NNNLO $t\bar{t}$ threshold cross section

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Talk based on collaborations;

- 2-loop EW x QCD hard-loop; D.Seidel(KA), M.Steinhauser(KA)
- NNNLO NR-QCD; M. Beneke(AC), A.Penin(Alberta), K.Schuller(AC)

LoopFest VII; Radiative Corrections for the LHC and ILC
University at Buffalo, 14-16 May 2008

Motivation

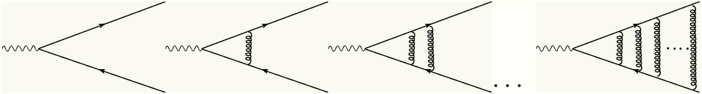
- QCD side:
  - m_t measurement at ILC requires precise theory calculation: $(\Delta m_t)_{\rm exp} \leq 50{\rm MeV} \rightarrow$ theory goal : $\delta\sigma/\sigma \leq 3\%$
  - NNLO was completed and compared in Top-WGR (2000) $\rightarrow$ NNLO result has large uncertainty ($\sim 20\%$)
    (Beneke-Signer-Smirnov, Hoang-Teubner, Melnikov-Yelkovsky, Nagano-Ota-Sumino, Penin-Pivovarov, Yakovlev)
  - RG improvement is being advanced
    (Hoang-Manohar-Stewart-Teubner(02), Pineda-Signer(06))
    At NNLL, uncertainty reduced to $\pm 6\%$ (Hoang,et al.), and it was pointed out that the main effect is NNNLO logarithm (Pineda et al.).
- EW side;
  - EW 1-loop known (5% at most) (Guth-Kühn ('92), Hoang-Reißer(05))
  - 2-loop $\mathcal{O}(\alpha\alpha_s)$ due to H/Z and g (Eiras-Steinhauser (06))

In this talk we present results: (1) EW QCD (W-g) mixed 2-loop corrections to hard matching coefficient; (2) NNNLO QCD corrections to the threshold cross section.

Part II

Threshold Cross section

Threshold cross section requires resummation of $\alpha_s/v \sim \mathcal{O}(1)$



- each gluon exchange yields Coulomb singularity, α_s/v

  $\mathrm{LO} \sim 1 + \frac{\alpha_s}{v} + (\frac{\alpha_s}{v})^2 + \cdots \sim \Sigma_n\, (\frac{\alpha_s}{v})^n$

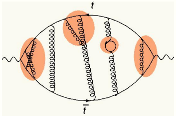
  $\mathrm{NLO} \sim \Sigma_n\, \{\alpha_s,\, v\} \times (\frac{\alpha_s}{v})^n$

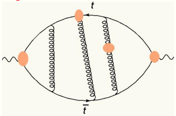
  $\mathrm{NNLO} \sim \Sigma_n\, \{\alpha_s^2,\, \alpha_s v,\, v^2\} \times (\frac{\alpha_s}{v})^n$

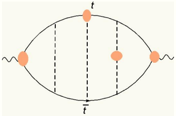
  $\mathrm{NNNLO} \sim \Sigma_n\, \{\alpha_s^3,\, \alpha_s^2 v,\, \alpha_s v^2,\, v^3\} \times (\frac{\alpha_s}{v})^n$

Threshold cross section $R_{t\bar{t}} \equiv \sigma_{t\bar{t}}/\sigma_{m=0} = \frac{4\pi e_t^2}{s} \,\mathrm{Im}\,\Pi(s)$

Principal quantity is $\Pi(q) = i \int d^4x e^{iqx} \langle 0| J^\mu(x) J_\mu(0) |0\rangle$







- Integrating hard mode $\longrightarrow$ NRQCD (Caswel-Lepage '86):

$$J^i(x) = \left[\bar{t}\gamma^i t\right] \quad \longrightarrow \quad c_v \left[\psi^\dagger \sigma^i \chi\right]$$

$$\mathcal{L}_{\mathrm{NRQCD}} = \psi^\dagger \left[iD_0 + \frac{\mathbf{D}^2}{2m} + \frac{\mathbf{D}^4}{8m^3} - \frac{d_1\, g_s}{2m} \sigma \mathbf{B} \right] \psi + (\psi \leftrightarrow \chi) + \cdots$$

$$\Pi(q) = i \int d^4x e^{iEx}\, c_v^2\, \langle 0| \left[\psi^\dagger \sigma^i \chi\right](x)\ \left[\chi^\dagger \sigma_i \psi\right](0) |0\rangle$$

- Integrating soft/potential modes $\longrightarrow$ PNRQCD

(Pineda-Soto '97/Luke - Manohar-Rothstein'99):

