NNLO corrections to jet rates and event shapes in e^+e^- annihilation

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$e^+e^- \to 3$ jets and event shapes

Classical QCD observable

- testing ground for QCD: perturbation theory, power corrections and logarithmic resummation
- precision measurement of strong coupling constant α_s
- current error on α_s from jet observables dominated by theoretical uncertainty:
  S. Bethke, 2006

$$\alpha_s(M_Z) \quad = \quad 0.121 \pm 0.001(\text{experiment}) \pm 0.005(\text{theory})$$

- theoretical uncertainty largely from missing higher orders
- previous status: NLO plus NLL resummation

Theoretical description

- easier than at hadron colliders, since coloured partons only in final state:
  no initial state emission, no parton distributions
- new calculational methods first developed for e^+e^-, then extended to hadronic processes

$e^+e^- \to 3$ jets and event shapes

Event shape variables

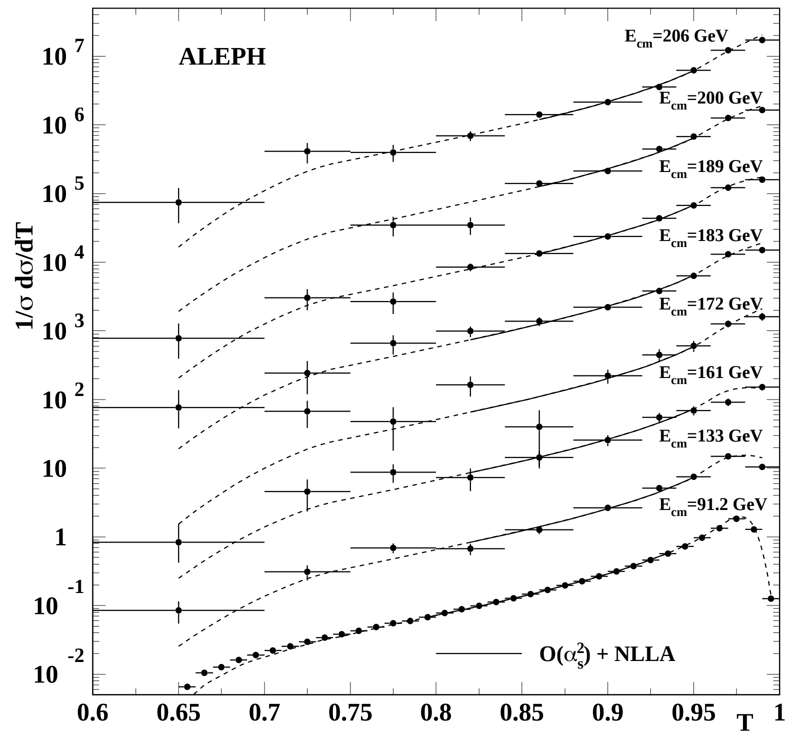
assign a number x to a set of final state momenta: $\{p\}_i \to x$

e.g. Thrust in e^+e^-

$$T = \max_{\vec{n}} \frac{\sum_{i=1}^{n} |\vec{p_i} \cdot \vec{n}|}{\sum_{i=1}^{n} |\vec{p_i}|}$$

limiting values:

- back-to-back (two-jet) limit: $T = 1$
- spherical limit: $T = 1/2$



$e^+e^- \to 3$ jets and event shapes

Standard Set of LEP

- Thrust (E. Farhi)

$$T = \max_{\vec{n}} \left(\sum_{i=1}^{n} |\vec{p_i} \cdot \vec{n}| \right) / \left(\sum_{i=1}^{n} |\vec{p_i}| \right)$$

- Heavy jet mass (L. Clavelli, D. Wyler)

$$M_i^2/s = \frac{1}{E_{\text{vis}}^2} \left(\sum_{k \in H_i} |\vec{p_k}| \right)^2$$

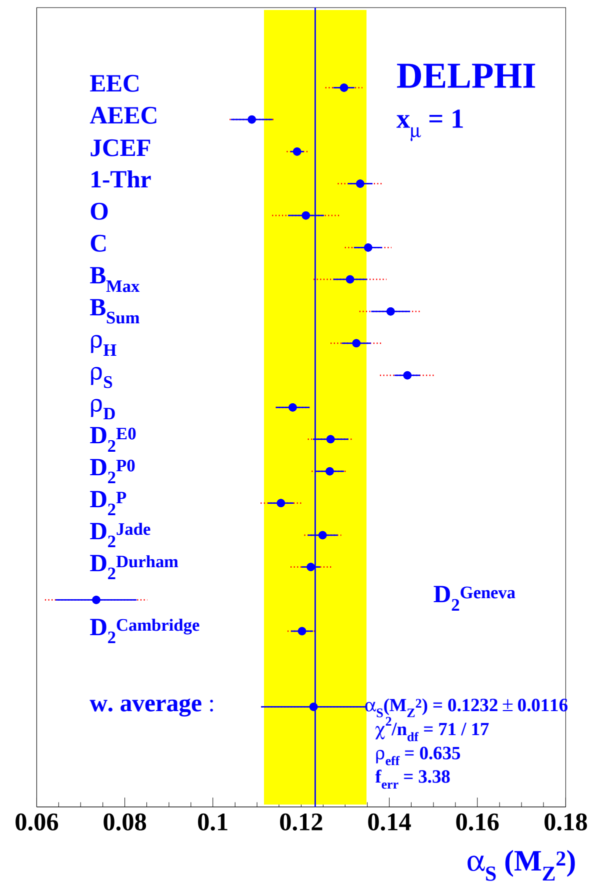
- C-parameter: eigenvalues of the tensor (G. Parisi)

$$\Theta^{\alpha\beta} = \frac{1}{\sum_k |\vec{p_k}|} \frac{\sum_k p_k^\alpha p_k^\beta}{\sum_k |\vec{p_k}|}$$

