Automating dipole subtraction for QCD NLO calculations

Tanju Gleisberg

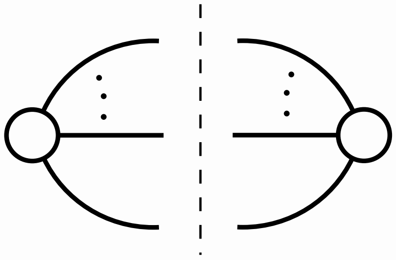
—

Stanford Linear Accelerator Center

Cross sections

- LO:

$$\sigma^{\mathrm{LO}} = \int_m d\sigma^{\mathrm{B}}$$



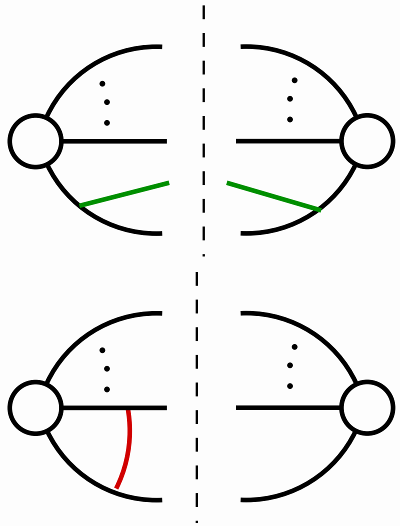
finite by definition: external partons well separated in phase space

➔ straightforward numerical evaluation of the matrix element

- Highly automated tools available, e.g.
  - Alpgen
  - MadGraph
  - Helac/Phegas
  - O'Mega/Whizard
  - AMEGIC++

➔ Typically processes with up to 6-8 final state particles feasible (exact tree level ME's)

Cross sections



- NLO correction:

$$\sigma^{\mathrm{NLO}} = \int d\sigma^{\mathrm{NLO}} = \int_{m+1} d\sigma^{\mathrm{R}} + \int_{m} d\sigma^{\mathrm{V}}$$

(Total cross section: $\sigma = \sigma^{\mathrm{LO}} + \sigma^{\mathrm{NLO}}$)

Difficulties:

  - Individual terms are infrared divergent
  - Evaluation of virtual corrections

- A number of tools with hard coded matrix elements available, e.g. MCFM, NLOJET, ...
covering $2 \to 2$, most $2 \to 3$ and very few $2 \to 4$ processes
- Goal: fully automated calculation of NLO cross sections

Subtraction method

$$\begin{aligned}\sigma^{\rm NLO} &= \int d\sigma^{\rm NLO} = \int_{m+1} d\sigma^{\rm R} + \int_m d\sigma^{\rm V} \\ &= \int_{m+1} d\sigma^{\rm R} - \int_{m+1} d\sigma^{\rm A} + \int_{m+1} d\sigma^{\rm A} + \int_m d\sigma^{\rm V} \\ &= \int_{m+1} \left[d\sigma^{\rm R}_{\varepsilon=0} - d\sigma^{\rm A}_{\varepsilon=0} \right] + \int_m \left[d\sigma^{\rm V} + \int_1 d\sigma^{\rm A} \right]_{\varepsilon=0}\end{aligned}$$

$d\sigma^{\rm A}$: approximation to $d\sigma^{\rm R}$ with following properties:

- $d\sigma^{\rm R} - d\sigma^{\rm A} \to 0$ in soft/collinear limit
  ($\to$ reproduces exactly the divergency structure)
- analytically integrable over the 1-parton phase space of the extra real emission

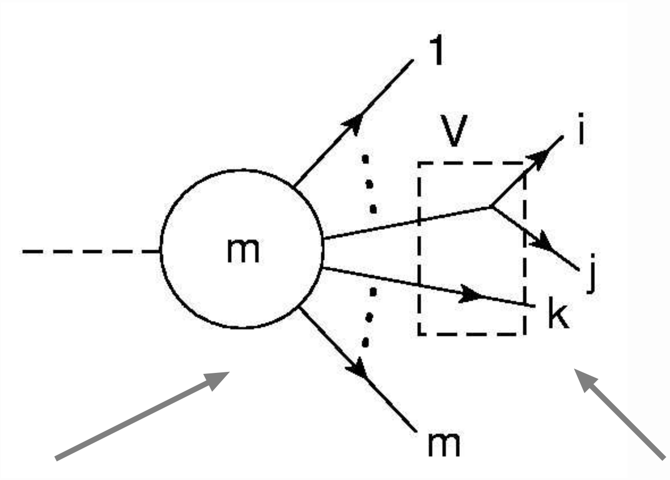
- real part can be directly (numerically) integrated in 4 dimensions
- virtual part integrable in 4 dimension only after cancelation of poles

The dipole subtraction method

Recipe to construct $d\sigma^{\mathrm{A}}$: dipole subtraction terms

[S. Catani, M. H. Seymour, 1997]

