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AUTOMATED COMPUTATION OF ONE-LOOP AMPLITUDES

Work in collaboration with C. Berger, Z. Bern, L. Dixon, F. Febres Cordero, H. Ita, D. Kosower, D. Maître.

arxiv: 0803.4180 [hep-ph]

OVERVIEW – NLO COMPUTATIONS

Maximising discovery potential of the LHC $\Rightarrow$ require NLO amplitudes,

- Computation of QCD Backgrounds.
- Many process required.
- Typically of high multiplicity.
- Automation needed.

Automate computation of one-loop amplitudes using *BlackHat*

- Generalised Unitarity.
- On-shell recursion.

WHAT DO WE NEED?

- One-loop high multiplicity processes,

Newest Les Houches list, (2007)

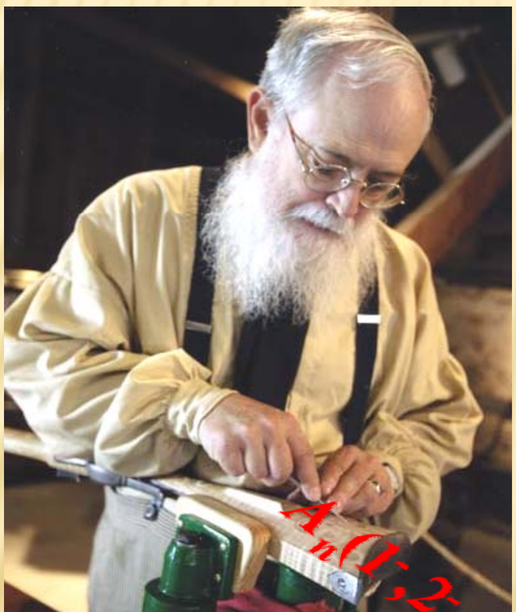
Process ($V \in \{Z, W, \gamma\}$)	Comments
4. $pp \to t\bar{t}\,b\bar{b}$	relevant for $t\bar{t}H$
5. $pp \to t\bar{t}$+2jets	relevant for $t\bar{t}H$
6. $pp \to VV\,b\bar{b}$,	relevant for VBF $\to H \to VV$, $t\bar{t}H$
7. $pp \to VV$+2jets	relevant for VBF $\to H \to VV$ VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld.
8. $pp \to V$+3jets	various new physics signatures
NLO calculations added to list in 2007	
9. $pp \to b\bar{b}b\bar{b}$	Higgs and new physics signatures

WHAT'S BEEN DONE?

- Using analytic and numerical techniques
  - QCD corrections to vector boson pair production (W^+W^-, $W\,Z$ & ZZ) via vector boson fusion (VBF). (Jager, Oleari, Zeppenfeld)+(Bozzi)
  - QCD and EW corrections to Higgs production via VBF. (Ciccolini, Denner, Dittmaier)
  - $pp \to WW+j+X$. (Campbell, Ellis, Zanderighi). (Dittmaier, Kallweit, Uwer)
  - $pp \to$ Higgs$+2$ jets. (Campbell, Ellis, Zanderighi), (Ciccolini, Denner, Dittmaier).
  - $pp \to$ Higgs$+3$ jets (leading contribution) (Figy, Hankele, Zeppenfeld).
  - $pp \to ZZZ$, $pp \to t\bar{t}H$, (Lazopoulos, Petriello, Melnikov) $pp \to t\bar{t}Z$ +(McElmurry)
  - $pp \to ZZZ$, WZZ, WWZ, ZZZ (Binoth, Ossola, Papadopoulos, Pittau),
  - $pp \to W/Zb\bar{b}$ (Febres Cordero, Reina, Wackeroth),
  - $gg \to gggg$ amplitude. (Ellis, Giele, Zanderighi)
  - 6 photons (Nagy, Soper), (Ossola, Papadopoulos, Pittau), (Binoth, Heinrich, Gehrmann, Mastrolia)

AUTOMATION

- Large number of processes to calculate (for the LHC),
  - Automatic procedure highly desirable.
- We want to go from



- Implement new methods numerically.

