NNLO corrections to heavy fermion decays

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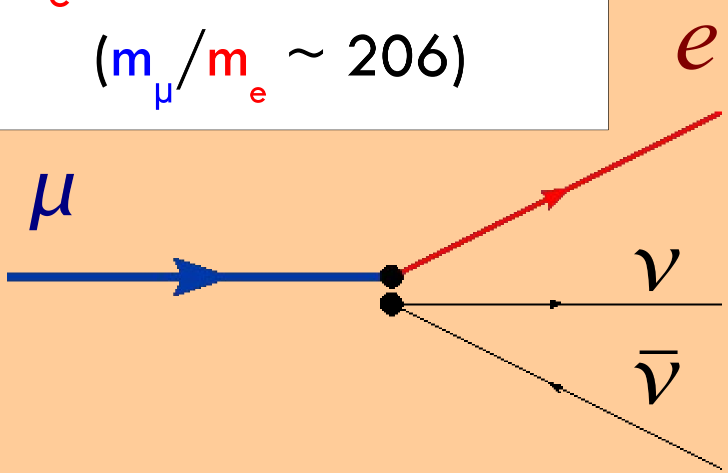
Muon decay: $\mu \to e, \bar{\nu}_e, \nu_\mu$

$m_\mu = 105.6$ MeV

$m_e = 0.511$ MeV

($m_\mu / m_e \sim 206$)



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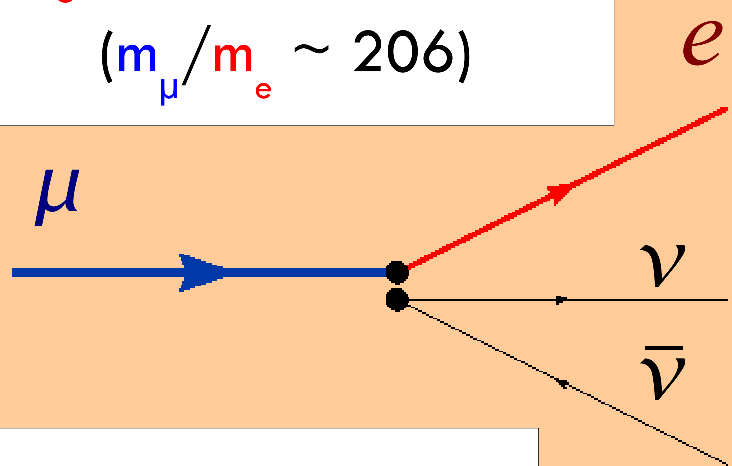

$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1 + \Delta_{QED}\right)$$

(no weak corrections)

- MuLan, PSI: decay rate $\lesssim$1ppm
- Results in

  G_F = 1.166... 10^{-5} GeV^{-2}

- $O(\alpha^2)$ corrections (m_e = 0):

  van Ritbergen, Stuart (2000)

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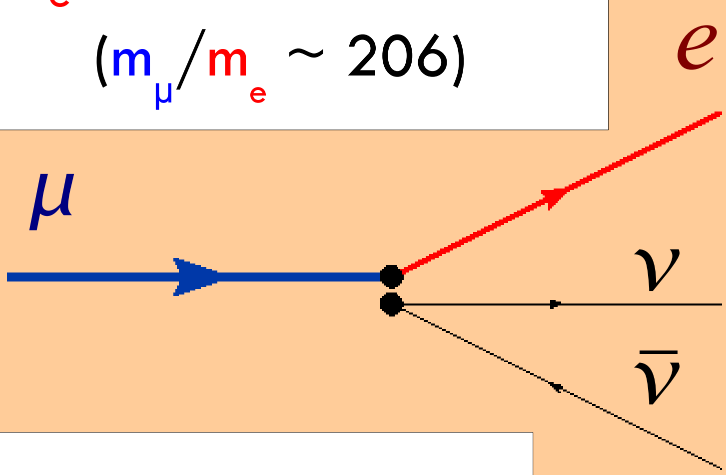

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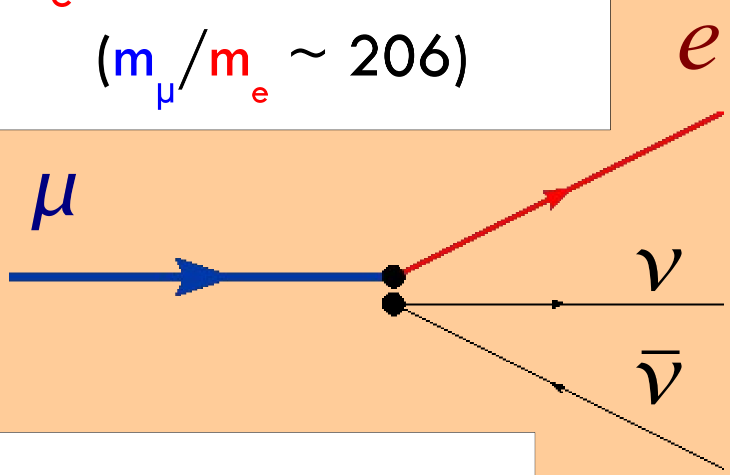

