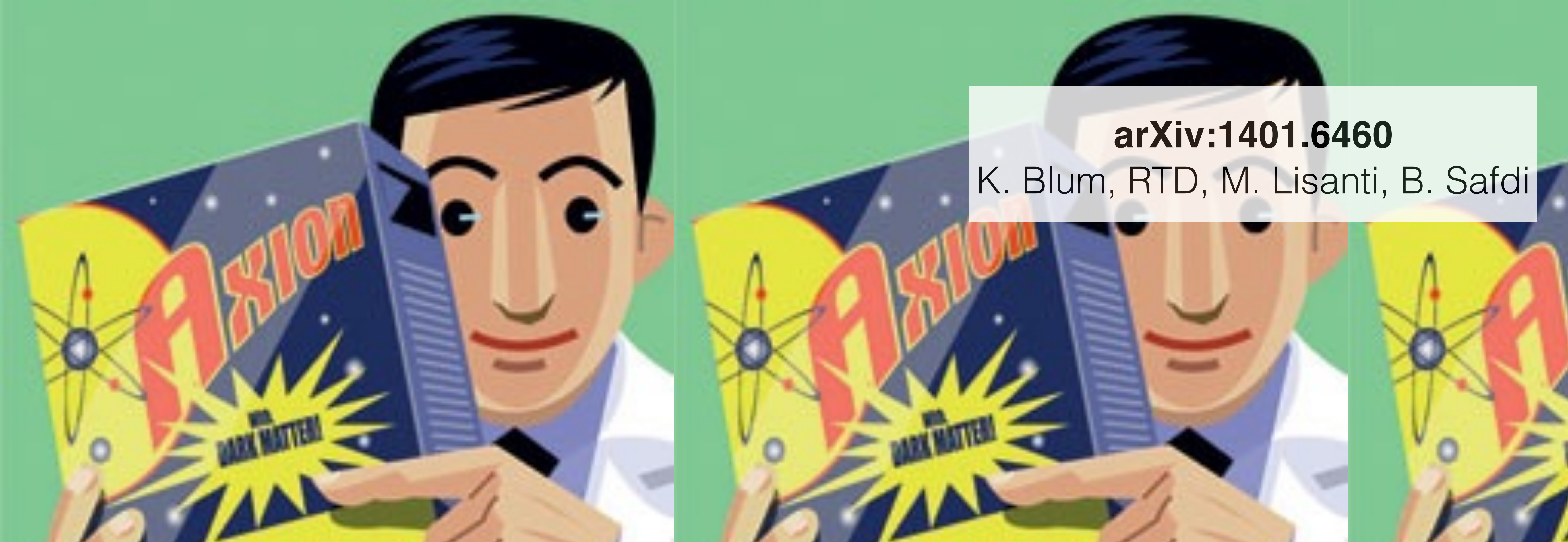


arXiv:1401.6460

K. Blum, RTD, M. Lisanti, B. Safdi

CONSTRAINTS ON AXION DARK MATTER FROM BIG BANG NUCLEOSYNTHESIS

Raffaele Tito D'Agnolo
IAS Princeton
5/14/2014

OUTLINE

- THE BASIC BUILDING BLOCKS
- THE THREE GRACES OF BBN
- THE TUNING IN OUR WAYS
- CONCLUSION AND FUTURE PROSPECTS

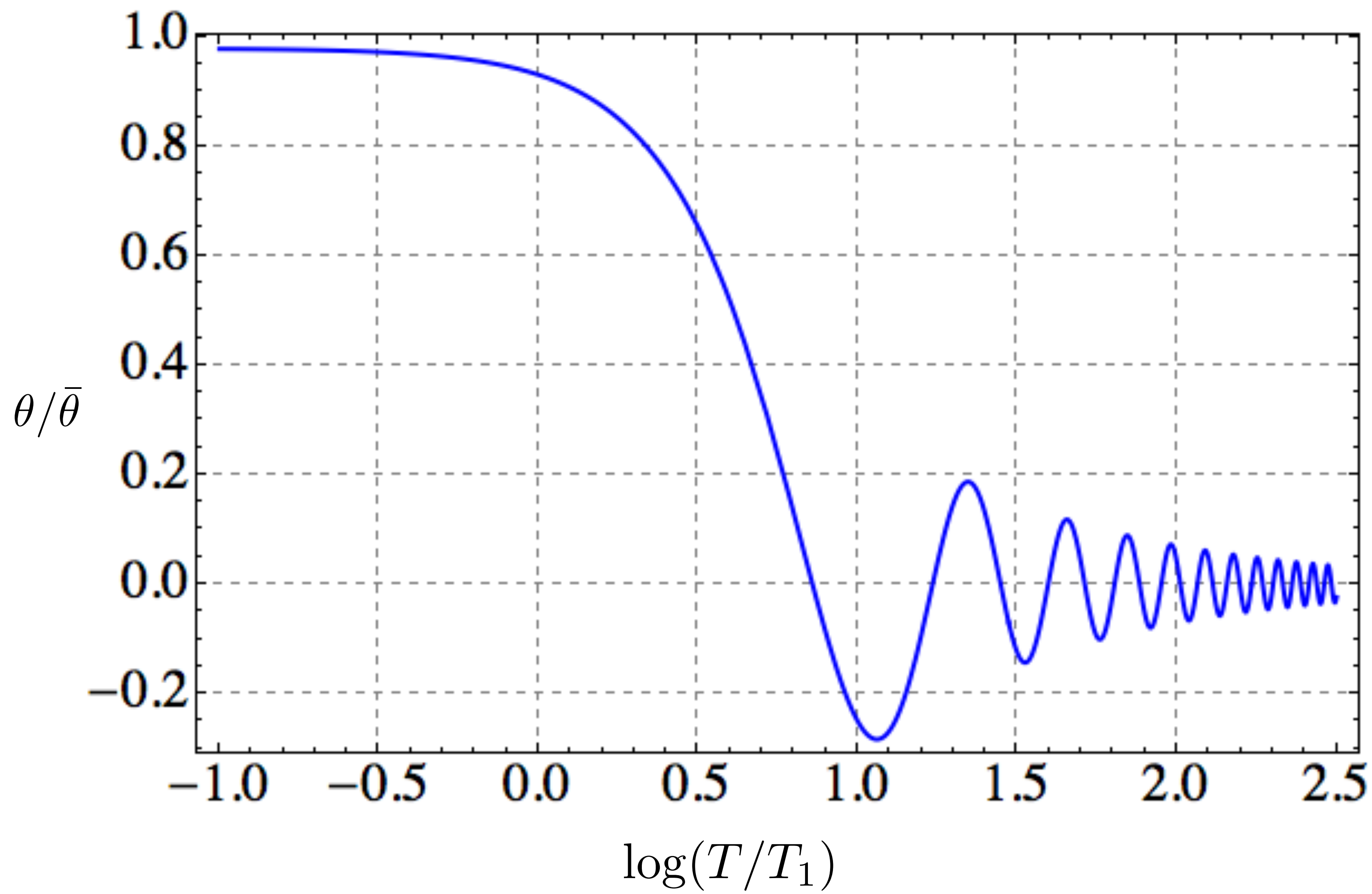
THE BASIC BUILDING BLOCKS

AXION POTENTIAL

$$\frac{a}{f_a}\frac{g_s^2}{32\pi}G_{a\mu\nu}\tilde{G}^{a\mu\nu} \longrightarrow V(a)=\frac{1}{2}m_a^2\left(a+\langle a\rangle\right)^2+\ldots$$

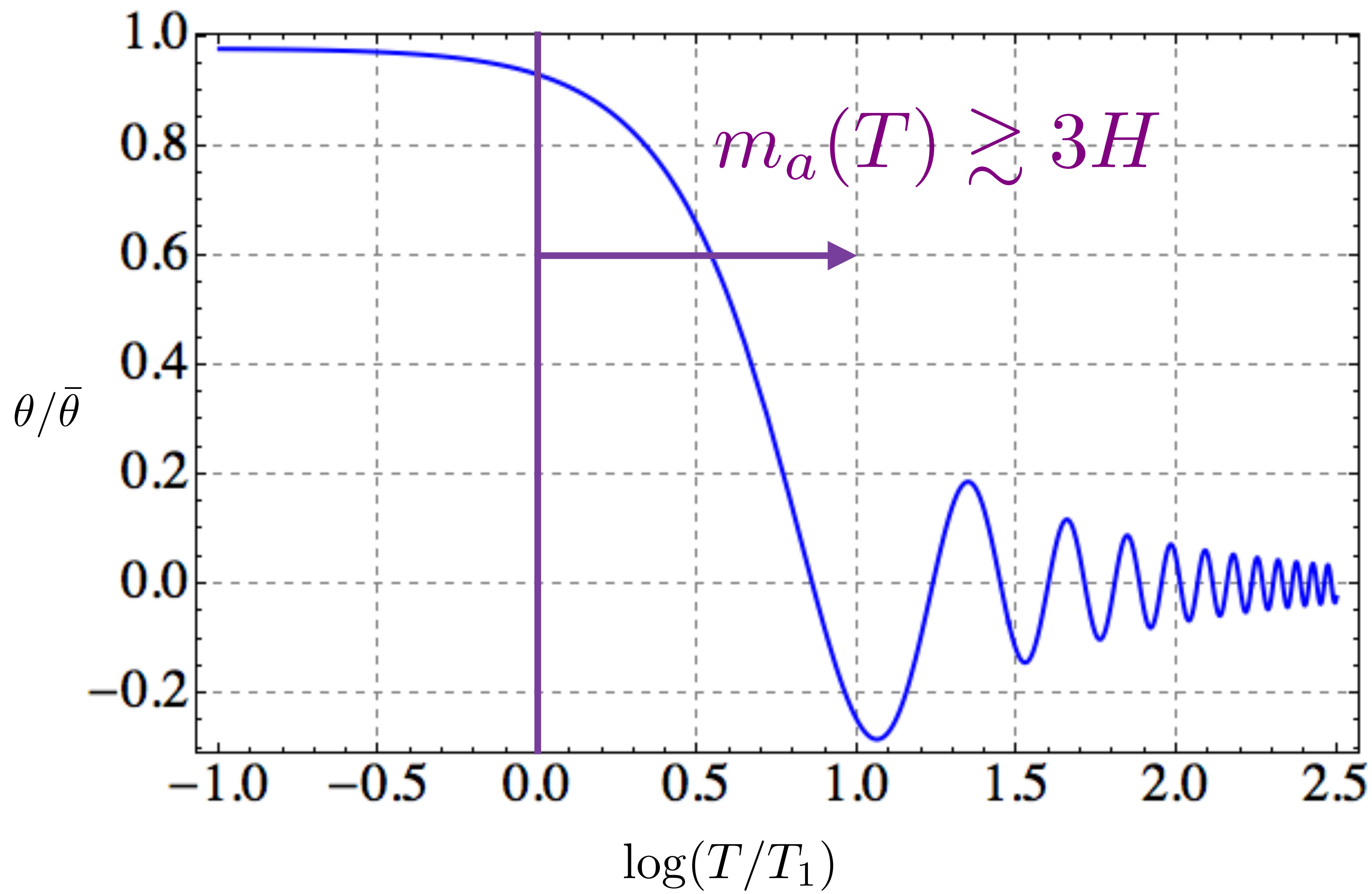
$$m_a=\frac{m_\pi f_\pi\sqrt{m_u m_d}}{(m_u+m_d)f_a}$$

