Recent progress in NNLO calculations

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Loopfest VII, Buffalo, may 2008

Introduction

The dynamics of hard scattering processes is nowadays remarkably well described by perturbative QCD

Until few years ago the standard for QCD theoretical predictions was essentially limited to NLO (plus possibly the all-order resummation of some logarithmically enhanced terms)

LO predictions can give only the order of magnitude for cross sections and distributions:

- the scale of α_S is not defined
- jets $\Leftrightarrow$ partons: jet structure starts to appear only beyond LO

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$\Rightarrow$ **NNLO is thus the first order at which a reliable estimate of the error can be given**

Does it mean that NNLO calculations are essential for every process ?

Well, we can say that NNLO predictions are desirable at least in the following cases:

- For those processes whose NLO corrections are comparable to the LO contributions

  ➡ e.g. Higgs production at hadron colliders

- For those benchmark processes measured with high experimental accuracy

  - α_S measurements from e^+e^- event shape variables

  ➡ - W, Z hadroproduction

  - heavy quark hadroproduction

  ....

- For some important background processes

  ➡ e.g. WW for Higgs boson searches

Ingredients of a NNLO calculation

Let us assume that the process involves n partons at LO ➡ we need:

- Double virtual contribution with n resolved partons

$$|\mathcal{M}_n^{(1)}|^2 + \langle \mathcal{M}_n^{(2)} | \mathcal{M}_n^{(0)} \rangle + \text{c.c.}$$

- Real-virtual contribution with 1 unresolved parton

$$\langle \mathcal{M}_{n+1}^{(1)} | \mathcal{M}_{n+1}^{(0)} \rangle + \text{c.c.}$$

- Double-real contribution with 2 unresolved partons

$$|\mathcal{M}_{n+2}^{(0)}|^2$$

Difficulty: they are affected by different kinds of singularities

- UV sing. affect only virtual corrections ➡ removed by renormalization
- IR singularities present in all the three contributions

Unfortunately the pattern of the cancellation of IR singularities is much more involved than at NLO !

(Fully) inclusive processes

In the case of one-scale quantities double real, real virtual and double virtual contributions can be analytically computed and the singularities explicitly cancelled

- DIS structure functions

E. Zijlstra, W. Van Neerven (1992)

- Single hadron production

P.J.Rijken, W.L.Van Neerven (1997)

- DY lepton pair production

R.Hamberg, W.Van Neerven, T.Matsuura (1991)

- Higgs boson production

R.Harlander, W.B. Kilgore (2002)

C. Anastasiou, K. Melnikov (2002)

V. Ravindran, J. Smith, W.L.Van Neerven (2003)

....

+

Vector boson rapidity distribution

➜ modelling the phase space constraint with an effective "propagator"

C.Anastasiou, K.Melnikov, L.Dixon,F.Petriello (2003)

But real experiments have finite acceptances !

What about more exclusive processes?

Many of the ingredients for NNLO corrections available since long time

Example: $e^+e^- \to 3$ jets

- Tree amplitude for $e^+e^- \to 5$ partons K. Hagiwara, D. Zeppenfeld (1989)
  F.A.Berends, W.Giele, H.Kuijf (1989)
- One-loop amplitude for $e^+e^- \to 4$ partons
  N. Glover, D. Miller (1996)
  Z.Bern, L.Dixon, D.Kosower,S.Weinzierl (1996,1997)
  J. Campbell, N. Glover, D. Miller (1997)
- Two-loop amplitude for $e^+e^- \to 3$ partons
  L.W. Garland et al. (2002)

Example: Drell-Yan

T.Matsuura, W.Van Neerven (1988)

R.Hamberg, W.Van Neerven, T.Matsuura (1991)

➡ Amplitudes known since more than 15 years !

Despite this fact until recently the computation of the corresponding NNLO corrections could not be performed

The IR singularity structure of the three contributions has now been understood

S. Catani (1998); J.Campbell, N. Glover (1998)
S. Catani, MG (1999); Z.Bern, V. Del Duca, W. Kilgore, C. Schmidt (1999), D. Kosower, P. Uwer (1999), S. Catani, MG (2000)
G.Sterman, M. Tejeda-Yeomans (2002)

However the organization of the calculation into finite pieces that can be integrated numerically is still a formidable task

Two main strategies have been followed:

- **Sector decomposition**
- **Subtraction method**

Sector decomposition

K. Hepp (1966)
T. Binoth, G.Heinrich (2000,2004)
C.Anastasiou, K.Melnikov, F.Petriello (2004)

Sector decomposition as implemented by Anastasiou and collaborators works by dividing the integration region into sectors each containing a single singularity that can be made explicit by expansion into distributions

➡ This leads to a fully automated procedure by which the coefficients of the poles as well as finite terms can be computed numerically

The method has been successfully applied to a number of important fully exclusive NNLO computations

- Higgs and vector boson production in hadron collisions

C.Anastasiou, K.Melnikov, F.Petriello (2004)
K.Melnikov, F.Petriello (2004)

- NNLO QED computation of muon decay

C.Anastasiou, K.Melnikov, F.Petriello (2005)

- Semileptonic decay $b \longrightarrow c\, l\, \bar{\nu}_l$

K.Melnikov (2008)

➡ see talk by Melnikov

Example: vector boson production

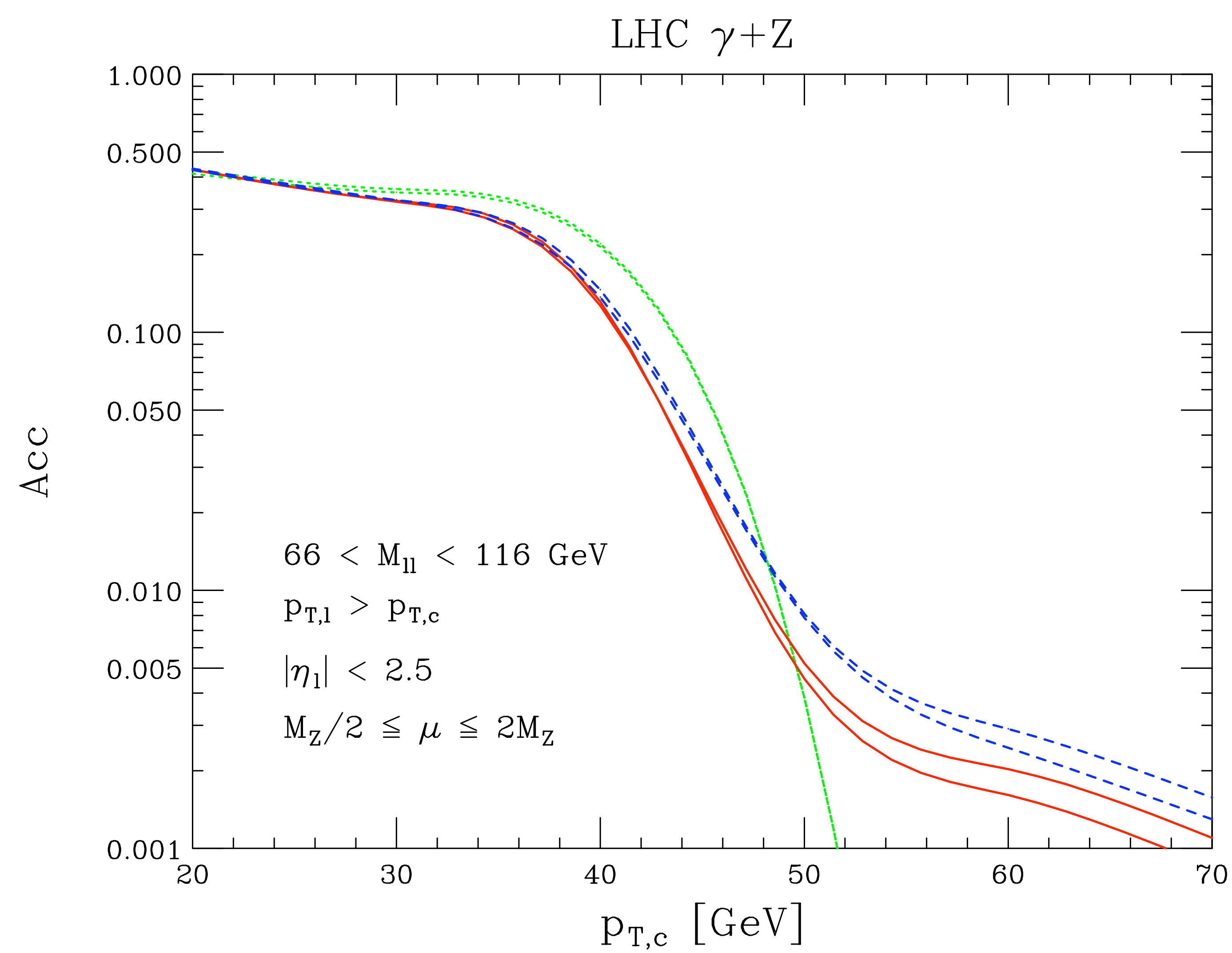
K.Melnikov, F.Petriello (2006)

