ESPROS DMAPS: Sensor characterization

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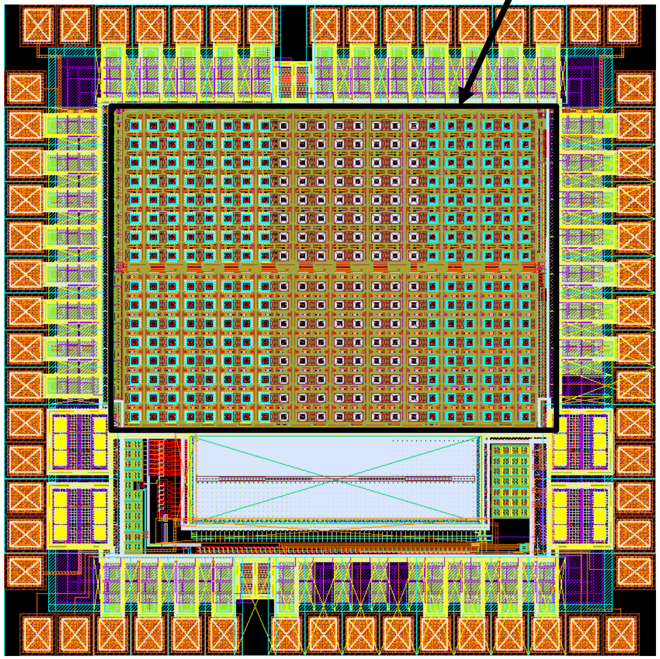
EPCB01 – Depleted Monolithic Pixel Sensor

Features

- Technology: ESPROS, 150 nm
- Deep p-type well to isolate full CMOS electronics
- 6 metal layers

- High resistive n-type bulk (> 2 kΩ cm)
- High voltage at sensor domain possible (~ 20 V)
- Backside p-implantation
- Thinned down to 50 µm



Chip Dim.

1.4×1.4 mm^2

352 pixels

Pixel array features

- Pixel size: 40 × 40 μm^2
- Charge collecting n-implantation area ~ 20 × 20 μm^2

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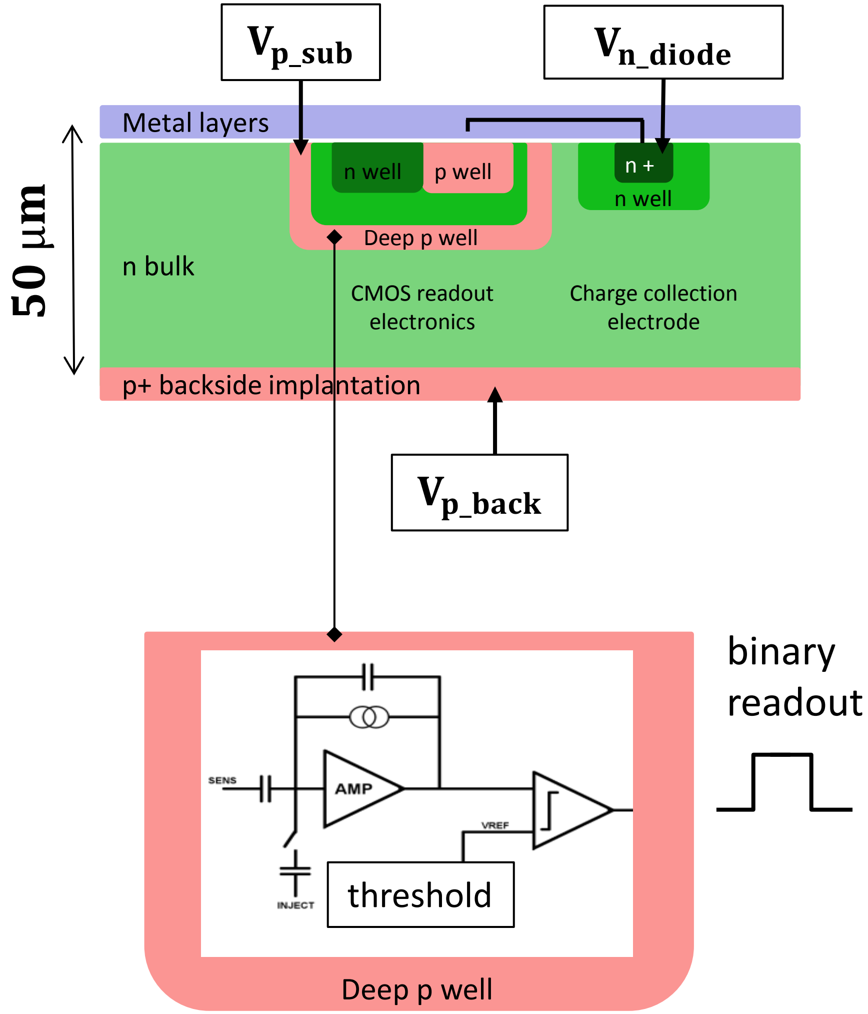
EPCB01 – Sensor and FE

Sensor

- N-implant is charge collecting electrode
- P-backside implantation
- Full CMOS electronics inside deep p-well



Front end

- CSA and comparator

EPCB01 – Sensor and FE

Sensor

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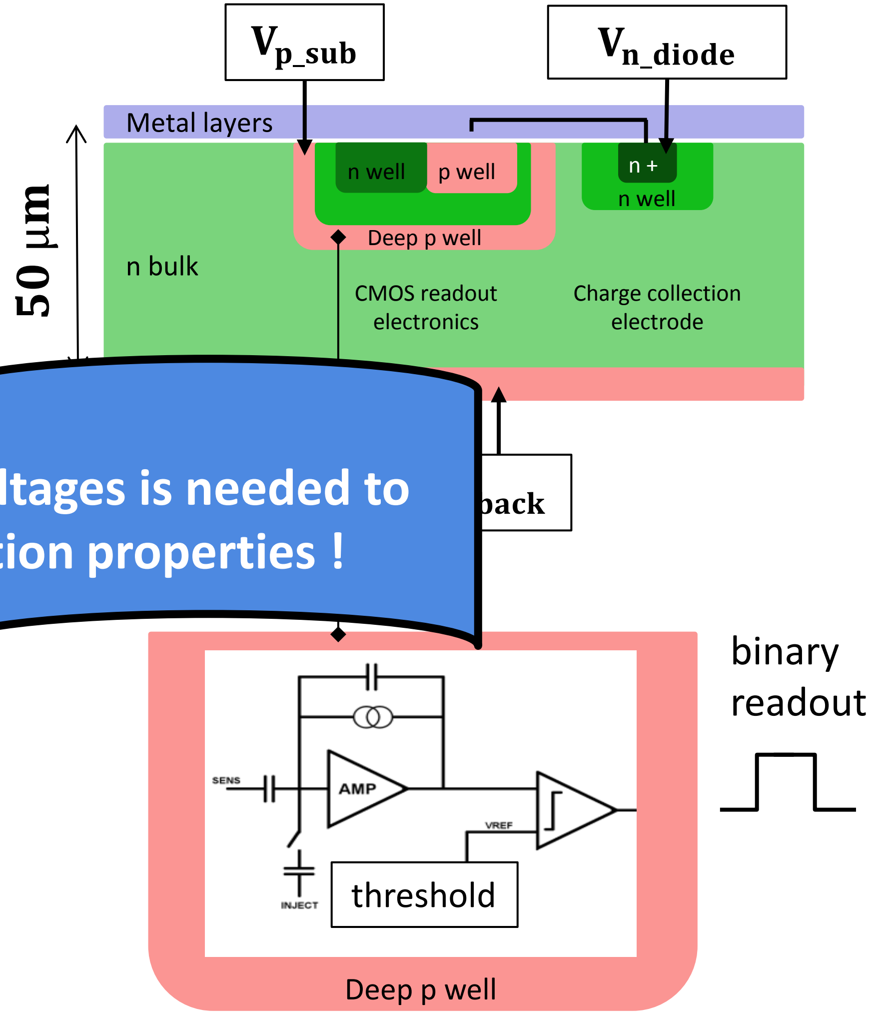
- P-backside implantation
- Full CMOS electronics inside deep p-well



→ Optimization of bias voltages is needed to characterize charge collection properties !

Front end

- CSA and comparator

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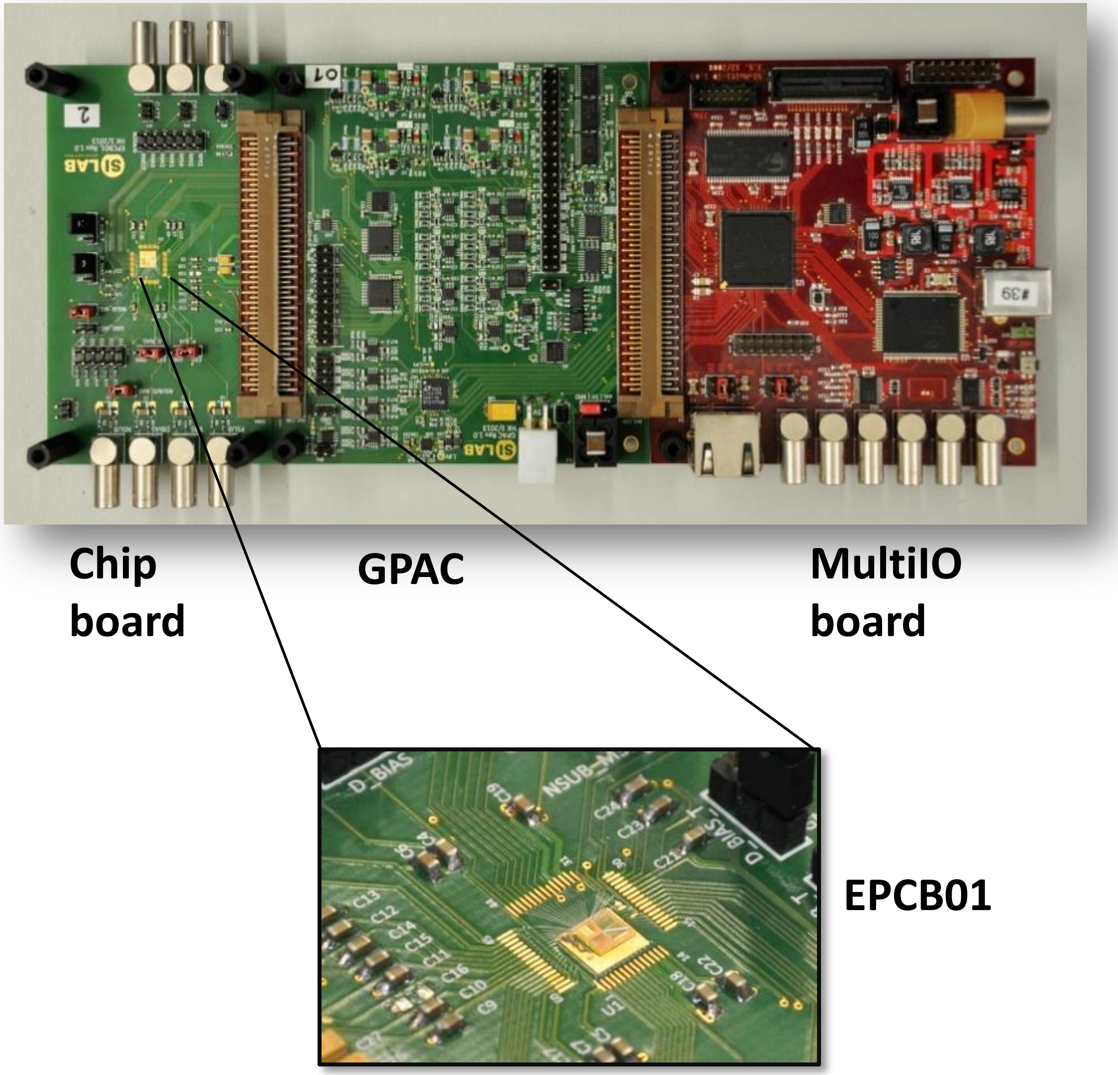
Test setup

Software

- Python project EPCB01 (based on modular test framework BASIL)

Hardware

- FPGA board: S3 MultiIO-board
- General purpose adapter card (analog and digital card with voltage sources, current sources, ADC, Injection circuitry)
- Chip carrier board



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Sensor characterization measurements

Laser

- Response homogeneity
- Bias parameter optimization

Sr 90 and electron beam

- Measure charge collected for MIP
- Cross check of laser results
- Relative CC dependence on bias parameters

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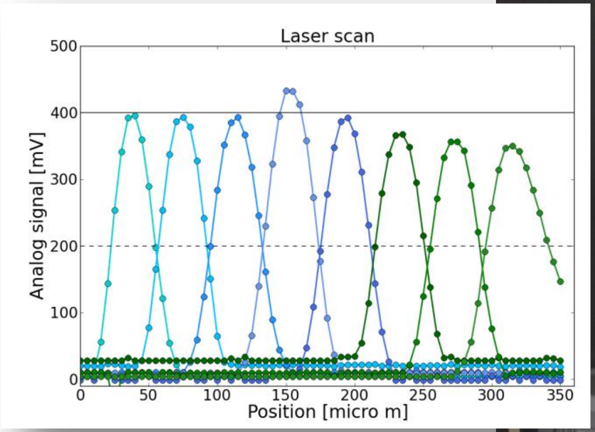
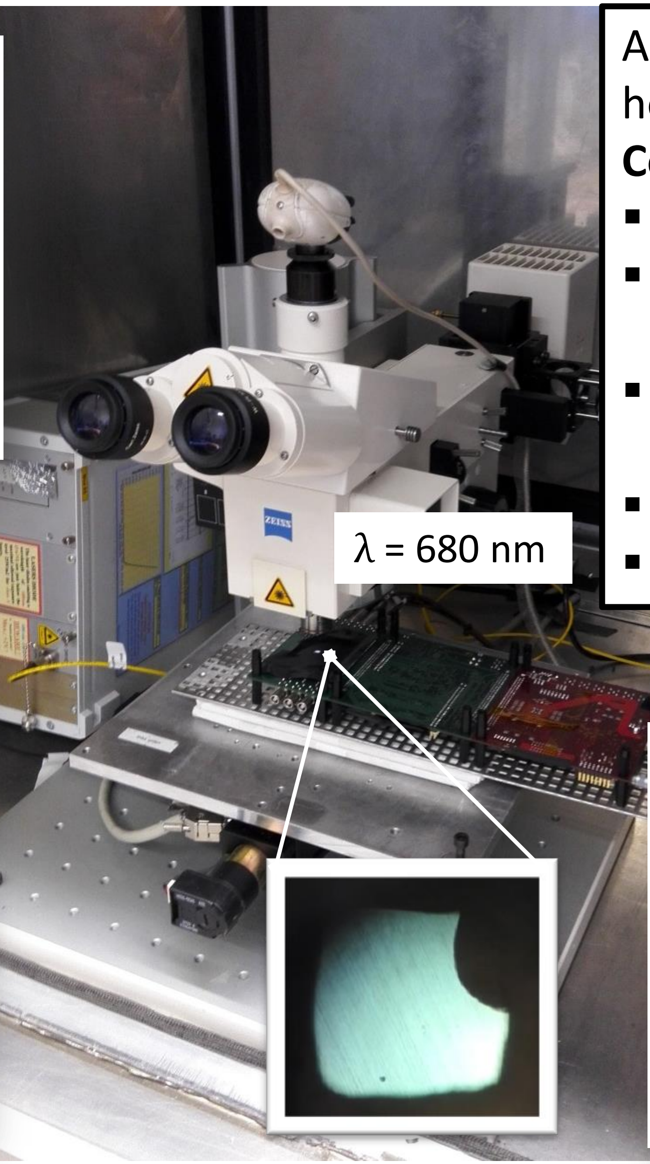
Laser measurements

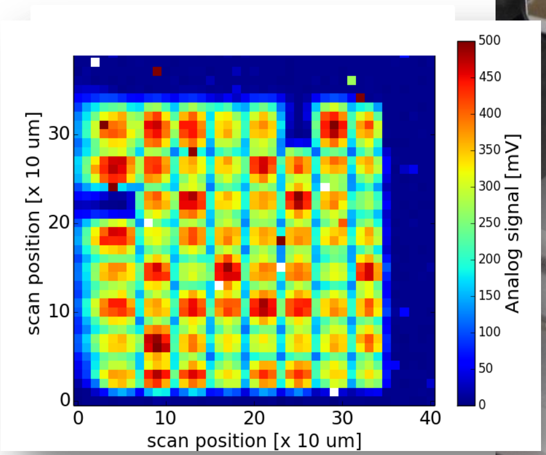


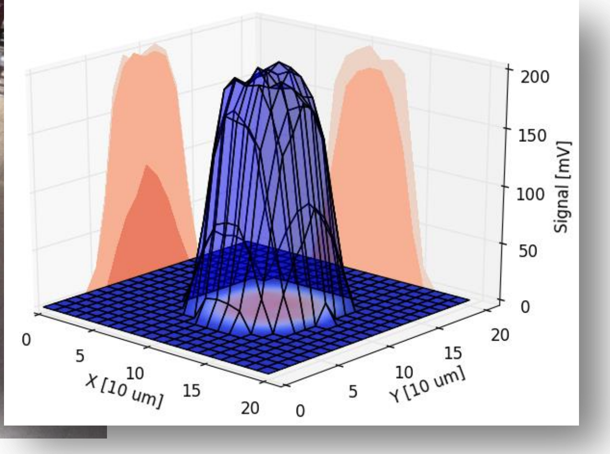


Advantages of laser system for homogeneity studies:

Controlled environment

- Laser spot size ~ 3 µm
- Movable in 2 µm steps → In pixel studies
- Easy repitition → Fast voltage scans
- Long term stability ± 100 e
- Backside illumination