MISALIGNMENT PRODUCTION



$$\ddot{\theta} + 3H\dot{\theta} + m_a^2(T)\theta = 0$$

MISALIGNMENT PRODUCTION



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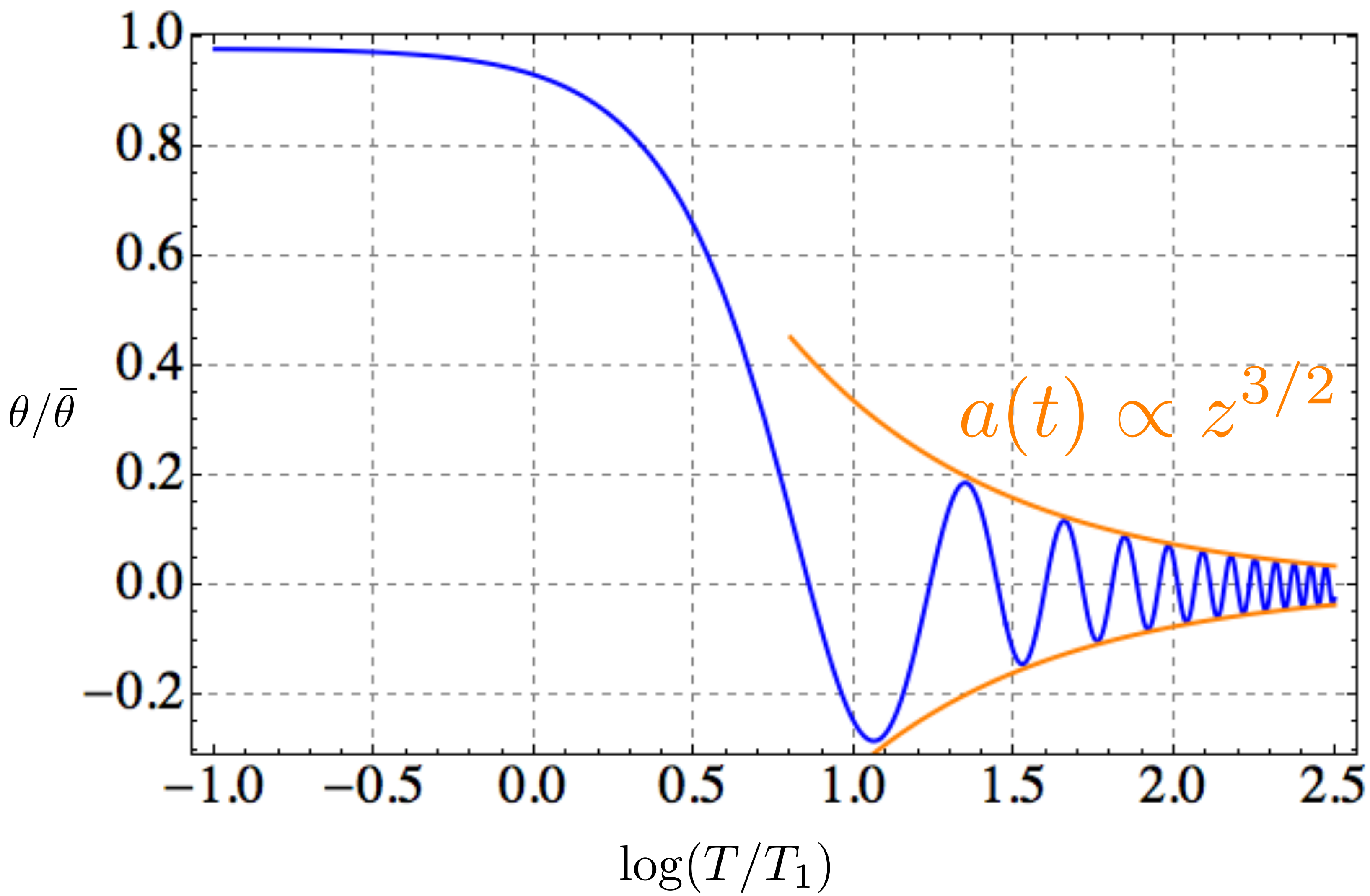


$$a(t) = f_a \theta(t) = \frac{\sqrt{2\rho_{DM}}}{m_a} \cos(m_a t)$$

THE SIZE OF THE EFFECT

$$g_a = \frac{a}{f_a} \sim \frac{\sqrt{\rho_{DM}}}{f_a m_a} \approx 10^{-22} \left(\frac{10^{-4}\ \mathrm{eV}}{m_a}\right) \left(\frac{10^{11}\ \mathrm{GeV}}{f_a}\right)$$

- HIGH MATTER DENSITIES (SN1987A)
- HIGH PRECISION + CP-odd EFFECT (EDM measurements)
- RESONANCE + LONG INTEGRATION TIME
- GOING BACK IN TIME

THE THREE GRACES OF BBN

THE THREE GRACES OF BBN

1. $z \approx 10^{10}$

2.

3.

THE THREE GRACES OF BBN

1. $z \approx 10^{10}$

2. $\delta Y_p / Y_p \approx 10\% \text{ at } 3\sigma$

3.

THE THREE GRACES OF BBN

1. $z \approx 10^{10}$

2. $\delta Y_p / Y_p \approx 10\%$ at 3σ

3. $\left(\frac{n}{p}\right)_F = e^{-(m_n - m_p)/T_F}, \quad m_n - m_p = \Delta m_0 + \delta m(a(T))$

$$\delta m \approx (0.37 \text{ MeV}) \left(\frac{a}{f_a}\right)^2$$

4He ABUNDANCE

$$\delta m \approx (0.37 \text{ MeV}) \left(\frac{a}{f_a}\right)^2$$

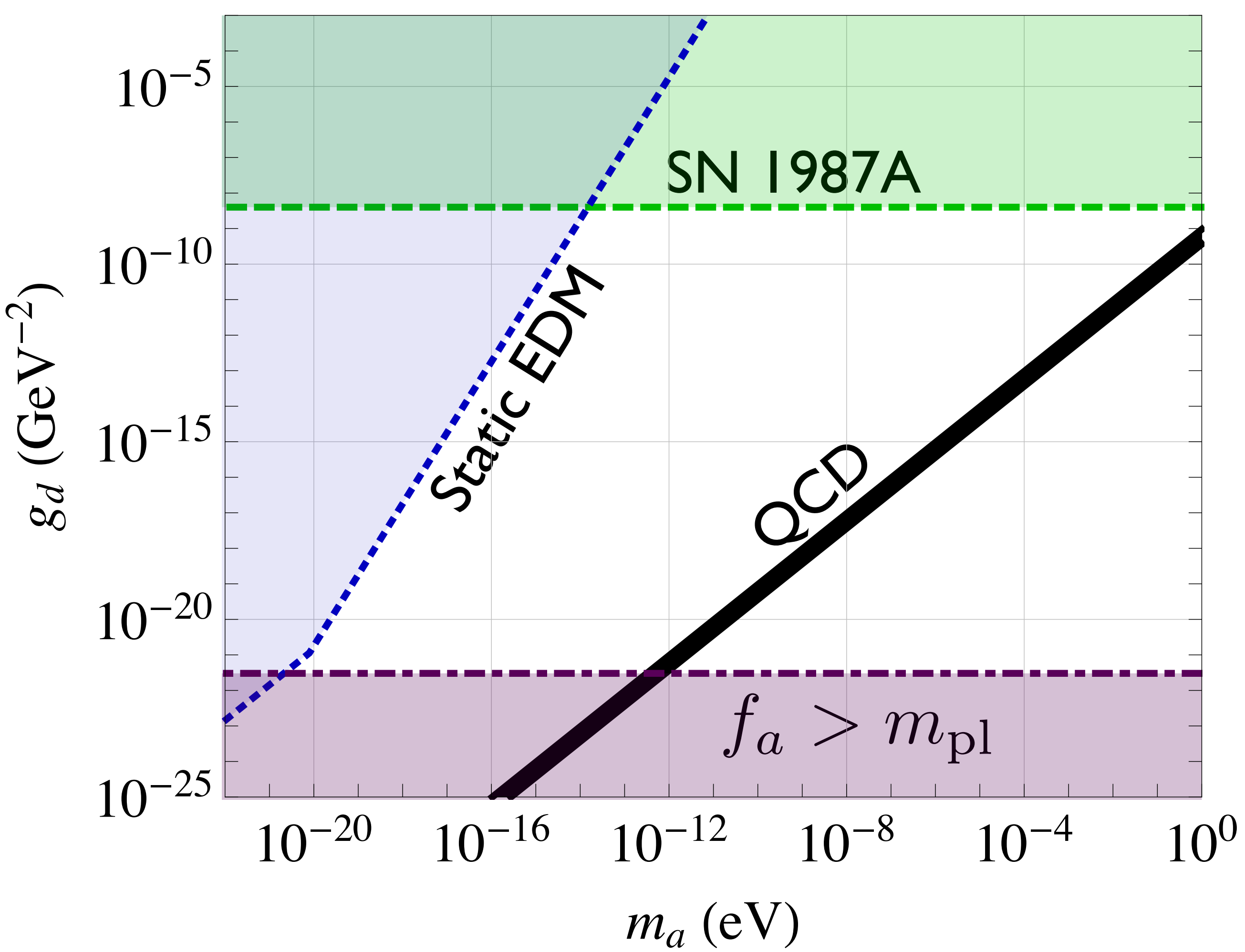
$$\frac{a(t)}{f_a} = (1 + z(t))^{3/2} \frac{\sqrt{2\bar{\rho}_{\mathrm{DM}}}}{f_a\, m_a} \cos(m_a t)$$