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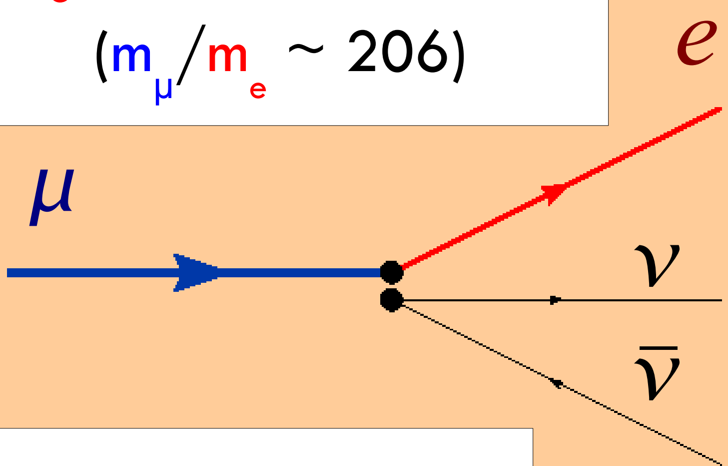

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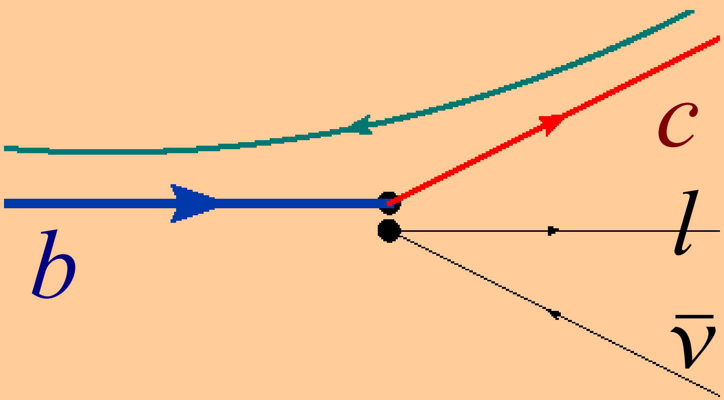
Is there a need to account for electron mass ? Yes!

Semi-leptonic B decays: $B \to X_c, \bar{\nu}, l$

m_b = 4.7…5.0 GeV

m_c = 1.5…1.8 GeV

(m_b/m_c ~ 3…4)



- Clean signature, dominant decay
- Non-perturbative part: HQE
- Need QCD process $b \to c\, \ell\, \nu$
- Fit Γ, ⟨E_ℓ⟩, ⟨E_ℓ^2⟩, … to extract

  $|V_{cb}| = (41.3 \pm 1.5)\cdot 10^{-3}$, m_b, …
- Massless $O(\alpha_s^2)$ corrections:

  same calculation as for muon
- m_c-dependent corrections are

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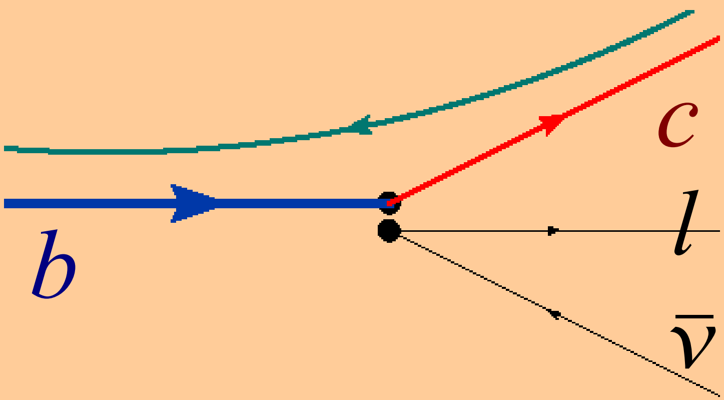

