# NLO QCD corrections to WW + 1-jet production at hadron colliders

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Work in collaboration with S.Dittmaier, S. Kallweit

Comparable results by [Campbell, Ellis, Zanderighi] and [Binoth, Guillet, Karg, Kauer, Sanguinetti]

*) Financed through Heisenberg fellowship and SFB-TR09

Deutsche Forschungsgemeinschaft DFG

# Contents

# WW + 1-Jet — Motivation



**Why is WW + 1 Jet important ?**

- Large fraction of WW with additional jet activity

  → Precise understanding important for tests of the SM at high scale, i.e. electro-weak gauge-boson coupling analysis

- Benchmark process for loop calculations ?

  No, not enough colored particles to be really difficult…

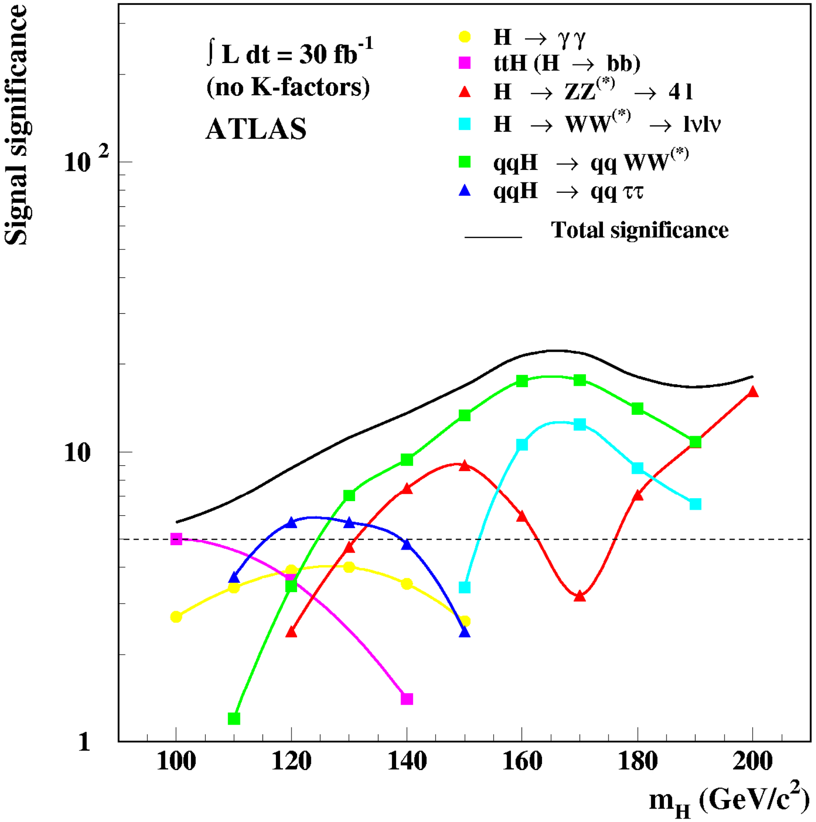
  (# diagrams, # divergencies, #exceptional regions,…)

- Important background process

# WW + 1-Jet — Motivation

Higgs search:

- For 155 GeV < $m_h$ < 185 GeV, H → WW is important channel
- For 130 GeV < $m_h$ < 190 GeV, **V**ector **B**oson **F**usion (VBF) dominates over gg→H as far as signal significance is concerned




[Atlas '03]

# WW + 1-Jet — Motivation

NLO corrections for Higgs production via VBF known:

- Total cross section [Han, Valencia, Willenbrock '92; Spira '98; Djouadi, Spira '00]
- Differential distributions [Figy, Oleari, Zeppenfeld ‘03, Berger,Campbell ‘04]
  → QCD uncertainty ~ 4%

## Experimental Signature:

Two forward tagging jets + “Higgs”

## Background reactions:

WW + 2 Jets, WW + 1 Jet

Top of the Les Houches list ‘07

NLO corrections unknown

If only leptonic decay of W´s and 1 Jet is demanded (→improved signal significance)
[Mellado, Quayle, Wu, ’05,’07]

