NLO QCD corrections to VVV production

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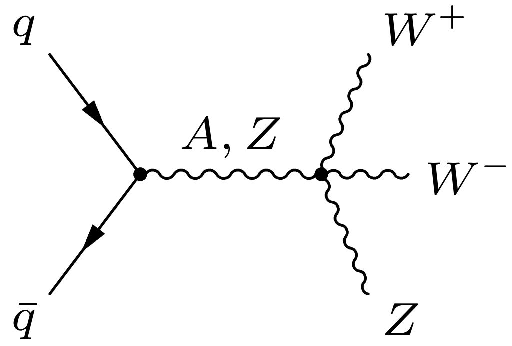
- **Motivation**
- **Calculation and Checks**
- **Cross section and distributions for W^+W^-Z**
- **Cross section and distributions for ZZW^+**
- **Summary**

* in collaboration with D. Zeppenfeld, S. Prestel, C. Oleari and F. Campanario

Motivation

physics motivation:

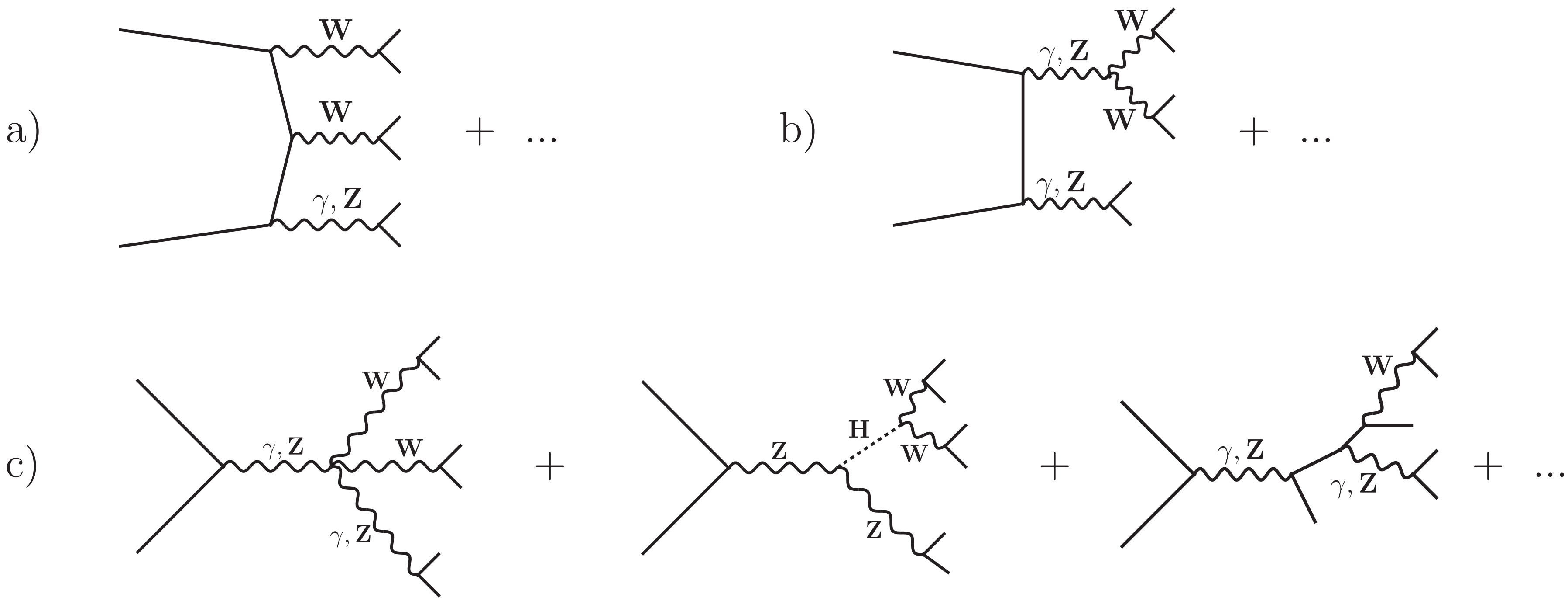
- SM background for SUSY processes with leptons + $\not{p}_T$ in the final state.
- Possibility to obtain information about quartic electroweak couplings.



status of the calculations:

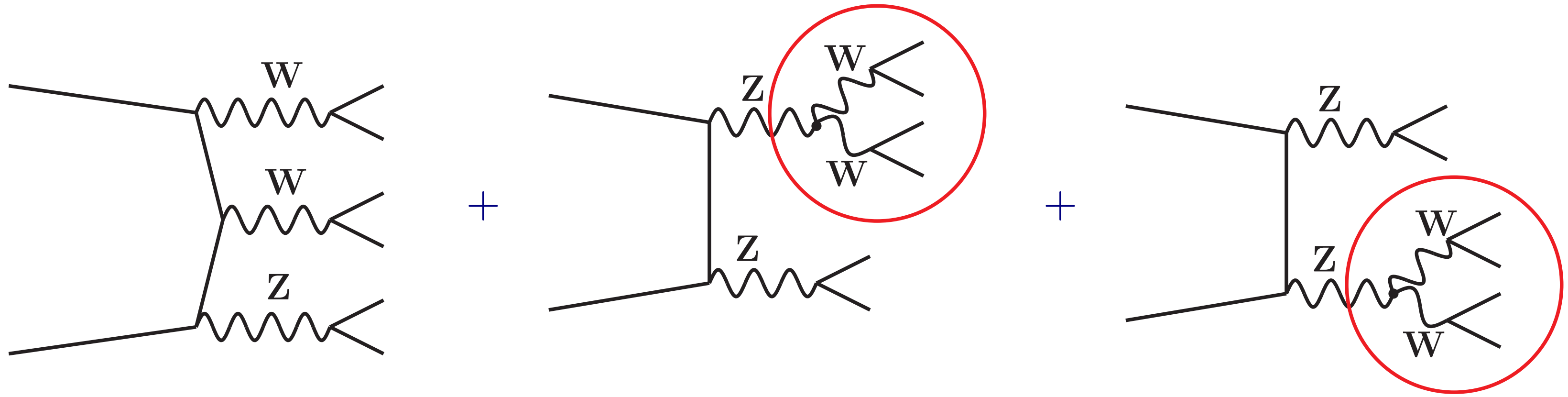
- Calculation of QCD corrections to ZZZ production without Higgs-contribution and leptonic decays.
  [Lazopoulos, Melnikov, Petriello; hep-ph/0703273]
- Calculation of QCD corrections to W^+W^-Z production with leptonic decays.
  [V.H., Zeppenfeld; arXiv:0712.3544]
- In Progress: $ZZW^\pm$ and $W^\pm W^\mp W^\pm$ production with leptonic decays.
  [In preparation: V.H., Zeppenfeld, Campanario, Prestel, Oleari]
- Calculation of QCD corrections to ZZZ, W^+W^-Z, $W^+W^-W^+$ and ZZW^+ production without Higgs-contribution and leptonic decays.
  [Binoth, Ossola, Papadopoulos, Pittau; arXiv:0804:0350]

Introduction to W^+W^-Z production



- All resonant and non-resonant matrix elements as well as spin correlations of final state leptons and Higgs contribution included.
- Interference terms due to identical particles in the final state have been neglected.
- All fermion mass effects neglected. ($H\tau\tau$-coupling $= 0$)

Matrix element calculation

$|M|^2 \sim \Big|$  $+ \ldots \Big|^2$

- In total 180 Feynman-Graphs ⇒ helicity amplitude method.
  [Hagiwara and Zeppenfeld; Nucl.Phys.B274:1,1986.]

- Additional increase in speed:
  Same building blocks appear in different Feynman-Graphs.
  They are calculated only once per phase space point.

  ⇒ Hand written code for LO process is more than 4 × faster than `MadGraph` generated code.

Catani Seymour Dipole Subtraction

$$\sigma^{NLO} = \int d\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V + \int_m d\sigma^C$$

- $\int_{m+1} d\sigma^R$ and $\int_m d\sigma^V$ are separately IR divergent in 4 dimensions.
- Introduce local counter-term $d\sigma^A$ with the same singular behavior as $d\sigma^R$.

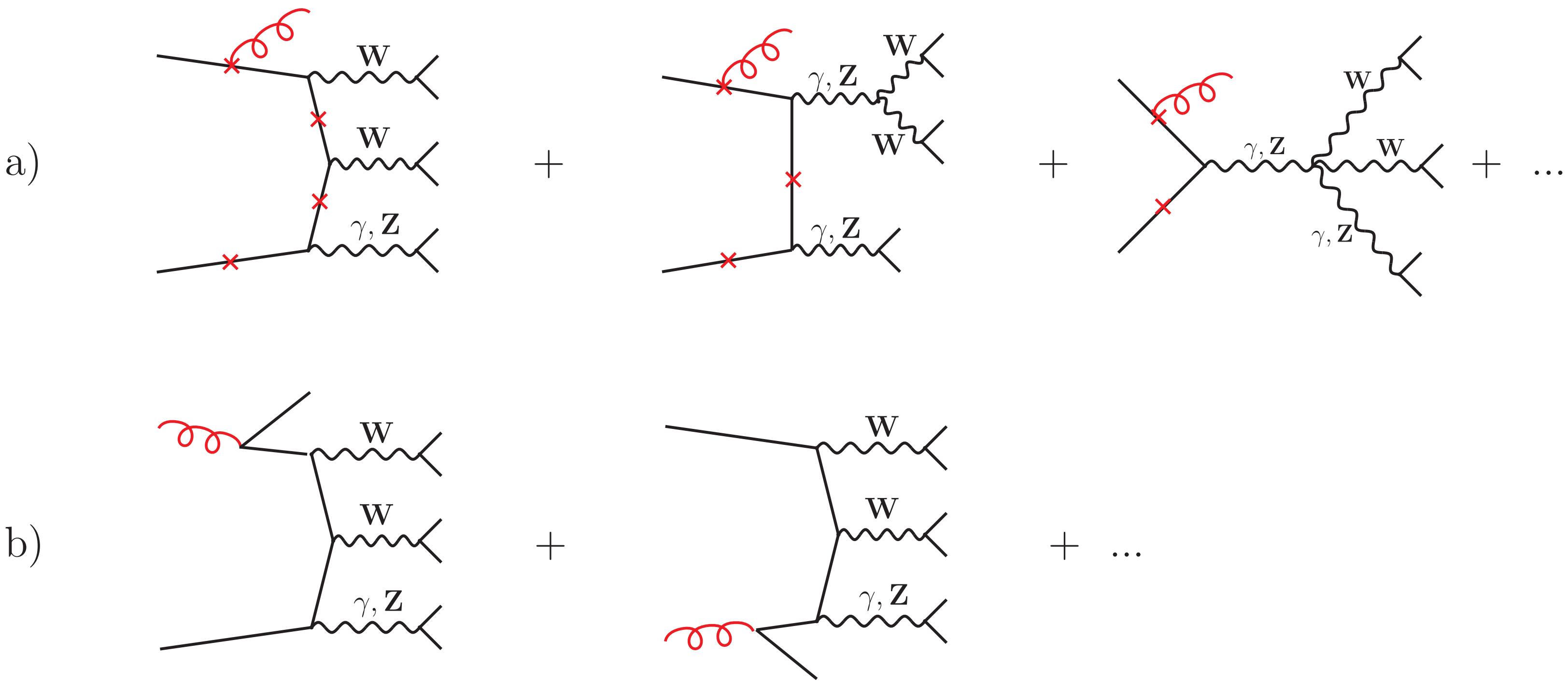
$$\sigma^{NLO} = \underbrace{\int_{m+1} \left[\left(d\sigma^R \right)_{\epsilon=0} - \left(d\sigma^A \right)_{\epsilon=0} \right]} + \underbrace{\int_m \left[d\sigma^V + \int_1 d\sigma^A \right]_{\epsilon=0}} + \underbrace{\int_m d\sigma^C}$$

Can be integrated numerically in 4 dimensions.