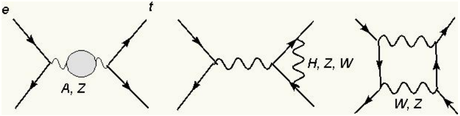
$$\mathcal{L}_{\mathrm{PNRQCD}} = \psi^\dagger \left[i\partial_0 + \frac{\partial^2}{2m} + \frac{\partial^4}{8m^3} - g_s \mathbf{x}\mathbf{E}(t, \mathbf{0}) \right] \psi + (\psi \leftrightarrow \chi)$$
$$+ \int d\mathbf{r} \left[\psi^\dagger \psi\right](x + \mathbf{r})\, V_{pot}(\mathbf{r}) \left[\chi^\dagger \chi\right](x) + \cdots$$

Part III

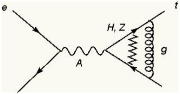
EW x QCD hard loop corrections

- One-loop EW is known since long

  Grzadkowski-Kühn-Krawczyk-Stuart('87), Guth-Kühn('92), Hoang-Reißer (05)

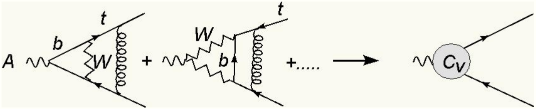


- 2-loop $\alpha\alpha_s$ corr (Z/H and g);Eiras-Steinhauser(06)



- This talk is on $\alpha\alpha_s$ (W and g) corrections to $At\bar{t}$-vertex

On threshold $t\bar{t}$ production, i.e. $s = 4m_t^2$

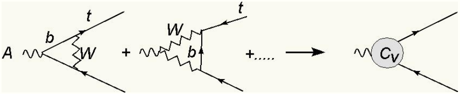


We match SM top pair production vertex to $c_v \psi^\dagger \sigma^i \chi$

- ψ, χ are NR 2-component spinor, e.g. $u(p) = \begin{pmatrix} \sqrt{\frac{E+m}{2m}}\psi \\ \frac{\sigma \mathbf{p}}{\sqrt{2m(E+m)}}\psi \end{pmatrix}$
- c_v is gauge dependent, but well-defined and one of building blocks for $R_{t\bar{t}}$
- Hard-loop is equivalent to $t\bar{t}$ on-threshold amplitude (+h.o.)
- Method:

  The result in expansion of small-(M_W^2/m_t^2) $(\sim 1/4)$
  Differential Eq.(Remid'97) + Mellin-Barnes Rep. are applied,
  $1/\epsilon$-poles canceled analytically, some finite parts numerically
  We used: QGRAF, q2e, exp, crusher ,MB, AMBRE, HypExp, Cuba
  + some implementation by ourselves

1-loop lesson: Convergence of z-expansion ($z = \frac{M_W^2}{m_t^2}$)



$$Q_t\, c_v^{(1)}|_{\rm Wboson} = \frac{\alpha}{4\pi s_w^2}\Big[\frac{0.201}{z} + (0.48 + 0.79i + 0.25\ln z)$$

$$+z(-0.0024 - 1.37 - 0.44\ln z) + z^2(-0.07 + 1.39i + 0.44\ln z) + \mathcal{O}(z^3)\Big]$$

- Inclusion of first five terms ≈ exact result (red line) (Guth-Kuhn)

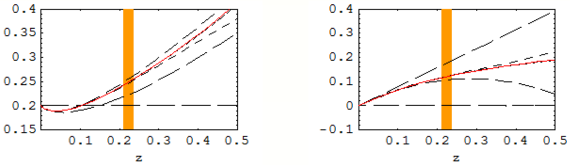
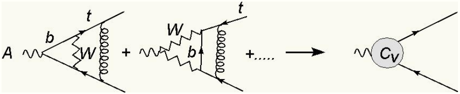


Fig.1: Real (left) and imaginary (right) parts of $C_v/[\frac{\alpha}{4\pi s_w^2}\,\frac{1}{z}]$. The shortest dashed-line is $\mathcal{O}(z^3)$ shitting on exact line.

- Leading (1/z)-term is due to ($\phi_W t\bar{t}$-) Yukawa coupling
- Imaginary part contains un-wanted $b\bar{b}, W^+W^-$ cuts (Hoang-Reisser)

2-loop EW x QCD in z-expansion ($z = \frac{M_W^2}{m_t^2}$)



$$Q_t\, C_V^{(2)}|_{\mathrm{W\times g}} = \frac{\alpha}{4\pi s_w^2}\frac{\alpha_s C_F}{4\pi}\Big[\frac{22.23-16.43i}{z} + (8.55 - 3.16i + 2.00\ln z)$$
$$+z(-19.28+3.59+(2.16+4.10i)\ln z)+z^2(4.00+20.10i-(4.53+2.91i)\ln z)+\mathcal{O}(z^3)\Big]$$

- Inclusion of successive terms shows a sign of convergence

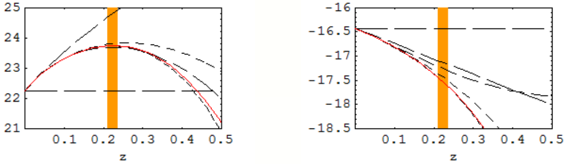
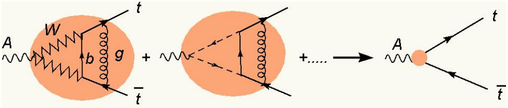