- Jet broadenings (P.E.L. Rakow, B. Webber)

$$B_i = \left(\sum_{k \in H_i} |\vec{p_k} \times \vec{n}_T| \right) / \left(2 \sum_k |\vec{p_k}| \right)$$

$$B_W = \max(B_1, B_2) \qquad B_T = B_1 + B_2$$



- $3j \to 2j$ transition parameter in Durham algorithm y_{23}^D

  S.Catani, Y.L.Dokshitzer, M.Olsson, G.Turnock, B.Webber

$e^+e^- \to 3$ jets and event shapes

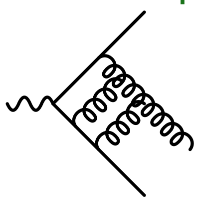
Current status: NLO and NLL

- NLO calculations of event shapes and $3j$
  R.K. Ellis, D.A. Ross, A.E. Terrano; Z. Kunszt
  J. Vermaseren, K.F. Gaemers, S.J. Oldham; L. Clavelli, D. Wyler
- NLO parton level event generators for $3j$
  EVENT: Z. Kunszt, P. Nason
  EERAD: W. Giele, E.W.N. Glover
  EVENT2: S. Catani, M. Seymour
- NLO parton level event generators for $4j$
  MenloParc: L.D. Dixon, A. Signer
  EERAD2: J. Campbell, M. Cullen, E.W.N. Glover
  Debrecen: Z. Nagy, Z. Trocsanyi
  Mercurito: D. Kosower, S. Weinzierl
- NLL resummation
  S. Catani, L. Trentadue, G. Turnock, B. Webber; Y.L. Dokshitzer, A. Lucenti, G. Marchesini, G.P. Salam; A. Banfi, G. Zanderighi
- Power corrections
  G. Korchemsky, G. Sterman; Y. Dokshitzer, B.R. Webber

Ingredients to NNLO $e^+e^- \to$ 3-jet

- Two-loop matrix elements

$|\mathcal{M}|^2_{\text{2-loop},3 \text{ partons}}$

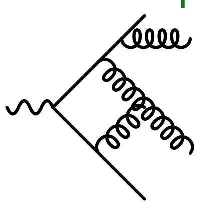


explicit infrared poles from loop integrals

L. Garland, N. Glover, A. Koukoutsakis, E. Remiddi, TG;
S. Moch, P. Uwer, S. Weinzierl

- One-loop matrix elements

$|\mathcal{M}|^2_{\text{1-loop},4 \text{ partons}}$

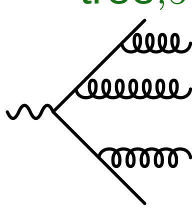


explicit infrared poles from loop integral and
implicit infrared poles due to single unresolved radiation

Z. Bern, L. Dixon, D. Kosower, S. Weinzierl;
J. Campbell, D.J. Miller, E.W.N. Glover

- Tree level matrix elements

$|\mathcal{M}|^2_{\text{tree},5 \text{ partons}}$



implicit infrared poles due to double unresolved radiation

K. Hagiwara, D. Zeppenfeld;
F.A. Berends, W.T. Giele, H. Kuijf;
N. Falck, D. Graudenz, G. Kramer

Infrared Poles cancel in the sum

NNLO Infrared Subtraction

Structure of NNLO m-jet cross section:

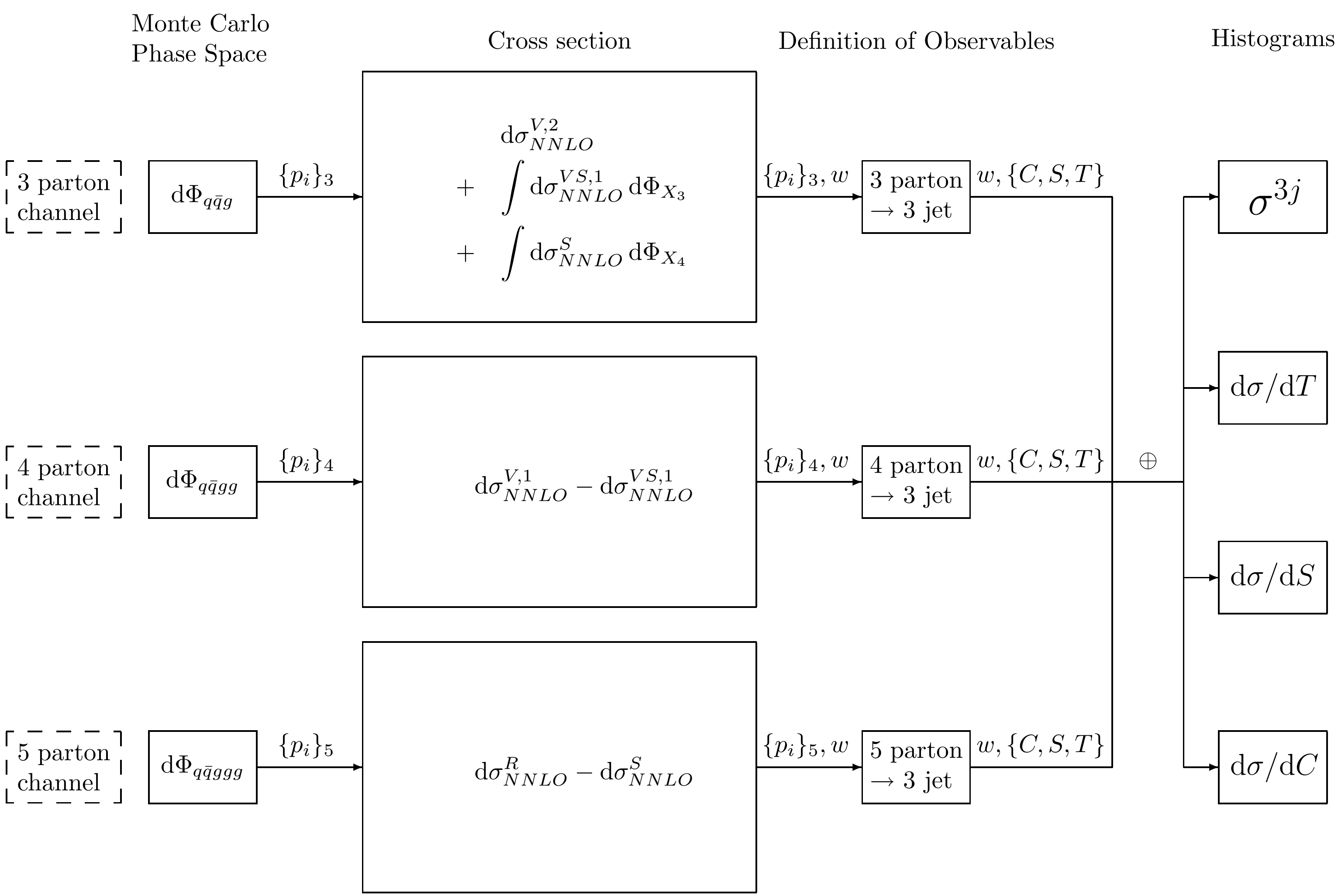
$$
\begin{aligned}
\mathrm{d}\sigma_{NNLO} &= \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\sigma^{R}_{NNLO} - \mathrm{d}\sigma^{S}_{NNLO}\right) \\
&+ \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}\right) \\
&+ \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{V,2}_{NNLO} + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^{S}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{VS,1}_{NNLO} \, ,
\end{aligned}
$$