Cancel poles analytically.

Additional finite collinear term.

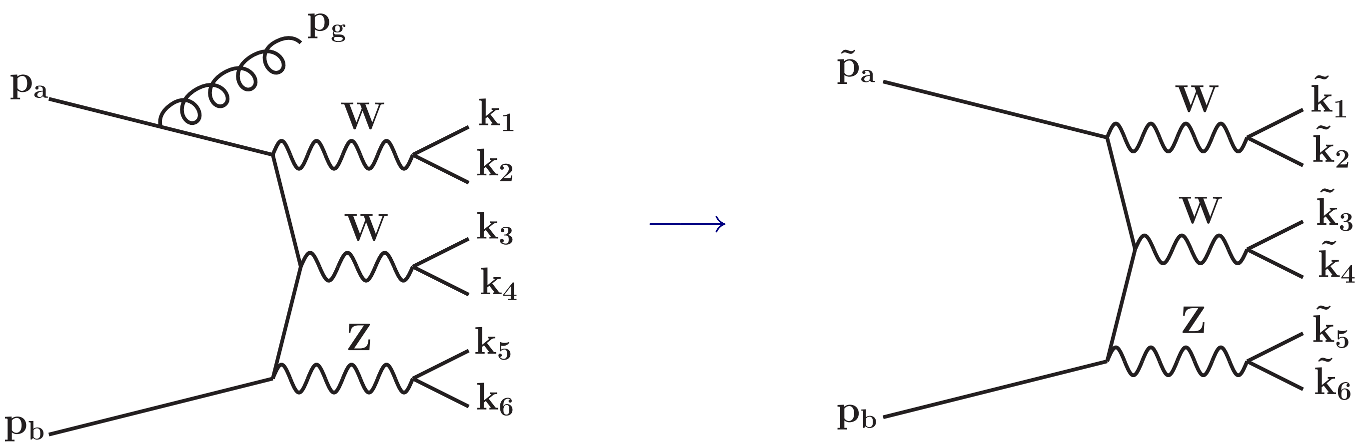
Real emission: matrix element



- In total 416 Feynman diagrams.
- Leptonic tensors can be calculated once per phase space point, stored and reused for different Feynman diagrams.

  ⇒ Hand written code for LO process is more than 12 × faster than MADGRAPH generated code.

Real emission: $\int_{m+1}\left[\left(d\sigma^R\right)_{\epsilon=0} - \left(d\sigma^A\right)_{\epsilon=0}\right]$



- Subtraction term for emission of a gluon from parton a:

$$d\sigma^A = \frac{1}{2\ x\ p_a \cdot p_g}\ 8\pi\alpha_S\ C_F\ \left[\frac{2}{1-x} - (1+x)\right] |M_B(\tilde{k}_1, \ldots, \tilde{k}_6; \tilde{p}_a, p_b)|^2$$

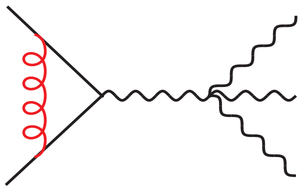
with

$$x = 1 - \frac{p_g \cdot (p_a + p_b)}{p_a \cdot p_b}$$

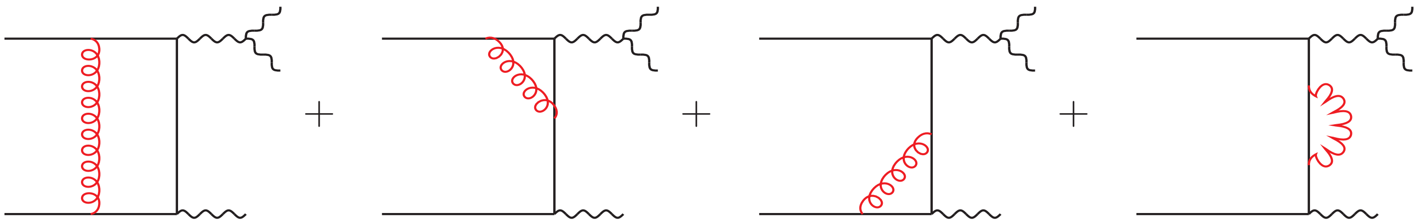
- And similar for gluons in the initial state.

Virtual contributions:

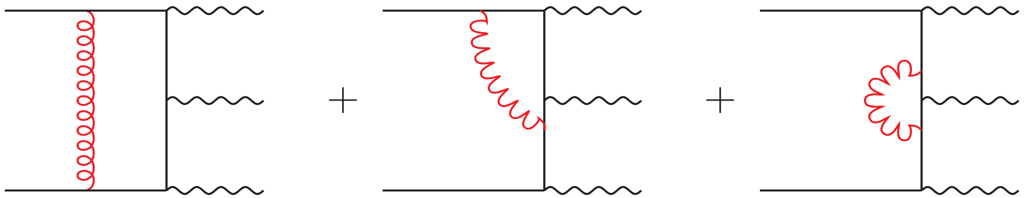
- Three different types of virtual contributions:

a) 

proportional to born matrix element

b) 

Boxline contribution

c)  $+$ 5 others

Pentline contribution

Virtual contributions:

- Boxline- and Pentline-contributions have same structure as in V + 2 Jet- and V V + 2 Jet-production. In dimensional regularization:

$$M_V = \tilde{M}_V + \frac{\alpha_S}{4\pi} C_F \left(\frac{4\pi\mu^2}{s}\right)^{\epsilon} \Gamma(1+\epsilon) \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \frac{4\,\pi^2}{3}\right] M_B.$$

- Calculation of finite contribution $\tilde{M}_V$:
  - Box-routine from [Oleari, Zeppenfeld: hep-ph/0310156].
  - Pentagon-routine from [Jäger, Oleari, Zeppenfeld: hep-ph/0603177], [Jäger, Oleari, Zeppenfeld, Bozzi: hep-ph/0701105].

- Actually needed: LO + virtual contribution:

$$\begin{aligned} |M_B + M_V|^2 &= (M_B + M_V)\cdot(M_B^* + M_V^*) = |M_B|^2 + M_V\, M_B^* + M_B\, M_V^* + \mathcal{O}(\alpha_S^2) \\ &= |M_B|^2 + 2\, Re[M_V\, M_B^*] + \mathcal{O}(\alpha_S^2) \end{aligned}$$

Virtual contribution + integrated dipole

- virtual contribution:

$$2\ Re[M_V\ M_B^*] = 2\ Re[\tilde{M}_V\ M_B^*] + \frac{\alpha_S}{2\pi}\, C_F \left(\frac{4\pi\mu^2}{s}\right)^{\epsilon} \Gamma(1+\epsilon) \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \frac{4\ \pi^2}{3}\right]\ |M_B|^2$$

- integrated dipole:

$$\langle\ p_a, p_b\ |\ \mathbf{I}\ |\ p_a, p_b\ \rangle = \frac{\alpha_S}{2\pi}\, C_F\ \left(\frac{4\pi\mu^2}{s}\right)^{\epsilon}\ \Gamma(1+\epsilon)\ \left[\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + 10 - \frac{4\ \pi^2}{3}\right]\ |M_B|^2$$

$\Rightarrow$ virtual contribution + integrated dipole:

$$\langle\ ..|\ \mathbf{I}|\ ..\ \rangle + 2\ Re\left[M_V \cdot M_B^*\right] = \frac{\alpha_S}{2\pi}\, C_F\ 2\ |M_B|^2 + 2\ Re\left[\tilde{M}_V \cdot M_B^*\right]$$

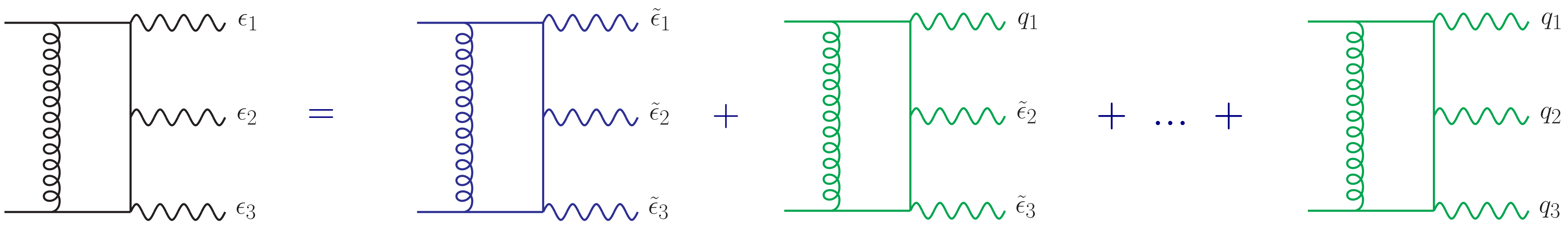
Additional trick to reduce pentagon contribution:

- Pentagon routine is quite time consuming.

⇒ shift polarization vectors in order to reduce magnitude of pentagon contribution:

$$\epsilon_V^\mu = x_V q_V^\mu + \tilde{\epsilon}_V^\mu .$$

- pentagons contracted with momenta instead of pol. vectors can be expressed in terms of boxes.



Full Pentagon contribution

"True Pentagon" contribution

Box-type contribution

- Reduction of Pentagon contribution to the cross section by more than a factor of 8.

Additional finite collinear term:

- For hadrons (partons) in the initial state: additional finite terms:

$$\sigma^{coll}(q\bar{q} \to \nu_e e^+ \mu^- \bar{\nu}_\mu) = \int_0^1 dx_a \int_0^1 dx_b \; d\Phi_6(k_1, \ldots k_6; p_a + p_b) \, \frac{1}{2\hat{s}} \mid M_{Born} \mid^2$$

$$* \left[f^c_{q/p}(x_a, \mu_F) \; f_{\bar{q}/p}(x_b, \mu_F) + f_{q/p}(x_a, \mu_F) \; f^c_{\bar{q}/p}(x_b, \mu_F) \right]$$

with the modified Parton Distribution Function

$$f^c_{q/p}(x_a, \mu_F) = \frac{\alpha_S}{2\pi} \int_{x_a}^1 \frac{dx}{x} \Big\{ f_{g/p}\left(\frac{x_a}{x}, \mu_F\right) \; A(x)$$

$$+ \left[f_{q/p}\left(\frac{x_a}{x}, \mu_F\right) - x f_{q/p}\left(x_a, \mu_F\right) \right] \; B(x) + f_{q/p}\left(\frac{x_a}{x}, \mu_F\right) \; C(x) \Big\}$$

$$+ \frac{\alpha_s}{2\pi} f_{q/p}(x_a, \mu_F) \; D(x_a)$$

where $A(x) = T_R \left[2 \, x(1-x) + (x^2 + (1-x)^2) \; \ln\left(\frac{(1-x)^2 Q^2}{x \mu_F^2} \right) \right]$

Checks for various VVV codes

- Matrix elements checked against `MadGraph`.
- LO cross sections checked against `MadEvent` and `HELAC`.
- Finite collinear terms are the same for W^+W^- production:
  We have implemented this process and compared against `MCFM`.
- Ward identity tests for boxes and pentagons.
- Comparison of ZZZ in narrow width approximation and without Higgs contribution with [Lazopoulos, Melnikov, Petriello; hep-ph/0703273]:
  Agreement at the level of the accuracy of the Monte Carlo runs.
- Comparison of W^+W^-Z and ZZW^+ in narrow width approximation and without Higgs contribution with [Binoth, Ossola, Papadopoulos, Pittau; arXiv:0804:0350]:
  Agreement for ZZW^+; Slight discrepancy of 2.5 % for W^+W^-Z (in progress)

Cuts, scales, etc.

