Towards the completion of the NNLO program for $\bar{B} \to X_s\gamma$: m_c-dependent matrix elements

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Buffalo May 2008

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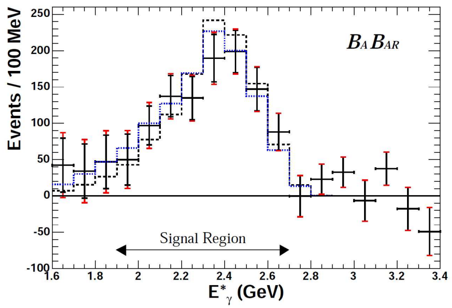
Motivations

- $\bar{B} \to X_s\gamma$ most precise short-distance information currently available for $\Delta B = 1$ FCNC

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm exp}_{E_\gamma > 1.6\,{\rm GeV}} = (3.55 \pm 0.26) \times 10^{-4}$$

[HFAG2006]



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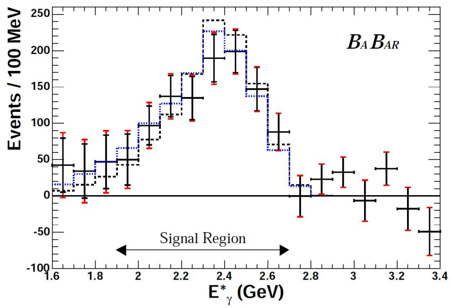

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm exp}_{E_\gamma > 1.6\,{\rm GeV}} = (3.55 \pm 0.26) \times 10^{-4}$$

[HFAG2006]

  - less sensitive to non-perturbative effects
    dominant ones: $\mathcal{O}(\Lambda^2/m_b^2)$, $\mathcal{O}(\Lambda^2/m_c^2)$, $\mathcal{O}(\alpha_s\Lambda/m_b)$

$$\Longrightarrow \quad \Gamma(\bar{B} \to X_s\gamma) \approx \Gamma(b \to X_s^{parton}\gamma)$$
$$= \Gamma(b \to s\gamma) + \Gamma(b \to s\gamma g) + \ldots$$

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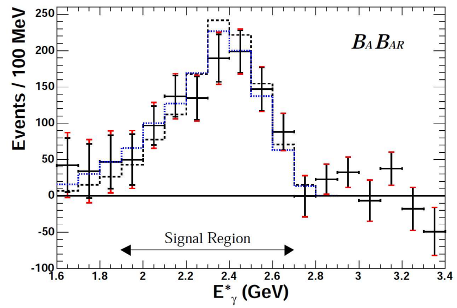

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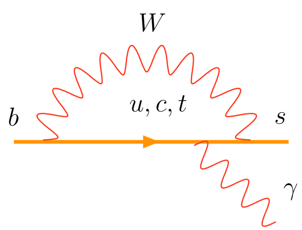
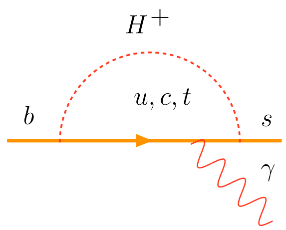
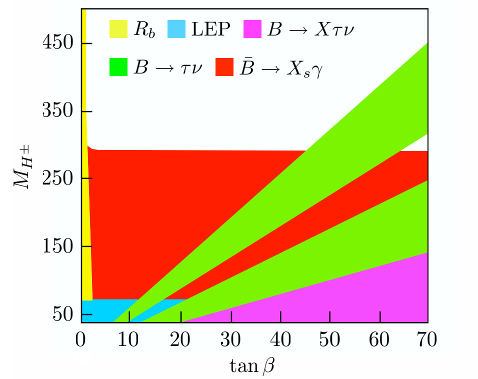
  - loop induced in SM and highly sensitive to new physics which is not suppressed by factors of α as compared to SM contributions







Motivations

- Theoretical error vs. experimental one:
  - $\mathcal{B}(\bar{B} \to X_s\gamma)^{\text{th,NLO}}_{E_\gamma > 1.6\,\text{GeV}} = (3.57 \pm 0.30) \times 10^{-4}$
    [Misiak et al 2001,Buras et al 2002]
  - $\mathcal{B}(\bar{B} \to X_s\gamma)^{\text{exp}} = (3.55 \pm 0.26) \times 10^{-4}$
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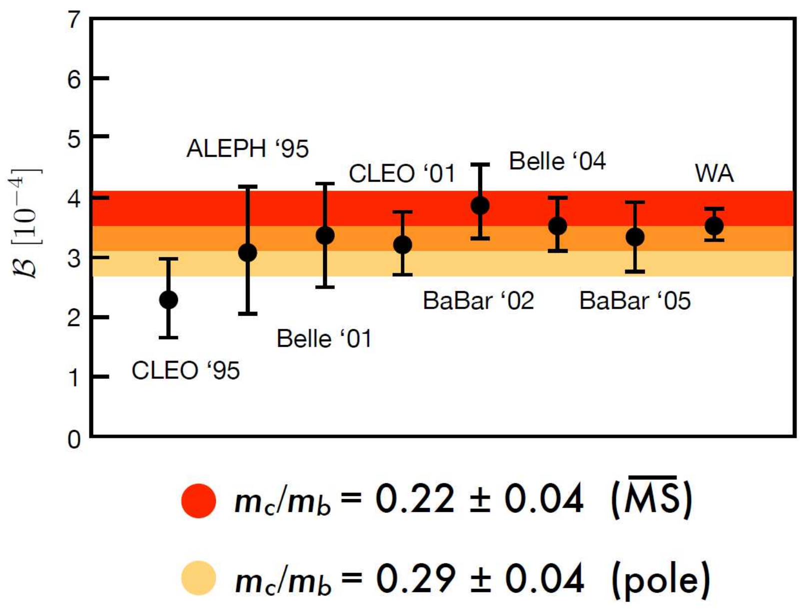
  Super-B factory: 5% uncertainty possible
  (more statistics, lower E_γ)



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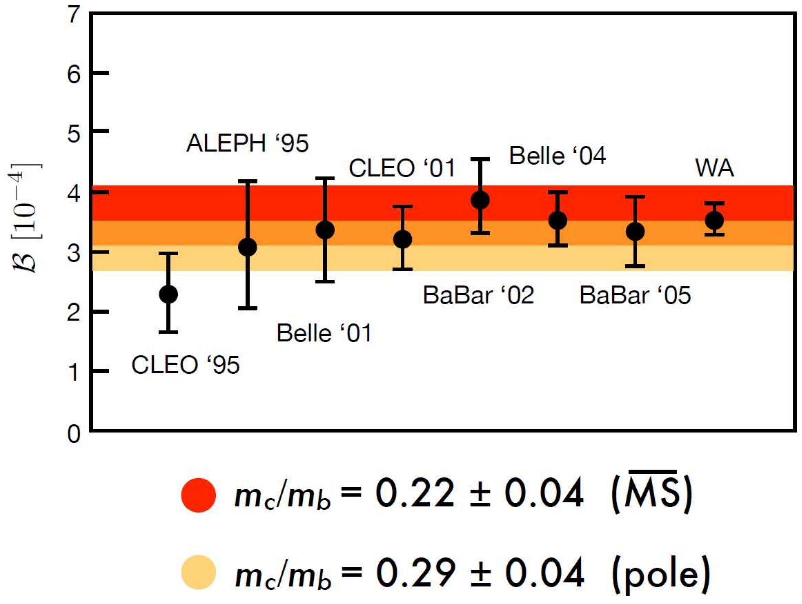