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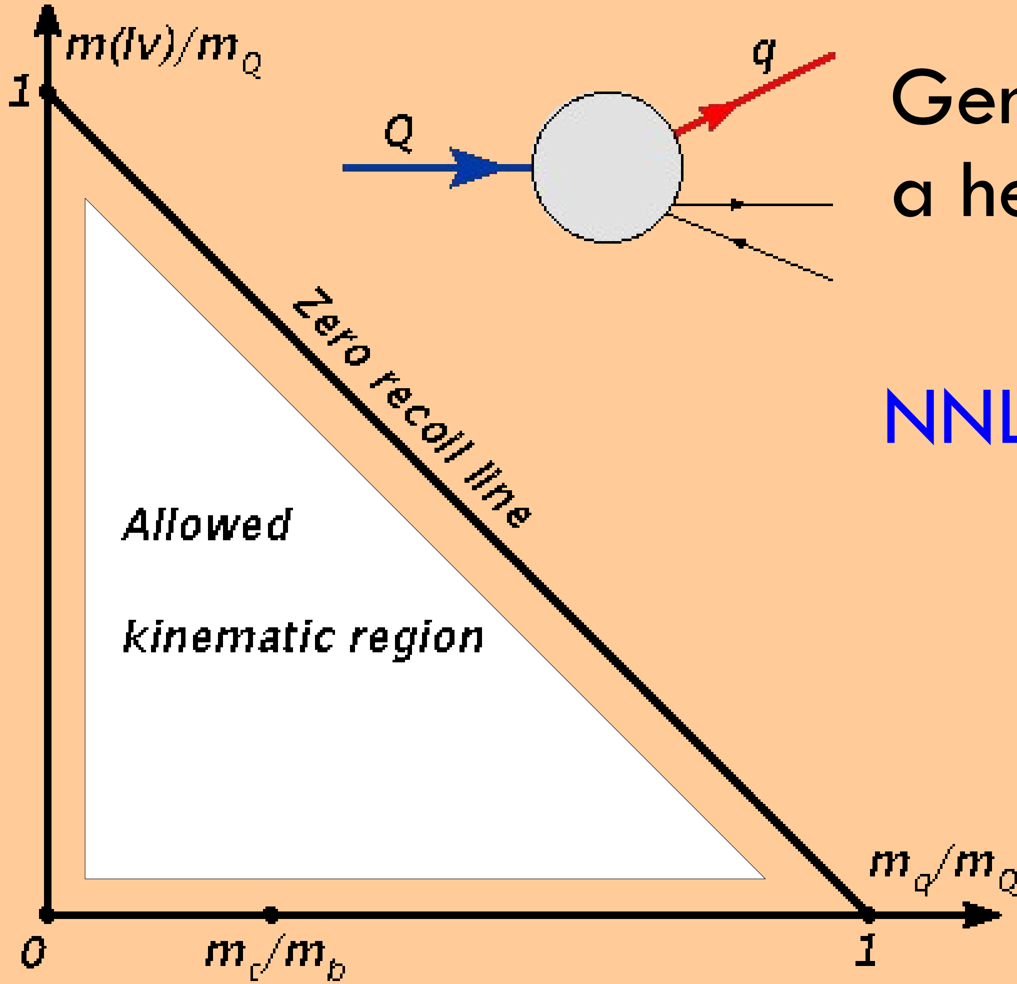
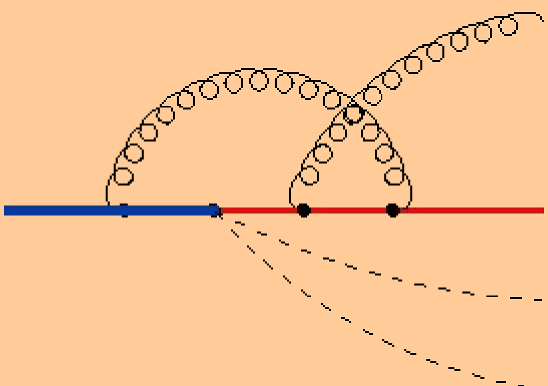
A very challenging double-scale calculation!

Studies of mass effects



Generic semi-leptonic decay of a heavy quark Q to a quark q

NNLO QCD corrections:



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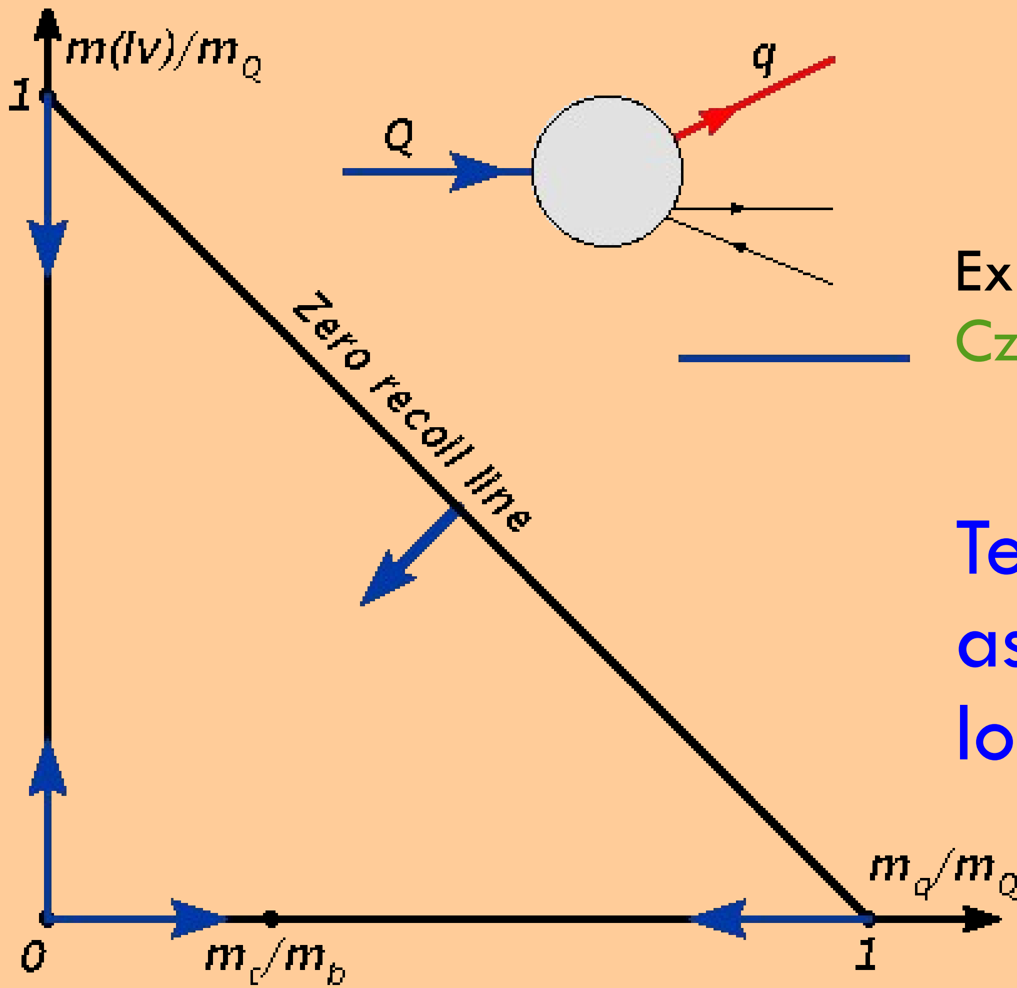
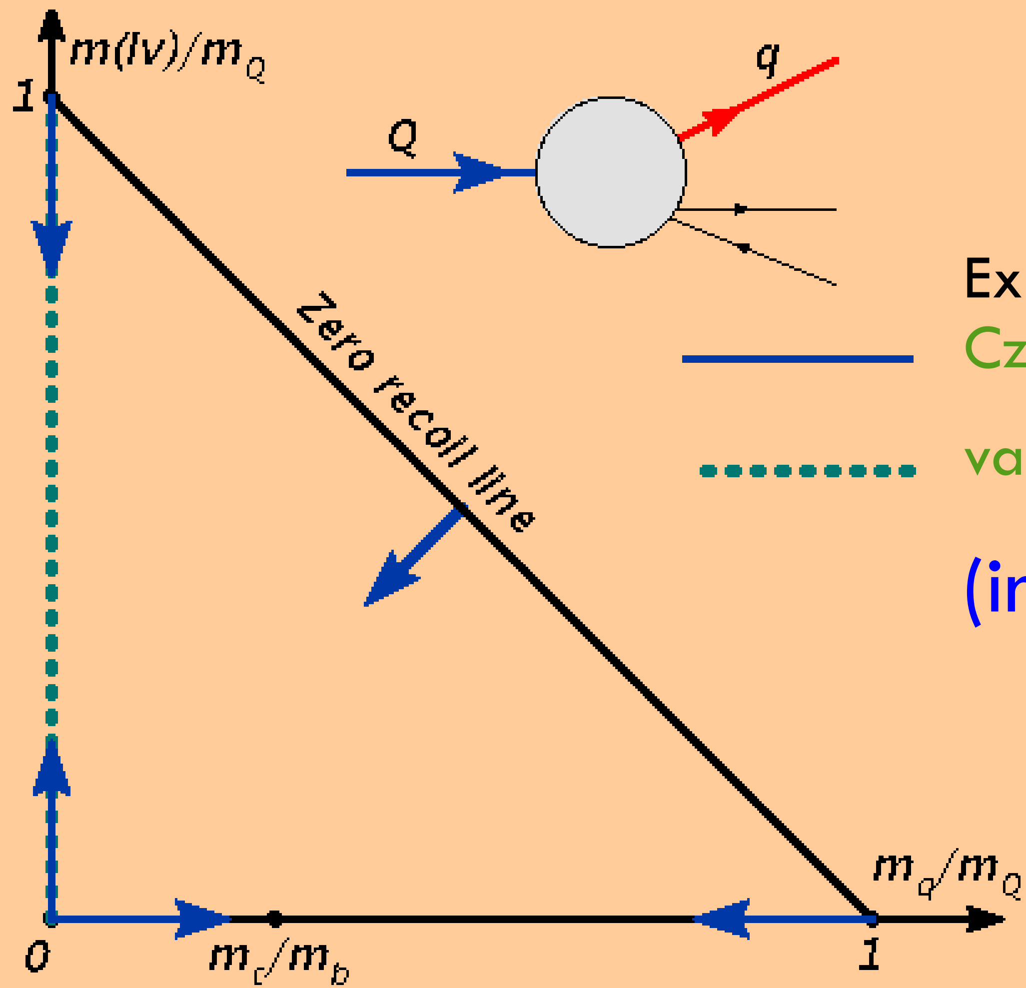


Expansions in special kinematics by Czarnecki, Melnikov, Blokland, ...

Techniques: optical theorem, asymptotic expansions of loop integrals, ...