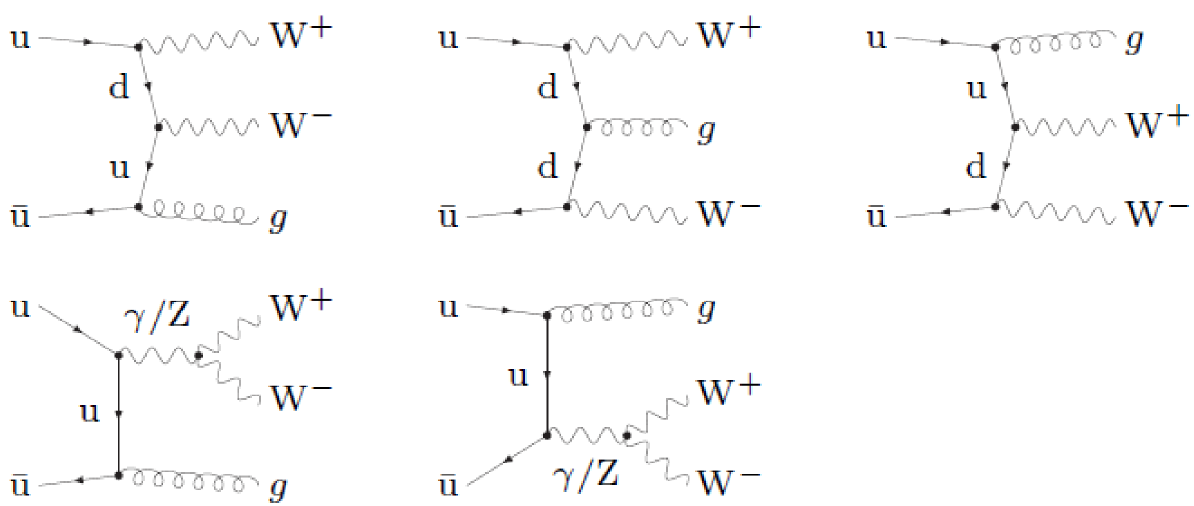
# Leading-order results

Basic process: $0 \to W^+W^-q\bar{q}g$

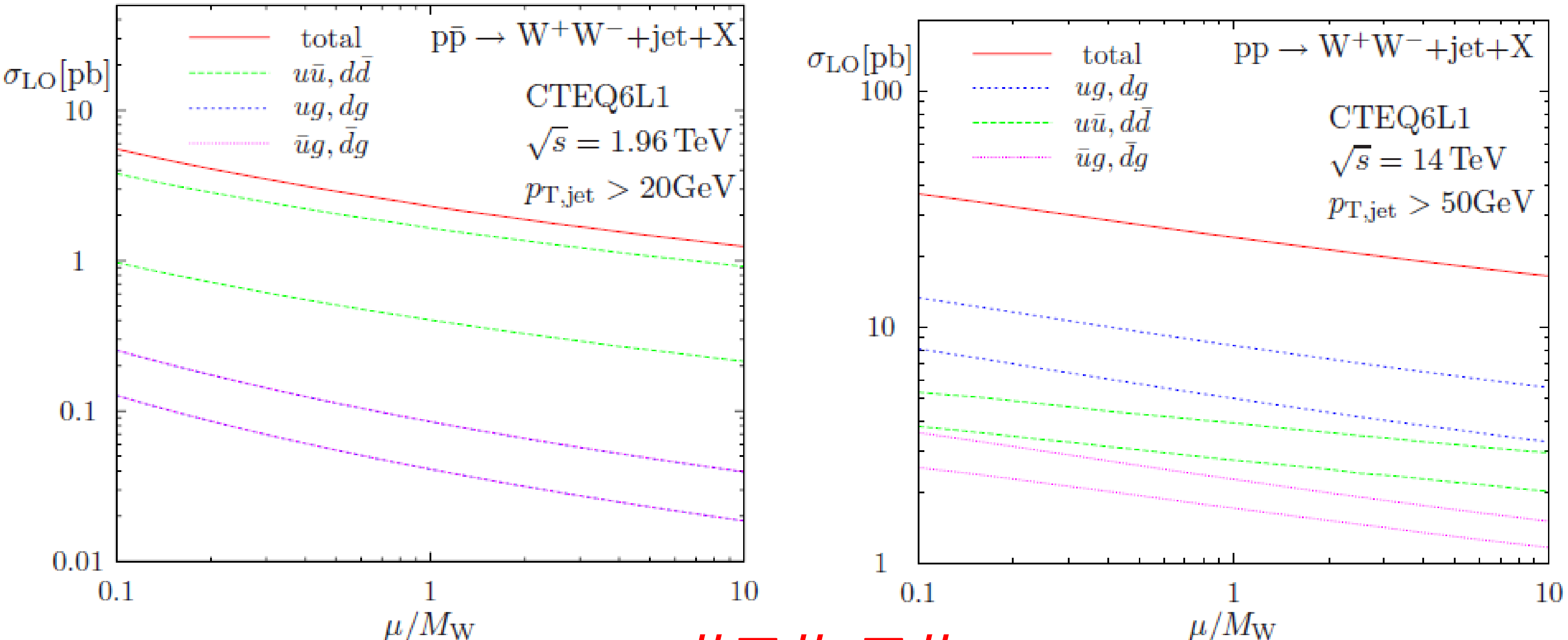
Different quark flavours + crossing

→ 3 x 4 = 12 different partonic channels

Diagrams for uu:



# Leading-order results




$\mu = \mu_f = \mu_r$

Some features:

- Jet algorithm required to render cross section finite → Ellis-Soper-Algorithm, no recombination at LO
- Dependence on 2x2-“CKM” matrix cancels (unitarity)
- Significance of individual channels due to PDF’s
- “Large” residual scale dependance

LHC: 12(30)% for change 2(5)
Tevatron: 25(75)% for change 2(5)

# Methods

# Next-to leading order corrections

$$\underbrace{\sigma_{\rm NLO} = \int_{m+1} \sigma_{\rm real} + \int_m \sigma_{\rm virt.} + \int dx \int_m \sigma_{\rm fact.}(x)}$$

Every piece is individually divergent,
only in the combination a finite result is obtained