$\Longrightarrow$ strong constraints on new physics require better theoretical precision

Motivations

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm exp}_{E_\gamma > 1.6\,{\rm GeV}} = (3.55 \pm 0.26) \times 10^{-4}$$

[HFAG 2006]

- Contributions to the theory prediction

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > 1.6\,{\rm GeV}} = \mathcal{B}(\bar{B} \to X_c e\bar{\nu})_{\rm exp} \left[\frac{\Gamma(b\to s\gamma)}{\Gamma(b\to ce\bar{\nu})}\right]_{\rm LO\ EW} f\left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)}\right) \times$$

$$\times \Bigg\{ 1 + \underbrace{\underset{\sim 25\%}{\underset{\textbf{NLO}}{\mathcal{O}(\alpha_s)}} + \underset{\sim 7\%}{\underset{\textbf{NNLO}}{\mathcal{O}(\alpha_s^2)}} + \underset{\sim 4\%}{\mathcal{O}(\alpha_{\rm em})}}_{\textbf{perturbative corrections}} + \underbrace{\underset{\sim 1\%}{\mathcal{O}\left(\frac{\Lambda^2}{m_b^2}\right)} + \underset{\sim 3\%}{\mathcal{O}\left(\frac{\Lambda^2}{m_c^2}\right)} + \underset{<\sim 5\%}{\mathcal{O}\left(\frac{\Lambda}{m_b}\alpha_s\right)}}_{\textbf{non-perturbative corrections}} \Bigg\}$$

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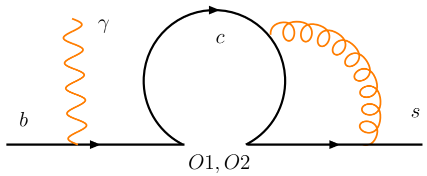
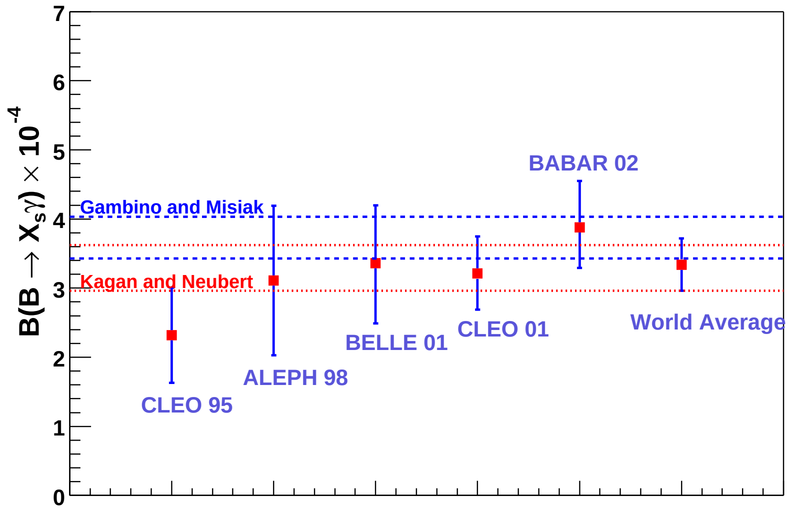
expected NNLO corrections to $\mathcal{B}$ ($\sim 7\%$) are of the same size as the experimental error

Motivations

- Charm quark mass definition ambiguity
  - dependence of $\mathcal{B}(\bar{B} \to X_s\gamma)^{theo}$ on m_c enters through the $\langle s\gamma|O_{1,2}|b\rangle$ which start contributing at $\mathcal{O}(\alpha_s)$



[Aubertetal 02]



  - $m_c^{pole}/m_b^{pole} = 0.29 \pm 0.02$
    $\mathcal{B}(\bar{B} \to X_s\gamma)^{theo} = (3.32 \pm 0.30) \times 10^{-4}$
  - $\overline{m}_c(m_b/2)/m_b^{pole} = 0.22 \pm 0.04$
    $\mathcal{B}(\bar{B} \to X_s\gamma)^{theo} = (3.70 \pm 0.30) \times 10^{-4}$

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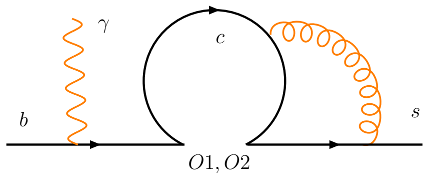

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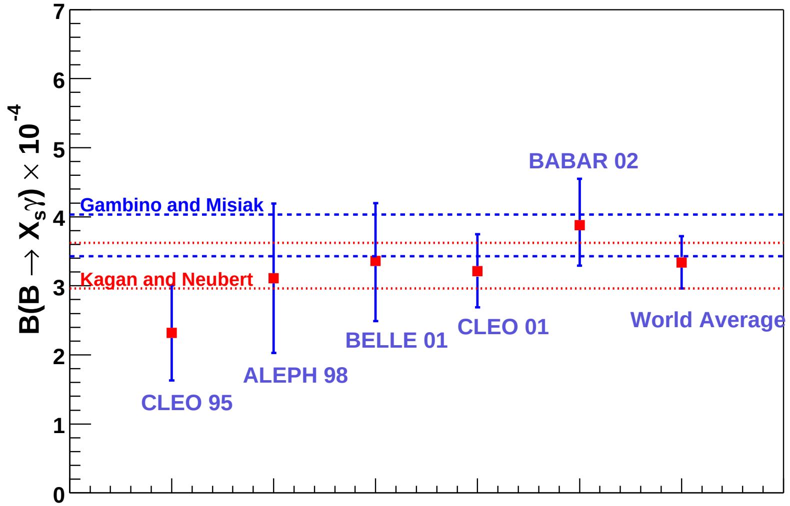

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- difference between using $\overline{m}_c(\mu)$ and m_c^{pole} is a NNLO effect in the branching ratio
  $\Longrightarrow$ resolving the ambiguity requires going to the NNLO level

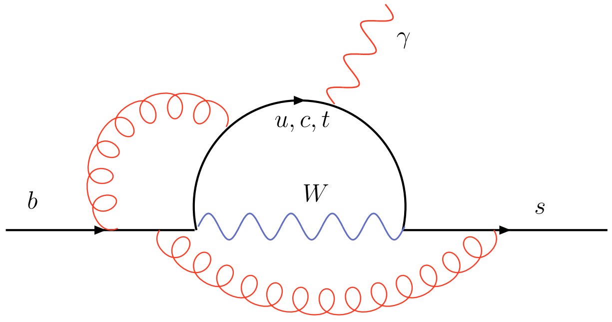
Theoretical framework

- diagrams involve scales with large hierarchy

  $M_W\,, M_t \gg m_b \gg m_s \implies$ large $\log\left(\frac{M_W^2}{m_b^2}\right)$

  $\longrightarrow$ resummation of $\alpha_s \log\left(\frac{M_W^2}{m_b^2}\right)$ is necessary

  using RG techniques



- start by introducing an effective theory without the heavy fields

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD\times QED}(u,d,s,c,b) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i C_i(\mu)\, O_i(\mu)$$