- based on universal (process independent) infrared (soft & collinear) limit
- single dipole term (for real correction with $(m+1)$-partons)



m-parton LO-ME splitting operator

momentum map: $p_i, p_j, p_k \longrightarrow \tilde{p}_{ij}, \tilde{p}_k$

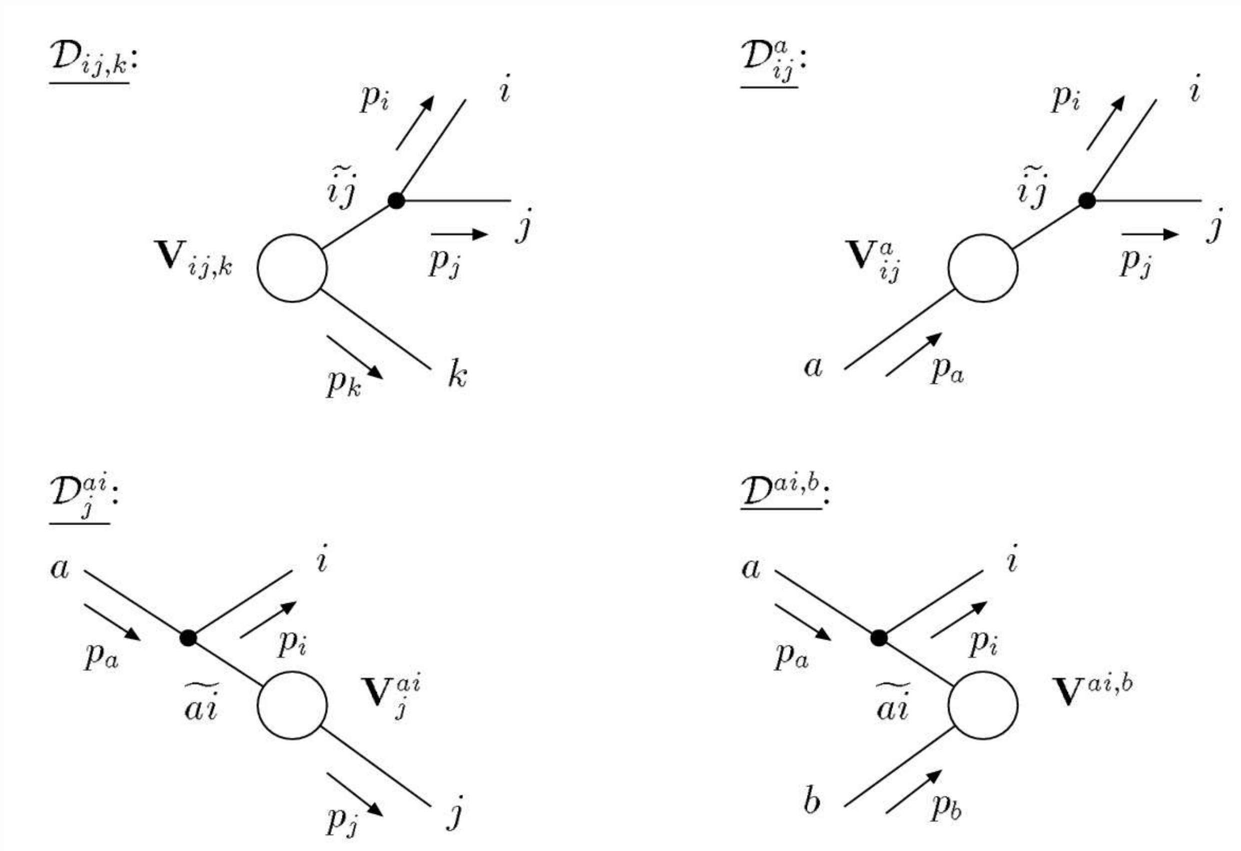
(momentum conservation and on-shellness)

$$d\sigma^{\mathrm{A}}_{ij,k} = d\sigma^{\mathrm{LO}}_{\tilde{ij},\tilde{k}} \otimes dV_{ij,k}$$

$$dV_{ij,k} \sim \frac{1}{p_i p_j} \mathbf{T}_{ij} \cdot \mathbf{T}_k \, \mathbf{V}_{ij,k} \, d\Phi^{(1)}$$

The dipole subtraction method

- 4 different types of dipoles:

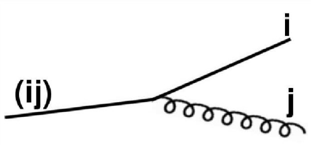


$$d\sigma^{\mathrm{A}} = d\Phi \sum_{k\neq i\neq j} \mathcal{D}_{ij,k} + \left\{ \sum_{i\neq j} \mathcal{D}_{ij}{}^{a} + \sum_{k\neq i} \mathcal{D}_{k}{}^{ai} + \sum_{i} \mathcal{D}^{ai,b} + (a \leftrightarrow b) \right\}$$