- PDFs: CTEQ6L1 at LO and CTEQ6M, $\alpha_S(m_Z) = 0.118$ at NLO.
- Cuts and Masses:

$$p_{T_\ell} > 10 \text{ GeV}, \qquad |\eta_\ell| < 2.5, \qquad m_{\ell\ell} > 15 \text{ GeV}, \qquad m_H = 120 \text{ GeV}.$$

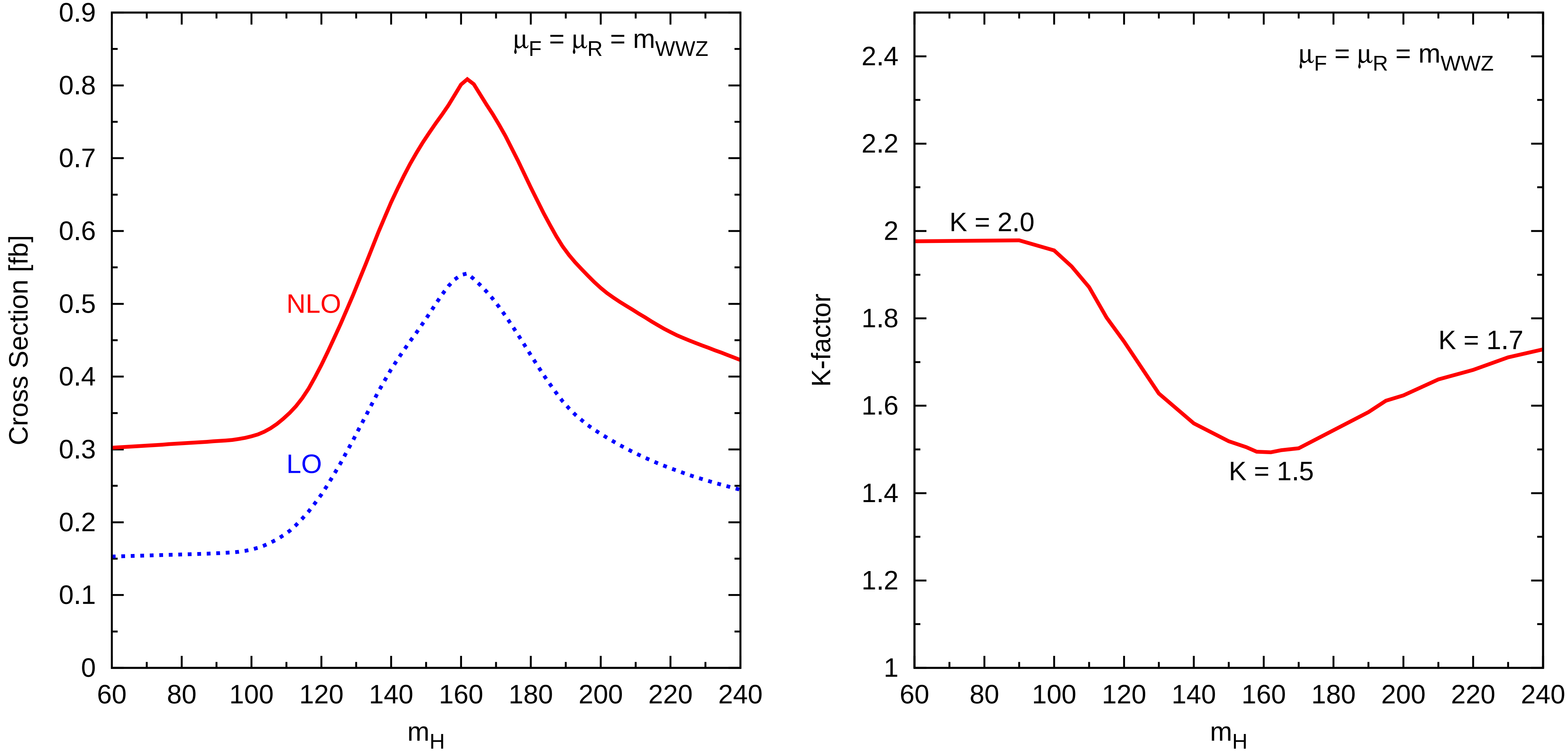
- Reference scale for μ_F and μ_R:

$$\mu = m_{WWZ} = \sqrt{(p_{\ell_1} + p_{\ell_2} + p_{\ell_3} + p_{\ell_4} + p_{\nu_1} + p_{\nu_2})^2}.$$

- Generated process: $pp \rightarrow \nu_e\, e^+\, \mu^-\, \bar{\nu}_\mu\, \tau^-\, \tau^+$.

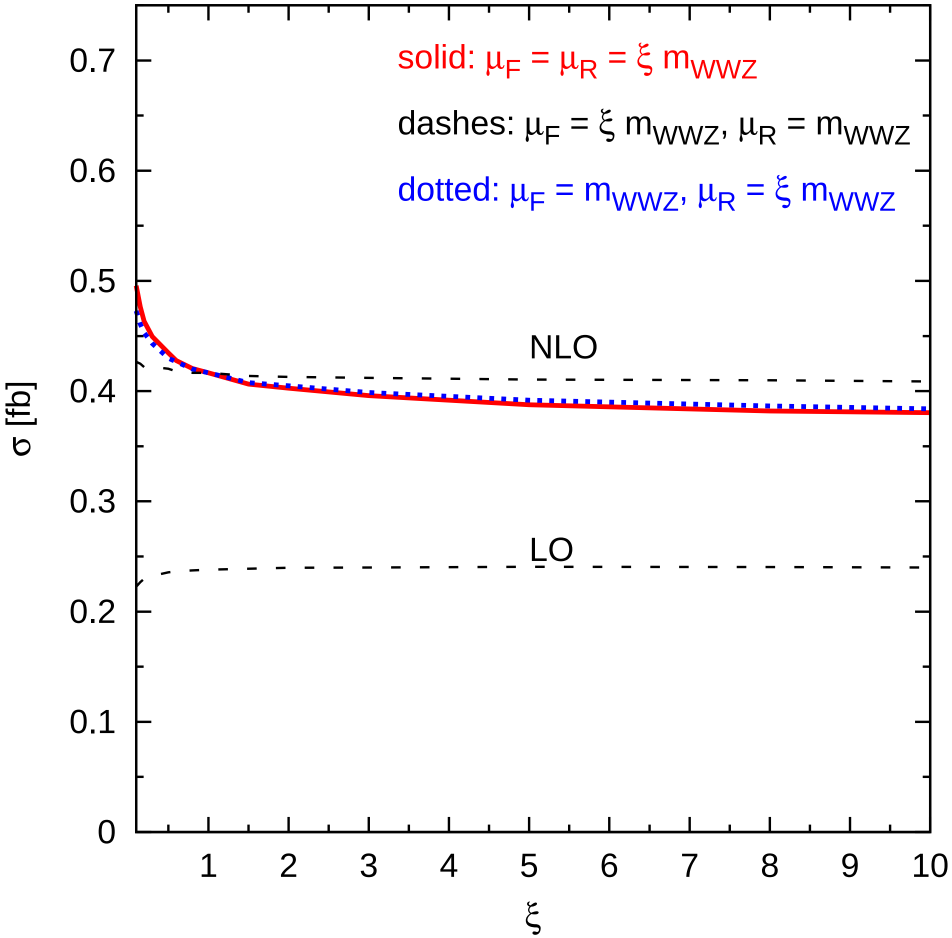
  Phenomenologically more interesting: final states with four electrons and/or muons
  $\Rightarrow$ we have multiplied the results by a combinatorial factor of 8 in all figures.

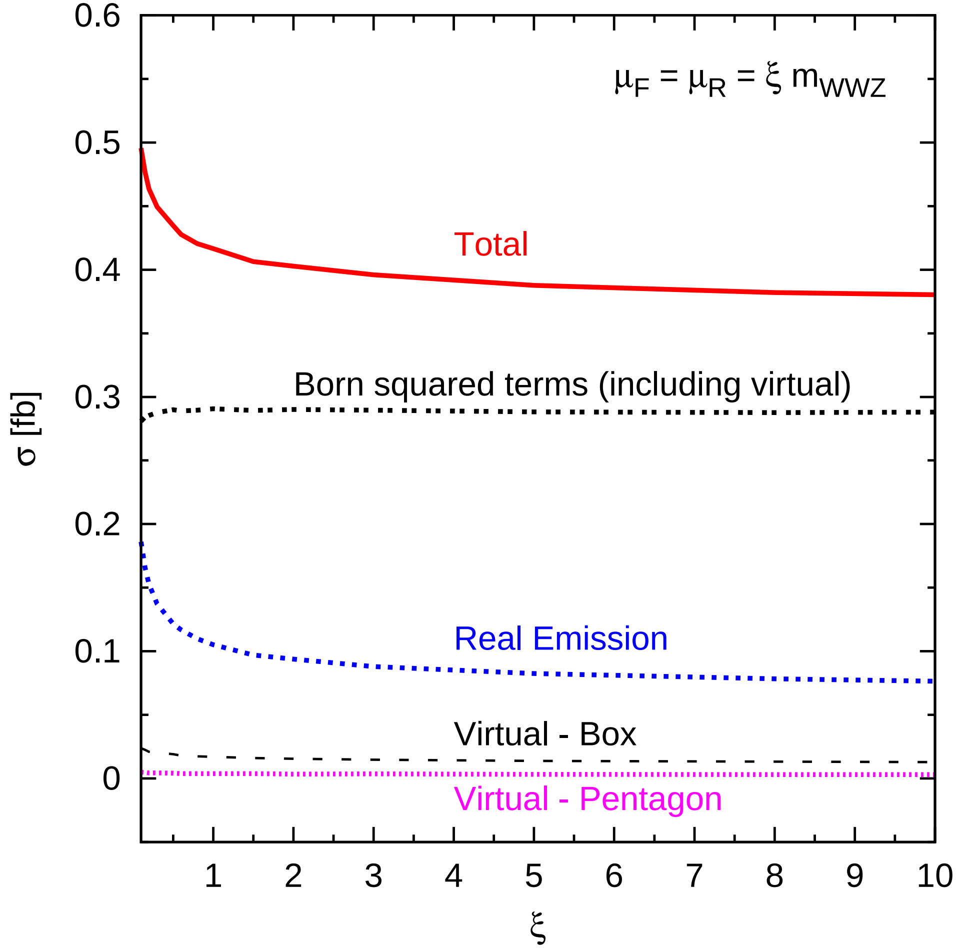
Higgs mass dependence:



- K-factor is reduced by Higgs contribution.
- K-factor for $pp \to ZH$ production is about K $= 1.3$
  [Han and Willenbrock, Phys. Lett. B **273** (1991) 167.]