- $\mathrm{d}\sigma^{S}_{NNLO}$: real radiation subtraction term for $\mathrm{d}\sigma^{R}_{NNLO}$
- $\mathrm{d}\sigma^{VS,1}_{NNLO}$: one-loop virtual subtraction term for $\mathrm{d}\sigma^{V,1}_{NNLO}$
- $\mathrm{d}\sigma^{V,2}_{NNLO}$: two-loop virtual corrections

Each line above is finite numerically and free of infrared ϵ-poles $\longrightarrow$ numerical programme

Numerical Implementation

Structure of $e^+e^- \to 3$ jets program:



Numerical Implementation

Antenna subtraction

NLO: M. Cullen, J. Campbell, E.W.N. Glover; D. Kosower; A. Daleo, D. Maitre, TG

NNLO: A. Gehrmann-De Ridder, E.W.N. Glover, TG

- construct subtraction terms from physical $1 \to 3$ and $1 \to 4$ matrix elements
- each antenna function interpolates between all limits associated to one or two unresolved partons
- integrated subtraction terms cancel infrared pole structure of two-loop matrix element

  S. Catani; G. Sterman, M.E. Yeomans-Tejeda

Checks

- cancellation of infrared poles in 3-parton and 4-parton channel
- convergence of subtraction terms towards matrix elements along phase space trajectories
- distributions in raw phase space variables
- independence on phase space cut y_0