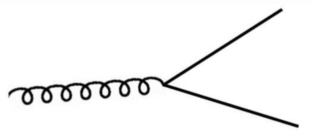
final-final (FF) dipole terms

$$\mathcal{D}_{ij,k}(p_1,\ldots,p_{m+1}) = -\frac{1}{2p_i \cdot p_j}$$

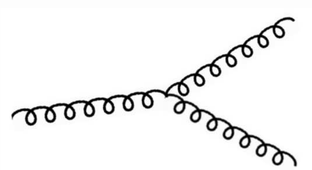
$$\cdot_m < 1,\ldots,\tilde{ij},\ldots,\tilde{k},\ldots,m+1|\frac{\mathbf{T}_k\cdot\mathbf{T}_{ij}}{\mathbf{T}_{ij}^2}\mathbf{V}_{ij,k}|1,\ldots,\tilde{ij},\ldots,\tilde{k},\ldots,m+1>_m$$



$$< s|V_{q_i g_j,k}(\tilde{z}_i;y_{ij,k})|s'>= 8\pi\alpha_S C_F\left[\frac{2}{1-\tilde{z}_i(1-y_{ij,k})}-(1+\tilde{z}_i)-\epsilon(1-\tilde{z}_i)\right]\delta_{ss'}$$



$$< \mu|V_{q_i\bar{q}_j,k}(\tilde{z}_i;y_{ij,k})|\nu>= 8\pi\alpha_S T_R\left[-g^{\mu\nu}-\frac{2}{p_ip_j}(\tilde{z}_ip_i-\tilde{z}_jp_j)^\mu(\tilde{z}_ip_i-\tilde{z}_jp_j)^\nu\right]$$



$$< \mu|V_{g_i g_j,k}(\tilde{z}_i;y_{ij,k})|\nu>= 16\pi\alpha_S C_A\left[-g^{\mu\nu}\left(\frac{1}{1-\tilde{z}_i(1-y_{ij,k})}+\frac{1}{1-\tilde{z}_j(1-y_{ij,k})}-2\right)\right.$$

$$\left.+(1-\epsilon)\frac{2}{p_ip_j}(\tilde{z}_ip_i-\tilde{z}_jp_j)^\mu(\tilde{z}_ip_i-\tilde{z}_jp_j)^\nu\right]$$

Integration over the phase space of the emitted parton

- Phase space for 1-parton emission can be factorized:

$$d\phi(p_i p_j p_k; Q) = d\phi(\tilde{p_{ij}}, \tilde{p_k}, Q)[dp_i(\tilde{p_{ij}}, \tilde{p_k})]$$

- Integral over dipole terms (performed once and for all):

$$\int_1 d\sigma^{\mathrm{A}}_{ij,k} = d\sigma^{\mathrm{LO}}_{\tilde{ij},\tilde{k}} \otimes \int d^d V_{ij,k} = d\sigma^{\mathrm{LO}} \otimes \mathbf{I}(\epsilon)$$

where

$$\begin{aligned}\mathbf{I}(\epsilon) &= -\frac{\alpha_S}{2\pi}\frac{1}{\Gamma(1-\epsilon)}\sum_i \frac{1}{\mathbf{T}_i^2}\mathcal{V}_i(\epsilon)\sum_{i\neq j}\mathbf{T}_i\cdot\mathbf{T}_j\left(\frac{4\pi\mu^2}{2p_i p_j}\right)^{\epsilon} \\ &= \mathbf{A}\epsilon^{-2} + \mathbf{B}\epsilon^{-1} + \mathbf{C} + \mathcal{O}(\epsilon)\end{aligned}$$

Full procedure for hadronic initial states

Cross sections for hadronic initial states:

$$\sigma(p,p') = \sum_{a,b} \int_0^1 d\eta f_a(\eta,\mu_F^2) \int_0^1 d\eta' f_b(\eta',\mu_F^2) \left[\sigma^{\text{LO}}(\eta p,\eta' p') + \sigma^{\text{NLO}}(\eta p,\eta' p',\mu_F^2)\right]$$

$$\sigma^{\text{NLO}}(p_a,p_b,\mu_F^2) = \int_{m+1} d\sigma_{ab}^R(p_a,p_b) + \int_m d\sigma_{ab}^V(p_a,p_b) + \int_m d\sigma_{ab}^C(p_a,p_b,\mu_F^2)$$

- Collinear counterterm $d\sigma_{ab}^C$ contains collinear IS pole terms and factorization scale & scheme dependent finite terms (to be canceled with dependence in PDF's)
- Result structure of counter-/subtraction-terms:

$$\int_{m+1} d\sigma_{ab}^A(p_a,p_b) + \int_m d\sigma_{ab}^C(p_a,p_b,\mu_F^2) = \int_m \left[d\sigma_{ab}^B(p_a,p_b) \times \mathbf{I}(\epsilon)\right]$$
$$+ \sum_{a'} \int_0^1 dx \int_m \left[\left(\mathbf{K}^{a,a'}(x) + \mathbf{P}^{a,a'}(xp_a,x;\mu_F^2)\right) \times d\sigma_{a'b}^B(xp_a,p_b)\right] + < a \leftrightarrow b >$$

