Production and evolution of dark matter axions in the early universe

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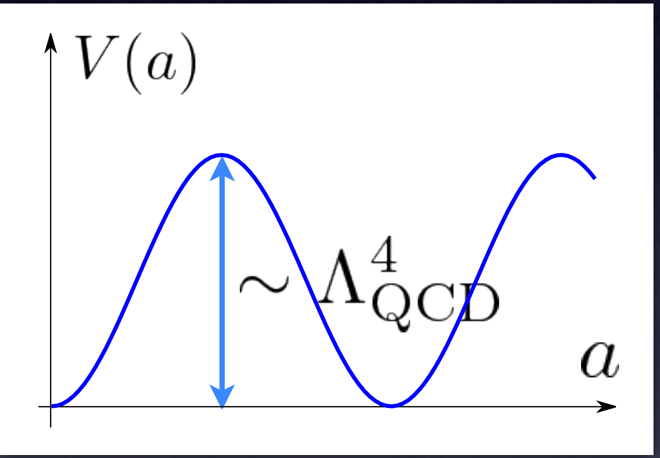
Axion

- Spontaneous breaking of continuous Peccei-Quinn symmetry at $T \simeq F_a \simeq 10^{9-12}\mathrm{GeV}$ "axion decay constant"
  - Nambu-Goldstone theorem
    → emergence of the (massless) particle ≡ axion
    Weinberg(1978), Wilczek(1978)
- Axion has a small mass (QCD effect)
  → pseudo-Nambu-Golstone boson

$$m_a \sim \frac{\Lambda_{\mathrm{QCD}}^2}{F_a} \sim 6 \times 10^{-6}\mathrm{eV}\left(\frac{10^{12}\mathrm{GeV}}{F_a}\right)$$

$$\Lambda_{\mathrm{QCD}} \simeq \mathcal{O}(100)\mathrm{MeV}$$

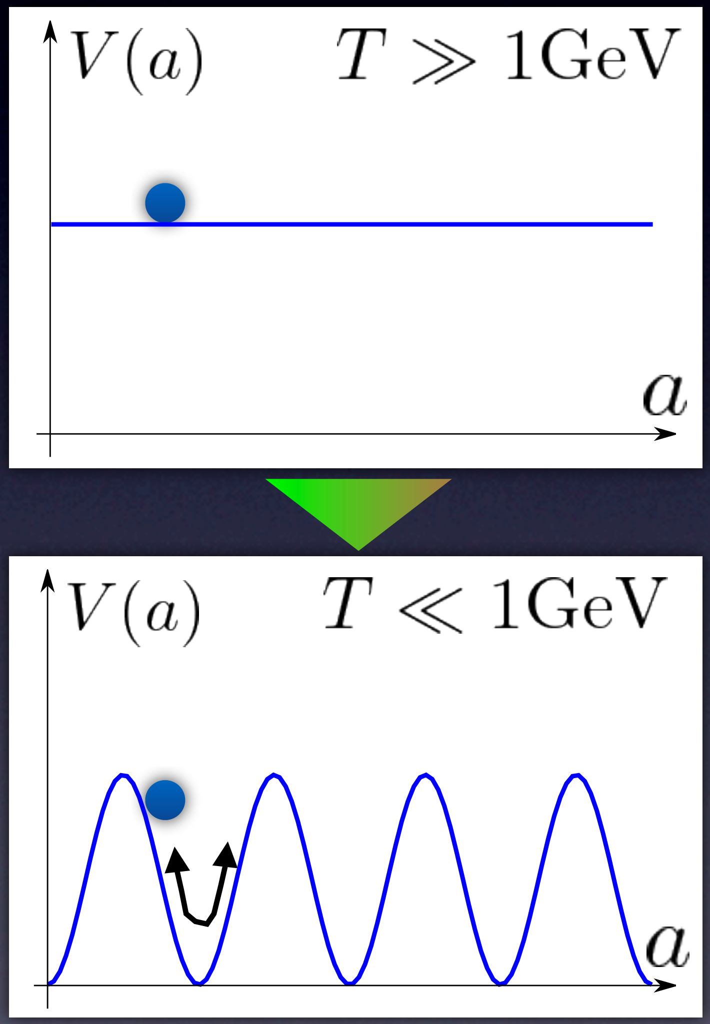


- Tiny coupling with matter + non-thermal production
  → good candidate of cold dark matter
- How they are produced, and how they evolved?
- Can we distinguish them from other dark matter candidate?

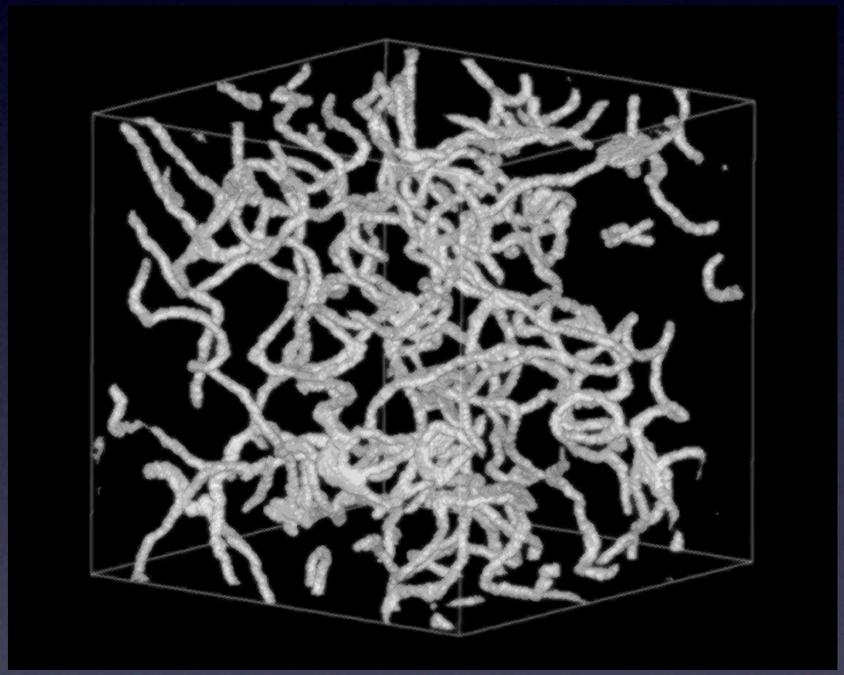
How axions are produced ?

Three mechanisms

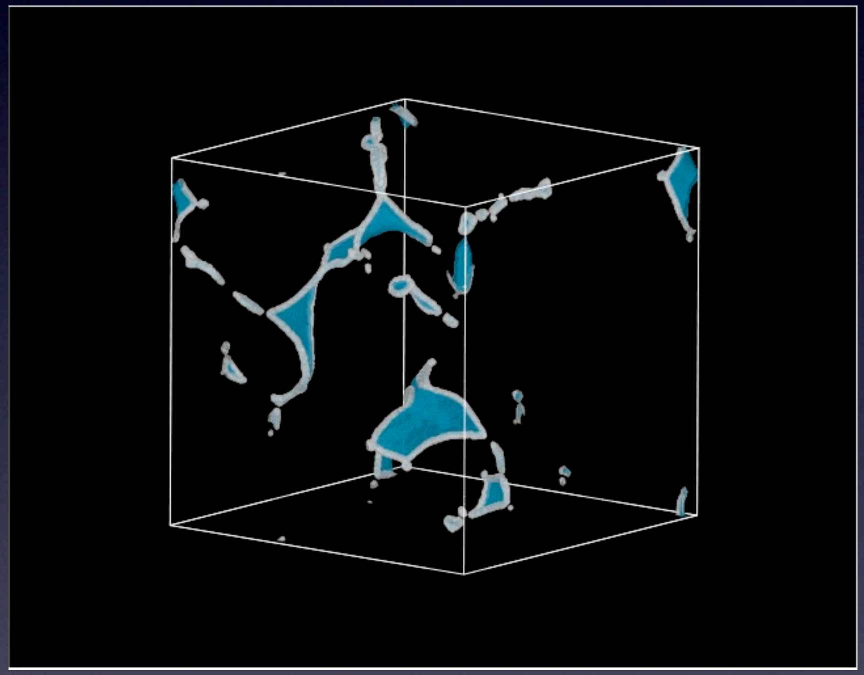
(1) misalignment mechanism



(2) radiation from strings



(3) collapse of string-wall systems



- Axions produced via (1) may be responsible for BEC dark matter
- Additional contributions (2) & (3) exist if Peccei-Quinn symmetry is broken after inflation

1. Axion production from topological defects

References:
Hiramatsu, Kawasaki, KS, JCAP08(2011)030 [1012.4558]
Hiramatsu, Kawasaki, KS, Sekiguchi, PRD85, 105020 (2012) [1202.5851]
Hiramatsu, Kawasaki, KS, Sekiguchi, JCAP01(2013)001 [1207.3166]
and refined work in progress [14xx.xxxx]

2. Evolution of axion dark matter in the condensed regime (possibility of axion BEC)

References:
KS, Yamaguchi, PRD87, 085010 (2013) [1210.7080]
Noumi, KS, Sato, Yamaguchi, PRD89, 065012 (2014) [1310.0167]

Axion production from topological defects

Axionic string and axionic domain wall

Peccei-Quinn field (complex scalar field)

$$\Phi = |\Phi| e^{ia(x)/F_a} \qquad a(x) : \text{axion field}$$

String formation $T \lesssim F_a$

Spontaneous breaking of $U(1)_{PQ}$

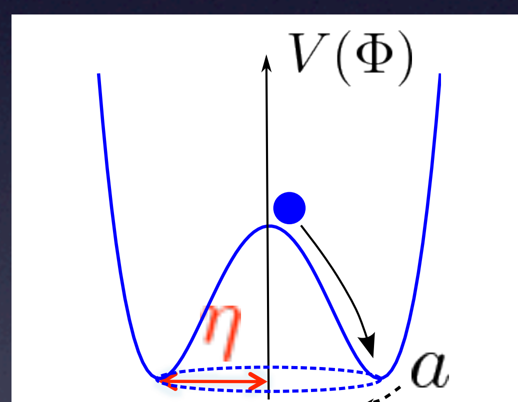
$$V(\Phi) = \frac{\lambda}{4}(|\Phi|^2 - \eta^2)^2$$

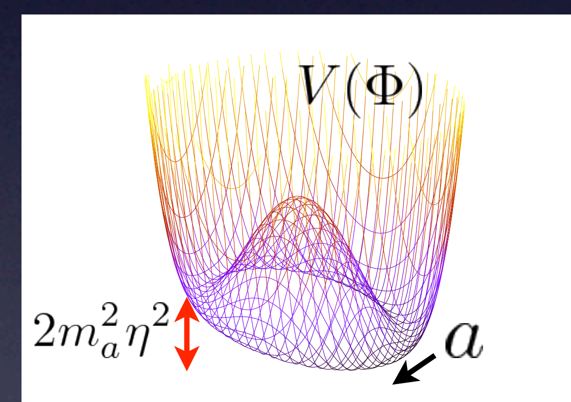
Domain wall formation $T \lesssim 1\text{GeV}$

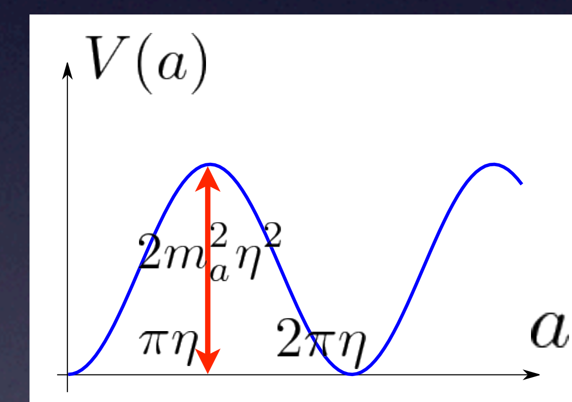
QCD effect

$$V(\Phi) = \frac{\lambda}{4}(|\Phi|^2 - \eta^2)^2 + \boxed{m_a^2 \eta^2 (1 - \cos(a/\eta))}$$

