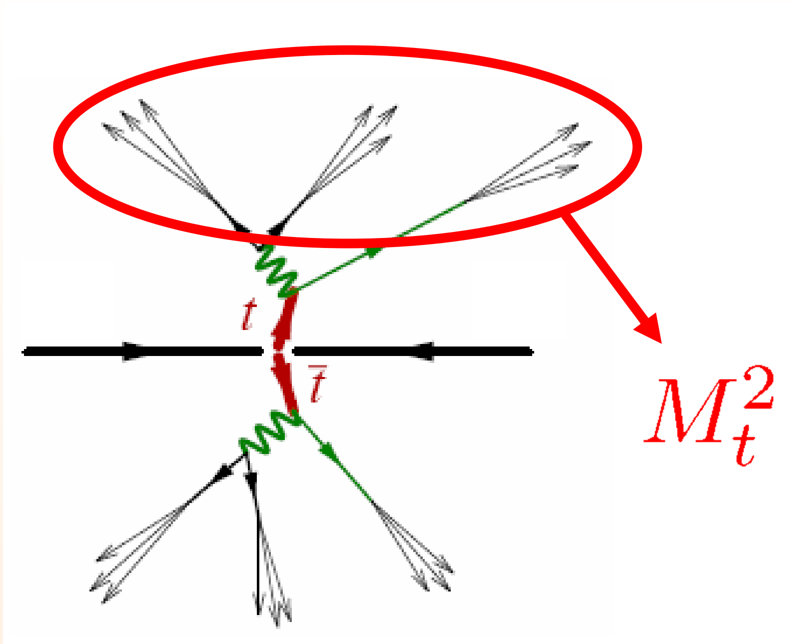
Λ_{QCD}

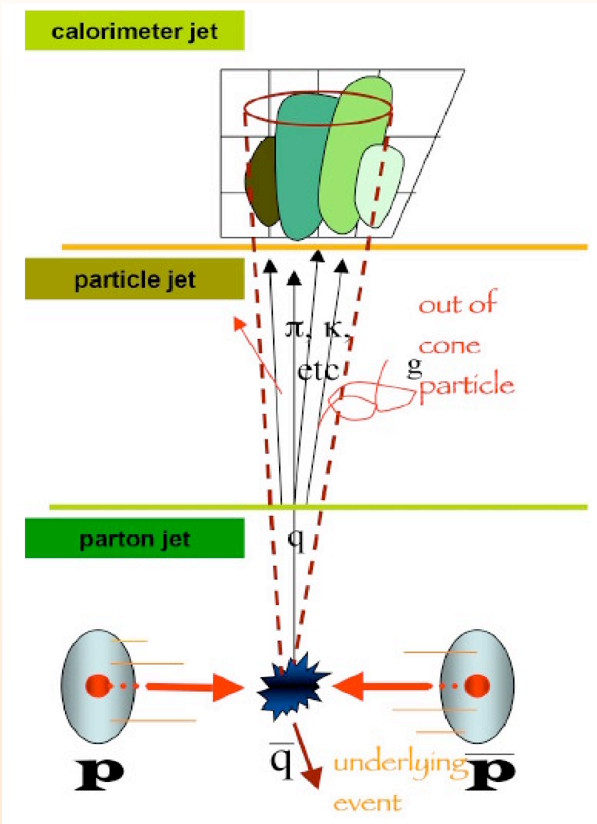
$$\begin{tikzcd}
pp \arrow[r, "\sigma"] & t\bar{t}X \arrow[d, "\Gamma"'] \arrow[r] & \cdots \\
& bW \arrow[d] \arrow[r] & q\bar{q}' \\
& e^{+}\nu &
\end{tikzcd}$$



CDF & D0:

exploit kinematic info (eg. template method) to achieve high sensitivity





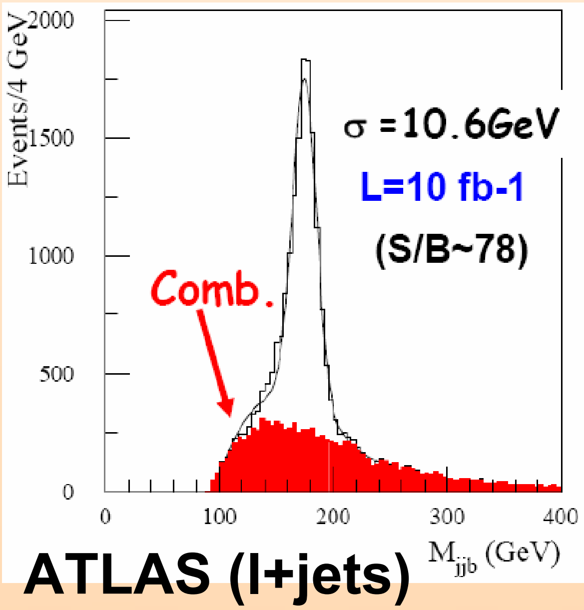
uncertainties: combinatorics, jet-energy scale, pdf's, kinematics with LO Monte Carlo, b-tagging, ...

$$m_t = 172.6 \pm 0.8(\mathrm{stat}) \pm 1.1(\mathrm{syst})\ \mathrm{GeV}$$

LHC: $pp \to t\bar{t}X$

top factory, 8 million $t\bar{t}$ / year

$\delta m_t \sim 1\,\mathrm{GeV}$ systematics dominated



Future -ILC: $e^+e^- \to t\bar{t}$

exploit threshold region $\sqrt{s} \simeq 2m_t$

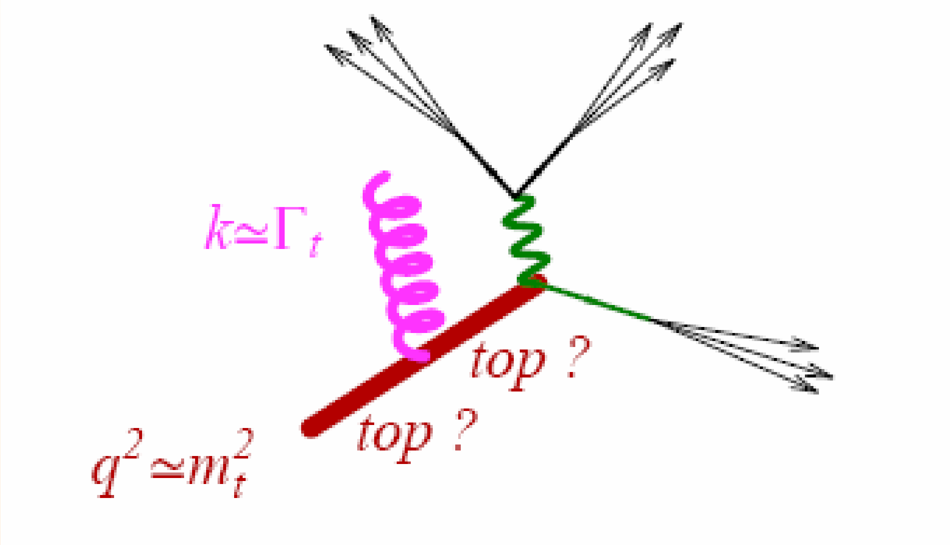
with high precision theory calculations $\delta m_t \sim 0.1\,\mathrm{GeV}$

What mass?

$m_t = 172.6 \pm 0.8(\text{stat}) \pm 1.1(\text{syst})\ \text{GeV}$

- pole mass?
  - ambiguity $\delta m \sim \Lambda_{\text{QCD}}$, linear sensitivity to IR momenta
  - poor behavior of α_s expansion
  - not used anymore for m_b, m_c

    e.g. $m_b^{1S} = (4.70 \pm 0.04)\ \text{GeV}$



$\delta m \sim \alpha_s(\Gamma)\Gamma$

quark masses are Lagrangian parameters, use a suitable scheme

$$m_t^{\text{pole}} = m_t^{\text{schemeA}}(1 + \alpha_s + \alpha_s^2 + \dots)$$

or

$$m_t^{\text{pole}} = m_t^{\text{schemeB}} + R\,(\alpha_s + \alpha_s^2 + \dots)$$

- top $\overline{\mathrm{MS}}$ mass? $\qquad m^{\mathrm{pole}} - m^{\overline{\mathrm{MS}}}(m) \sim 8\,\mathrm{GeV}$

If top-decay is described by Breit-Wigner, the answer is NO

When we switch to a short-distance mass scheme we must expand in α_s

$$\delta\overline{m} \sim \alpha_s \overline{m} \gg \Gamma$$

$$\frac{\Gamma}{\left[\frac{(M_t^2 - m^{\mathrm{pole}\,2})^2}{m^{\mathrm{pole}\,2}} + \Gamma^2\right]} = \frac{\Gamma}{\left[\frac{(M_t^2 - \overline{m}^2)^2}{\overline{m}^2} + \Gamma^2\right]} + \frac{(4\,\hat{s}\,\Gamma)\,\delta\overline{m}}{\left[\frac{(M_t^2 - \overline{m}^2)^2}{\overline{m}^2} + \Gamma^2\right]^2}$$

$$\sim 1/\Gamma \qquad\qquad \sim \alpha_s \overline{m}/\Gamma^2$$

not a correction!
it swamps the 1st term

- must be a "top-resonance mass scheme" $\qquad R \sim \Gamma$

$$m^{\mathrm{pole}} - m \sim \alpha_s \Gamma$$

Lesson: some schemes are more appropriate than others

Theory Issues for $pp \to t\bar{t}X$

- jet observable ★★
- suitable top mass for jets ★
- initial state radiation
- final state radiation ★
- underlying events
- color reconnection ★
- beam remnant
- parton distributions
- sum large logs ★

Here we'll study

$e^+e^- \to t\bar{t}X$

and the issues ★

Top Mass from Jets far above threshold at the ILC

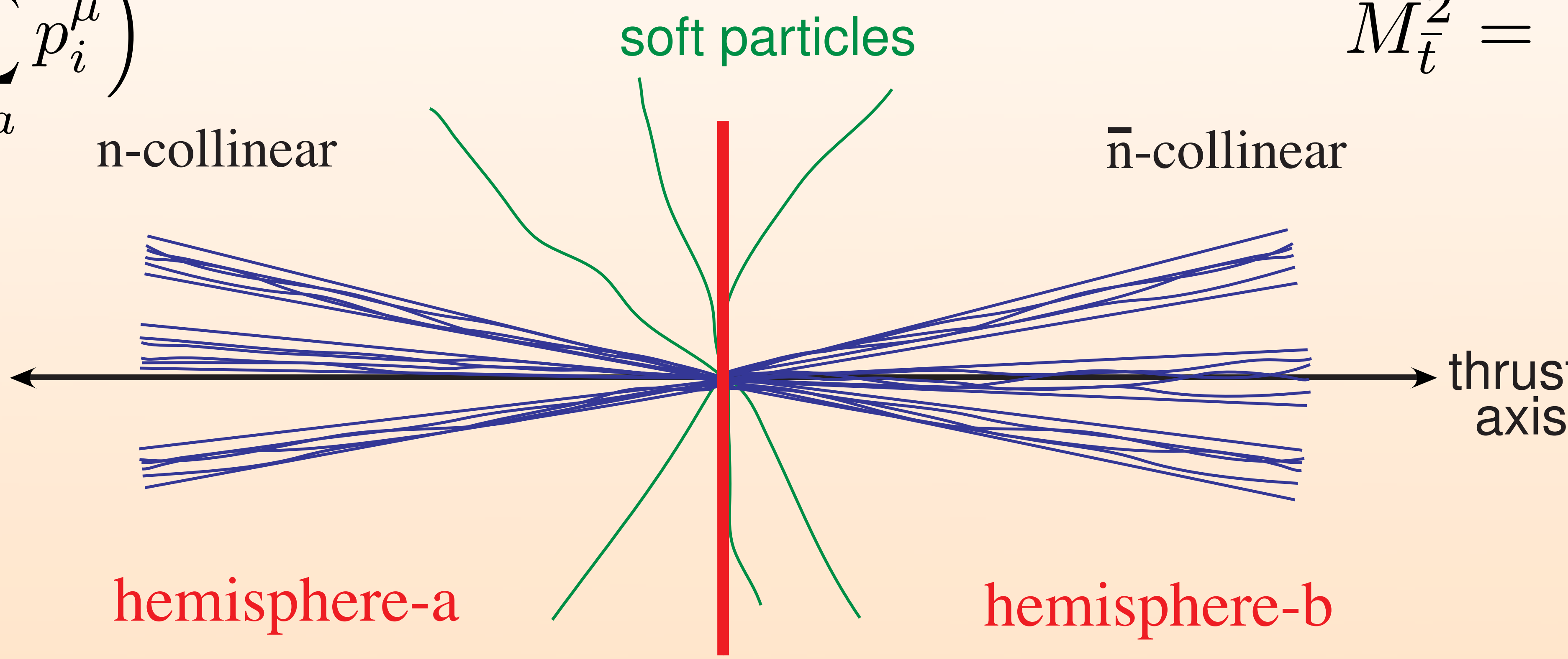
$$Q \gg m_t \gg \Gamma_t$$

Measure what observable?

$$\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}$$

Hemisphere Invariant Masses

$$M_t^2 = \Big(\sum_{i\in a} p_i^\mu\Big)^2 \qquad\qquad M_{\bar t}^2 = \Big(\sum_{i\in b} p_i^\mu\Big)^2$$