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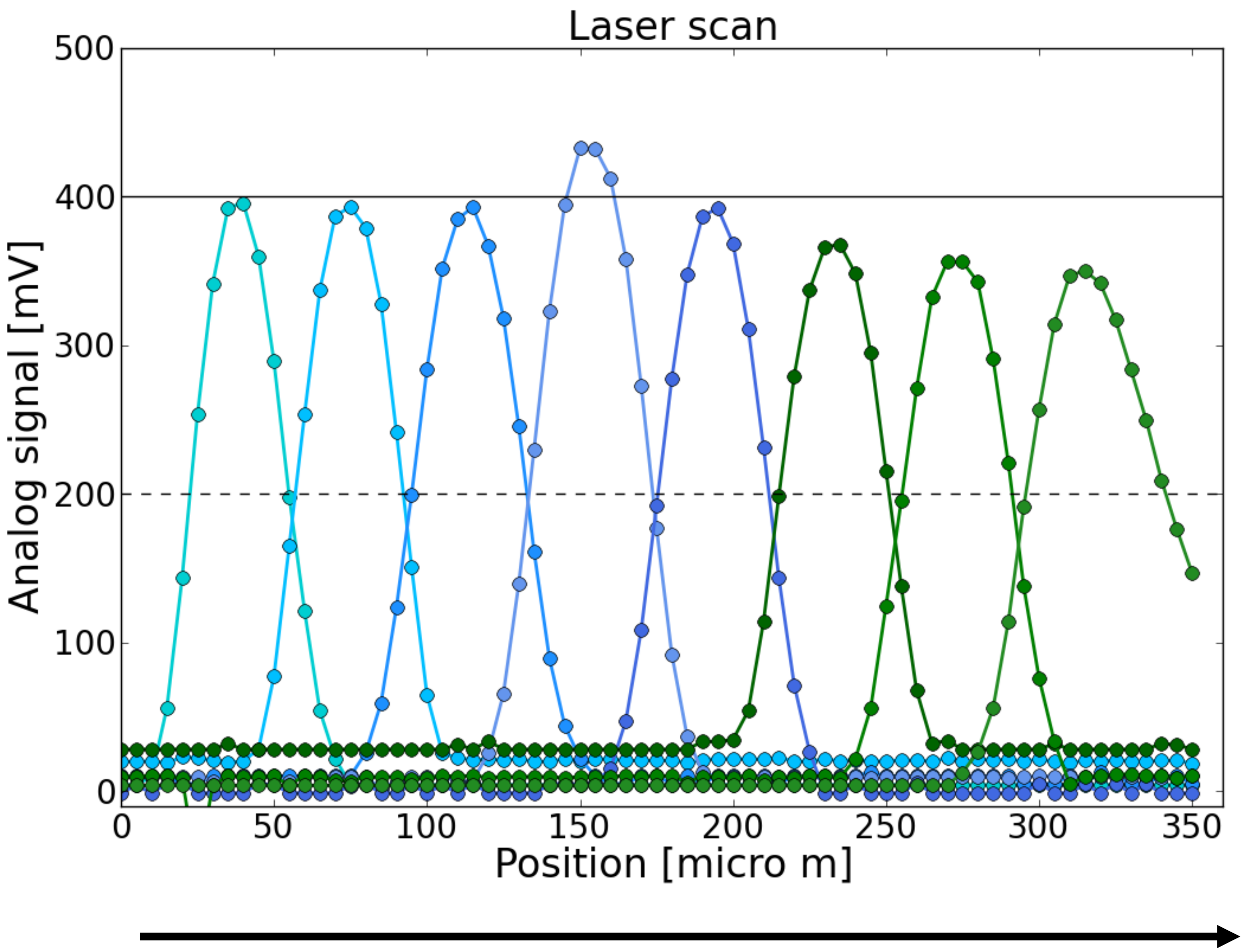
Laser scan 1D



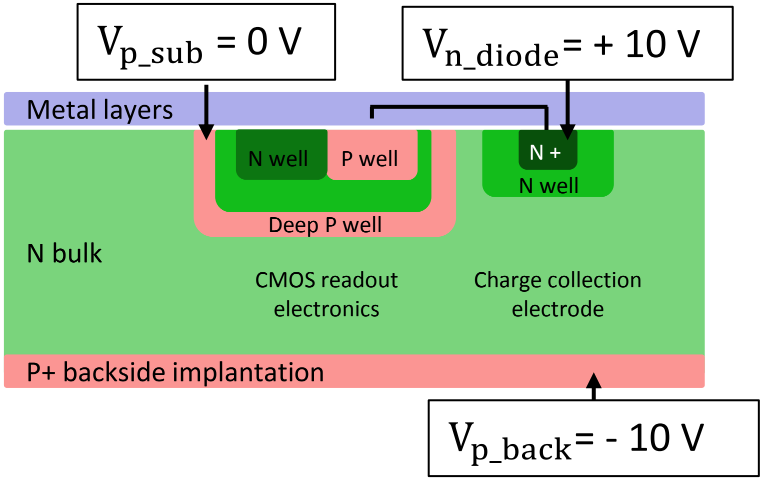
Laser moving direction

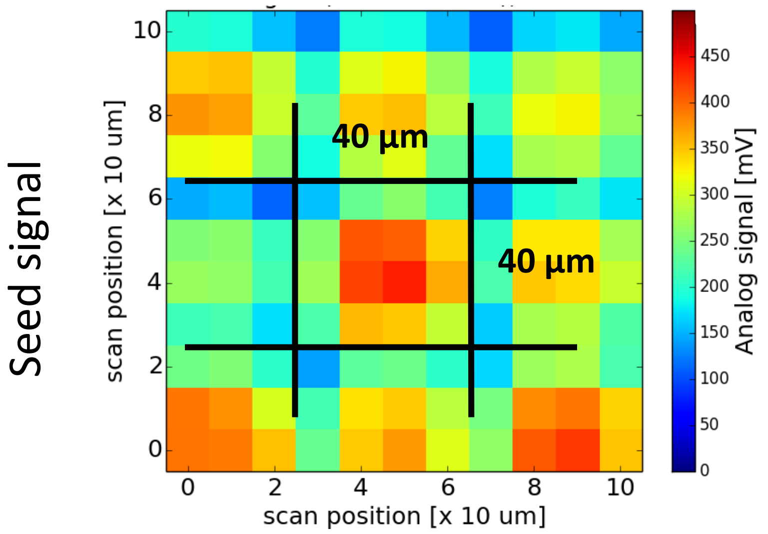
Laser scan over nine pixels

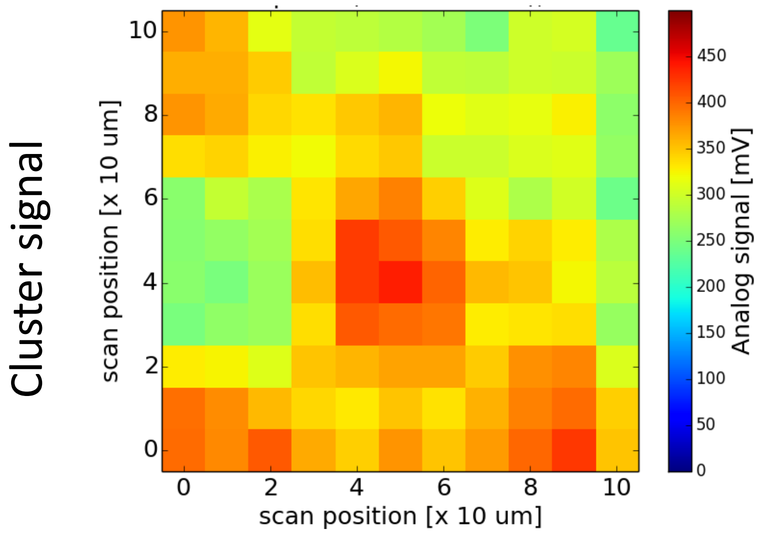
Scan settings

- Step size: 10 µm, 10 x 10 steps

Remark: No fixed pattern noise correction (worst case)







→ Figure of merit: $\Delta \text{Signal} / \overline{\text{Signal}}$

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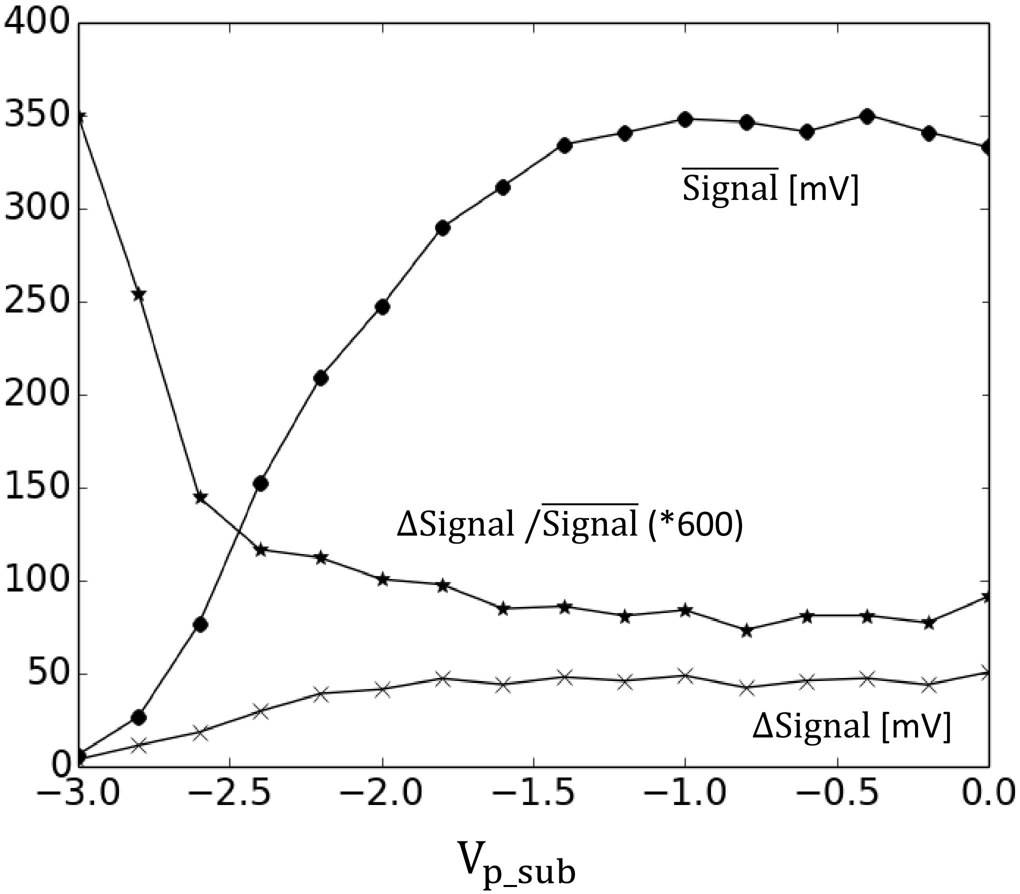
Parameter scan of V_{p_sub} over nine pixels

ΔSignal and $\overline{\text{Signal}}$ versus V_{p_sub}

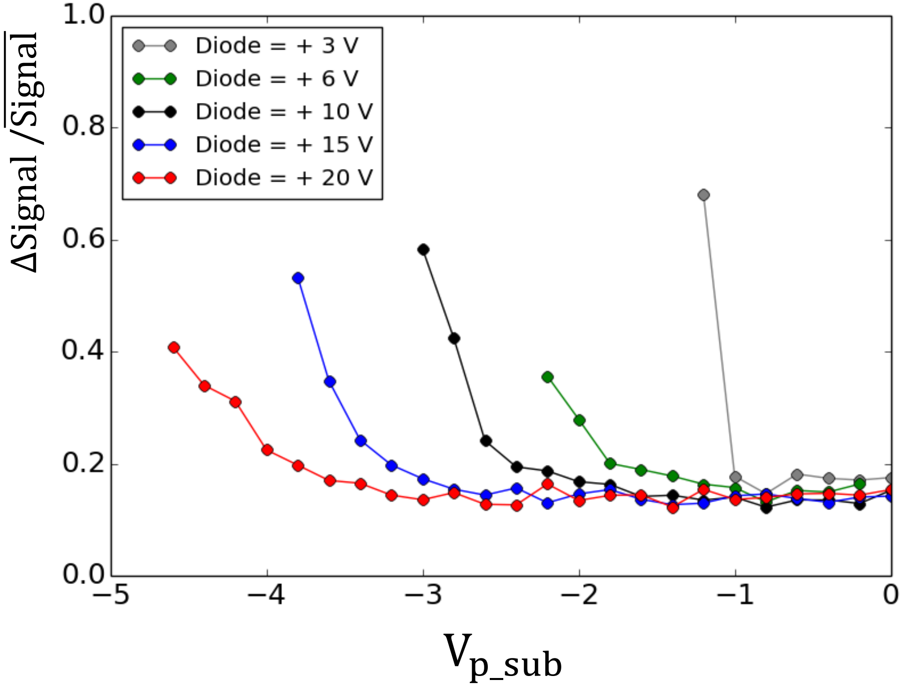


→ Best homogeneity ΔSignal /$\overline{\text{Signal}}$ = 15 % for V_{p_sub} from 0 to - 1.5 V

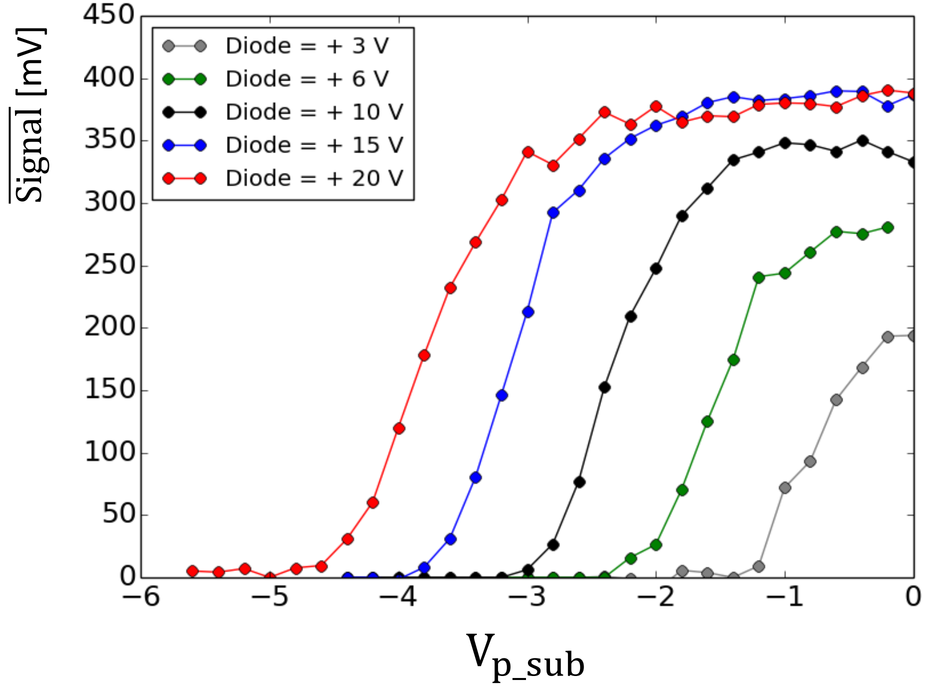
→ No charge collection below V_{p_sub} = - 3 V

Parameter scan of V_{p_sub} and V_{n_diode} over nine pixels

Inhomogeneity



Amplitude

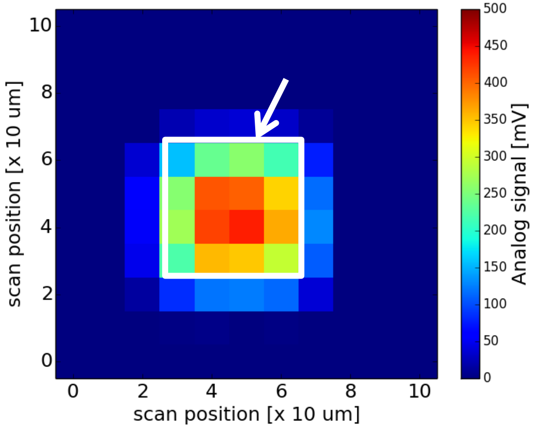


→ Depleted sensor at V_{n_diode} > + 15 V

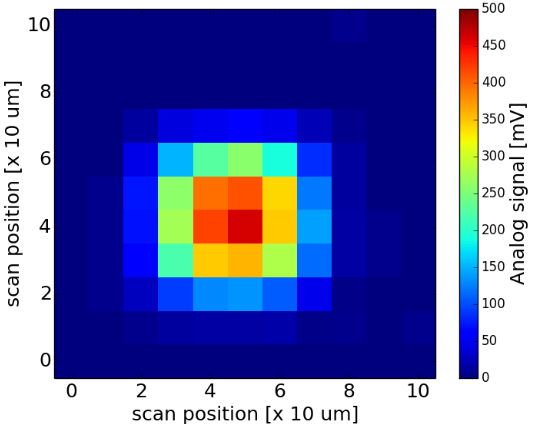
→ With increasing V_{n_diode} , the range for V_{p_sub} with signal increases

Signal of central pixel for different V_{p_sub}

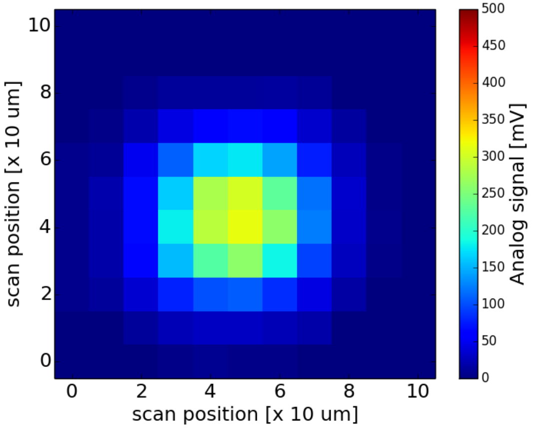
V_{p_sub} = 0 V



V_{p_sub} = - 1 V

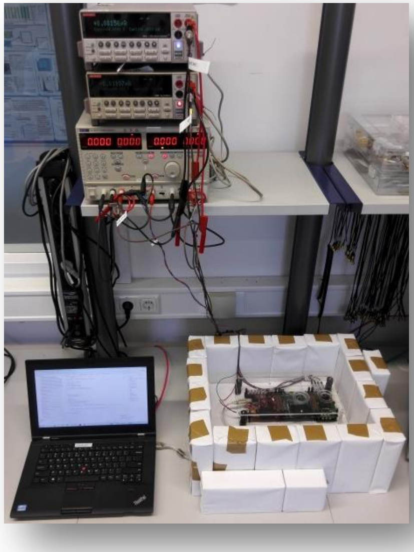


V_{p_sub} = - 2 V



→ Lowering V_{p_sub} the charge spread is larger and the seed signal is decreased