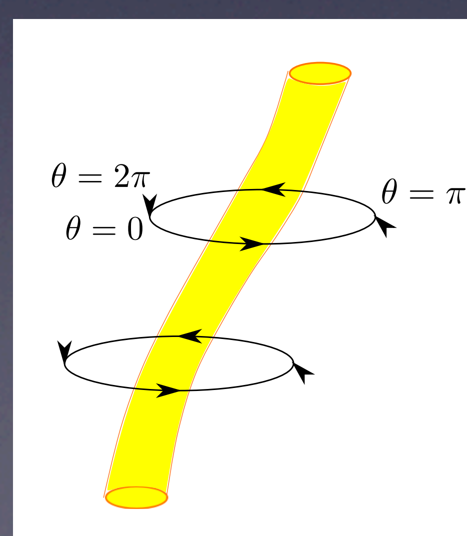
field space

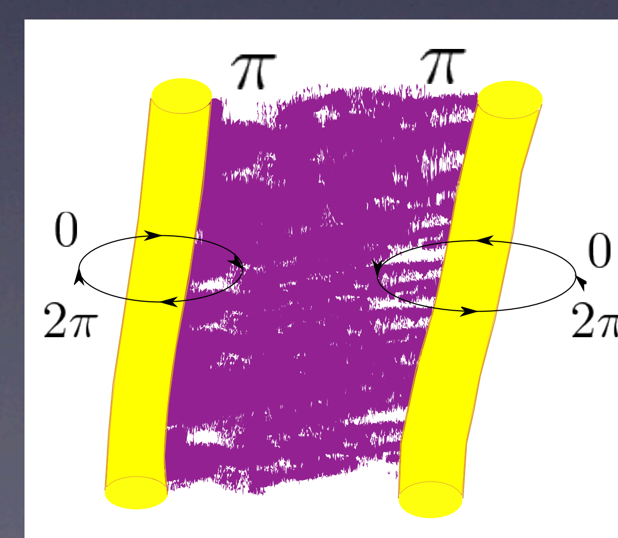






coordinate space





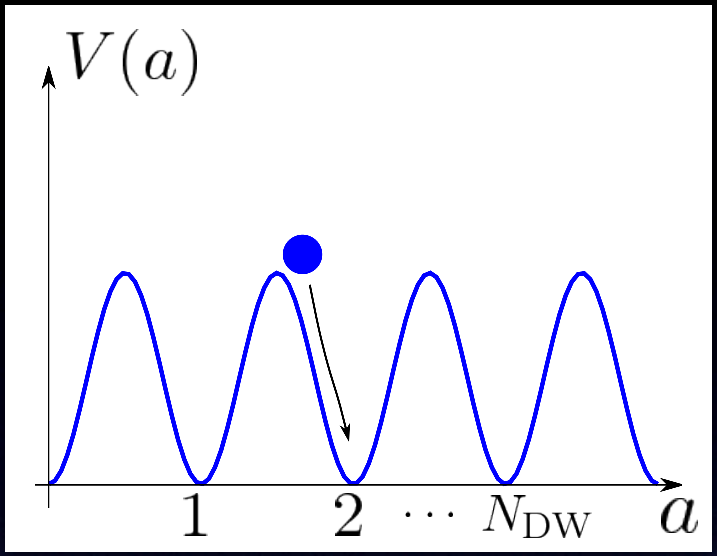
strings attached by domain walls

Domain wall problem

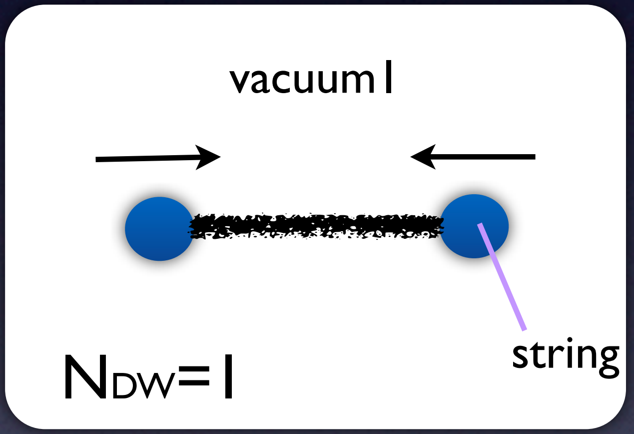
- Domain wall number NDW
    - NDW degenerate vacua

$$V(a) = \frac{m_a^2 \eta^2}{N_{\rm DW}^2}(1 - \cos(N_{\rm DW} a/\eta))$$

$N_{\rm DW}$: integer determined by QCD anomaly

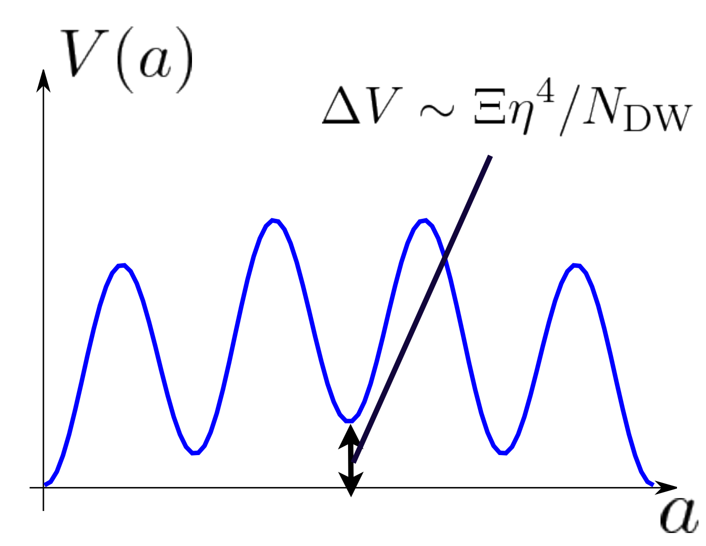


- If NDW=1
  String-wall systems are unstable
    - Decay → production of axions



- If NDW>1
  String-wall systems are stable
    - come to overclose the universe (domain wall problem)
      Zel'dovich, Kobzarev and Okun, JETP 40, 1 (1975)
    - We may avoid this problem by introducing an explicit symmetry breaking term (walls become unstable)
      Sikivie, PRL 48, 1156 (1982)

$$V(\Phi) = \frac{m_a^2 \eta^2}{N_{\rm DW}^2}(1 - \cos(N_{\rm DW} a/\eta)) - \boxed{\Xi \eta^3 (\Phi e^{-i\delta} + {\rm h.c.})}$$



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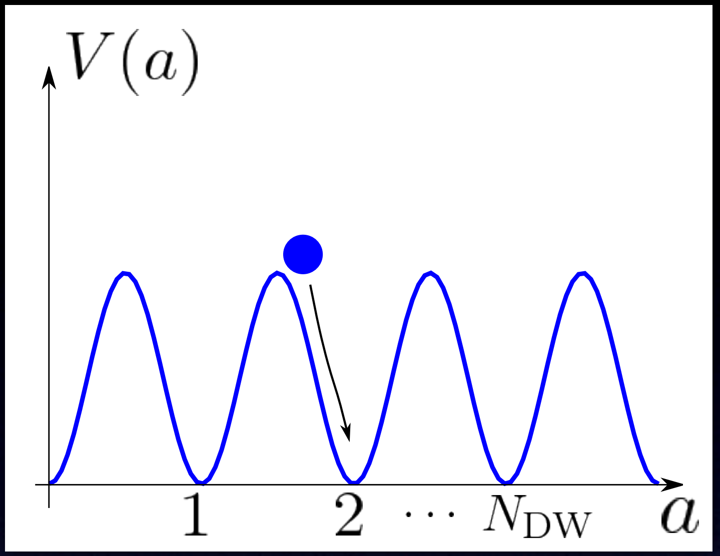

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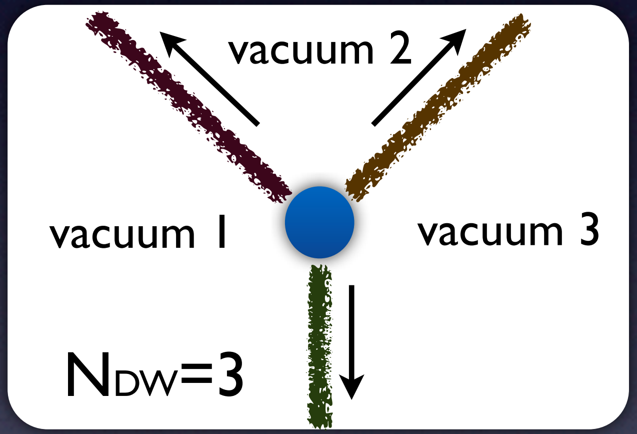

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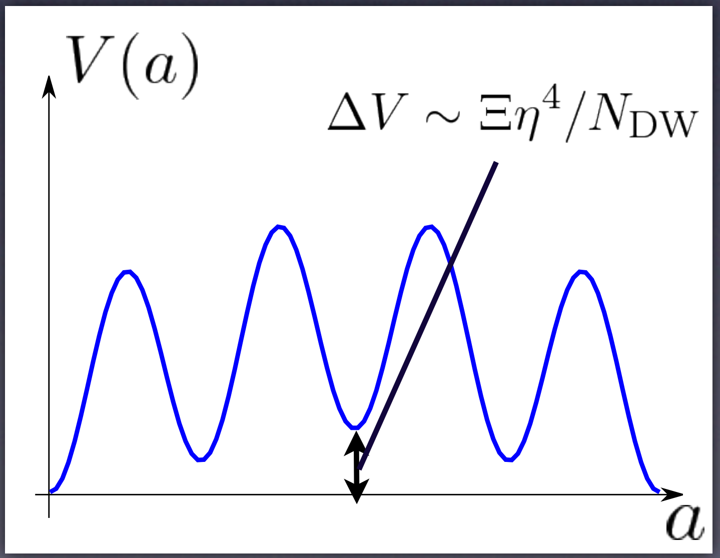
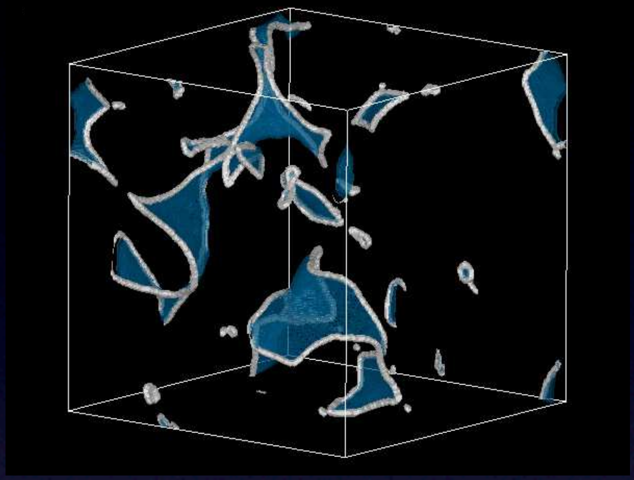


Recent studies

- Lattice simulation of PQ field to estimate the spectrum of axions produced from string-wall systems



- $N_{DW}=1$: wall decay contribution is significant

$$\Omega_a < \Omega_{\rm CDM} \Rightarrow \boxed{F_a \lesssim (4\text{-}6) \times 10^{10}\,\mathrm{GeV}}$$

Hiramatsu, Kawasaki, KS, Sekiguchi, PRD85, 105020 (2012)