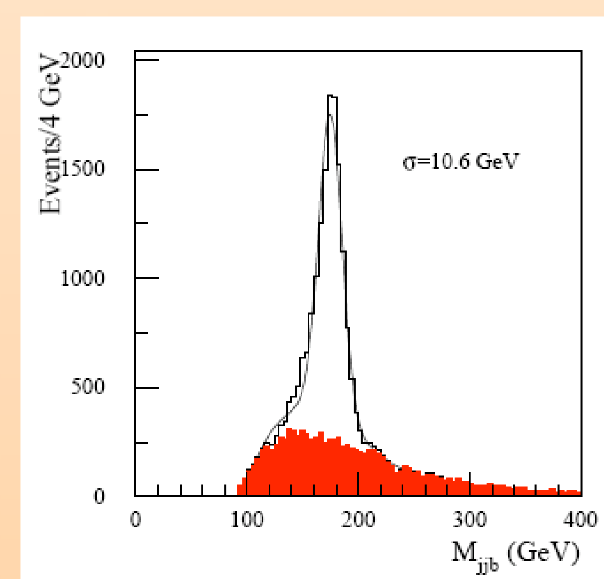


Peak region:

$$s_t \equiv M_t^2 - m^2 \sim m\Gamma \ll m^2$$

$$\boxed{\hat{s}_t \equiv \frac{M_t^2 - m^2}{m} \sim \Gamma \ll m}$$

Breit Wigner: $$\frac{m\Gamma}{s_t^2 + (m\Gamma)^2} = \Big(\frac{\Gamma}{m}\Big)\frac{1}{\hat{s}_t^2 + \Gamma^2}$$



- $Q \gg m$ "dijets" dominate, inclusive in decay products

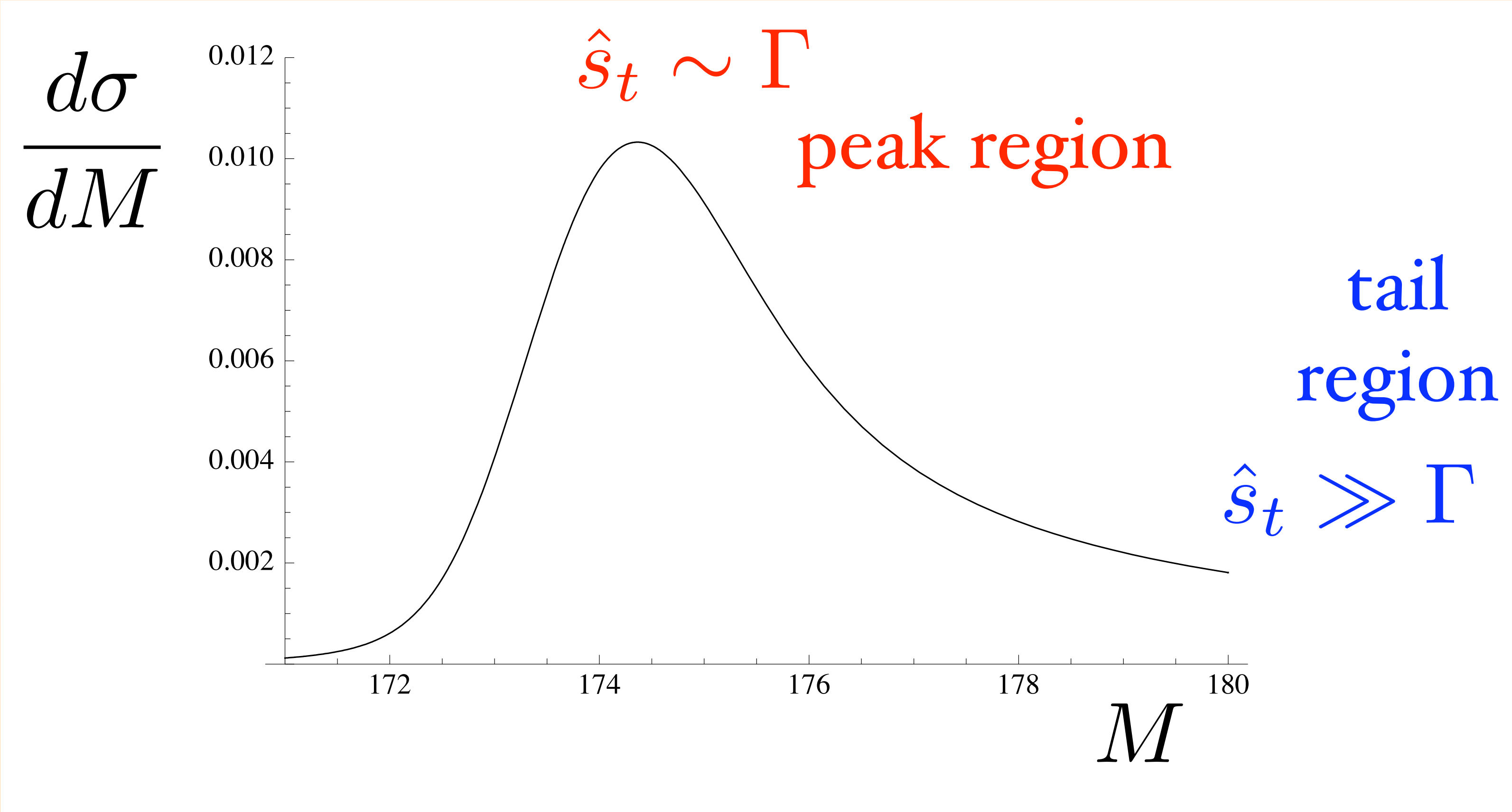
- $m \gg \Gamma$ = physical width

$$\Gamma = \Gamma_t + \dots$$

- $m \gg \hat{s}_t$

- $\Gamma > \Lambda_{\mathrm{QCD}}$

$$\hat{s}_t \equiv \frac{M_t^2 - m^2}{m}$$



$$Q \gg m \gg \Gamma \sim \hat{s}_{t,\bar{t}}$$

Disparate Scales ➡ Effective Field Theory

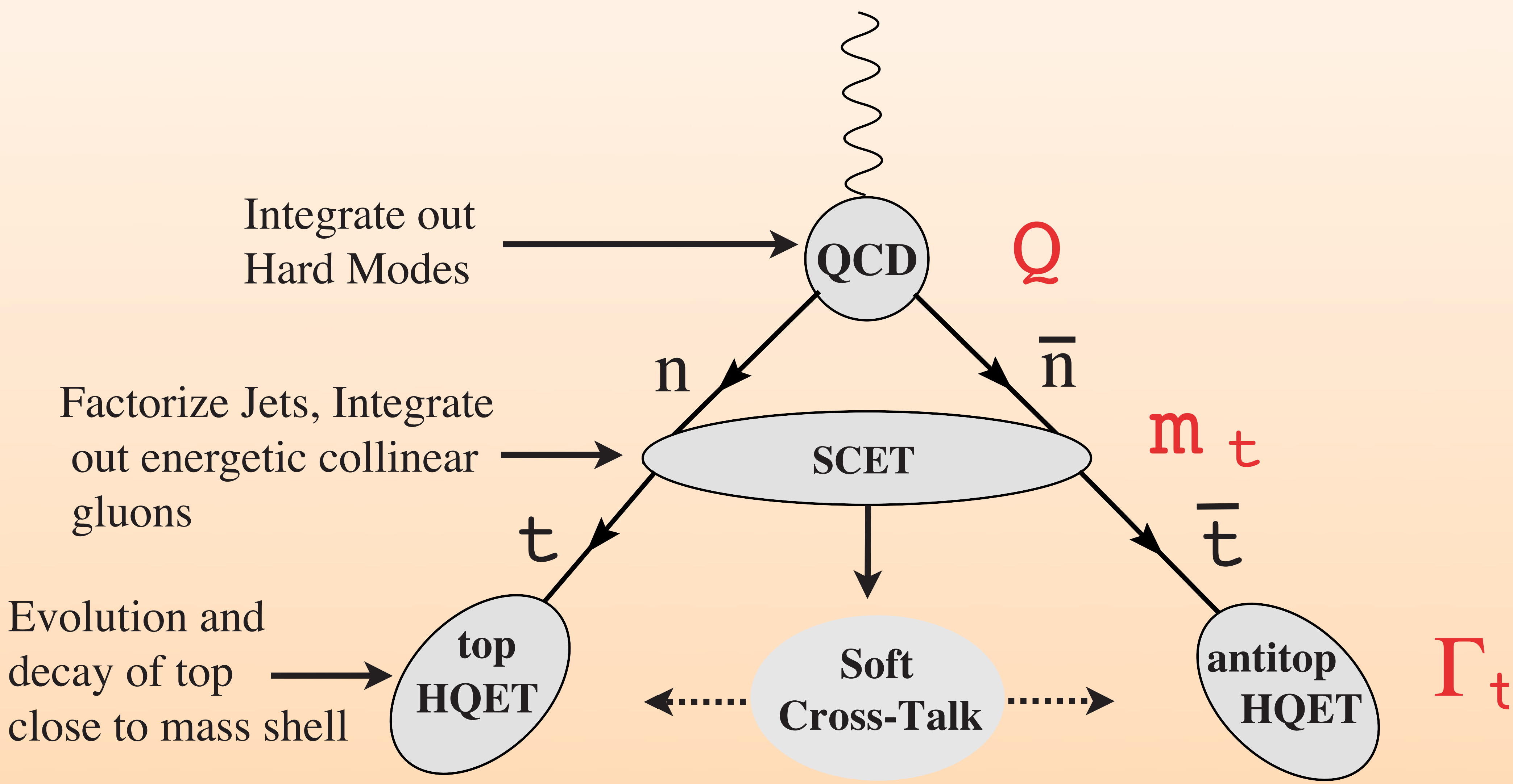
QCD

↓

SCET

↓

HQET



Derive a Factorization Theorem:

Fleming, Hoang, Mantry, I.S.

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right)$$
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(\ell^+, \ell^-, \mu).$$

Answer

$$+\,\mathcal{O}\left(\frac{m\alpha_s(m)}{Q}\right) + \mathcal{O}\left(\frac{m^2}{Q^2}\right) + \mathcal{O}\left(\frac{\Gamma_t}{m}\right) + \mathcal{O}\left(\frac{s_t, s_{\bar t}}{m^2}\right)$$

Valid to all orders in α_s

Compare to factorization theorem for massless dijets:

$$\left(\frac{d^2\sigma}{dM_a^2 dM_b^2}\right) = \sigma_0 H(Q,\mu) \int d\ell^+ d\ell^-\, J_+(M_a^2 - Q\ell^+, \mu)\, J_-(M_b^2 - Q\ell^-, \mu)\, S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

Korchemsky & Sterman

Hard Production modes integrated out

"Hard" collinear gluons integrated out

$$\left(\frac{d^2\sigma}{dM_t^2\,dM_{\bar t}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\; B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\text{hemi}}(\ell^+, \ell^-, \mu).$$

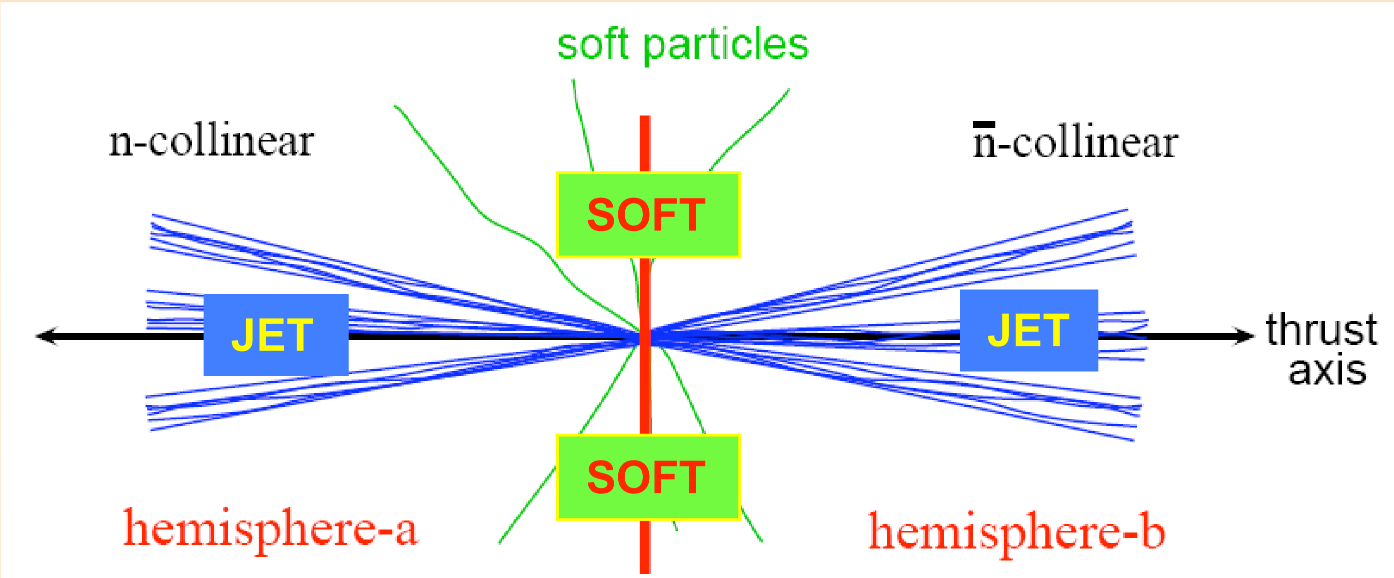
Answer

Jet Functions

Evolution and decay of top quark close to mass shell

Soft Function

Non-perturbative Cross talk



Hard Production modes integrated out

"Hard" collinear gluons integrated out

Answer

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\text{hemi}}(\ell^+, \ell^-, \mu).$$

Evolution and decay of top quark close to mass shell

Non-perturbative Cross talk

At tree level in α_s expansion this is a Breit-Wigner.

