Resonance Enhancement in Axion Tube Experiments

arXiv:1403.1576

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oscillating axion as DM

$$V(a) \approx \frac{1}{2} m_a^2 a^2$$

- Below T~1 GeV, axion develops a potential:
- Axion oscillates coherently with energy density: $\rho = \langle m_a^2 a^2 \rangle$
- halo made of axions: $\rho \le \rho_{halo} \sim 0.3\ GeV / cm^3$
- Maxwell distribution with small virial velocity: $v \simeq 10^{-3}$

detecting axion oscillation

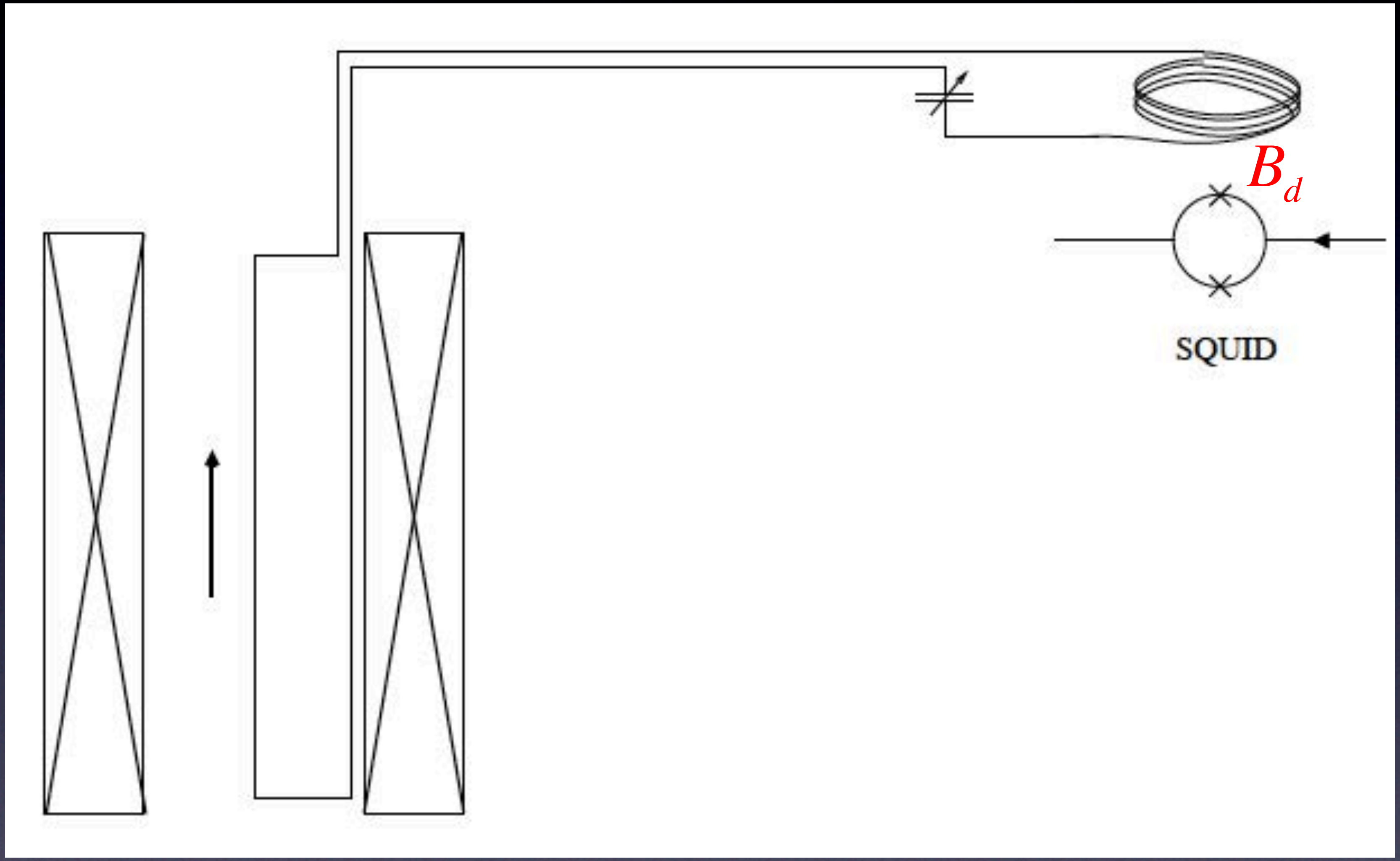
- axion-photon interaction $L = -ga\vec{E}\cdot\vec{B}$
- axion mass of μeV~meV : microwave rad. $10^{-6}\ \mathrm{eV} \le m_a \le 10^{-3}\ \mathrm{eV}$
- Sikivie haloscope : high-Q cavity + strong magnetic field
- oscillating electric displacement $\vec{D}(t) = -ga(t)\vec{B}_0$

(JH,Kim '91)

(Sikivie, Sullivan, Tanner '13)