Three-jet cross section at NNLO

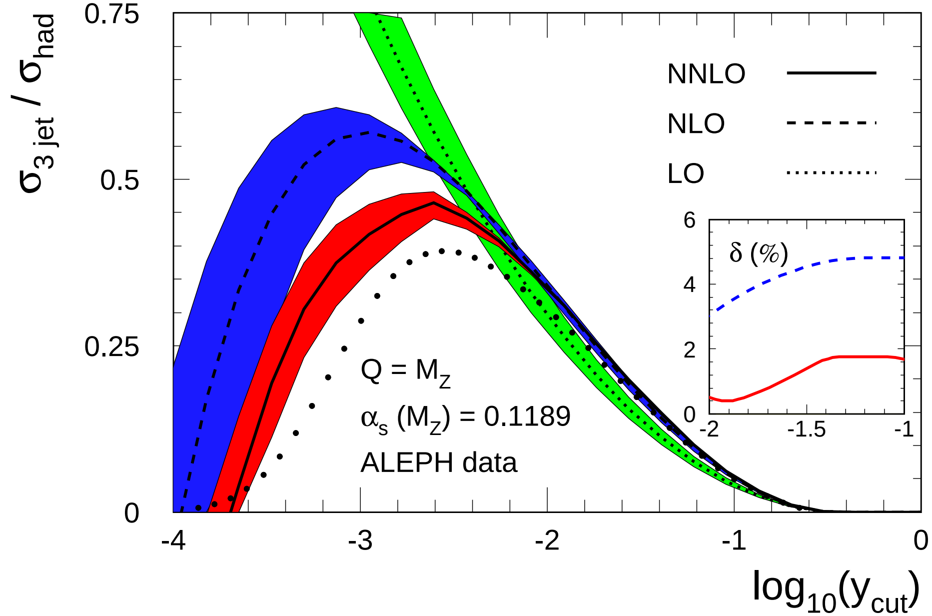
NNLO corrections: jet rates

Three-jet fraction in Durham jet algorithm

$$y_{i,j,D} = \frac{2\min(E_i^2, E_j^2)\,(1-\cos\theta_{ij})}{E_{vis}^2}$$

- vary $\mu = [M_Z/2\,;\,2\,M_Z]$
- determine minimal and maximal values

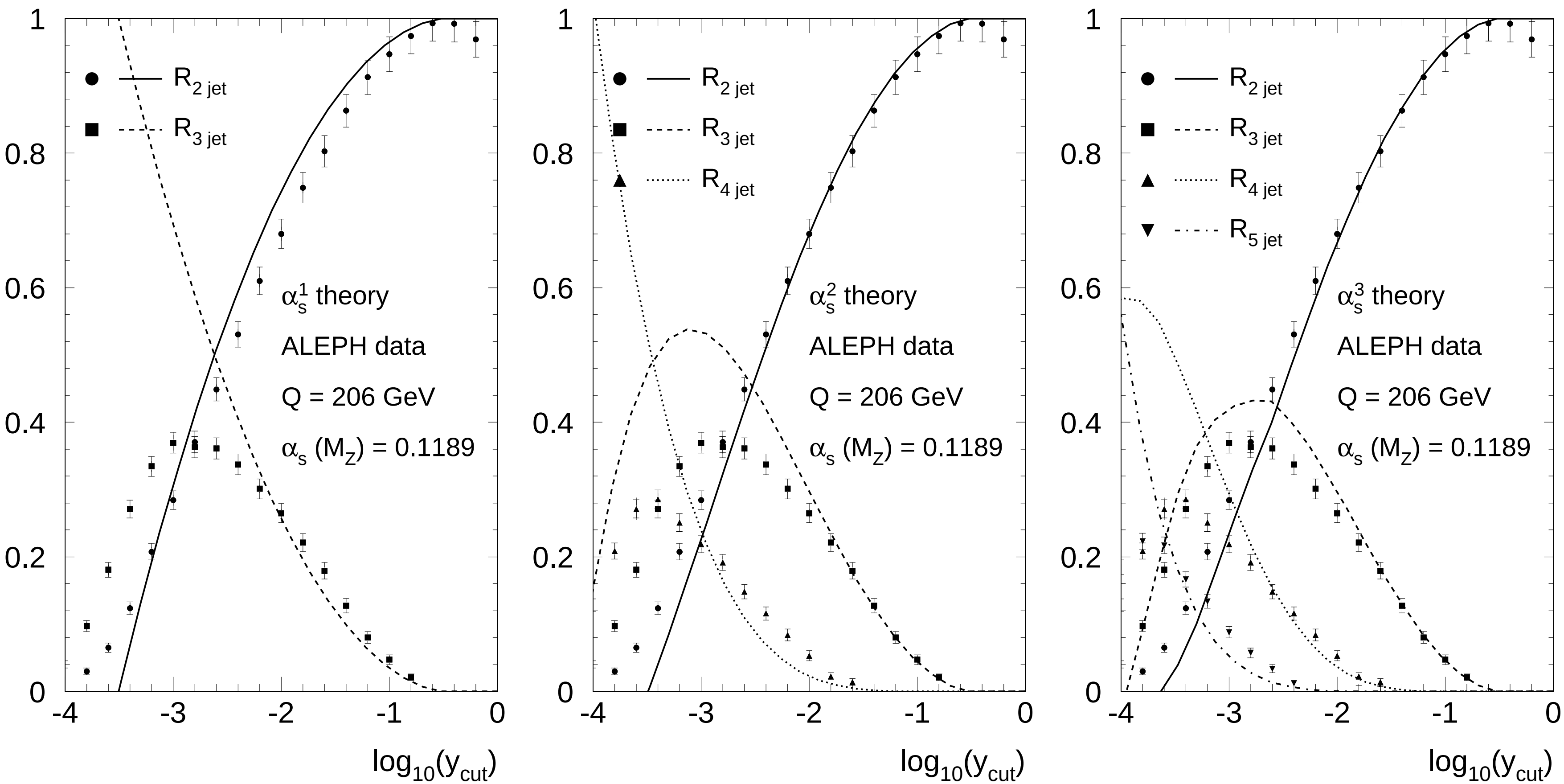
$$\delta = \frac{\max(\sigma) - \min(\sigma)}{2\sigma(\mu = M_Z)}$$



- NNLO corrections small
- substantial reduction of scale dependence
- better description towards lower jet resolution