- B.W. receives calculable perturbative corrections

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\text{hemi}}(\ell^+, \ell^-, \mu).$$

Answer

- cross-section depends on a hadronic soft function, not just B.W.'s
  ** the B.W. is only a good approx. for collinear top & gluons **

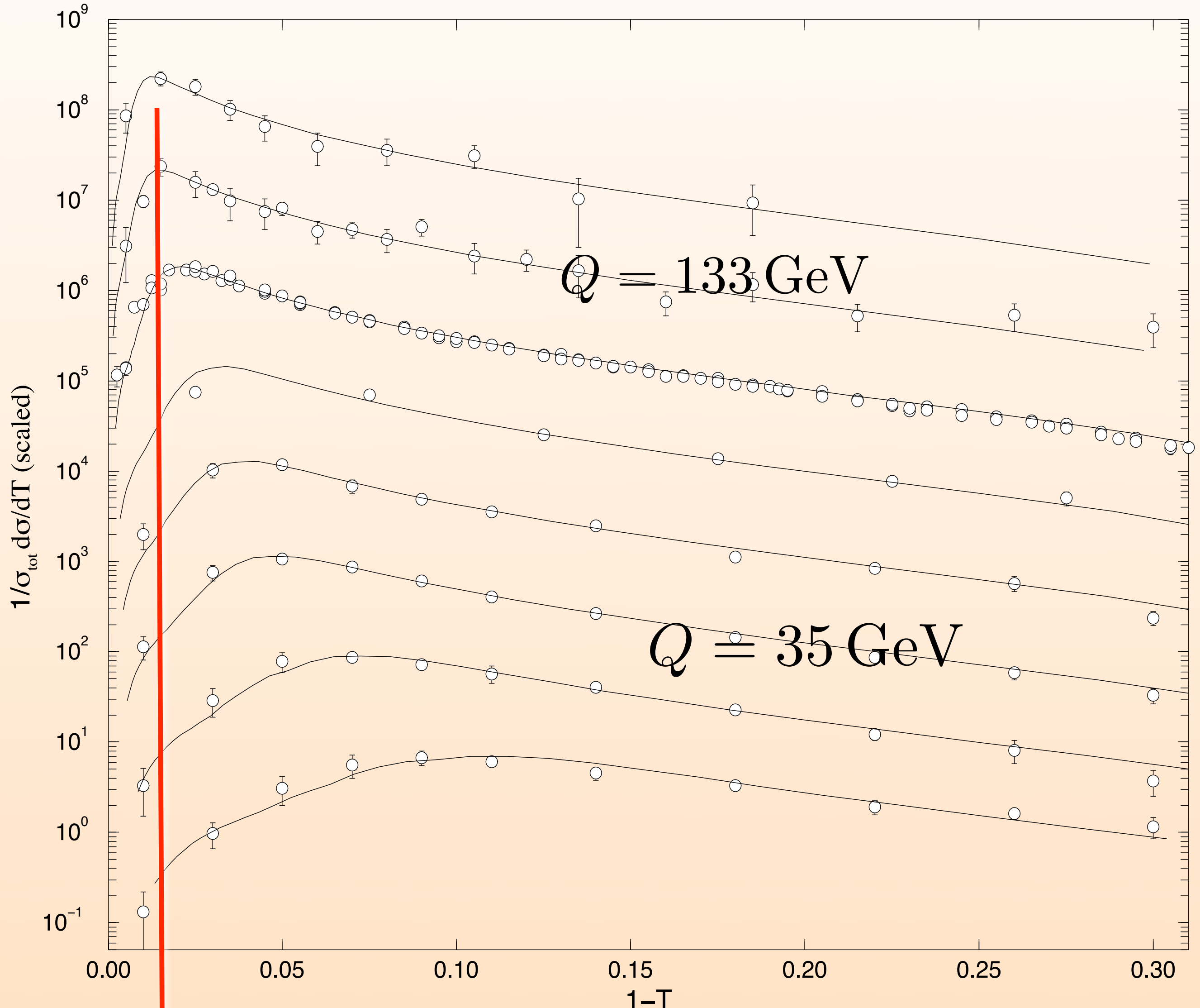
- the formula removes the largest component of soft momentum to get the correct argument for evaluating the B.W. functions

$$\hat s_t = \frac{M_t^2 - m^2}{m}$$

Everything but the soft function is calculable in perturbation theory.

S_hemi is universal, & measured in massless jet event shapes (at LEP!)

Eg. Thrust data from massless quark jets at LEP



$$1 - T$$

peaks at $\dfrac{2\Lambda_{\mathrm{QCD}}}{Q}$

$$T = \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \mathbf{p}_i|}{Q}$$

Korchemsky & Sterman

$$1 = \int dT\, \delta\Big(1 - T - \frac{M_a^2 + M_b^2}{Q^2}\Big)$$

$$\left(\frac{d^2\sigma}{dM_a^2 dM_b^2}\right) = \sigma_0 H(Q,\mu) \int d\ell^+ d\ell^-\; J_+(M_a^2 - Q\ell^+,\mu)\; J_-(M_b^2 - Q\ell^-,\mu)\; S_{\mathrm{hemi}}(\ell^+,\ell^-,\mu)$$

For our event shape for massive quarks:

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right)$$
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(\ell^+, \ell^-, \mu).$$

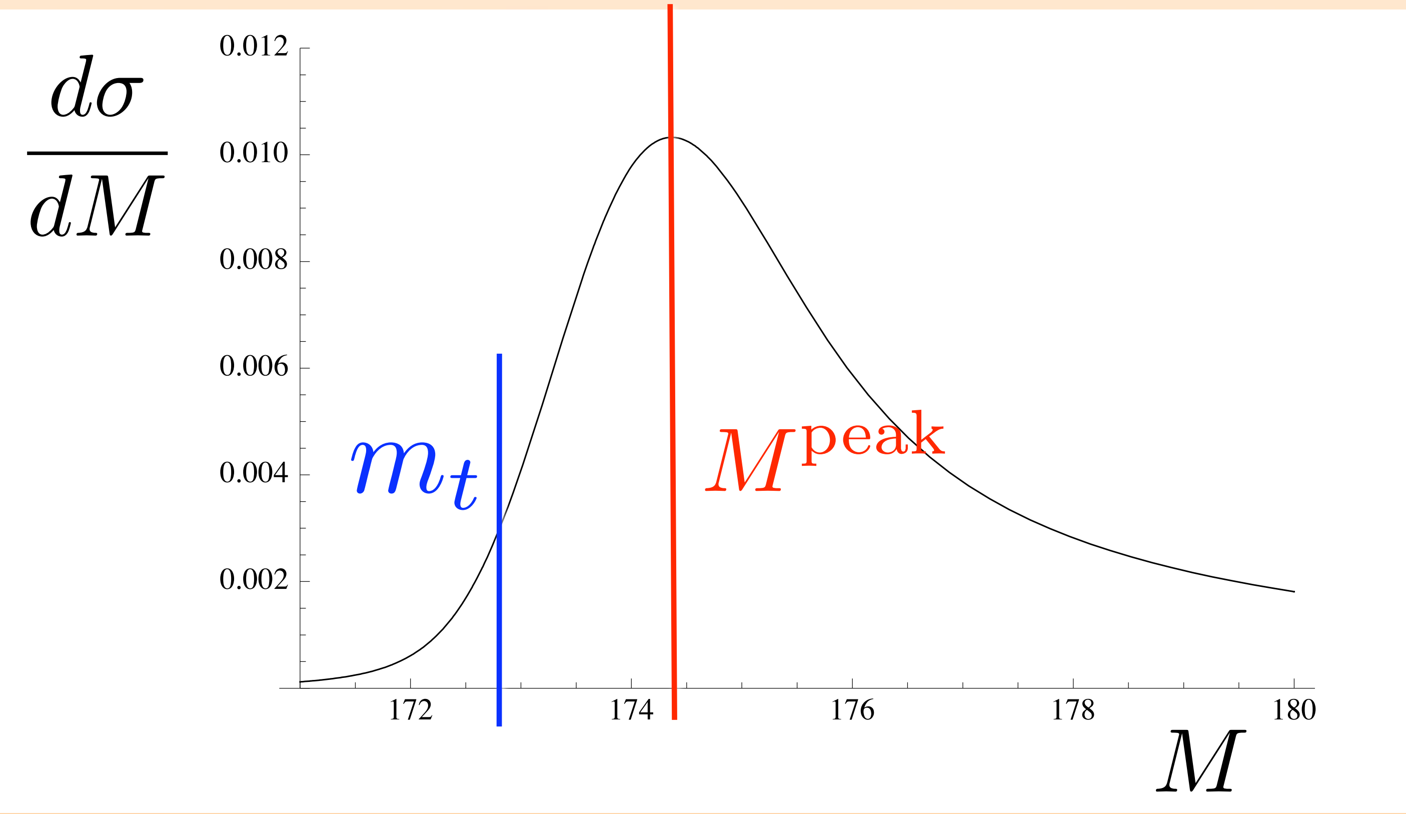
Answer

$$M^{\text{peak}} = m_t + \Gamma_t(\alpha_s + \alpha_s^2 + \ldots) + \frac{Q\Lambda_{\text{QCD}}}{m_t}$$

measure this

extract this

Short distance m_t can (in principle) be determined to better than Λ_{QCD}

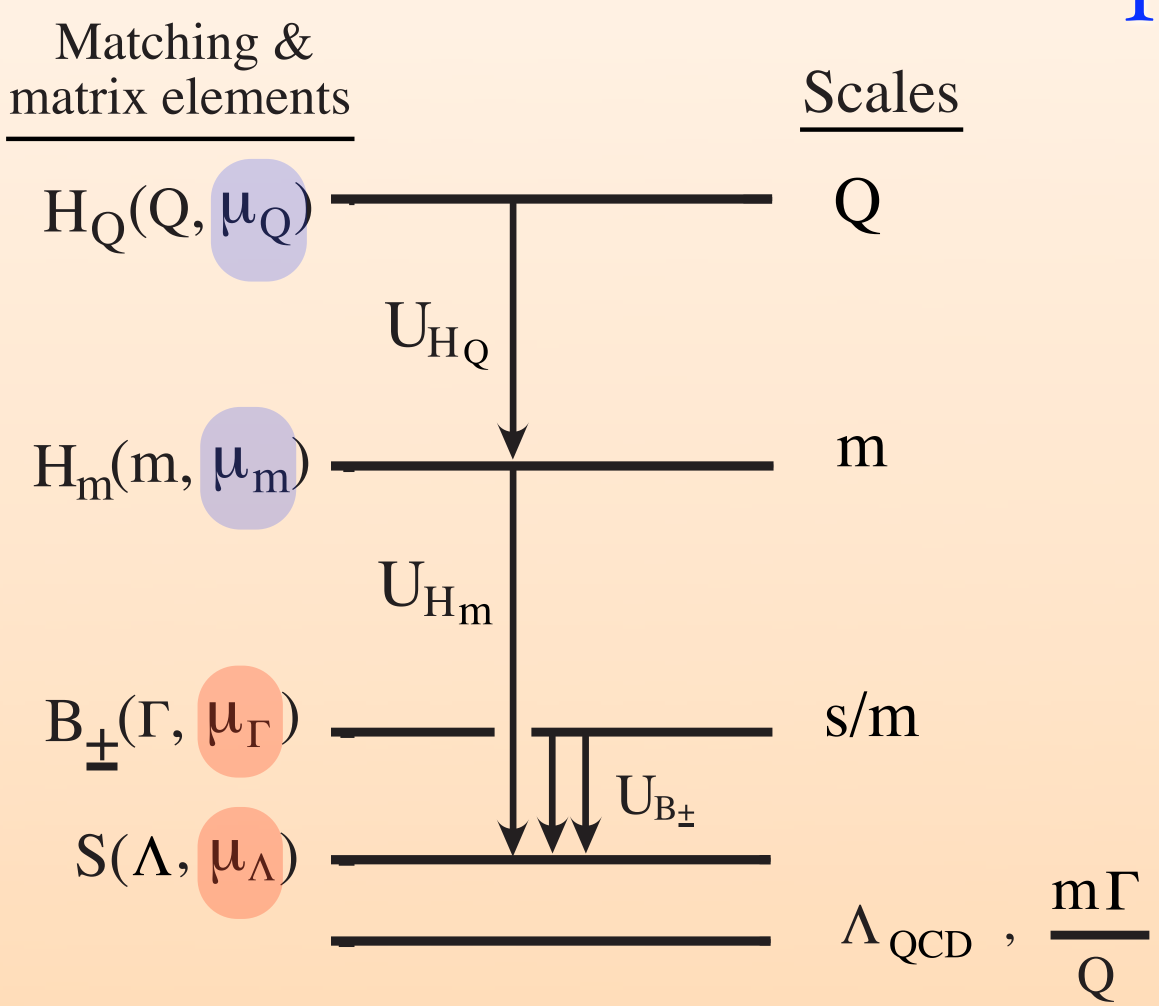


Summing the Large Logs

$$\frac{d\sigma}{dM_t^2\, dM_{\bar t}^2} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m_J, \frac{Q}{m_J}, \mu_m, \mu\Big)$$
$$\times \int d\ell^+ d\ell^- B_+\Big(\hat s_t - \frac{Q\ell^+}{m_J}, \Gamma_t, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m_J}, \Gamma_t, \mu\Big) S(\ell^+, \ell^-, \mu)$$