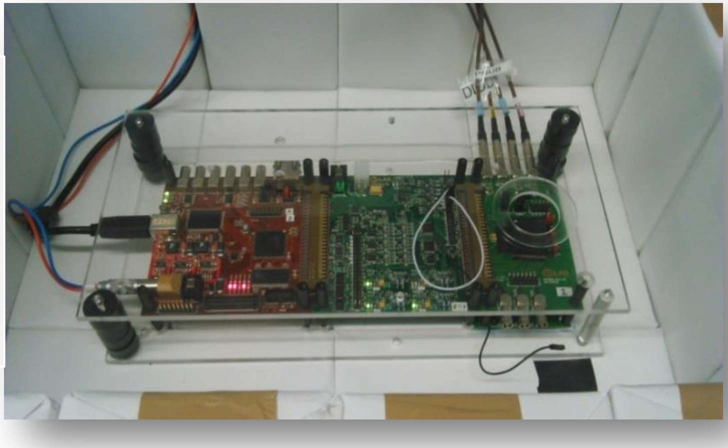
Sr 90 and electron beam measurements

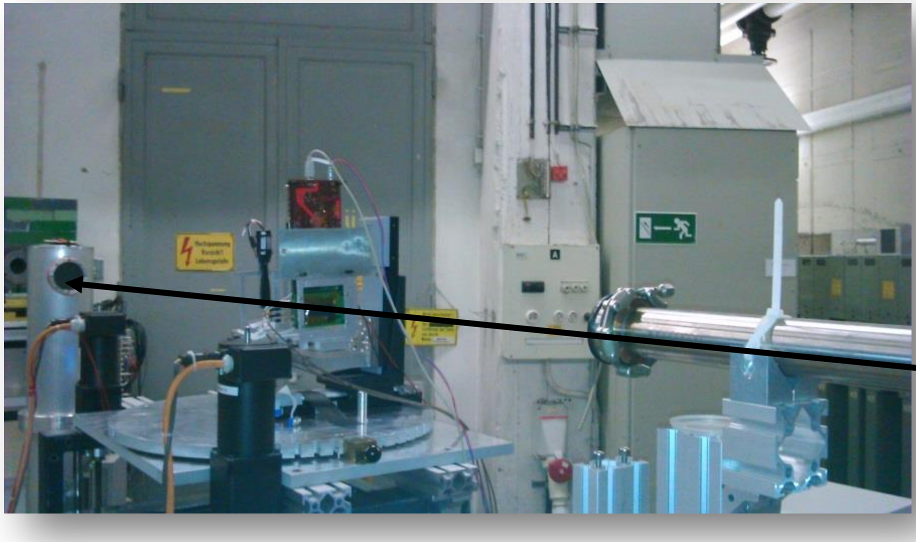


← **Source setup** →

Sr90

Expect $Q_{Sr90}(\mathrm{MPV}) = 3.86$ ke
in 50 µm silicon





← **Test beam setup**

3.2 GeV electron beam

Expect $Q_{MIP}(\mathrm{MPV}) = 3.34$ ke
in 50 µm silicon

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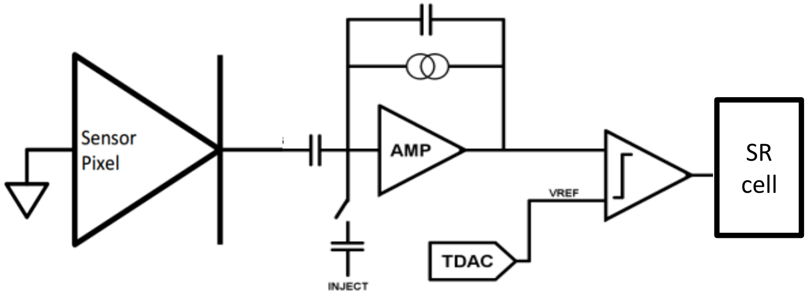
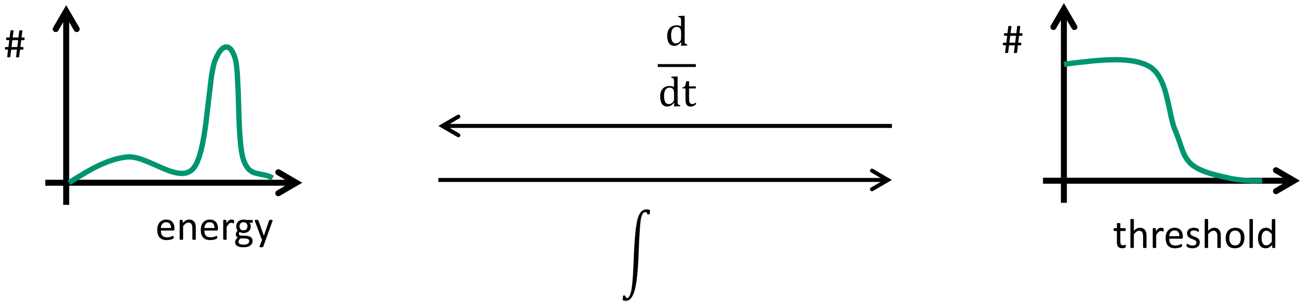
Energy spectrum from a binary output

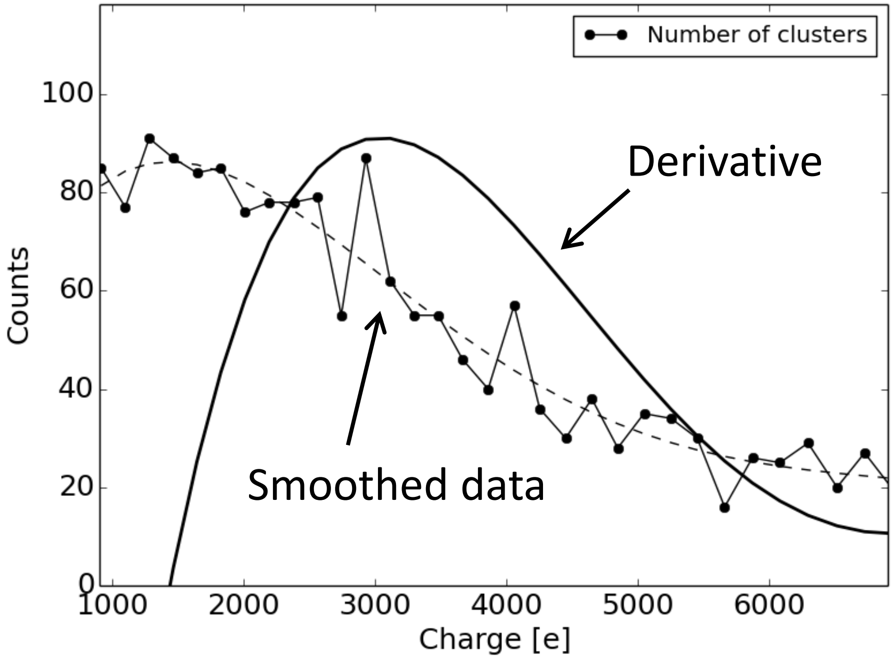


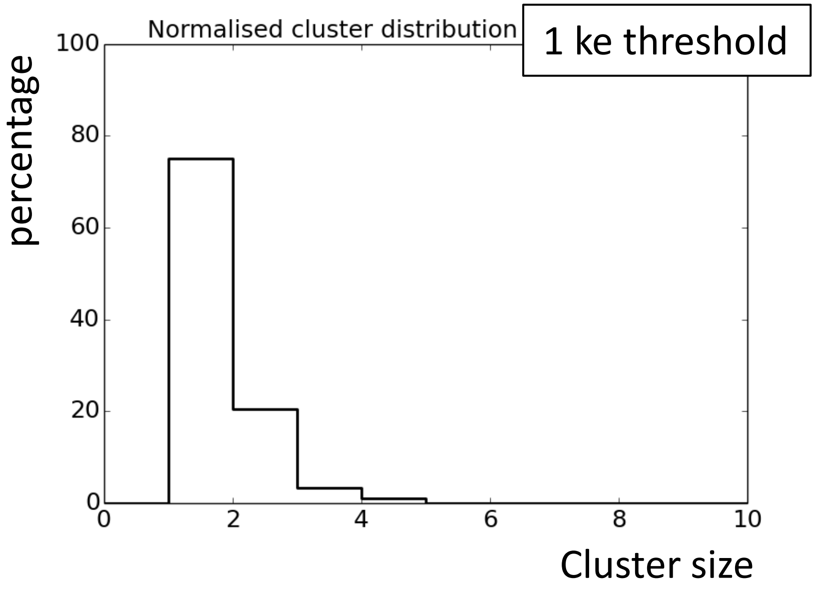
[Pohl, A method for precise single pixel charge reconstruction for counting pixel detectros, To be submitted]

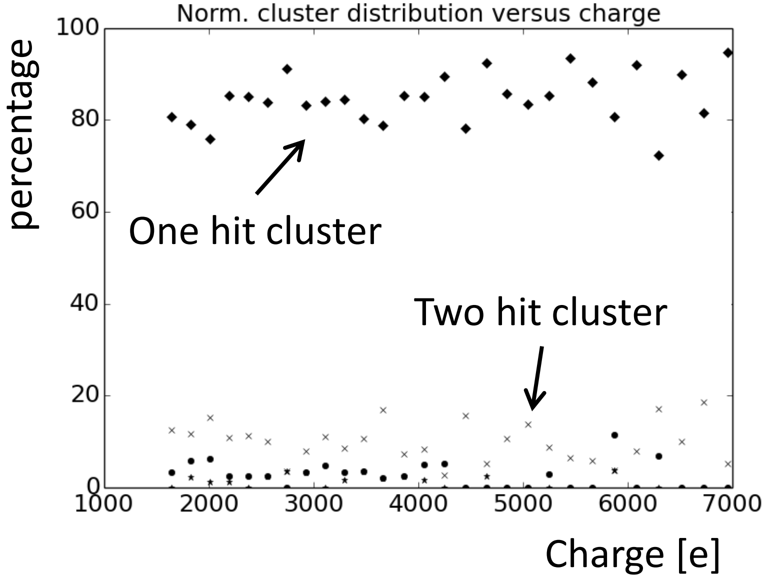
- Count cluster versus thresold voltage
- No trigger
- Analysis with 36 pixels
- Calibrate x-axis with internal injection, global calibration with cut at 7 ke



Results of Sr90 measurement







Reconstructed charge:

- 3.1 ke
- 15% less than expected (3.7 ke)

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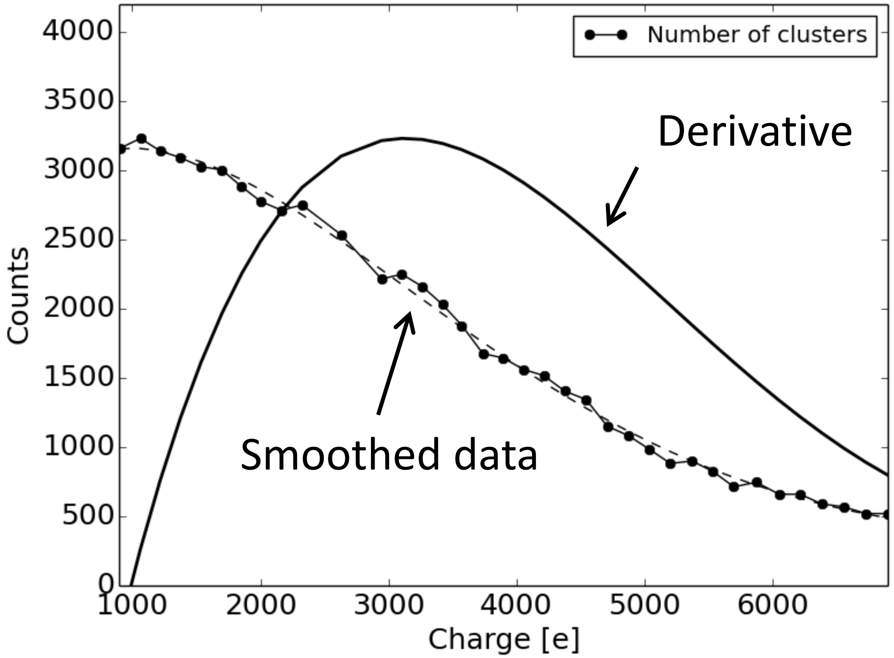
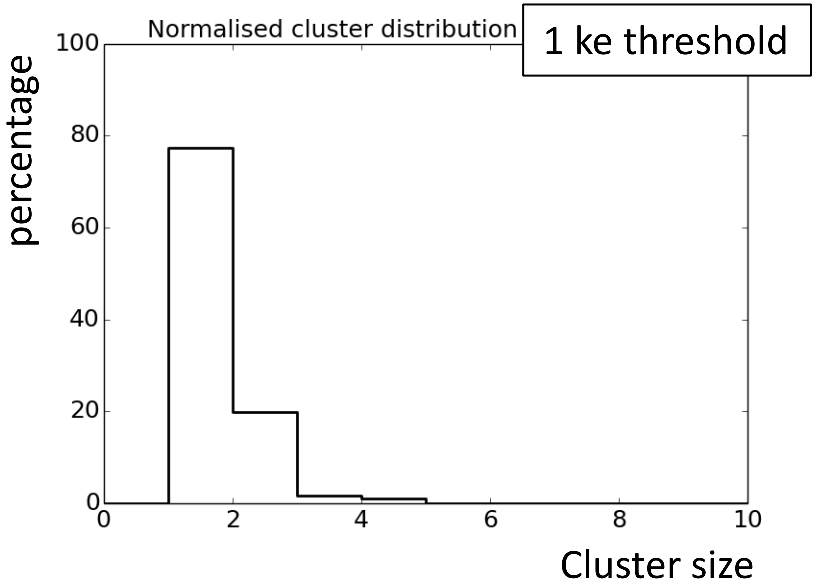
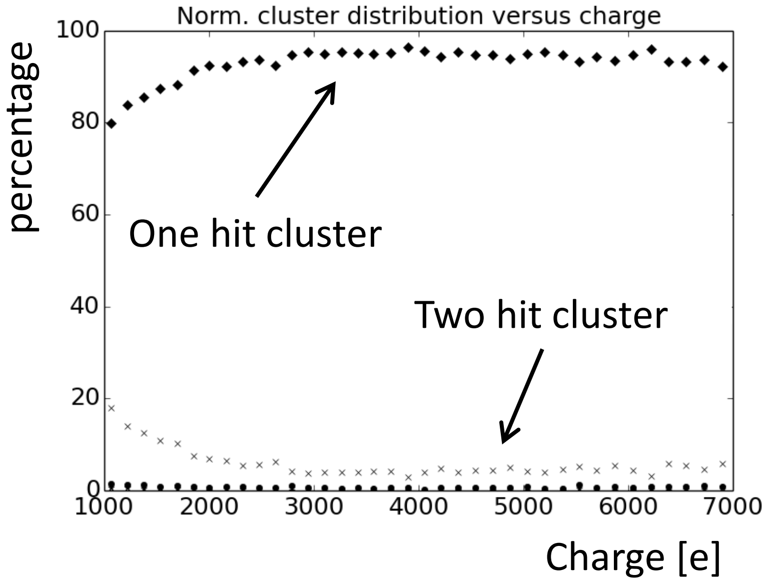
Results of 3.2 GeV electron beam (ELSA) measurement







Reconstructed charge:

- 3.1 ke
- 7% less than expected (3.34 ke)
- At highest threshold rate is not zero (delta electrons)

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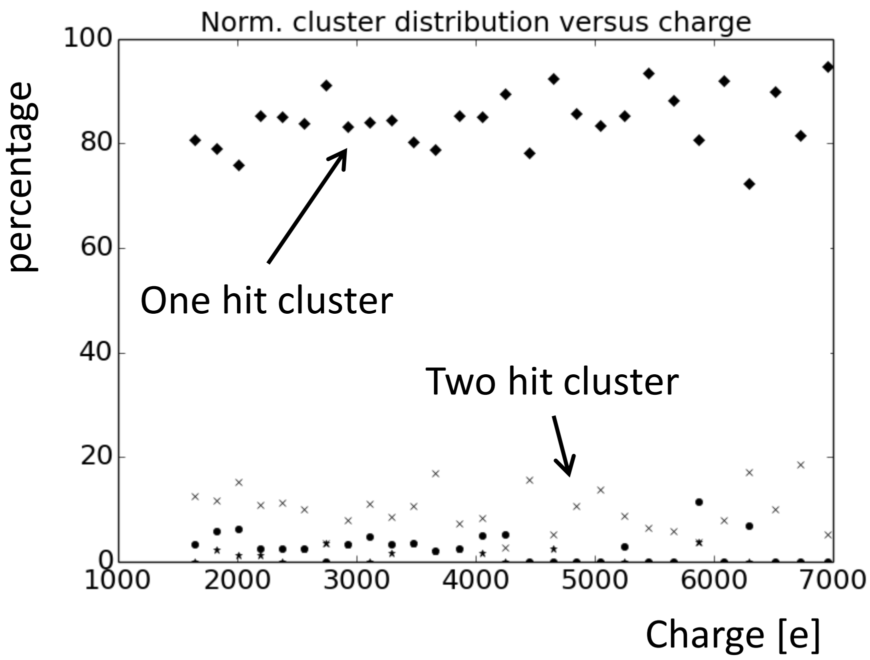
Comparison of normalised cluster distributions Sr90/beam