Three-jet cross section at NNLO

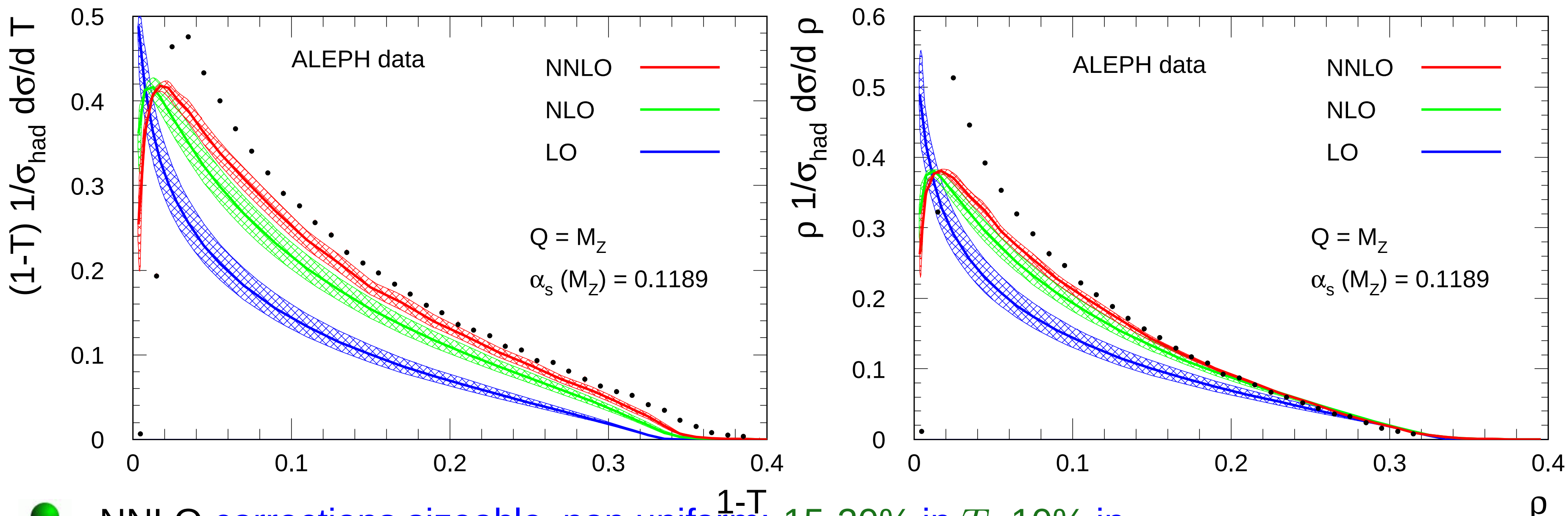
NNLO corrections: jet rates



- substantial improvement towards lower y_{cut}
- two-jet rate now NNNLO

Event shapes at NNLO

NNLO thrust and heavy mass distributions

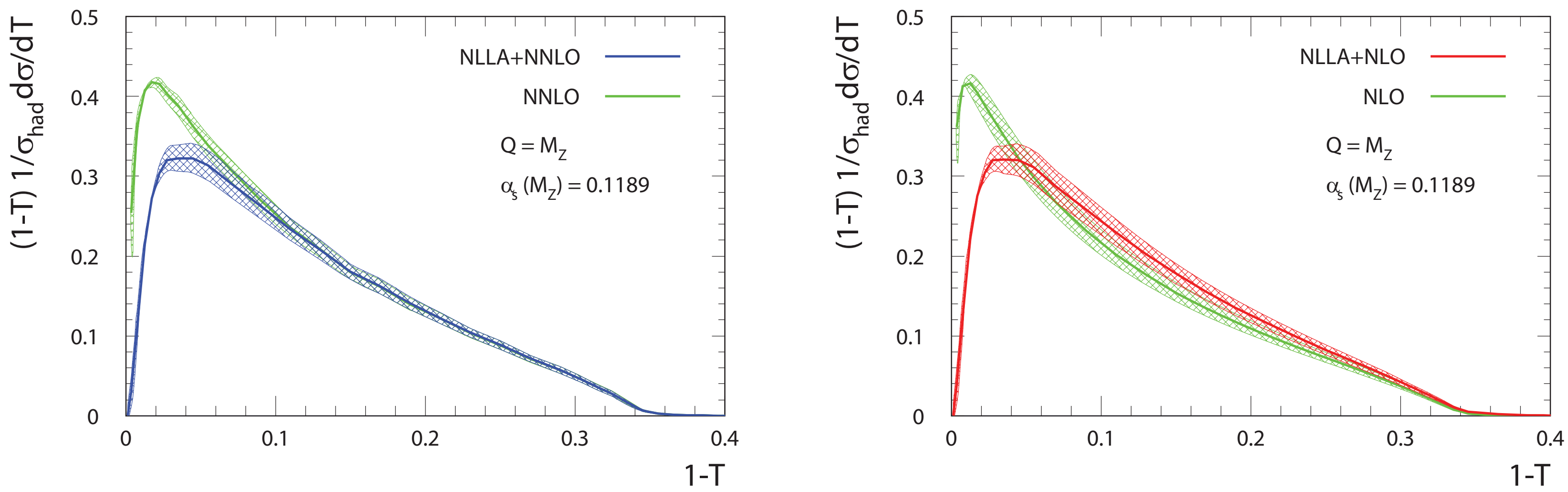


- NNLO corrections sizeable, non uniform: 15-20% in T, 10% in ρ
- theory uncertainty reduced by about 40 %
- large $1 - T, \rho > 0.33$: kinematically forbidden at LO
- small $1 - T, \rho$: two-jet region, need matching onto NLL resummation
  G. Luisoni, H. Stenzel, TG
- need to include hadronization corrections