The various functions are sensitive to different scales

To minimize the logs we need several stages of matching and running



$$\mu_Q \simeq Q$$

$$\mu_m \simeq m$$

$$\mu_\Gamma \simeq \mathcal{O}\Big(\Gamma_t + \frac{Q\Lambda}{m} + \frac{s_{t,\bar t}}{m}\Big)\,,$$

$$\mu_\Lambda \simeq \mathcal{O}\Big(\Lambda + \frac{m\Gamma_t}{Q} + \frac{s_{t,\bar t}}{Q}\Big)\,.$$

so typically $\dfrac{\mu_\Gamma}{\mu_\Delta} \sim \dfrac{Q}{m}$

Result with resummation:

$$\frac{d^2\sigma}{dM_t^2 dM_{\bar t}^2} = \sigma_0 \;\; H_Q(Q,\mu_h)\, U_{H_Q}(Q,\mu_h,\mu_m)\, H_m(m,\mu_m)\, U_{H_m}\Big(\frac{Q}{m_J},\mu_m,\mu_\Lambda\Big)$$
$$\times \int_{-\infty}^{\infty} d\hat s_t'\, d\hat s_{\bar t}'\; U_{B_+}(\hat s_t - \hat s_t', \mu_\Lambda, \mu_\Gamma)\, U_{B_-}(\hat s_{\bar t} - \hat s_{\bar t}', \mu_\Lambda, \mu_\Gamma)$$
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^- \; B_+\Big(\hat s_t' - \frac{Q\ell^+}{m}, \Gamma, \mu_\Gamma\Big)\, B_-\Big(\hat s_{\bar t}' - \frac{Q\ell^-}{m}, \Gamma, \mu_\Gamma\Big)\, S(\ell^+,\ell^-,\mu_\Lambda)$$

Here: sum double logs $\quad \mathrm{LL} = \sum_k [\alpha_s \ln^2]^k$

$$\mu\frac{d}{d\mu} H_m\Big(m,\frac{Q}{m},\mu\Big) = \gamma_{H_m}\Big(\frac{Q}{m},\mu\Big)\, H_m\Big(m,\frac{Q}{m},\mu\Big)$$

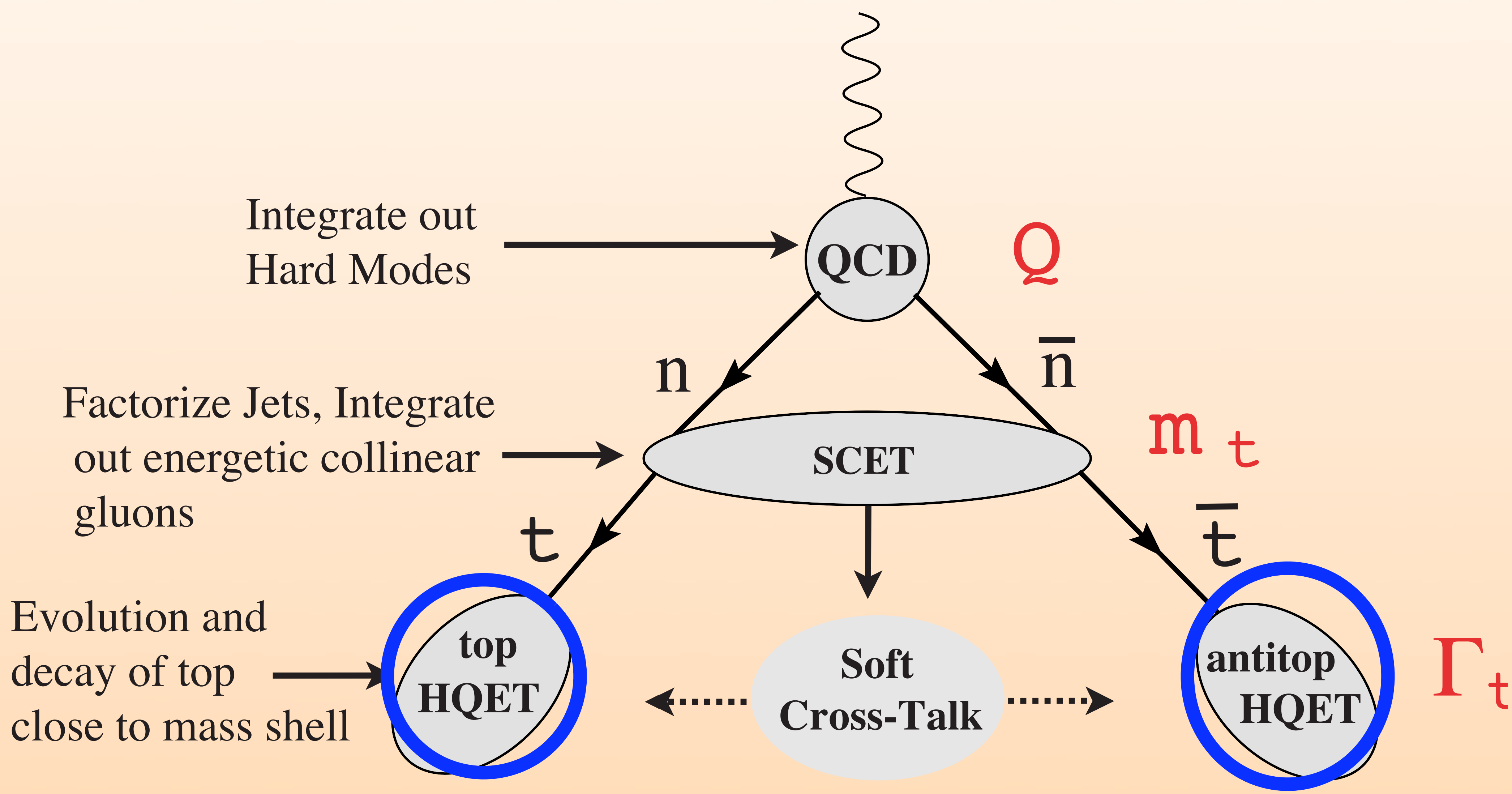
$$H_m\Big(m,\frac{Q}{m},\mu_m,\mu\Big) = H_m(m,\mu_m)\, U_{H_m}\Big(\frac{Q}{m},\mu_m,\mu\Big)$$

$$\mu\frac{d}{d\mu} B_\pm(\hat s,\mu) = \int d\hat s'\; \gamma_{B_\pm}(\hat s - \hat s',\mu)\, B_\pm(\hat s',\mu)$$

$$B_\pm(\hat s,\mu) = \int d\hat s'\; U_B(\hat s - \hat s',\mu,\mu_\Gamma)\, B_\pm(\hat s',\mu_\Gamma)$$

Only the logs between μ_Γ and μ_Λ can modify the shape of the invariant mass distribution (the rest just modify normalization)

Heavy Quark Jet Function



unstable boosted HQET

$$v_+^\mu = \left(\frac{m}{Q}, \frac{Q}{m}, \mathbf{0}_\perp\right) \sim (\lambda, \lambda^{-1}, 0)$$

fluctuations beneath the mass

$$p^\mu = m v_+^\mu + k^\mu$$

collinear, but with smaller overall scale

one HQET for top

one HQET for antitop

bHQET $[\Gamma/m \ll 1]$	
n-ucollinear $(h_{v_+},\, A^\mu_{v_+})$	$k^\mu \sim \Gamma(\lambda, \lambda^{-1}, 1)$
$\bar{n}$-ucollinear $(h_{v_-},\, A^\mu_{v_-})$	$k^\mu \sim \Gamma(\lambda^{-1}, \lambda, 1)$
same soft $(q_s,\, A^\mu_s)$	$p^\mu_s \sim (\Delta, \Delta, \Delta)$

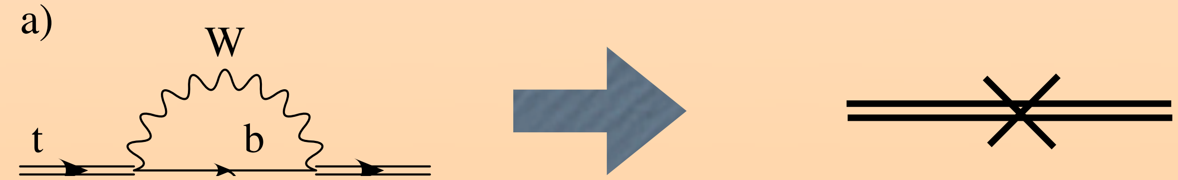
$$\mathcal{L}_+ = \bar{h}_{v_+}\Big(iv_+\cdot D_+ - \delta m + \frac{i}{2}\Gamma_t\Big)h_{v_+}, \qquad \mathcal{L}_- = \bar{h}_{v_-}\Big(iv_-\cdot D_- - \delta m + \frac{i}{2}\Gamma_t\Big)h_{v_-}$$

mass scheme choice

$$\delta m = m^{\text{pole}} - m$$

our observable is inclusive in top decay products

a)

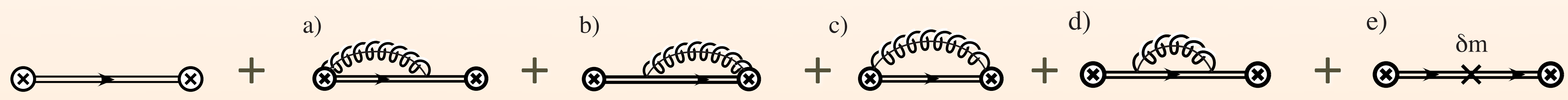


Heavy Quark Jet Function

Can be computed perturbatively

$$B(\hat{s}, \delta m, \Gamma_t, \mu) = \mathrm{Im}\big[\mathcal{B}(\hat{s}, \delta m, \Gamma_t, \mu)\big]$$

$= \mathrm{Im}\Big[$



$+ \dots \Big]$

$$\mathcal{B}(2v_+ \cdot r, \delta m, \Gamma_t, \mu) = \frac{-i}{4\pi N_c m} \int d^4x \, e^{ir\cdot x} \, \langle 0 \,| T\{\bar{h}_{v_+}(0) W_n(0) W_n^\dagger(x) h_{v_+}(x)\} |\, 0 \rangle$$

shift property $\quad \mathcal{B}(\hat{s}, \delta m, \Gamma_t, \mu) = \mathcal{B}(\hat{s} - 2\delta m + i\Gamma_t, \mu)$

Renormalization and RGE:

convolutions $\mathcal{B}(\hat{s},\mu) = \int d\hat{s}'\ Z_B^{-1}(\hat{s}-\hat{s}',\mu)\ \mathcal{B}^{\mathrm{bare}}(\hat{s}')$

$$\mu\frac{d}{d\mu}\mathcal{B}(\hat{s},\mu) = \int d\hat{s}'\ \gamma_B(\hat{s}-\hat{s}',\mu)\,\mathcal{B}(\hat{s}',\mu)$$

$$\gamma_B(\hat{s},\mu) = -2\Gamma^{\mathrm{c}}[\alpha_s]\frac{1}{\mu}\left[\frac{\mu\,\theta(\hat{s})}{\hat{s}}\right]_+ + \gamma[\alpha_s]\delta(\hat{s})$$

cusp anom.dim. — known to 3 loops

non-cusp term — now known to 2 loops

Position space:

$$\tilde{\gamma}_B(y,\mu) = 2\Gamma^{\mathrm{c}}[\alpha_s]\ \ln\left(ie^{\gamma_E}y\,\mu\right) + \gamma[\alpha_s]$$

solution: $\tilde{B}(y,\mu) = e^{K(\mu,\mu_0)}\left(ie^{\gamma_E}y\,\mu_0\right)^{\omega(\mu,\mu_0)}\ \tilde{B}(y,\mu_0)$

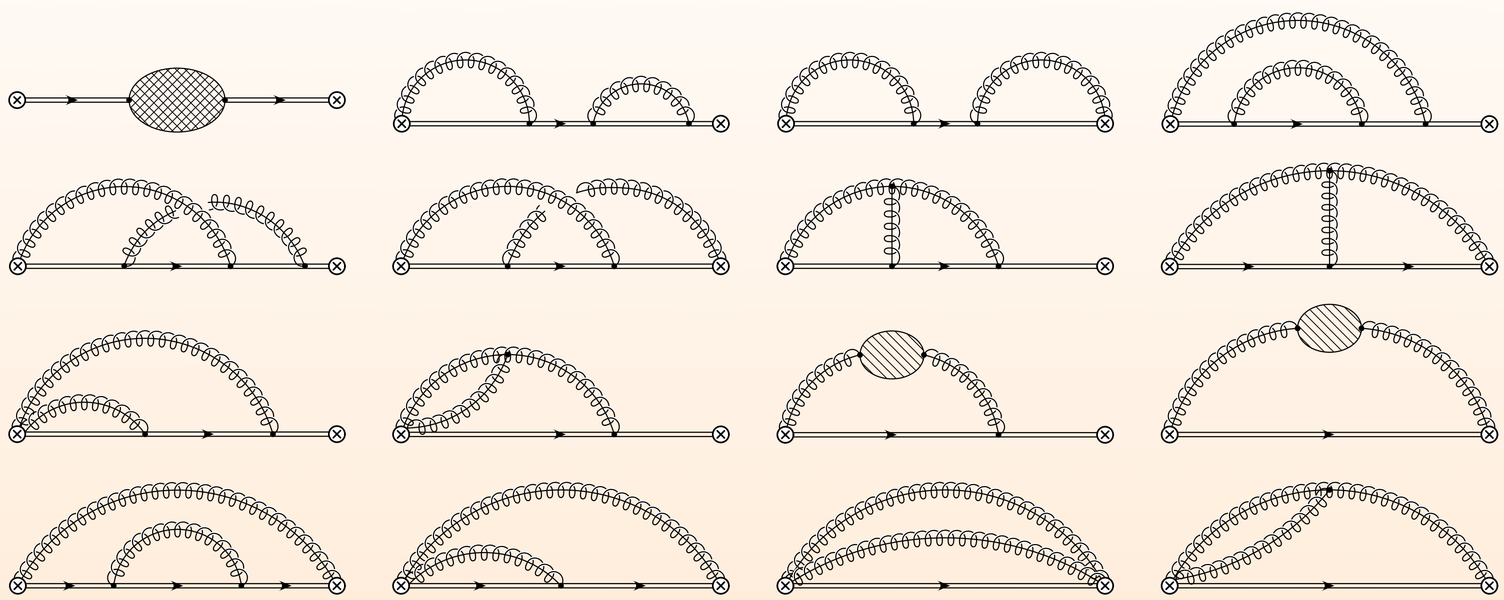
$$\omega(\mu,\mu_0) = 2\int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)}\frac{d\alpha}{\beta[\alpha]}\,\Gamma^c[\alpha]\ ,\quad K(\mu,\mu_0) = \ldots$$