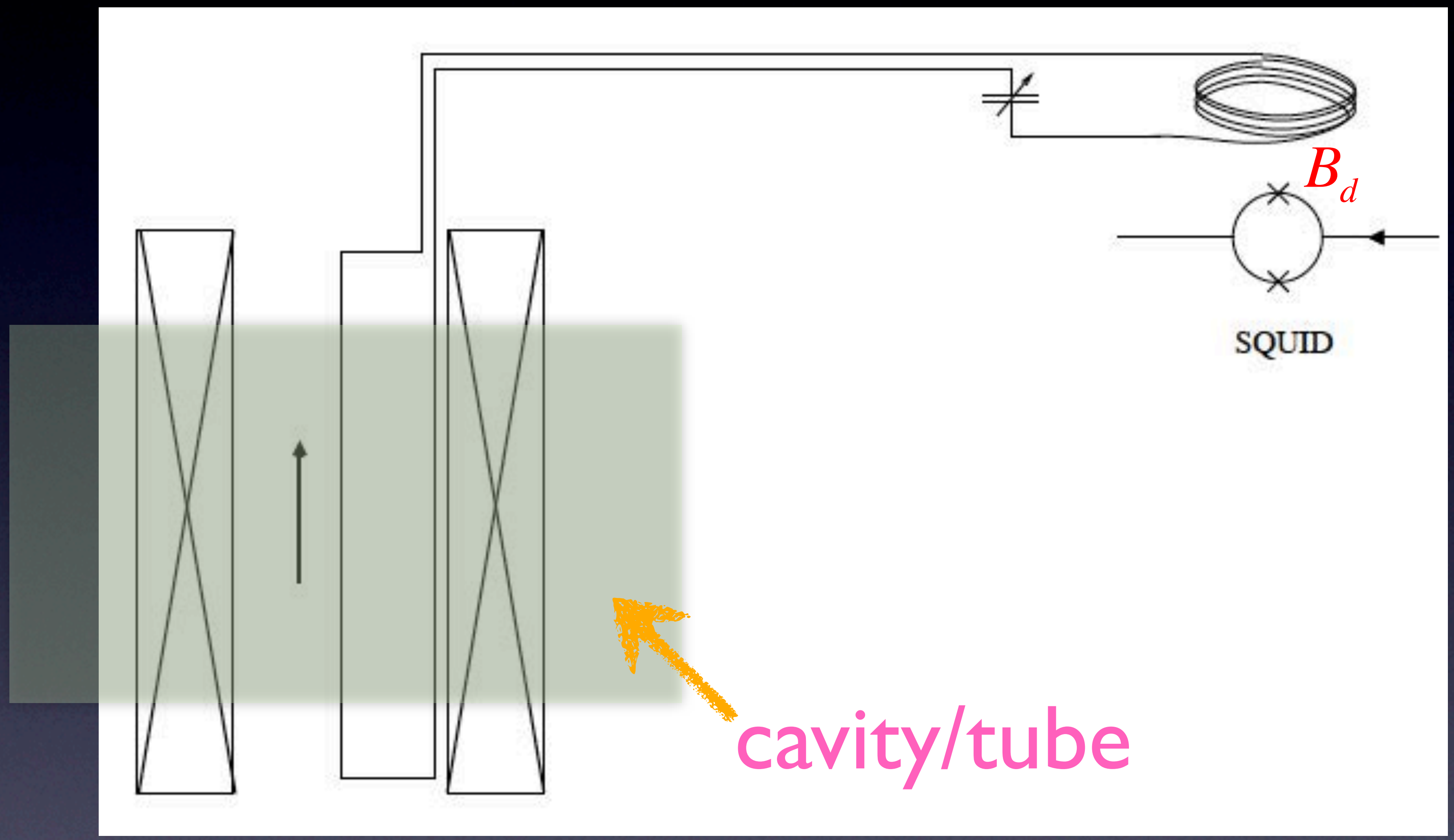
- Direct detection of oscillating EM field amplified by resonance in cavity/tube along with LC resonance looks promising.
- Explicit mode calculation on/off resonance is needed for optimal positioning of a pick-up loop.
- Liberate ourselves from conventional thinking of cavity volume=solenoidal volume.

- Sikivie-Sullivan-Tanner's proposal



- For B_0 of 10 T, $B_d \simeq 1 \times 10^{-15}\ T$

- our proposal



- For B_0 of 10 T, $B_d \simeq 1\times 10^{-15}\ Q_J\ f\ T$

filling factor

enhancement factor

mode calculation(TM)

- long round metal tube of radius R
- strong longitudinal external magnetic field $\vec{B}_0 = B_0 \hat{z}$
- I. tube volume = solenoid volume
- II. tube volume > solenoid volume

axion electromagnetism

$$L = -ga\vec{E}\cdot\vec{B} + \ldots \quad (g \equiv g_\gamma \frac{\alpha_{em}}{\pi f_a})$$

- axion-photon interaction

$$\vec{\nabla}\cdot\vec{E} = \rho + g\vec{\nabla}a\cdot\vec{B},$$

$$\vec{\nabla}\times\vec{B} - \partial_t\vec{E} = \vec{j} - g\vec{B}\partial_t a - g\vec{\nabla}a\times\vec{E},$$

$$\vec{\nabla}\cdot\vec{B} = 0,$$

$$\vec{\nabla}\times\vec{E} + \partial_t\vec{B} = 0,$$

$$(\partial_t^2 - \vec{\nabla}^2)a = -m_a^2 a - g\vec{E}\cdot\vec{B} + \rho_a.$$

halo axions

(Krauss et al ’85)