Event shapes at NLLA+NNLO

Matching onto resummation

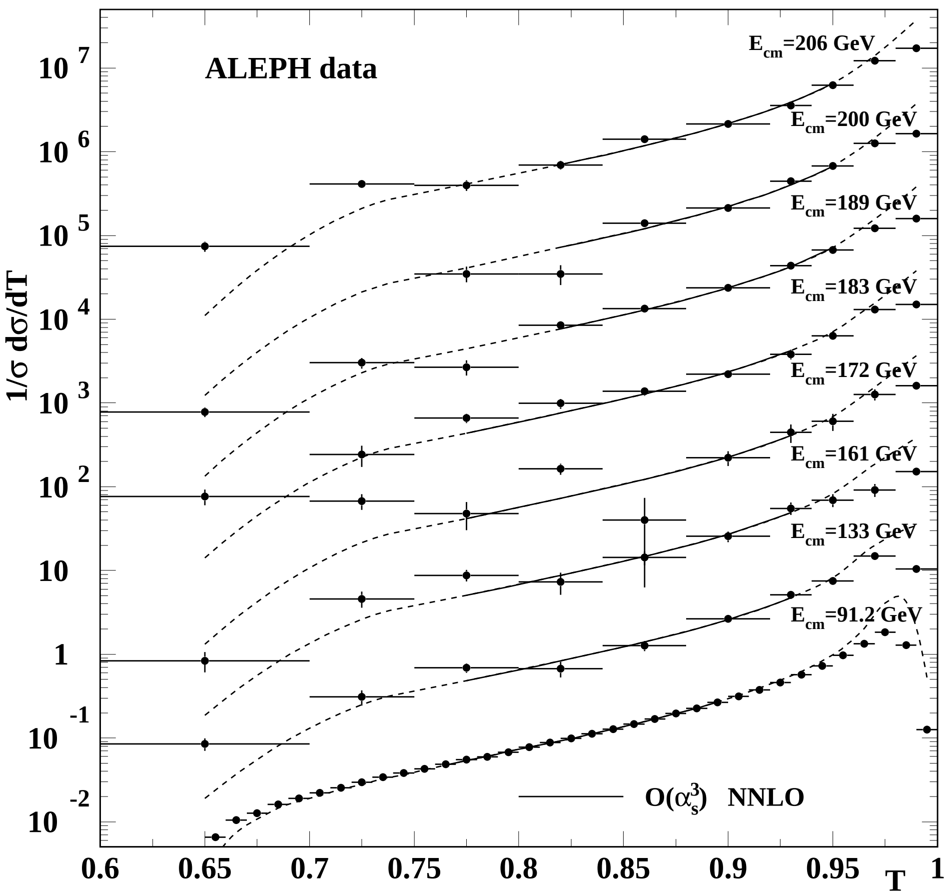
G. Luisoni, H. Stenzel, TG



- resummation to NLLA (S. Catani, L. Trentadue, G. Turnock, B. Webber; Y.L. Dokshitzer, A. Lucenti, G. Marchesini, G.P. Salam; A. Banfi, G. Zanderighi)
- NLO and NLLA+NLO differ in normalisation throughout the full kinematical range
- difference between NNLO and NLLA+NNLO restricted to the two-jet region
- improved scale-dependence in three-jet region
- scale-dependence of NLLA dominant $\longrightarrow$ need higher orders in resummation
  T. Becher, M. Schwartz: thrust beyond NLLA

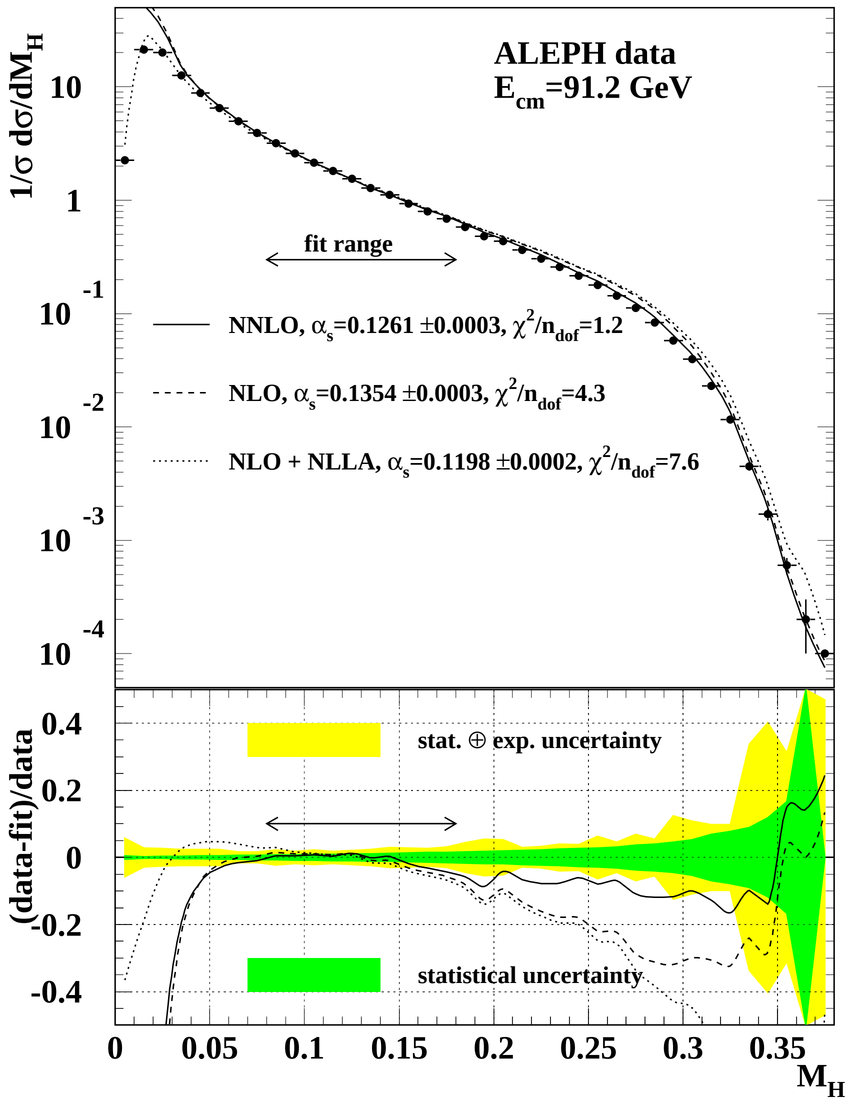
Comparison with data

High precision data from all LEP experiments, compare here to ALEPH



- include quark mass effects to NLO
  P. Nason, C. Oleari
  W. Bernreuther, A. Brandenburg, P. Uwer
  G. Rodrigo, A. Santamaria
- include hadronization corrections
  HERWIG: B. Webber et al.
  ARIADNE: T. Sjostrand et al.
- try new fit of α_s, based on ALEPH analysis
  G. Dissertori, A. Gehrmann-De Ridder,
  G. Heinrich, H. Stenzel, TG