TOWARDS AUTOMATION

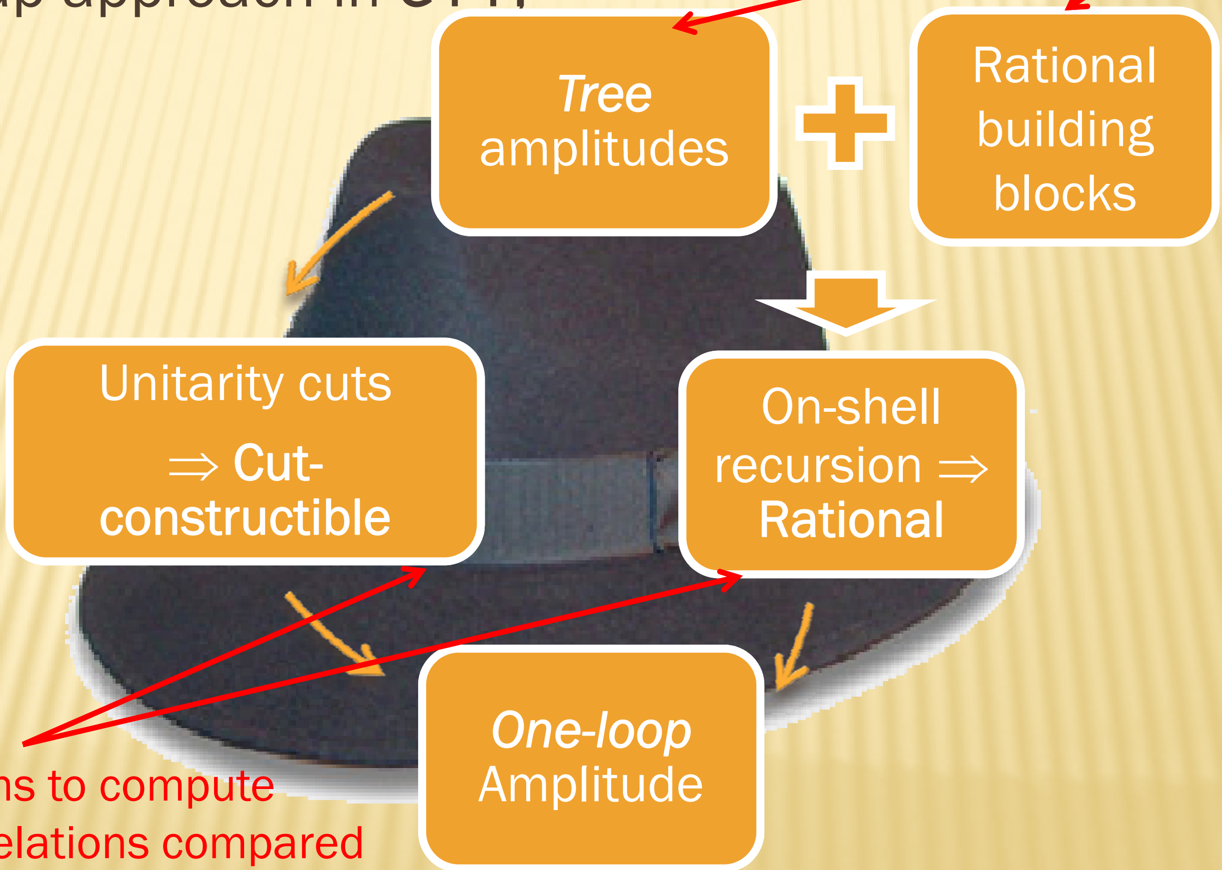
- Many different one-loop computational approaches,
  - OPP approach – solving system of equations numerically, gives integral coefficients (Algorithm implemented in `CutTools`) (Ossola, Papadopoulos, Pittau), (Mastrolia, Ossola, Papadopoulos, Pittau)
  - D-dimensional unitarity + alternative implementation of OPP approach (Ellis, Giele, Kunszt), (Giele, Kunszt, Melnikov)
  - General formula for integral coefficients (Britto, Feng) + (Mastrolia) + (Yang)
  - Computation using Feynman diagrams (Ellis, Giele, Zanderighi) (GOLEM (Binoth, Guffanti, Guillett, Heinrich, Karg, Kauer, Pilon, Reiter))
- `BlackHat` (Berger, Bern, Dixon, Febres Cordero, DF, Ita, Kosower, Maître)

BlackHat

(Berger, Bern, Dixon, Febres Cordero, DF, Ita, Kosower, Maître)

- Numerical implementation of the unitarity bootstrap approach in **c++**.

"Compact" On-shell inputs



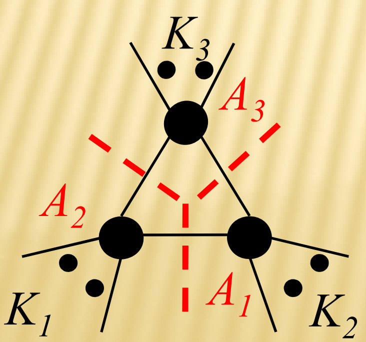
Much fewer terms to compute & no large cancelations compared with Feynman diagrams.

THE UNITARITY BOOTSTRAP

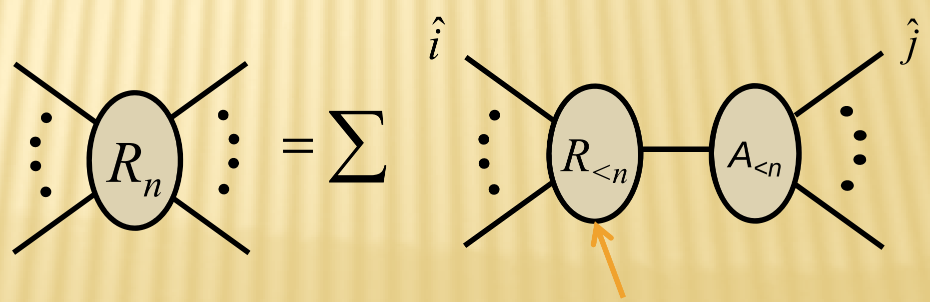
- Use the most efficient approach for each piece,