- homogeneous axion field

$$a(t) = \sqrt{T}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\mathcal{A}(\omega)e^{-i\omega t}$$

$$\mathcal{A}(\omega) = \frac{1}{\sqrt{T}}\int_{-T/2}^{T/2} dt\ a(t)e^{i\omega t}$$

- time average

$$\langle a^2(t)\rangle \equiv \frac{1}{T}\int_{-T/2}^{T/2} dt\ a^2(t) = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}|\mathcal{A}(\omega)|^2$$

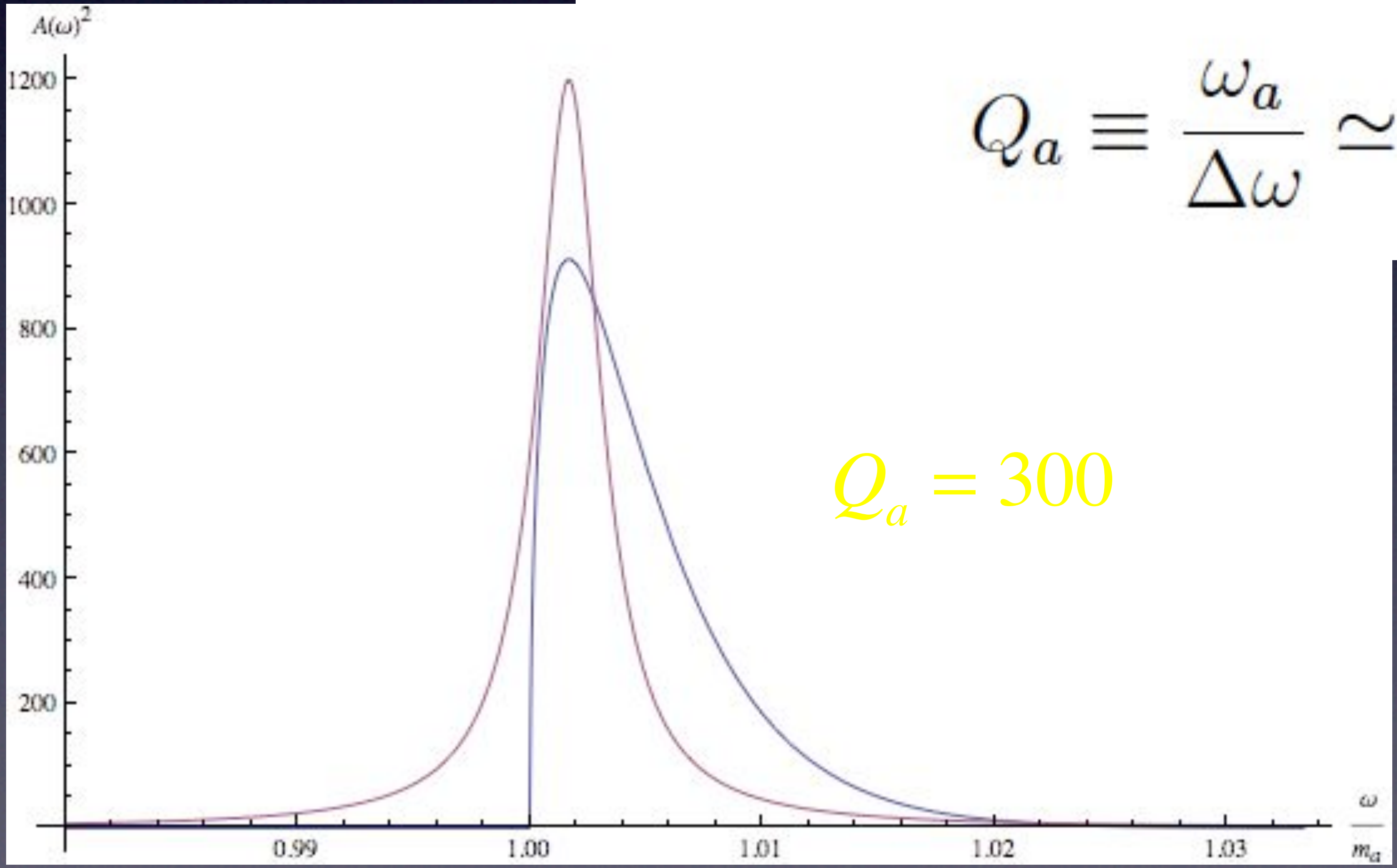
- Maxwell distribution → power spectrum

$$|\mathcal{A}(\omega)|^2 = \frac{4\sqrt{\pi}\rho_{halo}}{m_a^2(k_BT)^{3/2}}e^{-(\omega-m_a)/k_BT}\sqrt{\omega-m_a}$$