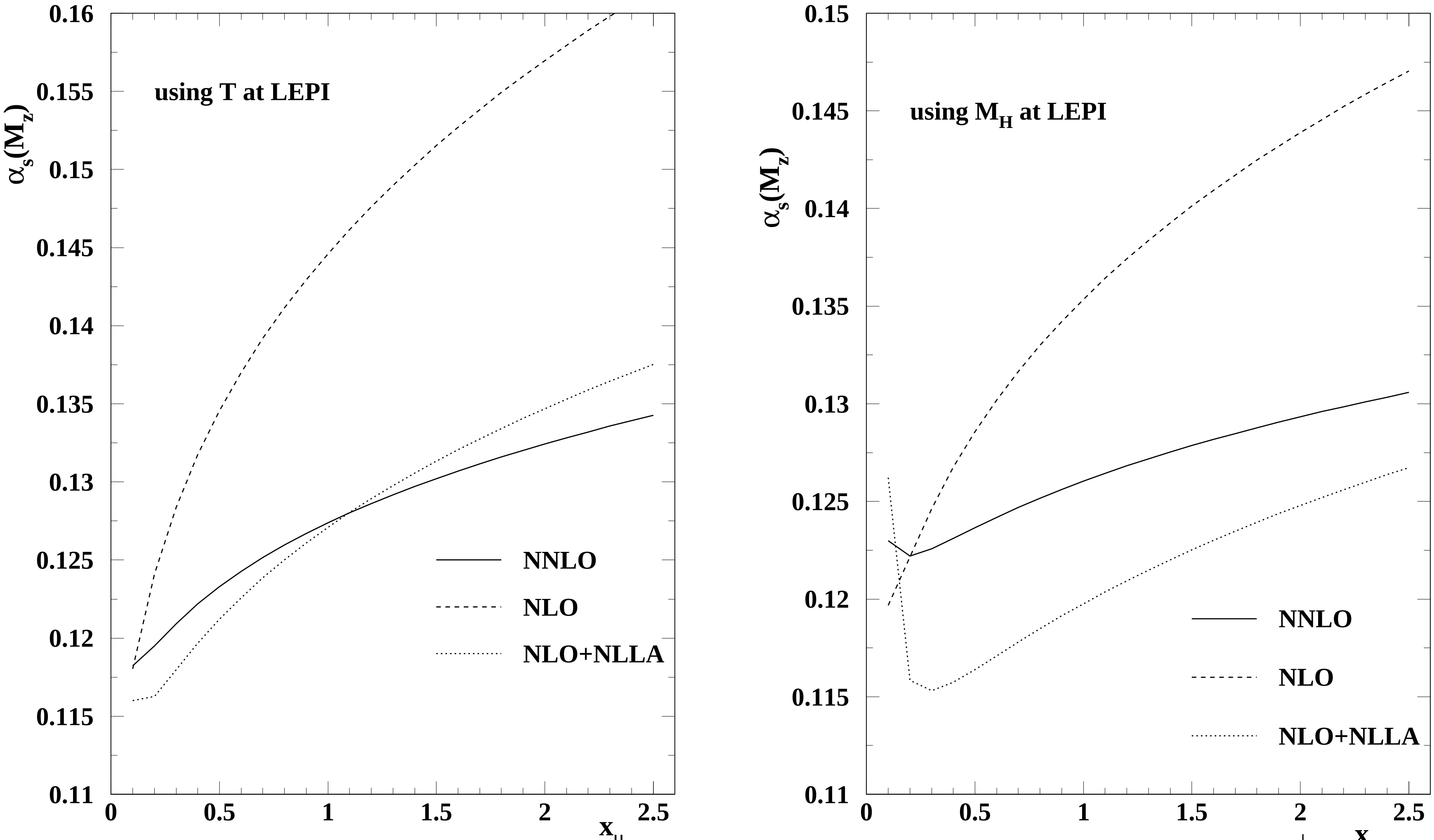
Extraction of α_s



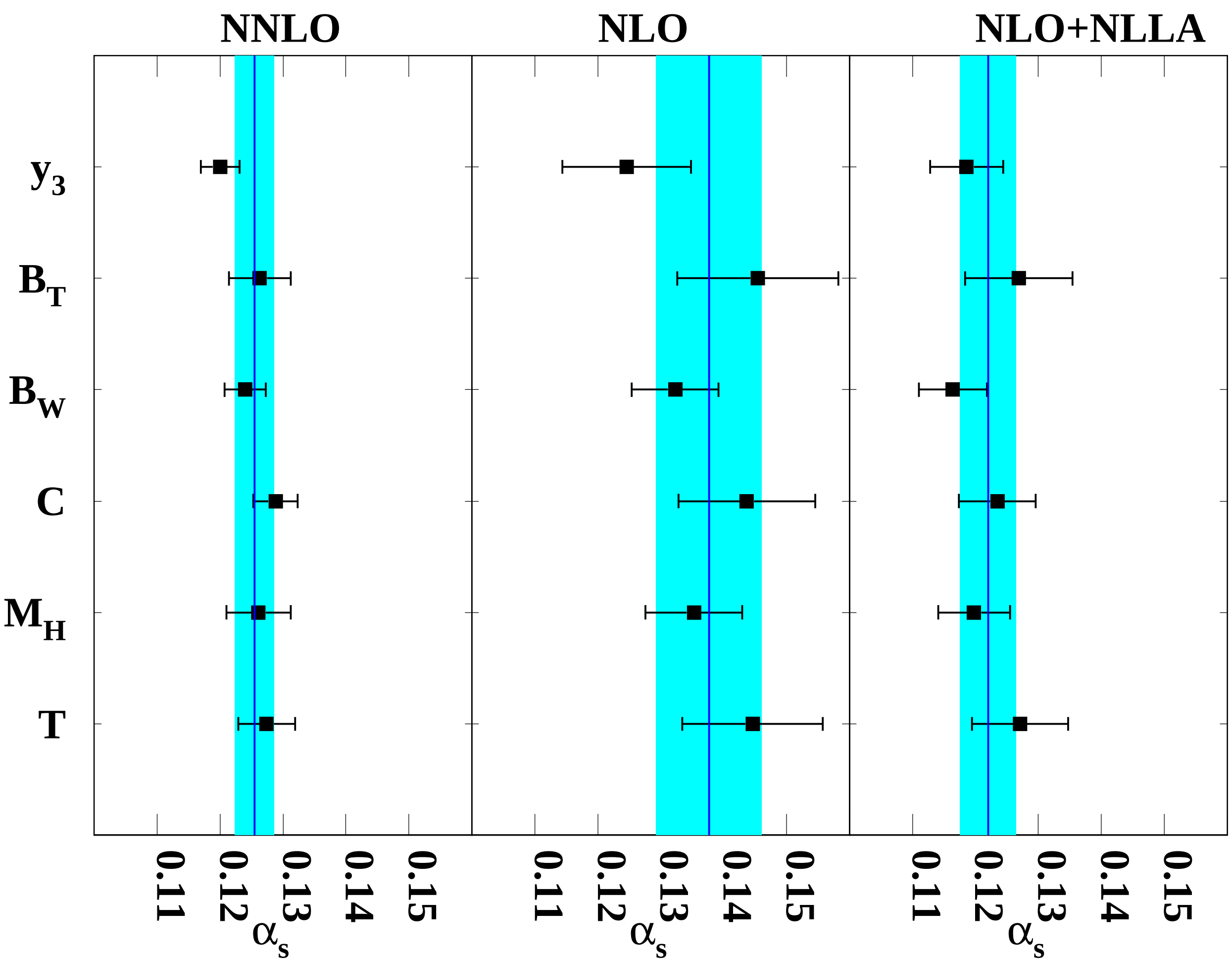
- clear improvement of NNLO over NLO
- good fit quality
- extended range of good description in 3-jet region
- matched NLO+NNLA still yields a better prediction in 2-jet region
- value of α_s lower than at NLO, but still rather high