Log's, Polylog's, etc. + Rational terms = Loop amplitude

Unitarity cuts in 4 dimensions



On-shell recurrence relations

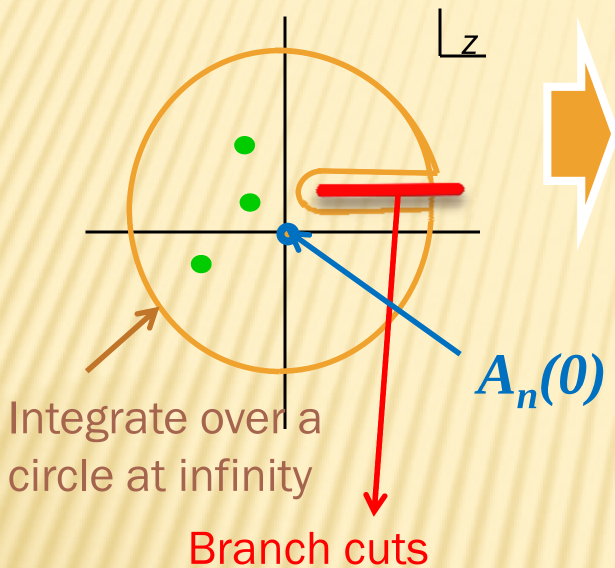


Recycle results of amplitudes with fewer legs

A SIMPLE IDEA

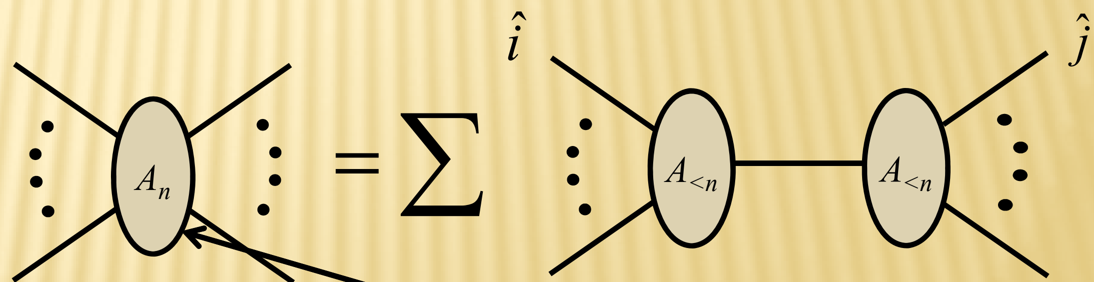
- Function of a complex variable containing only simple poles.

(Britto, Cachazo, Feng, Witten) (Bern, Dixon, Kosower)+(Berger, DF)

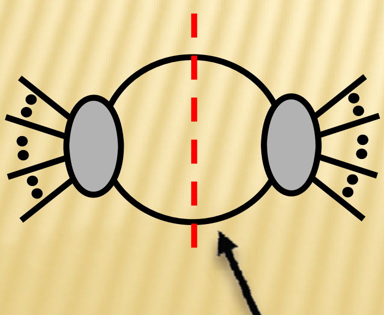


$$\frac{1}{2i\pi}\oint_C dz\,\frac{A_n(z)}{z} = 0 \quad\Rightarrow\quad A_n(0) = -\sum_{\text{poles}} \mathrm{Res}_z\,\frac{A_n(z)}{z}$$

- Factorisation properties of amplitude give on-shell recursion.



- Loop level?

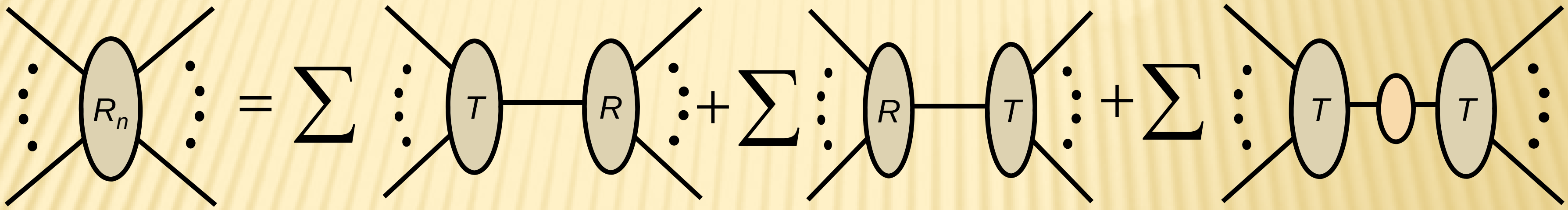


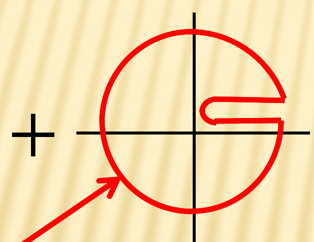
Unitarity techniques, gives loop cut pieces, C

Gives Rational pieces of loop, R

ON-SHELL RECURSION RELATIONS

- At one-loop recursion using on-shell tree amplitudes, T, and rational pieces of one-loop amplitudes, R,





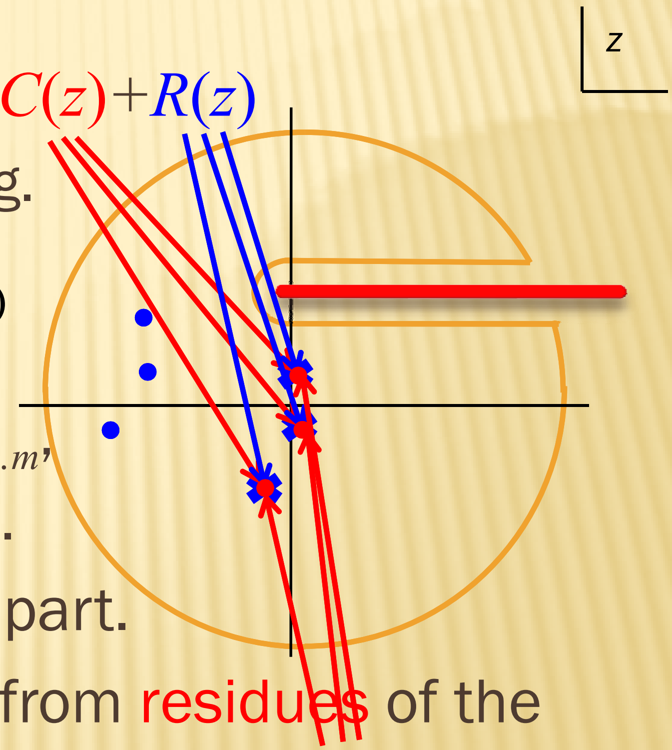
- Sum over all factorisations.
- “Inf” term from auxiliary recursion.
- Not the complete rational result, missing “Spurious” poles.