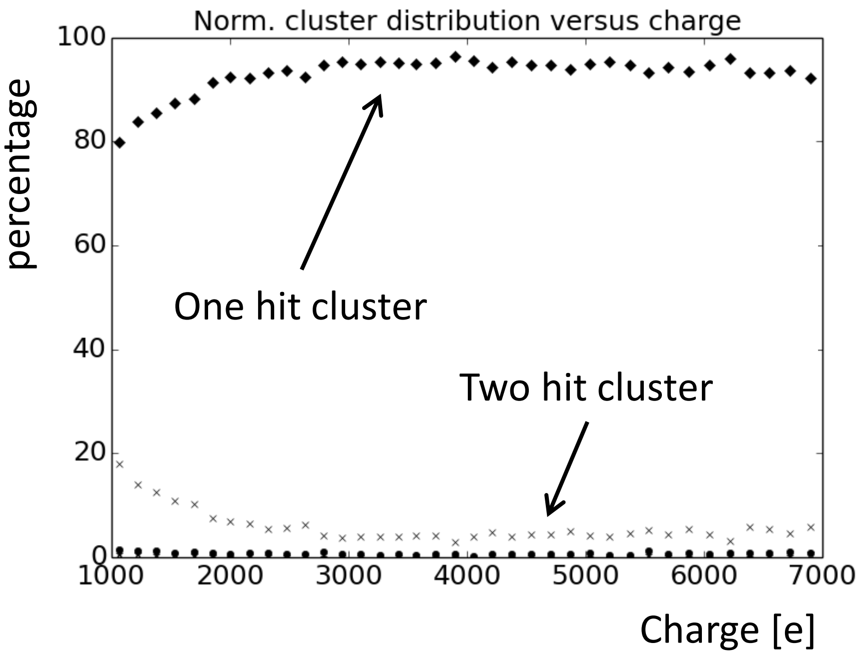
Sr 90 collimated



One hit cluster

Two hit cluster

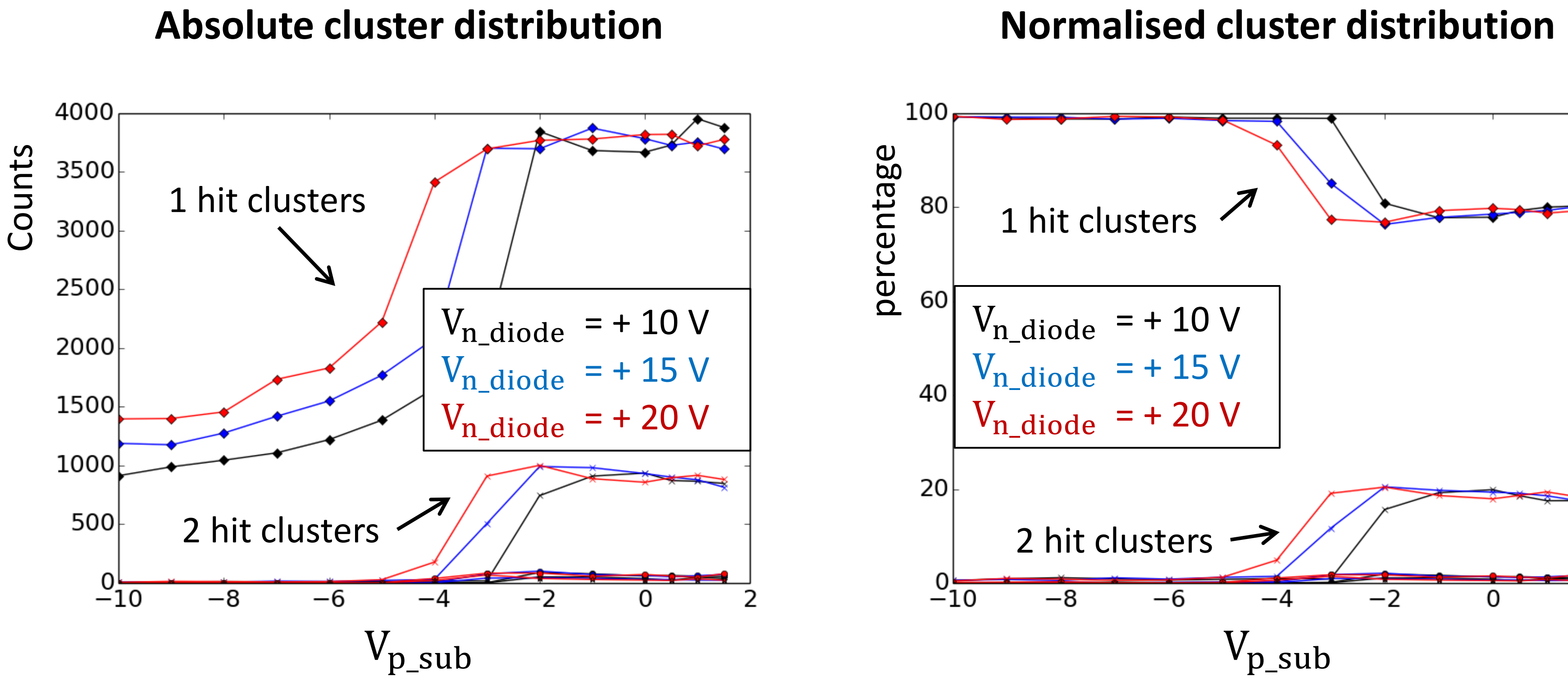
3.2 GeV electron beam



One hit cluster

Two hit cluster

Parameter scan of V_{p_sub} and V_{n_diode} with electron beam



- At low V_{p_sub} the one hit cluster count rate drops to ~ ¼
- At low V_{p_sub} the cluster distribution changes from 80 % to ~ 100% one hit clusters
- Count rate decrease starts later with higher V_{n_diode}

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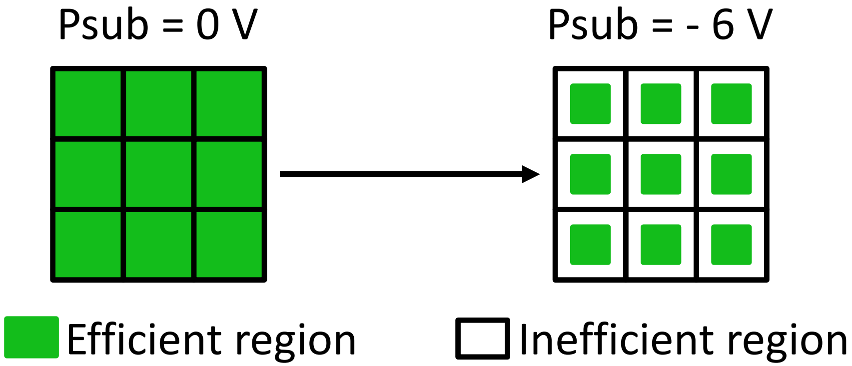
Explanation of bias parameter scans

Observations:

- Count rate decreases to ¼ with decreasing V_{p_sub}
- Only one hit clusters at lower V_{p_sub}
- Count rate decrease starts ‚later‘ with increasing V_{n_diode}



→ Inefficient regions start to built up with decreasing V_{p_sub}

Fraction of count rate loss corresponds roughly to the fraction of n-implantation area to total area (~¼)

→ The inefficient regions are likely to be below the psubstrate.

Charge is partially lost.

Possible reasons for charge loss:

- Trapping → Unlikely in unirradiated sensor
- Slower charge collection → Finite integration time of CSA of ~ 10 ns
- Competing electrode → Unlikely as psub implant is not attractive to electrons

→ **Next step:** TCAD Simulation (in progress)

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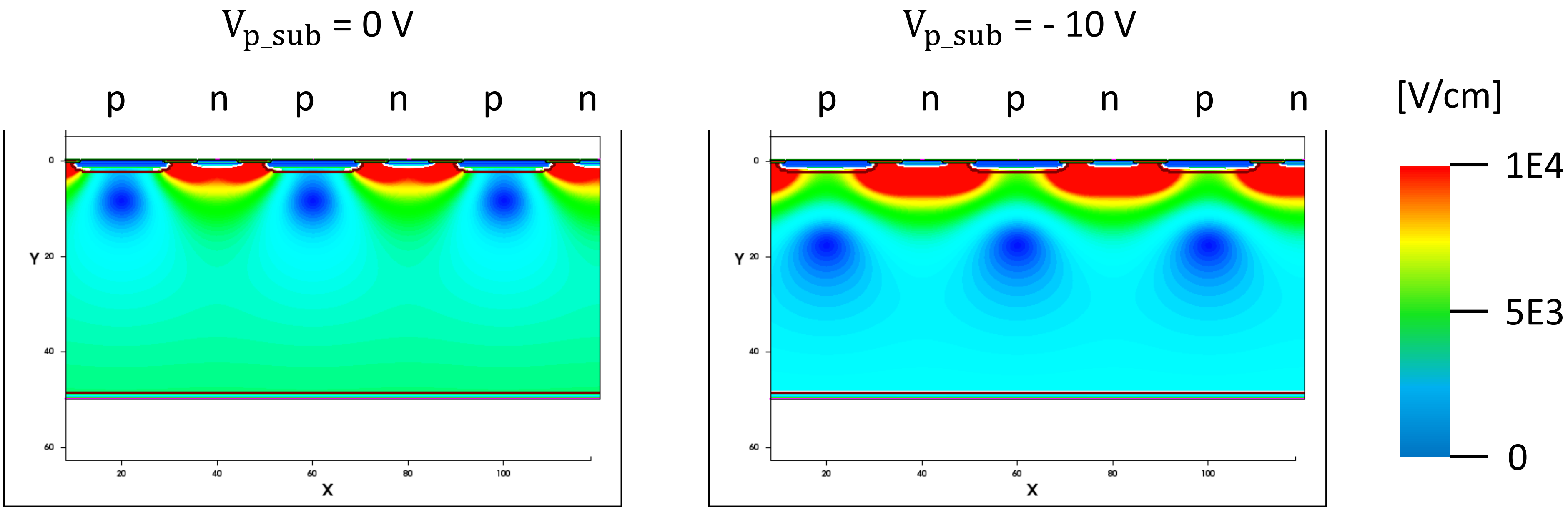
TCAD simulation



- Minimum of electric field shifts with lower V_{p_sub} towards backside
- Electric field is close to zero at the backside for V_{p_sub} = - 10 V

→ Effect on charge collection time is work in progress

Puzzling observation

Charge collection should depend only on

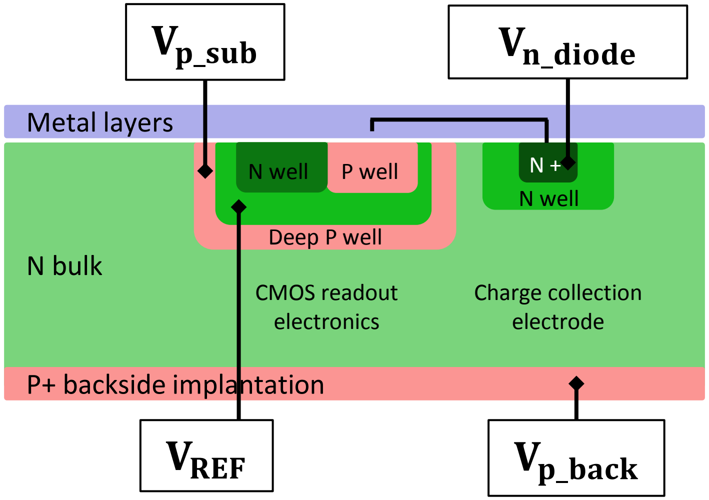
$$V_{n\_diode} \leftrightarrow V_{p\_sub} \leftrightarrow V_{p\_back}$$

and be independent of the reference V_{REF}

Test for two sets of bias parameters:

	Set A	Set B
▪ V_{n_diode}	+ 14 V	+ 12 V
▪ V_{p_sub}	- 1 V	- 3 V
▪ V_{p_back}	- 11 V	-13 V

	Set A	**Set B**
One hit cluster count	3800	2900
One hit cluster fraction	< 80 %	95 %



→ Charge collection depends on the difference between bias voltages and system ground

→ Needs to be studied in more detail

→ Could be due to process problem, implants are maybe not as we assume

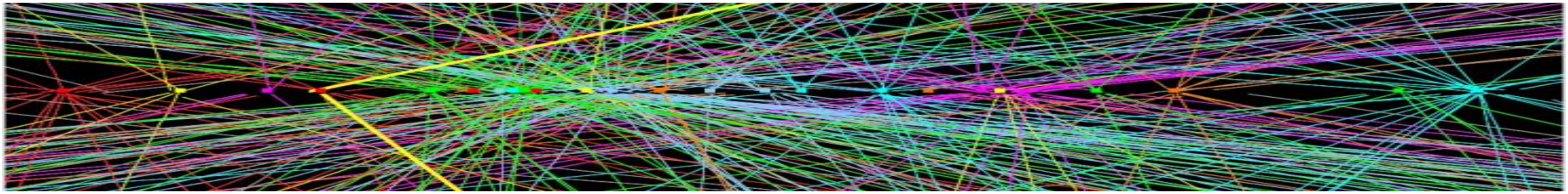
Summary

Sensor characterization of the first DMAPS prototype in ESPROS technology has been performed

- Homoegeneity studies and bias parameter optimization with laser system
- Sr 90 and electron beam used for measurement of charge collection
- Biasing of the sensor is partially understood

Radiation hardness studies are work in progress

- Xray irradiation studies show transistors are radiation tolerant up to 50 Mrad TID
- Neutron irradiated samples arrived recently (up to 5E+14 neq)



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Thank you

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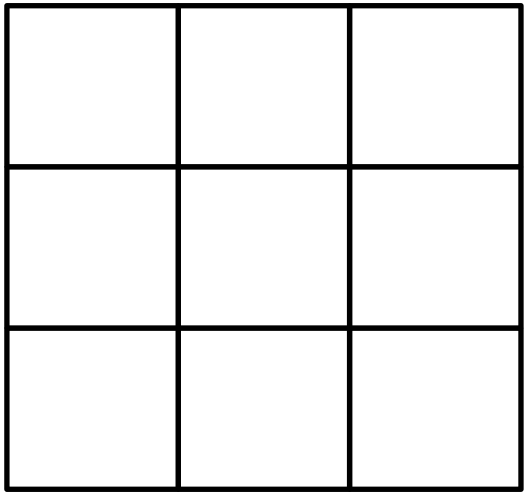
Backup



#

Threshold

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Front end performance

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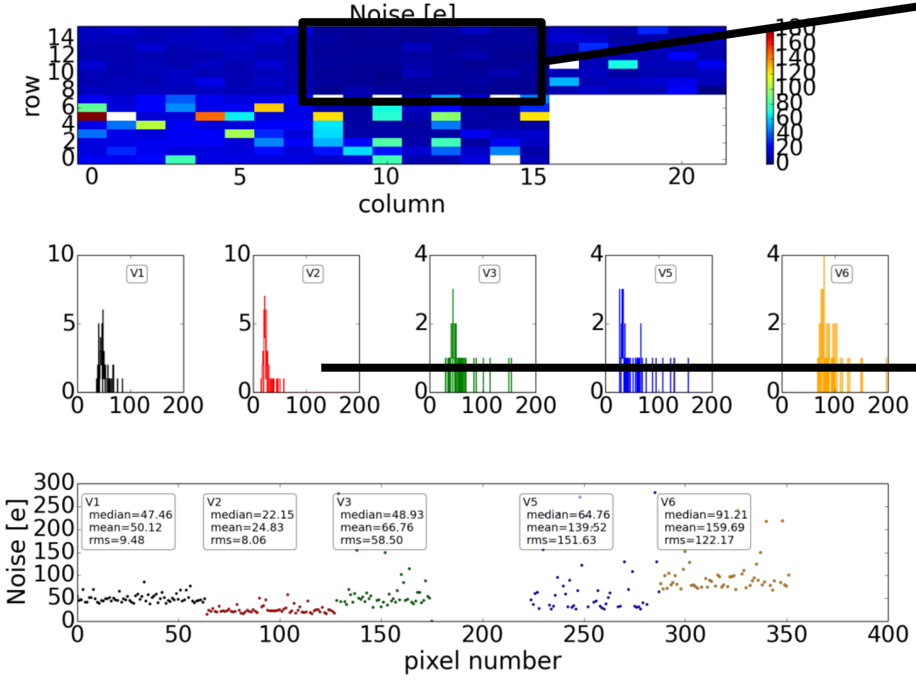
Noise

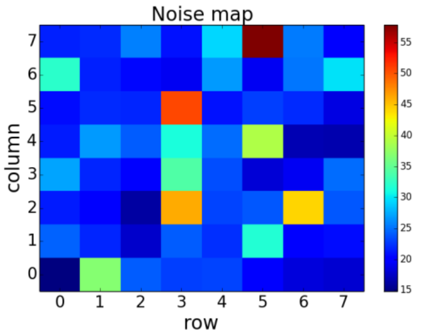
Noise measurement:

- Threshold and noise determined with threshold scan and scurve fit
- Charge injection from 0 to 5 ke⁻ through C_{inj} = 2 fF (1mV ⬄ 12.5 e⁻)

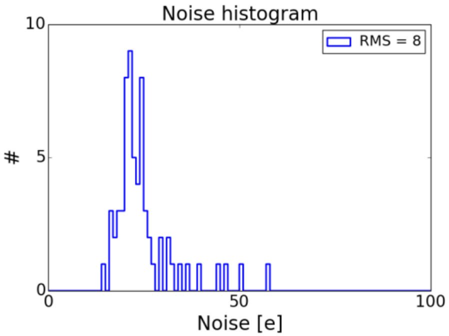
Noise of all matrices (V4 excluded, no inj. circuitry)



Zoom V2



V2



→ Lowest noise of 25 e⁻ of V2

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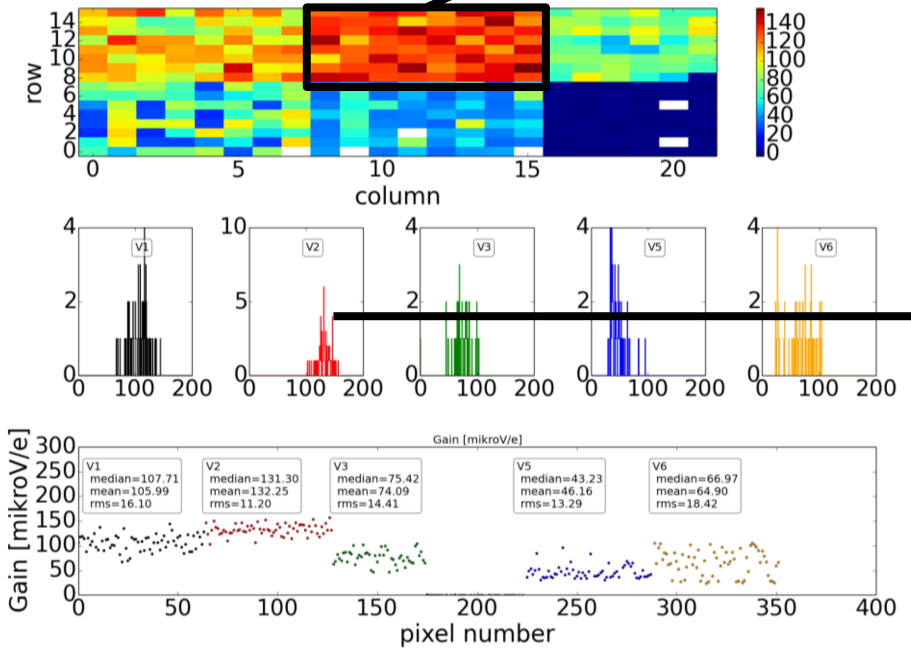
Gain

Gain measurement:

- Measure analog output with threshold scan for different injected charges
- Gain is slope in linear region

Gain of all matrices (V4 excluded, no inj. circuitry)