- power spectrum mimicked by Breit-Wigner function (axionic Q-factor)

$$|\mathcal{A}(\omega)|^2 = \frac{\omega_a \rho_{halo}}{m_a^2 Q_a} \frac{1}{(\omega - \omega_a)^2 + (\omega_a/2Q_a)^2}$$

$$Q_a \equiv \frac{\omega_a}{\Delta\omega} \simeq \frac{m_a}{m_a \langle v^2 \rangle /3} \sim 3 \times 10^6$$



$$\vec{B}_0 \otimes a(t) \Rightarrow \vec{E}_1 \oplus \vec{B}_1$$

perturbation up to 1st-order

$$\vec{\nabla} \cdot \vec{E}_1 = 0,$$

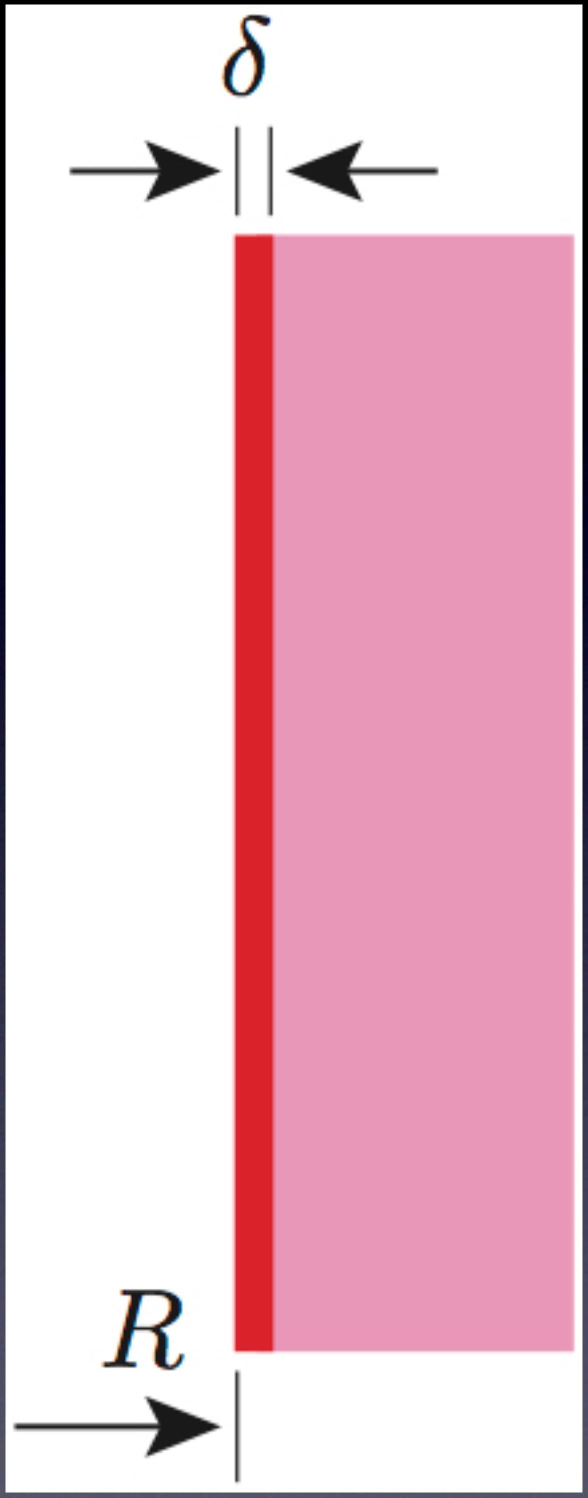
$$\vec{\nabla} \times \vec{B}_1 - \partial_t \vec{E}_1 = \vec{j}_1 - g\vec{B}_0 \partial_t a,$$

$$\vec{\nabla} \cdot \vec{B}_1 = 0,$$

$$\vec{\nabla} \times \vec{E}_1 + \partial_t \vec{B}_1 = 0$$

- current in conductor (conductivity σ)

$$\vec{j}_1 = \sigma \vec{E}_1 \theta(r - R) \qquad \sigma \gg m_a$$



$$\vec{E}_1 = E_z(r,t)\hat{z} \qquad \vec{B}_1 = B_\theta(r,t)\hat{\theta}$$

$$B_\theta(r,t) = \sqrt{T}\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} B(r,\omega) e^{-i\omega t}$$

$$E_z(r,t) = \sqrt{T}\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} E(r,\omega) e^{-i\omega t}$$

tube

$$B' + B/r + i\omega E = i\omega g B_0 \mathcal{A}(\omega)$$

$$-E' - i\omega B = 0$$

(solutions)

$$E(r,\omega) = g B_0 \mathcal{A}(\omega) + C J_0(\omega r),$$

$$B(r,\omega) = -iC J_1(\omega r),$$

skin depth

$$\delta \equiv \sqrt{\frac{2}{\sigma\omega}}$$

conductor

$$B' + B/r + i\omega E = \sigma E$$

$$-E' - i\omega B = 0$$

$$B^{cond}(r,\omega) = B^{cav}(R,\omega) e^{-(1-i)(r-R)/\delta}$$

$$= -iC J_1(\omega R) e^{-(1-i)(r-R)/\delta}$$

$$E^{cond}(r,\omega) = (1/\sigma) B'^{cond}(r,\omega)$$

$$= i(1-i)(C/\sigma\delta) J_1(\omega R) e^{-(1-i)(r-R)/\delta}$$

- boundary conditions (continuity of fields) determines the coefficient C

$$C = -\frac{gB_0\mathcal{A}(\omega)}{J_0(\omega R) - ie^{-i\frac{\pi}{4}}\sqrt{\frac{\omega}{\sigma}}J_1(\omega R)} \equiv -gB_0\mathcal{A}(\omega)\ Q_J(\omega)e^{-i\phi_J(\omega)}$$

- resonance enhancement factor Q_J

$$Q_J \equiv \frac{1}{\sqrt{J_0^2(\omega R) - \sqrt{2\omega/\sigma}J_0(\omega R)J_1(\omega R) + (\omega/\sigma)J_1^2(\omega R)}}$$

$$\phi_J \equiv -\tan^{-1}\frac{\sqrt{\omega/2\sigma}J_1(\omega R)}{J_0(\omega R) - \sqrt{\omega/2\sigma}J_1(\omega R)}$$