SPURIOUS POLES

- Shifting the amplitude by $z \Rightarrow A(z) = C(z) + R(z)$
  - Poles in C as well as branch cuts e.g.

$$bI_2 = \frac{\tilde{b}}{K_1^2 - K_2^2} \ln(-K_1^2) \to \frac{\tilde{b}}{K_1^2 - K_2^2 - zY} \ln(-K_1^2 - z\tilde{Y})$$

  - Not related to factorisation poles $s_{l \ldots m}$, i.e. do not appear in the final result.
- Cancel against poles in the rational part.
- Use this to compute spurious poles from residues of the cut terms.



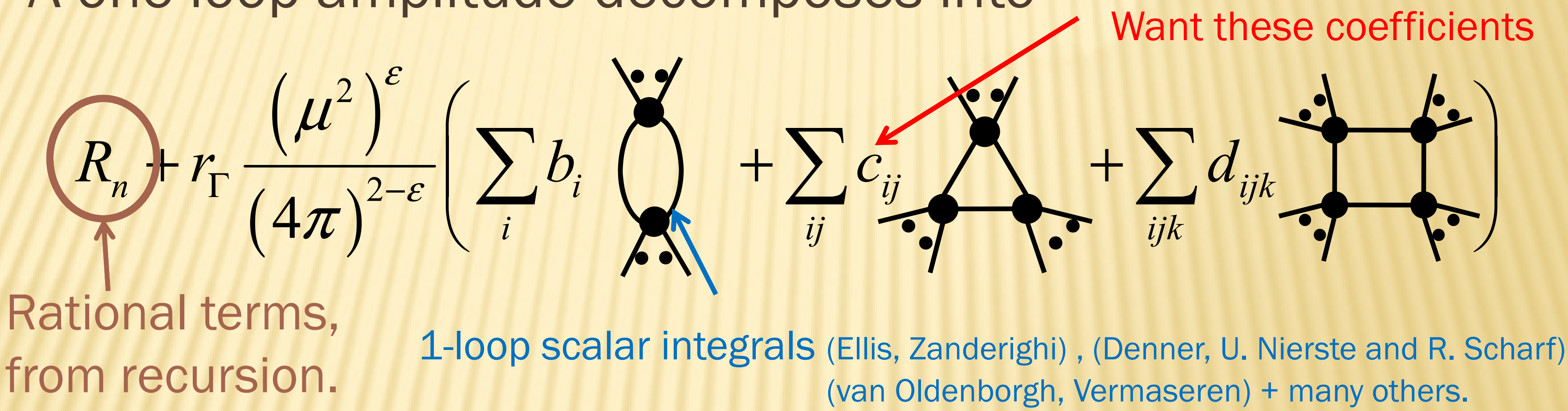
$$-\sum \mathrm{Res}_{z=z_s} \frac{C(z)}{z} = R_s$$

- Location of all spurious poles, z_s, is know
  - Poles located at the vanishing of shifted Gram determinants of boxes and triangles.

ONE-LOOP INTEGRAL BASIS

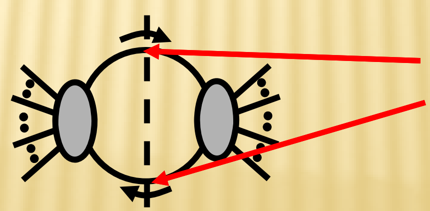
- Numerical computation of the “cut terms”.
- A one-loop amplitude decomposes into

$$R_n + r_\Gamma \frac{(\mu^2)^\varepsilon}{(4\pi)^{2-\varepsilon}} \left(\sum_i b_i \, [\text{bubble}] + \sum_{ij} c_{ij} \, [\text{triangle}] + \sum_{ijk} d_{ijk} \, [\text{box}] \right)$$



- Compute the coefficients from unitarity by taking cuts

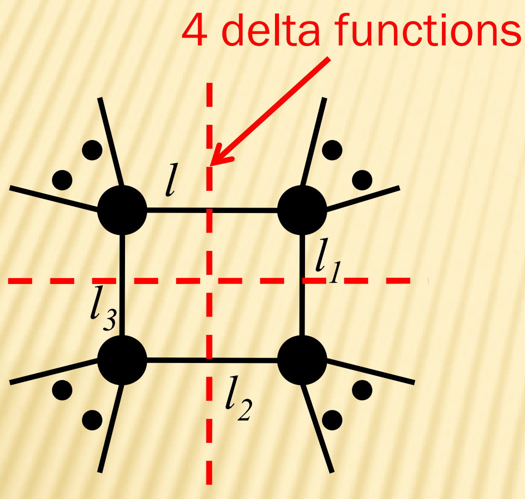
$$\frac{1}{(l-K_i)^2 + i\varepsilon} \rightarrow (2\pi)\,\delta((l-K_i)^2)$$



- Apply multiple cuts, generalised unitarity. (Bern, Dixon, Kosower) (Britto, Cachazo, Feng)

BOX COEFFICIENTS

- Quadruple cuts freeze the integral $\Rightarrow$ coefficient (Britto, Cachazo, Feng)



In 4 dimensions 4 integrals

$$\Rightarrow\ l_1^2 = 0, l_2^2 = 0, l_3^2 = 0, l_4^2 = 0$$

$$l_1^{\pm\mu} = \frac{\langle 1^\pm | \not{K}_2 \not{K}_3 \not{K}_4 \gamma^\mu | 1^\pm \rangle}{2\langle 1^\mp | \not{K}_2 \not{K}_4 | 1^\pm \rangle}, l_2^{\pm\mu} = \frac{\langle 1^\pm | \gamma^\mu \not{K}_2 \not{K}_3 \not{K}_4 | 1^\pm \rangle}{2\langle 1^\mp | \not{K}_2 \not{K}_4 | 1^\pm \rangle},$$

$$l_3^{\pm\mu} = \frac{\langle 1^\pm | \not{K}_2 \gamma^\mu \not{K}_3 \not{K}_4 | 1^\pm \rangle}{2\langle 1^\mp | \not{K}_2 \not{K}_4 | 1^\pm \rangle}, l_4^{\pm\mu} = \frac{\langle 1^\pm | \not{K}_2 \not{K}_3 \gamma^\mu \not{K}_4 | 1^\pm \rangle}{2\langle 1^\mp | \not{K}_2 \not{K}_4 | 1^\pm \rangle}.$$