V1
median=107.71
mean=105.99
rms=16.10

V2
median=131.30
mean=132.25
rms=11.20

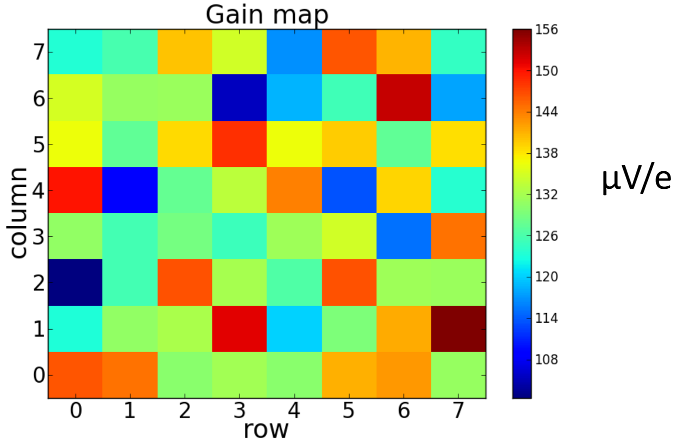
V3
median=75.42
mean=74.09
rms=14.41

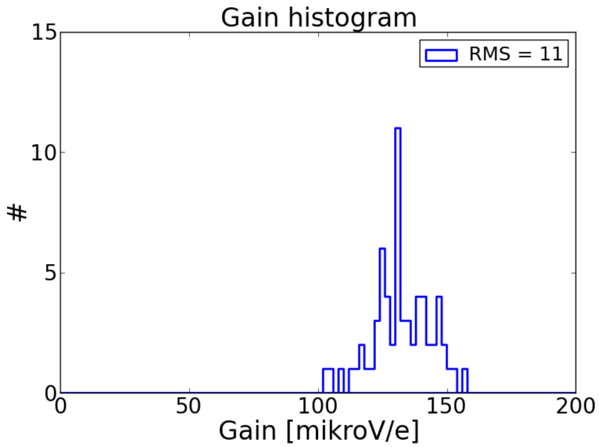
V5
median=43.23
mean=46.16
rms=13.29

V6
median=66.97
mean=64.90
rms=18.42

Zoom V2

V2





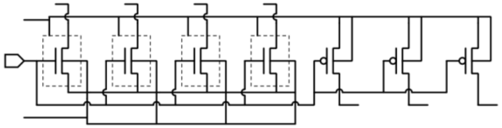
V2

→ Gain ~ 130 µV/e

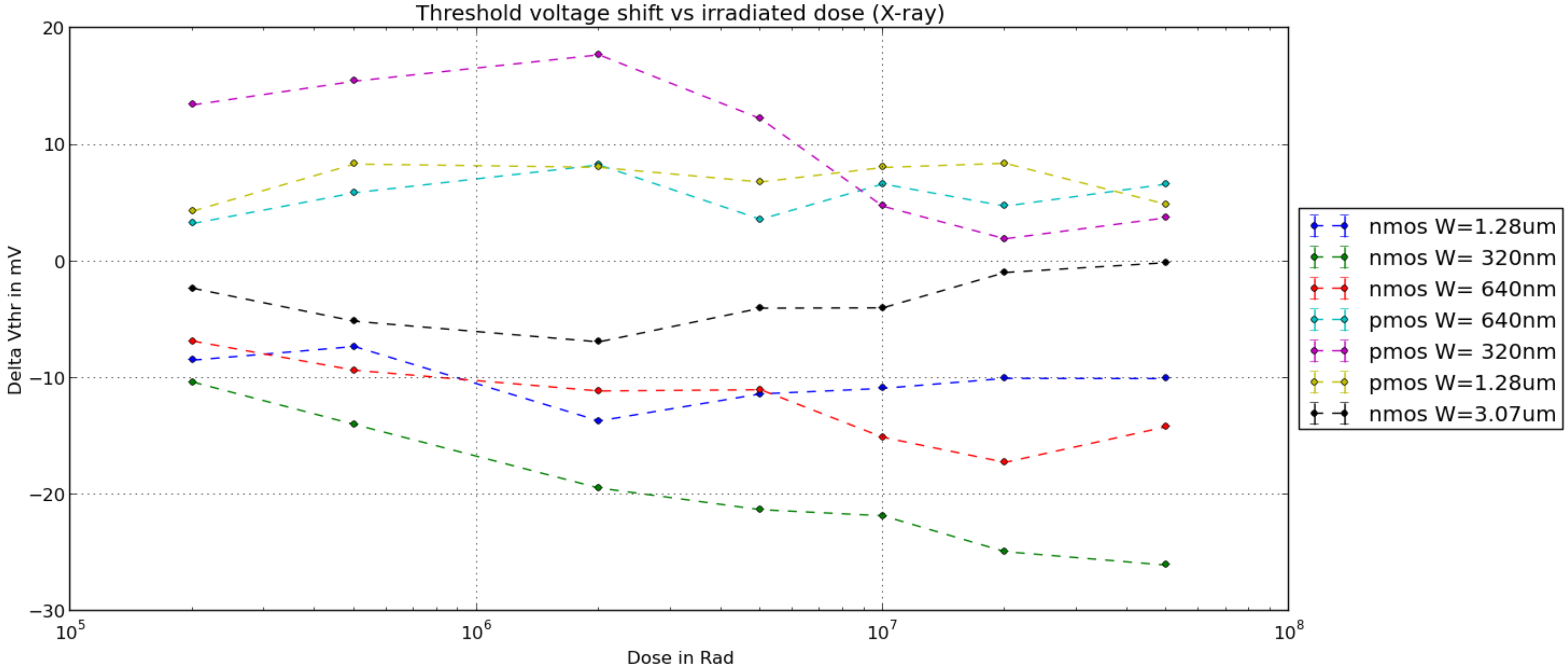
→ Gain spread ~ 11 µV/e

Irradiation

X-Ray Irradiation Test Result



- Irradiation with X-Ray (60 keV) and Vanadium filter (peak ~ 5keV) up to 50 Mrad



- Less than 25 mV shift after 50 Mrad of Total Ionizing Dose (TID)

- Enclosed layout has lower threshold shift and recovers at 50 MRad

Source measurements clusters

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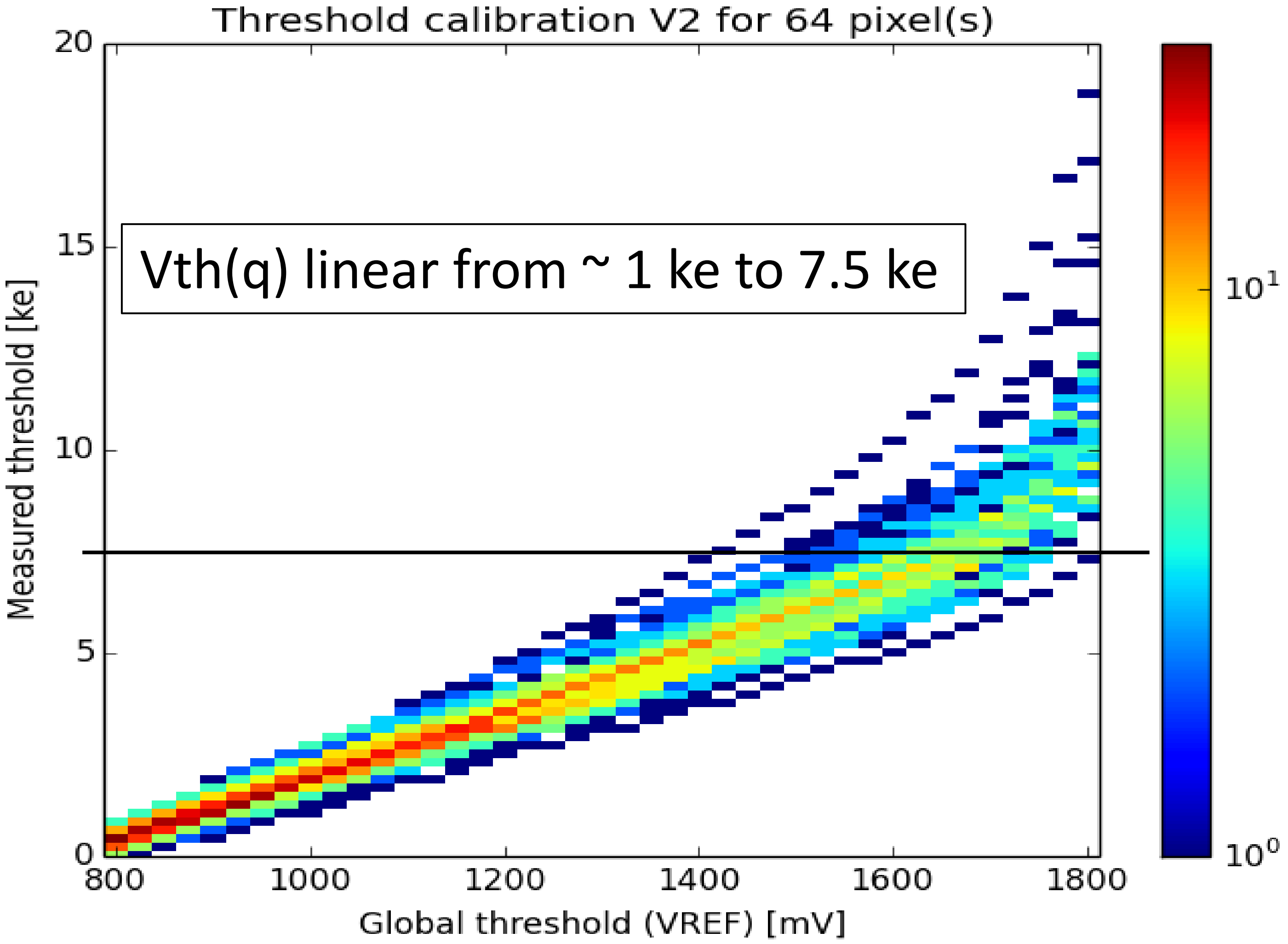
Threshold calibration tb and source data



Vth(q) linear from ~ 1 ke to 7.5 ke

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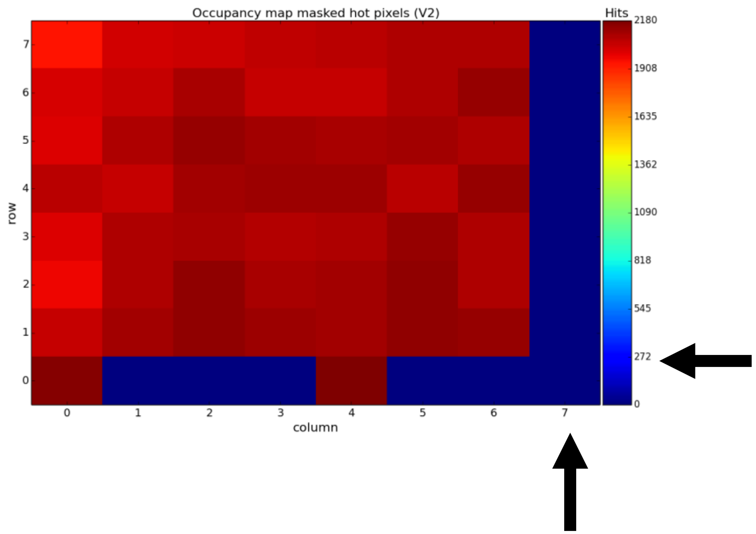
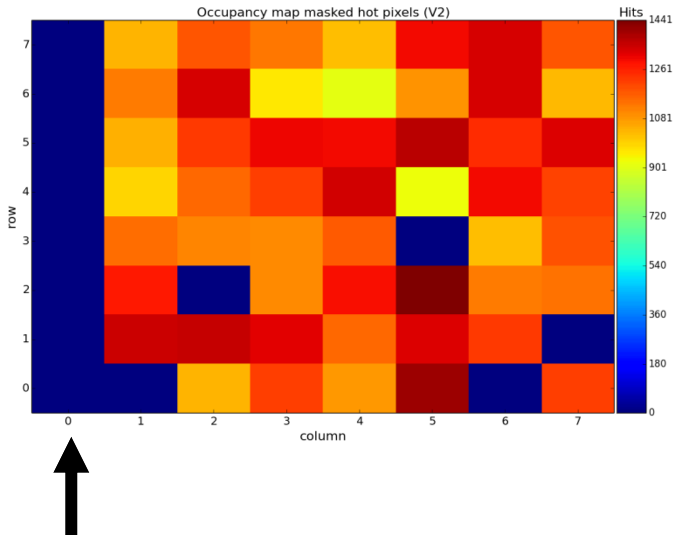
Noisy pixels - psub effects on edge rows/columns

Typical occupancy maps of V2

PSUB from +1.5 V to -4 V

PSUB from -5 V to -10 V





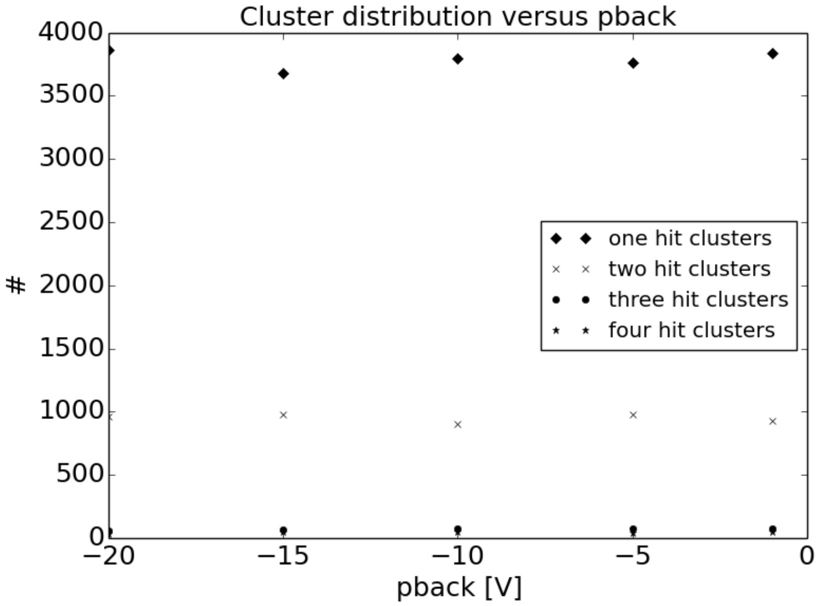
Noisy columns at V3, V5 edge

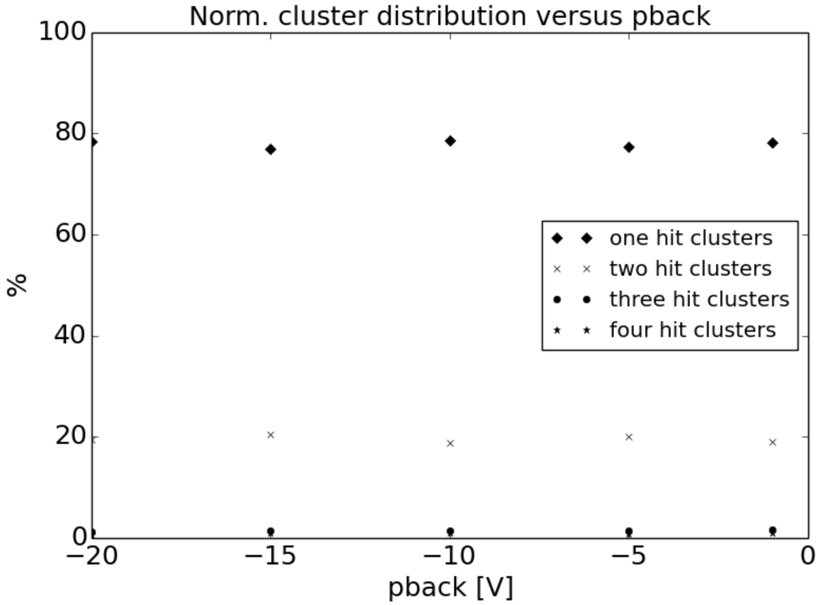
Can be due to nsub guard ring?

Noisy column at V1

PBACK Scan – Cluster distribution

- Clustersize distribution for different pback voltages
- Psub -1 V, Diode bias = Nsub = + 14 V





- No influence on count rate and clustersize.

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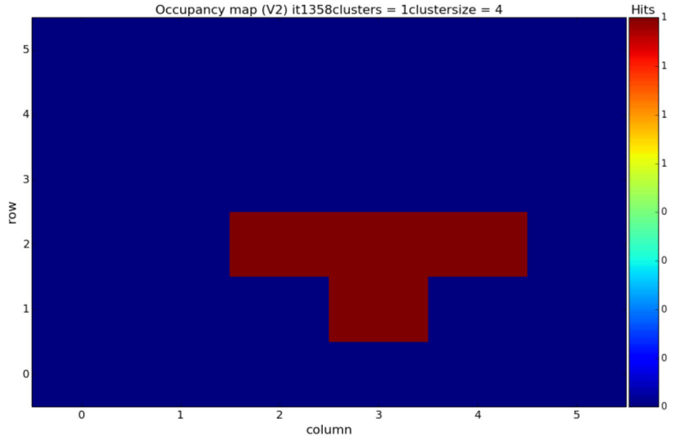
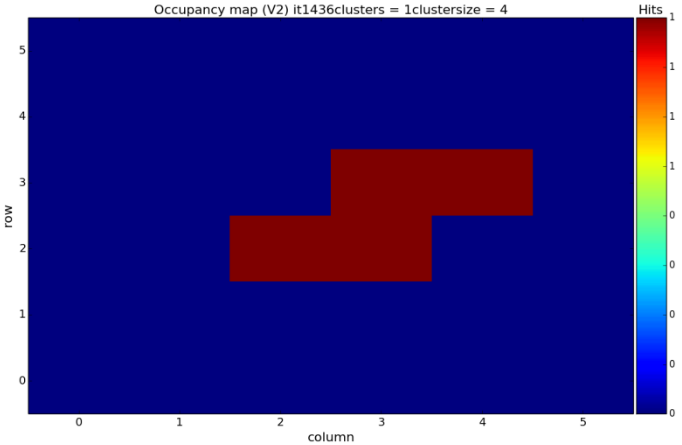
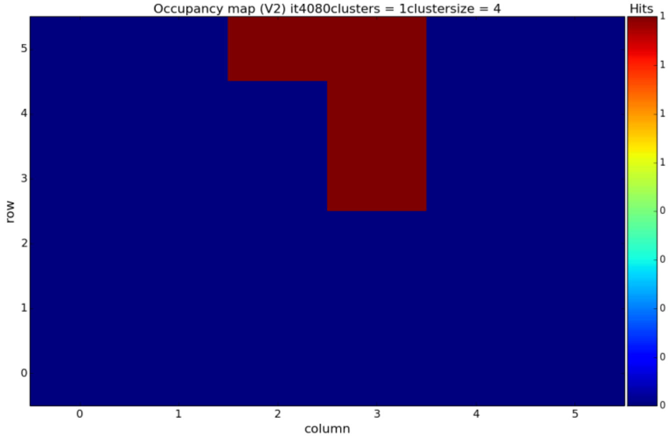
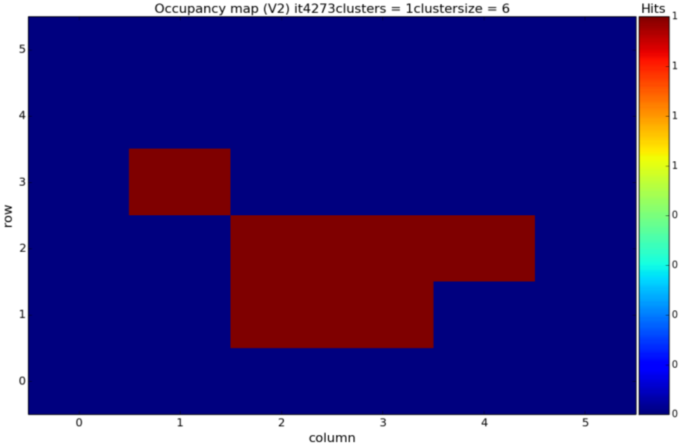
Exotic cluster examples