$Z + \gamma^*$ production at the LHC

Acceptance as the function of lepton p_T cut

The effect is at the percent level except for $p_{T,c} \gtrsim 30$ GeV

This is because LO result has a kinematical boundary for $p_{T,c} = M_V/2$



➡ The calculation becomes NLO beyond this region

Calculation implemented in the public program FEWZ

➡ Useful tool to validate MC event generators

Subtraction method

R.K. Ellis, D.A.Ross, A.E.Terrano (1981)
S.Frixione, Z.Kunszt, A. Signer (1995)
S.Catani, M. Seymour (1996)

$$d\sigma = \int_{n+1} r d\Phi_{n+1} + \int_n v d\Phi_n$$

$$d\sigma = \int_{n+1} \left(r d\Phi_{n+1} - \tilde{r} d\tilde{\Phi}_{n+1} \right) + \int_{n+1} \tilde{r} d\tilde{\Phi}_{n+1} + \int_n v d\Phi_n$$

Add and subtract a local counterterm with the same singularity structure of the real contribution that can be integrated analytically over the phase space of the unresolved parton

How to extend this procedure to NNLO ?

This absolutely non trivial issue has attracted quite an amount of work

Goal ➡ Formulate a general scheme that can be possibly applied to any process

D. Kosower (1998,2003,2005)
S. Weinzierl (2003)
S. Frixione, MG (2004)
A. & T. Gehrmann, N. Glover (2005)
G, Somogyi, Z. Trocsanyi, V. Del Duca (2005, 2007)

At present the only approach that has been proven to work is the antenna subtraction method by A. & T. Gehrmann and Glover

Counterterms constructed from antennae extracted from physical matrix elements

It led to the successful completion of the NNLO calculation of $e^+e^- \to 3$ jets

Impressive achievement of a five years project !

A. & T. Gehrmann, N. Glover,
G. Heinrich (2007)

Important impact on α_S measurement

➜ see talk by Gehrmann

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→ see talk by Gehrmann

For such a tough calculation an independent check would be welcome

The approach by Trocsanyi et al. is based on the subtraction of counterterms constructed by the direct combination of the universal kernels controlling the soft and collinear singularities

 fully local counterterms → better numerical convergence

 analytic integration over unresolved partons much more difficult

→ no complete result yet !

A shortcut

S. Catani, MG (2007)

Let us consider a specific, though important class of processes: the production of colourless high-mass systems F in hadron collisions (F may consist of lepton pairs, vector bosons, Higgs bosons......)

At LO it starts with $c\bar{c} \rightarrow F$

Strategy: start from NLO calculation of F+jet(s) and observe that as soon as the transverse momentum of the F $q_T \neq 0$ one can write:

$$d\sigma^F_{(N)NLO}|_{q_T \neq 0} = d\sigma^{F+\text{jets}}_{(N)LO}$$

Define a counterterm to deal with singular behaviour at $q_T \rightarrow 0$

But.....
the singular behaviour of $d\sigma^{F+\text{jets}}_{(N)LO}$ is well known from the resummation program of large logarithmic contributions at small transverse momenta

G. Parisi, R. Petronzio (1979)
J. Collins, D.E. Soper, G. Sterman (1985)
S. Catani, D. de Florian, MG (2000)

choose $d\sigma^{CT} \sim d\sigma^{(LO)} \otimes \Sigma^F(q_T/Q)$

where $\Sigma^F(q_T/Q) \sim \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \sum_{k=1}^{2n} \Sigma^{F(n;k)} \frac{Q^2}{q_T^2} \ln^{k-1} \frac{Q^2}{q_T^2}$

Then the calculation can be extended to include the $q_T = 0$ contribution:

$$d\sigma^F_{(N)NLO} = \mathcal{H}^F_{(N)NLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+\text{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO}\right]$$

where I have subtracted the truncation of the counterterm at (N)LO and added a contribution at $q_T = 0$ to restore the correct normalization