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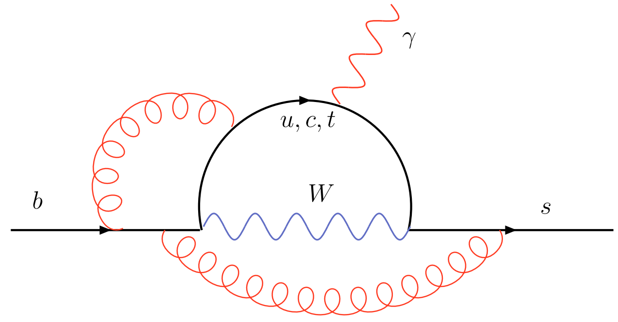

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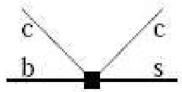
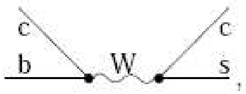
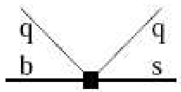
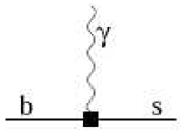
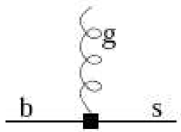
$O_{1,2} =$  $= (\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b),$ from , $|C_i(m_b)| \sim 1$

$O_{3,4,5,6} =$  $= (\bar{s}\Gamma_i b)\sum_q(\bar{q}\Gamma_i' q),$ $|C_i(m_b)| < 0.07$

$O_7 =$  $= \frac{e m_b}{16\pi^2}\, \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu},$ $C_7(m_b) \simeq -0.3$

$O_8 =$  $= \frac{g m_b}{16\pi^2}\, \bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu},$ $C_8(m_b) \simeq -0.15$

Theoretical framework

Calculation done in three steps:

- Matching find the Wilson coefficients $C_i(\mu)$ by comparing the full and the effective theory at the mass scale $\mu \approx M_W$
  $\Rightarrow$ no large logarithms and only vacuum diagrams

- Mixing compute the anomalous dimensions of the operators and solve the renormalization group equations to go down with the Wilson coefficients to $\mu \approx m_b$

$$\frac{d}{d\mu} C_j(\mu) = C_i(\mu)\gamma_{ij}(\mu)$$

- Matrix elements calculate the matrix elements of all the operators at $\mu \approx m_b \Rightarrow$ no large logarithms as no heavy masses are present

Current state of the art for NNLO corrections

1. Matching
   - 2-loop matching for (O_1, . . . ,O_6) [Bobeth,Misiak,Urban 00]
   - 3-loop matching for O_7 and O_8 [Misiak,Steinhauser 04]
2. Mixing
   - 3-loop: $(O_1, \ldots, O_6)$ and $(O_7,\ O_8)$ sectors [Gorbahn,Haisch 05] [Gorbahn,Haisch,Misiak 05]
   - 4-loop $(O_1, \ldots, O_6) \longrightarrow (O_7,\ O_8)$ [Czakon,Haisch,Misiak 06]

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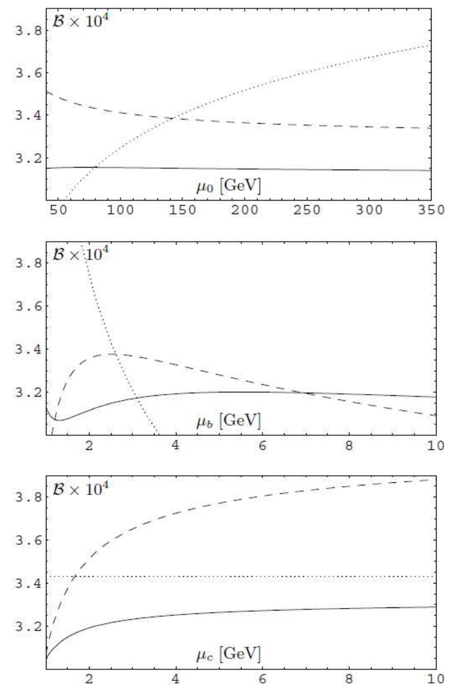
3. Matrix elements
   - O_1, O_2, O_7, O_8 large β_0 [Bieri,Greub,Steinhauser 03]
   - O_7 [Blokland,Czarnecki,Misiak,Slusarczyk,Tkachov 05] [Asatrian,Hovhannisyan,Poghosyan,Ewerth,Greub,Hurth 06]
   - O_7, photon spectrum [Melnikov,Mitov 05] [Asatrian,Ewerth,Ferroglia,Gambino,Greub 06]
   - O_1, O_2 leading term for $m_c \gg m_b$ [Misiak,Steinhauser 06]

The NNLO estimated Branching Ratio

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm theo}_{E_\gamma > 1.6\,{\rm GeV}} = (3.15 \pm 0.23) \times 10^{-4}$$

[Misiak et al 06] [Misiak,Steinhauser 06]

- Decomposition of Uncertainty
  - non-perturbative 5% $\mathcal{O}(\alpha_s \Lambda / m_b)$
  - parametric 3% $\alpha_s(M_Z)$, $\mathcal{B}^{exp}_{SL}$, $m_c \ldots$
  - m_c interpolation 3% ($O_{1,2}$ matrix elements)
  - higher order 3% (μ_b, μ_c, μ_0 dependence)



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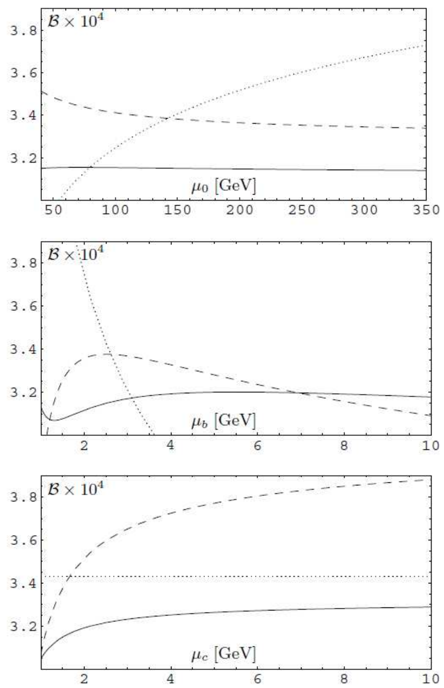

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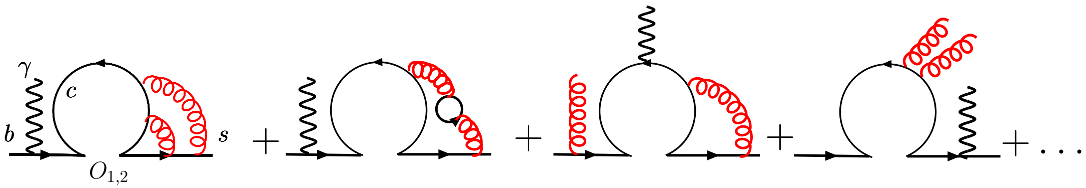
- source of the interpolation uncertainty is the missing $\mathcal{O}\left(\alpha_s^2\right)$ correction to $\langle s\gamma | O_{1,2} | b \rangle$