- on resonances :

$\omega R = \chi_{0l}$

zeroes of J_0

$$Q_J = \sqrt{\frac{\sigma}{\omega}}\frac{1}{|J_1(\chi_{0l})|}$$

$$\phi_J = l\pi + \frac{\pi}{4}$$

- near a resonance $$Q_J(\omega)e^{-i\phi_J(\omega)} \simeq -\frac{1}{R\ J_1(\chi_{0l})} \cdot \frac{1}{(\omega-\omega_l)+i(\omega_l/2Q_l)}$$
- quality factor for j-th res. $$\omega_l \equiv \frac{\chi_{0l}}{R}, \quad Q_l \equiv (\chi_{0l}\sigma R/2)^{1/2} = \frac{R}{\delta}$$
- fields inside tube

$$B_\theta(r,t) = \sqrt{T}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi} igB_0\mathcal{A}(\omega)J_1(\omega r)Q_J(\omega)e^{-i\phi_J(\omega)}e^{-i\omega t}$$
$$E_z(r,t) = \sqrt{T}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi} gB_0\mathcal{A}(\omega)(1-J_0(\omega r)Q_J(\omega)e^{-i\phi_J(\omega)})e^{-i\omega t}$$

- fields within conductor

$$B_\theta(r,t) = \sqrt{T}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi} igB_0\mathcal{A}(\omega)J_1(\omega R)Q_J(\omega)e^{-i\phi_J(\omega)}e^{-i\omega t}e^{i(r-R)/\delta}e^{-(r-R)/\delta}$$
$$E_z(r,t) = \sqrt{T}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{-i(1-i)}{\sigma\delta} gB_0\mathcal{A}(\omega)J_1(\omega R)Q_J(\omega)e^{-i\phi_J(\omega)}e^{-i\omega t}e^{i(r-R)/\delta}e^{-(r-R)/\delta}$$

energy inside tube

$$U = \int_V d^3r \, \langle (\vec{E}_1^2 + \vec{B}_1^2)/2 \rangle$$

$$\begin{aligned} U &\simeq (gB_0)^2 \frac{V}{R^2} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} |\mathcal{A}(\omega)|^2 \frac{1}{(\omega-\omega_l)^2 + (\omega_l/2Q_l)^2} \\ &\simeq (gB_0)^2 \frac{V}{R^2} \frac{\omega_a \rho_{halo}}{m_a^2 Q_a} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{(\omega-\omega_a)^2 + (\omega_a/2Q_a)^2} \cdot \frac{1}{(\omega-\omega_l)^2 + (\omega_l/2Q_l)^2} \end{aligned}$$