(all poles contained in the $\mathbf{I}(\epsilon)$-term)

Automatic dipole subtraction

- Basis: automatic tree level ME generation with AMEGIC++

  (the built-in ME generator of the event generator SHERPA)

$$
\begin{aligned}
|M|^2 &= {}_m< 1,\ldots,m|1,\ldots,m >_m \\
&= \sum_{i,j} \left[A_i(\{p_k\})A_j^*(\{p_k\})\right] \left[\mathbf{C}_i\mathbf{C}_j^\dagger\right]
\end{aligned}
$$

  - Feynman amplitudes A_i are computed in a helicity method
  - automatic analytic simplification, storage of the final ME as a C++-library
  - automatic generation of appropriate phase-space maps
  - provides structures to generate all parton level processes that contribute to a jet XS at once
  - implemented models: SM, MSSM, ADD, ...

Automatic dipole subtraction

[TG, F. Krauss, 2007]

$$\sigma^{\mathrm{NLO}} = \int_{m+1} \left[d\sigma^{\mathrm{R}} - d\sigma^{\mathrm{A}} \right] + \int_{m} \left[d\sigma^{\mathrm{V}} + \int_{1} d\sigma^{\mathrm{A}} \right]$$

Extensions of AMEGIC++:

- fully automatic generation of the real correction and all dipole subtraction terms
- coefficients for ϵ^{-2}, ϵ^{-1} and ϵ^{0} of the integrated subtraction term

➔ Independence of the subtraction function/method

- Monte-Carlo integration methods for the m- and $(m+1)$-parton phase spaces are provided
- automatic organization of parton level subprocesses (and reusage of intermediate results)
- Completed: SM with massless partons (+BSM as long as no new colored particles involved)
- Work in progress: subtraction for massive partons

  [S. Catani, S. Dittmaier, M. H. Seymour, Z. Trocsanyi, 2002]

  (implemented, but not fully tested yet)

Tests & simple examples

Test of numeric stability:

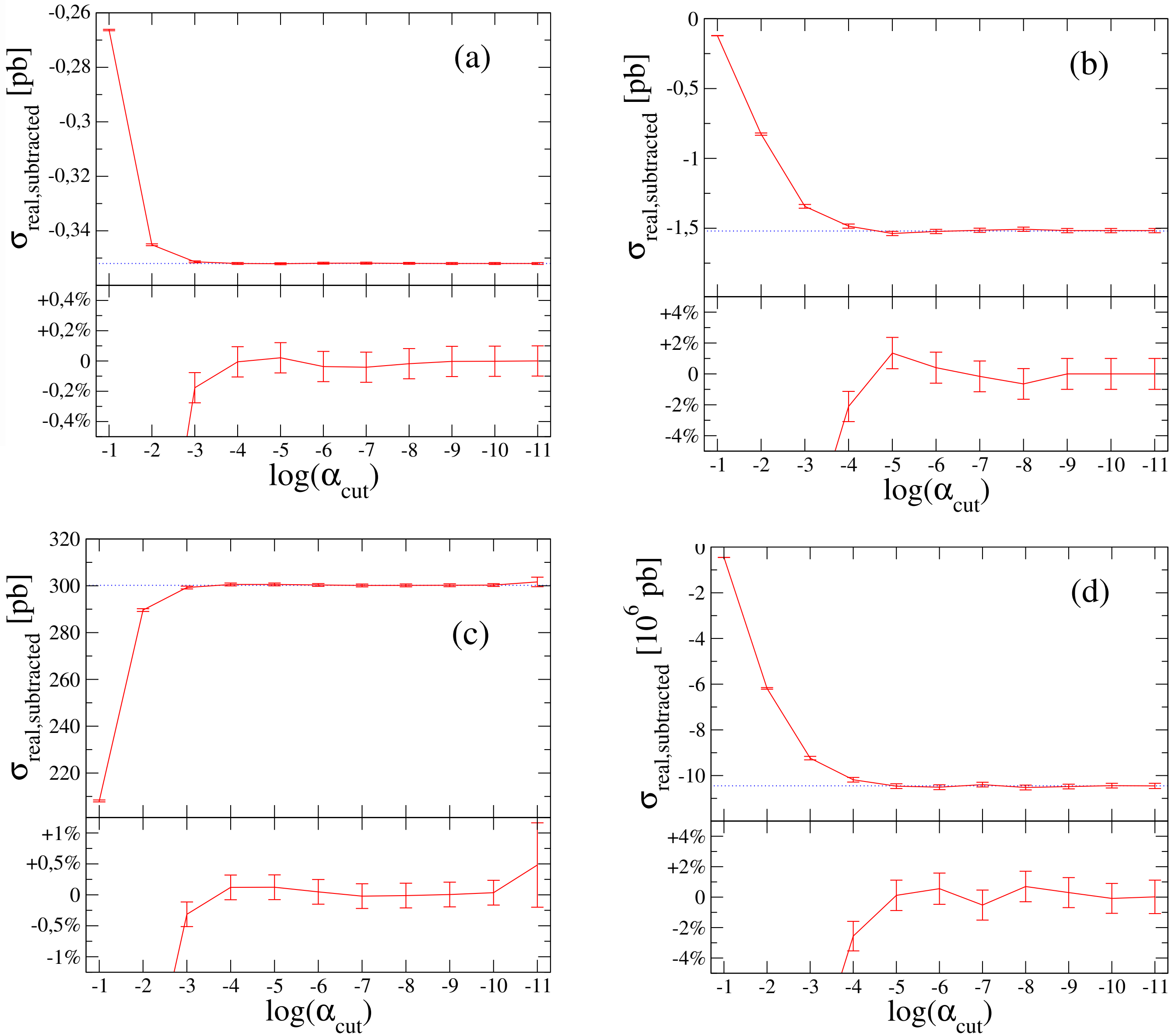
Dependence of the subtracted real correction XS on α_{cut}