- $N_{DW}>1$: Constraint becomes more severe but there still exists some loophole (i.e. fine-tuning of the parameter)

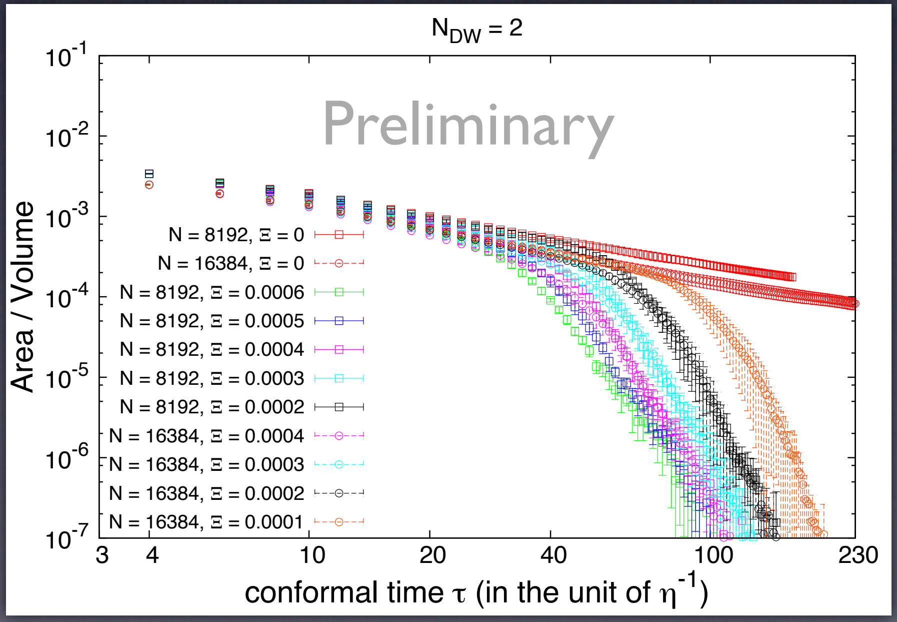
Hiramatsu, Kawasaki, KS, Sekiguchi, JCAP01(2013)001

- Large systematic uncertainty on the determination of the wall-lifetime in the model with $N_{DW}>1$

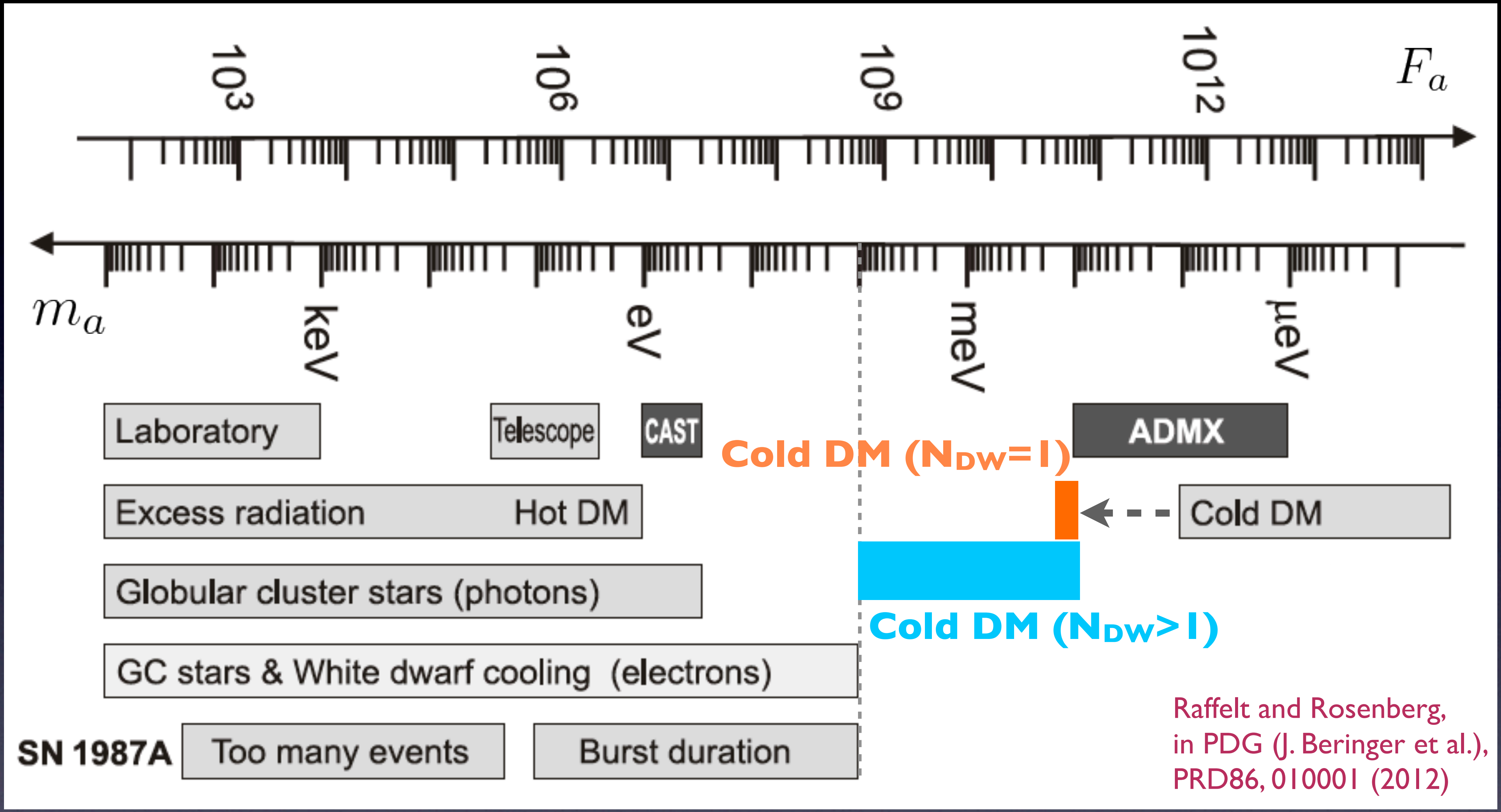
Refined calculation in larger simulation box is in process now

Previous study: 4096^2 grids $\Rightarrow$ Recent study: 16384^2 grids

Hiramatsu, Kawasaki, KS, JCAP08(2011)030



Prospects



- Additional contribution from string-wall systems

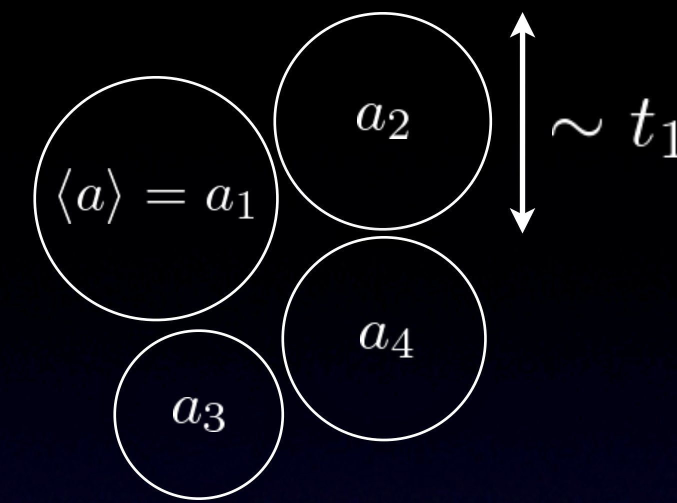
  → axions can be CDM at low F_a (high m_a)

- Large uncertainty for models with N_{DW}>1 due to the poor estimation of the life time of domain walls

Evolution of axion dark matter in the condensed regime (possibility of axion BEC)

Axion BEC dark matter ?

- Peculiarities of axion dark matter
  - Non-thermal production



$$H \lesssim m_a \quad (t = t_1) \qquad t_1 \sim 10^{-7}\mathrm{sec}$$

$$\delta v \sim \frac{\delta p}{m_a} \sim \frac{R(t_1)}{R(t_0)} \frac{1}{m_a t_1} \sim 3 \times 10^{-17} \left(\frac{F_a}{10^{12}\mathrm{GeV}} \right)^{0.81}$$

small velocity dispersion
("cold" dark matter)

  - Large occupation number

$$\boxed{\mathcal{N} \sim n_a \frac{(2\pi)^3}{\frac{4\pi}{3}(m_a \delta v)^3} \sim 10^{61} \left(\frac{F_a}{10^{12}\mathrm{GeV}} \right)^{2.75}}$$

($n_a \sim m_a F_a^2 (R(t_1)/R(t_0))^3$: number density of axions)

- A possibility that axions exist in the form of Bose-Einstein condensate (BEC) due to gravitational self-interaction

Sikivie, Yang, PRL103, 111301 (2009)

- Observable signatures (distinction between axions and WIMPs) ?
  - Effects on phase space structure of galactic halo (?)

Sikivie, Phys. Lett. B695, 22 (2011); Banik, Sikivie, PRD88, 123517 (2013)

  - Effects on cosmological parameters (?)

Erken, Sikivie, Tam, Yang, PRL108, 061304 (2012)

Can axions form a BEC ?

- BEC can be defined in thermal equilibrium state

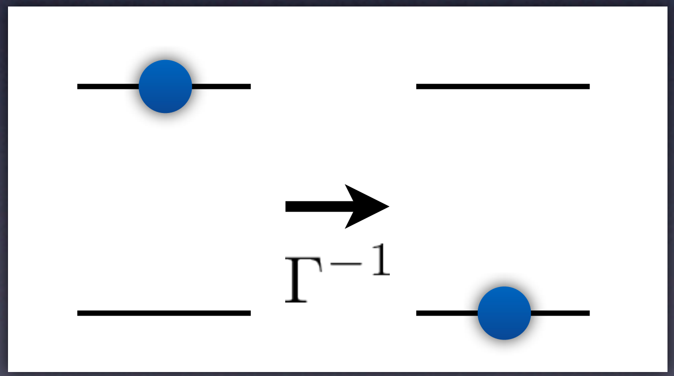
- Can axions thermalize in the expanding universe ?

- Naive expectation :
  They develop toward thermal equilibrium if
  the transition rate Γ exceeds the expansion rate H

$$\boxed{\Gamma \sim \dot{\mathcal{N}}_{\mathbf{p}}/\mathcal{N}_{\mathbf{p}} > H}$$

$\mathcal{N}_{\mathbf{p}}$: occupation number

(state labeled by three momentum $\mathbf{p}$)



- Identify elementary processes to estimate Γ !