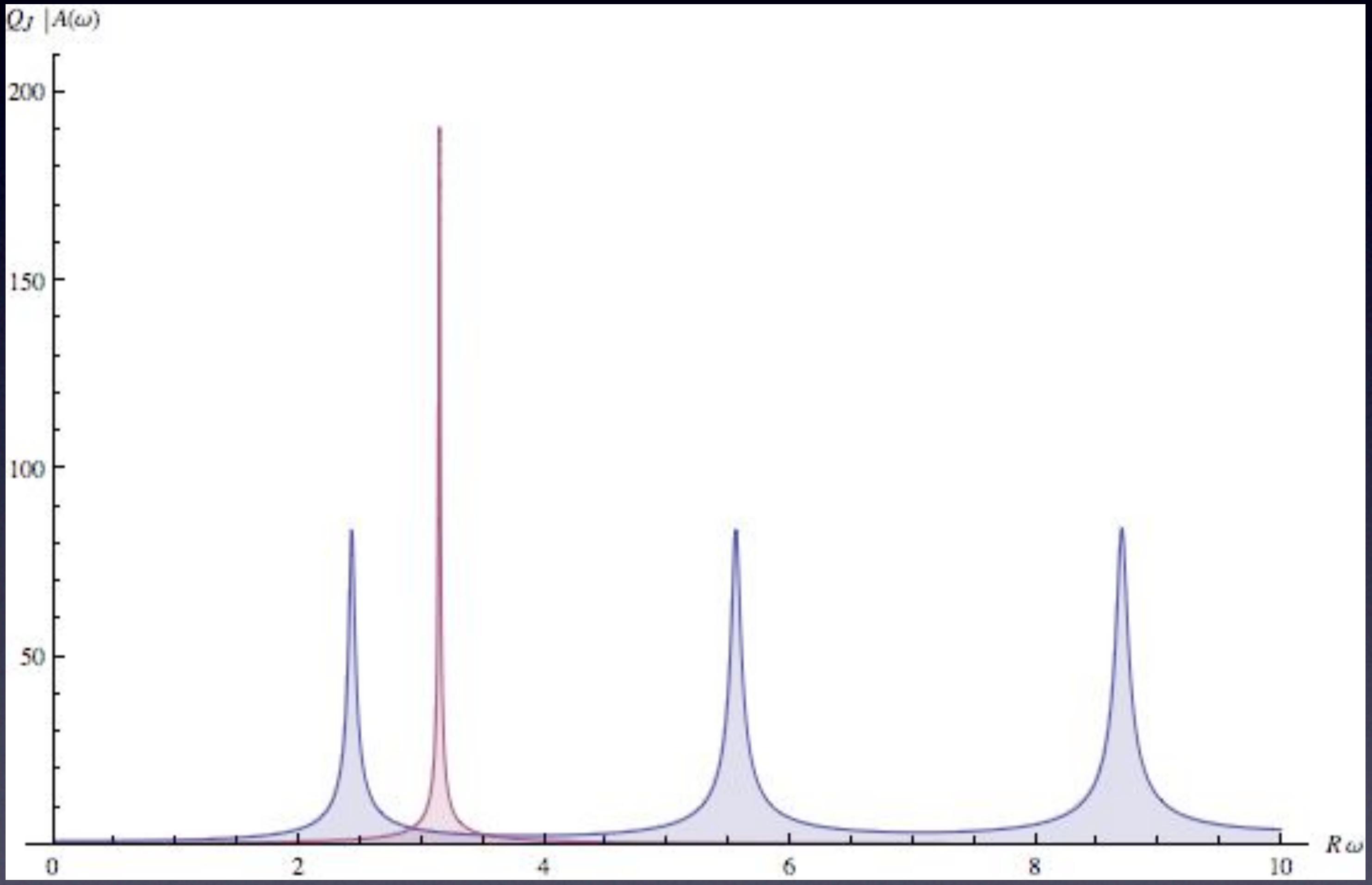
- off resonance

$$U \simeq (gB_0)^2 \frac{V}{R^2} \frac{\omega_a \rho_{halo}}{m_a^2 Q_a} \frac{1}{(\omega_a - \omega_l)^2} \left(\frac{Q_a}{\omega_a} + \frac{Q_l}{\omega_l} \right)$$

- on resonance

$$U \simeq (gB_0)^2 V \frac{\omega_a \rho_{halo}}{(m_a R)^2 Q_a} \begin{cases} \frac{4 Q_a Q_l^2}{\omega_a^3} & (Q_a \gg Q_l), \\ \frac{4 Q_l Q_a^2}{\omega_a^3} & (Q_a \ll Q_l). \end{cases}$$

$$Q_a = 300,\ Q_l = 100,\ \omega_a \simeq 1.3\,\omega_l$$



power loss

$$P_{loss} = \int \langle \vec{j} \cdot \vec{E}^c \rangle d^3 r$$
$$\simeq (gB_0)^2 \frac{S}{R^2} \frac{\omega_a \rho_{halo}}{\sqrt{2\sigma} m_a^2 Q_a} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sqrt{\omega} \frac{1}{(\omega - \omega_a)^2 + (\omega_a/2Q_a)^2} \cdot \frac{1}{(\omega - \omega_l)^2 + (\omega_l/2Q_l)^2}$$

$$Q_{tube} = \omega_a \frac{\text{Stored energy}}{\text{Power loss}} = \omega_a \frac{U}{P_{loss}} = \frac{2V}{S\delta} = \frac{R}{\delta} = Q_l$$

$$P = \frac{\omega_a U}{Q_l} = (gB_0)^2 \, V \frac{\omega_a^2 \rho_{halo}}{(m_a R)^2 Q_a Q_l} \cdot \begin{cases} \frac{4 Q_a Q_l^2}{\omega_a^3} & (Q_l \ll Q_a) \\ \frac{4 Q_a^2 Q_l}{\omega_a^3} & (Q_a \ll Q_l) \end{cases}$$