□ $\sqrt{\Delta_4}$

$$d_{ijk} = \frac{1}{2}\sum_{a=1}^{2} A_1(l_{ijk;a}) A_2(l_{ijk;a}) A_3(l_{ijk;a}) A_4(l_{ijk;a})$$

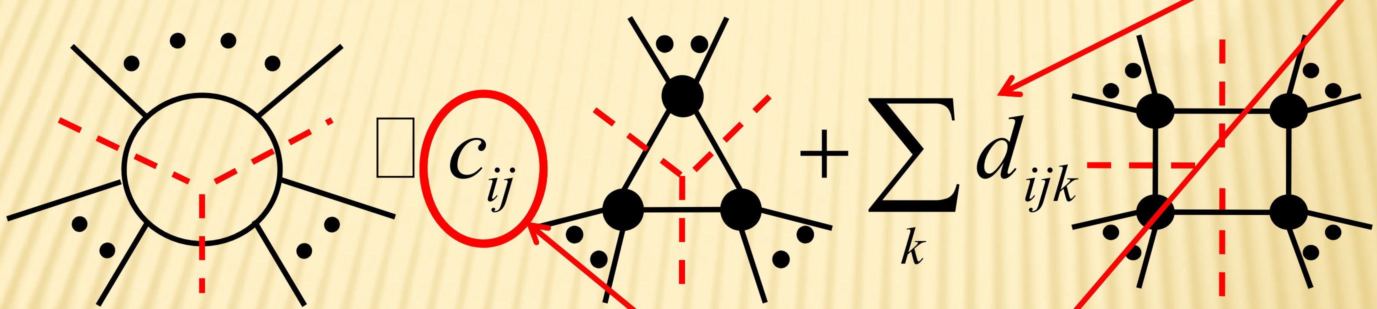
- Box Gram determinant appears in the denominator.

$$\Delta_4 = -2\langle 1^- | \not{K}_2 \not{K}_4 | 1^+ \rangle \langle 1^+ | \not{K}_2 \not{K}_4 | 1^- \rangle$$

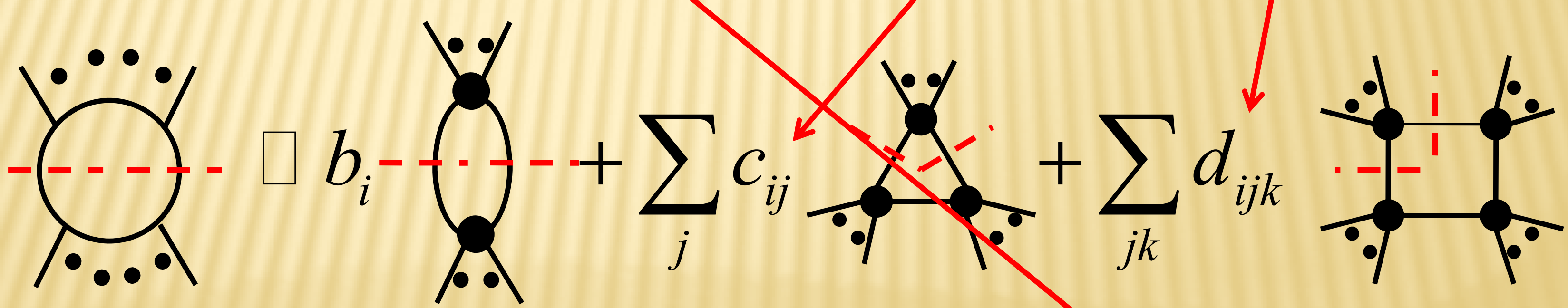
- Spurious poles will go as the power of l^μ in the integrand.

TWO-PARTICLE AND TRIPLE CUTS

- What about bubble and triangle terms?
- Triple cut ⇒ Scalar triangle coefficients?



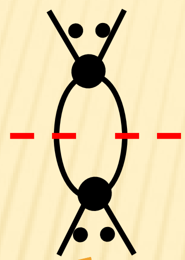
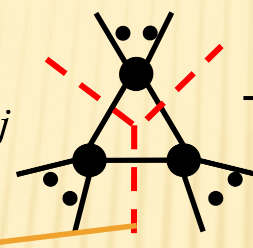
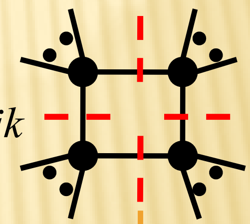
- Two-particle cut ⇒ Scalar bubble coefficients?



- Disentangle these coefficients.

BUBBLES & TRIANGLES

- Compute the coefficients using different numbers of cuts

$\sum_i b_i$  $+ \sum_{ij} c_{ij}$  $+ \sum_{ijk} d_{ijk}$ 

Depends upon unconstrained components of loop momenta.

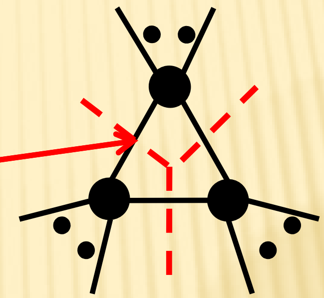
Quadruple cuts, gives box coefficients

- Analytically examining the large value behaviour of the integrand in these components gives the coefficients (DF) (extension to massive loops (Kilgore))
- Modify this approach for a numerical implementation.