Momentum space:

$$B(\hat{s},\mu) = \int_{-\infty}^{+\infty} d\hat{s}'\ U_B(\hat{s}-\hat{s}',\mu,\mu_0)\ B(\hat{s}',\mu_0)\,,\qquad U_B(\hat{s}-\hat{s}',\mu,\mu_0) = \frac{e^K\left(e^{\gamma_E}\right)^\omega}{\mu_0\,\Gamma(-\omega)}\left[\frac{\mu_0^{1+\omega}\,\theta(\hat{s}-\hat{s}')}{(\hat{s}-\hat{s}')^{1+\omega}}\right]_+$$

Two-Loop Result

Jain, Scimemi, I.S.



$$m\,\mathcal{B}_2(\hat{s},\delta m,\mu) = C_F^2\Big[\frac{1}{2}L^4 + L^3 + \Big(\frac{3}{2}+\frac{13\pi^2}{24}\Big)L^2 + \Big(1+\frac{13\pi^2}{24}-4\zeta_3\Big)L^1 + \Big(\frac{1}{2}+\frac{7\pi^2}{24}+\frac{53\pi^4}{640}-2\zeta_3\Big)L^0\Big]$$
$$+ C_F C_A\Big[\Big(\frac{1}{3}-\frac{\pi^2}{12}\Big)L^2 + \Big(\frac{5}{18}-\frac{\pi^2}{12}-\frac{5\zeta_3}{4}\Big)L^1 + \Big(-\frac{11}{54}+\frac{5\pi^2}{48}-\frac{19\pi^4}{960}-\frac{5\zeta_3}{8}\Big)L^0\Big]$$
$$+ C_F\beta_0\Big[\frac{1}{6}L^3 + \frac{2}{3}L^2 + \Big(\frac{47}{36}+\frac{\pi^2}{12}\Big)L^1 + \Big(\frac{281}{216}+\frac{23\pi^2}{192}-\frac{17\zeta_3}{48}\Big)L^0\Big]$$
$$- 2\delta m_2 (L^0)' + 2(\delta m_1)^2 (L^0)'' - 2\delta m_1\, C_F\Big[L^2 + L^1 + \Big(1+\frac{5\pi^2}{24}\Big)L^0\Big]'.$$

Still need to specify a suitable mass scheme

$$L^k = \frac{1}{\pi(-\hat{s}-i0)}\,\ln^k\Big(\frac{\mu}{-\hat{s}-i0}\Big)$$

$$\delta m = \frac{\alpha_s(\mu)}{\pi}\,\delta m_1(\mu) + \frac{\alpha_s^2(\mu)}{\pi^2}\,\delta m_2(\mu) + \ldots\,.$$

Mass Scheme should:

- be renormalon free $(\text{not } m^{\text{pole}})$
- be a top-resonance mass scheme $\delta m \sim \alpha_s \Gamma_t$ $(\text{not } \overline{\text{MS}})$
- have a RGE in μ

$\delta m = m_{pole} - m$

3 possibilites for scheme with stable peak position:

"peak" a) $$\frac{d}{d\hat{s}} B(\hat{s}, \delta m^{\text{peak}}, \Gamma_t, \mu)\Big|_{\hat{s}=0} = 0\,,$$

"moment" b) $$\int_{-\infty}^{R} d\hat{s}\ \hat{s}\ B(\hat{s}, \delta m^{\text{mom}}, \mu) = 0\,,$$ $R \sim \Gamma_t$

"position" c) $$\delta m_J = \frac{-i}{2\,\tilde{B}(y,\mu)}\,\frac{d}{dy}\,\tilde{B}(y,\mu)\Big|_{y=-ie^{-\gamma_E}/R} = e^{\gamma_E}\,\frac{R}{2}\,\frac{d}{d\ln(iy)}\ln\tilde{B}(y,\mu)\Big|_{iye^{\gamma_E}=1/R}\,.$$

Only c) has a consistent anomalous dimension equation.

"top jet mass scheme"

(two loop conversion to $\overline{\text{MS}}$ is now known)

Position scheme is nice:

$$\frac{dm_J(\mu)}{d\ln\mu} = -e^{\gamma_E} R\, \Gamma^{\rm c}[\alpha_s(\mu)]$$

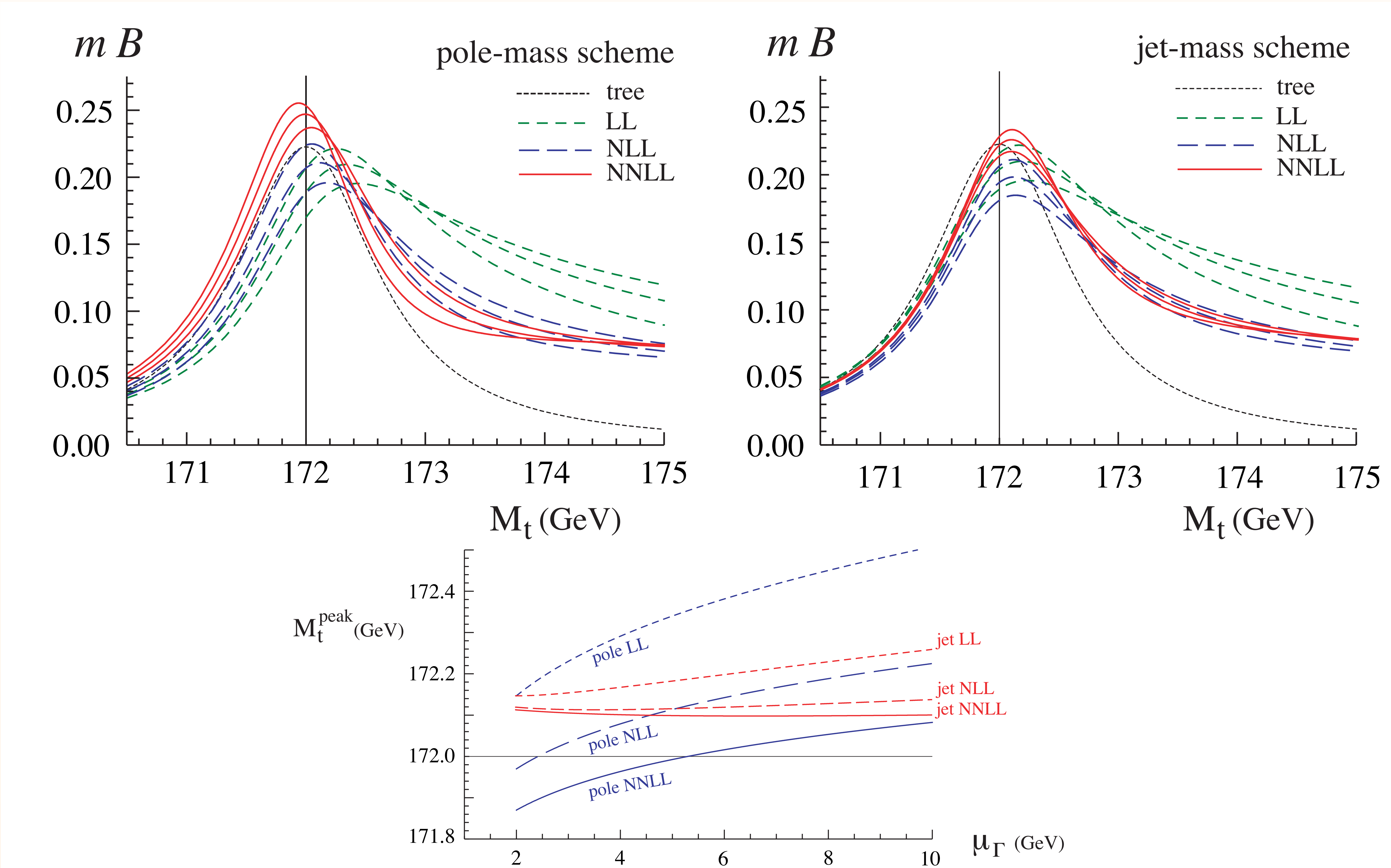
anom.dim. is determined by cusp term, and therefore is known to 3 loops

Result is jet-function with resummation:

$$\begin{aligned} B(\hat{s}, \delta m_J, \Gamma_t, \mu_\Lambda, \mu_\Gamma) &\equiv \int d\hat{s}'\; U_B(\hat{s}-\hat{s}', \mu_\Lambda, \mu_\Gamma)\; B(\hat{s}', \delta m_J, \Gamma_t, \mu_\Gamma) \\ &= \int d\hat{s}'\, d\hat{s}''\; U_B(\hat{s}-\hat{s}', \mu_\Lambda, \mu_\Gamma)\; \underbrace{B(\hat{s}'-\hat{s}'', \delta m_J, \mu_\Gamma)}\; \frac{\Gamma_t}{\pi(\hat{s}''^{\,2}+\Gamma_t^2)}\,. \end{aligned}$$

convolute result from the previous page to sum logs and include width effects

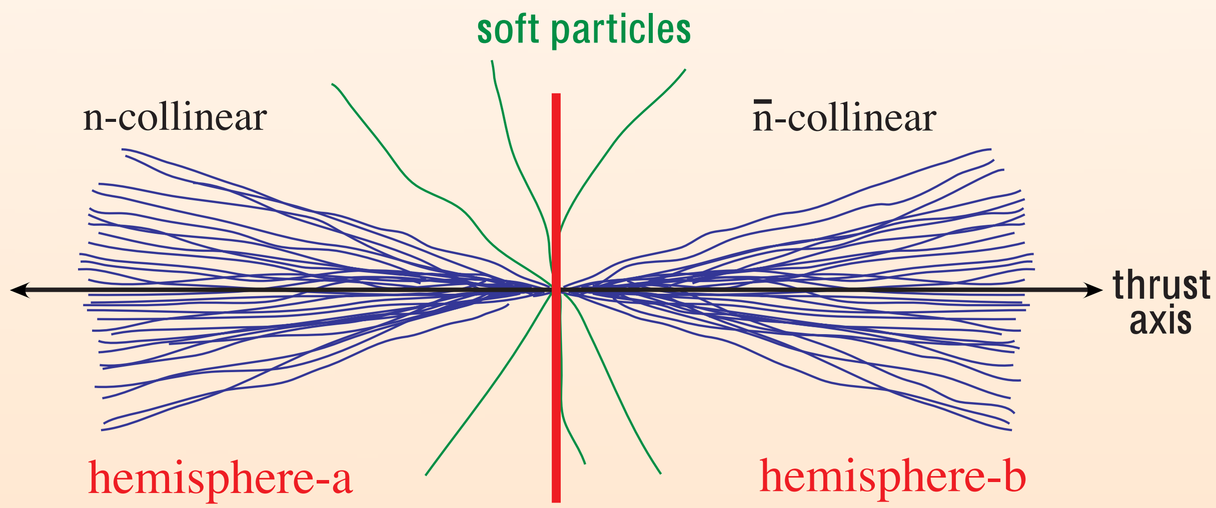
Jet Function Results up to NNLL: (3 curves vary μ_Γ)

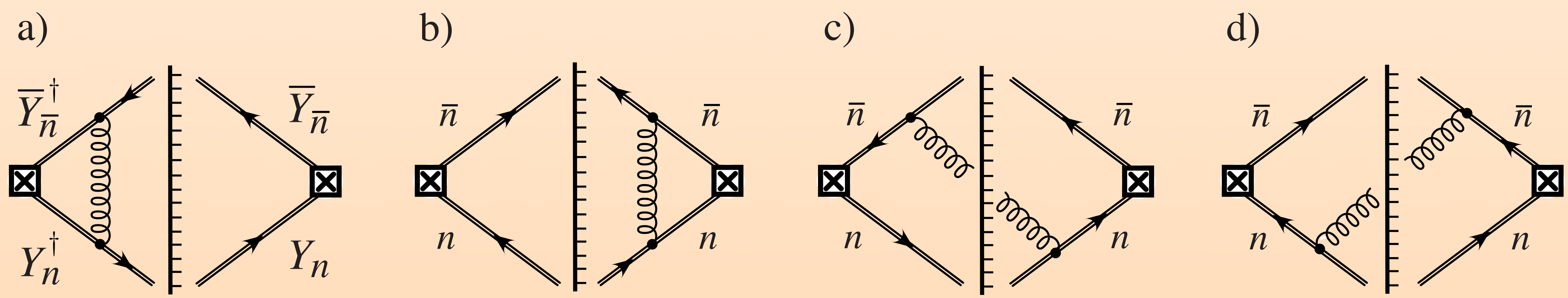


Soft Function

$$S_{\rm hemi}(\ell^+,\ell^-,\mu) = \frac{1}{N_c}\sum_{X_s}\delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b})\langle 0|\underbrace{\overline{Y}_{\bar n}\, Y_n}(0)|X_s\rangle\langle X_s|\underbrace{Y_n^\dagger\, \overline{Y}_{\bar n}^\dagger}(0)|0\rangle$$

soft Wilson lines





$$S_{\rm part}(\ell^+,\ell^-,\mu) = \delta(\ell^+)\delta(\ell^-) + \delta(\ell^+)S^1_{\rm part}(\ell^-,\mu) + \delta(\ell^-)S^1_{\rm part}(\ell^+,\mu)\,,$$

$$S^1_{\rm part}(\ell,\mu) = \frac{C_F\alpha_s(\mu)}{\pi}\Big[\frac{\pi^2}{24}\delta(\ell) - 2\mathcal{L}^1(\ell)\Big]$$

$$\mathcal{L}^1(\ell) = \frac{1}{\mu}\Big[\frac{\theta(\ell)\ln(\ell/\mu)}{\ell/\mu}\Big]_+$$

$S(\ell^+, \ell^-, \mu)$

- Anomalous dimension determined by partonic calculation.

  it has cusp anom.dim.