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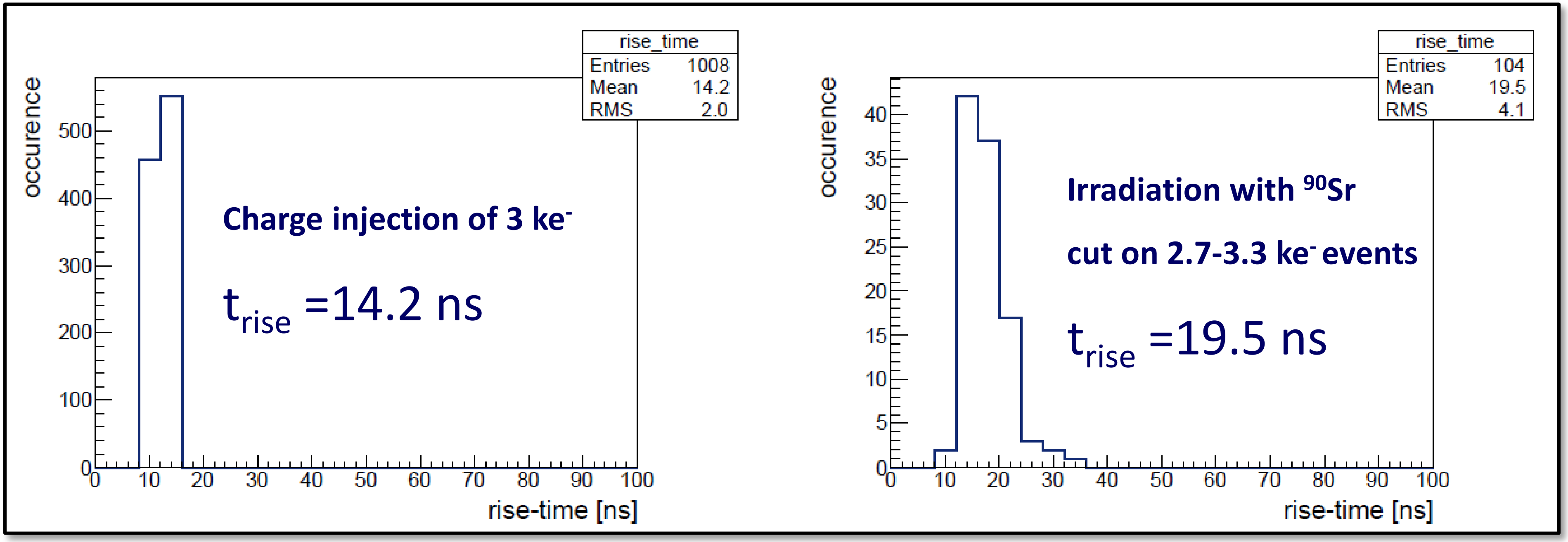
Signal rise-time

- Rise-time => 25% - 75% signal amplitude
- Upper limit on charge collection time



Charge injection of 3 ke$^-$

t_{rise} =14.2 ns

Irradiation with ^{90}Sr

cut on 2.7-3.3 ke$^-$ events

t_{rise} =19.5 ns

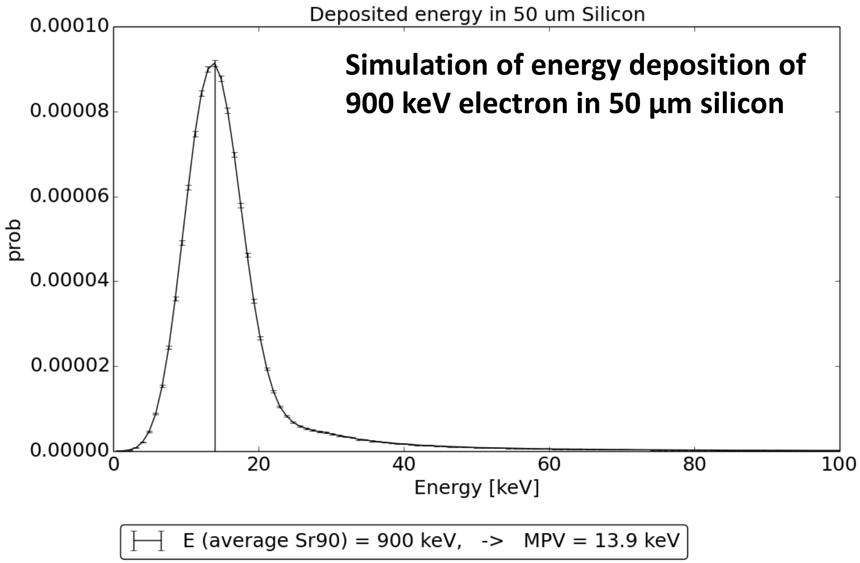
- Fast charge collection
- 50 % of the signal charge is collected for less than 19.5 ns

Expectation energy deposition, multiple scattering and charge sharing

Expectation of energy deposition in 50 µm Si

Literature:

- The most probable value of energy deposition (MPV) for a **MIP** in 50 µm silicon is: MPV = 12 keV [1]
- → $Q_{\mathrm{MIP}}(\mathrm{MPV}) = 3.34\ \mathrm{ke}$



Simulation of MPV for Sr90 source with pyPENELOPE for average energy of emitted energy spectrum Sr90 (900 keV) [2]:
MPV = 13.9 keV

- $Q_{\mathrm{Sr90}}(\mathrm{MPV}) = 3.86\ \mathrm{ke}$

- $Q_{\mathrm{generated,\,MIP}}(\mathrm{MPV}) < Q_{\mathrm{generated,Sr90}}(\mathrm{MPV})$

[1] Bichsel, Straggling in thin silicon detectors, Reviews of Modern Physics 60 (1988) 663-699 (page: 685 and 691)
[2] Sempau, Monte Carlo simulation of electron beams from an accelerator head using PENELOPE, Phys. Med. Biol. 46 (2001) 1163–1186

Effect of charge sharing and multiple scattering on the reconstructed charge

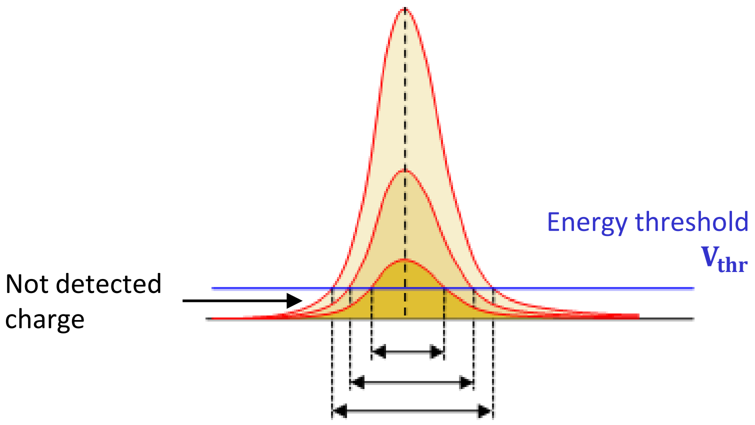
Multiple scattering → Due to higher energy → higher fraction of one hit clusters with beam as compared to Sr90

Charge sharing → Due to smallest angular spread → higher fraction of one hit clusters with beam

Effects on reconstructed charge:

Hit detection only if $\mathbf{V_q} > \mathbf{V_{thr}}$

→ The bigger clusters have a higher fraction of not detected charge which is below threshold (more charge is not detected)



Cluster diameter depends on $\mathrm{Q_{generated}}$ and $\mathbf{V_{thr}}$

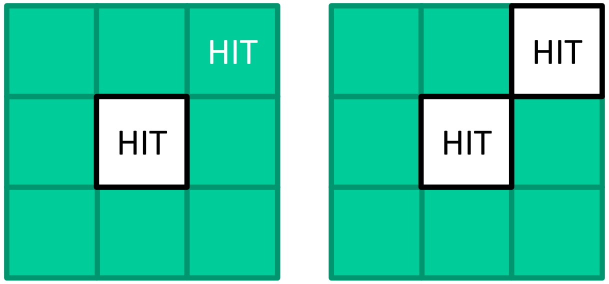
→ The reconstructed charge is expected to be less than the prediction for both sources

→ The effect is expected to be more pronounced for the Sr 90 measurement

Cluster algorithm

1. Start with a cluster of size one.

2. Check if neighbours to cluster of one pixel distance are hit
    - Yes: add them to cluster → 2.
    - No: nothing → 3.



3. Store cluster and remove all clustered hits from hitmap

4. Start with 1.) until no hits are left

```
def clustering(data=None, pdf_name=None):
    """ Cluster data and return clusterized data with one pixel per cluster """
    """ data is of shape [frame_number][x_coord][y_coord] """
    """ pdf_name is the full path to the pdf output file """

    '''  Make copy of data for plotting '''
    data_raw = np.copy(data)

    print 'Shape of data to be clustered = ', data.shape
    '''  clusters is a one dimensional array and holds the number of clusters versus clustersize '''
    clusters =np.zeros(shape=(64))
    '''  cluster_pixel_data_for_thr_calib is output array containing clustered frames '''
    cluster_pixel_data_for_thr_calib = np.zeros(shape=(data.shape[0], data.shape[1], data.shape[2]))

    with PdfPages(pdf_name + '.pdf') as pdf_name:

        '''  Loop over all frames '''
        for it in range(0,data.shape[0]):
            print 'Frame it ........................................................... ', it, '/', data.shape[0]
            print 'Frame[it] = \n', data[it]
            coords = np.where(data[it][:][:] == 1)
            x = coords[0]
            y = coords[1]
            cluster_count = 0
            print 'x = ', x
            print 'y = ', y

            '''  Start to search for all clusters within one frame '''
            ''' Go over all hit x coordintes, make cluster and remove these from the hit x coordintes and then start over until nothing left '''
            while len(x)>0:

                cluster_count += 1
                '''  Define the first entry of the array holding all x coordinates of hit pixels to be the start for clustersearch '''
                x_y_cluster = [(x[0],y[0])]
                print 'Start values of cluster ', cluster_count ,' = ', x_y_cluster

                keep_searching = True

                iti = 0
                while keep_searching is True:
                    print 'Number of hits to be clustered = ', len(x)
                    iti += 1
                    start_len = len(x_y_cluster)
                    print 'Iterator of while Loop = \t', iti ,'\t Clustersize of current cluster candidate = \t', start_len
                    for c_it in range(0,len(x_y_cluster)):
#                        print 'c_it = ', c_it
                        for x_it in range(0,len(x)):
                            x_diff_cluster_member = x_y_cluster[c_it][0]-x[x_it]
                            y_diff_cluster_member = x_y_cluster[c_it][1]-y[x_it]
                            if (x_diff_cluster_member == 0 or x_diff_cluster_member == 1 or x_diff_cluster_member == -1) and (y_diff_cluster_member == 0 or
y_diff_cluster_member == 1 or y_diff_cluster_member == -1):
                                # Here all neighbours are found, also those which already belong to cluster
                                x_y_cluster.append((x[x_it],y[x_it]))

                    x_y_cluster = sorted(set(x_y_cluster)) # remove duplicates from cluster data
                    print 'Iterator of while Loop = \t', iti ,'\t Clustersize after search = \t', len(x_y_cluster)
                    if start_len == len(x_y_cluster):
                        keep_searching=False
                    if len(x_y_cluster) == len(x):# means all hits of this frame belong to only one cluster
                        keep_searching=False
                    print 'Keep searching bool = ', keep_searching

                print 'Finished a cluster with coordinates = ', x_y_cluster

                '''  Cluster finished, store it '''
                clustersize = len(x_y_cluster)
                clusters[clustersize] = clusters[clustersize] + 1
                cluster_pixel_data_for_thr_calib[it][x[0]][y[0]]+=1

                print 'Clustered frame[it] = \n', cluster_pixel_data_for_thr_calib[it]

                '''  Remove clustered hits from data '''
                for hit in range(0,len(x_y_cluster)):
                    data[it][x_y_cluster[hit][0]][x_y_cluster[hit][1]]=0
#                print 'Mod hitmap after reduction of hits from ', cluster_count , ' clusters = ', data[it]

                '''  Make new x and y hit coordinates for next cluster '''
                print '... continue with next cluster'
                coords = np.where(data[it][:][:] == 1)
                x = coords[0]
                y = coords[1]
                print 'next x', x
                print 'next y', y

                if clustersize > 3:
                    plotting.plot_2D_hitmap(hist2d=data_raw[it], title='Occupancy map (V2) it' + str(it) + 'clusters = ' + str(cluster_count)+ 'clustersize =
' + str(clustersize), zlabel='Hits', z_max=None, filename=pdf_name, savename='test')

            print 'Found clusters: ', cluster_count
            if cluster_count > 1:
                plotting.plot_2D_hitmap(hist2d=data_raw[it], title='Occupancy map (V2) it' + str(it) + 'clusters = ' + str(cluster_count), zlabel='Hits',
z_max=None, filename=pdf_name, savename='test')

        plot_cluster_distribution(data=clusters, title='Cluster distribution 6x6 V2', filename=pdf_name, savename='test')

    print 'Clustering done.'
```

Charge sharing (geometrical effects)

Charge cloud spreads laterally due to diffusion and therefore charge can be shared between neighbouring pixels → clusters

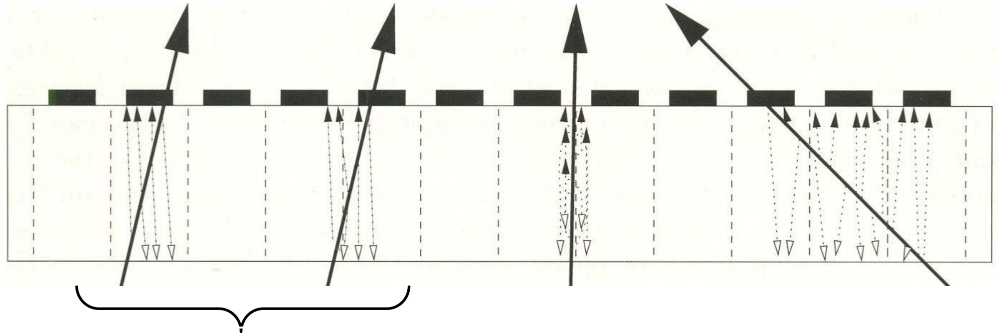
Charge sharing is geometrically affected by:

- Position of track with respect to sensor surface
- Geometry of pixel cell
- Angle of track with respect to sensor surface

Electrons: ⇡

Holes: ⇣

Incident particle: ↑



Same angle but different positions

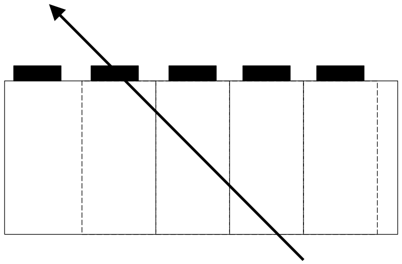
→ Left: Clustersize: 1

→ Right: Clustersize: 2

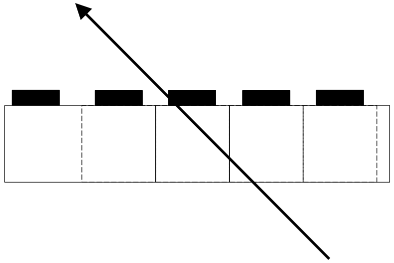
Track exactly on pixel edge

Clustersize: 2

Clustersize: 3



Clustersize: 3



Clustersize: 2

Effects of charge sharing on cluster size distribution

- Position of track with respect to sensor surface → Not known, same for Sr90 and electron beam
- Geometry of pixel cell → almost cubic, 40 µm x 40 µm x 50 µm, fixed
- **Angle** of track with respect to sensor surface → Different for Sr 90 and electron beam

→ The bigger the angular spread, the higher the fraction of bigger clusters.
→ The smaller the angular spread, the higher the fraction of one hit clusters.

Angular spread of Sr 90 > Angular spread Sr 90 collimated > focussed electron beam

Expectation:
→ Highest fraction of one hit clusters for electron beam measurement
→ Smallest fraction of one hit clusters with uncollimated Sr 90 source

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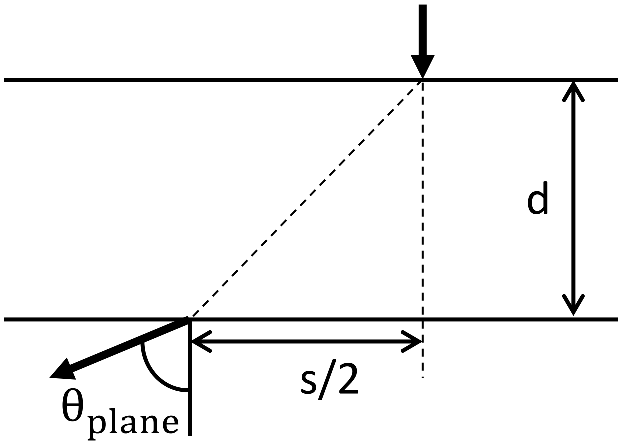
Effect of multiple scattering on the cluster size

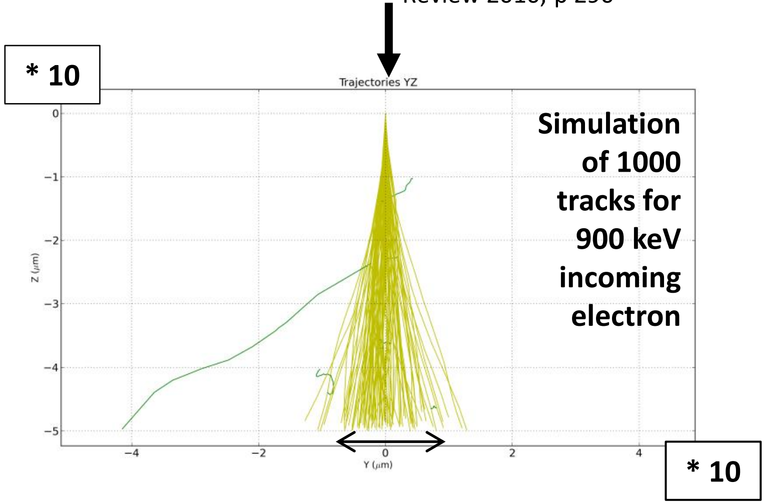
Multiple scattering

Deflection of particle in matter by coulomb scattering, can be calculated:

- $\theta_{plane} = \frac{13.6}{\beta p[\mathrm{MeV/c}]} z \sqrt{\frac{x}{X_0}}\left(1 + 0.038 \ln \frac{x}{X_0}\right)$
- θ_{plane} = sigma of Gauss fit to central 98% of the scattering angles [3]
- $s/2 = d \cdot \tan \theta_{plane} \approx d \cdot \theta_{plane}$; d = 50 µm
- s(1 sigma) **= 20 µm for Sr 90 (2.28 MeV)**
- **s = 0.003 µm for 3.2 GeV electron beam**



[3] Bichsel, Groom, Klein, PDG Review 2010, p 290



*** 10**

Simulation of 1000 tracks for 900 keV incoming electron

*** 10**

Simulation of electron trajectories for 2.28 MeV incident particle agrees with calculation

→ Expectation: Higher fraction of one hit clusters with electron beam measurement as compared to Sr 90 measurement

- β, p, z properties incoming particle
- $\frac{x}{X_0}$ traveled distance in units of radiation length