Fig.1: Real (left) and imaginary (right) parts of $C_V/[\frac{\alpha}{4\pi s_w^2}\frac{\alpha_s C_F}{4\pi}\frac{1}{z}]$.
Orange band is physical mass range for top quark $m_t = 165 - 175$ GeV.
$\mathcal{O}(z^3) \approx 0.3\%$ (the shortest dash line) ; $\mathcal{O}(z^4) \approx 0.03\%$ (red line)

- Leading $1/z$ due to Yukawa coupling

Result for 2-loop EW x QCD corrections



- Obtained 2-loop corrections give 0.9 % shift to R (relevant for m_t measurement at ILC)
- The result is dominated ($\approx 90\%$) by a top-Yukawa enhanced term

2-loop EW(W) x QCD corrections (YK-Seidel-Steinhauser (Preliminary))

$$Q_t C^{(2)}_{EWQCD} = \frac{C_F \alpha_S}{4\pi} \frac{\alpha}{4\pi \sin^2\theta_W} \Bigg[\frac{22.23 - 16.43i}{z} + (8.55 - 3.16i - 2.00 \ln z)$$

$$-z(19.27 - 3.59i) + z \ln z(2.16 + 4.10) + z^2(4.00 + 20.10i) - z^2 \ln z(4.53 + 2.91i) \Bigg]$$

$$= \Big[(3.15_{1/z} + 0.36_0 - 0.15_1 + 0.02_2 - 0.01_3 + 0.001_4$$

$$+ i(-2.33_{1/z} - 0.10_0 - 0.02_1 - 0.02_2 - 0.01_3 - 0.001_4) \Big] \times 10^{-3}$$

Part IV

QCD NNLO part

We use PNRQCD to compute $\Pi(q) \sim \langle j^i(x) j_i(x) \rangle$.

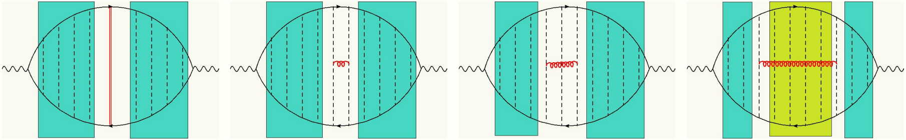
PNRQCD Lagrangian (Pineda-Soto('98); Luke-Manohar-Rothstein('99))

$$\mathcal{L}_{\mathrm{PNRQCD}} = \psi^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_t} \right) \psi + \int d\vec{r} \left[\psi^\dagger \psi \right] V_{pot}(r) \left[\chi^\dagger \chi \right]$$

$$+ig\, \psi^\dagger \left[A_{0,us} + \frac{\nabla \vec{A}_{us}}{m} \right] \psi - \frac{1}{4} F_{us}^2 + \cdots$$

- e.g. $\delta \tilde{V}_C = -\frac{4\pi\alpha_s}{\mathbf{q}^2} \left(\frac{\alpha_s(\mathbf{q})}{4\pi} \right)^3 \left[a_3 + 8\pi^2 C_A^3 \left(\frac{1}{3\epsilon} + \ln \frac{\mu_{US}^2}{\mathbf{q}^2} \right) \right]$
- Remaining Mode is Ultra Soft gluon: $k \sim m(v^2, \vec{v}^2)$

Ultrasoft renormalization I: Hamiltonian (static case)

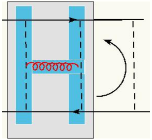


- quark-gluon vertex is $1/m$ suppressed; $\psi^\dagger\left(iD_0 + \frac{\vec{D}^2}{2m}\right)\psi$
- n_g, number of potential exchange $\sim \Delta t$; $n_g > 1 \Leftrightarrow$ UV finite
- ADM $1/\epsilon$ of the Coulomb pot is a counter term for us corr

$$\delta V_C^{\mathrm{ADM}} = \frac{1}{\epsilon}\frac{C_A^3\alpha_s^4}{\mathbf{q}^2}$$

$$\mathrm{H}_0 = -\frac{\alpha_s^3 C_A^2}{\epsilon}\left(\frac{1}{mq} + \frac{2(p^2/m-E)}{\mathbf{q}^3} + \epsilon L_{\mathrm{Bethe}}\right) + O(\epsilon)$$

$$\mathrm{H}_1 = -\frac{1}{\epsilon}\frac{C_A^2(C_A-2C_F)\alpha_s^4}{\mathbf{q}^2} + L_{\mathrm{Bethe}} + O(\epsilon)$$



- UV cancelation happens tricky way:

$$\frac{(p^2/m-E)}{\mathbf{q}^3} \Rightarrow \frac{C_F\alpha_s}{\mathbf{q}^2} \text{ (Eq. of motion)}$$