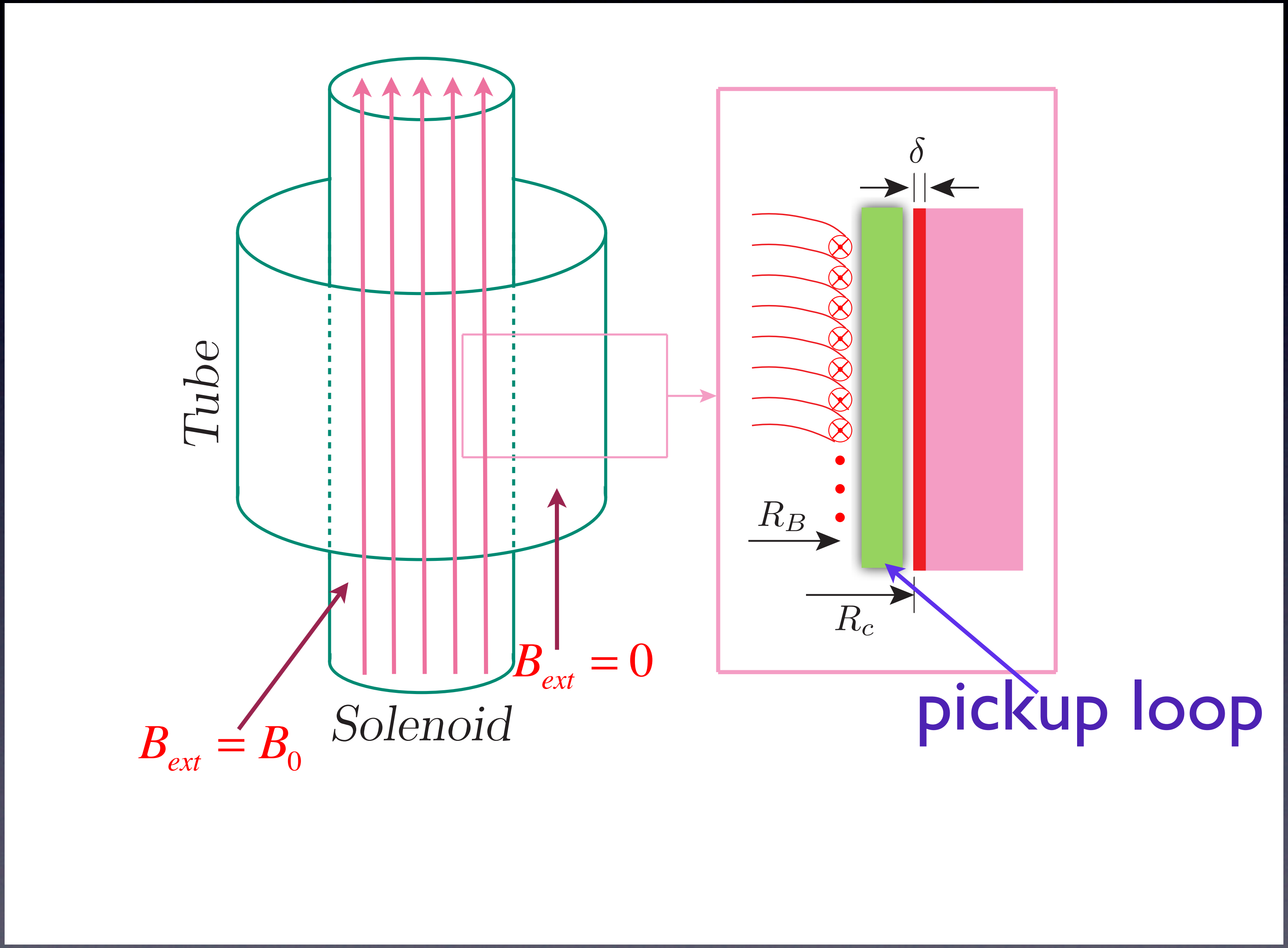
$$P = \frac{(gB_0)^2 \rho_{halo} V}{m_a} \cdot \frac{4}{\chi_{0l}^2} \min(Q_l, Q_a)$$

Sikivie ('83, '85)

redesigning cavity/tube

- separating solenoid from cavity
- cavity mode excited by partially applied magnetic field
- pick-up loop placed exterior to strong magnetic field domain and connected to a circuit coupled with a magnetometer (SQUID or SERF)

solenoid within cavity



TM mode

$$(0 \le r \le R_B)\begin{cases} E(r,\omega) = gB_0\mathcal{A}(\omega) + CJ_0(\omega r), \\ B(r,\omega) = -iCJ_1(\omega r), \end{cases}$$

$$(R_B \le r \le R_c)\begin{cases} E(r,\omega) = FJ_0(\omega r) + GN_0(\omega r), \\ B(r,\omega) = -i(FJ_1(\omega r) + GN_1(\omega r)), \end{cases}$$

$$(r \ge R_c)\begin{cases} E(r,\omega) = \frac{i(1-i)}{\sigma\delta}(FJ_1(\omega R_c) + GN_1(\omega R_c))e^{-(1-i)(r-R_c)/\delta} \\ B(r,\omega) = -i(FJ_1(\omega R_c) + GN_1(\omega R_c))e^{-(1-i)(r-R_c)/\delta}. \end{cases}$$

$$C = gB_0\mathcal{A}(\omega)\frac{\pi x_B}{2}\left(N_1(x_B) - J_1(x_B)\frac{N_0(x_c) - \frac{i(1-i)}{\sigma\delta}N_1(x_c)}{J_0(x_c) - \frac{i(1-i)}{\sigma\delta}J_1(x_c)}\right),$$

$$F = C - gB_0\mathcal{A}(\omega)\frac{\pi x_B}{2}N_1(x_B),\ G = gB_0\mathcal{A}(\omega)\frac{\pi x_B}{2}J_1(x_B),$$

TM mode on resonance

$$(0 \le r \le R_B) \begin{cases} E(r,\omega) = gB_0\mathcal{A}(\omega) + CJ_0(\omega r), \\ B(r,\omega) = -iCJ_1(\omega r), \end{cases}$$

$$(R_B \le r \le R_c) \begin{cases} E(r,\omega) = \cancel{F}C J_0(\omega r) + GN_0(\omega r), \\ B(r,\omega) = -i(\cancel{F}C J_1(\omega r) + GN_1(\omega r)), \end{cases}$$

$$(r \ge R_c) \begin{cases} E(r,\omega) = \frac{i(1-i)}{\sigma\delta}(\cancel{F}C J_1(\omega R_c) + GN_1(\omega R_c))e^{-(1-i)(r-R_c)/\delta} \\ B(r,\omega) = -i(\cancel{F}C J_1(\omega R_c) + GN_1(\omega R_c))e^{-(1-i)(r-R_c)/\delta}. \end{cases}$$

$$C = gB_0\mathcal{A}(\omega)\frac{\pi x_B}{2}\left(\cancel{N_1(x_B)} - J_1(x_B)\frac{N_0(x_c) - \frac{i(1-i)}{\sigma\delta}\cancel{N_1(x_c)}}{\cancel{J_0(x_c)} - \frac{i(1-i)}{\sigma\delta}J_1(x_c)}\right),$$

$$F = C - gB_0\mathcal{A}(\omega)\frac{\pi x_B}{2}N_1(x_B), \; G = gB_0\mathcal{A}(\omega)\frac{\pi x_B}{2}J_1(x_B),$$