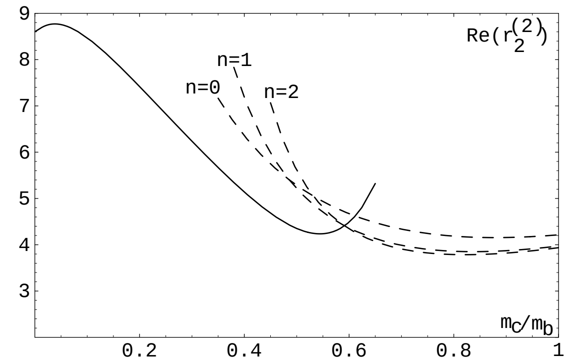


More about the interpolation uncertainty

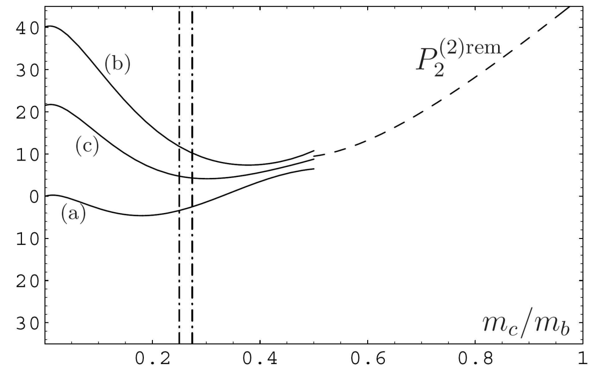
- $\mathcal{O}\left(\alpha_s^2\right)$ perturbative contribution to $\mathcal{B}(\bar{B} \to X_s\gamma)$: $\qquad P_2^{(2)} = \sum_{i,j=1}^{8} C_i^{(0)} C_j^{(0)} \left(n_f\, A_{ij} + B_{ij}\right)$

- using large β_0 approx. $\qquad P_2^{(2)} = \sum_{i,j=1}^{8} C_i^{(0)} C_j^{(0)} \left(\frac{-3}{2}\beta_0\, A_{ij} + B'_{ij}\right) = P_2^{(2),\beta_0} + P_2^{(2),rem}$
  - $P_2^{(2),\beta_0}$ known for $\langle s\gamma|O_{1,2,7,8}|b\rangle$

- expansions in limits $m_c/m_b \to 0$ and $m_c \gg m_b$ match nicely for Re$\langle s\gamma|O_2|b\rangle^{\beta_0}$
- good approximation already for $n = 0$
- no large $c\bar{c}$ threshold effects at $m_c = m_b/2$



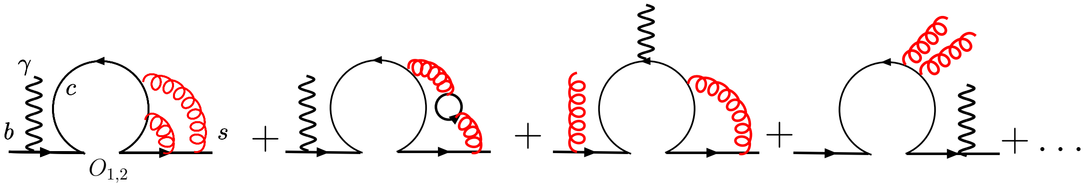
- calculate the leading term of large m_c expansion for $P_2^{(2),rem}$ and interpolate to physical m_c
- making assumptions for $P_2^{(2),rem}$ at $m_c = 0$ is the source of the interpolation uncertainty



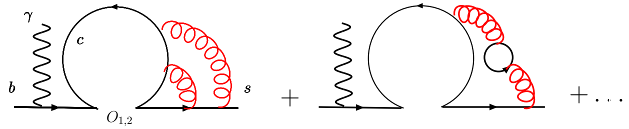
Reducing the overall uncertainty of $\mathcal{B}(\bar{B} \to X_s\gamma)^{\text{theo,NNLO}}_{E_\gamma > 1.6\,\text{GeV}}$

- removing the interpolation uncertainty

  $\Longrightarrow$ need a complete calculation of $\langle s\gamma|O_{1,2}|b\rangle$ at $m_c \neq 0$



$\longrightarrow$ working on the virtual part [R. B, Czakon, Schutzmeier]



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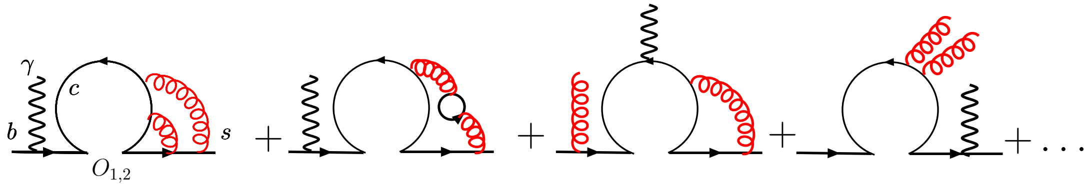

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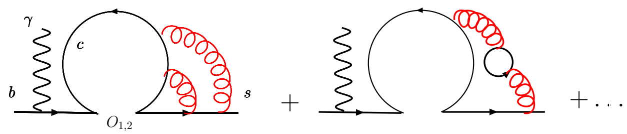

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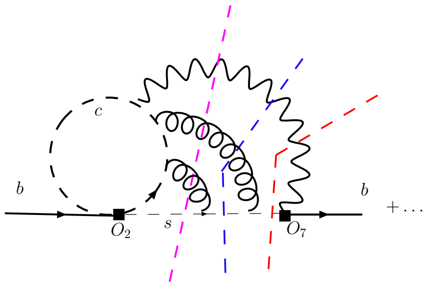

in progress [R. B, Czakon, Schutzmeier]

Removing the interpolation uncertainty: virtual part

- approx. 400 3-loop on-shell vertex diagrams with two scales m_b & m_c
- around 500 masters are involved in the bare amplitude
- symbolic reduction down to masters is not yet complete for the full 3-loop vertex
- $\mathcal{O}\left(\alpha_s^2 n_f\right)$ correction to $\langle s\gamma|O_{1,2}|b\rangle$:

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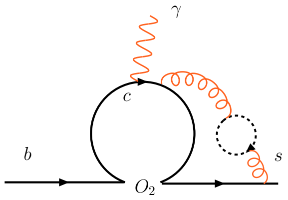

- masters were calculated with Mellin Barnes
  - first way: a numerical integration of the MB representations is performed for specific values of z using the `MB` package [MB : Czakon 05] , [MBrepresentation : Chachamis, Czakon 06]
  - second way:
    - perform an expansion in $z = m_c^2/m_b^2$ by closing contours
    - coefficients of the expansion are given by at most a 1-dimensional MB integral expressed as a sum over residues
    - sum these infinite series using **XSummer** [Moch & Uwer 05]

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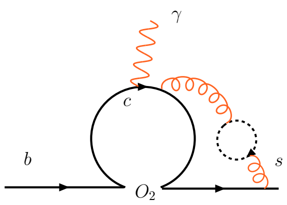
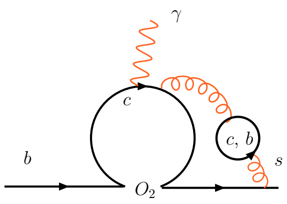
- $\mathcal{O}\left(\alpha_s^2 n_f\right)$ correction to $\langle s\gamma|O_{1,2}|b\rangle$:

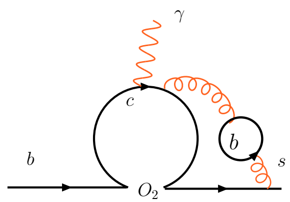


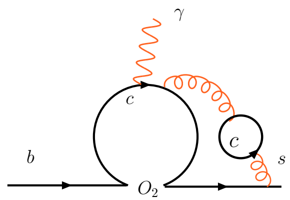


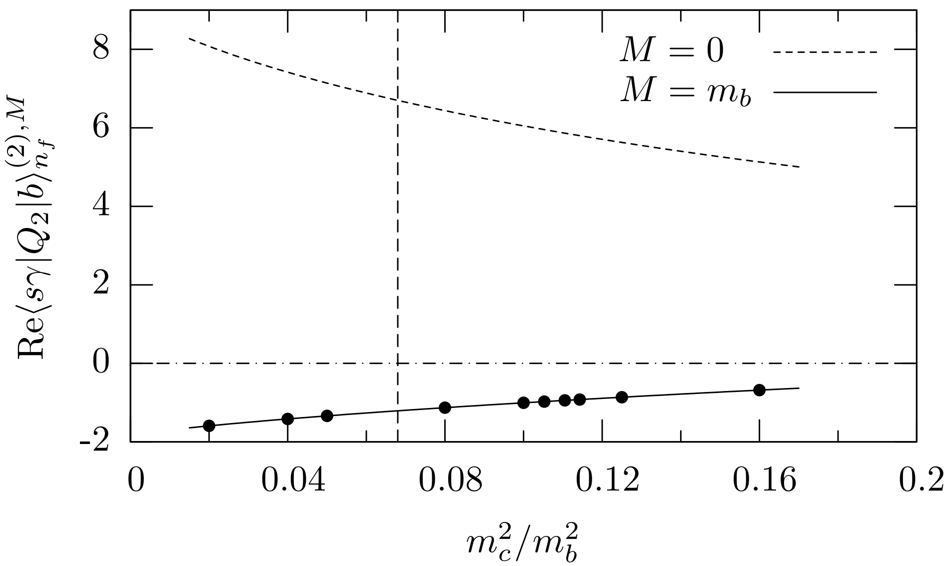
- masters were calculated with Mellin Barnes
  - first way: a numerical integration of the MB representations is performed for specific values of z using the MB package [MB : Czakon 05] , [MBrepresentation : Chachamis, Czakon 06]
  - second way:
    - perform an expansion in $z = m_c^2/m_b^2$ by closing contours
    - coefficients of the expansion are given by at most a 1-dimensional MB integral expressed as a sum over residues
    - sum these infinite series using **XSummer** [Moch & Uwer 05]
- MB alone was not enough to calculate all the masters due to poor convergence
- use differential equations solved numerically
  - boundaries were obtained using diagrammatic large mass expansion for $m_c \gg m_b$
    $\longrightarrow$ more about this method later

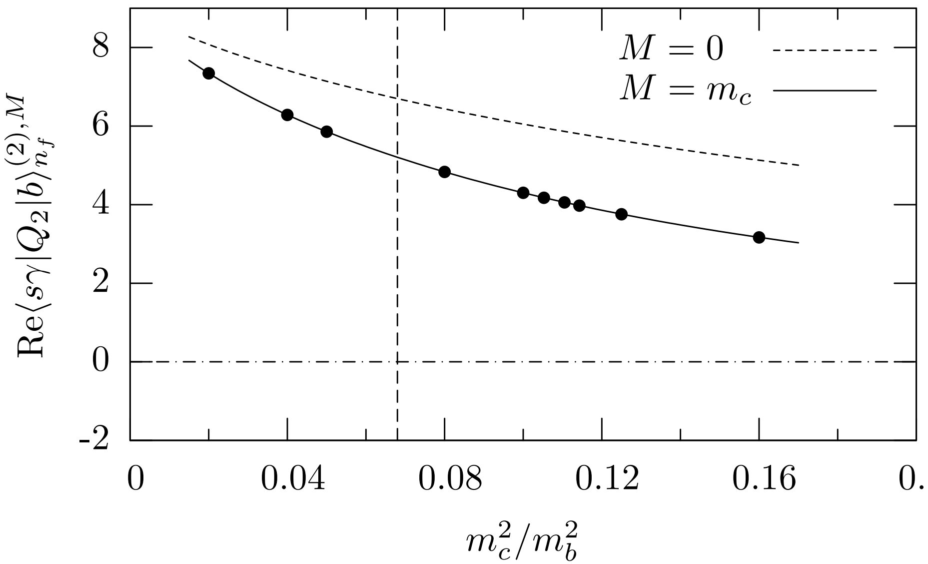
$\langle s\gamma|O_2|b\rangle_{\mathcal{O}(\alpha_s^2 n_f)}$

- Results for the massive fermionic contributions:









- massless approximation overestimates the massive b result and has the opposite sign !

- less pronounced differences for the c-quark $\longrightarrow$ moderate negative corrections wrt. massless approximation

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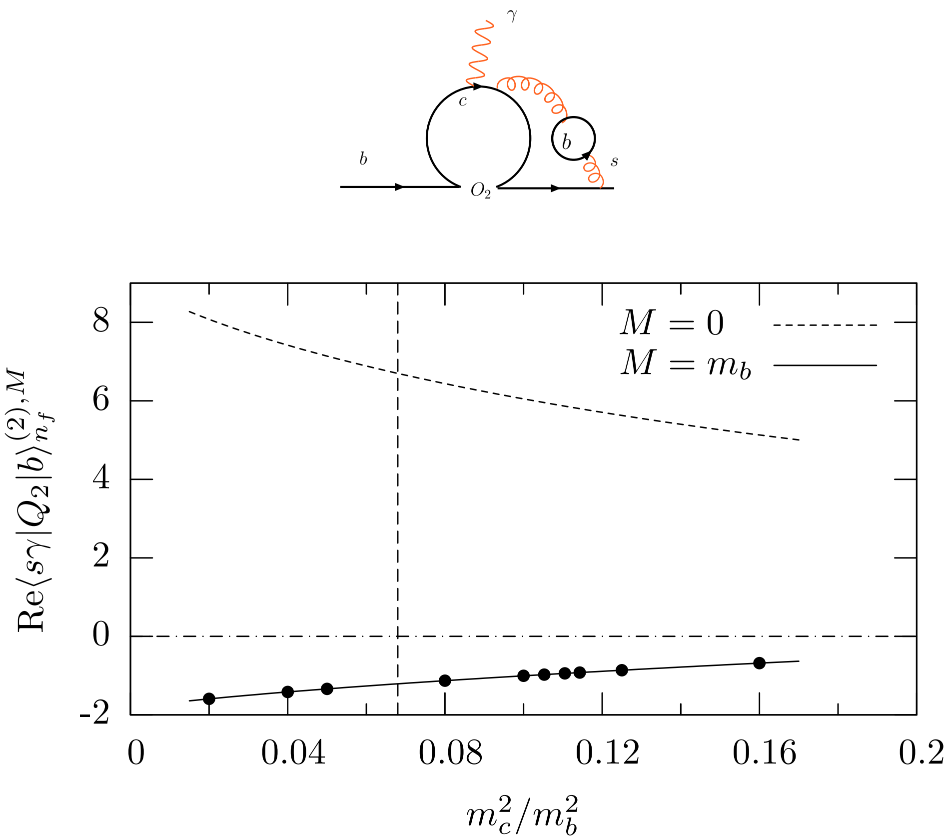