Ultrasoft renormalization II: vertex

$$\delta V_C^{\mathrm{ADM}} = \frac{1}{\epsilon}\frac{C_A^3\alpha_s^4}{\mathbf{q}^2}$$

$$\mathrm{H}_0 = -\frac{\alpha_s^3 C_A^2}{\epsilon}\left(\frac{1}{mq} + \frac{2(p^2/m-E)}{\mathbf{q}^3} + \epsilon L_{\mathrm{Bethe}}\right) + O(\epsilon)$$

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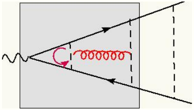

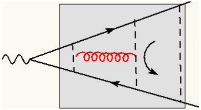

- Loop near photon vertices are more singular
  $\Leftrightarrow$ Vertex Renormalization
- $1/\epsilon$ cancelation does not happen exactly anymore
  - due to vertex divergence, (3+1) dimensional Eq. of motion is invalid
  - diagrams with different loop order get mis-match of Eq. of motion

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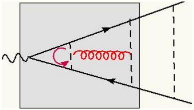

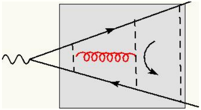

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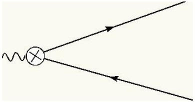
Needs external current renormalization



Ultrasoft corrections to Green Function

- All the logarithmic part were obtained analytically (Beneke-YK 08)
- Constant part numerically, a function of one dimensionless variable $\hat{E} \equiv (E + i\Gamma_t)/(m_t \alpha_s^2)$

$$\delta^{us} G(E) = \frac{2m^2\alpha_s^4}{9\pi^2} \Bigg\{ \left[\frac{17\, i\hat{\Gamma}_t}{24} + \frac{527\,\hat{G}_C}{72} \right] \frac{1}{\epsilon^2} + \left[\frac{17\, i\hat{\Gamma}_t}{12} + \frac{221\,\hat{G}_C}{36} \right] \frac{L_\mu}{\epsilon} + \Bigg[\left(\frac{19}{12} \ln 2 - \frac{91}{72} \right) i\hat{\Gamma}_t$$

$$+ \left(-\frac{119}{12} \ln 2 + \frac{2059}{108} \right) \hat{G}_C \Bigg] \frac{1}{\epsilon} + \left[-\frac{34\, i\hat{\Gamma}_t}{3} - \frac{595\,\hat{G}_C}{9} \right] L_{\alpha_s}^2 + \left[-\frac{17\, i\hat{\Gamma}_t}{12} - \frac{833\,\hat{G}_C}{36} \right] L_\mu^2$$

$$+ \left[\frac{34\, i\hat{\Gamma}_t}{3} + \frac{748\,\hat{G}_C}{9} \right] L_{\alpha_s} L_\mu + \Bigg[\frac{2380\,\mathcal{P}^2}{27} + \left(\frac{272 \ln 2}{9} - \frac{23483}{162} + \frac{2380}{27\lambda} + \frac{272}{27\lambda^2} \right) \mathcal{P}$$

$$+ \left(\frac{27\lambda}{2} - \frac{16}{3\lambda} \right) \psi' + \frac{64}{27\lambda^3} + \frac{4\,(-1331 + 306 \ln 2)}{81\lambda} + \frac{4\,(-199 + 114 \ln 2)}{81\lambda^2} \Bigg] L_{\alpha_s}$$

$$+ \Bigg[-\frac{1496\,\mathcal{P}^2}{27} + \left(-\frac{34 \ln 2}{3} + \frac{5065}{72} - \frac{1496}{27\lambda} - \frac{136}{27\lambda^2} \right) \mathcal{P} + \left(\frac{8}{3\lambda} - \frac{81\lambda}{8} \right) \psi'$$

$$- \frac{32}{27\lambda^3} + \frac{163 - 114 \ln 2}{27\lambda^2} + \frac{271 - 51 \ln 2}{9\lambda} \Bigg] L_\mu + \delta^{us}(\hat{E}) \Bigg\},$$

$$L_\mu = \ln \frac{\mu}{m_t}, \quad L_{\alpha_s} = \ln \alpha_s, \quad \lambda = \frac{C_F}{2\sqrt{-\hat{E}}}, \quad \mathcal{P} = \ln \left(\frac{C_F}{\lambda} \right) + \gamma_E + \psi(1 - \lambda),$$

Scale dependence of ultrasoft corrections to R

$E = \sqrt{s} - 2m_t$, $\Gamma_t = 1.4$ GeV, $m_t = 175$ GeV, $\alpha_s = 0.14$

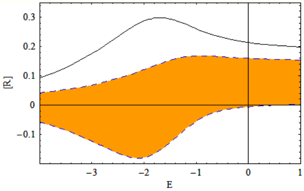


Fig.1: Ultrasoft correction only. Constant (solid line), log+cons (orange band) with $\mu = 32.6$ GeV (upper dashed), $\mu = 175$ GeV (lower dashed)

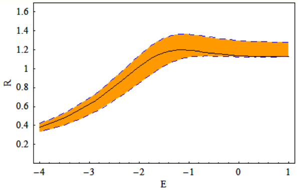


Fig.2: $R_{\rm LO} + [R]_{us}$; LO (solid line), LO+ ultrasoft (orange band) with $\mu = 32.6$ GeV (upper dashed) and $\mu = 175$ GeV (lower dashed)

- Ultrasoft contribution itself is not physical (scale dependent)
- Constant part is $+25\%$ in Fig.1 around peak position.