Analytic methods

KS, Yamaguchi, PRD87, 085010 (2013)

- Develop analytic methods to calculate the time evolution of the expectation value of the operator

$$\langle \mathrm{in}|\mathcal{O}(t)|\mathrm{in}\rangle = \langle \mathcal{O}\rangle + i\int_{t_0}^{t} dt_1 \langle [H_I(t_1), \mathcal{O}]\rangle + (i)^2 \int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 \langle [H_I(t_1), [H_I(t_2), \mathcal{O}]]\rangle + \dots$$

$\mathcal{O} = \mathcal{N}_n = \frac{a_n^\dagger a_n}{V}$: number operator $\quad H_I(t)$: interaction Hamiltonian

- Specify the "in" (initial) state

For axions "wavy fields" rather than "point particles"

V : volume of the 3-dim space

α_i : some complex value

use a coherent state $|\mathrm{in}\rangle = |\{\alpha\}\rangle = \prod_i e^{-\frac{1}{2}|\alpha_i|^2} \sum_{n=0}^{\infty} \frac{\alpha_i^n}{n!\sqrt{V^n}} (a_i^\dagger)^n |0\rangle$

with $a_i|0\rangle = 0$

- Gravitational quartic interaction (Newtonian approx.)

$$H_I = -\frac{G}{2}\int d^3x d^3x' \frac{\rho(\mathbf{x},t)\rho(\mathbf{x}',t)}{|\mathbf{x}-\mathbf{x}'|} \Rightarrow \Lambda_{kl}^{ij} = -4\pi G m_a^2 \left(\frac{1}{|\mathbf{p}_k-\mathbf{p}_i|^2} + \frac{1}{|\mathbf{p}_k-\mathbf{p}_j|^2}\right) V\delta_{i+j,k+l}$$

$\rho(\mathbf{x},t)$: energy density of axions

Evolution of occupation number

KS, Yamaguchi, PRD87, 085010 (2013)

$$\langle \mathrm{in}|\mathcal{N}_p(t)|\mathrm{in}\rangle = \langle \mathcal{N}_p\rangle + i\int_{t_0}^{t} \langle [H_I(t_1), \mathcal{N}_p]\rangle + \mathcal{O}(H_I^2) + \ldots$$

$$i\int_{t_0}^{t} dt_1 \langle [H_I(t_1), \mathcal{N}_p]\rangle \xrightarrow{t-t_0\to\infty} -\frac{1}{2V^2}\sum_j\sum_k\sum_l \left[\Lambda_{kl}^{pj}\frac{e^{-i\Omega_{kl}^{pj}t}}{\Omega_{kl}^{pj}}\alpha_k^*\alpha_l^*\alpha_j\alpha_p + \mathrm{c.c.}\right]$$

for $\quad |\mathrm{in}\rangle = \prod_i e^{-\frac{1}{2}|\alpha_i|^2}\sum_{n=0}^{\infty}\frac{\alpha_i^n}{n!\sqrt{V^n}}(a_i^\dagger)^n|0\rangle$

coherent state

$$i\int_{t_0}^{t} dt_1 \langle [H_I(t_1), \mathcal{N}_p]\rangle = 0$$

for $\quad |\mathrm{in}\rangle = \prod_k \frac{1}{\sqrt{\mathcal{N}_k! V^{\mathcal{N}_k}}}(a_k^\dagger)^{\mathcal{N}_k}|0\rangle$

number state

- First order term is relevant if

  (1) condensed regime $\Omega_{kl}^{pj}t \ll 1$

  (c.f. $e^{-i\Omega_{kl}^{pj}t} \approx 0$ for particle kinetic regime $\Omega_{kl}^{pj}t \gg 1$)

  (2) coherent state representation $|\mathrm{in}\rangle = |\{\alpha\}\rangle$

$\Omega_{ij}^{kl} \equiv \omega_k + \omega_l - \omega_i - \omega_j$

: energy exchanged in the transitions

- For number ("particle-like") state, first order term exactly vanishes

Transition rate

Erken, Sikivie, Tam, Yang, PRD85, 063520 (2012); KS, Yamaguchi, PRD87, 085010 (2013)

- Transition rate of axions in coherent state

$$\Gamma \equiv \frac{1}{\mathcal{N}_p(t)} \frac{d\mathcal{N}_p(t)}{dt} \simeq \Lambda n_a$$

n_a : number density of axions

$\Lambda_{pj}^{kl} = \Lambda V \delta_{k+l,p+j}$: coefficient in the interaction term

From gravitational self-interaction

$$H_I = -\frac{G}{2} \int d^3x d^3x' \frac{\rho(\mathbf{x},t)\rho(\mathbf{x}',t)}{|\mathbf{x}-\mathbf{x}'|}$$

$R(t)$: scale factor of the universe

$$\Rightarrow \quad \boxed{\Gamma_g \simeq \frac{4\pi G m_a^2 n_a}{(\delta p)^2} \qquad \propto 1/R(t)}$$

$$\delta p \sim m_a \delta v \propto 1/R(t)$$

- Exceed the expansion rate at

$$\Gamma_g \gtrsim H \quad \Rightarrow \quad \boxed{T \simeq 2 \times 10^3 \mathrm{eV} \left(\frac{F_a}{10^{12}\mathrm{GeV}}\right)^{0.56}}$$

Formation of BEC at $T \sim \mathrm{keV}$?

General relativistic formulation

Noumi, KS, Sato, Yamaguchi, PRD89, 065012 (2014)

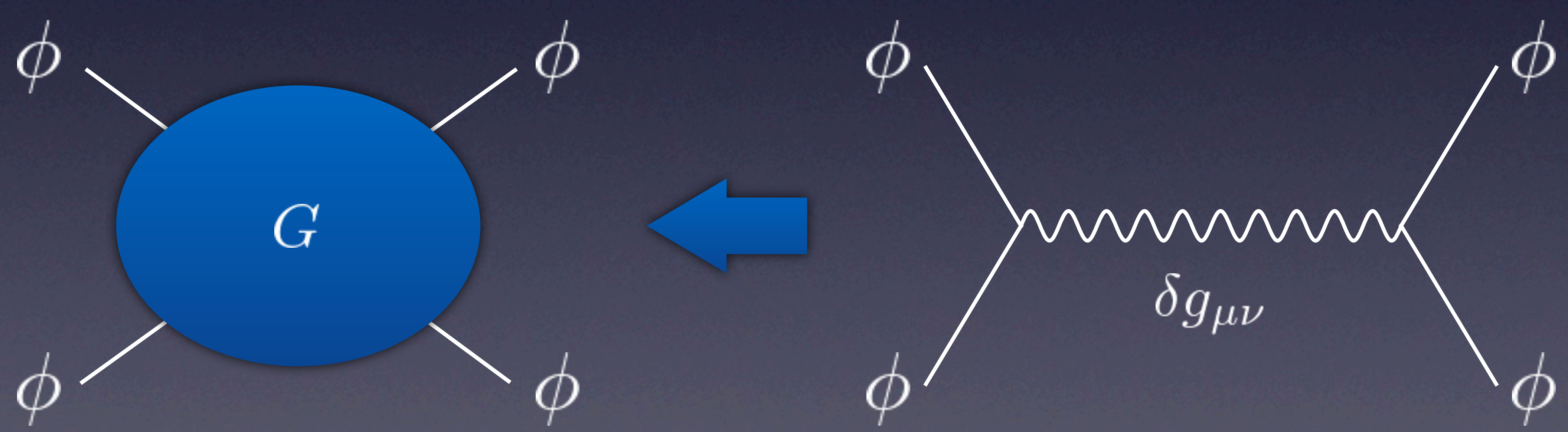
- Calculations in previous works : assumed Newtonian approximation

  → It breaks down for the modes $k/R \lesssim H$

- Reformulate in general relativistic framework

- Schematics:
  Effective quartic interaction from graviton exchange
  (contraction of cubic $\delta g_{\mu\nu}\phi^2$ interactions)

ϕ : axion field



- Reproduces the result in Newtonian approx.

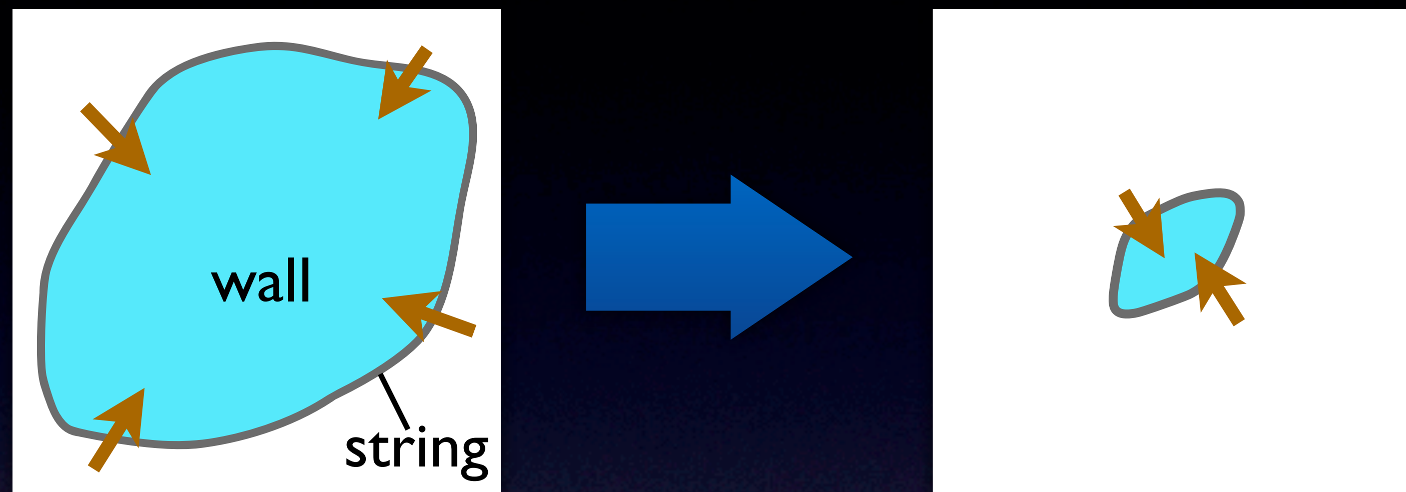
Summary

- Axion dark matter = coherent osc. + string + wall
- Wall decay contribution can be dominant
  - Model with $N_{\rm DW} = 1$: short-lived walls
    More severe constraint on axion decay constant
    $$4 \times 10^8 \mathrm{GeV} < F_a < (4\text{-}6) \times 10^{10} \mathrm{GeV}$$
  - Model with $N_{\rm DW} > 1$: long-lived walls
    It tends to overproduce axions, but can be responsible for CDM with meV mass range (need a tuning (?))
- Gravitational self-interaction of coherently oscillating axions becomes relevant at $T \sim \mathcal{O}(1)\mathrm{keV}$
  - Need further discussion to interpret the effect on structure formation

Backup

Annihilation mechanism of domain walls

- $N_{DW}=1$: rapidly decay due to the tension of walls (short-lived)



- $N_{DW}>1$: decay when the tension becomes comparable with the pressure (long-lived)

lifetime $\propto 1/\Xi$

tension $p_T \sim \sigma_{\rm wall}/R \sim m_a \eta^2/N_{\rm DW}^2 R$

pressure $p_V \sim \Delta V \sim \Xi \eta^4/N_{\rm DW}$

R : curvature radius of walls

$\sigma_{\rm wall}$: surface mass density of walls

$$t_{\rm dec} \sim R \quad \text{at} \quad p_V \simeq p_T$$

$$\simeq 7\times 10^2 {\rm sec}\ N_{\rm DW}^{-3} \left(\frac{10^{-58}}{\Xi}\right)\left(\frac{10^{10}{\rm GeV}}{F_a}\right)^3$$

Axion cosmology

- Parameters of the models
  - Axion decay constant (scale of PQ symmetry breaking) F_a
  - Domain wall number (number of degenerate vacua) $N_{\rm DW}$
  - Bias (lifetime of domain walls) Ξ
- Cosmological scenarios

F_a?

Large F_a (scenario I)
PQ symmetry is broken before inflation

Small F_a (scenario II)
PQ symmetry is broken after inflation

$N_{\rm DW}$?