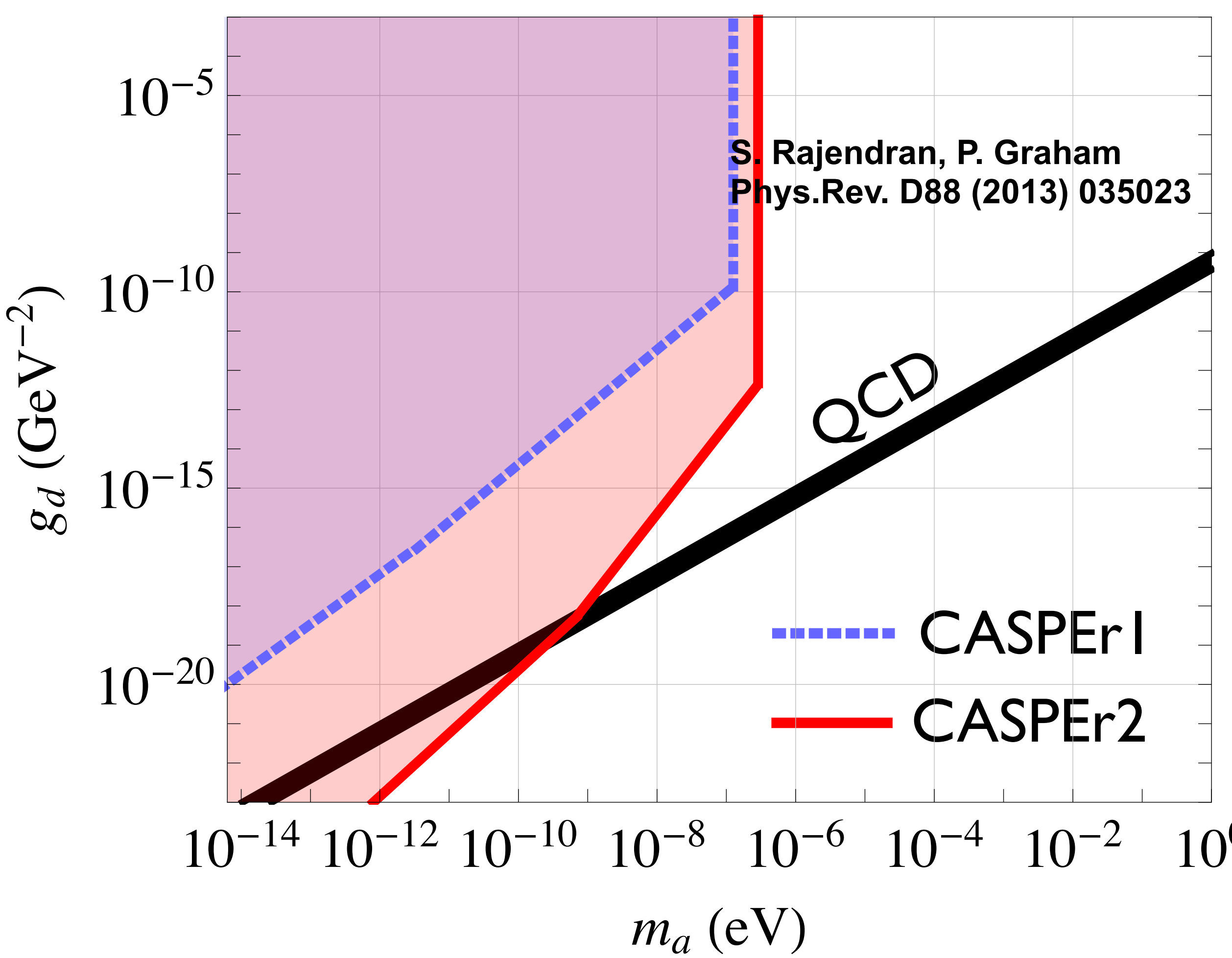
$$\approx 5 \times 10^{-9} \left(\frac{\mathrm{GeV}^2}{f_a m_a}\right) \left(\frac{1 + z(t)}{10^{10}}\right)^{3/2} \cos(m_a t)$$