We are combining all the QCD effect to build up scale invariant quantity for phenomenology. Following parameters in (P)NRQCD will be used;

$$J^i = c_v \psi^\dagger \sigma^i \chi + d_v \frac{1}{6m_t^2} \psi^\dagger \sigma^i \mathbf{D}^2 \chi \qquad \mathcal{L}_{\rm QCD} \Leftrightarrow \mathcal{L}_{\rm PNRQCD}$$

$c^{(2)}$: Beneke-Signer-Smirnov('97), Czanecki-Melnikov('97)

$c^{(3)}_{n_f}$: Marquard-Piclum- Seidel-Steihauser(06)

$d_v^{(1)}$: Luke-Savage('97)

a_2: Schröder('98)

$a_{3,pade}$: Chishtie-Elias (01)

$\delta\mathcal{L}^{(1)}$: Manohar('97), Beneke-Signer-Smirnov('99), Wüster-Schuller('03)

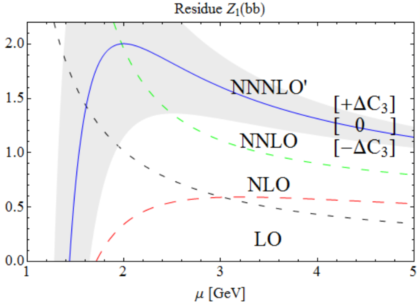
$\delta\mathcal{L}^{(2)}$: Kniehl-Penin- Smirnov-Steinhauser(02) ($\delta\mathcal{L}^{(2)} = \mathcal{O}(\epsilon)$ not known)

$\delta\mathcal{L}^{(us)}$: Brambilla-Pineda-Soto-Vairo('99), Kniehl-Penin- Smirnov-Steinhauser(02)

- We use $a_{3,pade}$ and set unknown $\mathcal{O}(\epsilon)$-potential terms zero (numerical difference is expected to be small)
- It will turn out that effect of $c^{(3)}$ is very important.
  We use $\pm c^{(3)}_{n_f}$ as an order estimate of unknown terms.
  Comment: At two loop non-n_f term of $c^{(2)}$ is larger than n_f-term in magnitude and its sign is opposite to n_f-term.

$\Upsilon(1S)$ residue: $\Pi(q) \overset{E\to E_n}{=} \frac{N_c}{2m_b^2}\frac{Z_n}{E_n - E - i\epsilon}$

Residue of $\Pi(q)$ is physical quantity, which can be extracted from leptonic decay width of Υ. Scale dependence of $Z_n(\mu)/Z_n(\mu_B)$ is plotted $(\mu_B = 2\text{GeV})$ (Beneke-YK-Penin-Schuller(07)).



$$Z_n = \left[c_v - \frac{E_n}{2m_b}\left(1 + \frac{d_v}{3}\right)\right]^2 |\Psi_n(0)|^2$$

$$\Gamma\left(\Upsilon(nS) \to l^+ l^-\right) = \frac{4\pi N_c Q_t^2 \alpha^2}{3m_b^2} Z_n$$

$m_b = 5\text{GeV}$ used

$\Gamma(\Upsilon(1S))|_{\text{exp}} = 1.4 \pm 0.1\text{KeV}$

$\rightarrow$ aqua band

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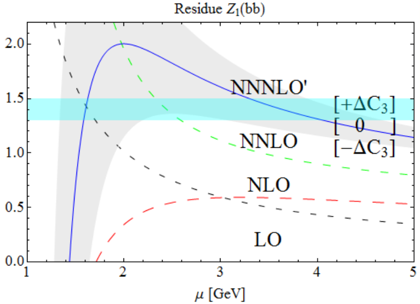


$$Z_n = \left[c_v - \frac{E_n}{2m_b}\left(1 + \frac{d_v}{3}\right)\right]^2 |\Psi_n(0)|^2$$

$$\Gamma\left(\Upsilon(nS) \to l^+l^-\right) = \frac{4\pi N_c Q_t^2 \alpha^2}{3m_b^2} Z_n$$

$m_b = 5\mathrm{GeV}$ used

$\Gamma(\Upsilon(1S))|_{\mathrm{exp}} = 1.4 \pm 0.1\mathrm{KeV}$

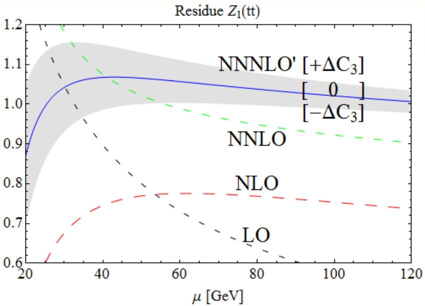
$\to$ aqua band

- Uncertainty (gray band) due to unknown $c_v^{(3)}$ is large.
- Scale-dependence can be reduced if $c_v^{(3)}$ is small (if non-n_f term has opposite sign to cancel n_f-term).