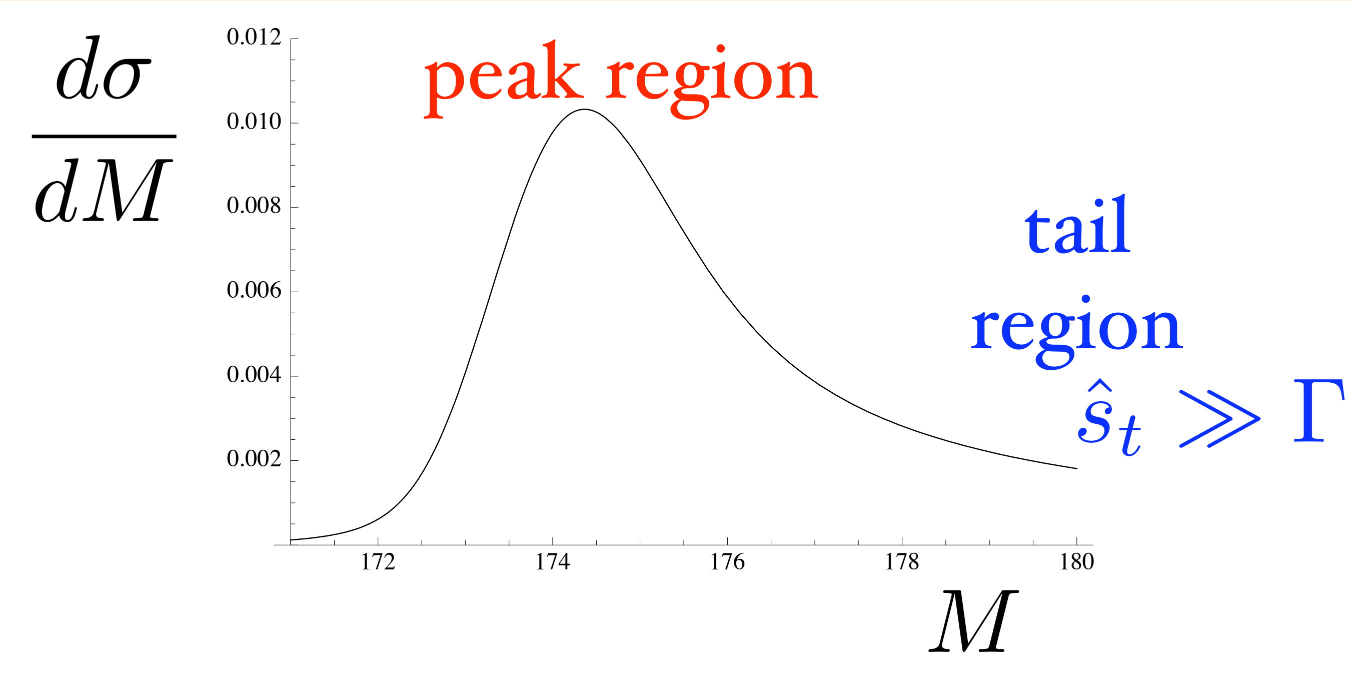
$$\int_{-\infty}^{L} d\ell^+ \int_{-\infty}^{L} d\ell^- \; S(\ell^+, \ell^-, \mu) \;= 1 + \frac{C_F \alpha_s(\mu)}{\pi} \left\{ \frac{\pi^2}{12} - 2 \ln^2 \left(\frac{L}{\mu} \right) \right\} + \ldots$$

- Cross-section in the tail region has $\ell^\pm \sim \dfrac{\hat{s}\, m}{Q} \gg \Lambda_{\mathrm{QCD}}$ and the

  soft function becomes perturbatively calculable

- In the peak region $\ell^\pm \sim \Lambda_{\mathrm{QCD}}$ $\longrightarrow$ nonperturbative soft function

these features should be built into S



A Convolution Formula does this

Hoang & I.S.

$$S(\ell^+,\ell^-,\mu) = \int_{-\infty}^{+\infty} d\tilde{\ell}^+ \int_{-\infty}^{+\infty} d\tilde{\ell}^- \; \underbrace{S_{\rm part}(\ell^+ - \tilde{\ell}^+, \ell^- - \tilde{\ell}^-, \mu)}_{\text{partonic soft function calculated at fixed order}} \; \underbrace{S_{\rm mod}(\tilde{\ell}^+, \tilde{\ell}^-)}_{\text{normalized model function (exponential fall off)}}$$

$$\int_{-\infty}^{+\infty} d\ell^+ d\ell^- \; S_{\rm mod}(\ell^+,\ell^-) \; = \; 1$$

- Soft-function has a $(u = 1/2)$ renormalon ambiguity

  implying that the partonic and model parts are sensitively tied together

- This is removed by introducing a minimum energy gap for the soft radiation

$$S(\ell^+,\ell^-,\mu) = \int_{-\infty}^{+\infty} d\tilde{\ell}^+ \int_{-\infty}^{+\infty} d\tilde{\ell}^- \; S_{\rm part}(\ell^+ - \tilde{\ell}^+, \ell^- - \tilde{\ell}^-, \mu) \, f_{\rm exp}(\tilde{\ell}^+ - \Delta, \tilde{\ell}^- - \Delta)$$

$$= \int_{-\infty}^{+\infty} d\tilde{\ell}^+ \int_{-\infty}^{+\infty} d\tilde{\ell}^- \; S_{\rm part}(\ell^+ - \tilde{\ell}^+ - \delta, \ell^- - \tilde{\ell}^- - \delta, \mu) \, f_{\rm exp}(\tilde{\ell}^+ - \bar{\Delta}, \tilde{\ell}^- - \bar{\Delta})$$

$$\Delta = \bar{\Delta} + \delta = \bar{\Delta} + (\alpha_s + \alpha_s^2 + \ldots)$$

$\bar{\Delta} = $ renormalon free

Fleming, Hoang,
Mantry, I.S.

Analysis at NLL order

(Next-to-Leading-Order with resummation to all orders of next-to-leading logarithms)

Analysis to NLL order

- One-loop matching

- One-loop matrix element for B+, and for the soft function:

$$S(\ell^+,\ell^-,\mu) = \int_{-\infty}^{+\infty} d\tilde{\ell}^+ \int_{-\infty}^{+\infty} d\tilde{\ell}^- \; S_{\rm part}(\ell^+ - \tilde{\ell}^+, \ell^- - \tilde{\ell}^-, \mu, \delta_i)\, S_{\rm mod}(\tilde{\ell}^+, \tilde{\ell}^-)$$

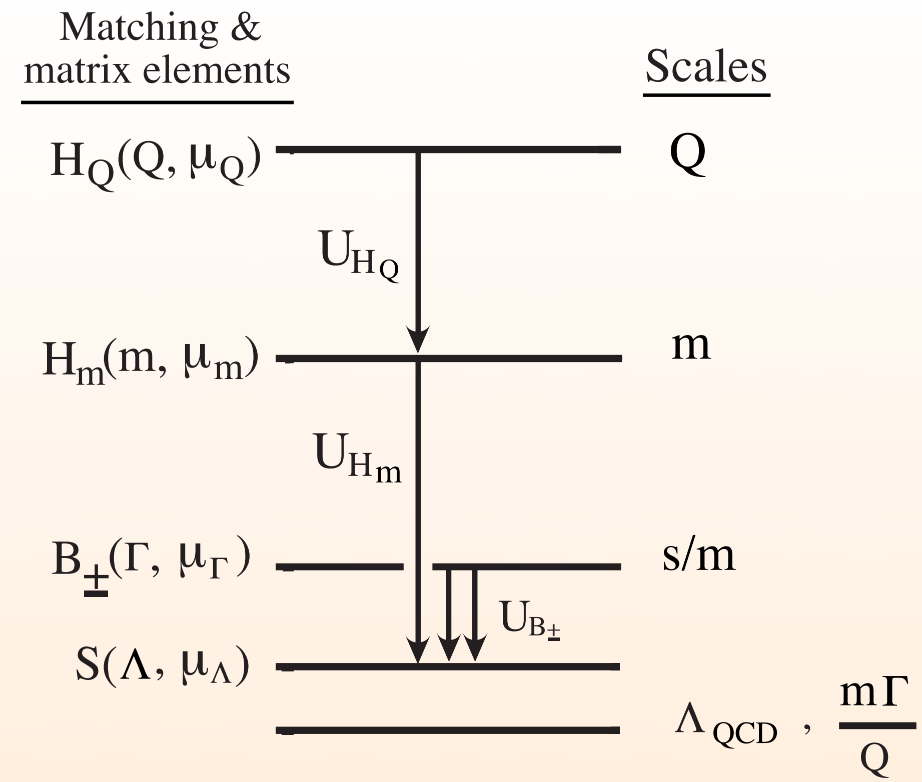
- Renormalon Free Schemes for top-mass and soft function parameters

- RGE evolution, sum large logs $Q \gg m \gg \Gamma \sim \hat{s}_{t,\bar{t}}$

  (Two-loop cusp anom.dims. & One-loop non-cusp)

- Proper choice for the scales

NLL Cross-Section Results



normalized cross-section

$$\frac{d^2\sigma}{dM_t\, dM_{\bar t}} = \frac{\sigma_0}{\Gamma_t^2}\, F\Big(M_t, M_{\bar t}, m_J, \frac{Q}{m_J}\Big)$$

numerical

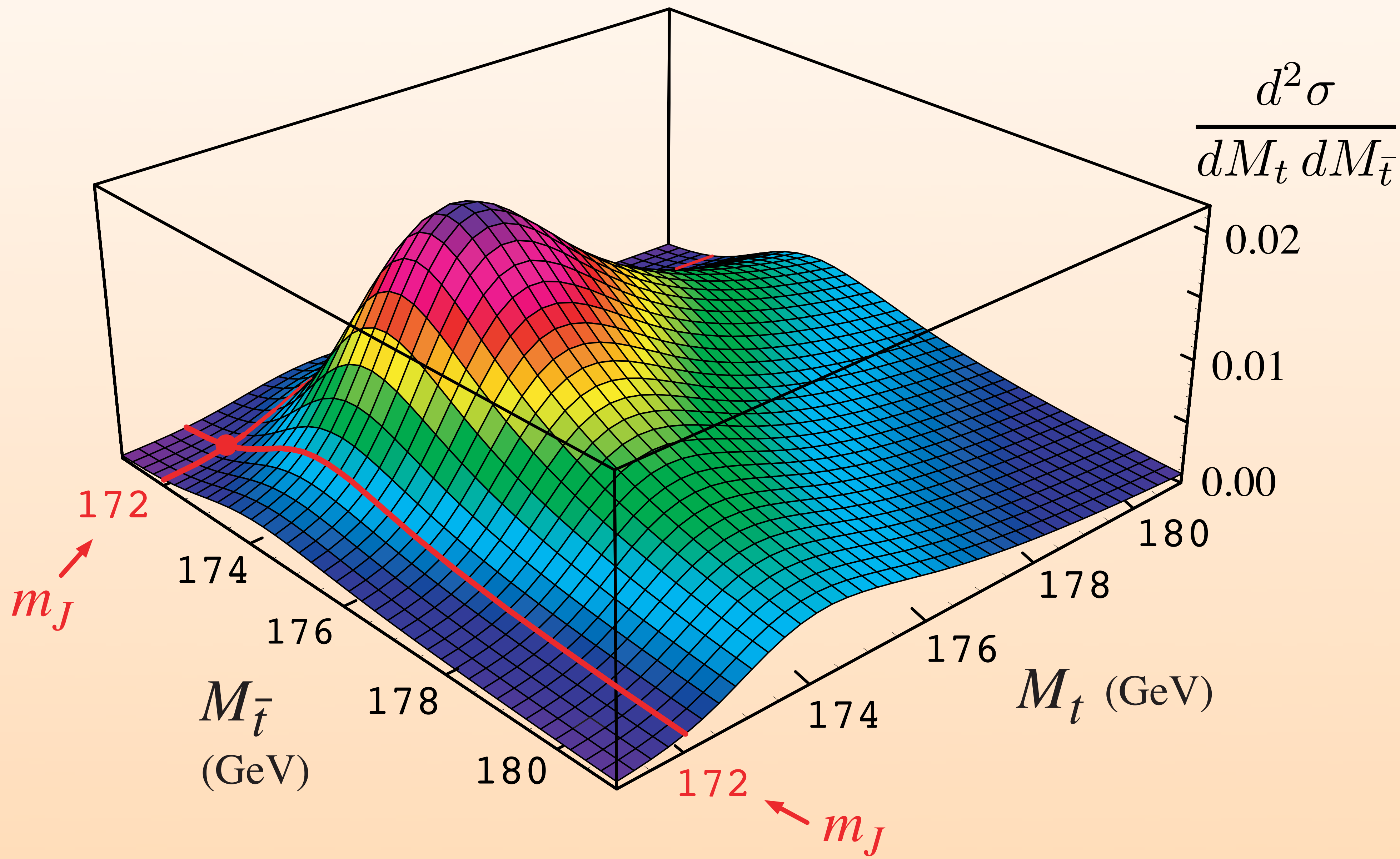
$$F\Big(M_t, M_{\bar t}, m_J, \frac{Q}{m_J}\Big) = \int_{-\infty}^{\infty} d\ell^+\, d\ell^-\; P\Big(\hat s_t - \frac{Q\ell^+}{m_J} - \frac{Q\bar\Delta(\mu_\Lambda)}{m_J}, \hat s_{\bar t} - \frac{Q\ell^-}{m_J} - \frac{Q\bar\Delta(\mu_\Lambda)}{m_J}, \mu_\Lambda\Big)\, S^{\rm mod}(\ell^+, \ell^-, 0)$$