4He ABUNDANCE IN THE RELEVANT PLANE

$$d_n = g_d a$$

$$g_d \approx \frac{4 \times 10^{-3}\ \mathrm{GeV}^{-1}}{f_a}$$





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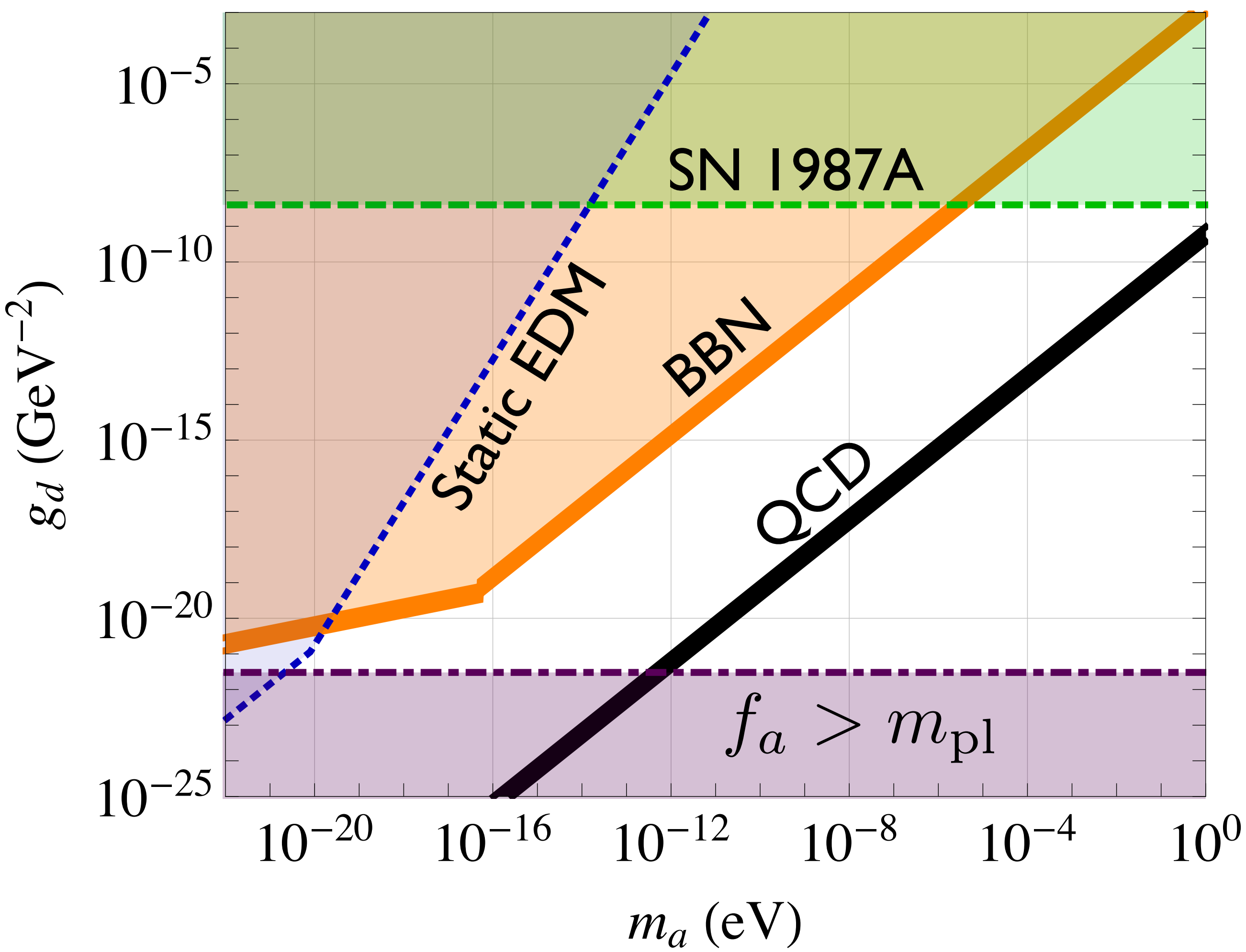
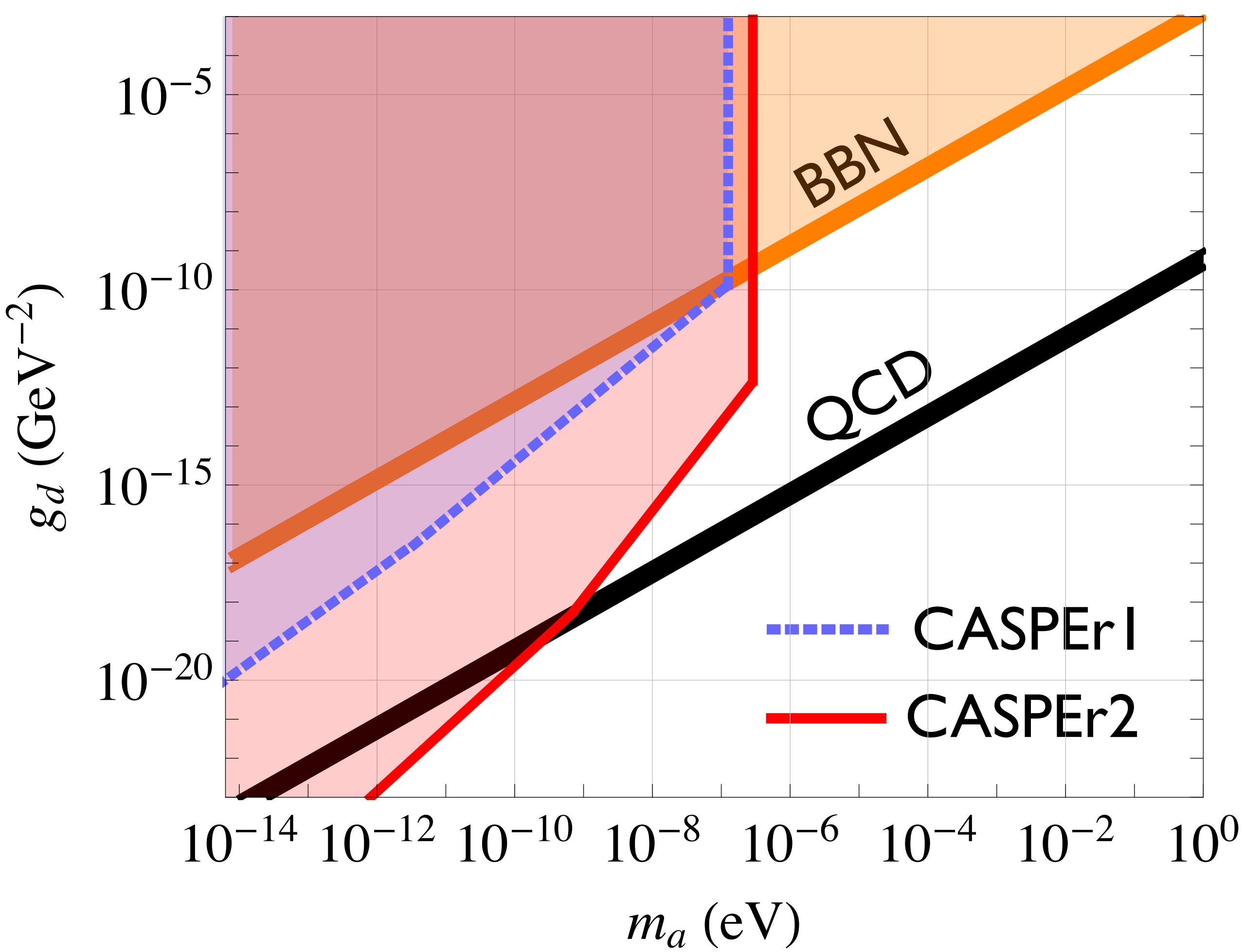
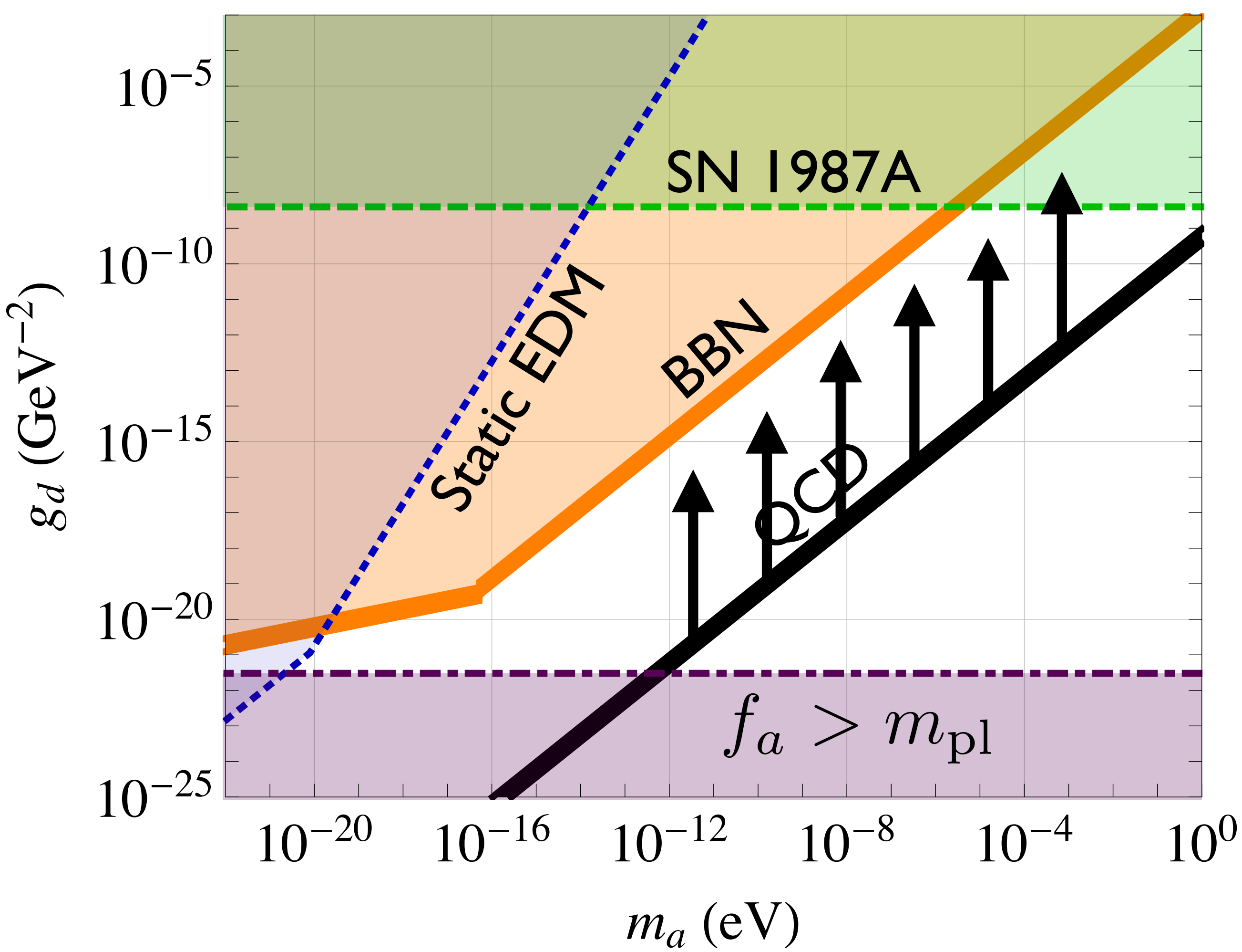




THE TUNING IN OUR WAYS

MASS TUNING

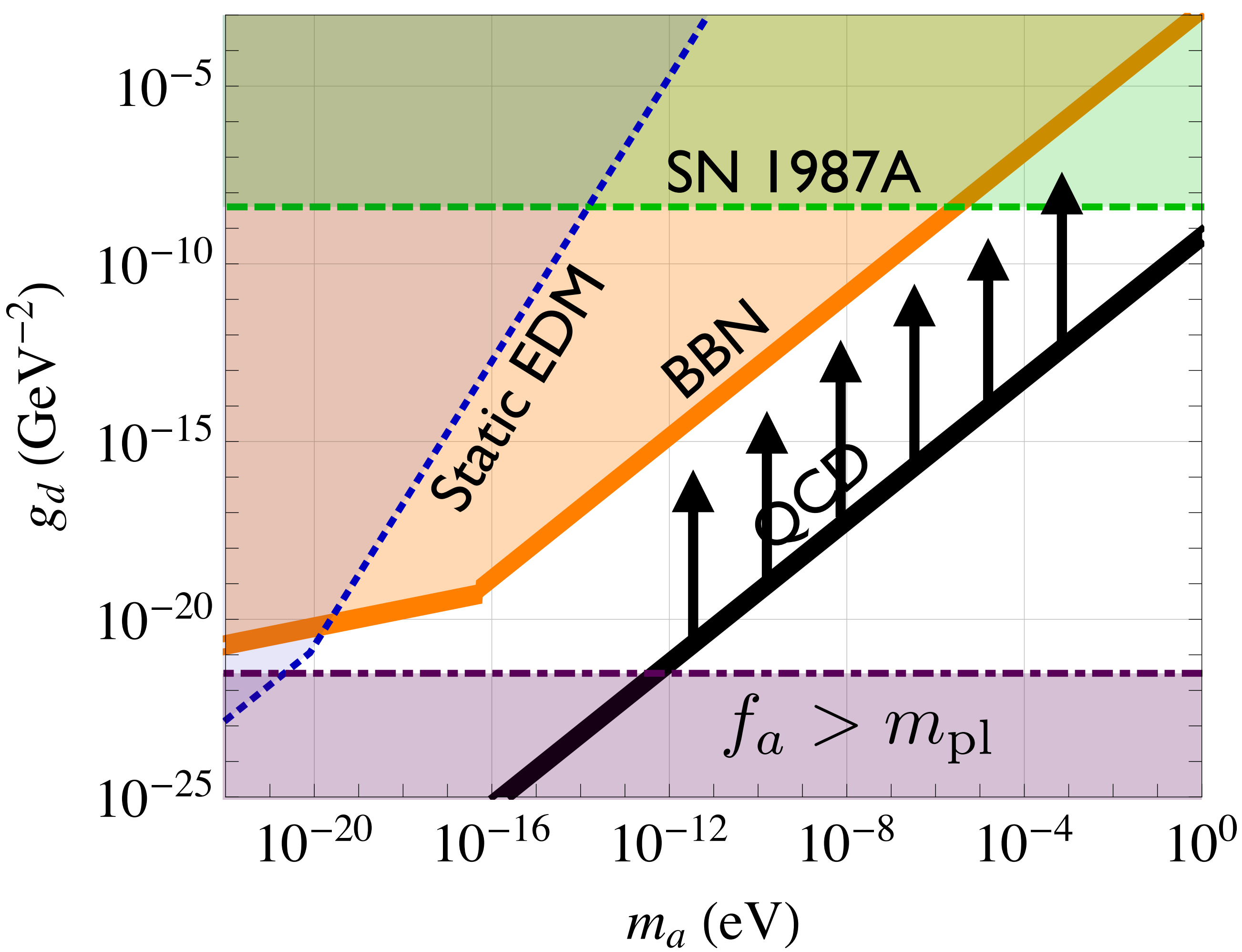


$$m_a \ll \frac{f_\pi m_\pi}{f_a}$$

$$\Delta\mathcal{L}(a) \propto \delta m^2 (a + \delta\theta)^2$$

$$\Delta_{mass} \sim \frac{f_a^2\, m_a^2}{f_\pi^2 m_\pi^2} \sim 10^{-14} \left(\frac{f_a\, m_a}{10^{-9}\ \mathrm{GeV}^2}\right)^2$$

