

Rare B -Decays as Probes of BSM Physics

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Workshop: Flavour as a Window to New Physics at the LHC

Interest in Rare B Decays

- Rare B Decays ($b \rightarrow s\gamma, b \rightarrow s\ell^+\ell^-, \dots$) are Flavour-Changing-Neutral-Current (FCNC) processes ($|\Delta B| = 1, |\Delta Q| = 0$); not allowed at the Tree level in the SM
- FCNC processes are governed by the GIM mechanism, which imparts them sensitivity to higher scales (m_t, m_W) and the CKM matrix elements, in particular, $V_{ti}; i = d, s, b$
- FCNC processes are sensitive to physics beyond the SM, such as supersymmetry, and the BSM amplitudes can be comparable to the (tW)-part of the GIM amplitudes
- Last, but not least, Rare B -decays enjoy great interest in the ongoing and planned experimental programme in heavy quark physics (CLEO, BABAR, BELLE, CDF, D0, LHC, Super-B factory)

Plan of Talk

- The standard candle of Rare B -decays: $B \rightarrow X_s \gamma$
- Benchmark exclusive radiative B -decays: $B \rightarrow K^* \gamma$ & $B_s \rightarrow \phi \gamma$
- Inclusive semileptonic Rare B -decays: $B \rightarrow X_s \ell^+ \ell^-$
- Benchmark exclusive semileptonic B -decays: $B \rightarrow K^{(*)} \ell^+ \ell^-$ & $B_s \rightarrow \phi \ell^+ \ell^-$
- Rarest of the measurable Rare B -decays: $B_s \rightarrow \mu^+ \mu^-$
- Summary

The Standard Candle: $B \rightarrow X_s \gamma$

Interest in the rare decay $B \rightarrow X_s \gamma$ transcends B Physics!

- Already well measured; more precise measurements anticipated at B- and SuperB-factories

Theoretical Interest:

- A monumental theoretical effort has gone in improving the perturbative precision; $B \rightarrow X_s \gamma$ in NNLO completed in 2006
 - First estimate of $\mathcal{B}(B \rightarrow X_s \gamma)$: Misiak et al. (17 authors), hep-ph/0609232
 - Analysis of $\mathcal{B}(B \rightarrow X_s \gamma)$ at NNLO with a cut on Photon energy, T. Becher and M. Neubert, hep-ph/0610067
- Non-perturbative effects under control thanks to HQET
- Sensitivity to new physics; hence constrains parameters of the BSM models such as Supersymmetry
- A crucial input in a large number of precision tests of the SM in $b \rightarrow s$ processes, such as $B \rightarrow X_s \ell^+ \ell^-$

The effective Lagrangian for $B \rightarrow X_s \gamma$ and $B \rightarrow X_s \ell^+ \ell^-$

$$\mathcal{L} = \mathcal{L}_{QCD \times QED}(q, l) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i$$

$(q = u, d, s, c, b, l = e, \mu)$

$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_i b), & i = 1, 2, \quad |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma'_i q), & i = 3, 4, 5, 6, \quad |C_i(m_b)| < 0.07 \\ \frac{em_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, \quad C_7(m_b) \sim -0.3 \\ \frac{gm_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} T^a b_R G_{\mu\nu}^a, & i = 8, \quad C_8(m_b) \sim -0.15 \\ \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{l} \gamma^\mu \gamma_5 l), & i = 9, 10 \quad |C_i(m_b)| \sim 4 \end{cases}$$

Three steps of the calculation:

Matching: Evaluating $C_i(\mu_0)$ at $\mu_0 \sim M_W$ by requiring equality of the SM and the effective theory Green functions

Mixing: Deriving the effective theory RGE and evolving $C_i(\mu)$ from μ_0 to $\mu_b \sim m_b$

Matrix elements: Evaluating the on-shell amplitudes at $\mu_b \sim m_b$

Wilson Coefficients of Four-Quark Operators

	$C_1(\mu_b)$	$C_2(\mu_b)$	$C_3(\mu_b)$	$C_4(\mu_b)$	$C_5(\mu_b)$	$C_6(\mu_b)$
LL	-0.257	1.112	0.012	-0.026	0.008	-0.033
NLL	-0.151	1.059	0.012	-0.034	0.010	-0.040

Wilson Coefficients of Other Operators

	$C_7^{\text{eff}}(\mu_b)$	$C_8^{\text{eff}}(\mu_b)$	$C_9(\mu_b)$	$C_{10}(\mu_b)$
LL	-0.314	-0.149	2.007	0
NLL	-0.308	-0.169	4.154	-4.261
NNLL	-0.290		4.214	-4.312

- Obtained for the following input:

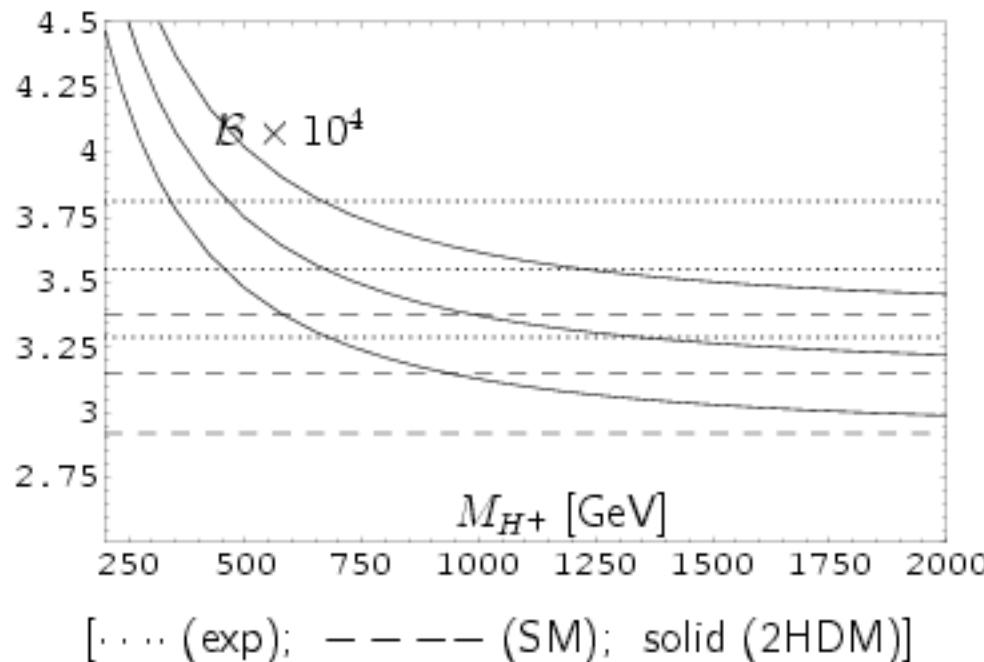
$$\mu_b = 4.6 \text{ GeV} \quad \bar{m}_t(\bar{m}_t) = 167 \text{ GeV}$$

$$M_W = 80.4 \text{ GeV} \quad \sin^2 \theta_W = 0.23$$

- Three-loop running is used for α_s coupling with $\Lambda_{\overline{\text{MS}}}^{(5)} = 220 \text{ MeV}$

$\mathcal{B}(\bar{B} \rightarrow X_s \gamma)$: Experiment vs. SM & 2HDM

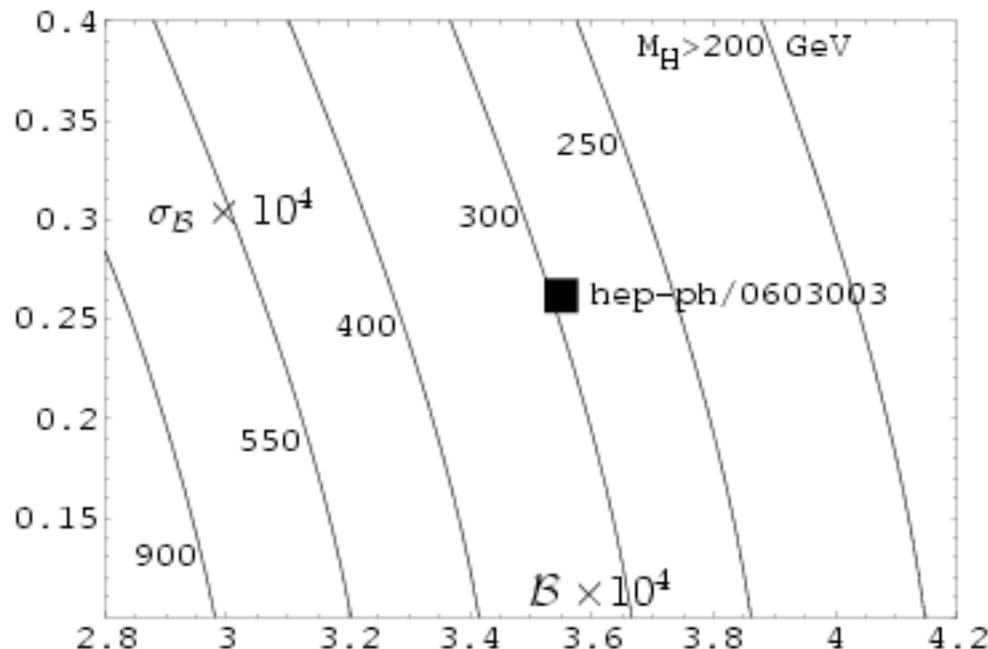
[Misiak et al., hep-ph/0609232]



- Experiment ($E_\gamma > 1.6$ GeV): $\mathcal{B}(\bar{B} \rightarrow X_s \gamma) = (3.52 \pm 0.23 \pm 0.09) \times 10^{-4}$
- NNLO SM: $\mathcal{B}(\bar{B} \rightarrow X_s \gamma) = (3.15 \pm 0.23) \times 10^{-4}$
(Parametric Update: $(3.30 \pm 0.24) \times 10^{-4}$ Gambino, Giordano [arXiv:0805.0271])
- SM is below the experiments by about 1σ
- In 2HDM, preferred value is $M_{H+} \simeq 650$ GeV