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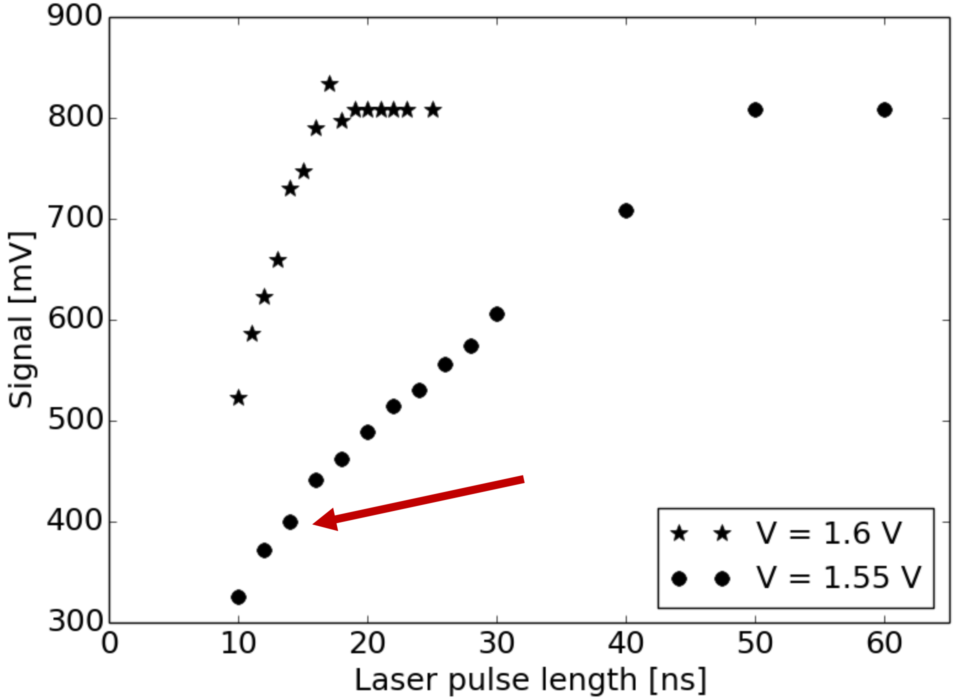
LASER measurements

Laser system linearity

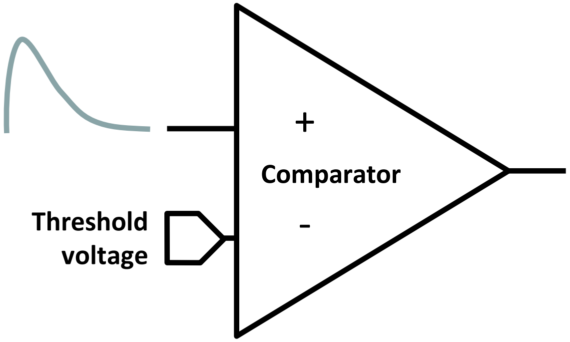
- Injection of laser can be configured by voltage and laser pulse width
- Linearity measured with complete system (Laser + EPCB01)



→ Typical setting: 14 ns, 1.55 V (linear region)

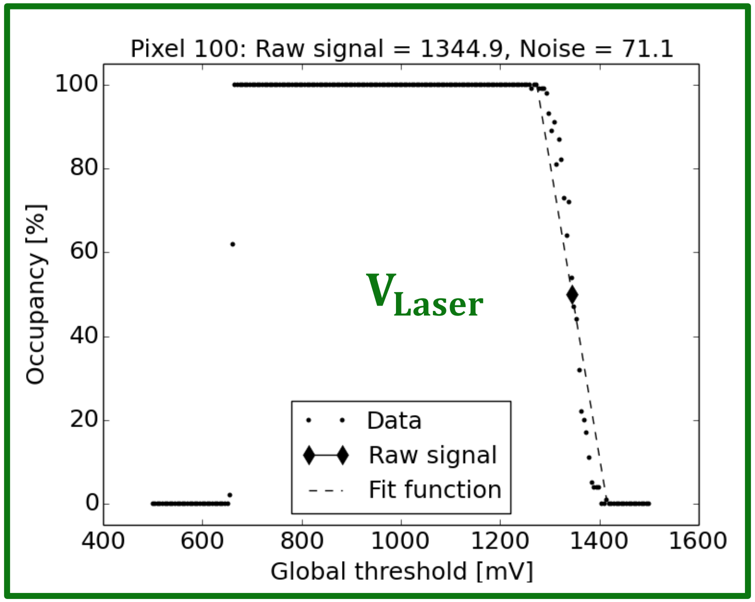
Method DAQ Laser scans

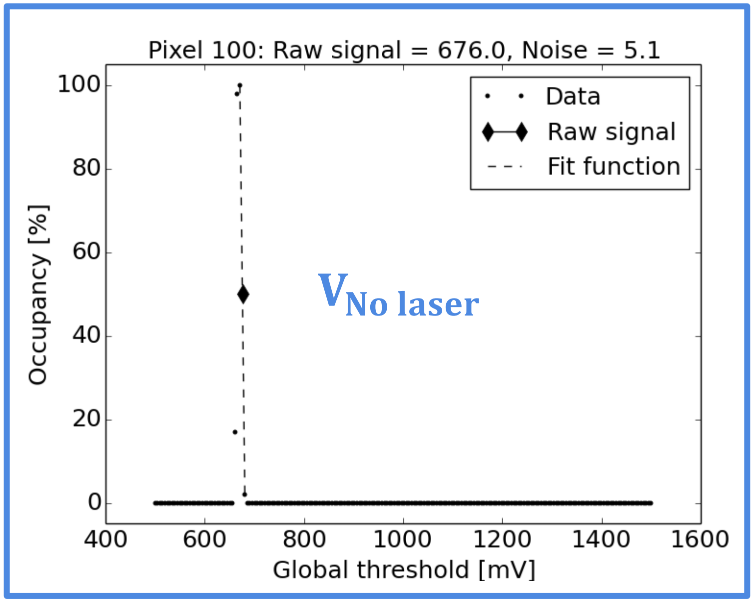
- Move laser over matrix and measure the signal



Signal:

- Threshold scan with laser as injection
- Scan range [500,1500] mV to cover signal and baseline
- V_{Laser} and o_{Laser} determined from line fit





Noise cut:

$V_{\text{Signal}} > V_{\text{Cut}}$

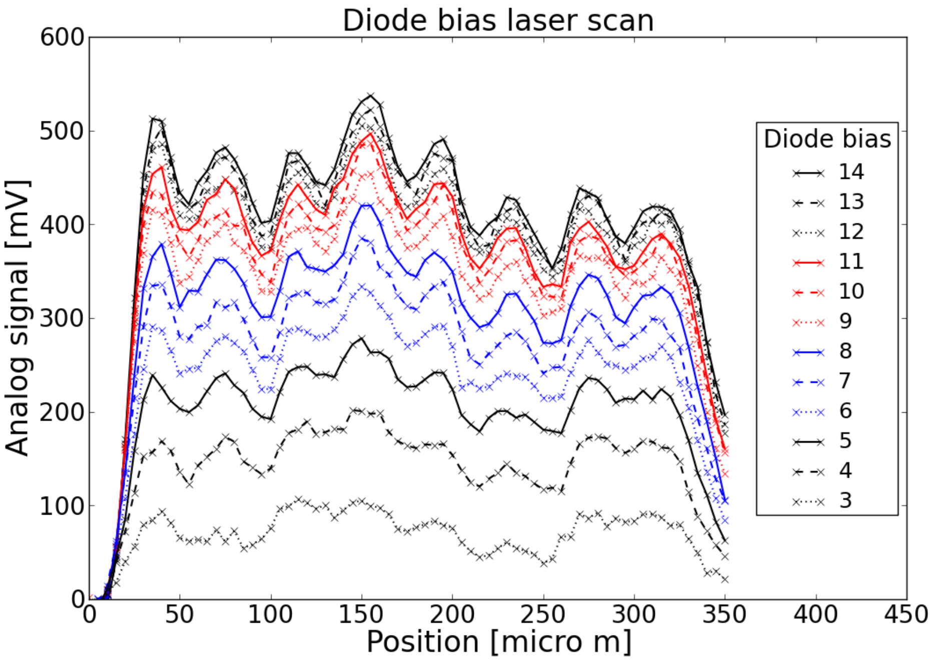
$V_{\text{Cut}} = 3 \cdot o_{\text{No laser}}$

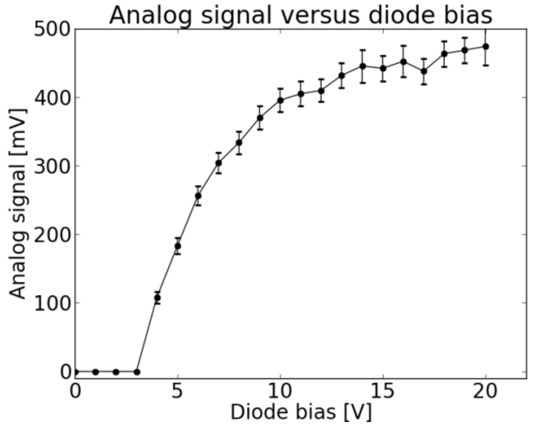
$$V_{\text{Signal}}[\mathbf{mV}] = (V_{\text{Laser}}\ [\mathbf{mV}\ ] - V_{\text{No laser}}\ [\mathbf{mV}]) \longrightarrow V_{\text{Signal}}[\mathbf{mV}] = 669\ \text{mV}$$

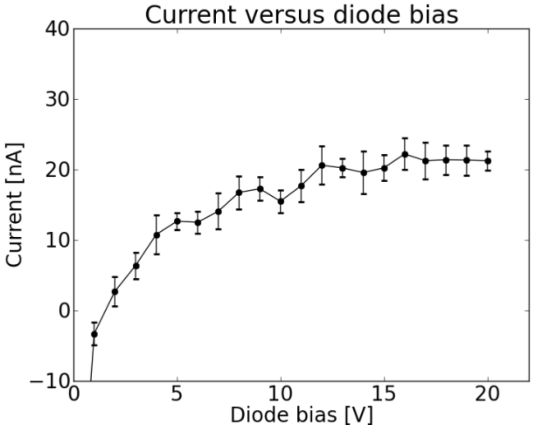
Diode bias scan with laser – V2

- Scan the same column with the laser for different diode bias settings

For one pixel and fixed laser position:







→ Increasing diode bias voltage results in higher signals, as expected

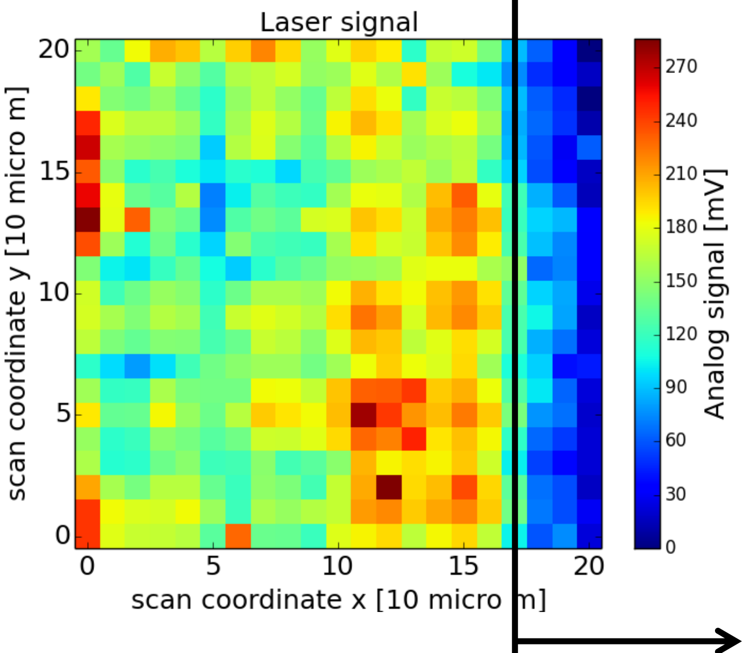
→ Saturation of signal above 10 V (→ depleted sensor)

→ Leakage current ~ 20 nA

V5 – Scan - Region 2.

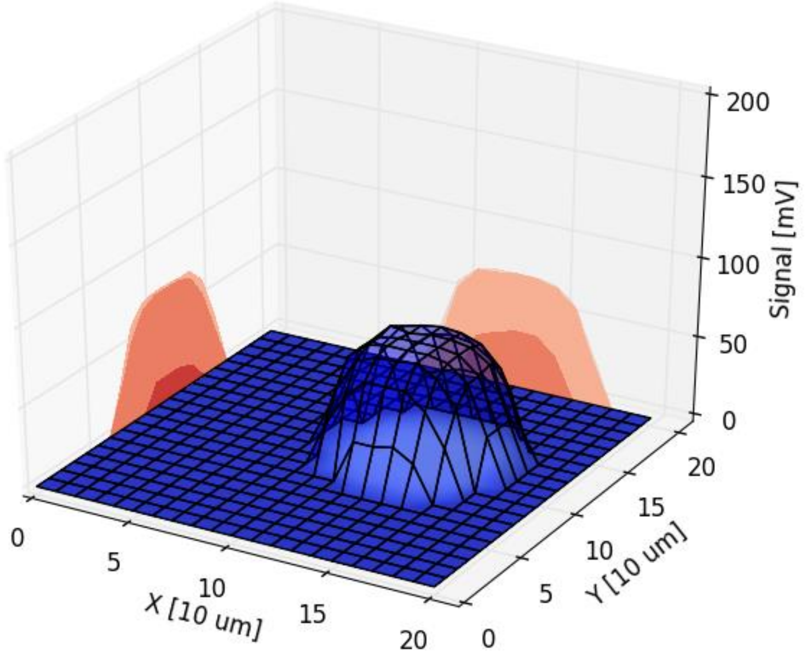
200 µm x 200 µm

10 µm step size



Other version

Pixel 276



→ With switched version inhomogeneities similar as with continous version are observed

→ Shape of single pixel response is box+gauss like, as expected

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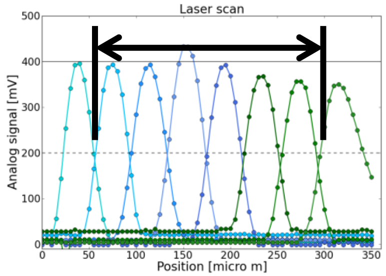
Voltage scans (analysis of old data)

Method:

- Analysis: Take average of sum in position range [50,300] µm
- Result is 2D plot of average signal



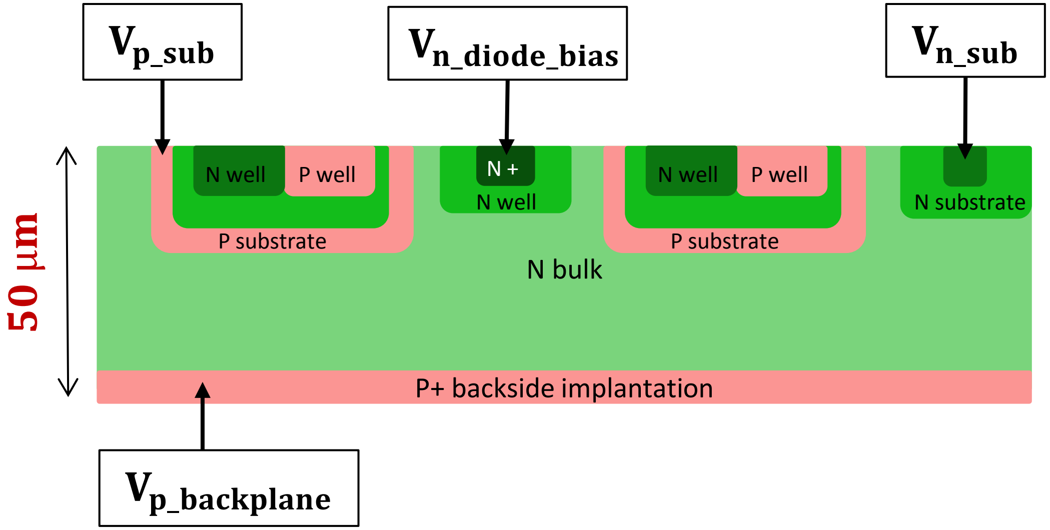
Scans:

Diode bias = Nsub = [+1, +2, +5]
Pback and Psub scan values:
[0, .. , -20], [0, .. , -20]

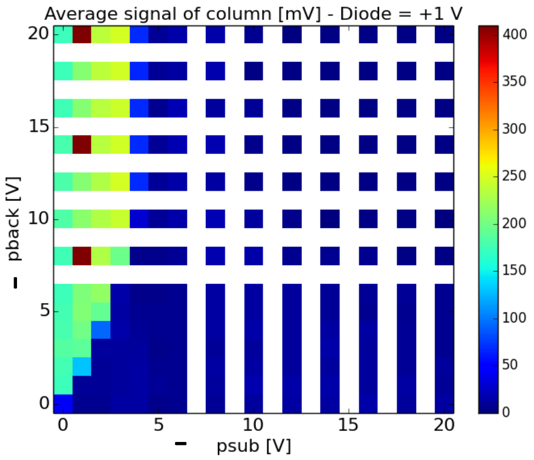
Diode bias = Nsub = [+3, +5]
Pback and Psub scan values:
Pback = [0, .. , -20]
Psub = [-5, .. , 4.5]

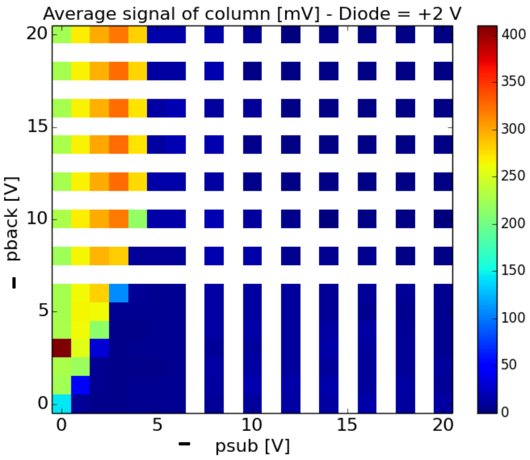
Nsub = +10
Diode bias = [4, 6, 8, 10, 12, 14, 16, 18, 20]
Pback = -1
Psub = [-2, .. , 2]

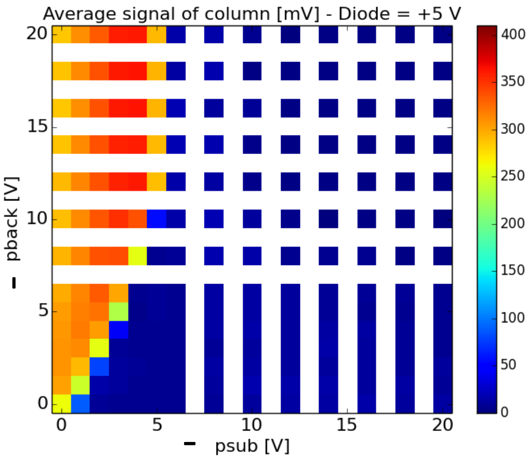
Nsub = +10
Diode bias = [4, 6, 8, 10, 12, 14, 16, 18, 20]
Psub = -1
Pback = [0, .. , -2]