$t\bar{t}(1S)$ residue: $\Pi(q) \overset{E\to E_n}{=} \frac{N_c}{2m_t^2}\frac{Z_n}{E_n - E - i\Gamma_t}$

The first residue Z_1 of $\Pi(q)$ governs magnitude of R.

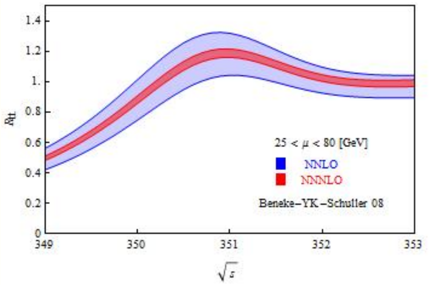
Below $Z_1(\mu)/Z_1(\mu_B)$ is plotted ($\mu_B = 32.62\mathrm{GeV}$) (Beneke-YK-Penin-Schuller(07)).



- Third order correction is 10-15% shift to NNLO (depends on value of $c^{(3)}$)
- Scale-dependence is mild (much better if non-n_f term is negative).

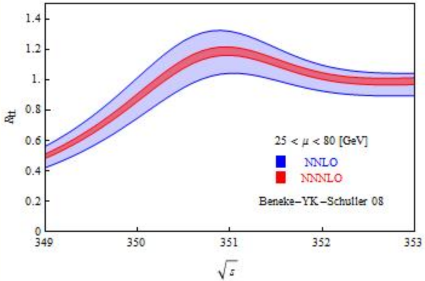
$$Z_{t\bar{t}(1S)}|_{\mu_B} = \frac{(C_F m_t \alpha_s)^3}{8\pi}\left[1 - 2.13\alpha_s + 22.7\alpha_s^2 + \left(- 38.8 + 5.8_{a3} + 37.6_{c3,n_f}\right)\alpha_s^3\right]$$

Top cross section



- $m_{t,\mathrm{PS}}(20\mathrm{GeV}) = 175\mathrm{GeV}$, $\Gamma_t = 1.4\mathrm{GeV}, \alpha_s(M_z) = 0.1189$.
  (We included all the known corrections, e.g. $c^{(3)}_{n_f}$. All the logarithm term of c_v is known)
- The scale-dependence is significantly reduced to 10% at NNNLO'.
- Constant part of NNNLO is also important:

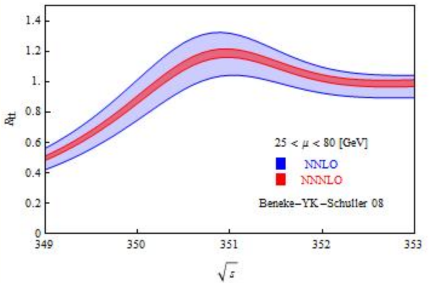
Top cross section



- $m_{t,\mathrm{PS}}(20\mathrm{GeV}) = 175\mathrm{GeV},\ \Gamma_t = 1.4\mathrm{GeV}, \alpha_s(M_z) = 0.1189.$
  (We included all the known corrections, e.g. $c^{(3)}_{n_f}$. All the logarithm term of c_v is known)
- The scale-dependence is significantly reduced to 10% at NNNLO'.
-

$$Z_1 = \frac{(m_t\alpha_s C_F)^3}{8\pi}\Big[1+\alpha_s\Big(-2.13+3.66L\Big)+\alpha_s^2\Big(8.38-7.26\ln\alpha_s-13.40L+8.93L^2\Big)$$
$$+\alpha_s^3\Big(11.01+[37.58]_{c_3,n_f}-9.79\ln\alpha_s-16.35\ln^2\alpha_s$$
$$+(53.17-44.27\ln\alpha_s)L-48.18L^2+18.17L^3\Big)\Big],\qquad (L=\ln(\mu/(m_t\alpha_s C_F)))$$

Top cross section



- $m_{t,\mathrm{PS}}(20\mathrm{GeV}) = 175\mathrm{GeV}$, $\Gamma_t = 1.4\mathrm{GeV}, \alpha_s(M_z) = 0.1189$.
  (We included all the known corrections, e.g. $c_{n_f}^{(3)}$. All the logarithm term of c_v is known)
- The scale-dependence is significantly reduced to 10% at NNNLO'.
- Constant part of NNNLO is also important:

Summary

QCD part:

- We have completed NNNLO QCD corrections to NR Green function.
- There are some missing coefficients for the threshold cross section. The most important piece is (probably) $c_v^{(3)}$.
- Remaining scale uncertainty is about 10% (if $c^{(3)}$ is NOT too large).