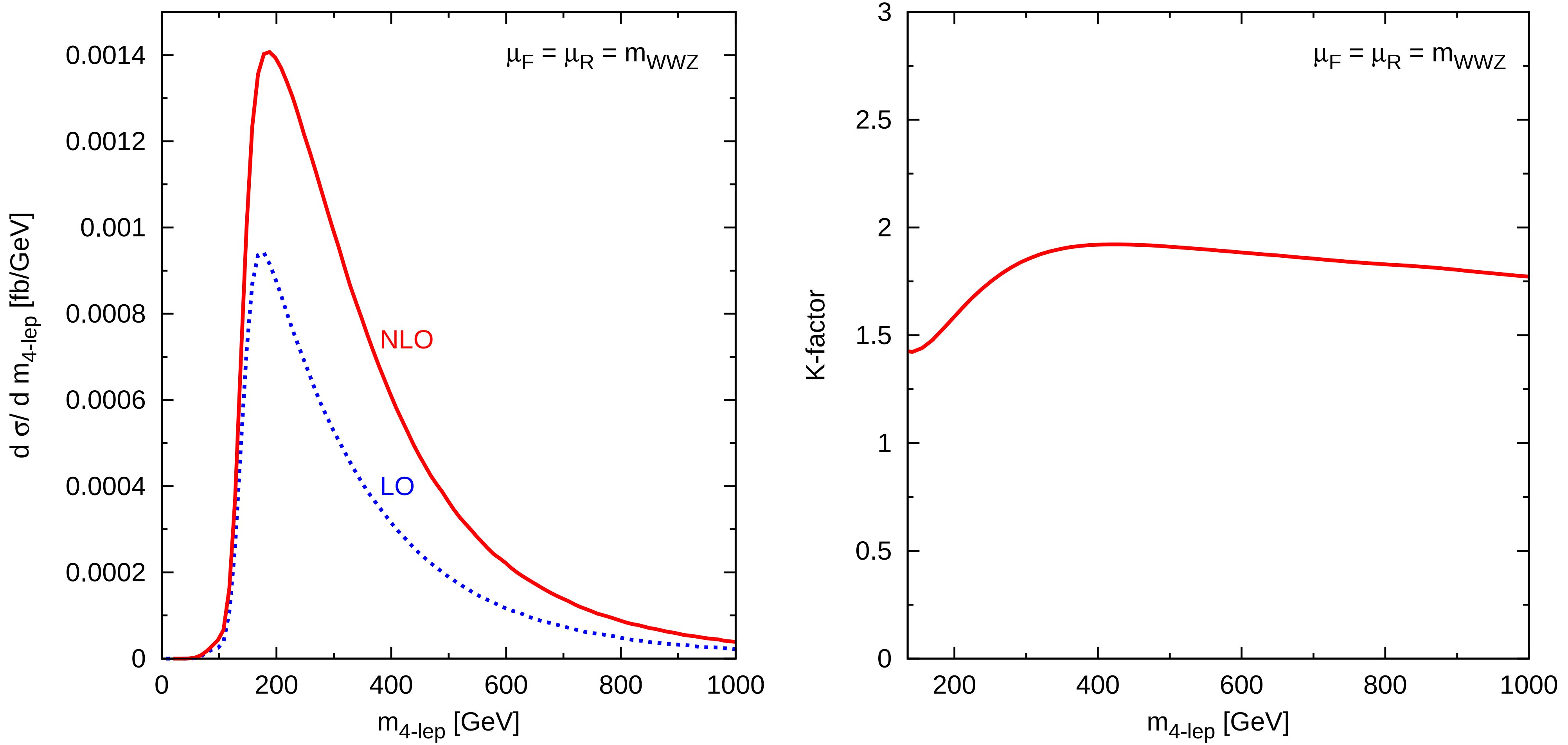
Scale dependence for m_{WWZ} as reference scale:





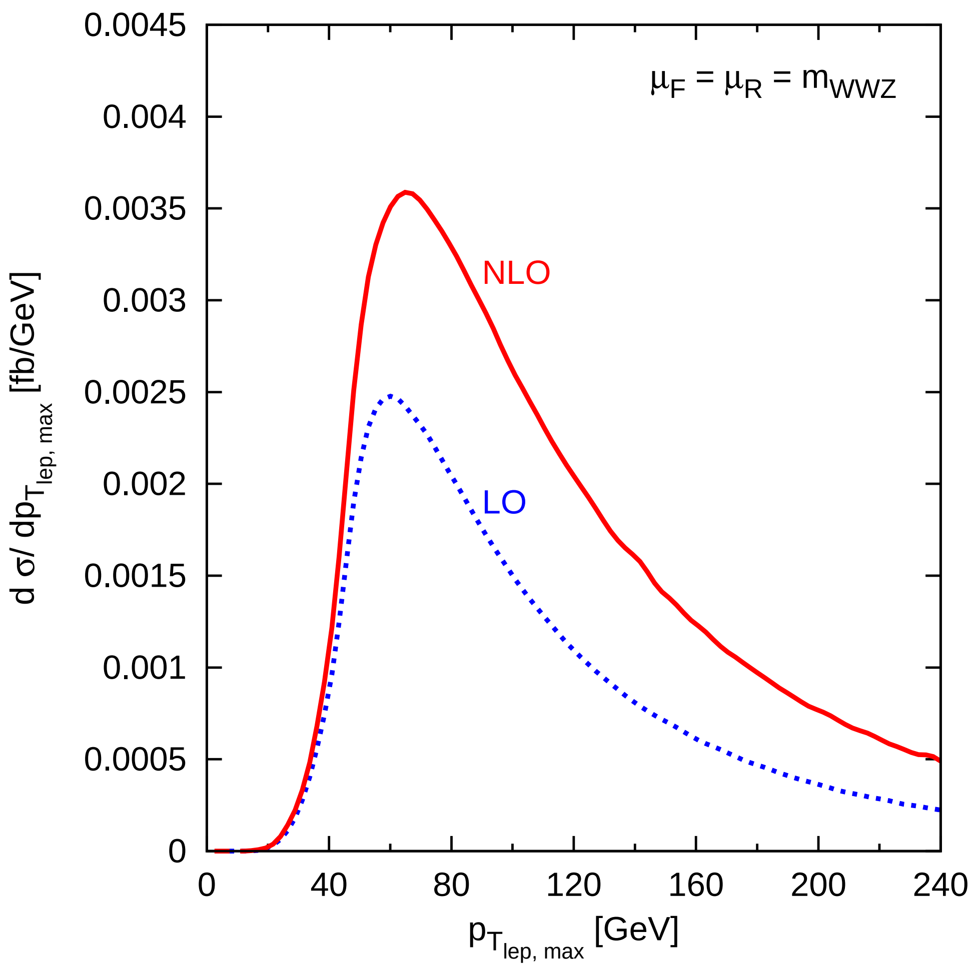
- **Variation of $0.5 < \xi < 2$:** LO: variation of 1.7% (-1% and +0.7%)
  NLO: variation of 7.7% (+4.4% and -3.3%).

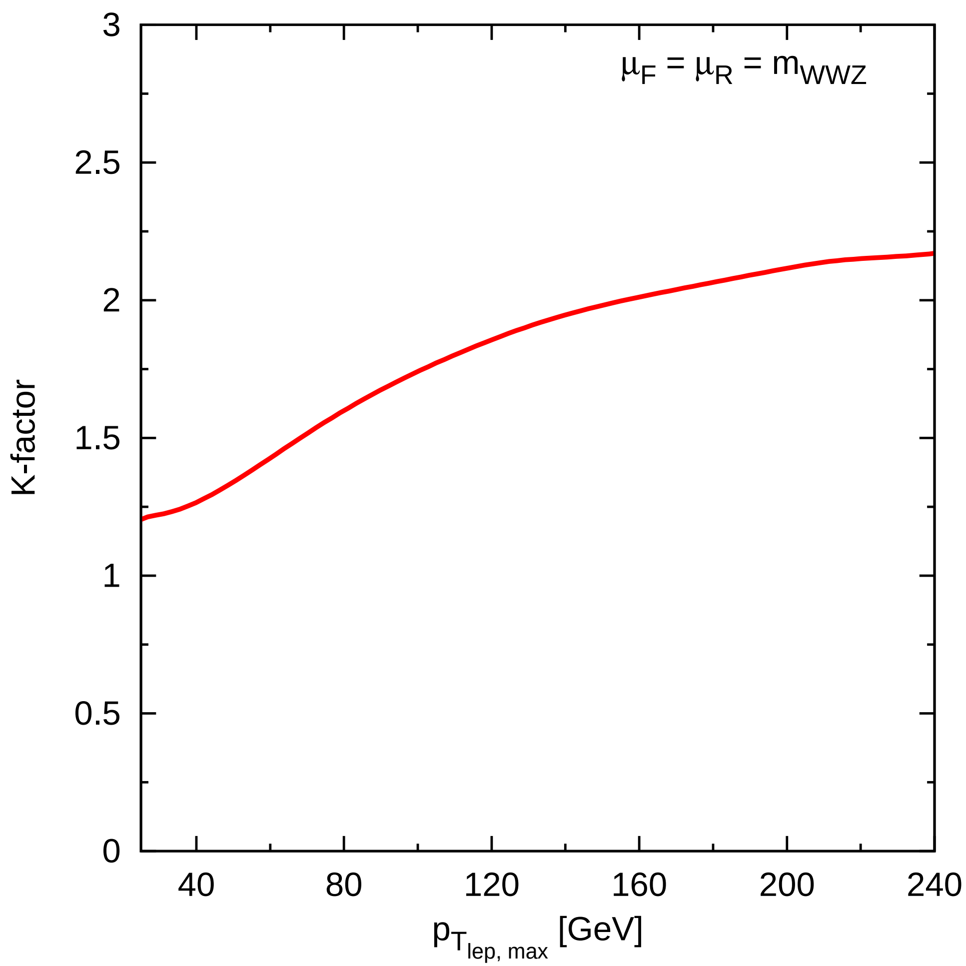
4-lepton invariant mass distribution:



- K-factor is almost constant for $m_{\rm WWZ} > 400$ GeV

p_T-distribution of the lepton with the highest p_T



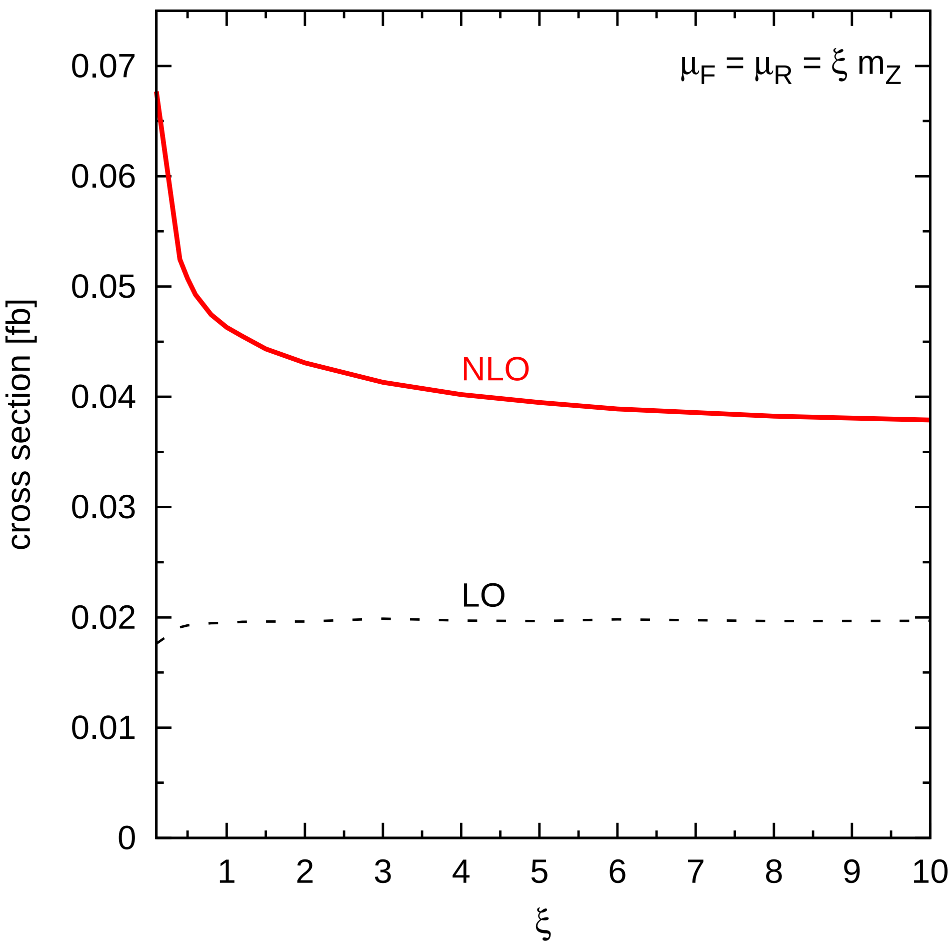


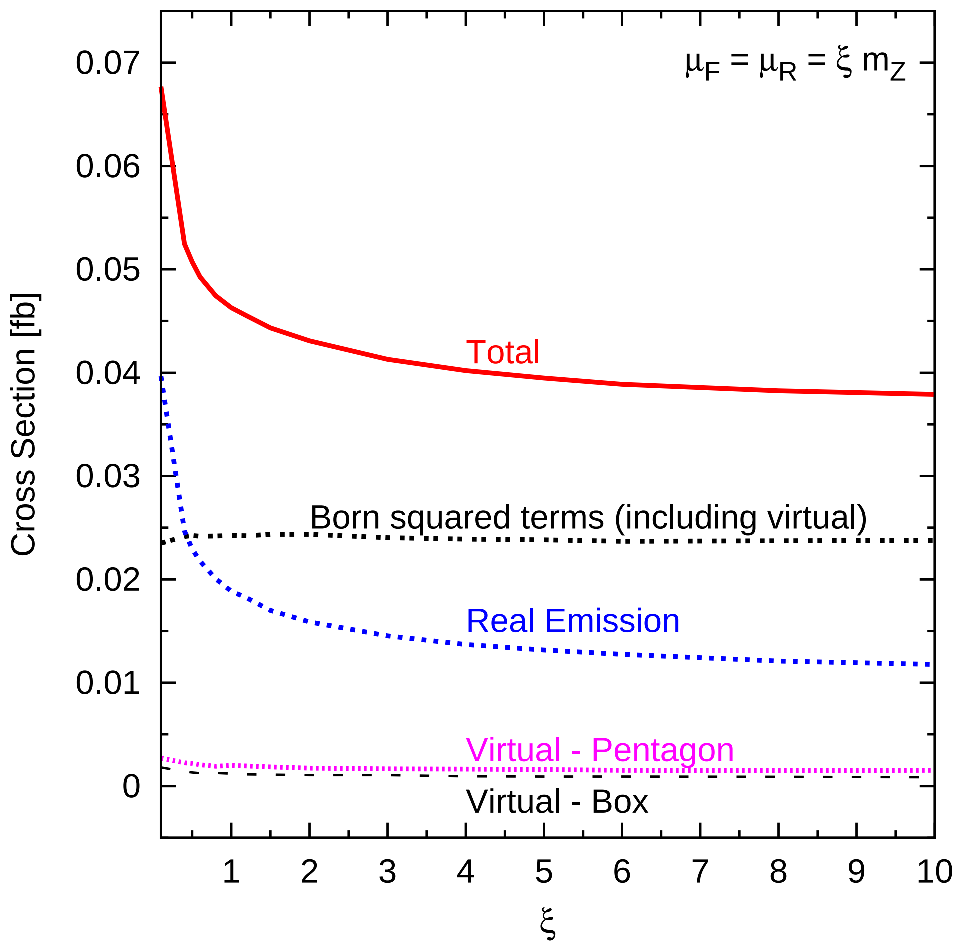
- K-factor increases with p_T
  $\Rightarrow$ simple multiplication of a constant overall K-factor would seriously change the shape.

Results for ZZW$^+$ production:

- All resonant and non-resonant matrix elements as well as spin correlations of final state leptons included.
- Interference terms due to identical particles in the final state and all fermion mass effects neglected.
- Calculation, cuts and PDFs completely analogous to W$^+$W$^-$Z case.
- Renormalization and factorization scale: $\mu_F = \mu_R = m_Z$.
- Checks against `MadGraph`, `MadEvent`, Ward identity tests, ...
- Comparison of ZZW$^+$ in narrow width approximation and without Higgs contribution with [Binoth, Ossola, Papadopoulos, Pittau; arXiv:0804:0350]:
  Agreement at the level of the accuracy of the Monte Carlo runs.
- Generated process: $pp \longrightarrow e^- \, e^+ \, \mu^- \, \mu^+ \, \nu_\tau \, \tau^+$.
  Phenomenologically more interesting: final states with four electrons and/or muons
  $\Rightarrow$ we have multiplied the results by a combinatorial factor of 8 in all figures.

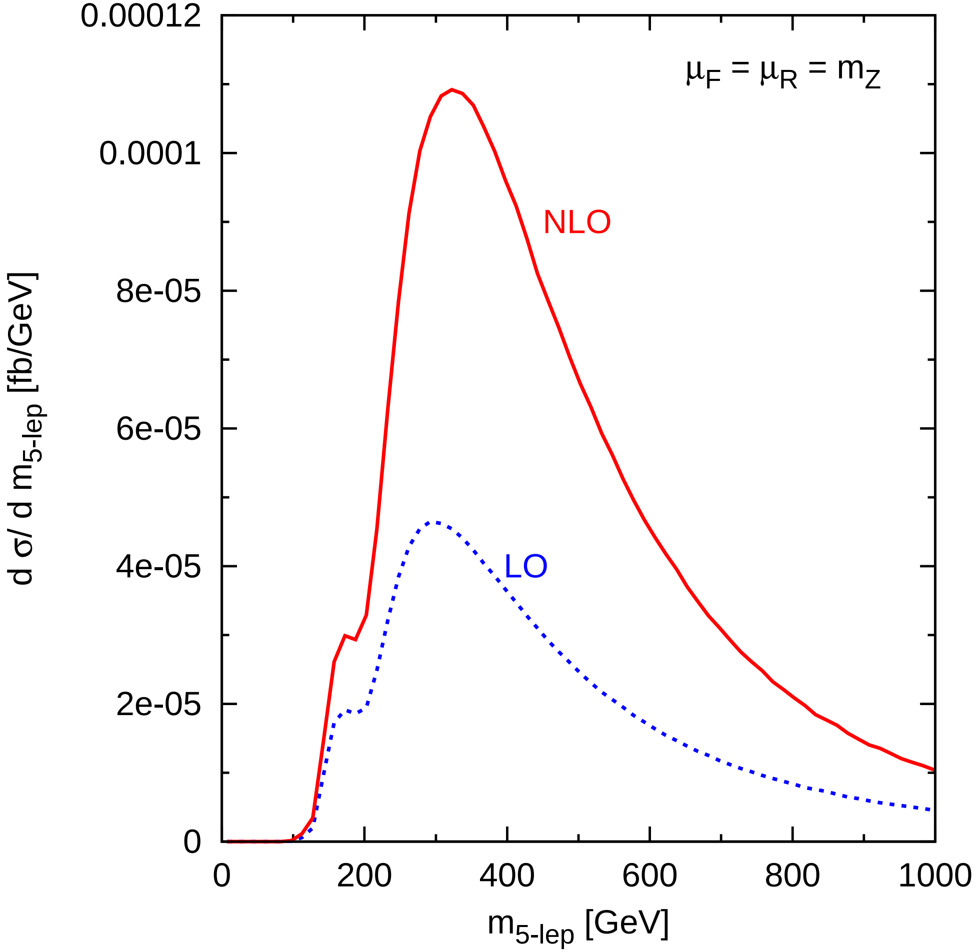
Scale variation for ZZW^+ production:

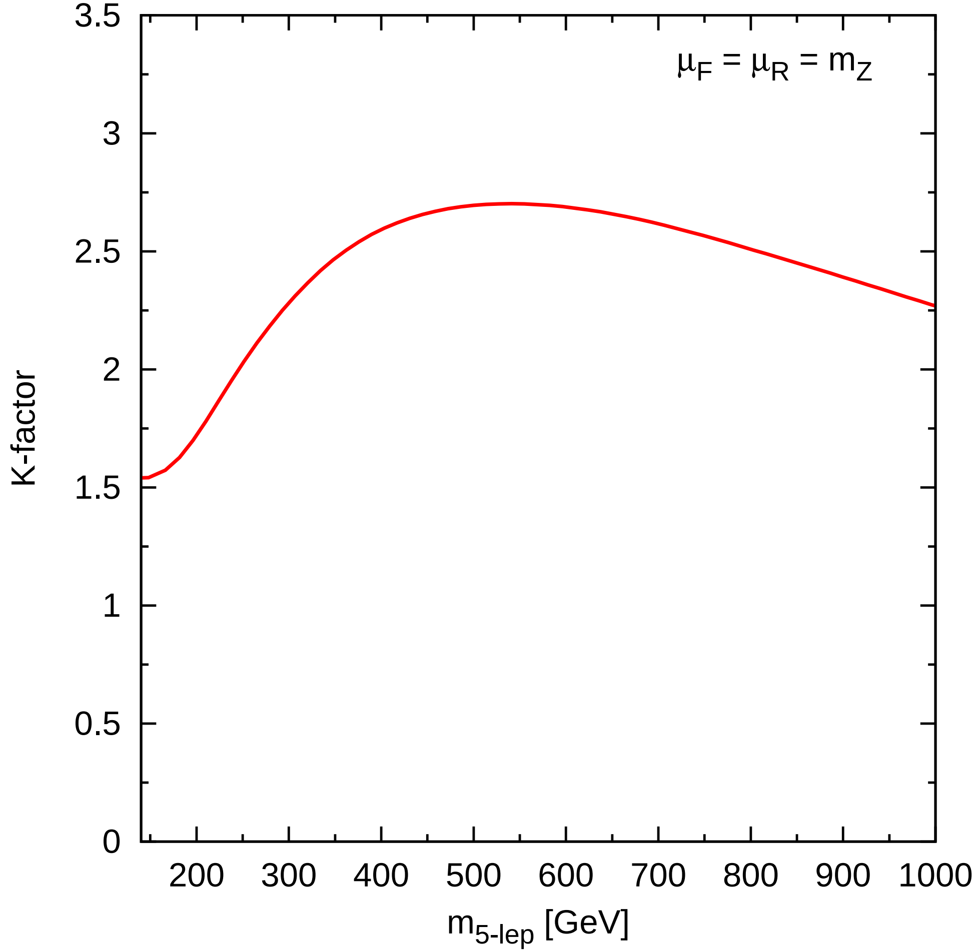




- **Variation of $1.5 < \xi < 6$:** LO: variation of 1.0% (-1.3% and -0.3%)
  NLO: variation of 13.2% (+7.3% and -5.9%).