CP TUNING

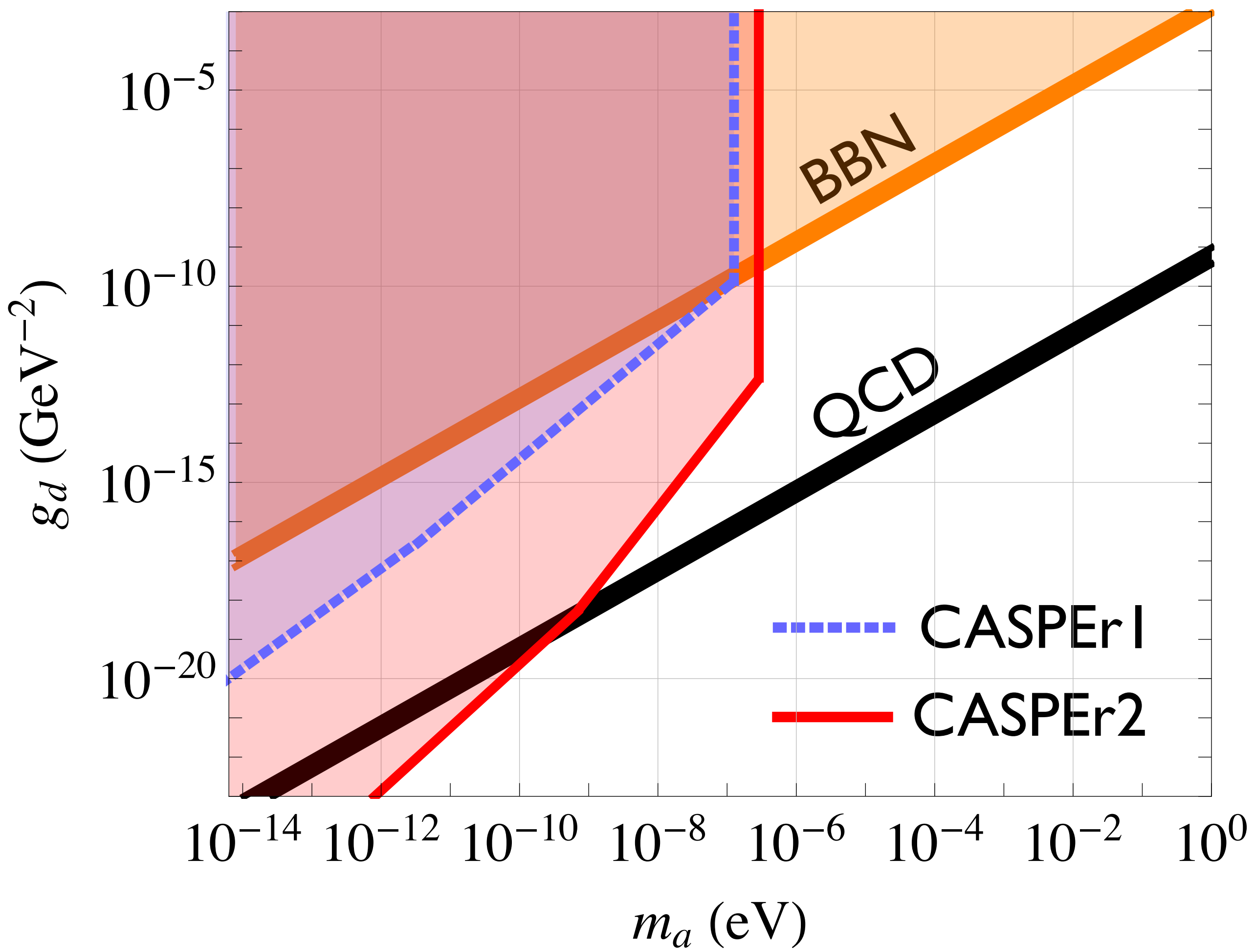


$$\Delta\mathcal{L}(a) \propto \delta m^2 (a + \delta\theta)^2$$

$$\frac{\partial V(a)}{\partial a} \to \langle a \rangle = -(\theta_{QCD} + \delta\theta)$$

$$|\delta\theta| \lesssim 10^{-10}$$

WHY BOTHERING WITH THE BOUND



- RELEVANT TO FUTURE AND PRESENT EXPERIMENTS
- GENERAL POINT ABOUT REDSHIFTING OPERATORS IN THE LAGRANGIAN
- RELATION BETWEEN GLUON-GLUON COUPLING, EDM AND NEUTRON-PROTON MASS DIFFERENCE

BEFORE THE QCD PHASE TRANSITION

$$\mathcal{L} = \frac{1}{2}\partial_\mu a \partial^\mu a + \sum_q \bar{q}\left(i\partial^\mu \gamma_\mu - m_q\right) q - \frac{a}{32\pi^2 f_a} G^a_{\mu\nu}\tilde{G}^{a\mu\nu} - \sum_q \lambda_q \frac{\partial_\mu a}{2 f_a} \bar{q}\gamma^\mu \gamma^5 q$$

MODEL DEPENDENT

A GLIMPSE AT THE ANSWER

$$\mathcal{L} \supset -\frac{1}{2}\frac{f_\pi^2 m_\pi^2 m_u m_d}{(m_u+m_d)^2}\left(\frac{a}{f_a}\right)^2$$
$$-\bar{N}\pi\cdot\sigma\left(i\gamma^5 g_{\pi NN} - 2\bar{g}_{\pi NN}\frac{a}{f_a}\right)N$$
$$+\frac{f_\pi \bar{g}_{\pi NN}}{2}\frac{m_d - m_u}{m_d+m_u}\left(\frac{a}{f_a}\right)^2 \bar{N}\sigma^3 N$$

AXIONS AND GRAVITATIONAL WAVES

COHERENT OSCILLATIONS

$$m_a = \omega_a \sim 10^{-4} - 10^{16}\ \mathrm{Hz}$$

TINY EFFECTIVE COUPLING (QCD LINE)

$$g_a = \frac{a}{f_a} \sim \frac{\sqrt{\rho^{\mathrm{l}}_{DM}}}{f_a m_a} \approx 10^{-19}$$

EDM vs LENGTH EFFECTS

$$d_n \sim g_a$$

$$\omega_g \sim 10^{-7} - 10^{11}\ \mathrm{Hz}$$

$$h \sim 10^{-20} - 10^{-30}$$

$$\delta \sim h$$

CONCLUSION

- WE HAVE SHOWN A NEW CONSTRAINT ON AXION DARK MATTER PARAMETER SPACE RELEVANT FOR PRESENT AND FUTURE EXPERIMENTS

- THE RELATION BETWEEN AXION INDUCED NEUTRON EDM AND NEUTRON-PROTON MASS DIFFERENCE IS MODEL INDEPENDENT

    - IT CAN BE EXPLOITED TO OBTAIN NEW ASTROPHYSICAL CONSTRAINTS AND AVENUES OF DETECTION

BACKUP

STRONG CP PROBLEM

$$\mathcal{L}_{\mathrm{QCD}} \supset \theta \frac{g_s^2}{32\pi^2} G_{a\mu\nu} \tilde{G}^{a\mu\nu}$$

$$d_n = 2.4 \times 10^{-16} |\theta| \ \mathrm{e} \cdot \mathrm{cm} \lesssim 10^{-26} \ \mathrm{e} \cdot \mathrm{cm}$$

$$|\theta| \lesssim 10^{-10}$$

STRONG CP PROBLEM

ANOMALOUS $U(1)_{PQ}$

SPONTANEOUSLY BROKEN AT A SCALE f_a

$$\mathcal{L}_{\rm QCD} \supset (\theta + C_a \frac{a}{f_a}) \frac{g_s^2}{32\pi^2} G_{a\mu\nu} \tilde{G}^{a\mu\nu}$$

$$\frac{\partial V(a)}{\partial a} = 0 \to \langle a \rangle = -\frac{f_a \theta}{C_a}$$


CHIRAL
LAGRANGIAN
HOMEWORKS

BEFORE THE QCD PHASE TRANSITION

$$\mathcal{L} = \frac{1}{2}\partial_\mu a \partial^\mu a + \sum_q \bar{q}\left(i\partial^\mu \gamma_\mu - m_q\right) q - \frac{a}{32\pi^2 f_a} G^a_{\mu\nu}\tilde{G}^{a\mu\nu} - \sum_q \lambda_q \frac{\partial_\mu a}{2 f_a}\bar{q}\gamma^\mu \gamma^5 q$$

GLUON-GLUON COUPLING AND QUARK MASSES

$$\mathcal{L} = \frac{1}{2}\partial_\mu a \partial^\mu a + \sum_q \bar{q}\left(i\partial^\mu \gamma_\mu - m_q\right) q - \frac{a}{32\pi^2 f_a} G^a_{\mu\nu}\tilde{G}^{a\mu\nu} - \sum_q \lambda_q \frac{\partial_\mu a}{2 f_a}\bar{q}\gamma^\mu\gamma^5 q$$

$$\boxed{q \to e^{-i\gamma^5 \alpha_q} q \quad \bar{q} \to \bar{q} e^{-i\gamma^5 \alpha_q}}$$

$$\frac{a}{f_a} \to \frac{a}{f_a} + 2\sum_q \alpha_q$$

QCD ANOMALY

$$m_q \to e^{-2i\alpha_q} m_q$$

GLUON-GLUON COUPLING AND QUARK MASSES

$$\mathcal{L} = \frac{1}{2}\partial_\mu a \partial^\mu a + \sum_q \bar{q}\left(i\partial^\mu \gamma_\mu - m_q\right) q - \frac{a}{32\pi^2 f_a} G^a_{\mu\nu}\tilde{G}^{a\mu\nu} - \sum_q \lambda_q \frac{\partial_\mu a}{2 f_a} \bar{q}\gamma^\mu \gamma^5 q$$

$$\alpha_q = -(ac_q)/2f_a \quad c_u + c_d = 1$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu a \partial^\mu a + \sum_q \bar{q}\left(i\partial^\mu \gamma_\mu - m_q\right) q - \sum_q \left(-c_q + \lambda_q\right) \frac{\partial_\mu a}{2 f_a} \bar{q}\gamma^\mu \gamma^5 q - \boxed{\sum_q m_q \bar{q} e^{i\frac{a c_q \gamma^5}{f_a}} q}$$