Standard procedure: [Frixione,Kunszt,Signer ´95, Catani,Seymour ´96, Nason,Oleari 98, Phaf, Weinzierl, Catani,Dittmaier,Seymour, Trocsanyi ´02]

**Dipole subtraction method**

$$\sigma_{\rm NLO} = \underbrace{\int_{m+1} \left[\sigma_{\rm real} - \sigma_{\rm sub}\right]}_{\rm finite} + \underbrace{\int_m \left[\sigma_{\rm virt.} + \bar{\sigma}^1_{\rm sub}\right]}_{\rm finite} + \underbrace{\int dx \int_m \left[\sigma_{\rm fact.}(x) + \bar{\sigma}_{\rm sub}(x)\right]}_{\rm finite}$$

With:

$$0 = -\int_{m+1} \sigma_{\rm sub} + \int_m \bar{\sigma}^1_{\rm sub} + \int dx \int_m \bar{\sigma}_{\rm sub}(x)$$

$\sigma_{\rm sub} \longrightarrow \sigma_{\rm real}$ in all single-unresolved regions

# Real corrections

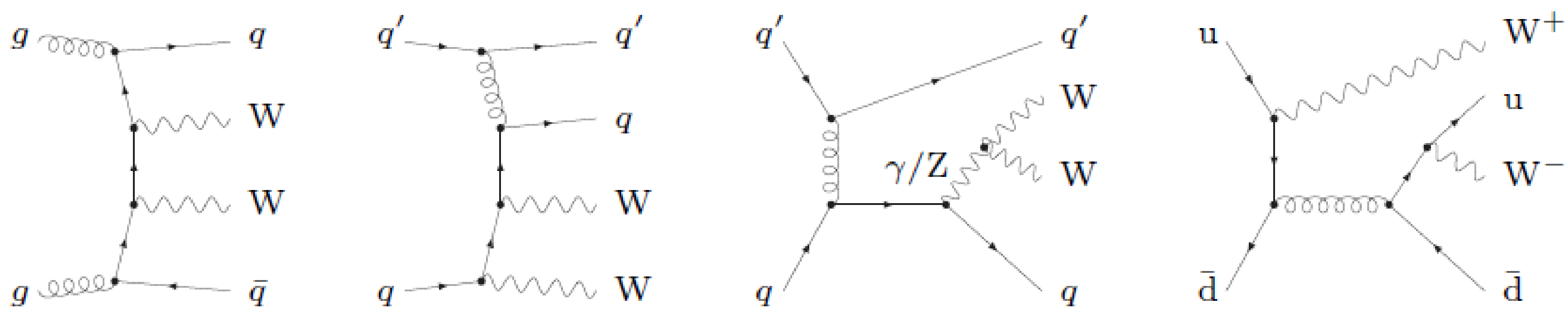
Generic amplitudes: $0 \rightarrow W^+W^-gg, \quad 0 \rightarrow W^+W^-q\bar{q}q'\bar{q}'$

Crossing + non-diagonal 2x2 flavour structure:

Some book keeping required

→ 136 different channels

Sample diagrams:




Two independent computer codes, based on:

- Short analytic expressions, using spinor helicity methods
- Madgraph [Stelzer, Long '94]

# Dipole subtraction method

$$\sigma_{\rm NLO} = \underbrace{\int_{m+1} \left[\sigma_{\rm real} - \sigma_{\rm sub}\right]}_{\rm finite} + \underbrace{\int_{m} \left[\sigma_{\rm virt.} + \bar{\sigma}^{1}_{\rm sub}\right]}_{\rm finite} + \underbrace{\int dx \int_{m} \left[\sigma_{\rm fact.}(x) + \bar{\sigma}_{\rm sub}(x)\right]}_{\rm finite}$$

Universal structure:

$$\sigma_{\rm sub} = \sum_{\rm dipoles} \mathcal{D}_{ij,k}(p_i, p_j, p_k)$$

Generic form:

$$\mathcal{D}_{ij;k} = -\frac{1}{(p_i + p_j) - m_{ij}^2} \langle \ldots, \widetilde{ij}, \ldots, \tilde{k}, \ldots \left| \underbrace{\frac{\mathbf{T}_a \cdot \mathbf{T}_{ij}}{\mathbf{T}_{ij}} V_{ij,k}}_{\rm universal} \right| \ldots, \widetilde{ij}, \ldots, \tilde{k}, \ldots \rangle$$

Leading-order amplitudes
Vector in color space

Color charge operators,
induce color correlation !

Spin dependent part,
induces spin correlation !