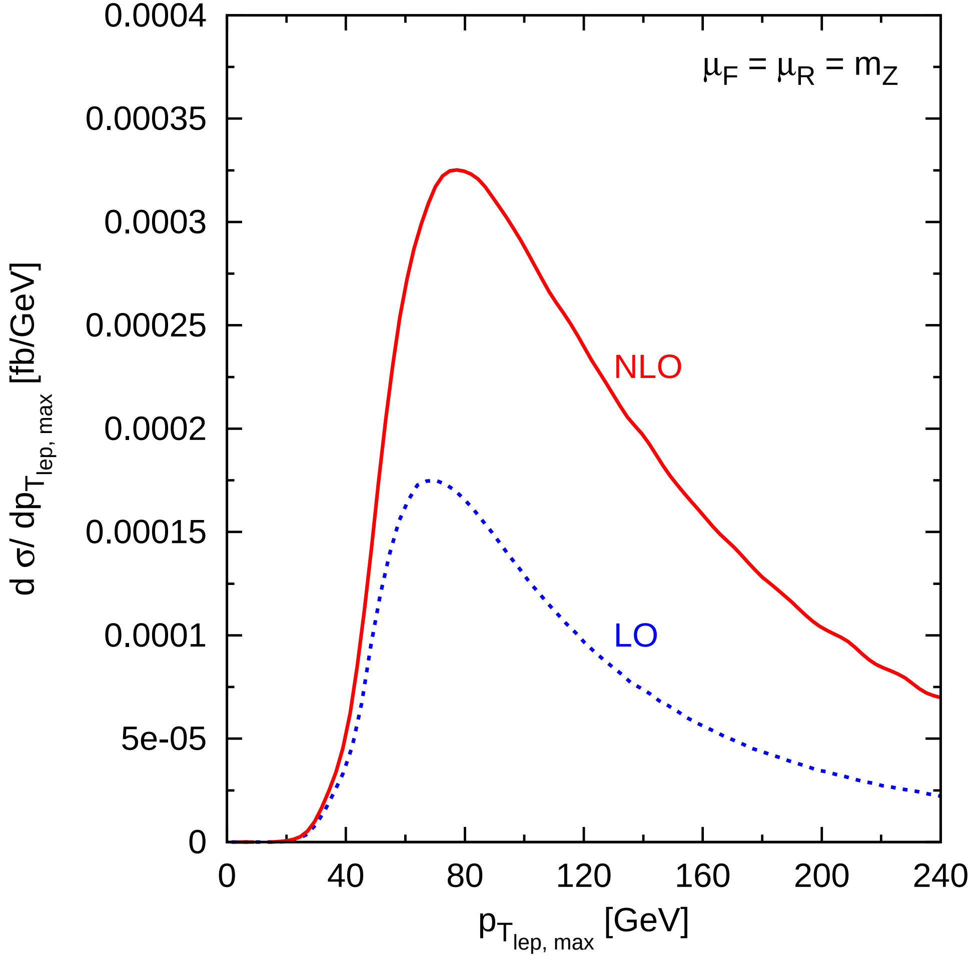
5-lepton invariant mass distribution:

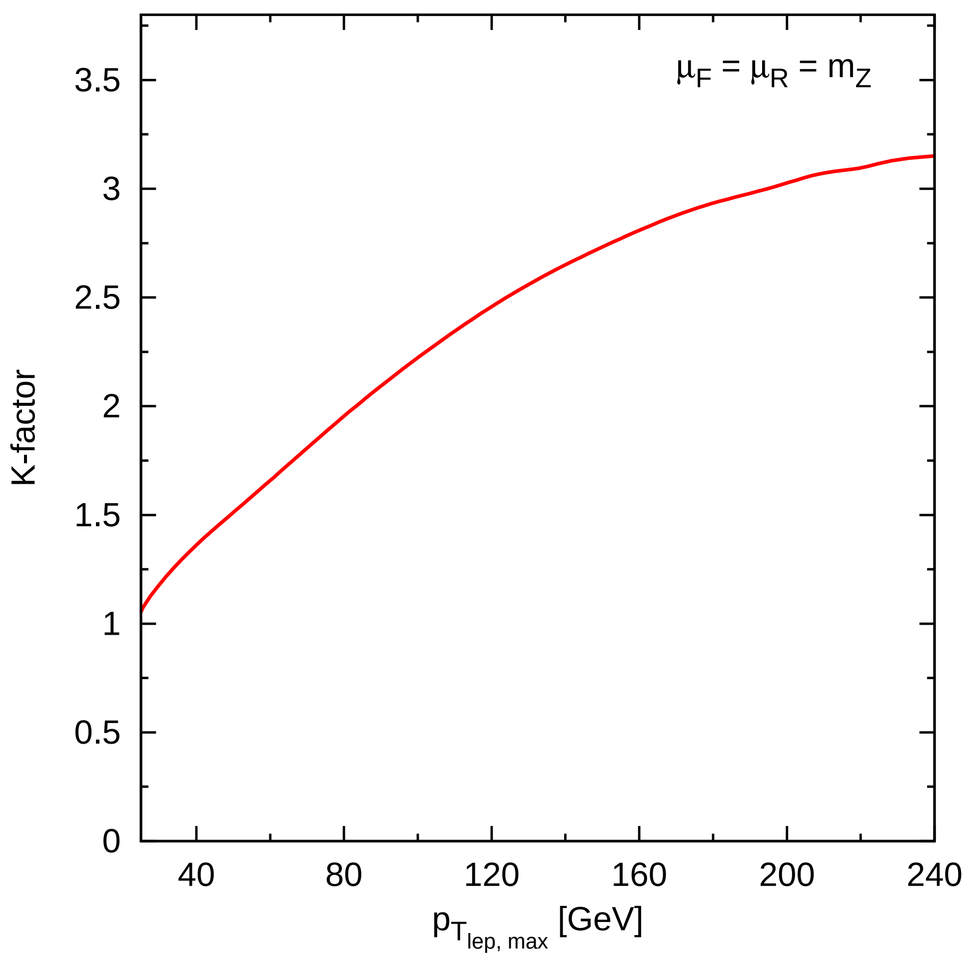




- The variation of the K-factor is larger as in the W^+W^-Z case.

Transverse momentum distribution for the highest-p_T lepton:





- The K-factor increases with p_T by almost a factor of 3.

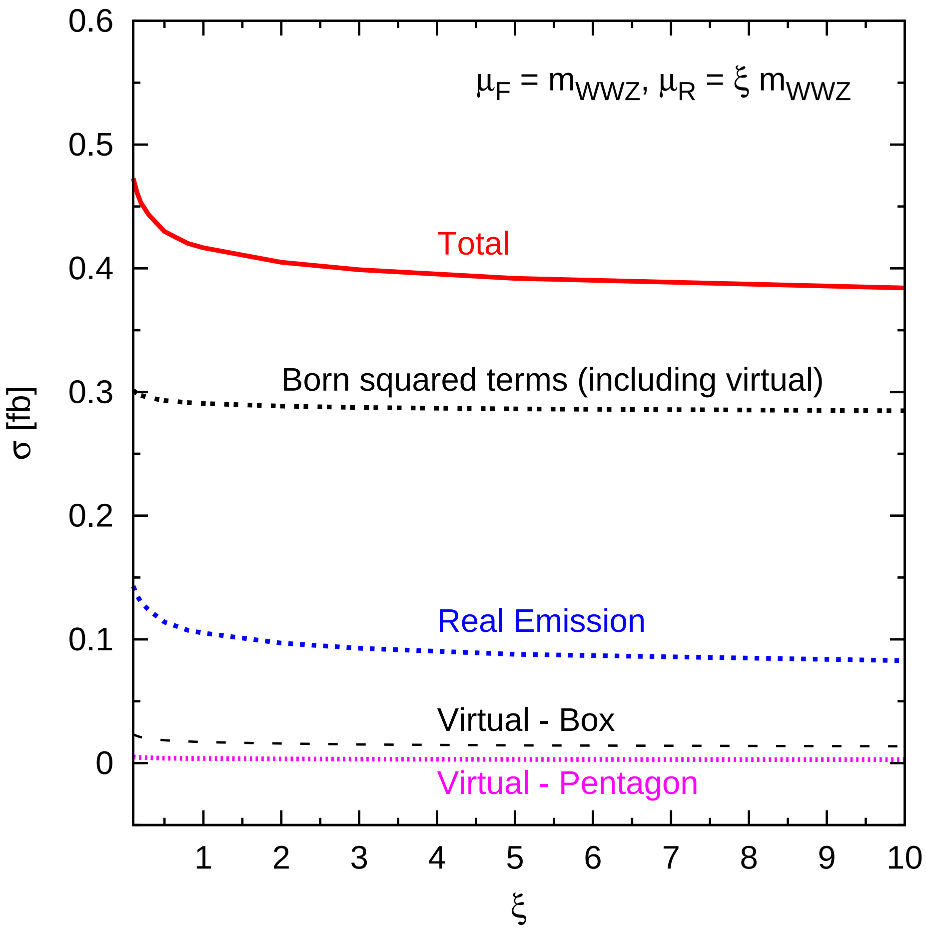
Summary

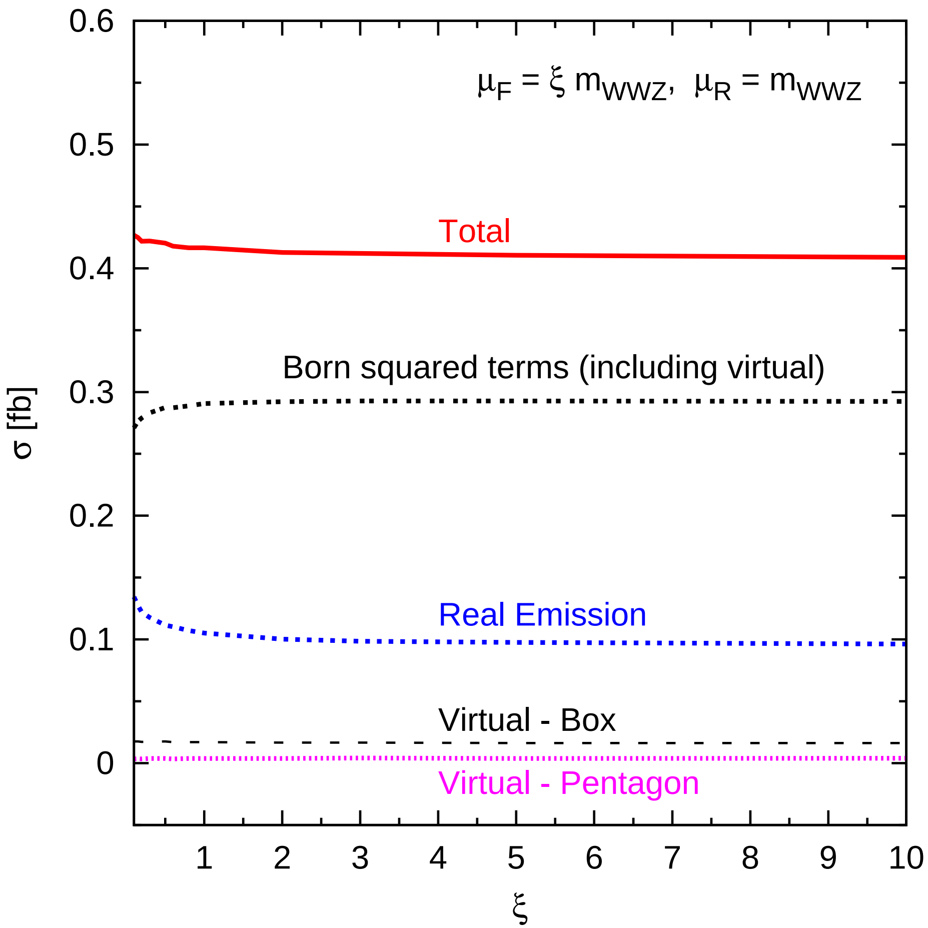
- NLO QCD corrections to W^+W^-Z and ZZW^+ production with leptonic decays have been evaluated.
- All off-shell diagrams as well as the Higgs-contributions have been considered.
- The K-factor is sizeable and NLO corrections lead to substantial shape changes of lepton distributions.
- Scale dependence of the NLO cross section is larger than the variation at LO, which is anomalously small.
- The NLO QCD corrections for W^+W^-Z, ZZW^+, ZZW^-, $W^+W^-W^+$ and $W^-W^+W^-$ production will soon be available in form of a fully flexible parton level Monte Carlo program in the `KITCup` collection, which is structured like `VBFNLO`.

  http://www-itp.particle.uni-karlsruhe.de/ vbfnloweb.