Studies of mass effects



Expansions in special kinematics by
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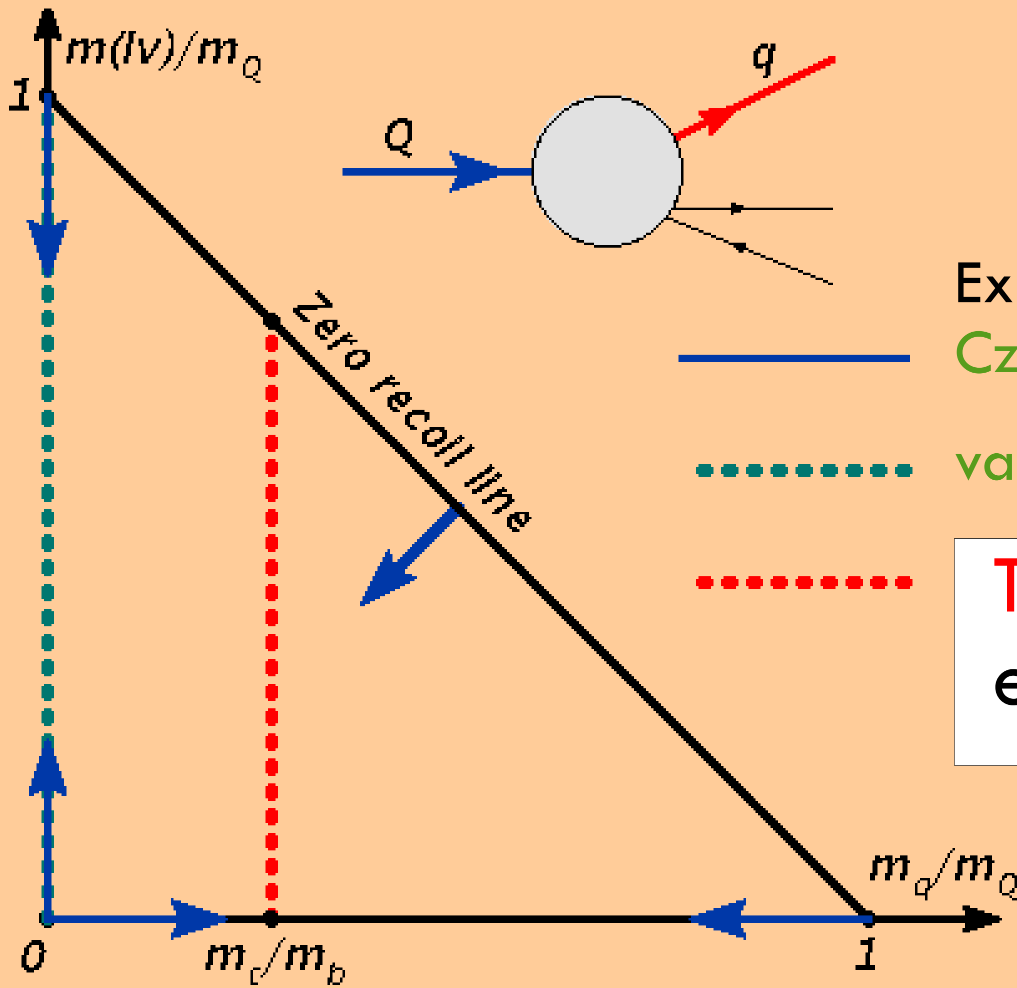
van Ritbergen, Stuart: $m_q = 0$

(integration over $m(l\nu)^2$)

Studies of mass effects



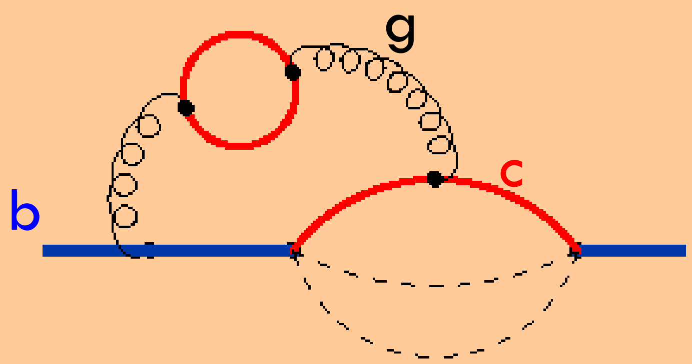
Expansions in special kinematics by Czarnecki, Melnikov, Blokland, ...

van Ritbergen, Stuart: $m_q = 0$

This work:
expansion in m_q/m_Q

- Agrees with Melnikov
- ~1% accuracy for $b \to c$
- Found rate, $\langle E_l^{1,2} \rangle$, $\langle E_H^{1,2} \rangle$

Structure of the result



Example: contribution of c-quark loops and pairs

$$
\begin{aligned}
X_C \quad = \quad & -\frac{1009}{288}+\frac{8}{3}\zeta_3+\frac{77\pi^2}{216} \\
- \quad & \rho \left\{\frac{5\pi^2}{4}\right\} \\
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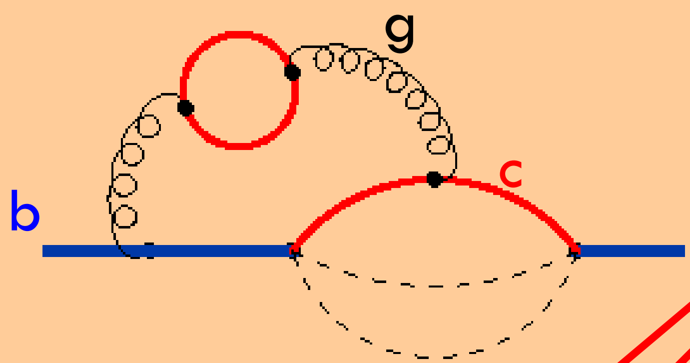

- Expand in $\rho = m_c/m_b$, exact coefficients

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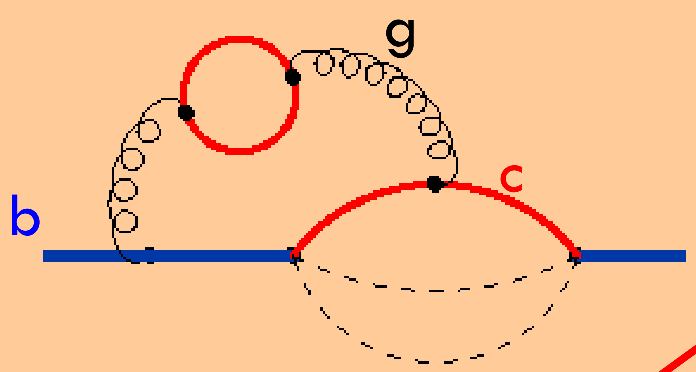