Example: $u\bar{u} \rightarrow W^+W^-gg$

→ 10 dipoles required

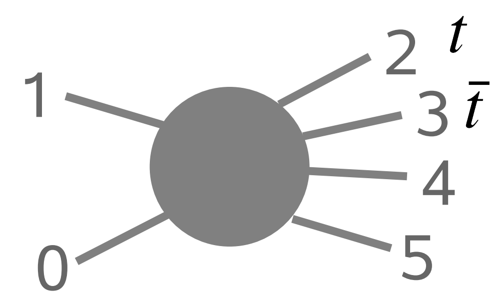
# Dipole subtraction method — implementation

```
emacs@pcth188.cern.ch
File  Edit  Options  Buffers  Tools  C++  Help

double ggttgg_counterterm(const vector<FourMomentum>& mmomenta) {

  static ggttg ggttgamplitude;
  static Correlator correlator(ggttgamplitude);

  static Dipole d[36] = {
    // FinalFinal:
    Dipole(2,4,3), Dipole(2,4,5), Dipole(2,5,3), Dipole(2,5,4),
    Dipole(3,4,2), Dipole(3,4,5), Dipole(3,5,2), Dipole(3,5,4),
    Dipole(4,5,2), Dipole(4,5,3),
    //FinalInitial:
    Dipole(2,4,0), Dipole(2,4,1), Dipole(2,5,0), Dipole(2,5,1),
    Dipole(3,4,0), Dipole(3,4,1), Dipole(3,5,0), Dipole(3,5,1),
    Dipole(4,5,0), Dipole(4,5,1),
    // InitialFinal:
    Dipole(0,4,2), Dipole(0,4,3), Dipole(0,4,5), Dipole(0,5,2),
    Dipole(0,5,3), Dipole(0,5,4), Dipole(1,4,2), Dipole(1,4,3),
    Dipole(1,4,5), Dipole(1,5,2), Dipole(1,5,3), Dipole(1,5,4),
    // IntialIntial:
    Dipole(0,4,1), Dipole(0,5,1), Dipole(1,4,0), Dipole(1,5,0)
  };


  SplittingKernels splittings(momenta,particles);


  double sum = 0.;

  for (int i = 0; i<36; i++) {
    splittings.Kernel(d[i]);
    correlator.EvalfDipole(d[i]);
    sum += d[i].value;
  }

  return( sum );

}
--:--  Dipoles.cpp    3:44PM 0.02    (C++ Abbrev)--L26--C0--All----------------------
```

$\longrightarrow \mathcal{D}_{15,0}$

LO – amplitude, with colour information, i.e. correlations

List of dipoles we want to calculate




reduced kinematics, “tilde momenta” + $V_{ij,k}$

Dipole $d_i$

# Dipole subtraction method — implementation

emacs@ttpodo.particle.uni-karlsruhe.de

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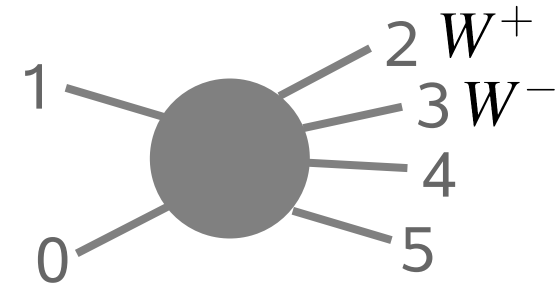
```
double uuwwggct(const vector<FourMomenta> & momenta ) {

  static uubWWg loamplitude;
  static Correlator correlator(obs);

  const int DMAX = 10;
  static Dipole d[DMAX] = {
    //FinalInitial:
    Dipole(4,5,0), Dipole(4,5,1),
    // InitialFinal:
    Dipole(0,4,5), Dipole(0,5,4),
    Dipole(1,4,5), Dipole(1,5,4),
    // IntialIntial:
    Dipole(0,4,1), Dipole(0,5,1), Dipole(1,4,0), Dipole(1,5,0)
  };

  SplittingKernels splittings(momenta,particles);

  double sum = 0.;

  for (int i = 0; i<DMAX; i++) {
    splittings.Kernel(d[i]);
    correlator.EvalfDipole(d[i]);
    sum += d[i].value;
  }

  return( sum );

}
```

-:-- Dipoles.cpp 11:31AM (C++ Abbrev)--L1--C0--All-----------
No diary entries for Monday, 12 May 2008