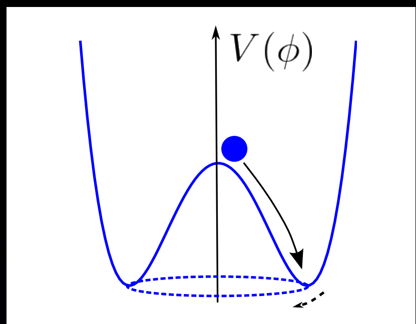
$N_{\rm DW} = 1$
Walls decay (scenario IIA) immediately after formation

$N_{\rm DW} > 1$
(scenario IIB)
Walls are long-lived

Investigate production and evolution of axion DM in each scenario

→ Derive constrains on the model parameters

Production of axions in the universe

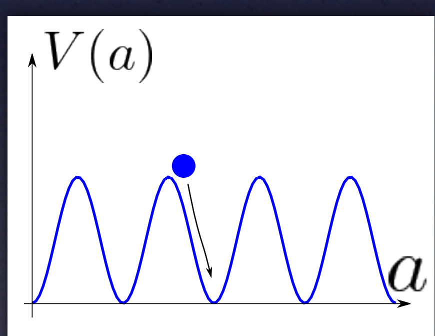


$T \simeq 10^{10-11}\mathrm{GeV}$
$(\simeq F_a \equiv \eta/N_{\mathrm{DW}})$

Inflation

PQ symmetry breaking
- formation of strings

$T \lesssim 1\mathrm{GeV}$



QCD phase transition
- axions acquire a mass
- formation of domain walls

(ii) string decay
Davis (1986)
Harari & Sikivie (1987)
Davis & Shellard (1988)
Dabholkar & Quoshnock (1989)
Battye & Shellard (1994)
Yamaguchi, Kawasaki & Yokoyama(1999) etc.

(i) coherent oscillation
Preskill & Wise (1983)
Abbott & Sikivie (1983)
Dine & Fischler (1983) etc.

(iii) wall decay
Lyth (1992)
Nagasawa & Kawasaki (1994)
Chang, Hagmann & Sikivie (1998)

$N_{\mathrm{DW}}=1$

immediately after formation ...

- collapse of string-wall networks

$N_{\mathrm{DW}}>1$

string-wall networks exist for a long time

annihilation of domain walls before they overclose the universe

Wall decay contribution to CDM abundance

- On the mean energy $\langle\omega_a\rangle$ of axions radiated from domain wall decay

Case A

$$\langle\omega_a\rangle \sim m_a$$

Nagasawa & Kawasaki (1994)

- Radiated axion is mildly relativistic
- Contribution for DM abundance can be large

Case B

$$\langle\omega_a\rangle \sim m_a \log(F_a/m_a)$$

Chang, Hagmann & Sikivie (1999)

- Spectrum is hard

$$dE/dk \sim 1/k$$

- Contribution for DM abundance is subdominant

$$\rho_a(t_{\rm today}) = m_a n_a(t_{\rm today}) = m_a \frac{\rho_a(t_{\rm decay})}{\langle\omega_a\rangle}\left(\frac{R(t_{\rm decay})}{R(t_{\rm today})}\right)^3$$

$R(t)$: scale factor of the universe

- This controversy can be resolved by simulation of defect networks

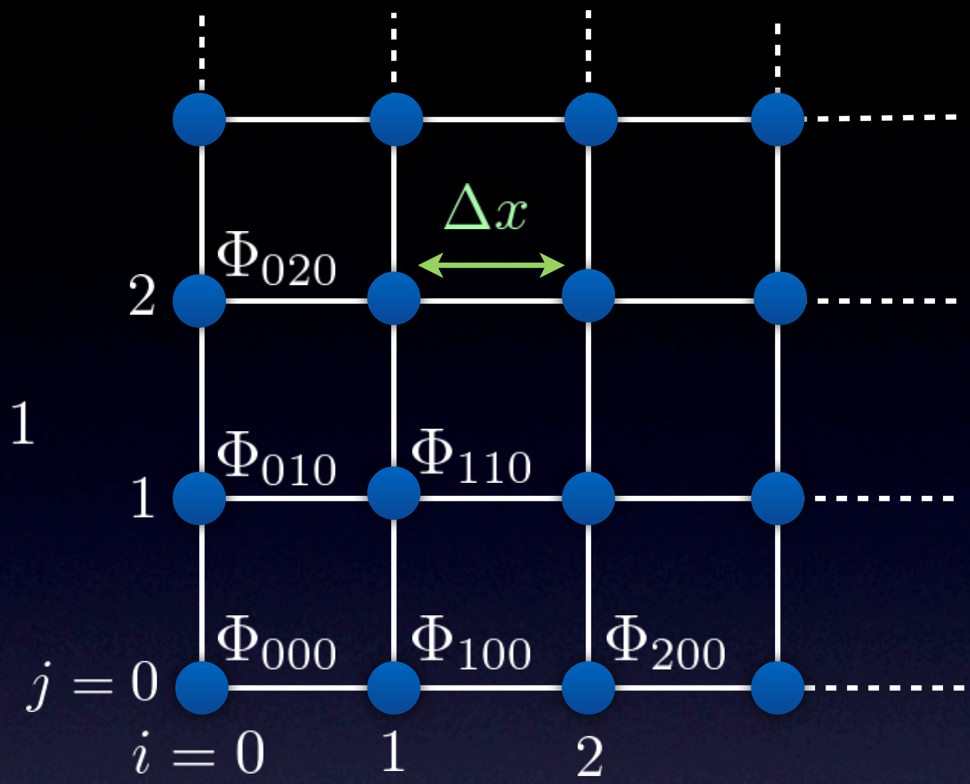
Numerical simulation

- Discretize the spatial coordinate

$$\vec{\mathbf{x}} \quad \rightarrow \quad (i, j, k)$$

$$i, j, k = 0, 1, \ldots, N-1$$

$$\Phi(\vec{\mathbf{x}}) \quad \rightarrow \quad \Phi_{i,j,k}$$



$$\nabla^2\Phi(\vec{\mathbf{x}}) \;\rightarrow\; (\nabla^2\Phi)_{i,j,k}$$

$$= \frac{1}{12(\Delta x)^2}\left[16(\Phi_{i+1,j,k} + \Phi_{i-1,j,k} + \Phi_{i,j+1,k} + \Phi_{i,j-1,k} + \Phi_{i,j,k+1} + \Phi_{i,j,k-1}) - (\Phi_{i+2,j,k} + \Phi_{i-2,j,k} + \Phi_{i,j+2,k} + \Phi_{i,j-2,k} + \Phi_{i,j,k+2} + \Phi_{i,j,k-2}) - 90\Phi_{i,j,k}\right]$$

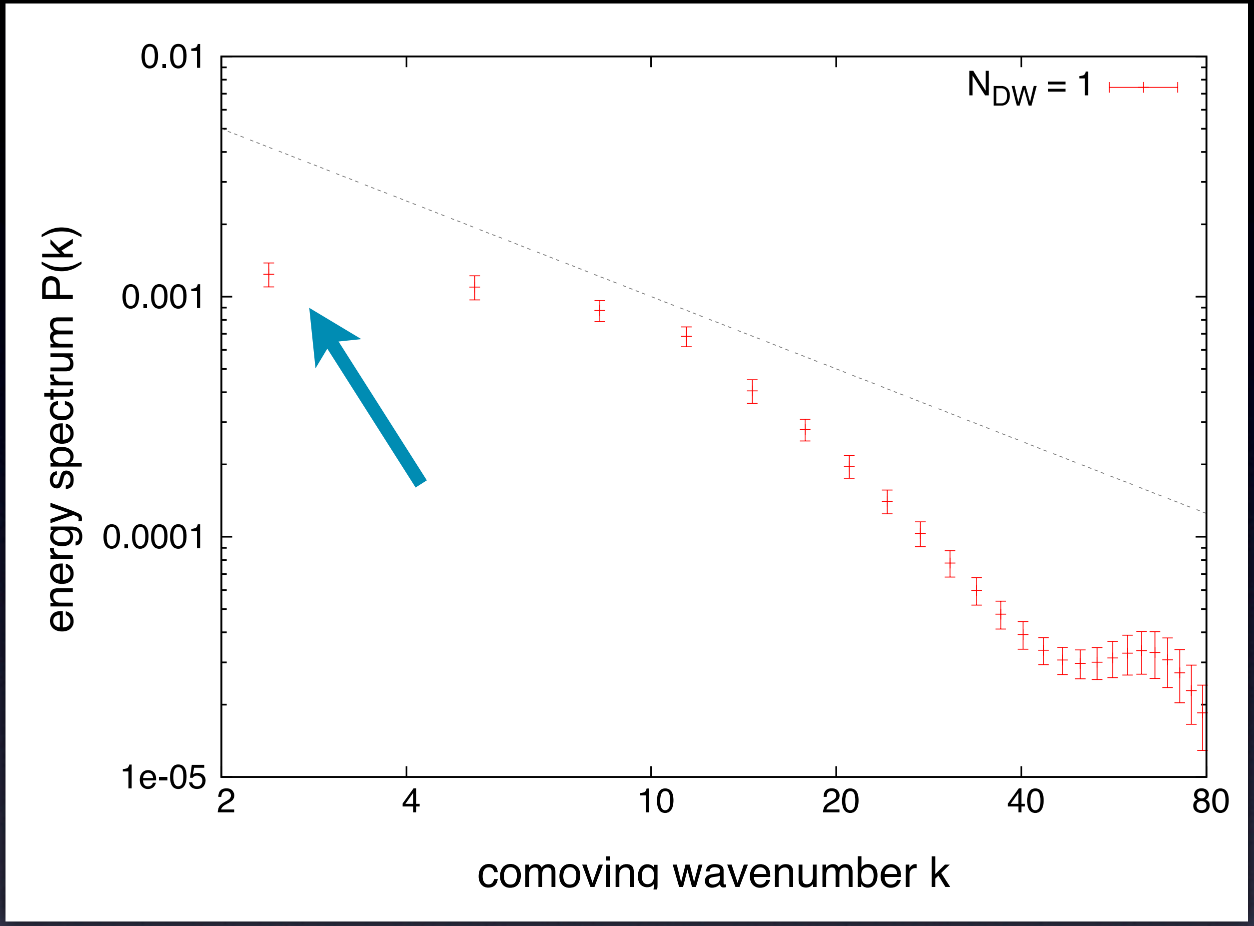
- Solve the classical EOM for complex scalar $\Phi = \phi_1 + i\phi_2$ on 3D lattice

$$\ddot{\phi}_i + 3H\dot{\phi}_i - \frac{\nabla^2}{R^2(t)}\phi_i = -\frac{\partial V}{\partial \phi_i} \qquad i = 1, 2$$