The function $\mathcal{H}^F$ can be computed in QCD perturbation theory

$$\mathcal{H}^F = 1 + \left(\frac{\alpha_S}{\pi}\right) \mathcal{H}^{F(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{F(2)} + \ldots\ldots$$

For a generic $pp \to F + X$ process:

- At NLO we need a LO calculation of $d\sigma^{F+\mathrm{jet(s)}}$ plus the knowledge of $d\sigma^{CT}_{LO}$ and $\mathcal{H}^{F(1)}$
  - the counterterm $d\sigma^{CT}_{LO}$ requires the resummation coefficients $A^{(1)}, B^{(1)}$ and the one loop anomalous dimensions
  - the general form of $\mathcal{H}^{F(1)}$ is known

D. de Florian, MG (2000)
G. Bozzi, S. Catani, D. de Florian, MG (2005)

- At NNLO we need a NLO calculation of $d\sigma^{F+\mathrm{jet(s)}}$ plus the knowledge of $d\sigma^{CT}_{NLO}$ and $\mathcal{H}^{F(2)}$
  - the counterterm $d\sigma^{CT}_{NLO}$ depends also on the resummation coefficients $A^{(2)}, B^{(2)}$ and on the two loop anomalous dimensions
  - the general form of $\mathcal{H}^{F(2)}$ is not known.....

........but we have computed $\mathcal{H}^{H(2)}$ for Higgs production !

S. Catani, MG (2007)

→ **since H+1 jet is known to NLO we have all the necessary ingredients to go to NNLO**

HNNLO

http://theory.fi.infn.it/grazzini/codes.html

HNNLO is a numerical program to compute Higgs boson production through gluon fusion in pp or $p\bar{p}$ collisions at LO, NLO, NNLO

- $H \to \gamma\gamma \quad (\texttt{higgsdec} = 1)$
- $H \to WW \to l\nu l\nu \quad (\texttt{higgsdec} = 2)$
- $H \to ZZ \to 4l$
  - $H \to e^+e^-\mu^+\mu^- \quad (\texttt{higgsdec} = 31)$
  - $H \to e^+e^-e^+e^- \quad (\texttt{higgsdec} = 32)$

→ includes appropriate interference contribution

The user can choose the cuts and plot the required distributions by modifying the `cuts.f` and `plotter.f` subroutines

Results: $gg \to H \to WW \to l\nu l\nu$

MG (2007)

Use *preselection cuts* as in Davatz. et al (2003)

see also C.Anastasiou, G. Dissertori, F. Stockli (2007)