CHIRAL LAGRANGIAN

$$M = \begin{pmatrix} m_u e^{ic_u a/f_a} & 0 \\ 0 & m_d e^{ic_d a/f_a} \end{pmatrix} \qquad U = u^2 \equiv e^{2i\pi^a \tau^a / f_\pi}$$

CHIRAL LAGRANGIAN

$$M = \begin{pmatrix} m_u e^{ic_u a/f_a} & 0 \\ 0 & m_d e^{ic_d a/f_a} \end{pmatrix} \qquad U = u^2 \equiv e^{2i\pi^a \tau^a/f_\pi}$$

$$\begin{aligned}\mathcal{L}_{Ch} = & \, i\bar{N}\gamma^\mu \partial_\mu N - m_N \bar{N}N \\ & - \frac{c_+}{2}\bar{N}\left(uMu + \text{h.c.}\right)N - \frac{c_-}{2}\bar{N}\left(uMu - \text{h.c.}\right)\gamma^5 N \\ & - c_3 \text{Tr}\left[MU + \text{h.c.}\right]\bar{N}N - c_4 \text{Tr}\left[MU - \text{h.c.}\right]\bar{N}\gamma^5 N \\ & + \ldots + (u^\dagger \partial u)\end{aligned}$$

CHIRAL LAGRANGIAN

$$c_+ \bar{N} \left(uMu + \text{h.c.} \right) N \supset c_1 \bar{N} \frac{\pi^a \tau^a}{f_\pi} \frac{a}{f_a} N + c_2 \bar{N} \tau_3 N \left(\frac{a}{f_a} \right)^2 + \ldots$$

THE MODEL DEPENDENT PIECE

$$\frac{\partial_\mu a}{2f_a}\bar{q}\gamma^\mu\gamma^5 q \supset \frac{f_\pi}{f_a}\partial_\mu a \partial^\mu \pi^0$$

KINETIC MIXING

$$\frac{\partial_\mu a}{2f_a}\bar{q}\gamma^\mu\gamma^5 q \supset \frac{\partial_\mu a}{2f_a}\bar{N}\gamma^\mu\gamma^5 N$$

EXTRA INTERACTIONS

THE MODEL DEPENDENT PIECE

$$\mathcal{L} \supset (c_d - c_u + \lambda_u - \lambda_d)\,\frac{f_\pi}{4f_a}\partial_\mu a \partial^\mu \pi^0$$

c_u, c_d FIELD REDEFINITION

WE FIXED $c_u + c_d = 1$

BUT WE STILL HAVE THE FREEDOM TO CHOOSE $c_u - c_d = \lambda_d - \lambda_u$

THE MODEL DEPENDENT PIECE

$$\mathcal{L}_{\mathrm{kin}} = \frac{1}{2}\left(\partial_\mu \pi^0 \partial^\mu \pi^0 + \partial_\mu a \partial^\mu a\right) - \epsilon \partial_\mu a \partial^\mu \pi^0$$

$$\epsilon \sim \frac{f_\pi}{f_a}$$

THE MODEL DEPENDENT PIECE

$$\mathcal{L}_{\rm kin} = \frac{1}{2}\left(\partial_\mu \pi^0 \partial^\mu \pi^0 + \partial_\mu a \partial^\mu a\right) - \epsilon \partial_\mu a \partial^\mu \pi^0$$

$$\downarrow$$

$$R = \begin{pmatrix} \cos\theta + \frac{\epsilon \sin\theta}{\sqrt{1-\epsilon^2}} & \frac{\epsilon\cos\theta}{\sqrt{1-\epsilon^2}} - \sin\theta \\ \frac{\sin\theta}{\sqrt{1-\epsilon^2}} & \frac{\cos\theta}{\sqrt{1-\epsilon^2}} \end{pmatrix}$$

$$\downarrow \quad \text{ANY } \theta$$

$$\mathcal{L}_{\rm kin} = \frac{1}{2}\left(\partial_\mu \Pi^0 \partial^\mu \Pi^0 + \partial_\mu A \partial^\mu A\right)$$

THE MODEL DEPENDENT PIECE

$$R = \begin{pmatrix} \cos\theta + \frac{\epsilon \sin\theta}{\sqrt{1-\epsilon^2}} & \frac{\epsilon\cos\theta}{\sqrt{1-\epsilon^2}} - \sin\theta \\ \frac{\sin\theta}{\sqrt{1-\epsilon^2}} & \frac{\cos\theta}{\sqrt{1-\epsilon^2}} \end{pmatrix}$$

$$\Big\downarrow \; \theta$$

DIAGONALIZES THE MASS MATRIX

$$R \approx \begin{pmatrix} 1 + \mathcal{O}(\epsilon^2) & \mathcal{O}(\epsilon^3) \\ \mathcal{O}(\epsilon) & 1 \end{pmatrix}$$

THE MODEL DEPENDENT PIECE

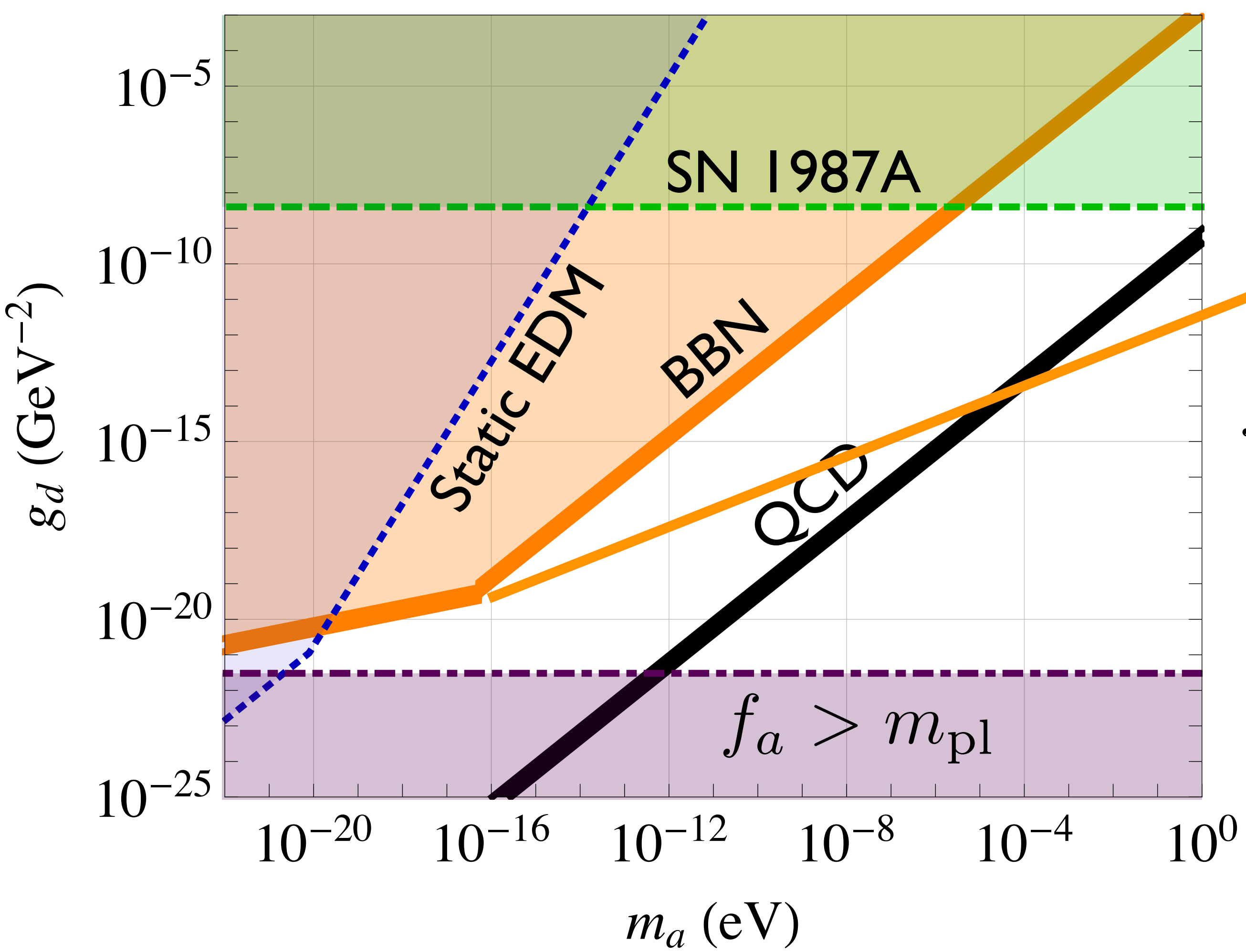
$$R = \begin{pmatrix} \cos\theta + \frac{\epsilon \sin\theta}{\sqrt{1-\epsilon^2}} & \frac{\epsilon\cos\theta}{\sqrt{1-\epsilon^2}} - \sin\theta \\ \frac{\sin\theta}{\sqrt{1-\epsilon^2}} & \frac{\cos\theta}{\sqrt{1-\epsilon^2}} \end{pmatrix}$$

$\Big\downarrow\ \theta$

DIAGONALIZES THE MASS MATRIX

$$R \approx \begin{pmatrix} 1+\mathcal{O}(\epsilon^2) & \mathcal{O}(\epsilon^3) \\ \mathcal{O}(\epsilon) & 1 \end{pmatrix} \longrightarrow \begin{array}{l} \Pi^0 \approx (1+\mathcal{O}(\epsilon^2))\pi^0 \\ A \approx a + \mathcal{O}(\epsilon)\pi^0 \end{array}$$