perturbative part is analytic

$$P\big(\hat s_t, \hat s_{\bar t}, \mu_\Lambda\big) = 4M_t M_{\bar t}\,\Gamma_t^2\, H_Q(Q,\mu_h)\, U_{H_Q}(Q,\mu_h,\mu_m)\, H_m(m,\mu_m)\, U_{H_m}\Big(\frac{Q}{m_J},\mu_m,\mu_\Lambda\Big)$$
$$\times\, G_+\Big(\hat s_t, \frac{Q}{m_J}, \Gamma_t, \mu_\Lambda\Big) G_-\Big(\hat s_{\bar t}, \frac{Q}{m_J}, \Gamma_t, \mu_\Lambda\Big)\,.$$

$$G_\pm\Big(\hat s, \frac{Q}{m_J}, \Gamma_t, \mu_\Lambda\Big) \equiv \int_{-\infty}^{+\infty} d\hat s'\, d\hat s''\, d\ell'\;\; U_B(\hat s - \hat s', \mu_\Lambda, \mu_\Gamma)$$
$$\times\, B_\pm^{\Gamma=0}\Big(\hat s' - \hat s'' - \frac{Q}{m_J}\,\ell', \mu_\Gamma, \delta m\Big)\, \tilde S_{\rm part}(\ell', \mu_\Lambda, \delta_1)\, \frac{\Gamma_t}{\pi(\hat s''^{\,2} + \Gamma_t^2)}$$

NLL Cross-Section Results

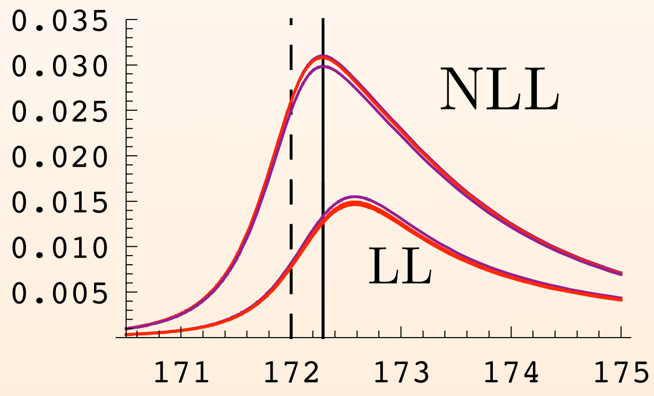


$\sim 2\,\mathrm{GeV}$ shift from the soft radiation

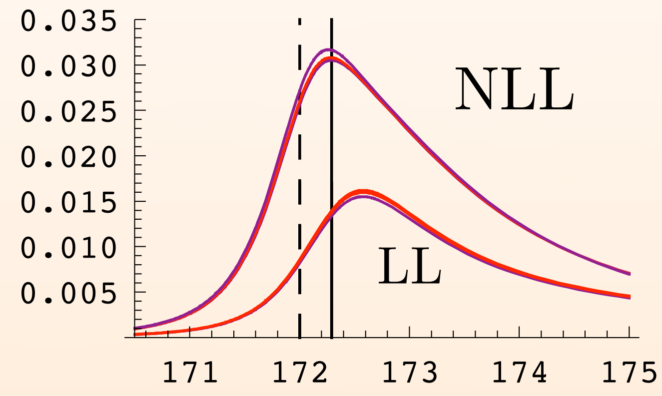
Perturbative corrections

$P(M_t, M_t)$ versus M_t

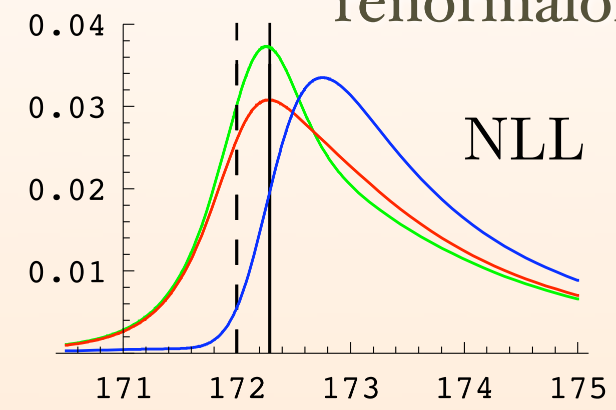
$\mu_m = 86, 172, 344\,\mathrm{GeV}$



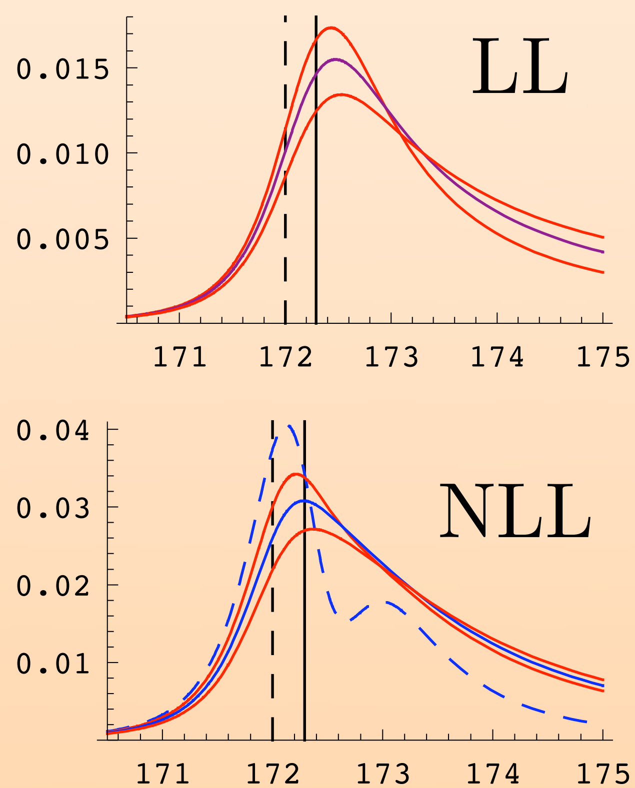
$\mu_Q = 430, 860, 1720\,\mathrm{GeV}$.



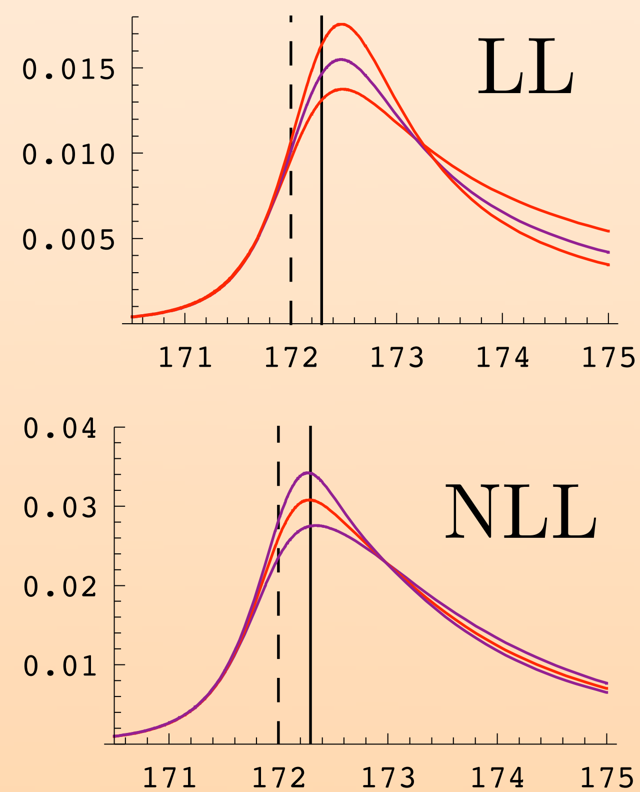
effect of removing renormalons (gives red)



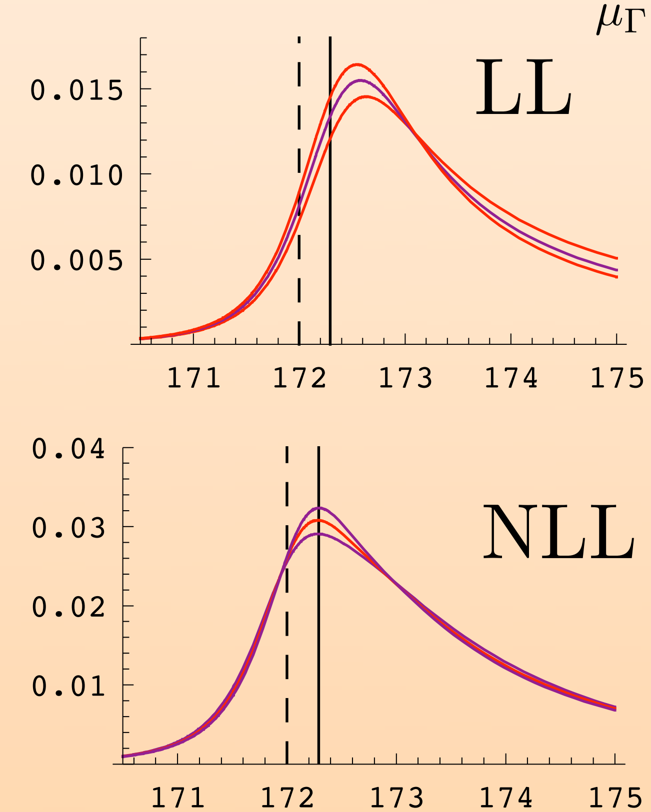
$\mu_\Gamma = 3.3, 5, 7.5\,\mathrm{GeV}$



$\mu_\Lambda = 0.8, 1.0, 1.2\,\mathrm{GeV}$



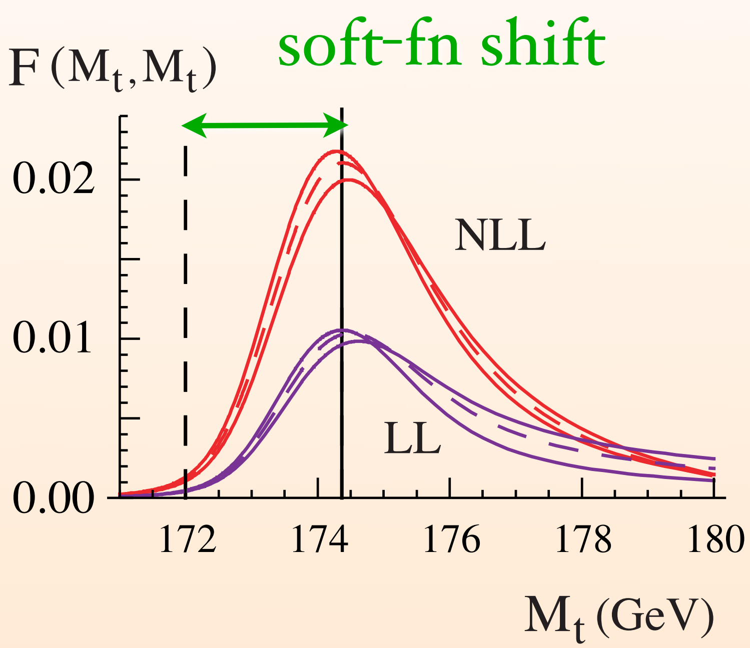
$\mu_\Lambda = 0.8, 1.0, 1.2\,\mathrm{GeV}$ with $\mu_\Gamma/\mu_\Lambda = Q/m$

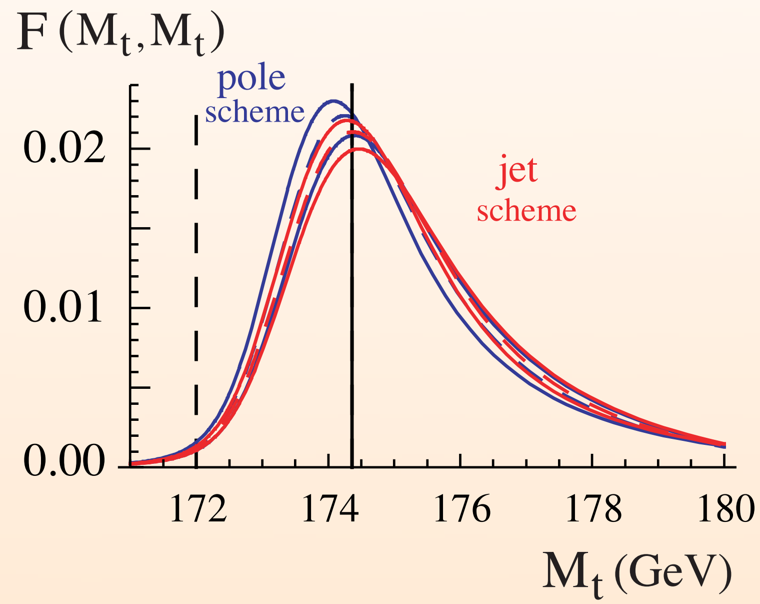


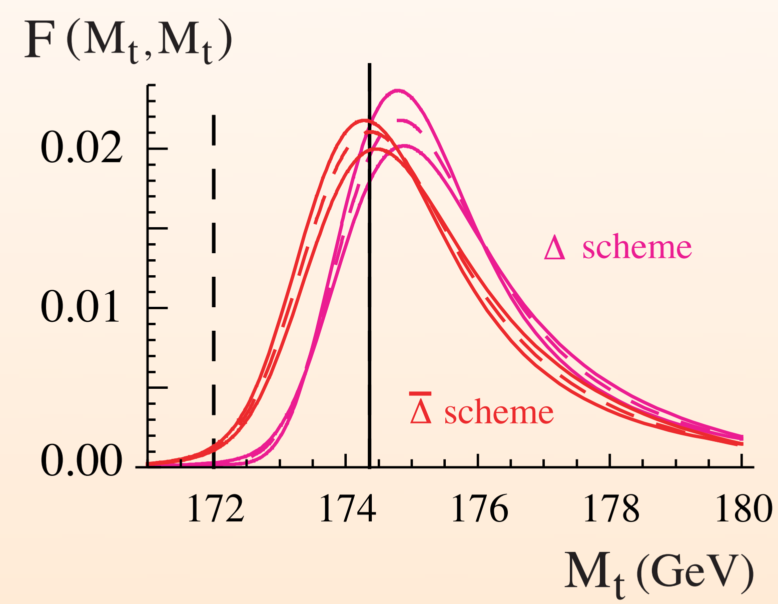
Normalized Cross-Section

$\mathrm{F}(M_t, M_t)$ versus M_t.