$\alpha_{\text{min}} = min(a_{\text{dipole}}) < a_{cut}$

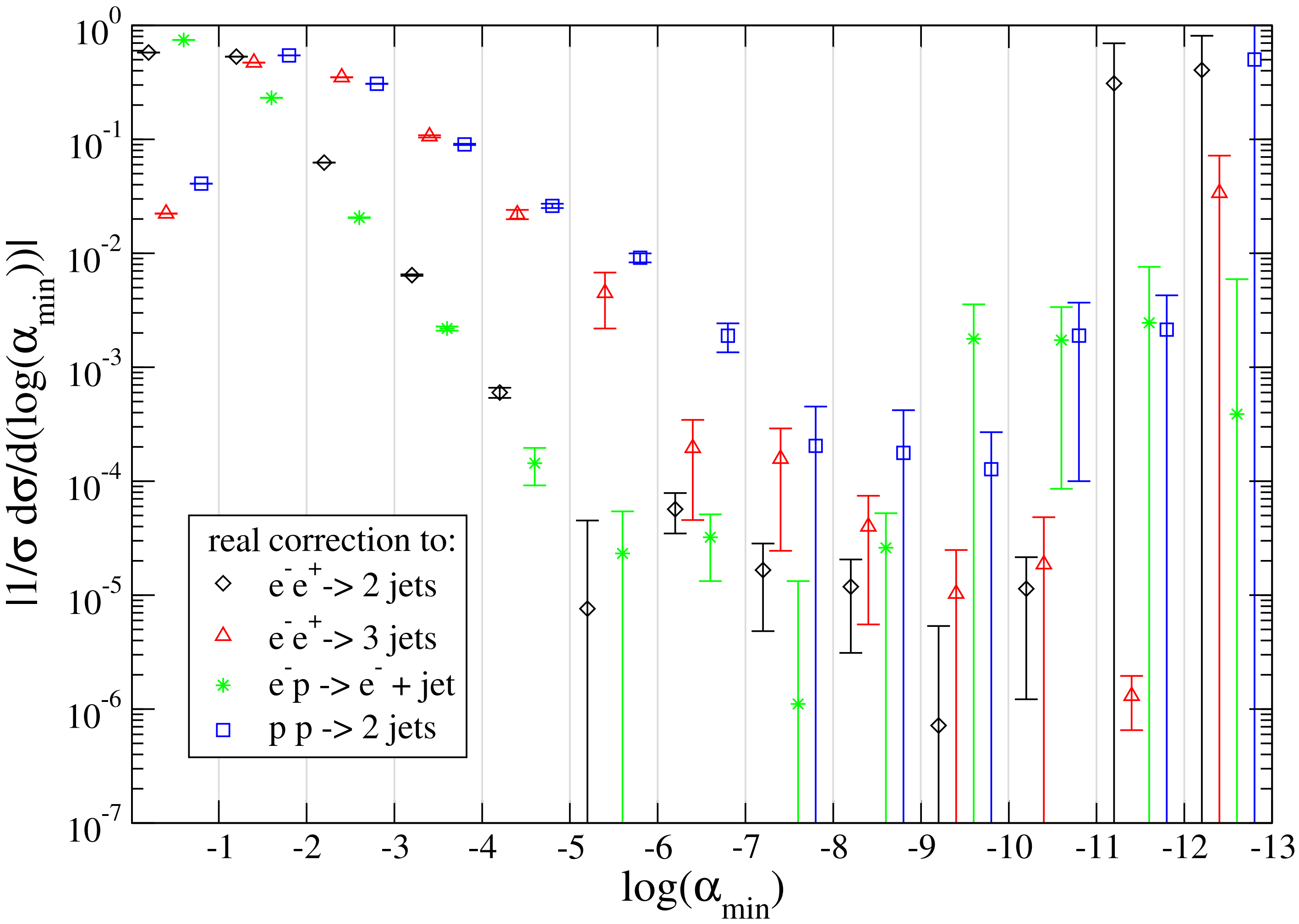
FF-dipole $\mathcal{D}_{ij,k}$:

$$a = y_{ij,k} = \frac{2p_i p_j}{(p_i+p_j+p_k)^2}$$

(a) $e^- e^+ \to 2jet$
(b) $e^- e^+ \to 3jet$
(c) $e^- p \to e^- + jet$
(d) $pp \to 2jet$