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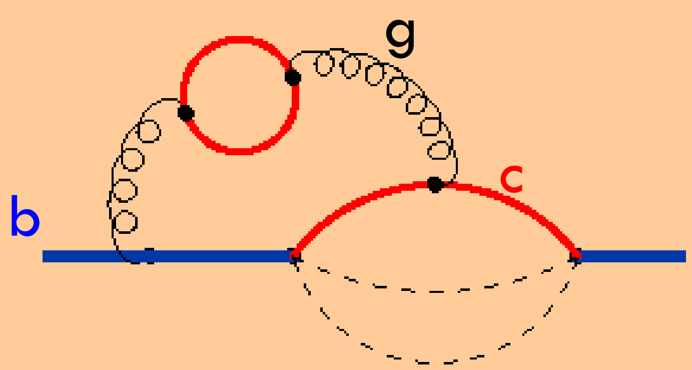

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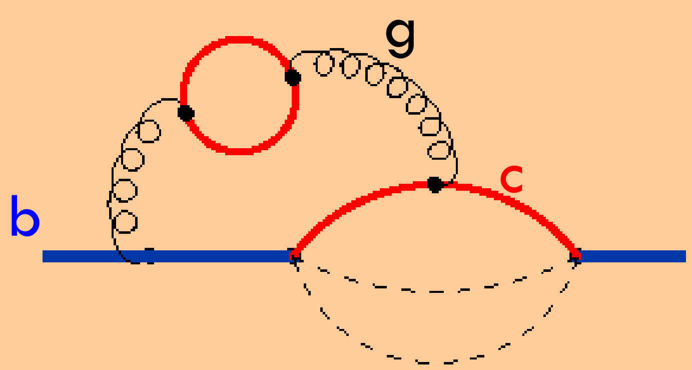

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Alexey Pak, LoopFest VII, 14 May 2008

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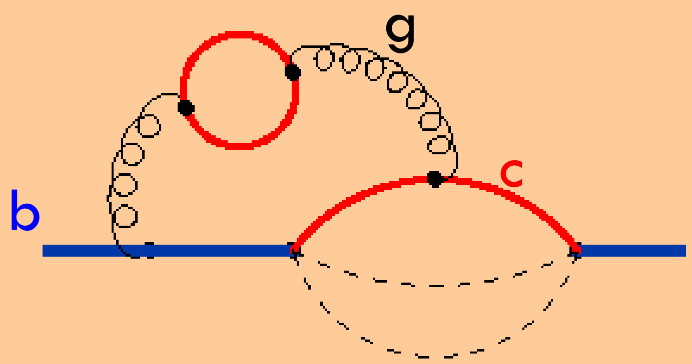

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found and explained a mistake

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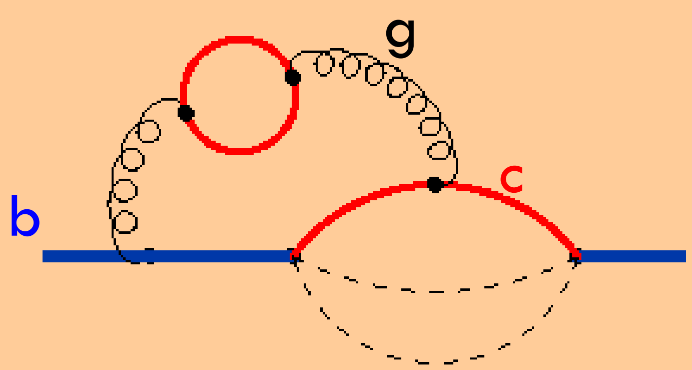

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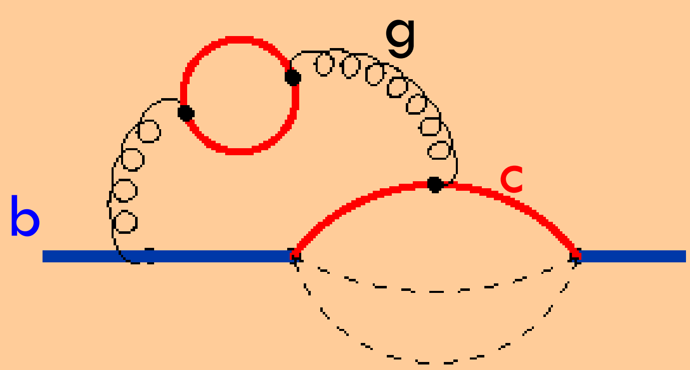

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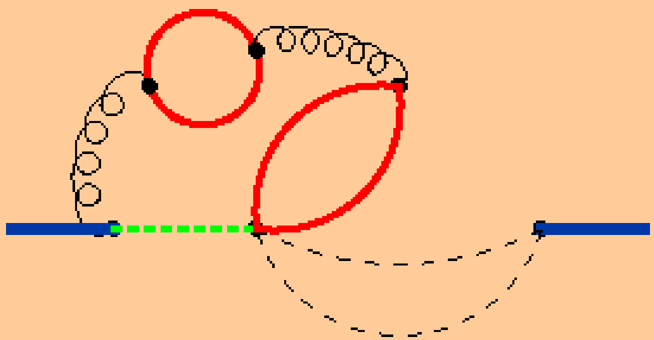
- **Linear term: unexpected!**

Mass definition issues



Origin of the linear term:
region with soft loop momenta
($k \sim m_q$)

- Calculation: pole mass scheme
- QCD: soft region $m_q \sim \Lambda_{QCD}$ cannot be treated perturbatively