Backup Slides:

Dependence of the W^+W^-Z production cross section on μ_R and μ_F:

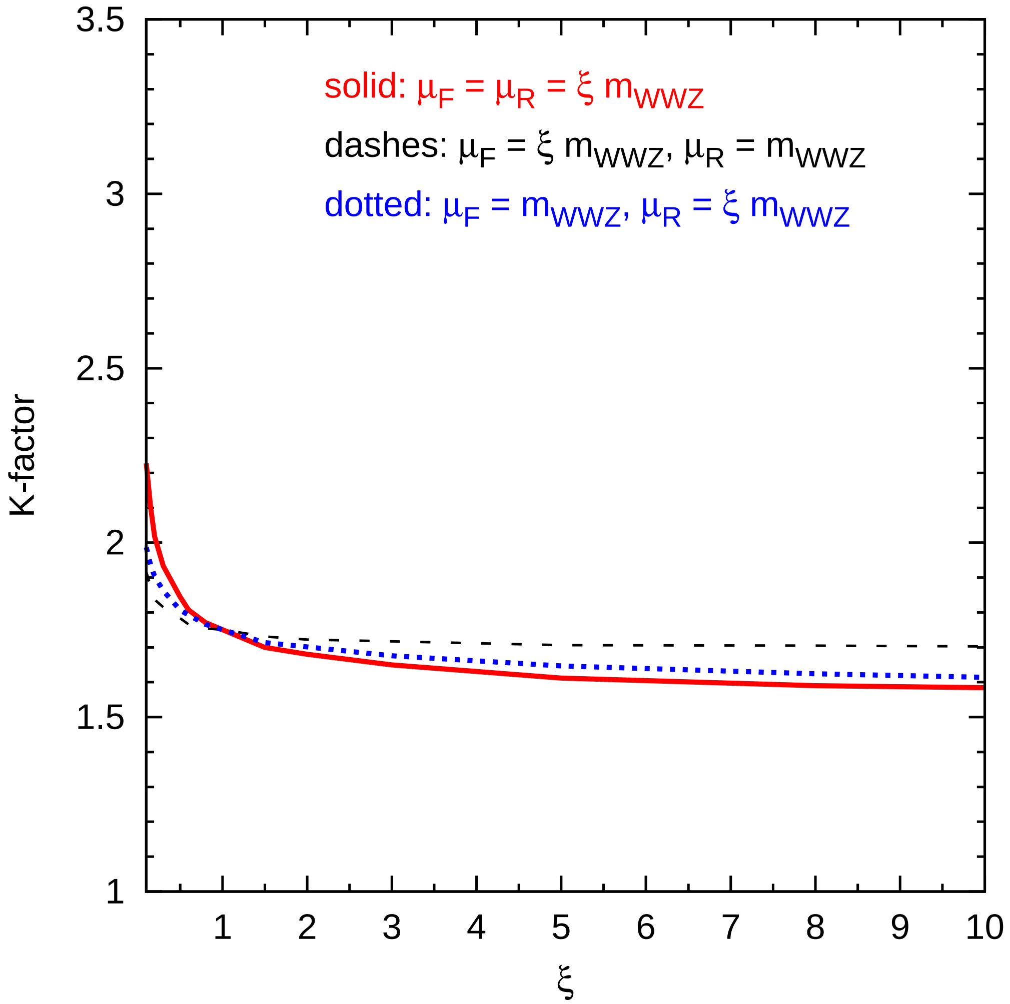




- **Variation of μ_R:**
  Cross section varies with $\alpha_S(\mu_R)$.

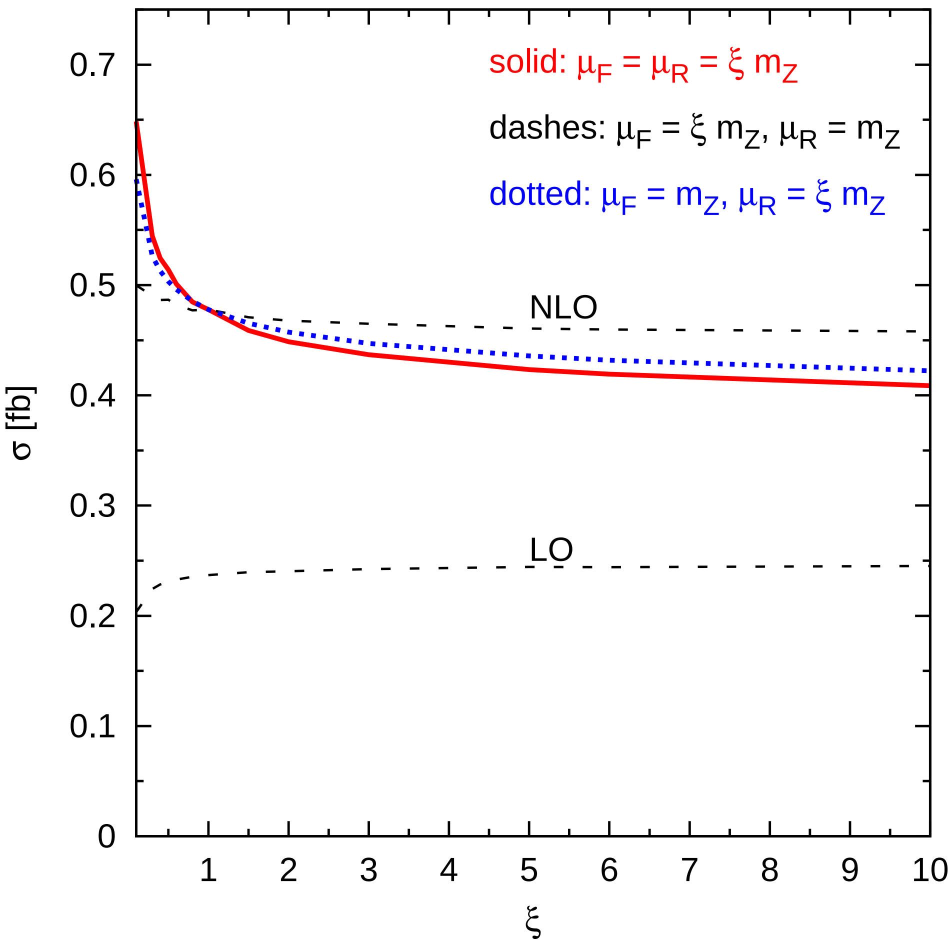
- **Variation of μ_F:**
  Additional collinear terms and PDFs depend on factorization scale.

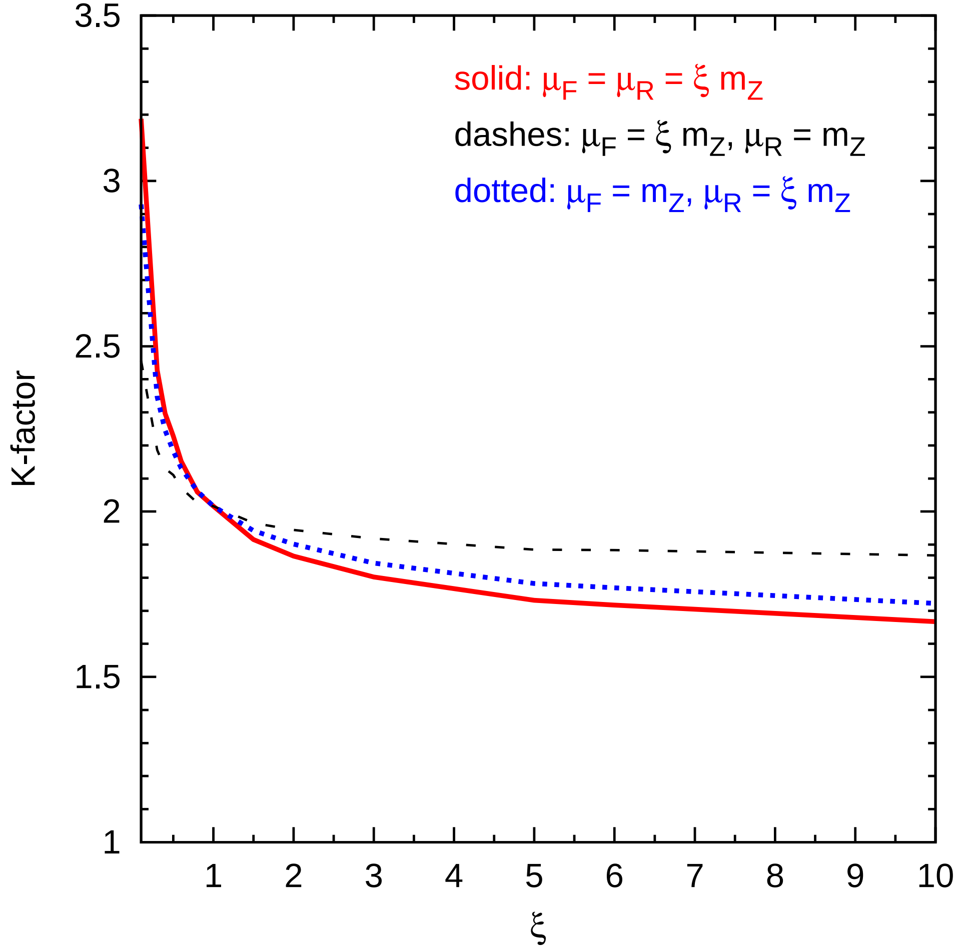
Variation of K-factor in W^+W^-Z production:



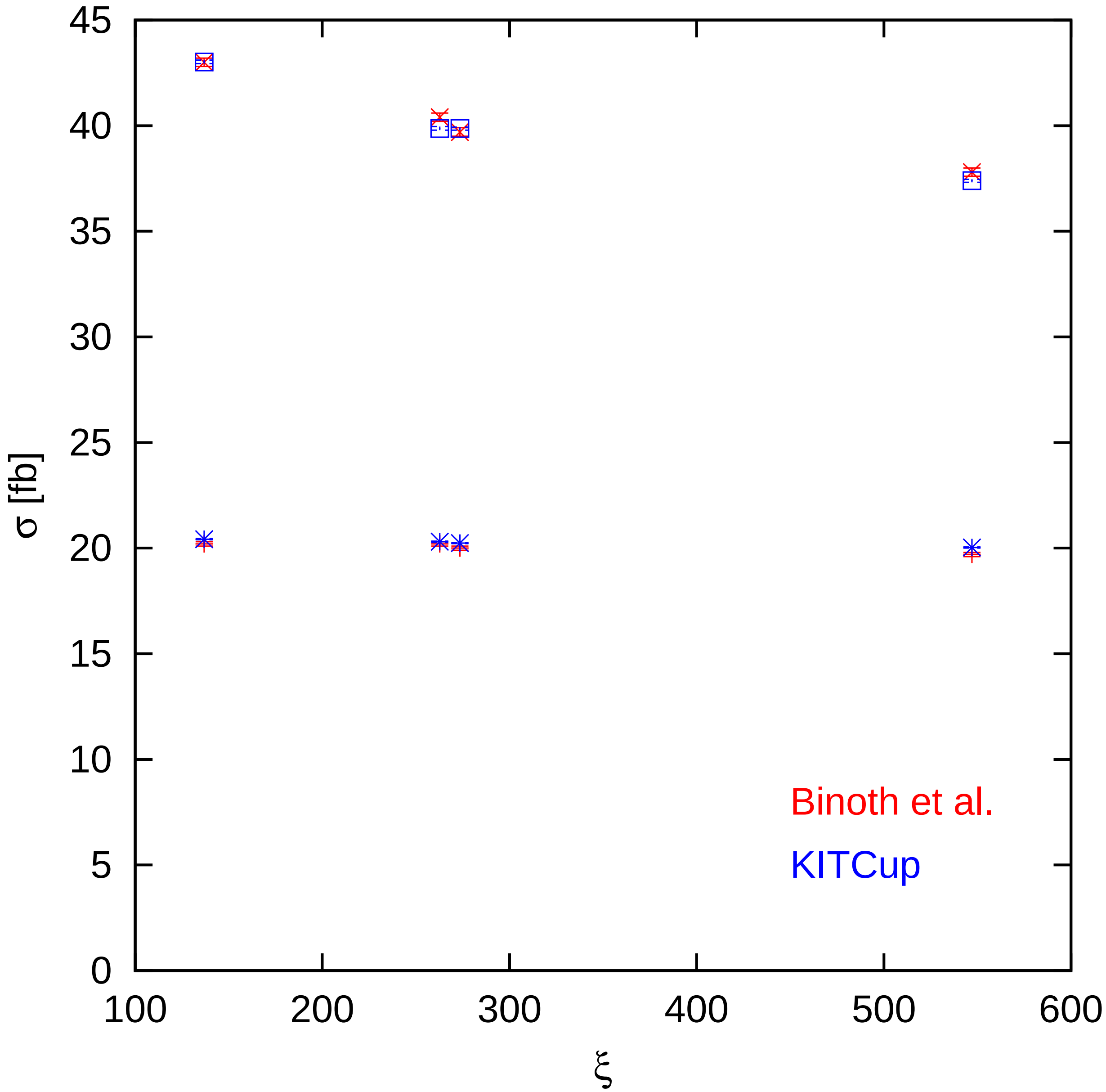
- K-factors varies between 2.2 and 1.6

Scale dependence of W^+W^-Z production cross section:





Comparison for ZZW^{+} production with Binoth et al.:



Comparison with Binoth et al.:

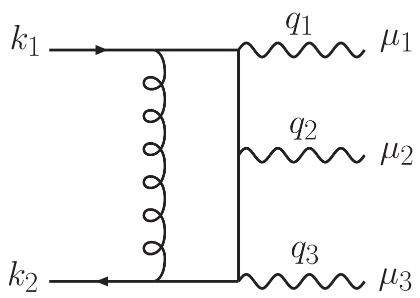
Scale	program	σ^{LO} [fb]	$\sigma^{NLO}[fb]$	K-factor
$0.5 \cdot (3\ m_Z)$	KITCup	20.42 ± 0.03	43.02 ± 0.08	2.11
	Paper	20.2 ± 0.1	43.0 ± 0.2	2.12
$(3\ m_Z)$	KITCup	20.24 ± 0.03	39.86 ± 0.07	1.98
	Paper	20.0 ± 0.1	39.7 ± 0.2	1.99
$2 \cdot (3\ m_Z)$	KITCup	20.03 ± 0.03	37.39 ± 0.07	1.87
	Paper	19.7 ± 0.1	37.8 ± 0.2	1.91

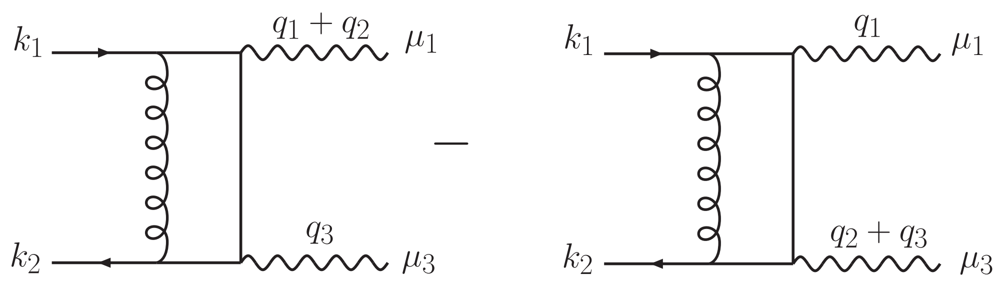
Comparison of cross sections between Binoth et al. and `KITCup` *for* ZZW^+ *production.*

Scale	program	process	σ^{LO} [fb]	$\sigma^{NLO}[fb]$
$2\ m_W + m_Z$	KITCup	W^+W^-Z	97.5 ± 0.1	186.5 ± 0.3
	Paper		96.8 ± 0.6	181.7 ± 0.8
$2\ m_Z + m_W$	KITCup	ZZW^+	20.30 ± 0.03	39.87 ± 0.08
	Paper		20.2 ± 0.1	40.4 ± 0.2

Comparison of cross sections between Binoth et al. and `KITCup` *for* W^+W^-Z *and* ZZW^+ *production.*

Contraction of a pentagon with external momenta:

a) 

b) 

$$\mathcal{P}_{\mu_1\mu_2\mu_3} = \int \frac{d^D l}{(2\,\pi)^D} \frac{\gamma^\rho(\not{l}+\not{k}_1-\not{q}_{123})\gamma_{\mu_3}(\not{l}+\not{k}_1-\not{q}_{12})\gamma_{\mu_2}(\not{l}+\not{k}_1-\not{q}_1)\gamma_{\mu_1}(\not{l}+\not{k}_1)\gamma_\rho}{l^2\,(l+k_1)^2\,(l+k_1-q_1)^2\,(l+k_1-q_{12})^2\,(l+k_1-q_{123})^2}$$

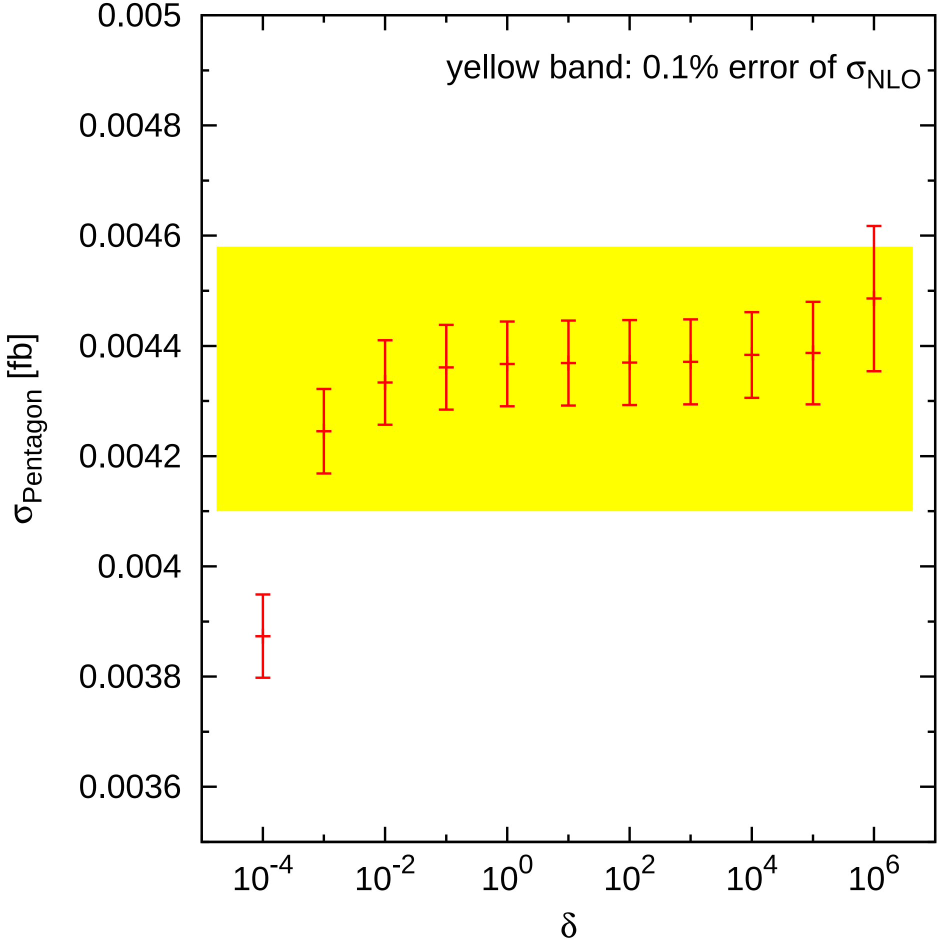
with $q_{12} = q_1 + q_2$ and $q_{123} = q_1 + q_2 + q_3$

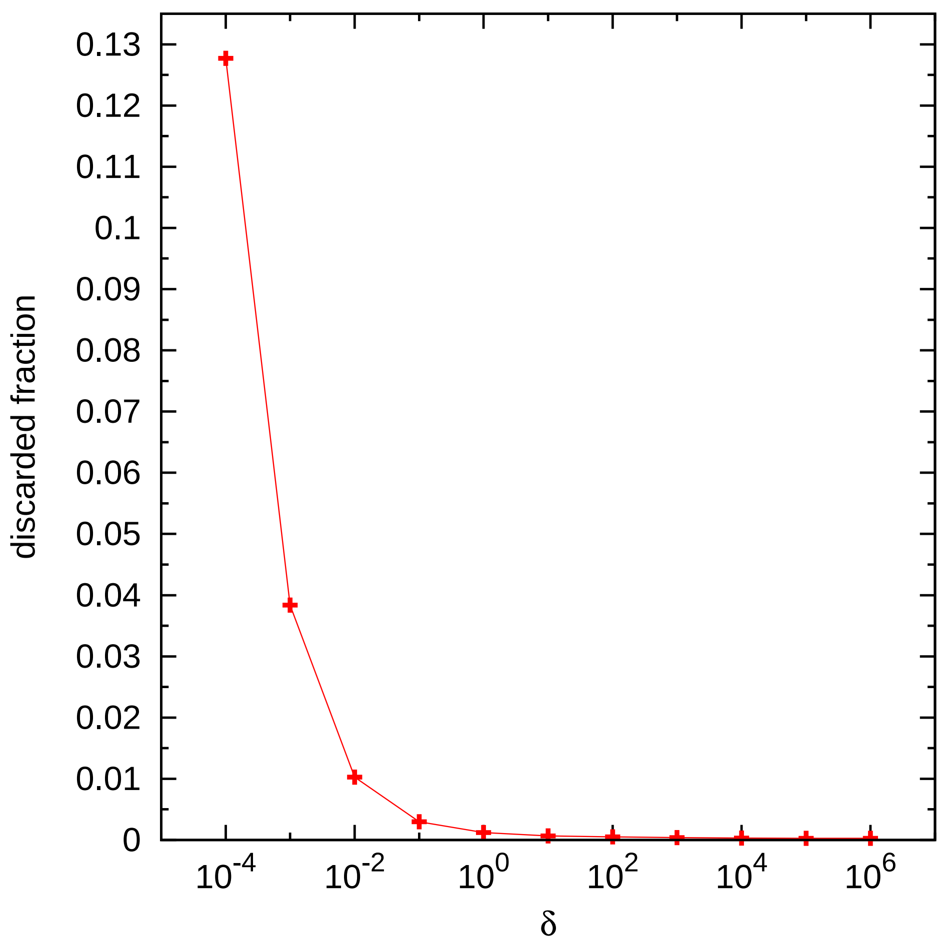
Contraction with $q_2^{\mu_2}$ gives a difference of two boxes

$$q_2^{\mu_2}\,\mathcal{P}_{\mu_1\mu_2\mu_3} = \int \frac{d^D l}{(2\,\pi)^D} \frac{\gamma^\rho\,(\not{l}+\not{k}_1-\not{q}_{123})\,\gamma_{\mu_3}\,(\not{l}+\not{k}_1-\not{q}_1)\,\gamma_{\mu_1}\,(\not{l}+\not{k}_1)\,\gamma_\rho}{l^2\,(l+k_1)^2\,(l+k_1-q_1)^2\,(l+k_1-q_{123})^2}$$
$$-\int \frac{d^D l}{(2\,\pi)^D} \frac{\gamma^\rho\,(\not{l}+\not{k}_1-\not{q}_{123})\,\gamma_{\mu_3}\,(\not{l}+\not{k}_1-\not{q}_{12})\,\gamma_{\mu_1}\,(\not{l}+\not{k}_1)\,\gamma_\rho}{l^2\,(l+k_1)^2\,(l+k_1-q_{12})^2\,(l+k_1-q_{123})^2}.$$

- Same relation for complete pentline and boxline contribution.

Numerical stability of the pentagons:





Pentagons contracted with an external momentum can be expressed in terms of boxes.
Pentline contributions are discarded when the two ways of calculating these terms differ by more than δ.