$$p_T^l > 20 \text{ GeV}$$

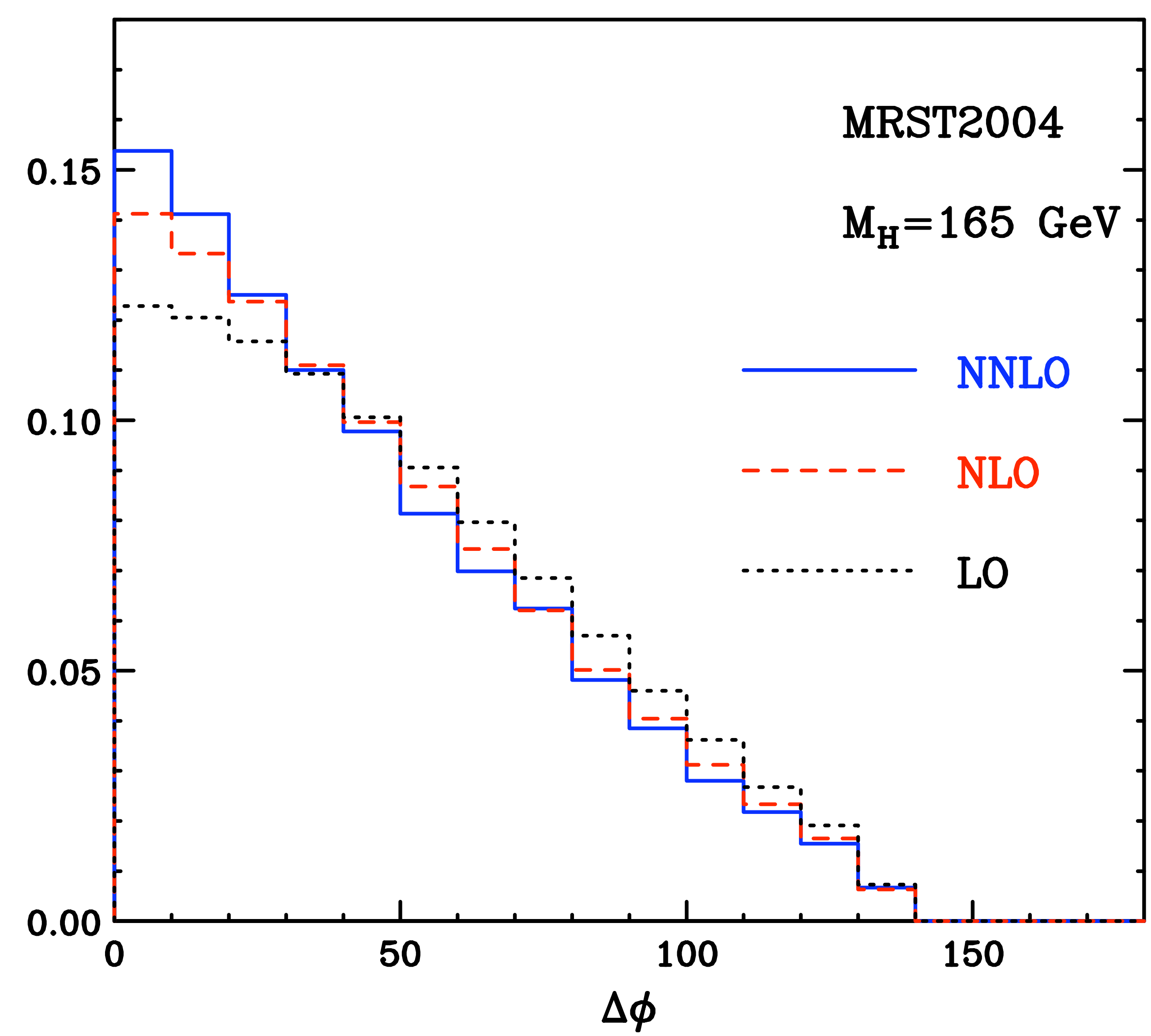
$$|y_l| < 2$$

$$p_T^{\text{miss}} > 20 \text{ GeV}$$

$$\Delta\phi < 135^o$$

$$m_{ll} < 80 \text{ GeV}$$

normalized $\Delta\phi$ **distribution**



The distributions appears to be steeper when going from LO to NLO and from NLO to NNLO

Use now *selection cuts* as in Davatz. et al (2003)

$$p_T^{\min} > 25\ \text{GeV} \qquad m_{ll} < 35\ \text{GeV} \qquad \Delta\phi < 45^o$$

$$35\ \text{GeV} < p_T^{\max} < 50\ \text{GeV} \qquad |y_l| < 2 \qquad p_T^{\text{miss}} > 20\ \text{GeV}$$