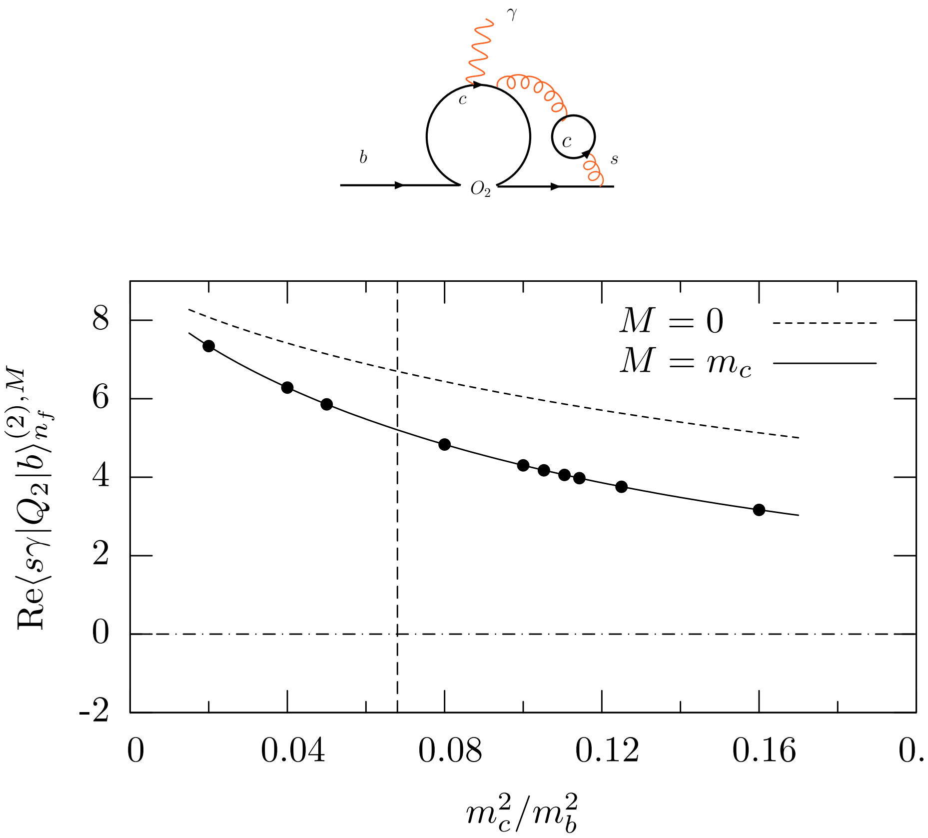

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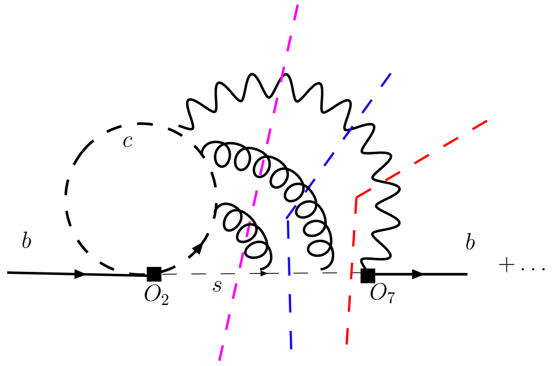
numerical impact of the mass corrections on $\mathcal{B}(\bar{B} \to X_s\gamma) = +$ 1.1% for $\mu_b = 2.5$ GeV

Reducing the interpolation uncertainty

- calculating $\mathcal{O}\left(\alpha_s^2\right)$ correction to $\langle s\gamma|O_{1,2}|b\rangle$ at $m_c = 0$ helps significantly in reducing the interpolation uncertainty

$\Longrightarrow$ 

up to 4-particle cuts: $\gamma s,\, \gamma sg,\, \gamma sgg,\, \gamma sq\bar{q}$

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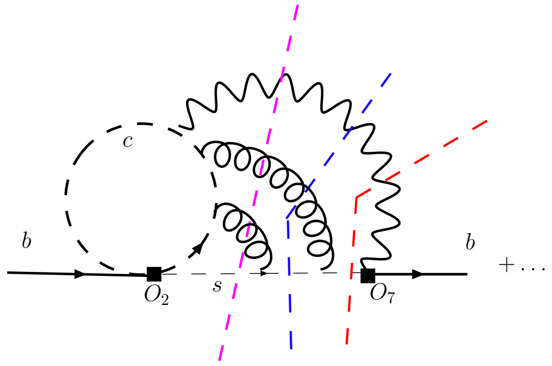
$\Longrightarrow$ 

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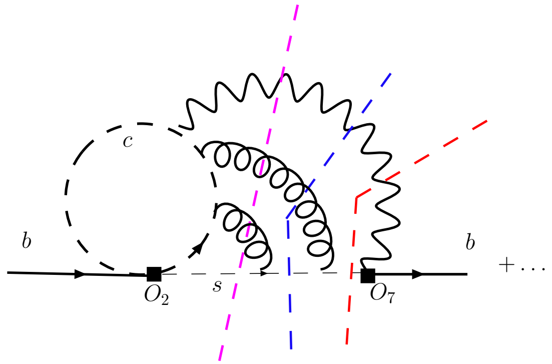
$\Longrightarrow$ 

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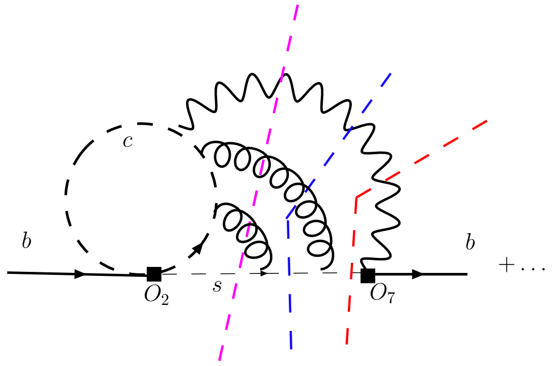
$\Longrightarrow$ 