4He ABUNDANCE IN THE RELEVANT PLANE

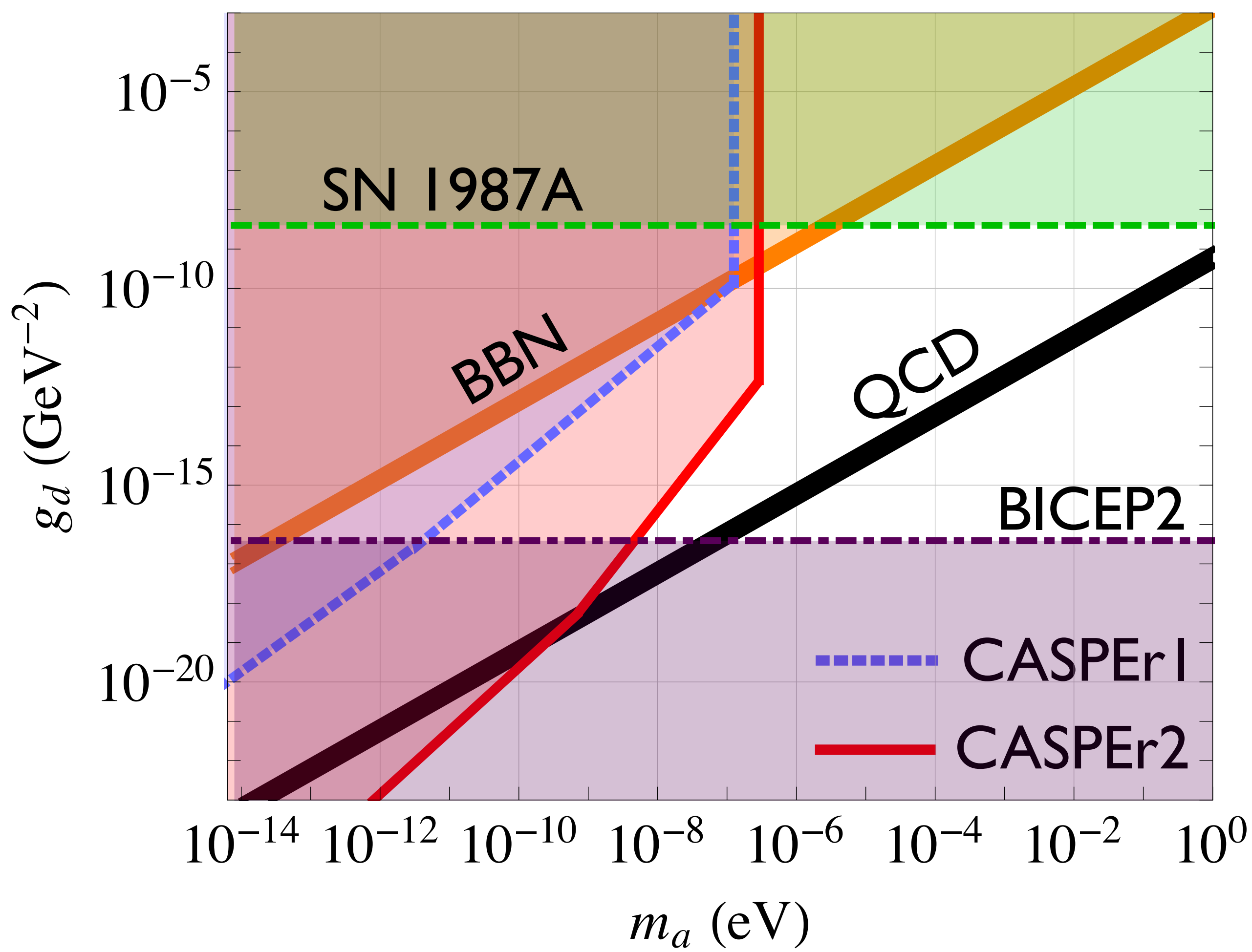


$$m_a \lesssim 3H_{\rm BBN}$$

$$f_a\, m_a \gtrsim \sqrt{2} \times \left(1.3 \times 10^{-9}\ {\rm GeV}^2\right) \left(\frac{1+z_m}{1+z_F}\right)^{3/2}$$

$$\approx \left(1.8 \times 10^{-9}\ {\rm GeV}^2\right) \left(\frac{m_a}{10^{-16}\ {\rm eV}}\right)^{3/4}$$

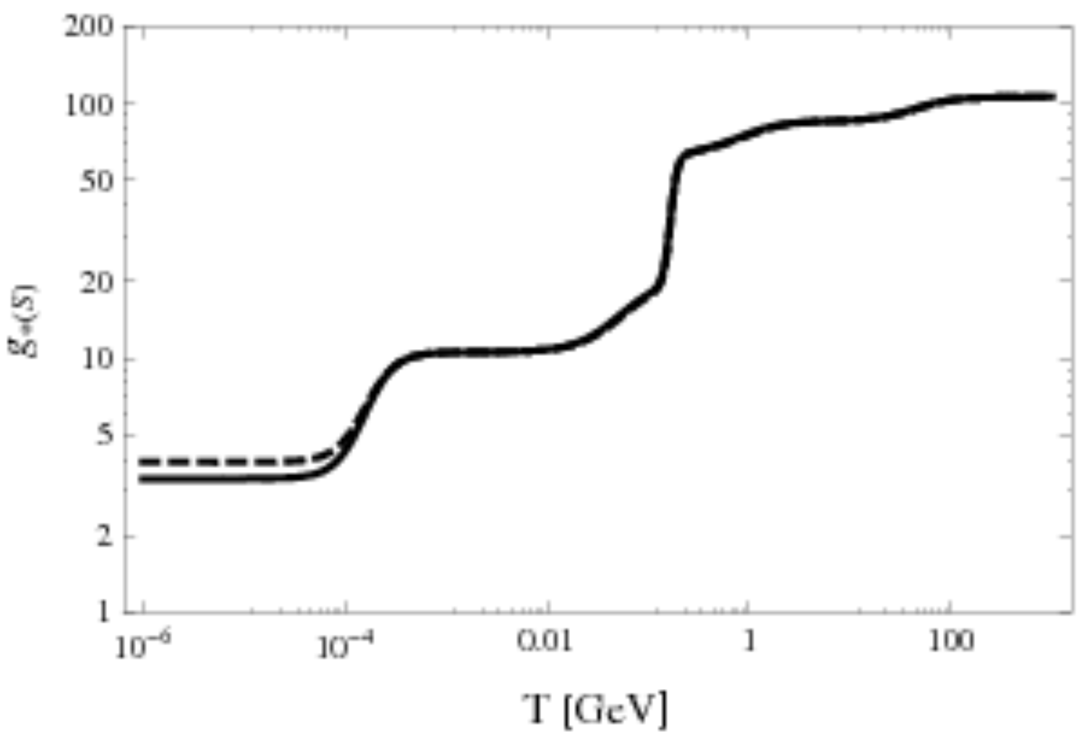
BICEP2



10^{-5}
10^{-10}
10^{-15}
10^{-20}
g_d (GeV^{-2})
SN 1987A
BBN
QCD
BICEP2
CASPEr1
CASPEr2
10^{-14} 10^{-12} 10^{-10} 10^{-8} 10^{-6} 10^{-4} 10^{-2} 10^{0}
m_a (eV)

RELATIVISTIC DEGREES OF FREEDOM

$$\Delta N_{\text{eff}} \sim 0.5 \left(\frac{g_*(T_\nu)}{g_*(T_a)} \right)^{4/3}$$



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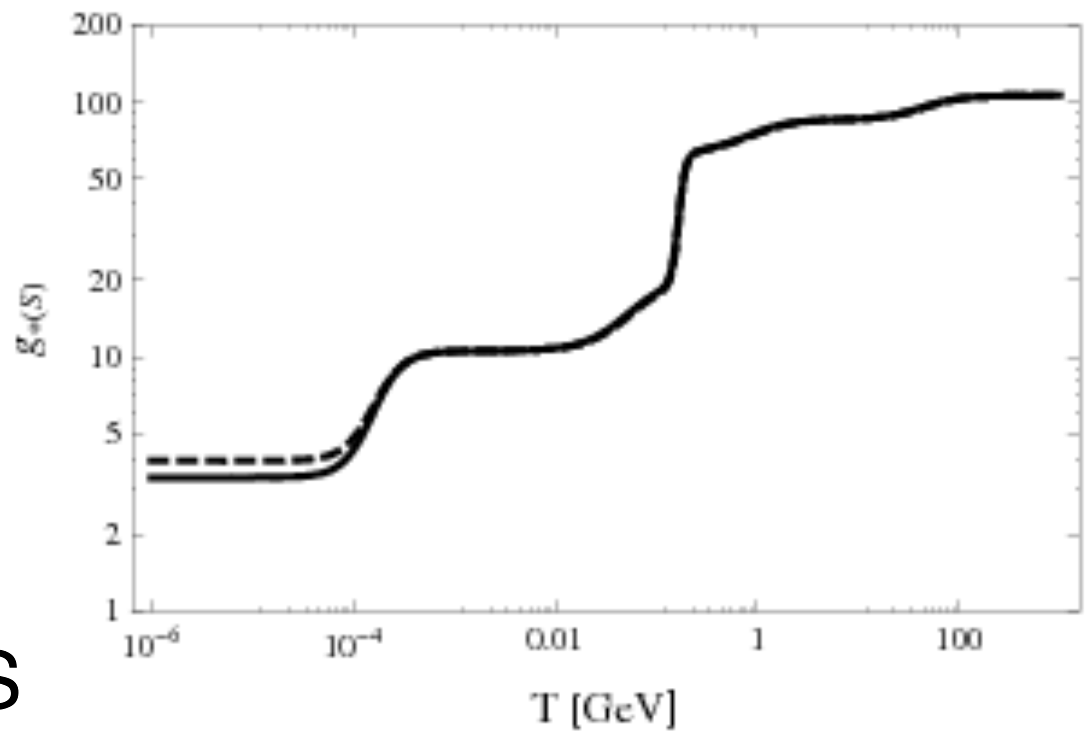


IF THERMALIZED BY PHOTONS

$$T_a \sim 100\ \text{GeV} \left(\frac{f_a}{10^7\ \text{GeV}} \right)^2 \longrightarrow f_a \gtrsim 10^{4-5}\ \text{GeV}$$