  Short-scale mass absorbs the linear term:

$$\frac{G_F^2 V_{cb}^2 m_Q^5}{192\pi^3}\left(\ldots \right) \quad vs \quad -\frac{5\pi^2}{4}\frac{m_q}{m_Q}$$

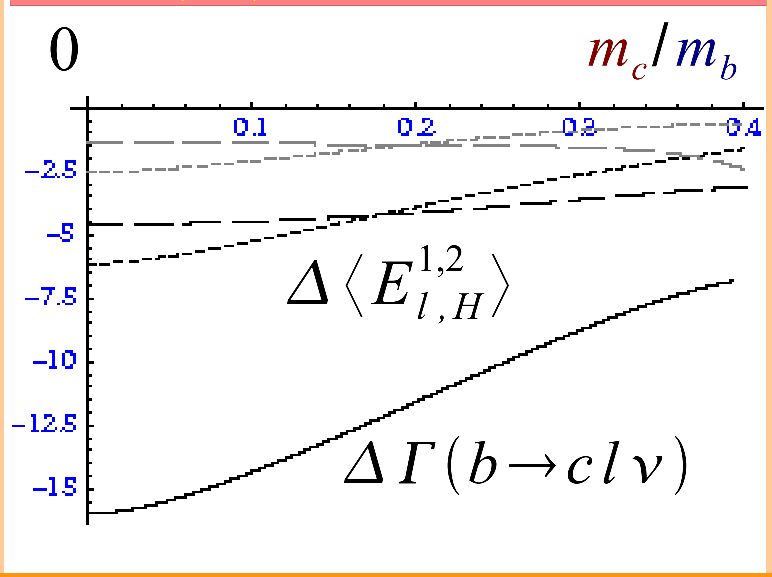
- QED: perturbative at any scale, pole masses of muon and electron

Correction of -0.43 ppm – relevant for MuLan experiment

Summary

- Analytical expansion of semi-leptonic decay rate and distribution moments in small quark mass
- From Melnikov's numerical results: @ a few %, effect of cuts can be modeled at the tree level (incl. central moments)
- We are working on differential quantities

$O(\alpha^2)$ corrections:



[arXiv: 0803.0960]
apak@phys.ualberta.ca

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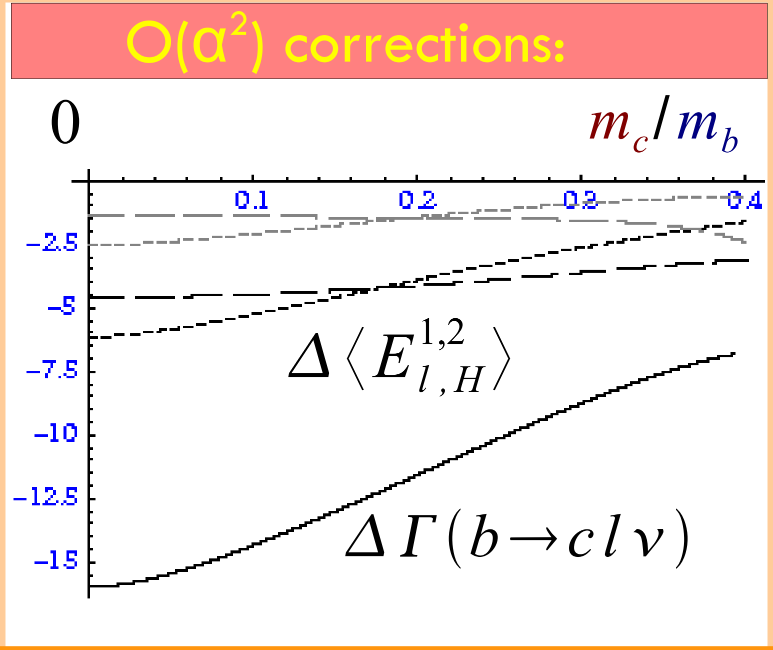
Now a little advertisement…



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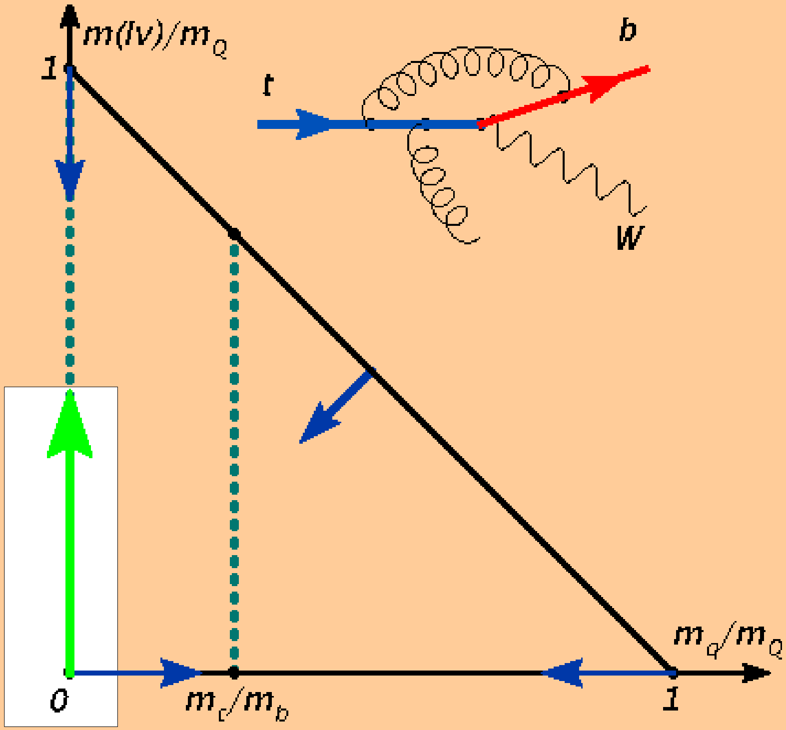
Top quark decay into polarized W