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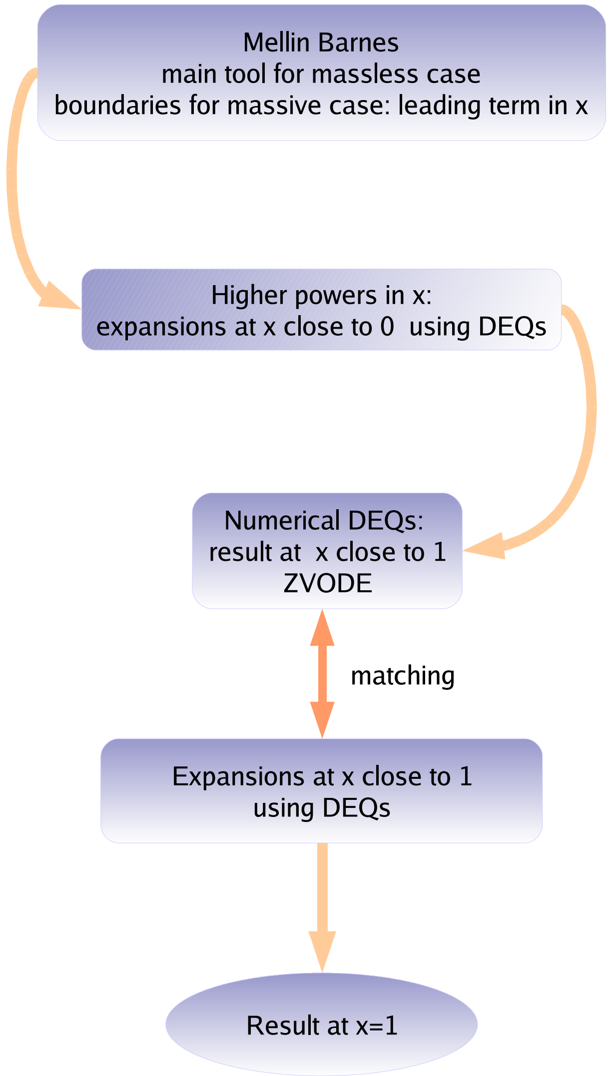
so what is the way out ?

Combining methods

Merging methods is the way to go, but a long chain of steps:

$$x = p_b^2/m_b^2$$



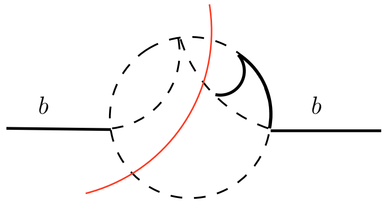
Boundaries for DEQs: 2- and 3-particle cuts

idea

- derive a MB representation for loops on the left and the right of the cut
- integrate over the phase space analytically
- perform an analytic continuation in ε for $\varepsilon \to 0$
- expand in $x = p_b^2/m_b^2$ where $p_b^2 \ll m_b^2$ by closing contours in the multi-fold MB integrals
- use Barnes Lemmas to remove some integrations if possible
- for multi-folds MB integrals (up to 3) integrate numerically
  $\longrightarrow$ up to 16 digits for 3-folds integrals

  useful packages:
  [MBasymptotics.m & quadprec.m, M.Czakon]

a simple example



- MB representation for this integral:

```
{-((E^((-I)*Pi-I*Pi*z1)*
s12^(-1-2*ep-z1)*Gamma[1-ep]^3*
Gamma[ep]*Gamma[-ep-z1]^2*
Gamma[-z1]*
Gamma[1+ep+z1])/(Gamma[2-2*ep]*
Gamma[1-2*ep-z1]*Gamma[1-ep-z1]))}
```

- after phase space integration, analytic continuation to $\varepsilon \to 0$ and ε expansion:

```
{MBint[((-2*I)*Pi*Gamma[-z1]^4*
  Gamma[1+z1])/(E^(I*Pi*z1)*ep*x^z1*
  Gamma[1-z1]^2*Gamma[2-z1]),
 {{ep -> 0},{z1 -> -1/2}}]}
```

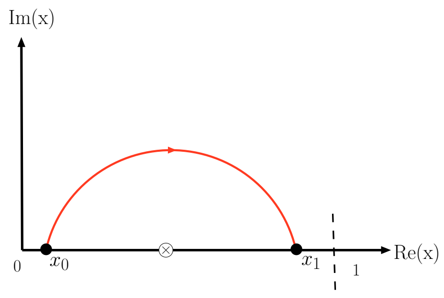
- after closing contour: $i\pi x/\varepsilon + \mathcal{O}(1)$

DEQs: expansions and numerical integration

- calculate the off-shell master integrals with the help of numerical differential eqts [Caffo, Czyz, Remiddi 98]
  - Our masters V_i are functions of ϵ and $x = p_b^2/m_b^2$
    $\Longrightarrow$ a system of differential eqts in x can be derived:
    $$\frac{d}{dx}V_i(x,\epsilon) = A_{ij}(x,\epsilon)V_j(x,\epsilon)$$
  - expand the masters in ϵ and x for $\epsilon,\ x \to 0$ using the ansatz:
    $$V_i(x,\epsilon) = \sum_{nmk} c^0_{inmk}\epsilon^n x^m \log^k x$$
  - solve recursively for c^0_{inmk} up to higher powers in x
  - use the boundary conditions to fix the left over constants
    $\rightarrow$ Mellin Barnes for 2- and 3-Pcuts,
    $\rightarrow$ diagrammatic large m_b expansion ($p_b^2 \ll m_b^2$) for 2-Pcuts

obtained high precision results for $x \approx 0$

  - use them as starting point for numerical integration in the complex plane to $x \approx 1 \rightarrow$ ZVODE, Hindmarsh et al



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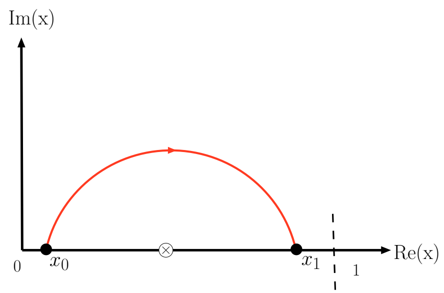


  - use them as starting point for numerical integration in the complex plane to $x \approx 1 \to$ ZVODE, Hindmarsh et al
  - perform an other power logarithmic expansion for $x \approx 1$ and solve recursively for c^1_{inmk}