TRIANGLE COEFFICIENTS

(del Aguila, Ossola, Papadopoulos, Pittau), (DF)

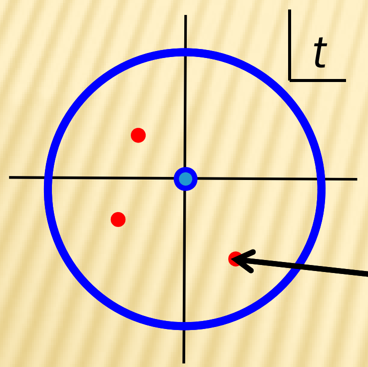
- Apply a triple cut.



Cut momentum parameterisation

$$l^{\mu} = K_1^{\flat\mu} + K_2^{\flat\mu} + \frac{t}{2}\langle K_1^{\flat-}|\gamma^{\mu}|K_2^{\flat-}\rangle + \frac{1}{2t}\langle K_1^{\flat+}|\gamma^{\mu}|K_2^{\flat+}\rangle$$

- Cut integrand, T_3, as a contour integral in terms of the single unconstrained parameter t



Previously computed from quadruple cut

$$T_3(t) = \frac{C_{-3}}{t^3} + \frac{C_{-2}}{t^2} + \frac{C_{-1}}{t} + C_0 + tC_1 + t^2C_2 + t^3C_3 + \sum_{i,\sigma=\pm} \frac{d_i^{\sigma}}{\xi_i^{\sigma}(t - t_i^{\sigma})}$$

Pole $\Rightarrow$ Additional propagator $\Rightarrow$ Box

Triangle Coeff

- Subtract box terms.
- Discrete Fourier projection to compute $C_0 = \frac{1}{2p+1}\sum_{j=-p}^{p} T_3(t_0 e^{2\pi i j/(2p+1)})$

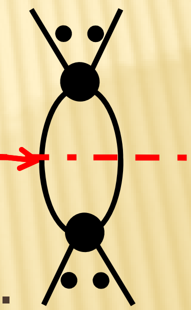
Different uses of DFP (Britto, Feng, Mastrolia), (Mastrolia, Ossola, Papadopoulos, Pittau)

BUBBLE COEFFICIENTS

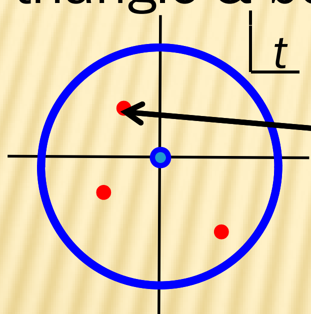
- Apply a two-particle cut.

  Cut momentum parameterisation

  $$l^\mu = y\bar{K}_1^\mu + (1-y)\chi^\mu + \frac{t}{2}\langle \bar{K}_1^- | \gamma^\mu | \chi^- \rangle + \frac{y(1-y)}{2t}\langle \chi^- | \gamma^\mu | \bar{K}_1^- \rangle$$



- Two free parameters y and t in the integrand B_2.
  - Contour integral in terms of t (with $y \in [0,1]$) now contains poles from triangle & box propagators.



Pole $\Rightarrow$ Additional propagator

$\Rightarrow$ Triangle/Box

- Subtract triple-cut terms (previously computed).
- Compute bubble coefficient using

$$B_0 = \frac{1}{20}\sum_{j=0}^{4}\left[B_2(0, t_0 e^{2\pi i j/5}) + 3B_2(2/3, t_0 e^{2\pi i j/5})\right]$$

NUMERICAL SPURIOUS POLE EXTRACTION

- Numerically extract spurious poles, use known pole locations.
- Expand Integral functions at the location of the poles, $\Delta_i(z)=0$, e.g.

$$I_{3m}^{3}(s_1,s_2,s_3) \to \frac{1}{6}\frac{\Delta_3}{s_1 s_2 s_3} - \frac{s_1+s_2+s_3}{120}\left(\frac{\Delta_3}{s_1 s_2 s_3}\right)^2 + \ldots$$

$$-\frac{1}{2}\sum_{i=1}^{3}\ln(-s_1)\frac{s_i - s_{i+1} - s_{i-1}}{s_{i+1}s_{i-1}}\left[1-\frac{1}{6}\frac{\Delta_3}{s_{i+1}s_{i-1}}+\ldots\right]$$

Box or triangle Gram determinant

- The coefficient multiplying this is also a series in $\Delta_i(z)=0$, e.g.

$$C_0(\Delta_3) \to \frac{a_3}{\Delta_3^3} + \frac{a_2}{\Delta_3^2} + \frac{a_1}{\Delta_3} + \ldots$$

- Numerically extract the $1/\Delta_i$ pole from the combination of the two, this is the spurious pole.

NUMERICAL STABILITY

- Use double precision for majority of points $\Rightarrow$ good precision.
- For a small number of exceptional points use higher precision (up to ~32 or ~64 digits.)
- Detect exceptional points using three tests,
  - Bubble coefficients in the cut must satisfy,

$$A_n^{\text{1-loop}}\Big|_{1/\varepsilon,\ \text{non-log}} = \sum_k b_k = -\left[\left(\frac{11}{3} - \frac{2}{3}\frac{n_f}{N_c}\right)\right] A_n^{\text{tree}}$$