LO – amplitude, with colour information, i.e. correlations

List of dipoles we want to calculate



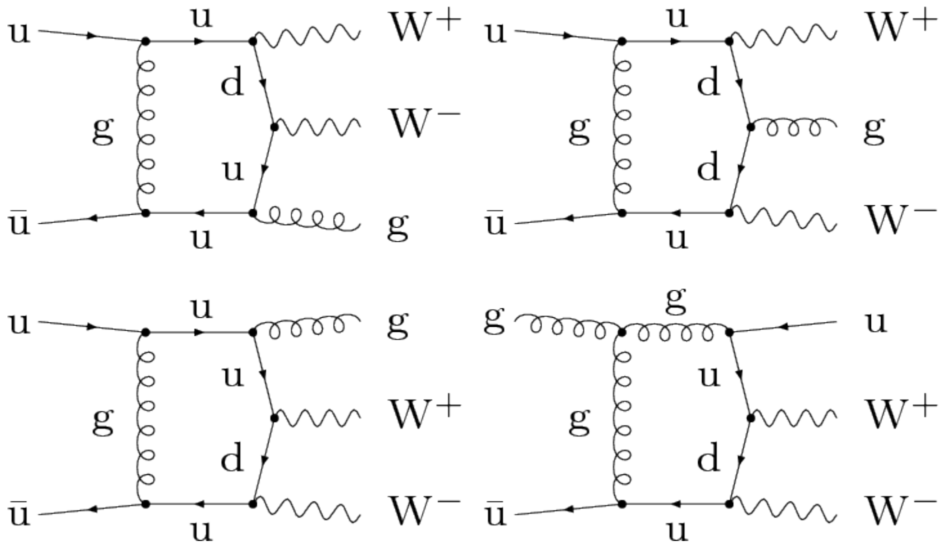

reduced kinematics, “tilde momenta” + $V_{ij,k}$

Dipole $d_i$

# Virtual corrections

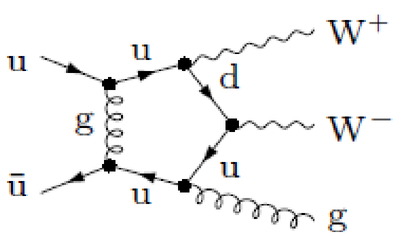


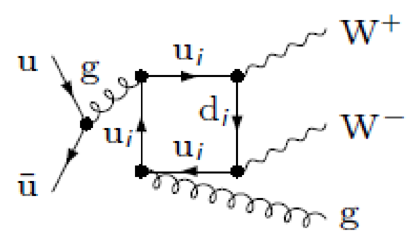
## Sample diagrams




Again many different channels!

Further decomposition possible:




“bosonic corrections” “fermionic corrections”

# Virtual corrections



Scalar integrals

$$\mathcal{A}_{1-\mathrm{loop}} = \sum_i a_i A_i + \sum_i b_i B_i + \sum_i c_i C_i + \sum_i d_i D_i + R$$

Issues:

- Scalar integrals ✓
- How to derive the decomposition ?

Traditional approach: Passarino-Veltman reduction

Large expressions → numerical implementation

Numerical stability and speed are important

# Reduction of tensor integrals — what we did…



## Four and lower-point tensor integrals:

Reduction à la Passarino-Veltman,
with special reduction formulae in singular regions,

## Five-point tensor integrals:

- Apply 4-dimensional reduction scheme, 5-point tensor integrals are reduced to 4-point tensor integrals

→ No dangerous Gram determinants! [Denner, Dittmaier '02]

Based on the fact that in 4 dimension 5-point integrals can be reduced to 4 point integrals [Melrose ´65, v. Neerven, Vermaseren '84]

Two independent computer codes based on:

- Feynarts 1.0 + Mathematica library + Fortran library
- Feynarts 3.2 [Hahn '00] + FormCalc/LoopTools [Hahn, Perez-Victoria '98]

# Checks of the NLO calculation



- Leading-order amplitudes checked with Madgraph
- Subtractions checked in singular regions
- Structure of UV singularities checked
- Structure of IR singularities checked

Most important:

- Two complete independent programs for all parts of the calculation, inparticular:

**complete numerics done twice !**

# Tuned comparison with other groups

Campbell, Ellis, Zanderighi (CEZ), JHEP 0712:056,2007

Binoth, Guillet, Karg, Kauer, Sanguinetti (in progress) (BGKKS)

Stefan Kallweit LL2008

- Integrated LO results checked:

| $pp \to W^+W^-$+jet+X | $\sigma_{LO}$ [fb] |
|---|---|
| DKU | 10371.7(12) |
| CEZ | 10372.26(97) |
| BGKKS | 10371.7(11) |

- Results for virtual corrections checked at one phase-space point:

$u\bar{u} \to W^+W^-g \quad |\mathcal{M}_{LO}|^2/e^4g_s^2 = 0.9963809154477200 \cdot 10^{-3}$

$$2\mathrm{Re}\{\mathcal{M}_V^* \cdot \mathcal{M}_{LO}\} = e^4 g_s^2 \Gamma(1+\varepsilon)\left(\frac{4\pi\mu^2}{M_W^2}\right)^\varepsilon \left(\frac{1}{\varepsilon^2}c_{-2} + \frac{1}{\varepsilon}c_{-1} + c_0\right)$$

All bosonic contributions:

| $u\bar{u} \to W^+W^-g$ | $c_{-2}[\mathrm{GeV}^{-2}]$ | $c_{-1}^{\mathrm{bos}}[\mathrm{GeV}^{-2}]$ | $c_0^{\mathrm{bos}}[\mathrm{GeV}^{-2}]$ |
|---|---|---|---|
| DKU | $-1.080699305508758 \cdot 10^{-4}$ | $7.842861905263072 \cdot 10^{-4}$ | $-3.382910915425372 \cdot 10^{-3}$ |
| CEZ | $-1.080699305505865 \cdot 10^{-4}$ | $7.842861905276719 \cdot 10^{-4}$ | $-3.382910915464027 \cdot 10^{-3}$ |
| BGKKS | $-1.080699305508814 \cdot 10^{-4}$ | $7.842861905263293 \cdot 10^{-4}$ | $-3.382910915616242 \cdot 10^{-3}$ |

Fermionic contributions for 2 light generations in the loop:

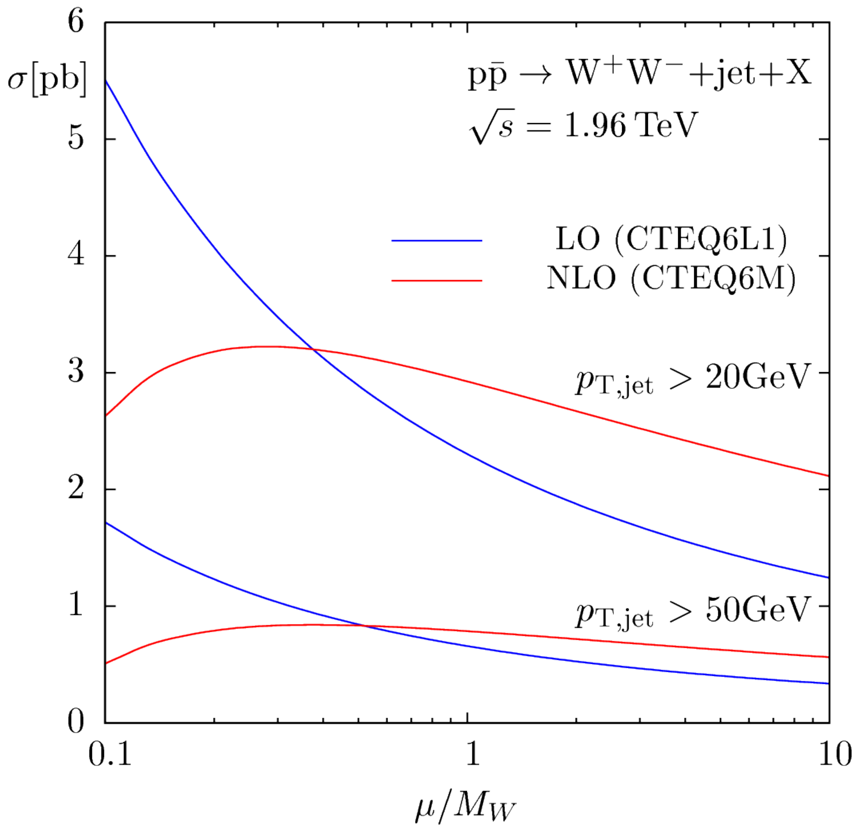
| $u\bar{u} \to W^+W^-g$ | $c_{-1}^{\mathrm{ferm1+2}}[\mathrm{GeV}^{-2}]$ | $c_0^{\mathrm{ferm1+2}}[\mathrm{GeV}^{-2}]$ |
|---|---|---|
| DKU | $2.542821895320379 \cdot 10^{-5}$ | $4.372323372044527 \cdot 10^{-7}$ |
| CEZ | $2.542821895311753 \cdot 10^{-5}$ | $4.372790378087550 \cdot 10^{-7}$ |
| BGKKS | $2.542821895314862 \cdot 10^{-5}$ | $4.372324288356448 \cdot 10^{-7}$ |

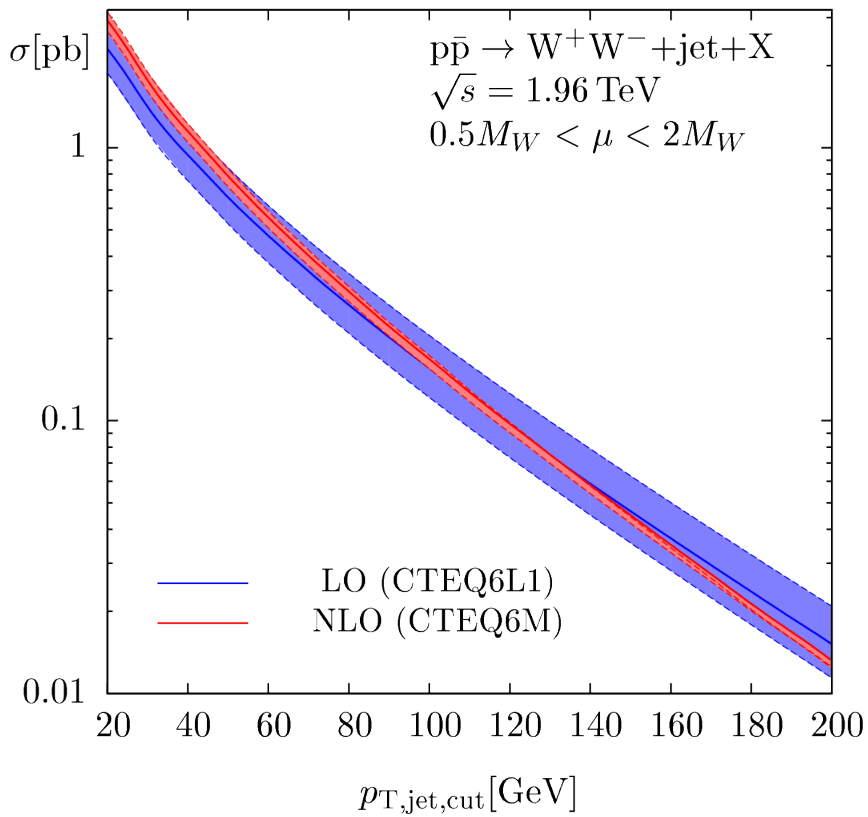
→ impressive agreement !