EW part:

- We have started 2-loop EW (W boson) and QCD mixed computations.
- (Hard-loop) Corrections to the $\gamma t\bar{t}$ vertex due to W and g shifts the threshold cross section about 1%. (Remaining most challenging part is $\mathcal{O}(\alpha\alpha_s)$ box diagrams)
- There are many to be done concerning to EW corrections $\Leftrightarrow$ Unstable top quark effect, etc.

Part VII

Backup

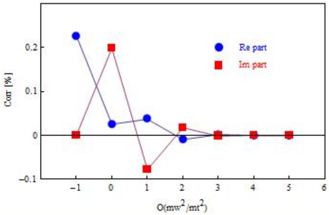
EW 1-loop

- W-loop corr to hard matching coefficient of $t\bar{t}$ X-Section:

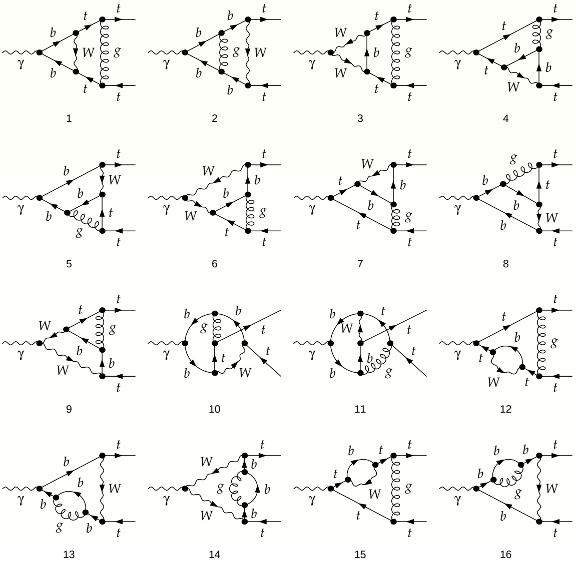
$$(C_V \times C_V) \sim 0.460 + \alpha(0.56 + 0.29\,i)_\triangle + \alpha(-2.61 - 3.46\,i)_\square + \text{GBCont}$$
$$(C_A \times C_A) \sim 0.022 + \alpha(0.19 + 0.08\,i)_\triangle + \alpha(-0.56 - 0.75\,i)_\square + \text{GBCont}$$

The imaginary part from (t $\longrightarrow$ W-b)-cut should be extracted (Hoang-Reißer)

- In fig. corr to tt-vertex by W-loop is shown in %



EW x QCD Feynman Diagrams



Potential insertion to the wave function

Quarkonium wave function at the origin: $\delta|\Psi(0)|^2 = |\Psi_C(0)|^2 \left(\frac{\alpha_s}{4\pi}\right)^3 f_3$

$$
\begin{aligned}
\frac{f_3^{nC}}{64\pi^2} = &\left[\frac{7}{6}C_F^3 + \frac{37}{12}C_A C_F^2 + \frac{4}{3}C_A^2 C_F + \beta_0\left(\frac{4}{3}C_F^2 + 2C_A C_F\right)\right] L^2 + \left[C_F^3\left(-\frac{3}{2} + \frac{14}{3n} - \frac{7S_1}{3}\right)\right. \\
&+ C_A C_F^2\left(\frac{226}{27} + \frac{8\ln 2}{3} + \frac{37}{3n} - \frac{5}{3n^2} - \frac{37S_1}{6} + C_A^2 C_F\left(\frac{145}{18} + \frac{4\ln 2}{3} + \frac{16}{3n} - \frac{8S_1}{3}\right)\right) \\
&+ C_F^2 T_F\left(\frac{2}{15} - \frac{59}{27}n_f\right) - \frac{109}{36}C_A C_F T_F n_f + \beta_0\left\{C_F^2\left(\frac{16}{3} + \frac{10}{3n} - \frac{75}{16n^2} - \frac{\pi^2 n}{9} - \frac{4S_1}{3} + \frac{2nS_2}{3}\right)\right. \\
&\left.\left.+ C_A C_F\left(\frac{15}{8} + \frac{5}{n} - \frac{\pi^2 n}{6} - 2S_1 + nS_2\right)\right\}\right] L + \left[\frac{1}{3}C_F^3 + \frac{1}{2}C_A C_F^2\right] L_m L + \left[\frac{1}{12}C_F^3 + \frac{1}{8}C_A C_F^2\right] L_m^2 \\
&+ \left[C_F^3\left(\frac{1}{12} + \frac{2}{3n} - \frac{S_1}{3}\right) + C_A C_F^2\left(-\frac{5}{9} + \frac{1}{n} - \frac{S_1}{2}\right) + \frac{1}{15}C_F^2 T_F\right] L_m + \frac{c_{\psi,3}^{nC}}{64\pi^2},
\end{aligned}
$$

(Beneke-YK-Schuller07)