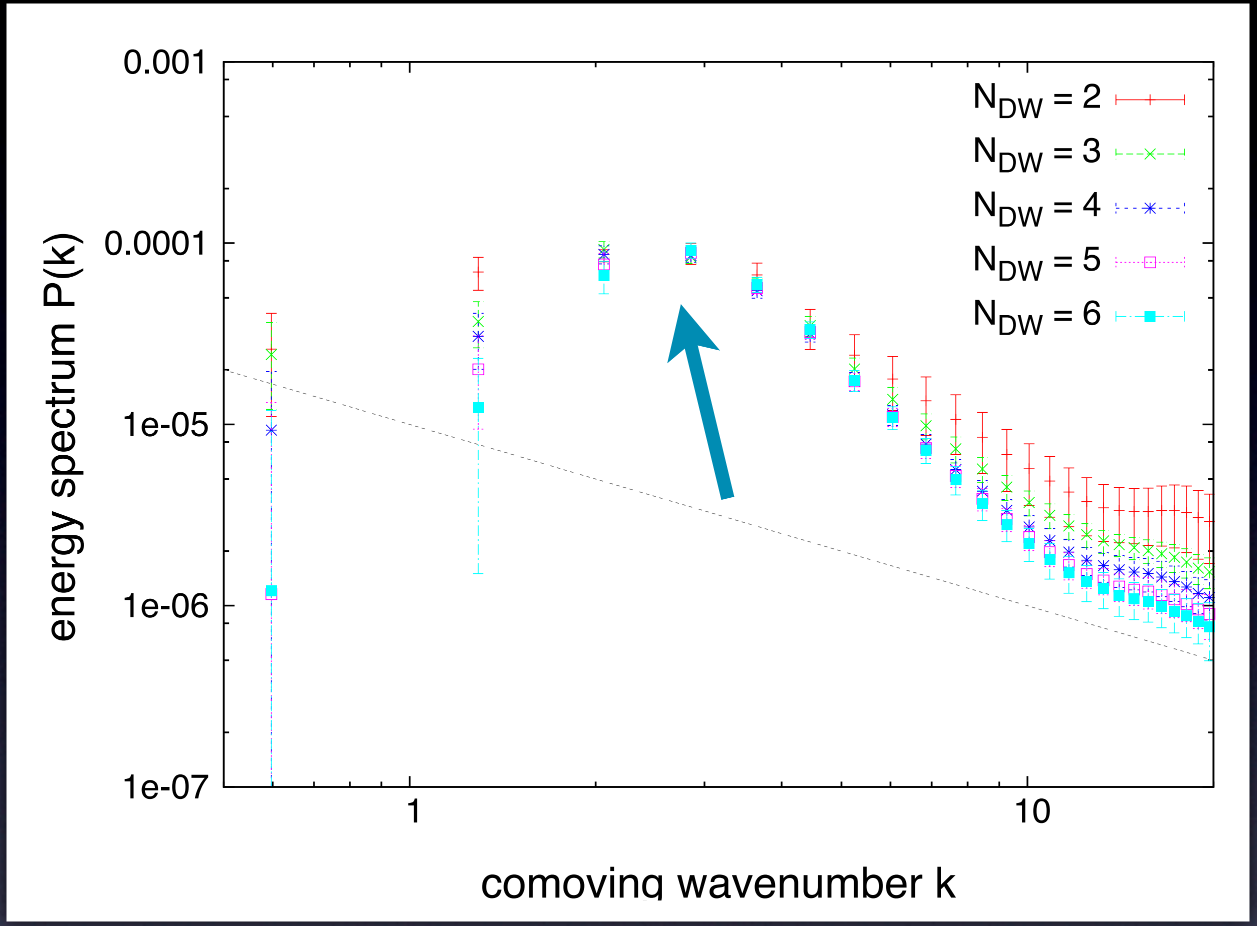
- Number of grids in simulation box : $N^3 = 512^3$

Spectrum of axions

$N_{DW}=1$



$N_{DW}>1$



Peaked at

$$\langle \omega_a \rangle \simeq \mathcal{O}(1) \times m_a$$

supports case A
(axions are mildly relativistic)

Contribution for relic
CDM abundance is large

$$\rho_a(t_{\text{today}}) = m_a \frac{\rho_a(t_{\text{decay}})}{\langle \omega_a \rangle} \left(\frac{R(t_{\text{decay}})}{R(t_{\text{today}})} \right)^3$$

$N_{DW}>1$ (long-lived walls)

- Cold dark matter abundance

$$\rho_{\rm wall}(t_{\rm dec}) \longrightarrow \rho_a \quad \Rightarrow \quad \Omega_a h^2 \le \Omega_{\rm CDM} h^2 \simeq 0.11$$

- Neutron electric dipole moment

Non zero value of Ξ $\qquad \Delta V = -\Xi \eta^3 (\Phi e^{-i\delta} + {\rm h.c.})$

$\Rightarrow$ shift the CP conserving minimum

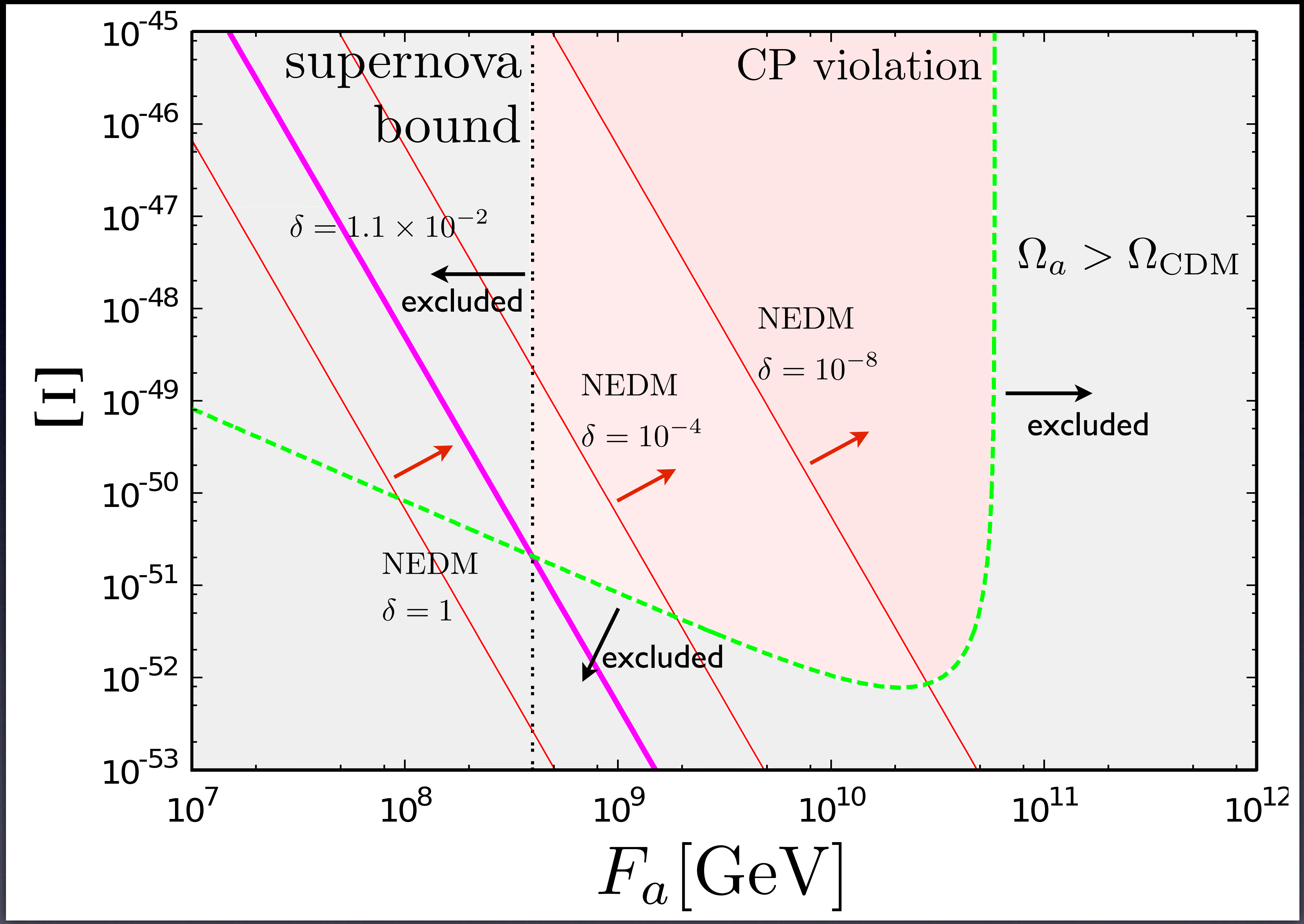
$$\bar{\theta} = \frac{\langle a \rangle}{F_a} = \frac{2 N_{\rm DW} \Xi \eta^2 \sin\delta}{m_a^2 + 2\Xi\eta^2\cos\delta} < 0.7 \times 10^{-11}$$

- Cooling rate of Supernova 1987A Raffelt (2008)

$$F_a > 4 \times 10^8 {\rm GeV}$$

$N_{DW}>1$ (long-lived walls)

Constraint on the bias term $\Delta V = -\Xi \eta^3 (\Phi e^{-i\delta} + \text{h.c.})$

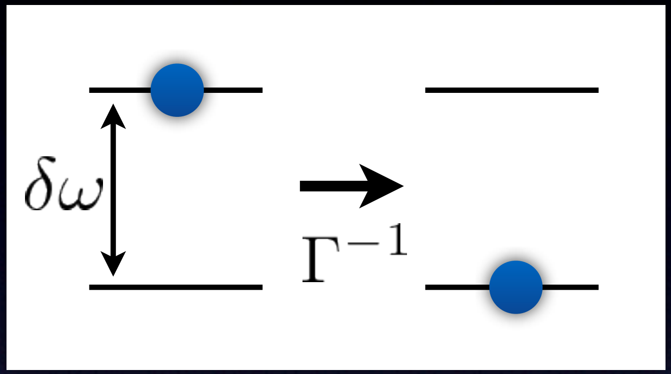


Hiramatsu, Kawasaki, KS and Sekiguchi, 1207.3166

δ must be highly suppressed

Transition in two different regimes

- Consider transitions between different quantum states.



- Two different regimes
- WIMPs $\delta\omega \to \text{large}$

$$\delta\omega \gg \Gamma$$

energy exchanged in the transitions

transition rate

"particle kinetic regime"

- axions $\delta\omega \to \text{small}$

$$\delta\omega \ll \Gamma$$

"condensed regime"

A transition makes sense if $\mathcal{N}\,\delta\omega \gg \Gamma$

Previous study

Erken, Sikivie, Tam, Yang, PRD85, 063520 (2012)

- Time evolution of quantum operators in the Heisenberg picture

$$H = \sum_i \omega_i a_i^\dagger a_i + \sum_{i,j,k,l} \frac{1}{4} \Lambda_{kl}^{ij} a_k^\dagger a_l^\dagger a_i a_j$$

l : label of the state (momentum)

$$\mathcal{N}_l = a_l^\dagger a_l$$

$$\dot{\mathcal{N}}_l = i[H, \mathcal{N}_l]$$

$$= i \sum_{i,j,k} \frac{1}{2} (\Lambda_{ij}^{kl} a_i^\dagger a_j^\dagger a_k a_l e^{-i\Omega_{ij}^{kl} t} - \text{H.c.})$$

Leading contribution in the condensed regime

$$\Omega_{ij}^{kl} t \ll 1$$

$$\dot{\mathcal{N}}_l \sim \mathcal{O}(\Lambda_{ij}^{kl})$$

$$+ \sum_{k,i,j} \frac{1}{2} |\Lambda_{ij}^{kl}|^2 [\mathcal{N}_i \mathcal{N}_j (\mathcal{N}_l + 1)(\mathcal{N}_k + 1)$$

$$- \mathcal{N}_l \mathcal{N}_k (\mathcal{N}_i + 1)(\mathcal{N}_j + 1)] \frac{2}{\Omega_{ij}^{kl}} \sin(\Omega_{ij}^{kl} t) + \ldots$$

reduce to Boltzmann eq. in the particle kinetic regime

$$\Omega_{ij}^{kl} t \gg 1$$

$$\dot{\mathcal{N}}_l \sim \mathcal{O}(|\Lambda_{ij}^{kl}|^2)$$

$$\Omega_{ij}^{kl} \equiv \omega_k + \omega_l - \omega_i - \omega_j$$

- Axions : condensed regime ($\Omega_{ij}^{kl} \sim m_a \delta v^2 < t^{-1}$)

  enhancement of interaction rate $\Gamma \sim \dot{\mathcal{N}}/\mathcal{N} \sim \mathcal{O}(\Lambda)$

- What about the quantum-mechanical averages $\langle \dot{\mathcal{N}}_l(t) \rangle$?