Results for

$p_T^{\text{veto}} = 30$ GeV

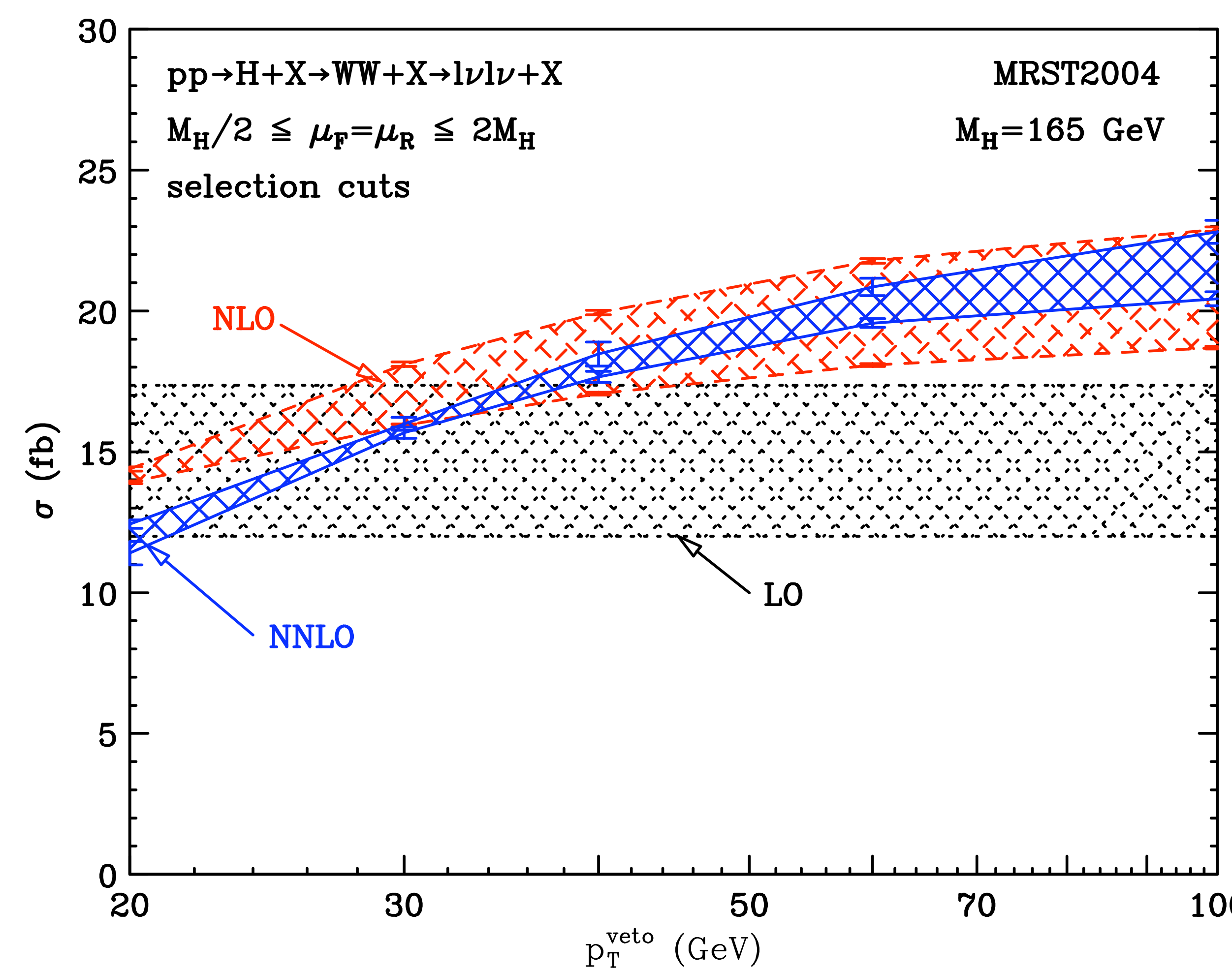
σ (fb)	LO	NLO	NNLO
$\mu_F = \mu_R = M_H/2$	17.36 ± 0.02	18.11 ± 0.08	15.70 ± 0.32
$\mu_F = \mu_R = M_H$	14.39 ± 0.02	17.07 ± 0.06	15.99 ± 0.23
$\mu_F = \mu_R = 2M_H$	12.00 ± 0.02	15.94 ± 0.05	15.68 ± 0.20

➡ **Impact of higher order corrections strongly reduced by selection cuts**

The NNLO band overlaps with the NLO one for $p_T^{\text{veto}} \gtrsim 30$ GeV

The bands do not overlap for $p_T^{\text{veto}} \lesssim 30$ GeV

NNLO efficiencies found in good agreement with MC@NLO



Anastasiou et al. (2008)

Summary & Outlook

- Fully exclusive NNLO calculations are important in many cases

  - they provide a precise estimate of higher order corrections when cuts are applied

  - the corresponding acceptances can be compared with those obtained with standard MC event generators

- After some years of work the first fully exclusive NNLO computations have appeared, most notably

  - Higgs and vector boson production in hadron collisions

  - $e^+e^- \rightarrow 3$ jets

- A new powerful method, based on sector decomposition complements the more traditional approach of the subtraction method

- What are the next NNLO calculations that could be performed ?

  - Vector boson pair production

    J. Campbell et al. (2007)
    S.Dittmaier et al. (2008)

    - $WW+\mathrm{jet}$ at NLO done
    - two loop correction known for $M_W^2 \ll s,t,u$ M. Chachamis et al. (2007)

  → important background for Higgs boson searches

  - Heavy quark production

    S. Dittmaier et al. (2007)

    - $t\bar{t}+\mathrm{jet}$ at NLO done
    - two loop amplitude computed M. Czakon (2008)

  - Jets in hadron collisions ?

- It will be interesting to see if the methods adopted so far will be applicable to these more involved computations

  - The subtraction method allows the direct implementation of the NNLO calculation into a parton level event generator

  - It could be numerically more efficient than sector decomposition for more complicated processes