Voltage scans of Pback and Psub







- → Psub < - 5 V no signal
- → Pback < Psub
- → Increase of Diode bias increases signal, as expected
- → For fixed Diode bias and Pback there is a maximum signal wrt Psub
- → For fixed Diode bias and Psub, decreasing Pback increases signal and saturates, as expected

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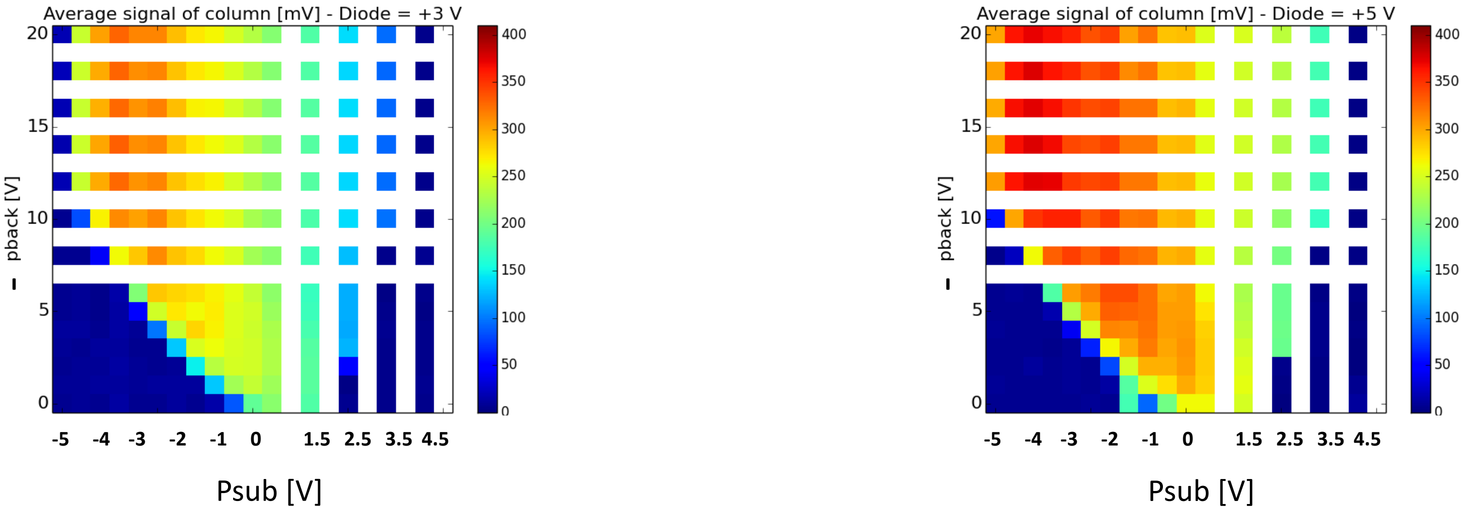
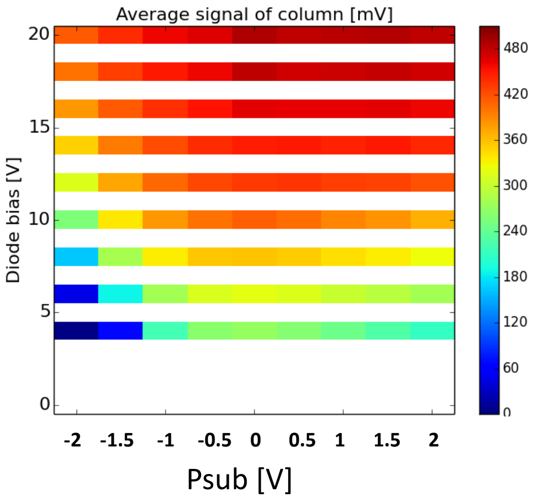
Voltage scans of **Pback and Psub**

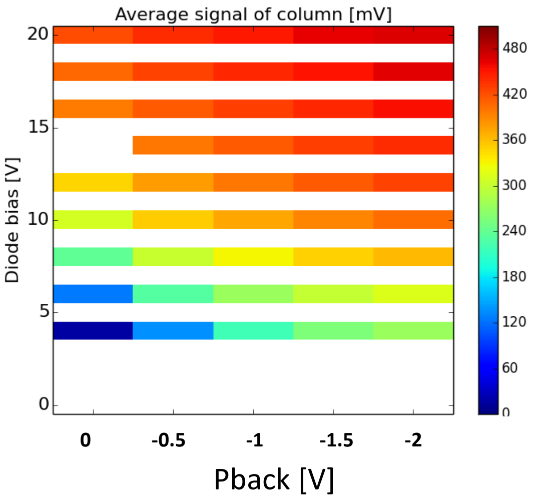


→ Psub operation range increases with decreasing Pback voltage

→ Is this expected ?

Voltage scans of Diode and Psub/Pback





→ Increasing diode bias increases signal

→ Decreasing Pback increases signal

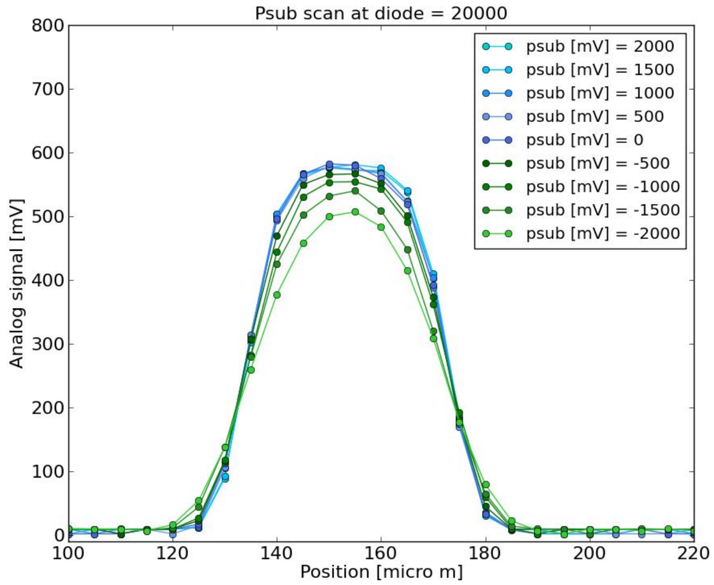
→ Signal maximum for psub around 0

→ Pback = Psub shows signal here!, difference to scan before: higher Diode bias and Nsub = +10 fixed

Voltage scans of Diode and Psub/Pback

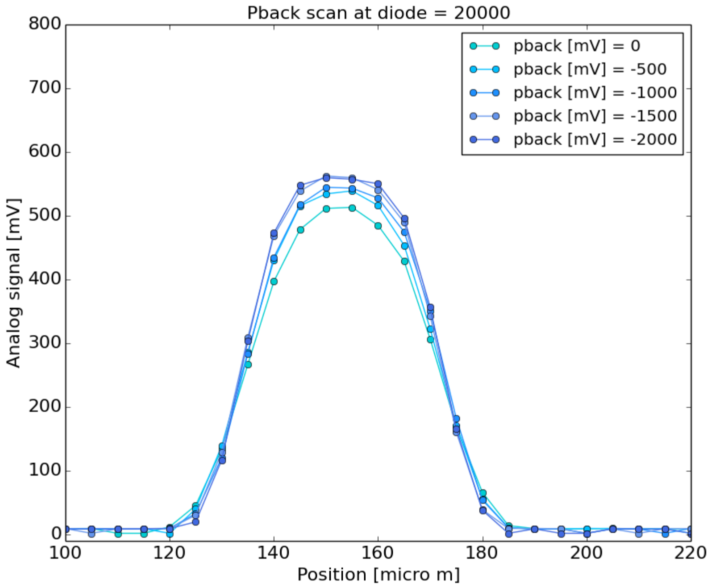
Pixel response shape

- Diode/Psub scan:
  - **Nsub = +10**
  - **Diode bias = [4, 6, 8, 10, 12, 14, 16, 18, 20]**
  - **Pback = -1**
  - **Psub = [-2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2]**
- Plot of response of one pixel:



→ Trend: More boxlike for Psub = 0
→ Between Psub = 0 and +2 no change

- Diode/Pback scan:
  - **Nsub = +10**
  - **Diode bias = [4, 6, 8, 10, 12, 14, 16, 18, 20]**
  - **Psub = -1**
  - **Psub = [0, -0.5, -1, -1.5, -2]**
- Plot of response of one pixel:



→ Trend: More boxlike for more negative Pback
→ Fit of Gauss convoluted with box to be implemented

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Mixed

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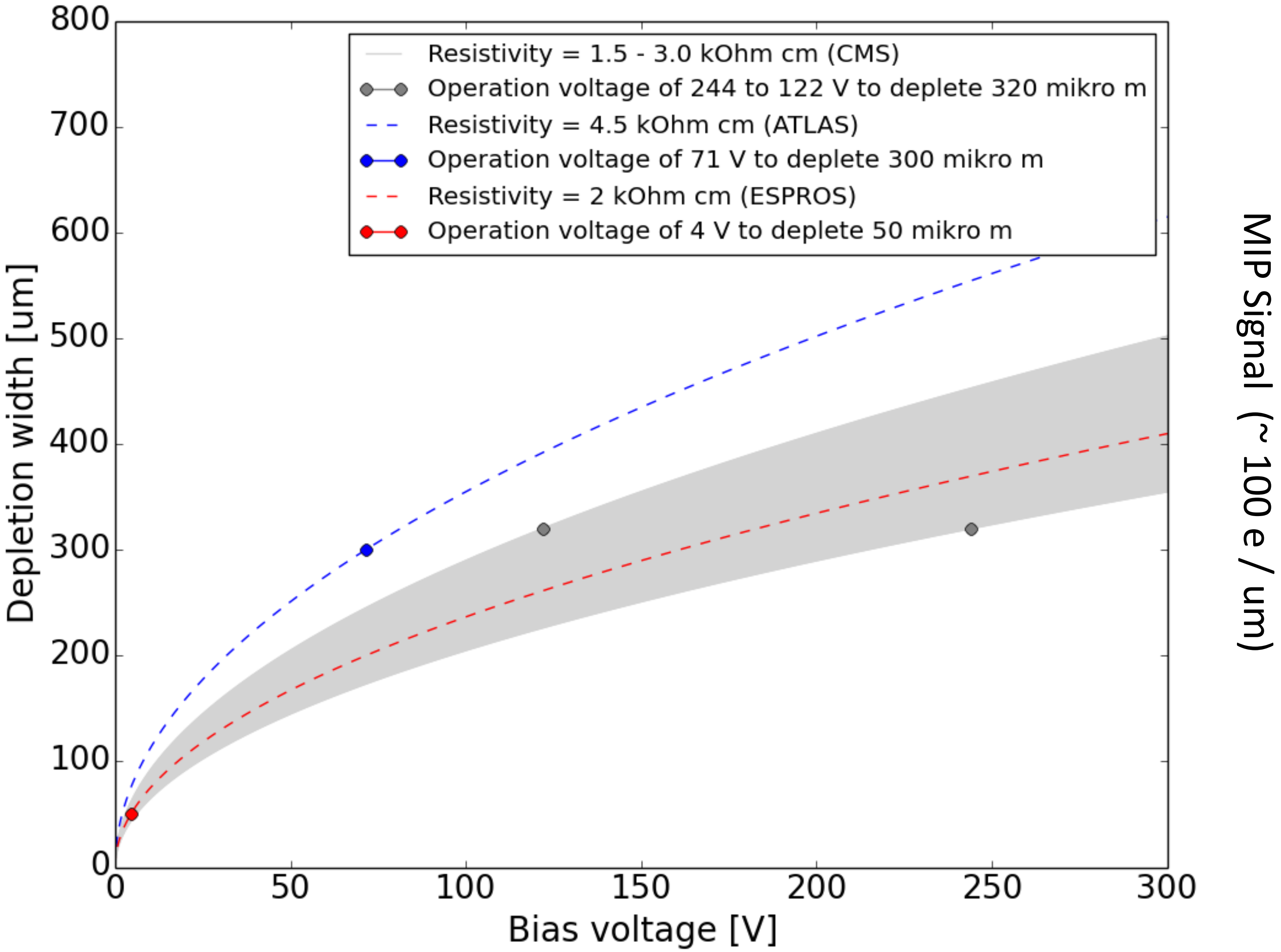
Depletion widths for different resistivities



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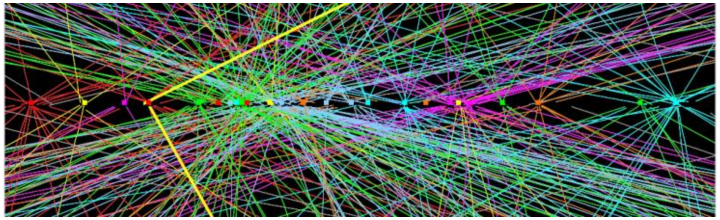
Motivation: Pixel trackers for HEP experiments



Pile up of interactions at High Luminosity LHC

→ More pile up

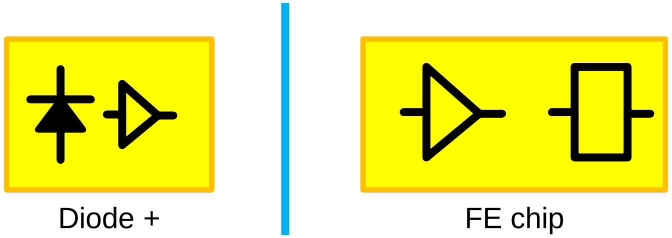
→ Increased radiation dose

Goal: Keep pixel tracker performance by using improved concepts!

This talk → CMOS active sensor test results

Concepts

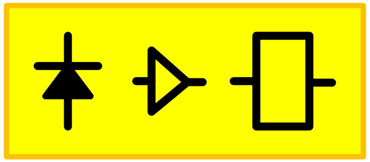
- Hybrid Pixels with CMOS active sensor:
  - **CMOS** (HV or HR) as a sensor (8”)
  - Standard (or digital) only FE chip
  - Example: CCPD (HVCMOS) on FE-I4



Diode + preamp

FE chip

- Monolithic Active Pixel Sensor
  - **MAPS** usually on non-depleted epi substrate → **diffusion signal**, not suited for HL-LHC
  - **Charge collection by drift → Fully depleted** MAPS (**DMAPS**) on high resistive substrate



Diode + Amp + Digital

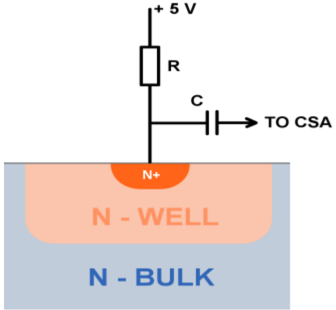
  - Slow r/o (known from MAPS) usually does not meet LHC timing requirements

→ We use this r/o scheme here for technology evaluation of test chips

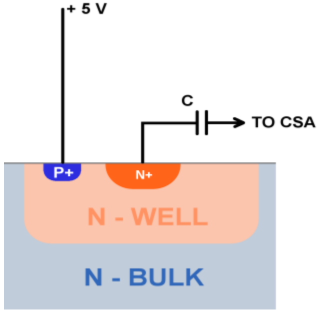
EPCB01 - Details

- Bias options:

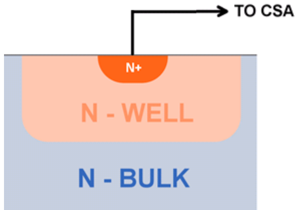
AC coupling
Resistor bias



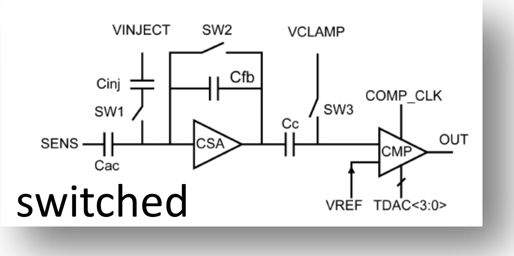
AC coupling
Diode bias



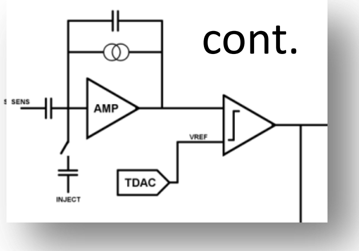
Direct coupling
Direct bias



- Pixel electronics options:



switched



cont.

- Matrix versions:

Matrix version	Biasing and coupling	Analog FE
V1	Resistor + AC	Continuous
V2	Diode + AC	Continuous
V3	Direct + DC	Continuous
V4	Direct + DC	Switched
V5	Diode + AC	Switched
V6	Resistor + AC	Switched

Lab test results:
→ FE characterization of all matrices
→ Sensor characterization of diode bias only

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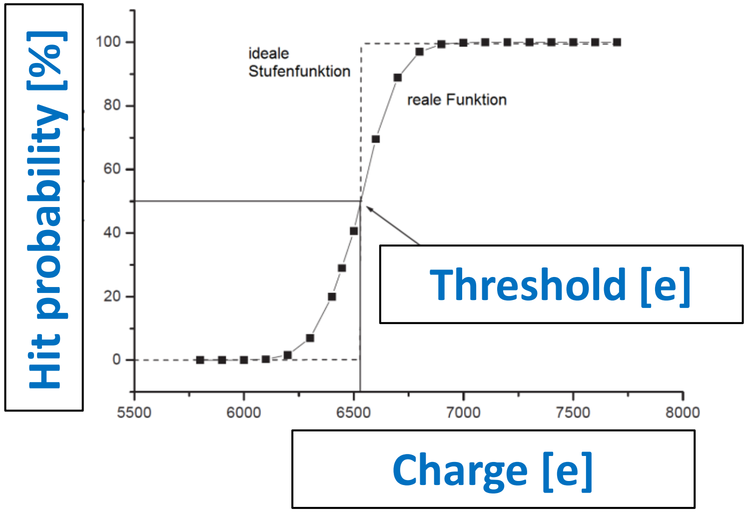
Scurve



Threshold

- Injection of different charges
- Plot hits versus charge injection
- Ideal case: step function
- Noise → Scurve (step + gauss)
- Threshold: x – value of fit function at y = 50 %
- $P_{\text{Treffer}}(Q) = \Theta(Q - Q_{\text{Schwelle}}) \cdot \exp\left(-\frac{Q^2}{2\sigma^2_{\text{Rauschen}}}\right)$

Amplitude

- Injection of the same charge
- Variation of the threshold
- Plot hits versus threshold
- Ideal case: step function
- Noise → Scurve (step + gauss)
- Amplitude: x – value of fit function at y = 50 %
- $P_{\text{Treffer}}(Q) = \Theta(Q - Q_{\text{Schwelle}}) \cdot \exp\left(-\frac{Q^2}{2\sigma^2_{\text{Rauschen}}}\right)$