RELATIVISTIC DEGREES OF FREEDOM

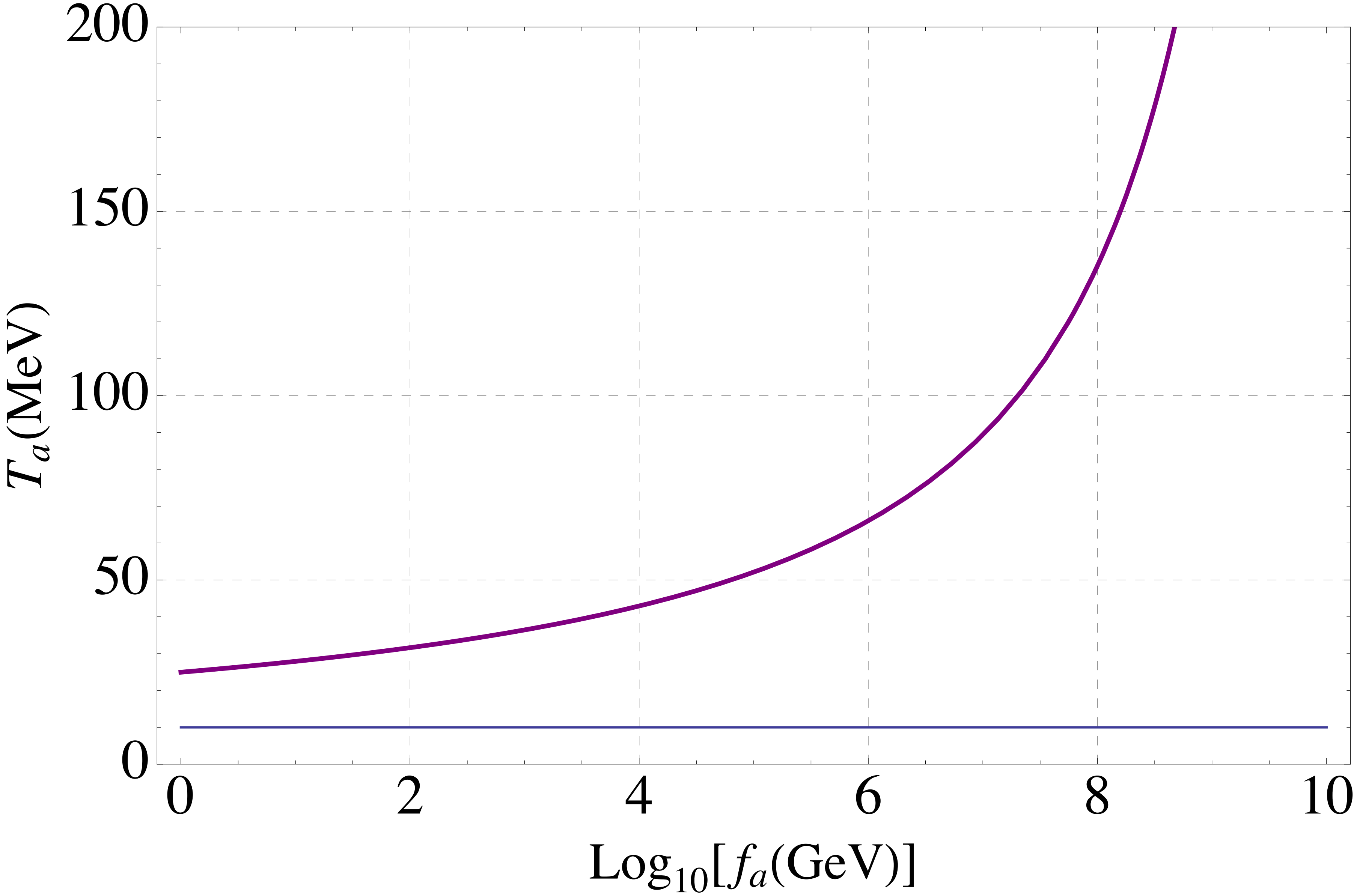
IF THERMALIZED BY HADRONS

$$N + a \rightarrow N + N$$
$$N + \pi \rightarrow N + a$$
$$\dots$$

$$\longrightarrow \quad \Gamma \sim e^{-\frac{m_N}{T}}$$

RELATIVISTIC DEGREES OF FREEDOM

IF THERMALIZED BY HADRONS



THERMAL AXION POPULATION

$$a_{\mathrm{Th}} \sim T$$

$$a_{\mathrm{Bkg}} \sim \sqrt{\rho_{DM}}/m_a \sim (T^2/m_a)\sqrt{z_{EQ}/z}$$

$$a_{\mathrm{Th}}/a_{\mathrm{Bkg}} \sim 10^{-3}\,\frac{m_a}{1\ \mathrm{eV}}$$

ANALYTICAL VS NUMERICAL

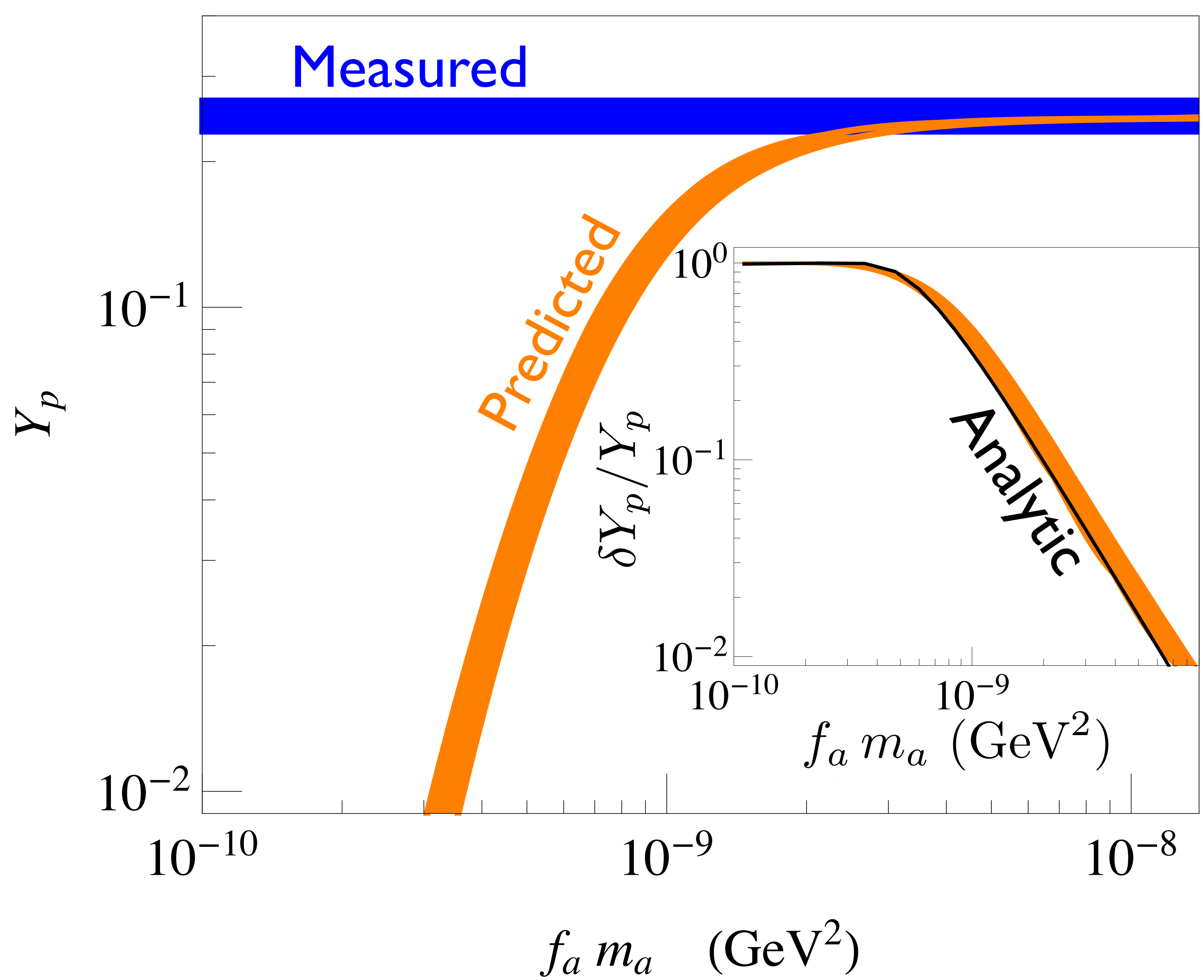
- SIMPLE ANALYTICAL ESTIMATE

$$f_a m_a \gtrsim 1.0 \times 10^{-9}\ \mathrm{GeV}^2$$

- FULL NUMERICAL CALCULATION
  - AXION CORRECTED BOLTZMANN EQUATIONS (CHANGE IN THE FREEZE-OUT TEMPERATURE)
  - AXION CORRECTED NEUTRON DECAY BETWEEN FREEZE-OUT AND BBN

$$f_a m_a \gtrsim 1.3 \times 10^{-9}\ \mathrm{GeV}^2$$

4He ABUNDANCE



A TWO AXIONS SYSTEM

$$\mathcal{L} \supset -\left(\frac{a}{f_a} + \frac{A}{F_A}\right)\frac{G^a_{\mu\nu}\tilde{G}^{a\mu\nu}}{32\pi^2}$$

$$V = \frac{1}{2}m_a^2\left(a + \frac{f_a}{F_A}A\right)^2 + \frac{1}{2}m_{\mathrm{UV}}^2A^2 + \ldots$$

A TWO AXIONS SYSTEM

$$\mathcal{L} \supset -\left(\frac{a}{f_a} + \frac{A}{F_A}\right)\frac{G^a_{\mu\nu}\tilde{G}^{a\mu\nu}}{32\pi^2}$$

$$V = \frac{1}{2}m_a^2\left(a + \frac{f_a}{F_A}A\right)^2 + \frac{1}{2}m_{\mathrm{UV}}^2 A^2 + \ldots$$

$$\alpha = f_a/F_A \quad \beta = m_{\mathrm{UV}}^2/m_a^2$$

$$\mathcal{M}^2 = m_a^2\left(\begin{array}{cc} 1 & \alpha \\ \alpha & \beta \end{array}\right)$$

A TWO AXIONS SYSTEM

- WITHOUT LOSS OF GENERALITY $\alpha \leq 1$
- NO TACHYONIC STATES $\beta > 0 \quad (|\alpha| < \sqrt{\beta})$
- THE TWO PREVIOUS CONDITIONS IMPLY

$$f_{1,2} m_{1,2} > \frac{f_\pi m_\pi \sqrt{m_u m_d}}{m_u + m_d}$$