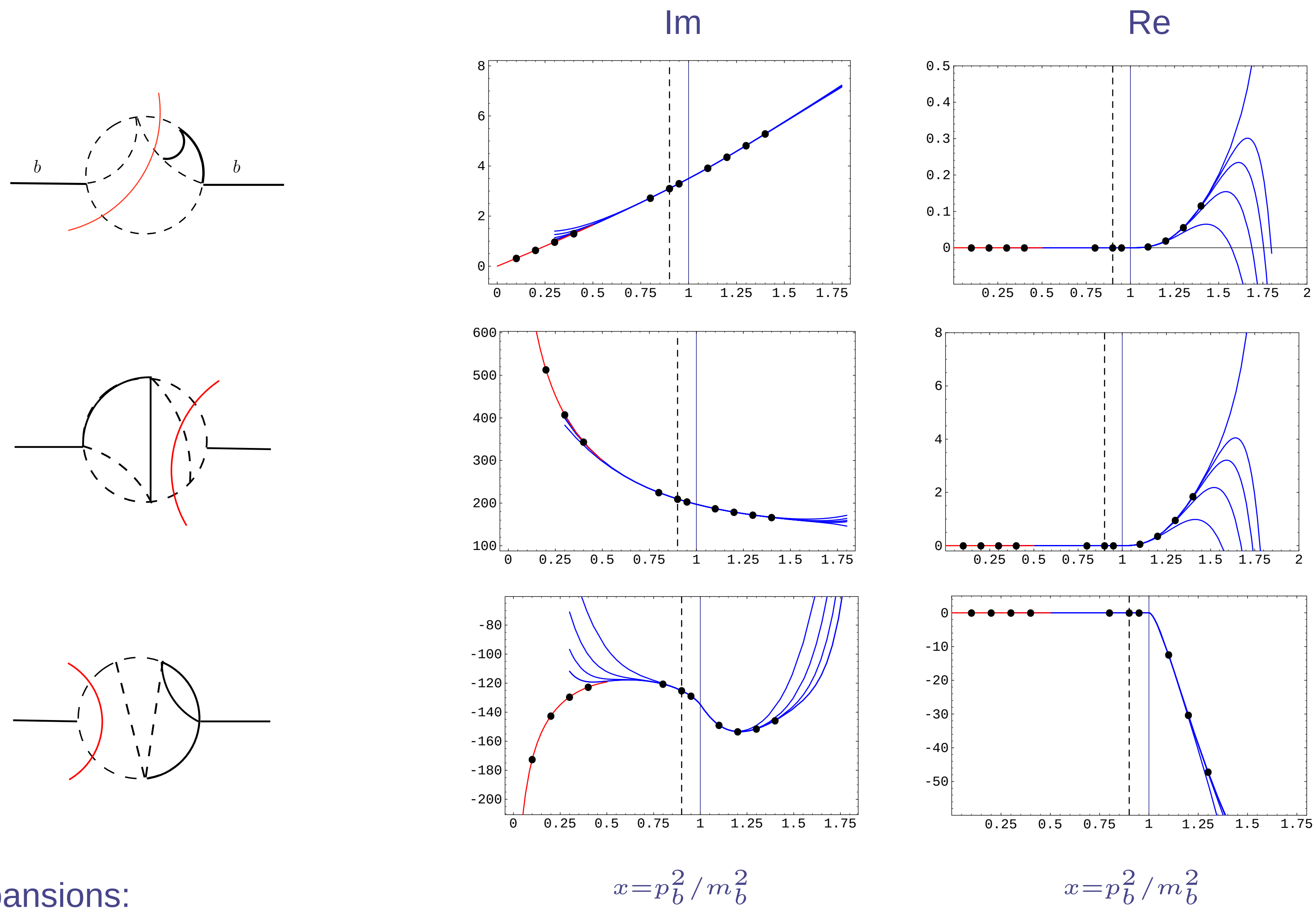
$$V_i(x,\epsilon) = \sum_{nmk} c^1_{inmk} \epsilon^n (1-x)^m \log^k (1-x)$$

  - match with the results of ZVODE to fix left over c^1_{inmk}

result for $x = 1$ is the leading term

Some Results for 2- and 3-particle cuts

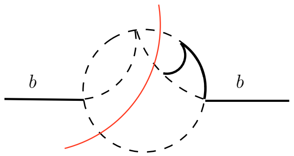
Preliminary results: sample masters with 2- and 3-particle cuts



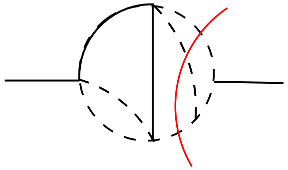
- Expansions:
  - $x \to 0$: up to x^{18}
  - $x \to 1$: up to $(1-x)^{12}$
- Numerical integration: starts at $x_0 = 0.02$
- Matching: done at $x_1 = 0.9$

Some Results for 2- and 3-particle cuts

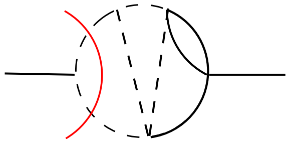
- Preliminary results at $x = 1$: sample masters with 2- and 3-particle cuts



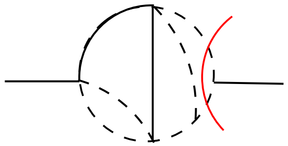
$$= \frac{3.10453\, i}{\varepsilon} + \mathcal{O}\left(1\right)$$



$$= \frac{3.14159\, i}{\varepsilon^3} + \frac{20.0142\, i}{\varepsilon^2} + \frac{77.1378\, i}{\varepsilon} + 209.713\, i + \mathcal{O}\left(\varepsilon\right)$$



$$= \frac{-2.0944\, i}{\varepsilon^3} - \frac{12.5778\, i}{\varepsilon^2} - \frac{35.6402\, i}{\varepsilon} - 125.153\, i + \mathcal{O}\left(\varepsilon\right)$$



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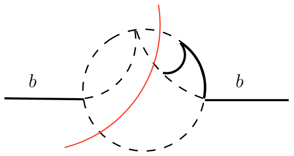
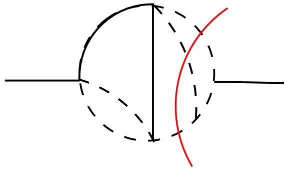


$$= \frac{3.10453\, i}{\varepsilon} + \mathcal{O}(1)$$



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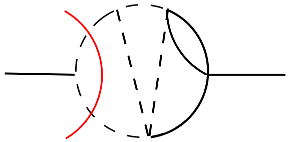

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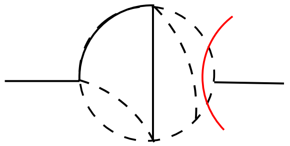

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masters with 2-particle cuts are obtained with two independent calculations
$\rightarrow$ cross checks will be done soon

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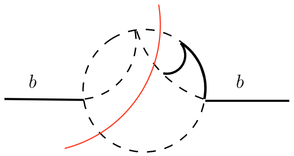

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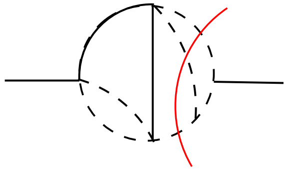

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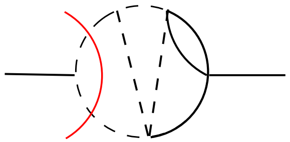

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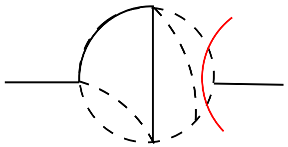

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- what we have:
  - masters with massless internal lines: all 2- and 3-particle cuts all 4-particle cuts but one
  - masters with b-quark internal lines: 2- and 3-particle cuts are almost there
- still to be calculated: masters with 4-particle cuts and internal b-lines

Summary

- Matching current and future experimental precision for $\bar{B} \to X_s \gamma$ decay necessitates NNLO corrections on the theory side
  crucial missing piece: $O(\alpha_s^2)$ correction to $\langle s\gamma|O_{1,2}|b\rangle$

- Reducing the interpolation uncertainty: needs $O(\alpha_s^2)$ correction to $\langle s\gamma|O_{1,2}|b\rangle$ at $m_c = 0$
  $\to 70\%$ of the project is completed

- Removing the interpolation uncertainty: needs $O(\alpha_s^2)$ correction to $\langle s\gamma|O_{1,2}|b\rangle$ at physical m_c
  $\longrightarrow$ completed the fermionic contribution

  $\to$ massless case: calculated in two ways and confirmed the findings of [Bieri, Greub, Steinhauser 03]

  $\to$ massive case: impact on the branching ratio $+1.1\%$ for $\mu_b = 2.5$GeV

  $\longrightarrow$ bosonic contribution: work in progress