  - For each spurious pole, z_s, the sum of all bubbles must be zero,

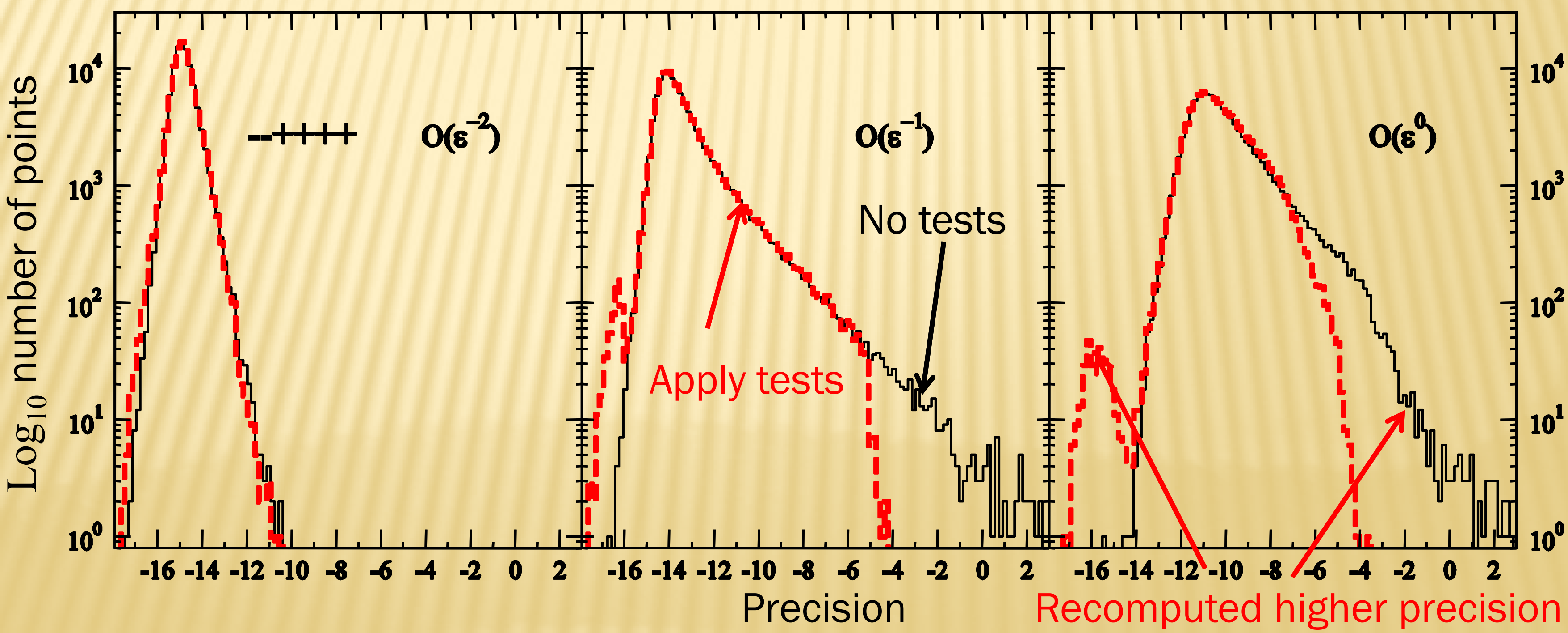
$$A_n^{\text{1-loop}}(z_s)\Big|_{1/\varepsilon,\ \text{non-log}} = \sum_k b_k(z_s) = 0$$

  - Large cancellation between cut and rational terms.
- Box and Triangle terms feed into bubble, so we test all pieces.

MHV RESULTS

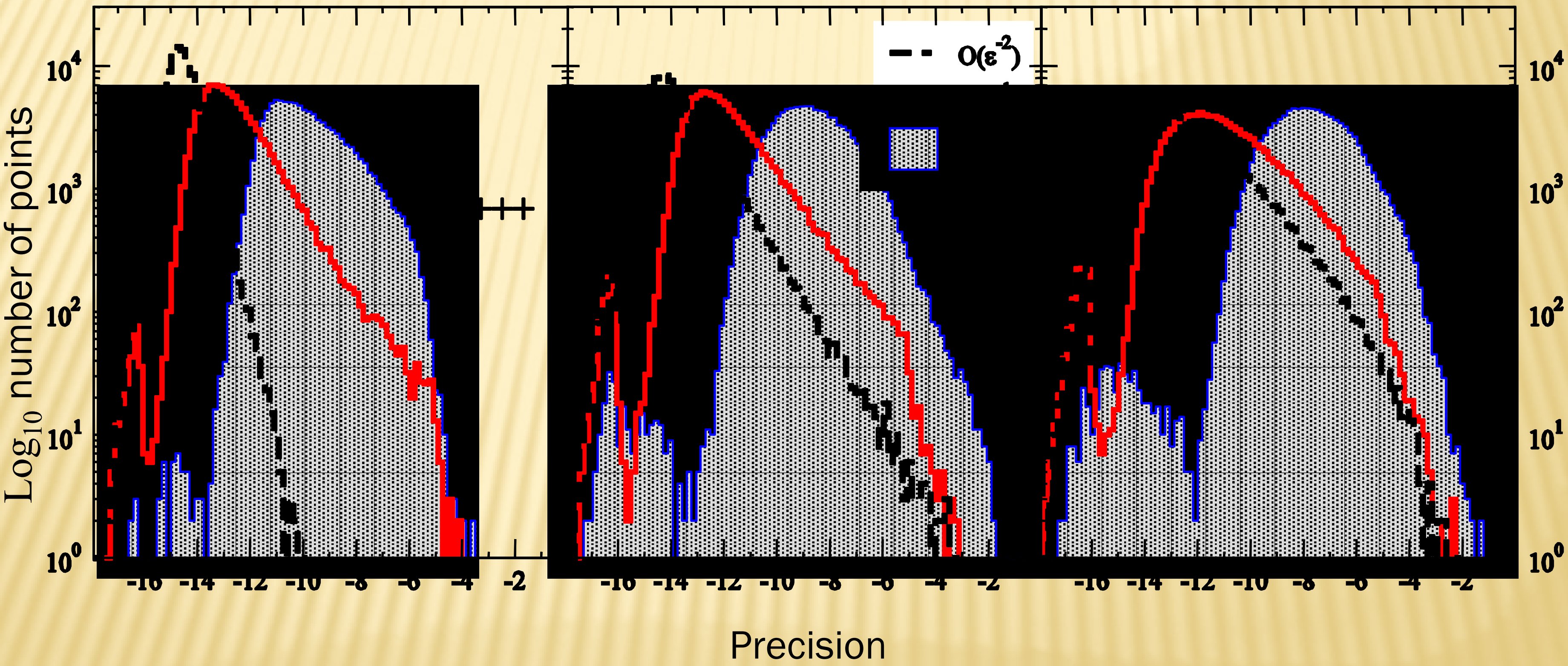
- Precision tests using 100,000 phase space points with cuts.
  - $E_T > 0.01\sqrt{s}$.
  - Pseudo-rapidity $\eta > 3$.
  - $\Delta_R > 4,\ \Delta_R = \sqrt{\Delta_\eta^2 + \Delta_\phi^2}$