Tests & simple examples



Tests & simple examples

[Z. Nagy, 2003]

Modification of the subtraction terms:

restriction of the 1-parton phase-space of the dipole functions

$$d\sigma^{\mathrm{A}'}_{ij,k} = d\sigma^{\mathrm{A}}_{ij,k}\, \theta\left(\alpha - x\right)$$

e.g. FF-dipoles: $x = y_{ij,k} = \frac{2p_i p_j}{(p_i+p_j+p_k)^2}$

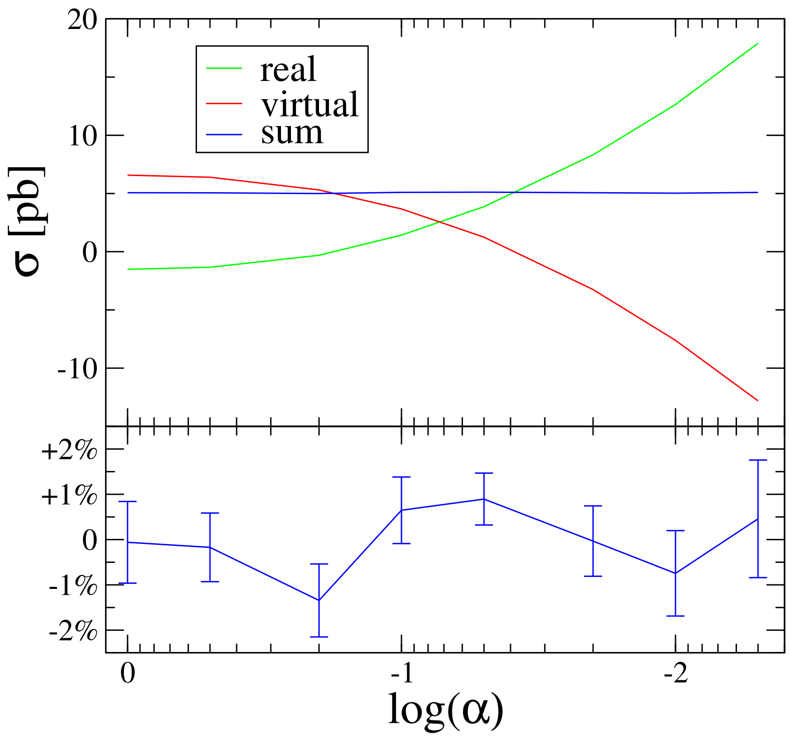
$\Rightarrow$ requires recalculation of $\int_1 d\sigma^{\mathrm{A}'}_{ij,k}$

- reduction of average number of dipole terms to be calculated
- improvement of the convergence of the numerical integration
- consistency check for the implementation

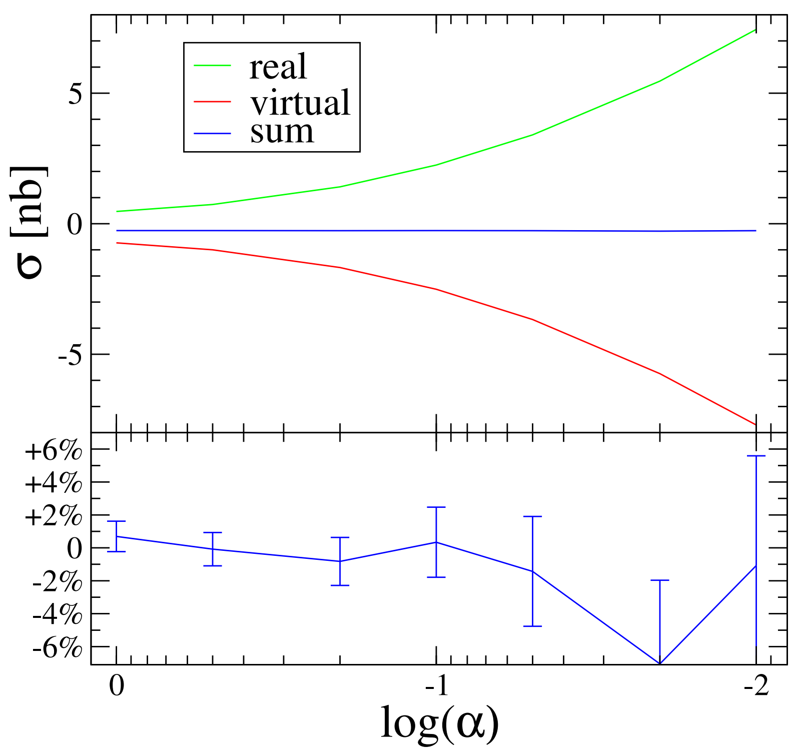
Tests & simple examples

Consistency checks with modified subtraction terms:

$e^- e^+ \to 3\text{jets}$

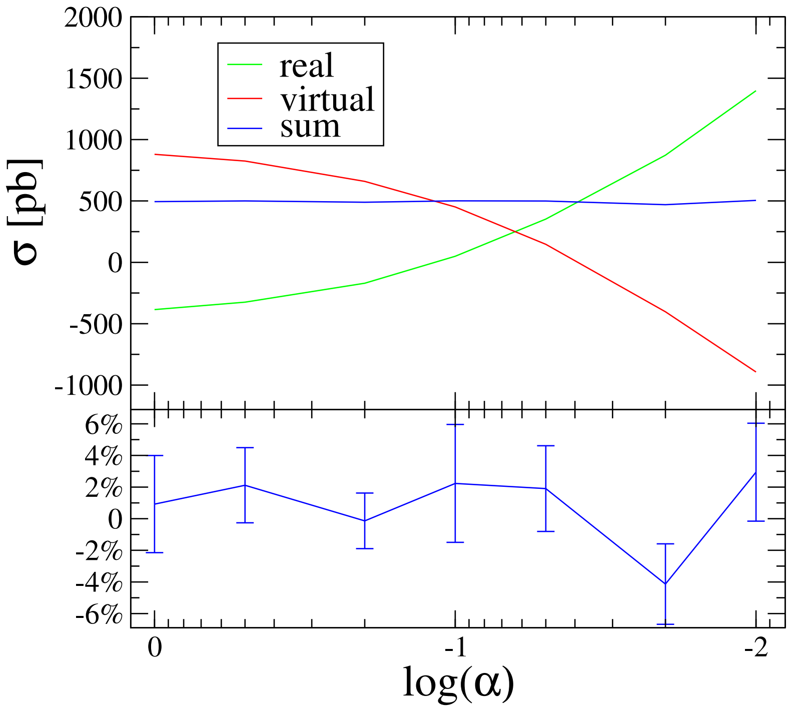


$e^- p \to e^- + \text{jet}$

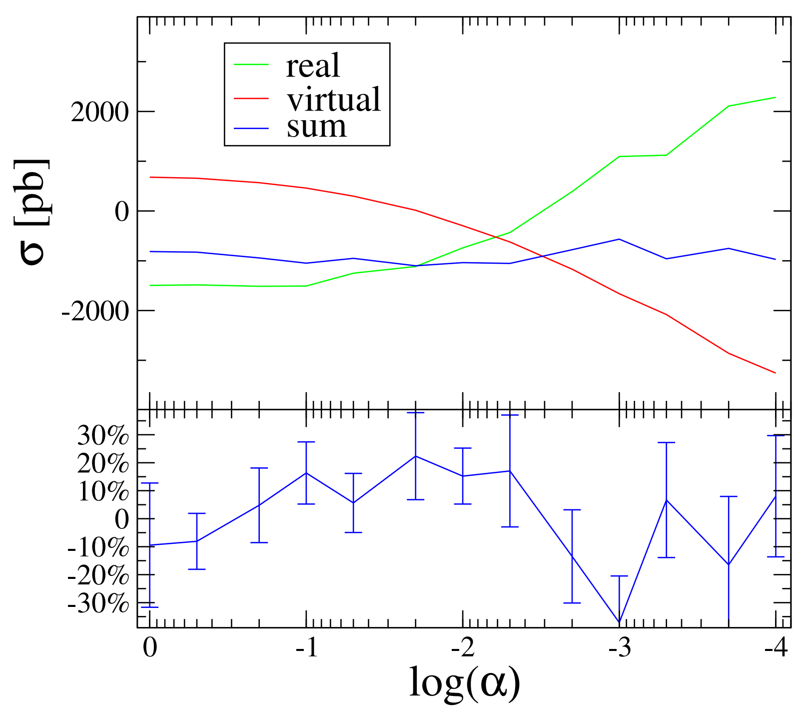


Consistency check with modified subtraction terms:

$pp \to W[\to ...] + 2\text{jets}$



$pp \to W[\to ...] + 3\text{jets}$



(includes 270 parton level processes and 5832 subtraction terms)