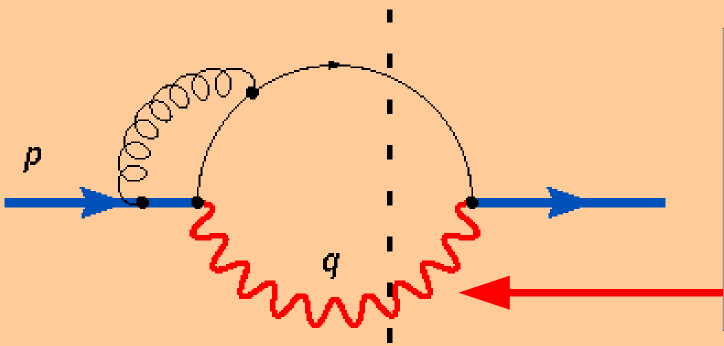
(Work by J. Piclum, A. Czarnecki, and J. Körner)



- LHC will produce many top quarks
- W polarization:
  tool to study the top decay
- Helicity fractions measurable at
  Tevatron and LHC

Can use optical theorem and expansions!

Top quark decay into polarized W



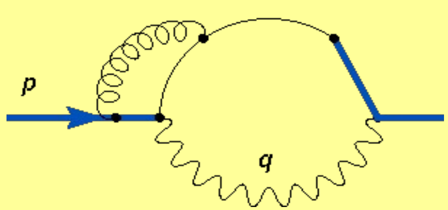
Unpolarized W:

$$P_0^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}$$

Projector on polarized W:

$$P_\pm^{\mu\nu} = \frac{i\,\epsilon^{\mu\nu\rho\sigma}\, p_\rho q_\sigma}{m_t |\vec{q}|}$$

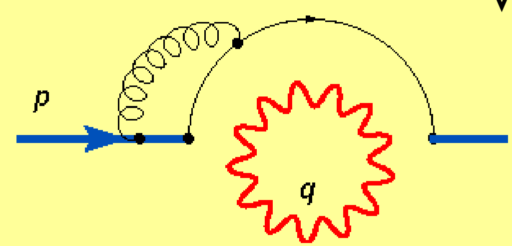
Hard region ($q \sim m_t$):



$$\frac{1}{|\vec{q}|} = \sum \frac{\ldots}{(q^2 + 2\,pq)^i}$$

- new massive propagators

Soft region ($q \sim m_W$):



$$\int \frac{d^D q}{(q^2 - m_W^2)(q_0^2 - m_W^2)}$$

- new vacuum bubbles

Laporta reduction, 12 new master integrals...

Top quark decay into polarized W

NNLO results (error from α_s scale):

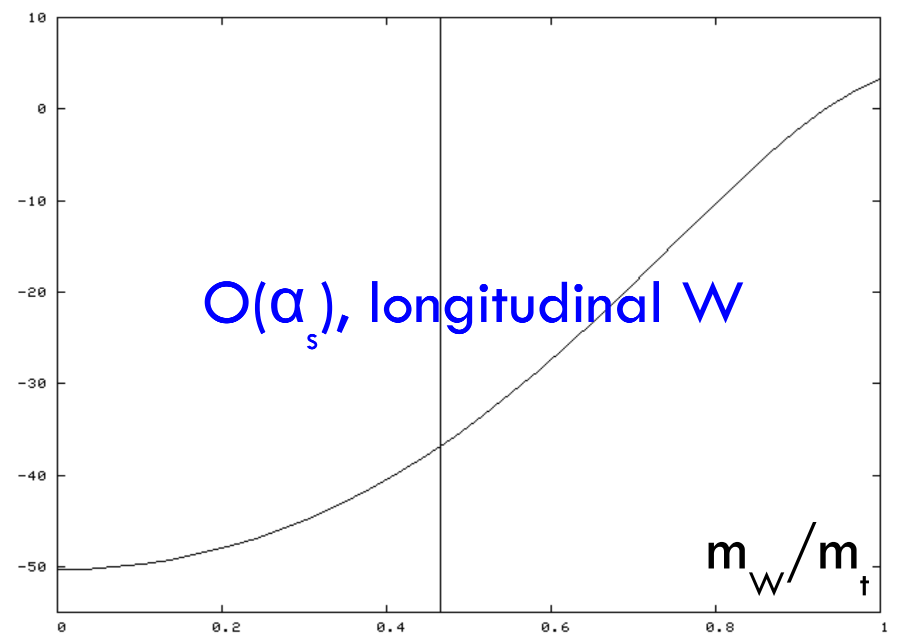
- $F_L = 0.6872(6)$
- $F_- = 0.3115(5)$
- $F_+ = 0.00126(6)$

Expected @ LHC:

1.9%

1.8%

0.21%



Aguilar-Saavedra et al '07

Thank you for your attention!