- On resonances C dominates over all other coefficients and the mode becomes that for a fully filled tube with a reduced amplitude:

$$E(r,\omega) \simeq f \; E_{filled}(r,\omega)$$

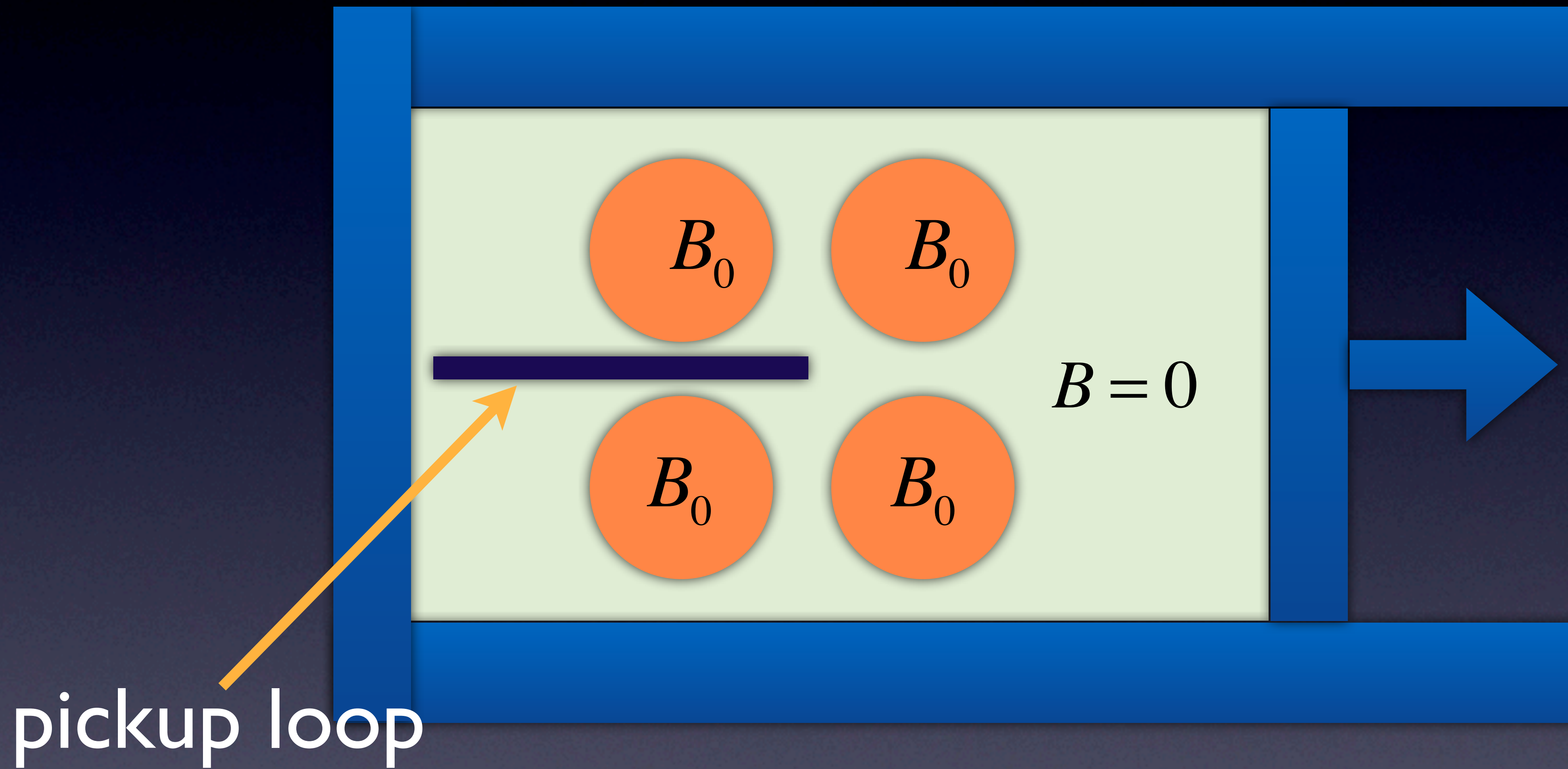
$$B(r,\omega) \simeq f \; B_{filled}(r,\omega)$$

- `filling factor`

$$f \equiv \frac{R_B J_1(\chi_{0l} R_B / R_c)}{R_c J_1(\chi_{0l})} \qquad\qquad f \simeq \frac{\chi_{0l}}{2J_1(\chi_{0l})} \frac{R_B^2}{R_c^2} \quad (R_B \ll R_c)$$

- Even for more complicated shape of magnetic field terrains, the following seems to be true: On resonances, EM fields would essentially be simple cavity modes with reduced amplitudes. (warning: It’s a conjecture yet!)

fail-proof cavity/tube?



quick & rough scan with low-Q tube

+

slow & thorough scan with high-Q tube

summary

- made explicit calculation of tube mode excited by oscillating axion + magnetic field
- cavity + superconducting LC circuit → potentially huge enhancement of signal
- discussed a way to redesign cavity exps