Potential insertion to the wave function

Quarkonium wave function at the origin: $\delta|\Psi(0)|^2 = |\Psi_C(0)|^2 \left(\frac{\alpha_s}{4\pi}\right)^3 f_3$

$$
\begin{aligned}
\frac{c^{nC}_{\psi,3}}{64\pi^2} &= \Big[-\frac{137}{36} - \frac{49\pi^2}{432} - \frac{25}{6n} + \frac{35}{12n^2} + S_1\left(\frac{3}{2} - \frac{14}{3n} + \frac{7S_1}{6}\right) - \frac{7S_2}{6}\Big] C_F^3 + \Big[\frac{7061}{486} - \frac{50\pi^2}{81} + \frac{1475}{108n} + \frac{\pi^2}{9n} \\
&- \frac{321}{32n^2} + \ln 2\left(\frac{353}{54} + \frac{16}{3n} - \frac{16\ln 2}{9}\right) - S_1\left(\frac{226}{27} + \frac{8\ln 2}{3} + \frac{37}{3n} + \frac{1}{n^2} - \frac{37S_1}{12}\right) - S_2\left(\frac{37}{12} + \frac{2}{3n}\right)\Big] C_A C_F^2 \\
&+ \Big[\frac{3407}{432} - \frac{5\pi^2}{18} + \frac{133}{9n} + \ln 2\left(\frac{187}{108} + \frac{8}{3n} - \frac{8\ln 2}{9}\right) - \frac{4S_2}{3} - S_1\left(\frac{145}{18} + \frac{4\ln 2}{3} + \frac{16}{3n} - \frac{4S_1}{3}\right)\Big] C_A^2 C_F \\
&+ \Big[\frac{1}{15} + \frac{4}{15n} - \frac{2S_1}{15}\Big] C_F^2 T_F + \Big[-\frac{361}{108} + \frac{49\ln 2}{108} - \frac{109}{18n} + \frac{109 S_1}{36}\Big] C_A C_F T_F\, n_f + \Big[-\frac{3391}{486} + \frac{5\pi^2}{648} \\
&- \frac{2\ln 2}{27} - \frac{118}{27n} + \frac{125}{24n^2} + \frac{59S_1}{27}\Big] C_F^2 T_F\, n_f + \beta_0 \Big[\Big\{\frac{1027}{648} + \frac{19}{6n} + \frac{25}{24n^2} - \frac{35\pi^2}{108} - \frac{11\pi^2 n}{27} + \frac{5\pi^2}{16n} \\
&+ \frac{4nS_3}{3} - \frac{2nS_{2,1}}{3} - S_1\left(\frac{10}{9} + \frac{1}{3n} + \frac{45}{16n^2} - \frac{\pi^2 n}{9} + \frac{2nS_2}{3}\right) + S_2\left(1 + \frac{22n}{9} - \frac{15}{8n}\right)\Big\} C_F^2 \\
&+ \Big\{\frac{7}{24} - \frac{91\pi^2}{144} - \frac{1}{4n} - \frac{5\pi^2 n}{24} - S_1\left(\frac{3}{8} + \frac{1}{2n} - \frac{\pi^2 n}{6} + nS_2\right) + S_2\left(\frac{3}{2} + \frac{5n}{4}\right) - nS_{2,1} + 2nS_3\Big\} C_A C_F\Big] \\
&+ \left(\frac{v_m^{(1,\epsilon)}}{8} + \frac{v_q^{(1,\epsilon)}}{12} + \frac{v_p^{(1,\epsilon)}}{12}\right) C_F^2 - \frac{C_F}{6}\, b_2^{(\epsilon)}.
\end{aligned}
$$

(Beneke-YK-Schuller07)

Potential insertion to the wave function

Quarkonium wave function at the origin: $\delta|\Psi(0)|^2 = |\Psi_C(0)|^2 \left(\frac{\alpha_s}{4\pi}\right)^3 f_3$

- "Toponium" wave function:

$$\frac{\delta_3|\psi_1(0)|^2_{nC}}{|\psi_1^{(0)}(0)|^2} = \frac{\alpha_s^3(\mu_B)}{\pi}\Big(-165.1 + 0.8\,\ln(\alpha_s C_F) + 0.9\,\ln^2(\alpha_s C_F)\Big) = -0.14$$

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- Bottomonium wave function:

$$\frac{\delta_3|\psi_1(0)|^2_{nC}}{|\psi_1^{(0)}(0)|^2} = \frac{\alpha_s^3(\mu_B)}{\pi}\Big(-162.0 + 0.8\ \ln(\alpha_s C_F) + 0.9\ \ln^2(\alpha_s C_F)\Big) = -1.4$$

Potential insertion to the wave function

Quarkonium wave function at the origin: $\delta|\Psi(0)|^2 = |\Psi_C(0)|^2 \left(\frac{\alpha_s}{4\pi}\right)^3 f_3$

- "Toponium" wave function:

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- However the wave function is not physical, namely μ-dependence.