Extraction of α_s

Uncertainty from renormalisation scale



Extraction of α_s



- scale uncertainty reduced by factor 2 compared to NLO; factor 1.3 compared to NLLA+NLO
- scatter among values from different observables reduced very substantially at NNLO $\longrightarrow$ genuine NNLO effect

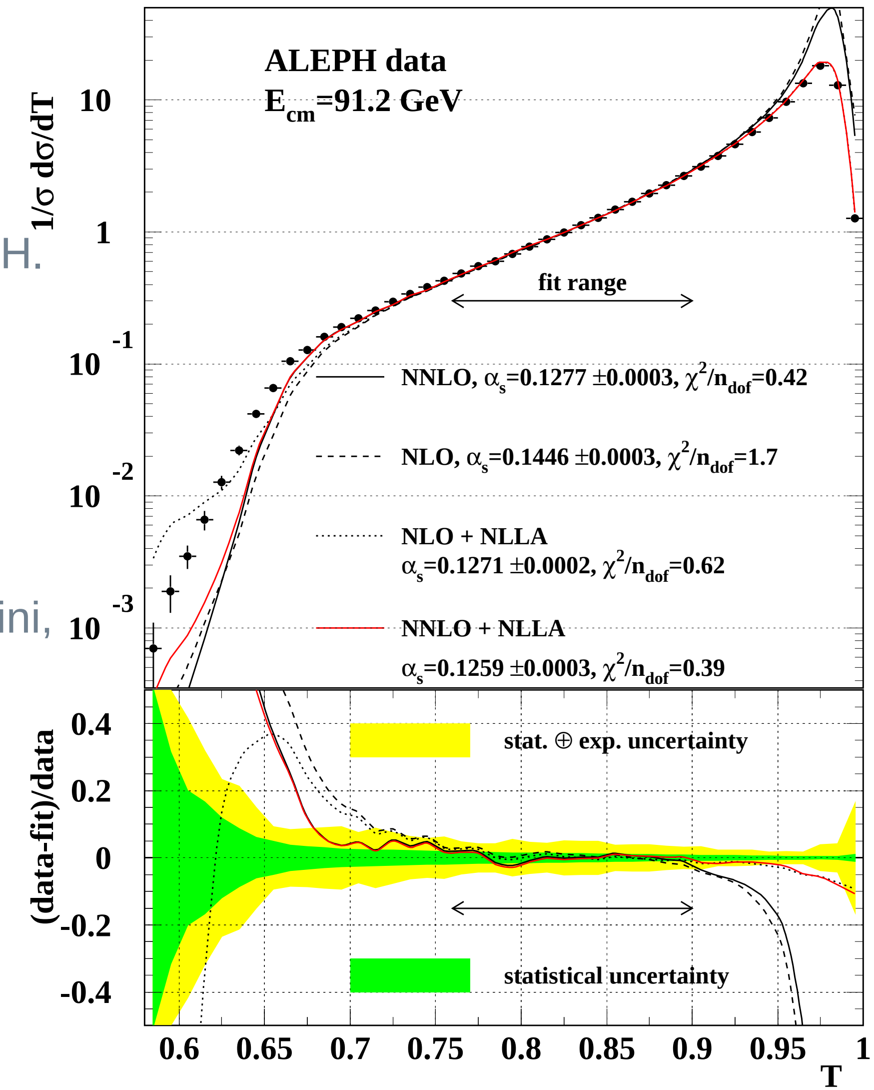
Result for all ALEPH event shapes of LEP1/LEP2

$$\alpha_s(M_Z) = 0.1240 \pm 0.0008(stat) \pm 0.0010(exp) \pm 0.0011(had) \pm 0.0029(theo)$$

Outlook

Next steps:

- α_s from NLLA+NNLO
  G. Dissertori, A. Gehrmann-De Ridder, E.W.N. Glover, G. Heinrich, G. Luisoni, H. Stenzel, TG
- study jet rates in different algorithms
- study moments of event shapes
- revisit analytic power corrections
  Y.L. Dokshitzer, A. Lucenti, G. Marchesini, G.P. Salam
- include electroweak corrections
  C. Carloni-Calame, S. Moretti, F. Piccinini, D. Ross
- resummation and matching at NNLLA



Summary and Outlook

- completed calculation of NNLO corrections to event shapes and $e^+e^- \to 3j$
- improved theory uncertainty
  - by 30% (T, C) to 60% R_{3j}
- new extraction of α_s:
  - improved consistency between different shape variables
  - lower theory uncertainty
- more phenomenology to come
- Precision calculations for jet observables at LHC in progress