“In” state

- $|\mathrm{in}\rangle$ = a state which represents the coherent oscillation of axions
- For axions “wavy fields”

use a coherent state $$|\{\alpha\}\rangle = \prod_i e^{-\frac{1}{2}|\alpha_i|^2} \sum_{n=0}^{\infty} \frac{\alpha_i^n}{n!\sqrt{V^n}} (a_i^\dagger)^n |0\rangle$$

$$a_i|\alpha_i\rangle = V^{1/2}\alpha_i|\alpha_i\rangle$$ with $a_i|0\rangle = 0$

- Field amplitude

hereafter, ϕ = axion field

$$\phi = \frac{1}{V}\sum_n \frac{1}{\sqrt{2E_{p_n}}}(e^{ip_n\cdot x}a_n + e^{-ip_n\cdot x}a_n^\dagger)$$

$$\langle\{\alpha\}|\phi|\{\alpha\}\rangle = \sum_n \frac{1}{\sqrt{2m_a V}}(e^{-im_a t + i\mathbf{p_n}\cdot\mathbf{x}}\alpha_n + e^{im_a t - i\mathbf{p_n}\cdot\mathbf{x}}\alpha_n^*)$$

$$= \sum_n \sqrt{\frac{2}{m_a V}}|\alpha_n|\cos(m_a t - \mathbf{p_n}\cdot\mathbf{x} - \beta)$$ classical field trajectory

- For other species “point particles” (photons, baryons, WIMPs,...)

use a number state $$|\{\mathcal{N}\}\rangle = \prod_k \frac{1}{\sqrt{\mathcal{N}_k! V^{\mathcal{N}_k}}}(a_k^\dagger)^{\mathcal{N}_k}|0\rangle$$

“Zero modes”

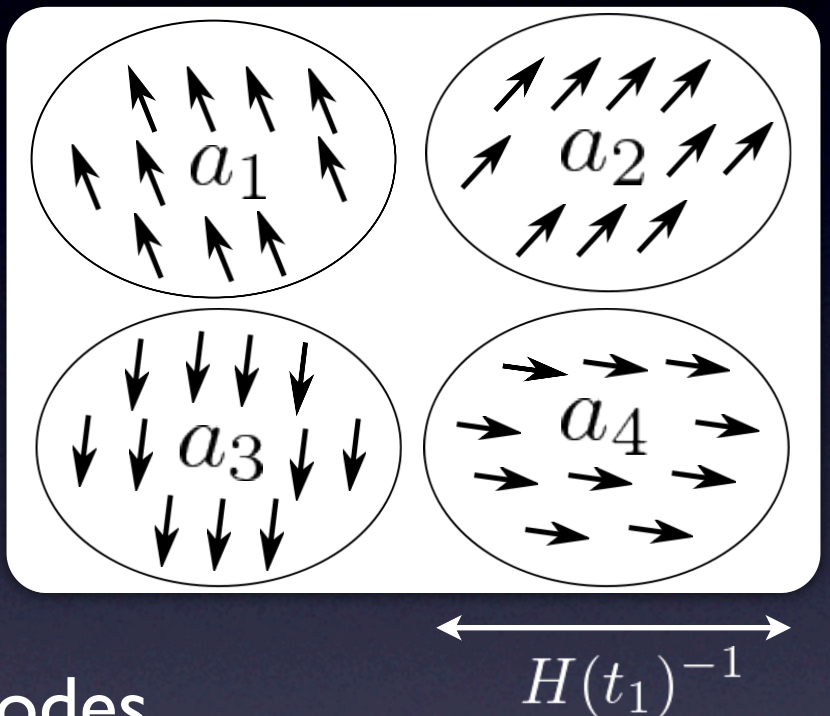
- Initial time t_1 (QCD phase transition) :

  amplitudes of oscillation might be uncorrelated

  beyond the horizon

  → axions have non-zero momenta

  $$p/R(t_1) \lesssim H(t_1) \sim m_a(t_1)$$



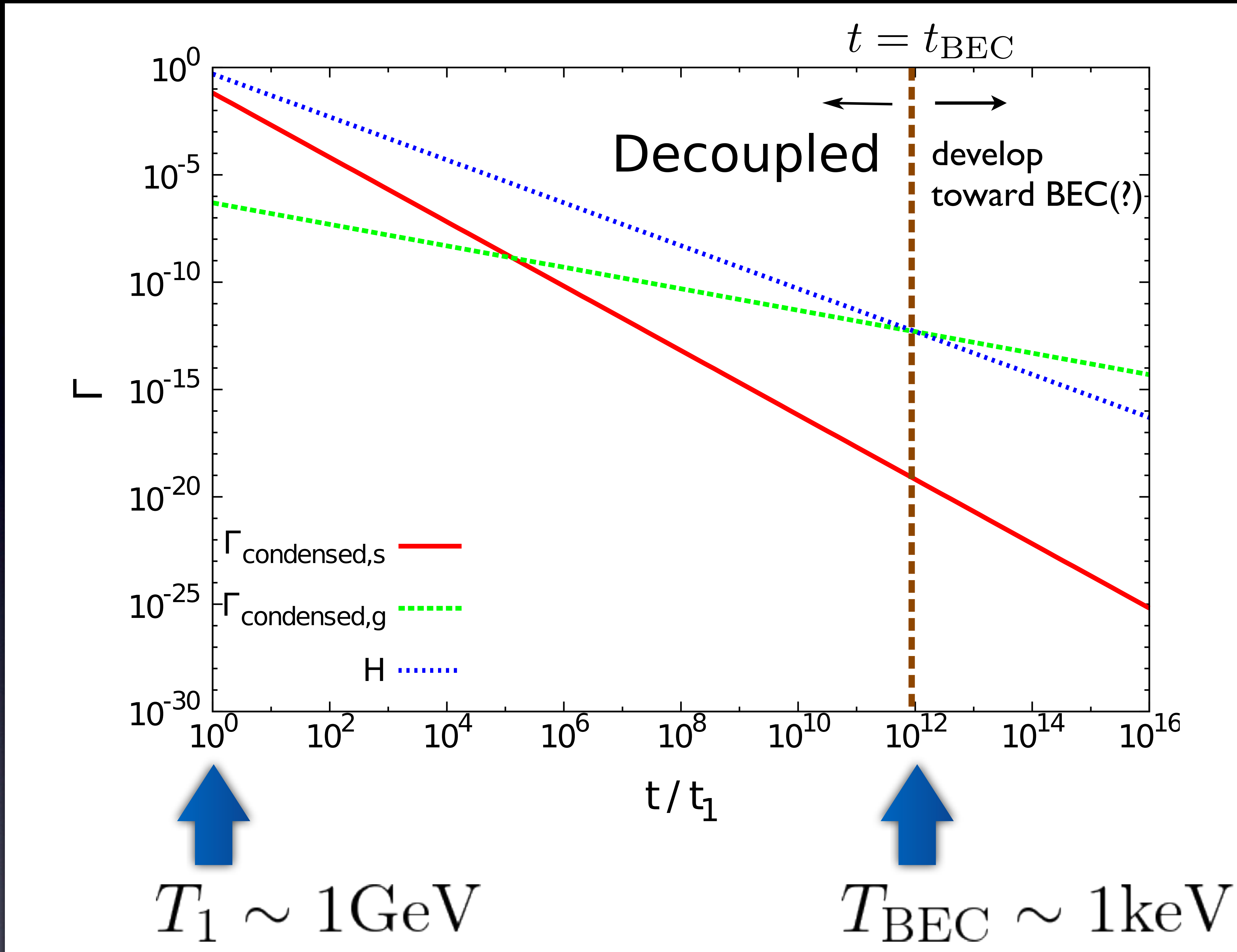
- Assume plural (say K) oscillating modes

$$|\{\alpha\}\rangle = \prod_i^K e^{-\frac{1}{2}|\alpha_i|^2} \sum_{n=0}^{\infty} \frac{\alpha_i^n}{n!\sqrt{V^n}} (a_i^\dagger)^n |0\rangle \qquad |\mathbf{p}_i|/R(t_1) \lesssim H(t_1) \sim m_a(t_1)$$

for $\quad i = 1, \ldots, K$

- Number density

$$n_a = \frac{1}{V} \sum_n \langle \{\alpha\} | \mathcal{N}_n | \{\alpha\} \rangle = \frac{1}{V} \sum_i^K |\alpha_i|^2 \equiv \sum_i^K n_{c,i}$$



scalar phi^4 $\quad H_I = -\int d^3x \frac{\lambda}{4!}\phi^4 \quad \Rightarrow \quad \Gamma_{\mathrm{condensed},s} \simeq \frac{\lambda n_a}{4m_a^2} \quad \propto 1/R^3(t)$

gravity $\quad H_I = -\frac{G}{2}\int d^3x d^3x' \frac{\rho(\mathbf{x},t)\rho(\mathbf{x}',t)}{|\mathbf{x}-\mathbf{x}'|} \quad \Rightarrow \quad \Gamma_{\mathrm{condensed},g} \simeq \frac{4\pi G m_a^2 n_a}{(\delta p)^2} \propto 1/R(t)$

Interaction with other species

- Interaction with other species b

$$H_{I,b}(t) = \frac{1}{V^4}\sum_{ijkl}\frac{1}{4}\Lambda_b{}^{ij}{}_{kl}e^{-i\Omega^{ij}_{kl}t}a^\dagger_k b^\dagger_l a_i b_j$$

- Assume b particles are represented as a number state

$$|\mathrm{in}\rangle = \prod_k \frac{1}{\sqrt{\mathcal{N}_k! V^{\mathcal{N}_k}}}(b^\dagger_k)^{\mathcal{N}_k}|\{\alpha\}\rangle$$

while $$|\{\alpha\}\rangle = \prod_i^K e^{-\frac{1}{2}|\alpha_i|^2}\sum_{n=0}^{\infty}\frac{\alpha_i^n}{n!\sqrt{V^n}}(a^\dagger_i)^n|0\rangle$$

- First order term exactly vanishes

$$\langle[H_{I,b}(t),\mathcal{N}_p]\rangle = 0$$

- Interaction with other species is second order effect.
    - Axions do not have thermal contact with other particles → does not conflict with standard cosmology

Action of the scalar-graviton system

- Assumption

  → FRW background geometry is supported by radiations

  dynamical d.o.f. : ϕ , $g_{\mu\nu}$, $\delta\rho_{\rm rad}$

- Take the unitary gauge ($\delta\rho_{\rm rad} = 0$, $\rho_{\rm rad} = \bar{\rho}_{\rm rad}(t)$)
  - Radiation perturbation $\delta\rho_{\rm rad}$ is "eaten" by the metric
  - The graviton (metric) has three d.o.f.