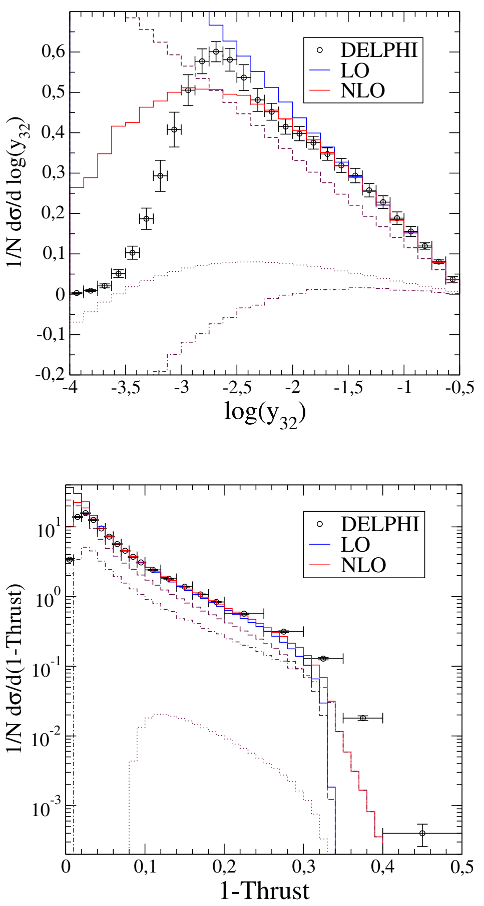
Tests & simple examples

Explicit checks with other implementations of the dipole subtraction:

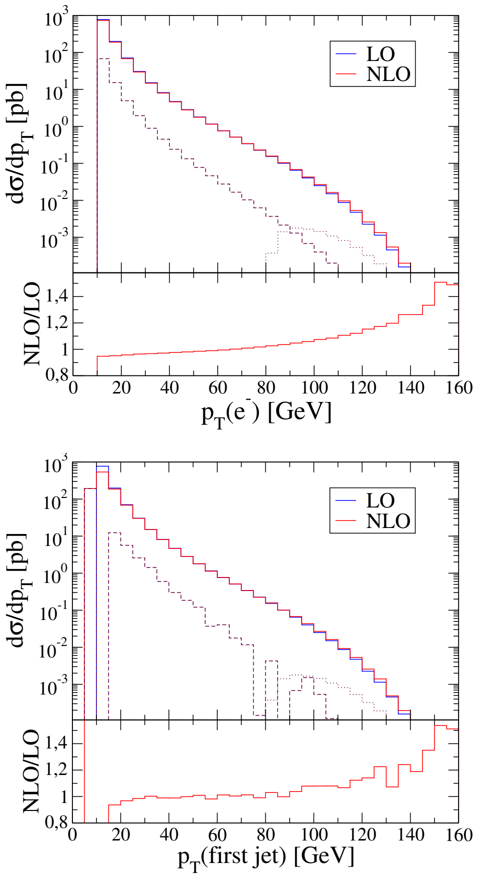
- term by term comparison with M. Seymour's code DISENT for $e^+e^- \to 2/3$ jets, $e^-p \to e^-1/2$ jets
- comparisons with integrated real subtracted corrections in MCFM ($pp \to W(+1\text{jet},)$, diboson production, ...)

Tests & simple examples

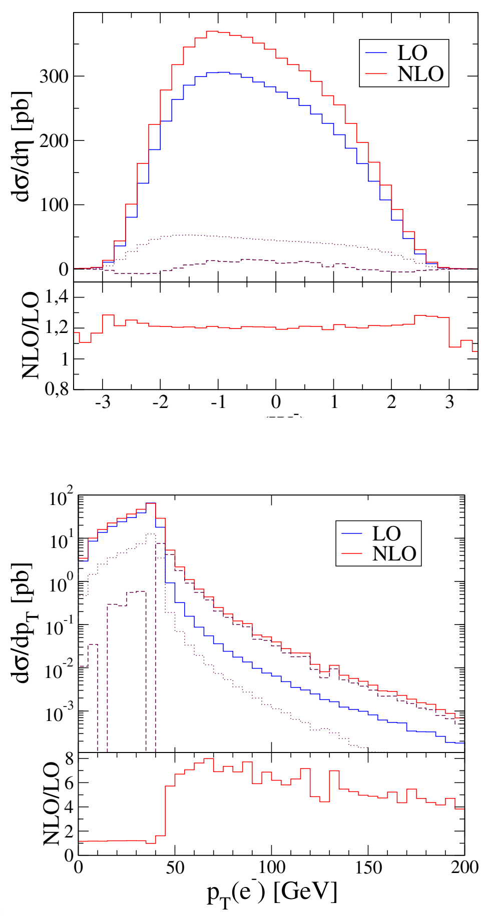
$e^- e^+ \to 3jets$



$e^- p \to e^- p + jet$



$p\bar{p} \to W^- \to e^- \bar{\nu}_e$



Summary and Outlook

- Real corrections and the dipole subtraction method are fully automated (some final work still to be done for massive partons)
- excellent numerical stability in all terms
- phase space integration methods are provided
- limits on particle multiplicities are comparable to limits of LO ME's
  ➔ interface to one-loop ME leads directly to a parton-level event generator

Future goal:

combination with automated one-loop calculation tools

towards a fully automated NLO-generator