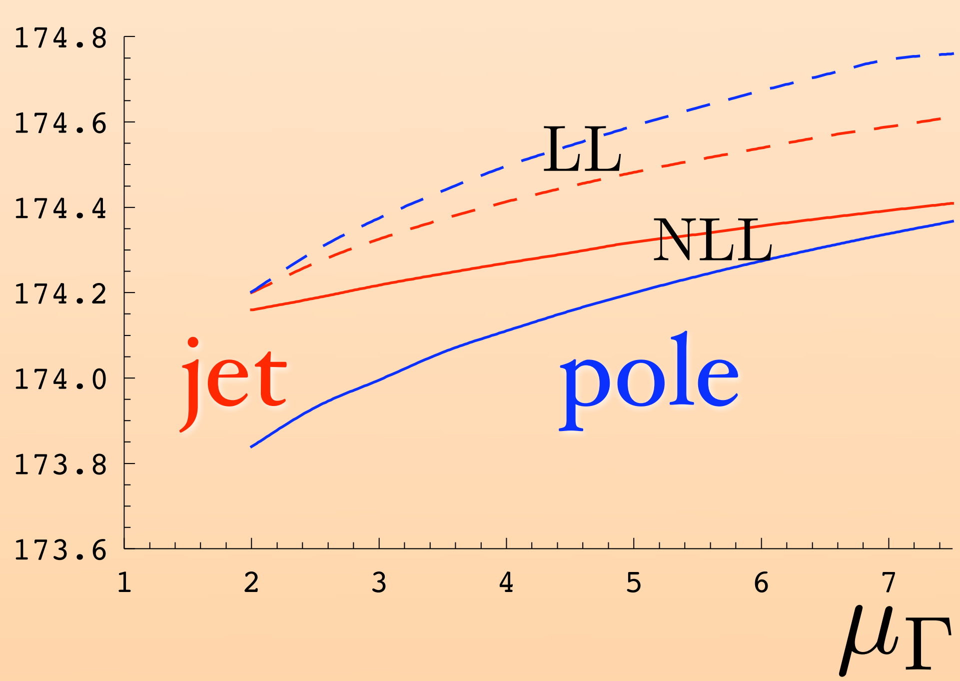
$\mu_\Gamma = 3.3, 5, 7.5\,\mathrm{GeV}$



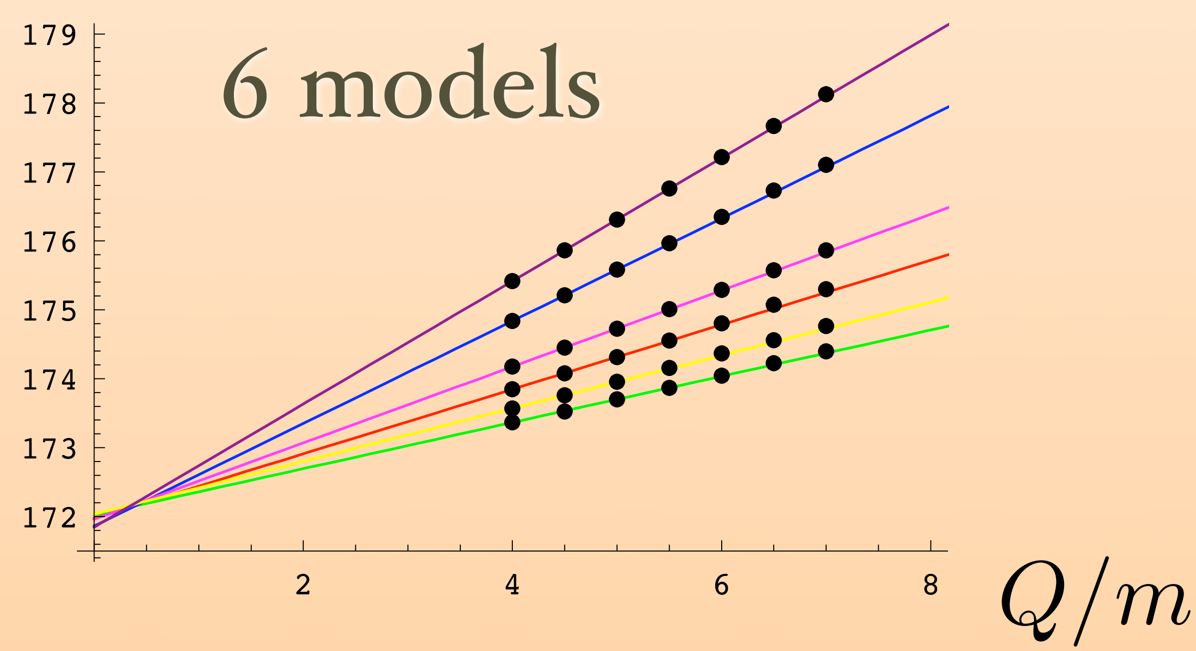




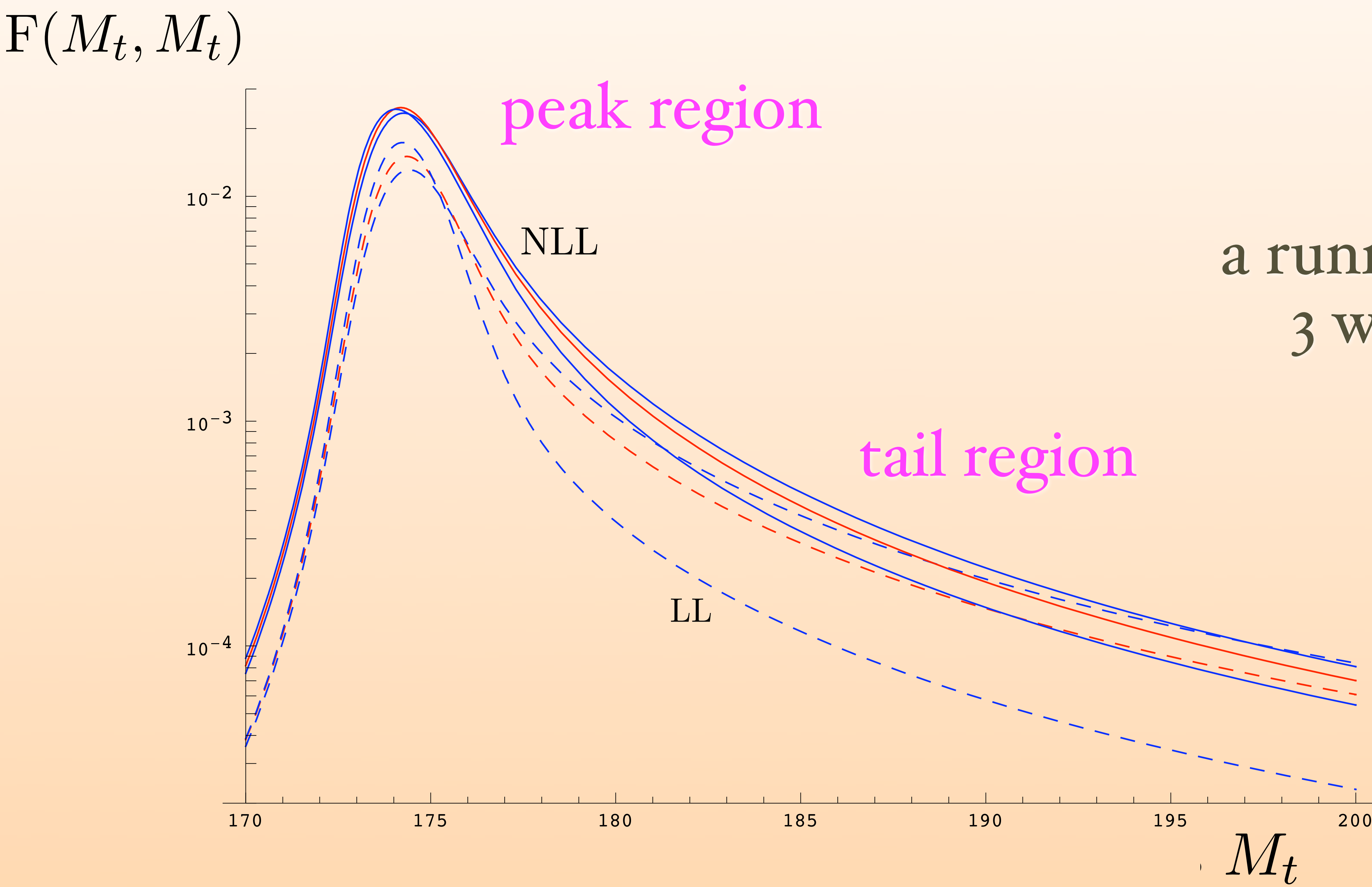
Peak Positions vs. μ_Γ



Peak Positions vs. Q/m



Beyond the peak region



a running μ_Γ
3 ways

What (if anything) can be said about the Tevatron mass?

- Given that top decay is described by a Breit-Wigner, we know that the mass should be close to a pole mass (top-resonance mass scheme)

$$m_{\mathrm{pole}} = m(R,\mu) + \delta m(R,\mu)\,, \qquad \delta m(R,\mu) = R \sum_{n=1}^{\infty} \sum_{k=0}^{n} a_{nk} \left[\frac{\alpha_s(\mu)}{4\pi}\right]^n \ln^k\left(\frac{\mu}{R}\right)$$

$$R \sim \Gamma$$

- Recently we derived an RGE for R, which allows us to smoothly connect these schemes to $\overline{\mathrm{MS}}$ where $R = \overline{m}(\mu)$

Hoang, Jain, Scimemi, I.S. (arXiv:0803.4214)

- We can estimate the scheme uncertainty of the Tevatron measurement by varying the initial $R = R_0 = 3^{+6}_{-2}\,\mathrm{GeV}$ (since any such mass is any equally good short distance scheme):

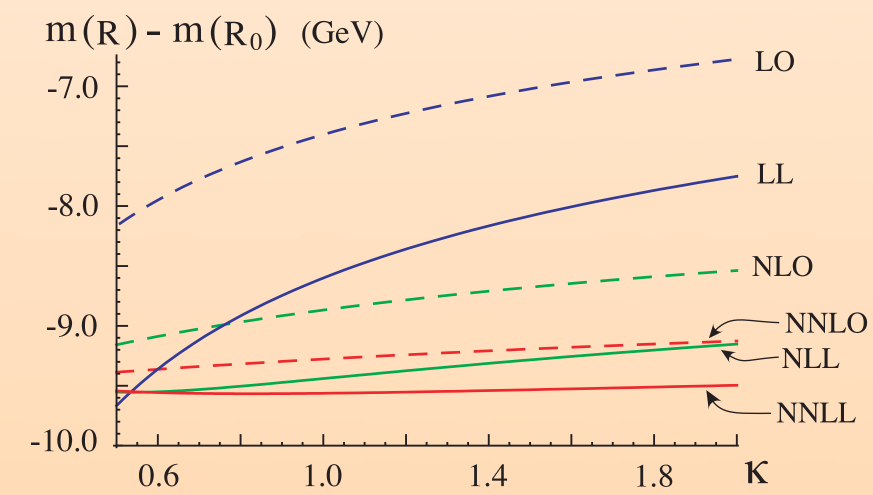
$$m_t(R_0) = 172.6 \pm 1.4\,\mathrm{GeV}$$

$$\overline{m}_t(\overline{m}_t) = 163.0 \pm 1.3 \begin{smallmatrix}+0.6\\-0.3\end{smallmatrix} \pm 0.05\,\mathrm{GeV}$$

scheme uncertainty

$R_0 = 3^{+6}_{-2}\,\mathrm{GeV}$

conversion uncertainty is small (3 loop with RGE)



Summary & Outlook

Top Jets

- Discussed a factorization theorem for invariant mass distributions for massive unstable particles: $e^+e^- \to t\bar{t}$ separation of perturbative and non-perturbative effects for ILC

- Systematic relation of peak to a Lagrangian mass parameter:

  What mass is measured? "Jet mass"

- Effective Field Theory: can be extended to higher orders in the power and perturbative expansions

Future:

- Reexamine LEP massless jet data for high precision soft function
- Extension to large pT events for LHC