    1 curvature perturbation ζ
    2 tensor perturbations γ_{ij}

- Time diffeo. is broken by $\bar{\rho}_{\rm rad}(t)$
  - Use residual spatial diffs to constrain the action for $g_{\mu\nu}$

Cheung, Fitzpatrick, Kaplan, Senatore, Creminelli, JHEP03(2008)014

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m_a^2\phi^2\right]$$
$$+ \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2\mathcal{R} + M_{\rm Pl}^2\dot{H}g^{00} - M_{\rm Pl}^2(3H^2+\dot{H}) + \frac{M_2^4}{2}(g^{00}+1)^2\right]$$

M_2 : a theoretical parameter related to the sound speed c_s $\quad c_s^2 = \dfrac{-M_{\rm Pl}^2\dot{H}}{-M_{\rm Pl}^2\dot{H}+2M_2^4} \simeq 1/3$

Effective quartic interactions

- ADM decomposition

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$h_{ij} = R^2(t)\underline{e^{2\zeta}(e^{\gamma})_{ij}} \quad \text{with} \quad \gamma_{ii} = \partial_i \gamma_{ij} = 0$$

fluctuations around FRW background

$\zeta, \quad \gamma_{ij}$: dynamical fields

$N, \quad N^i$: Lagrange multipliers

- Relevant interactions in the regime $m_a \gg H, \ k/R$

(Eliminating auxiliary fields $N, \ N^i$)

$$H_{I,\zeta\phi^2} \simeq \int d^3x R^3 \left[\frac{1}{2H}\dot{\zeta}(\dot{\phi}^2 + m_a^2\phi^2) - \frac{3}{2}\zeta(\dot{\phi}^2 - m_a^2\phi^2) \right] + \mathcal{O}(H^2\zeta\phi^2)$$

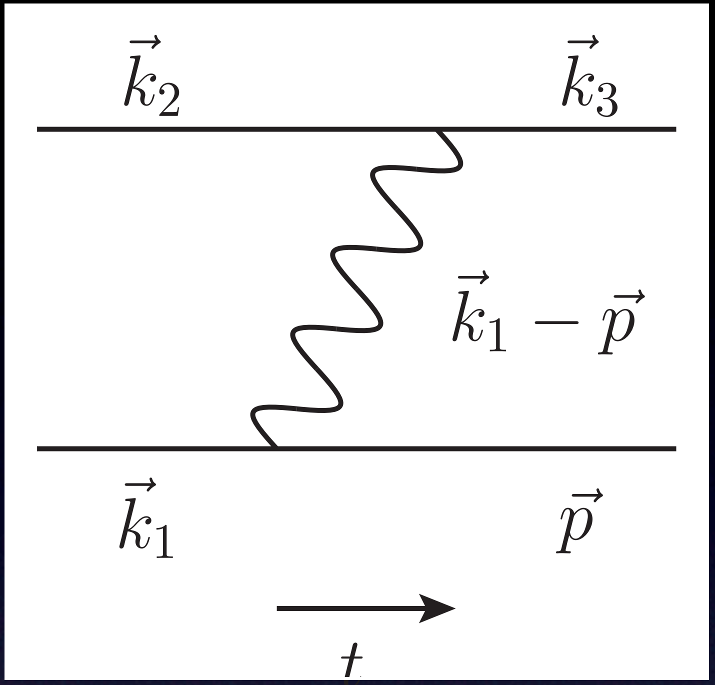
$$H_{I,\phi^4} \simeq \int d^3x R^3 \left[\frac{1}{16 M_{\rm Pl}^2 H^2 \tilde{\epsilon}}(\dot{\phi}^2 + m_a^2\phi^2)^2 \right] + \mathcal{O}(m_a^2\phi^4/M_{\rm Pl}^2)$$

where $\tilde{\epsilon} = 2c_s^{-2}$

$$H_{I,\gamma\phi^2} = \int d^3x R^3 \left[-\frac{1}{2}\gamma_{ij}\frac{\partial_i\phi\partial_j\phi}{R^2} \right] \sim \mathcal{O}(H^2\gamma\phi^2) \qquad \text{(subdominant)}$$

Contributions for $\langle \mathcal{N}_{\mathbf{p}}(t) \rangle$

$$\langle \mathcal{N}_{\mathbf{p}}(t) \rangle \simeq \langle \mathcal{N}_{\mathbf{p}}(t_0) \rangle + i \int_{t_0}^{t} dt_1 \langle [H_{I,\phi^4}(t_1), \mathcal{N}_{\mathbf{p}}] \rangle$$
$$+ i^2 \int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 \underbrace{\langle [H_{I,\zeta\phi^2}(t_1), [H_{I,\zeta\phi^2}(t_2), \mathcal{N}_{\mathbf{p}}]] \rangle}_{\text{contraction of cubic interactions}}$$
$$\simeq \langle \mathcal{N}_{\mathbf{p}}(t_0) \rangle + i \int_{t_0}^{t} dt_1 \langle [H_{\text{eff}}(t_1), \mathcal{N}_{\mathbf{p}}] \rangle$$



- Effective Hamiltonian for tree-level analysis

$$H_{\text{eff}}(t) = \int \left(\prod_{i=1}^{4} \frac{d^3 k_i}{(2\pi)^3} \right) (2\pi)^3 \delta^{(3)}(\mathbf{k_1} + \mathbf{k_2} - \mathbf{k_3} - \mathbf{k_4}) F(t; |\mathbf{k_1} - \mathbf{k_3}|) a^\dagger_{\mathbf{k_1}} a^\dagger_{\mathbf{k_2}} a_{\mathbf{k_3}} a_{\mathbf{k_4}}$$

$$F(t;k) = -\frac{2\pi G m_a^2}{R^3(t)} \frac{R^2(t)}{k^2} f\left(\frac{k}{k_H(t)} \right), \qquad f(x) = 1 - \cos x - x \sin x$$

$k_H(t) = R(t)H(t)/c_s$: sound horizon

- $f(k/k_H) \to 1 + (\text{highly oscillating terms})$ for $k/k_H \gg 1$

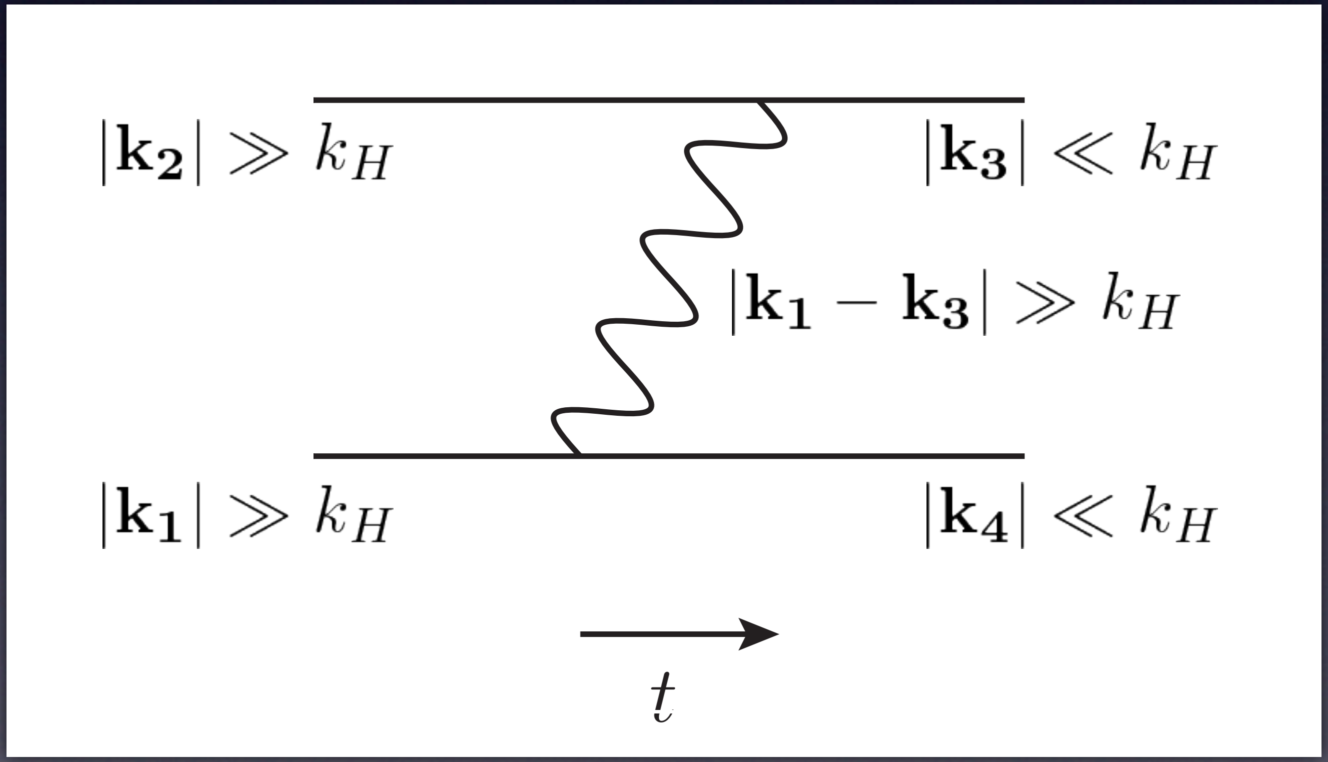
- Reproduces the result in Newtonian approx.

$$F(t; |\mathbf{k_1} - \mathbf{k_3}|) \to -\frac{2\pi G m_a^2 / R^3(t)}{|\mathbf{k_1} - \mathbf{k_3}|^2 / R^2(t)} \Rightarrow \boxed{\Gamma_g \sim \frac{G m_a^2 n_a}{(\delta p)^2}}$$

Transition into BEC ?

- Time scale of the process whose momentum transfer satisfies $|\mathbf{k_1} - \mathbf{k_3}| \gg k_H(t) = R(t)H(t)/c_s$ can be estimated as $\sim \Gamma_g^{-1}$
- Allows transition between sub-horizon & super-horizon modes

example :



- Relevance to the gravitational thermalization ?

Interaction rate for modes outside the horizon

- For the opposite limit $|\mathbf{k_1} - \mathbf{k_3}| \ll k_H$ (possible only for $k_1, k_2, k_3, k_4 \ll k_H$)

$$F(t; |\mathbf{k_1} - \mathbf{k_3}|) \to \frac{\pi G m_a^2 c_s^2}{R^3(t) H^2(t)} \quad \Rightarrow \quad \Gamma \sim \frac{G m_a^2 n_a}{H^2}$$

- Similar result can be obtained when we consider the evolution of the background field $\phi_{\rm cl} = \langle \alpha_{\mathbf{p=0}} | \phi | \alpha_{\mathbf{p=0}} \rangle$

From the effective action for $\phi_{\rm cl}$

$$e^{i\Gamma_{\rm eff}[\phi_{\rm cl}]} = \int \mathcal{D}\phi' \mathcal{D}\zeta \mathcal{D}\gamma_{ij} e^{iS[\phi', \zeta, \gamma_{ij}; \phi_{\rm cl}]}$$

$$\Gamma_{\rm eff}[\phi_{\rm cl}] = \int d^4x R^3 \left[\frac{1}{2}\dot{\phi}_{\rm cl}^2 - \frac{1}{2} m_a^2 \phi_{\rm cl}^2 - \frac{1}{16 H^2 M_{\rm Pl}^2 \tilde{\epsilon}} \left(\dot{\phi}_{\rm cl}^2 + m_a^2 \phi_{\rm cl}^2 \right)^2 + \dots \right]$$

$$\Rightarrow \quad \ddot{\phi}_{\rm cl} + 3H\dot{\phi}_{\rm cl} + m_a^2 \phi_{\rm cl} \approx -\frac{c_s^2 m_a^2}{4 H^2 M_{\rm Pl}^2} \phi_{\rm cl} \left(\dot{\phi}_{\rm cl}^2 + m_a^2 \phi_{\rm cl}^2 \right)$$

$$\Rightarrow \quad \Gamma \sim \frac{\dot{N}}{N} \sim -\frac{c_s^2 m_a^3 \phi_{\rm cl}^2}{2 H^2 M_{\rm Pl}^2} \sim \frac{G m_a^2 n_a}{H^2}$$

where $N = \dfrac{R^3 \rho_{\rm free}}{m_a}, \qquad \rho_{\rm free} = \dfrac{1}{2}\dot{\phi}_{\rm cl}^2 + \dfrac{1}{2} m_a^2 \phi_{\rm cl}^2$

- Not the "transition" between superhorizon modes, but the "correction" to the evolution of the background field

Present situation

- Gravitational self-interaction of axions becomes non-negligible

  ➡ Yes (for axions represented by coherent state; irrelevant for those produced by string-wall systems)

- Does it mean the formation of BEC ?

  (here BEC = perfect thermal equilibrium)

  ➡ Perhaps not exactly thermal equilibrium,
  but "quasi-BEC" can be formed in the out-of-equilibrium state

  Berges, Jaeckel, hep-ph/1402.4776

- Do axions have thermal contact with other particle species ?

  ➡ No (?)

  - At least the interaction between coherent state (axions) and number state (other "particles") vanishes at leading order.

    KS, Yamaguchi, PRD87, 085010 (2013)

  - Effects on cosmological parameters might be fictitious

    Erken, Sikivie, Tam, Yang, PRL108, 061304 (2012)

- Is there any distinct signatures on observations ?

  ➡ Still under debate → talk by Prof. S. Davidson