$$\text{Precision} = \log_{10}\left(\frac{|A^{\text{num}} - A^{\text{ref}}|}{|A^{\text{ref}}|}\right)$$



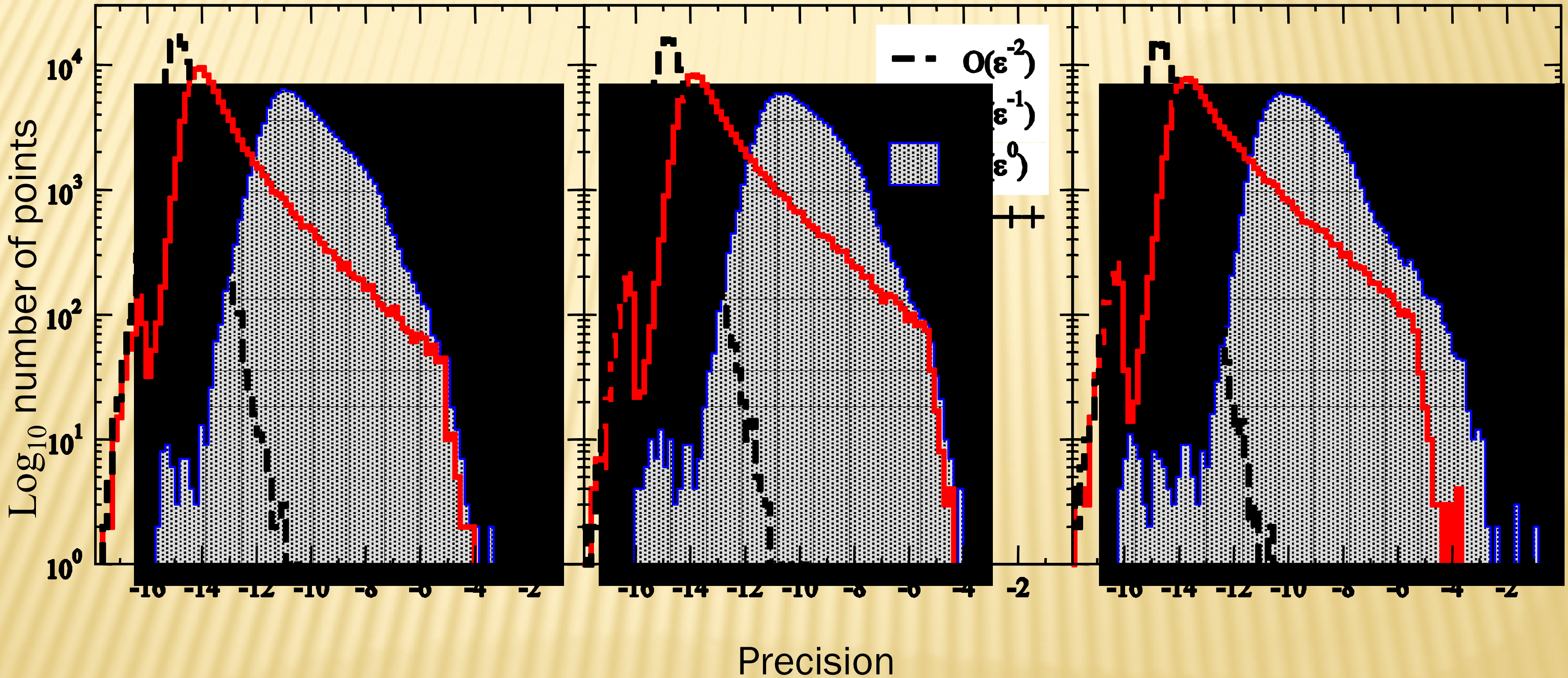
NMHV RESULTS

- Other 6-pt amplitudes are similar



MORE MHV RESULTS

- Again similar results when increasing the number of legs



TIMING

- On a 2.33GHz Xenon processor we have

Helicity	Cut part Only	Full double prec.	Full Multi prec.
--++++	2.4ms	6.8ms	8.3ms
--+++++	4.2ms	10.5ms	14ms
--++++++	6.1ms	28ms	43ms
-+-+++	3.1ms	17.3ms	24ms
-++-++	3.3ms	60ms	76ms
---+++	4.4ms	12ms	16ms
--+-++	5.9ms	42ms	48ms
-+-+-+	6.9ms	62ms	80ms

- The effect of the computation of higher precision at exceptional points can be seen.
- Computation of spurious poles is the dominant part.

CONCLUSION

BlackHat

- *Uses the unitarity bootstrap to numerically compute one loop amplitudes.*

First results

- *Multiple gluon amplitudes.*
- *Good control of numerical instabilities.*
- *Control of exceptional points.*
- *Reasonable speed.*

Future work

- *Include fermions and vector bosons.*
- *Massive particles.*
- *Combine into full NLO corrections.*