# Results

# Results WW+1-Jet — Tevatron

[Dittmaier, Kallweit, Uwer '07]
Phys.Rev.Lett.100:062003,2008



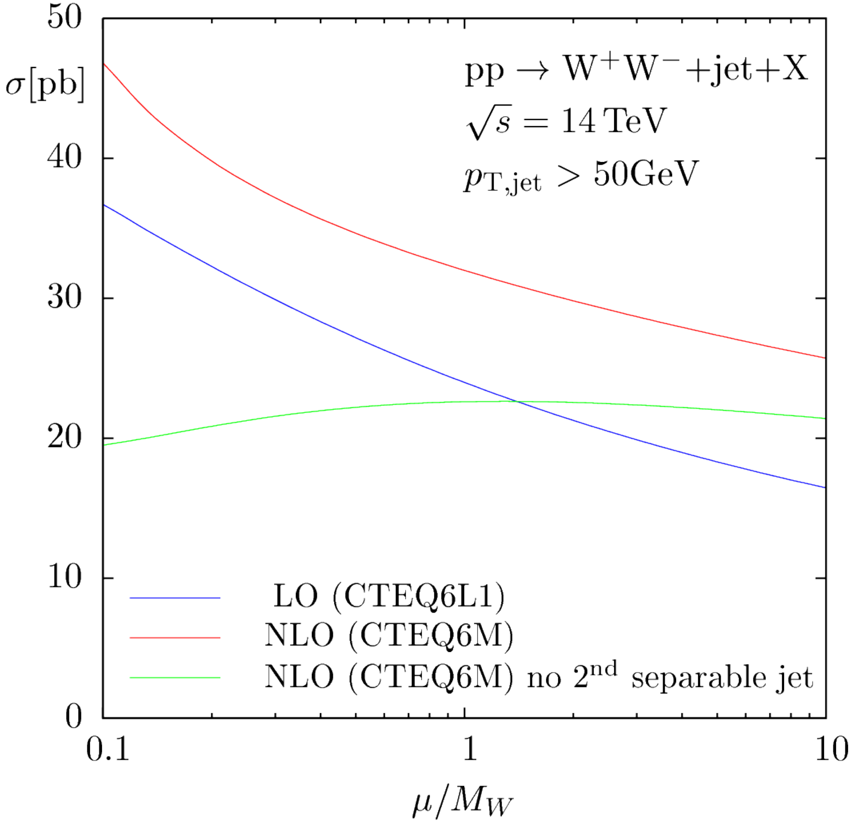



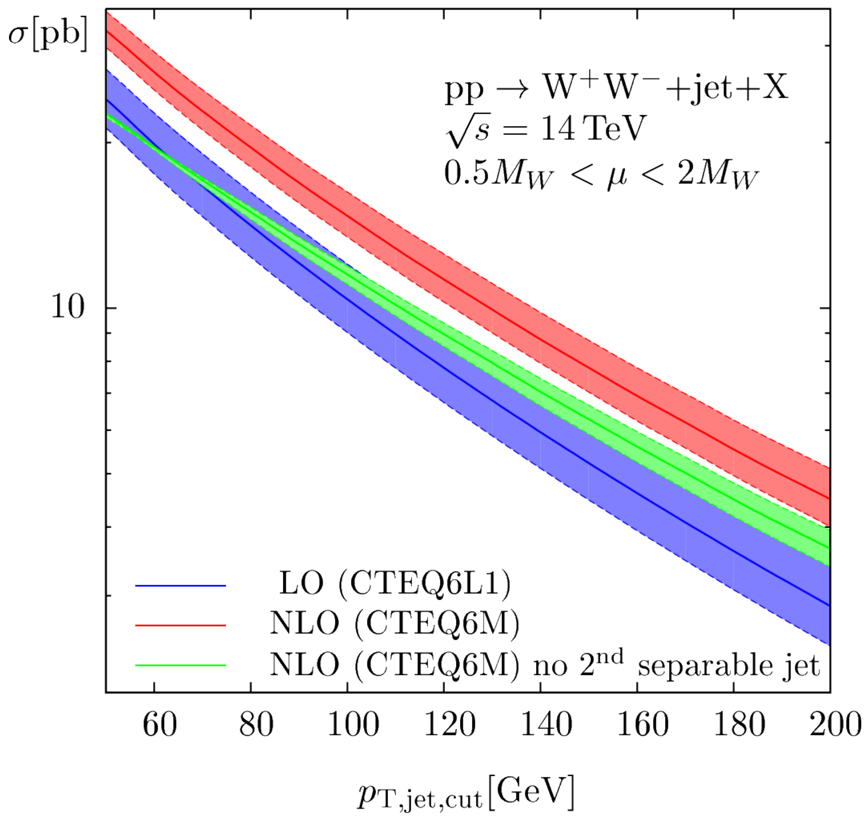

…as it should be…

Note: shown results independent from the decay of the W´s

# Results WW + 1-Jet — LHC

[Dittmaier, Kallweit, Uwer '07]
Phys.Rev.Lett.100:062003,2008



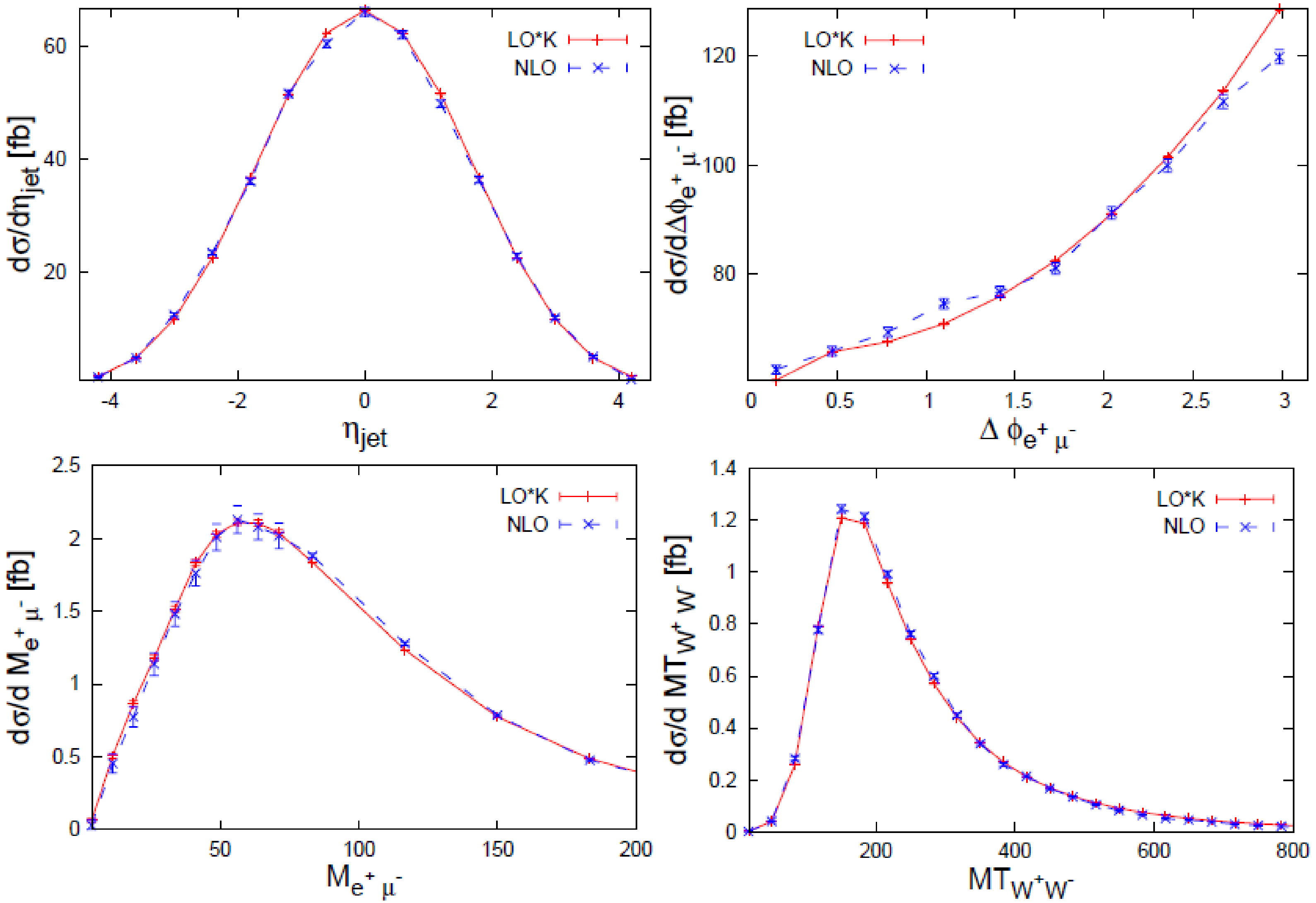




scale dependence only improved after jet-veto !

Note: shown results independent from the decay of the W´s

# Distributions including decay (LHC)

[Campbell, Ellis, Zanderighi, JHEP 0712:056,2007]




LO rescaled by appropriate K-factor

# Conclusions



- Our group: Two complete independent calculations
- In addition: perfect agreement with two other groups
- Scale dependence is improved (→LHC jet-veto)
- Corrections are important
- NLO has only mild effect on the shape of distributions [CEZ]

**Les Houches 07 wishlist**

| process ($V \in \{Z, W, \gamma\}$) | # groups working on |
|---|---|
| 1. $pp \to V\,V$ jet | 2 |
| 2. $pp \to t\bar{t}\,b\bar{b}$ | 1 |
| 3. $pp \to t\bar{t} + 2$ jets | |
| 4. $pp \to W\,W\,W$ | 1 |
| 5. $pp \to V\,V\,b\bar{b}$ | |
| 6. $pp \to V\,V + 2$ jets | |
| 7. $pp \to V + 3$ jets | |
| 8. $b\bar{b}b\bar{b}$ | 1 |
| 9. $gg \to W^*W^*$ (NLO, 2 loops) | ? |
| 10. EW corrections to VBF | 1 |
| 11. NNLO to VBF, $t\bar{t}$, $Z/\gamma$+jet, $W$+jet | |

Summary of activities in NLO multi-leg working group – p.

[Gudrun Heinrich ]

# Outlook



- Further improvements possible (and underway…)
  (remove redundancy, further tuning, except. momenta,…)
- Distributions
- Include leptonic decay of the W´s
- Apply tools to other processes