95% C.L. Lower Bound on M_{H^+} in 2HDM from $\mathcal{B}(\bar{B} \rightarrow X_s \gamma)$

[Misiak et al., hep-ph/0609232]



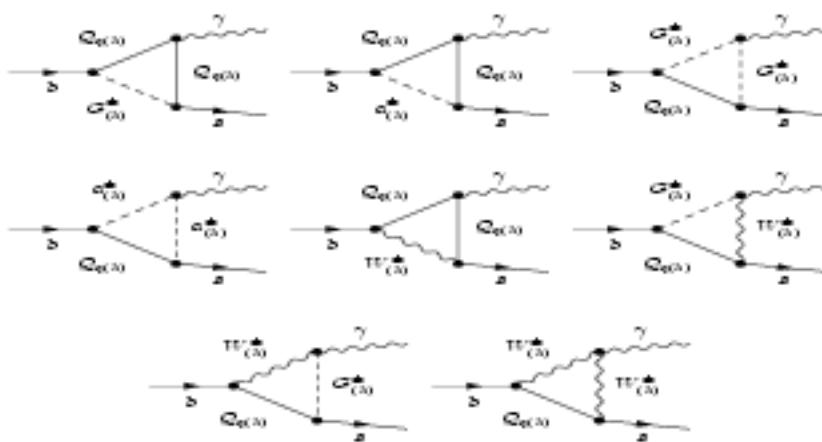
- 95% C.L. lower bound is around 295 GeV

Minimal Universal Extra Dimension Scenario and $\mathcal{B}(\bar{B} \rightarrow X_s \gamma)$

Buras et al. [hep-ph/0306158]; Haisch, Weiler [hep-ph/0703064]

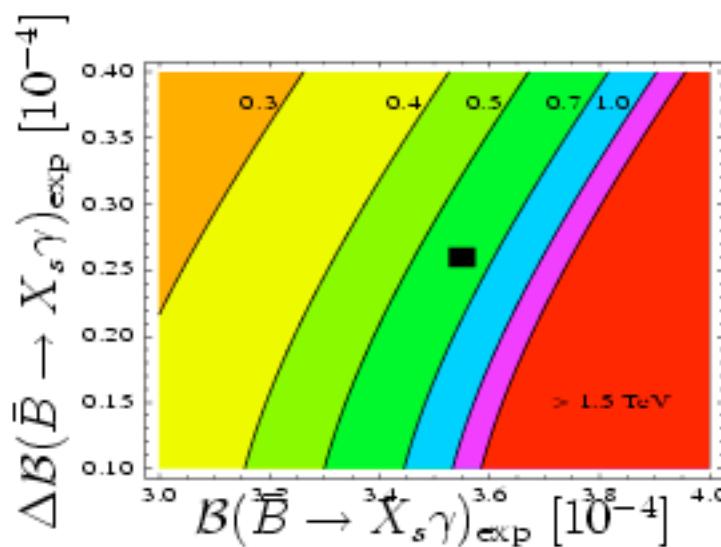
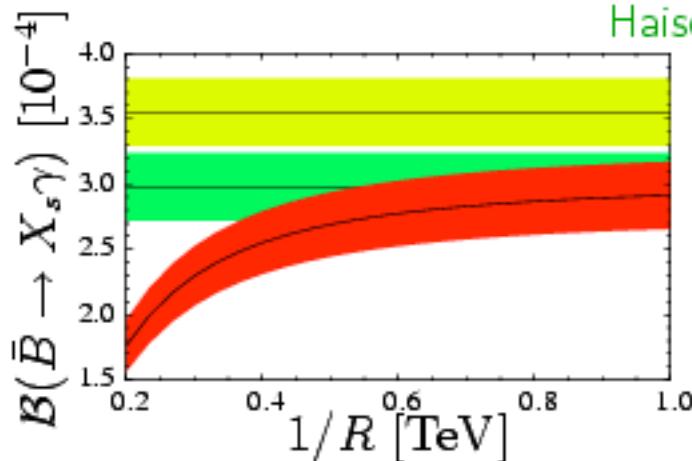
- In the simplest of these scenarios, the Appelquist-Cheng-Dobrescu (ACD) model, the SM is extended to $4+1$ dimensions, with the extra dimension compactified on an orbifold with radius R ; for $R \rightarrow 0$, SM is recovered
- For each SM field (denoted as zero mode, $n = 0$), there are infinite towers of KK modes ($n \geq 1$); also additional physical scalars exist $a_{(n)}^0$ and $a_{(n)}^\pm$
- For $SU(2)_L$ doublets, the KK modes are Q ; for $U(1)$ -singlet fields u_R and d_R , the KK modes are U and D ; $G_{(n)}^\pm$ are the KK partners of W^\pm
- $\mathcal{H}_{\text{eff}}^{\text{mUED}}$ has the same operator basis as $\mathcal{H}_{\text{eff}}^{\text{SM}}$; hence, only Wilson coefficients (Inami-Lim functions) are modified; $C_i(x_t) \rightarrow C_i(x_t, 1/R)$

One-loop corrections to $B \rightarrow X_s \gamma$ in mUED model



Minimal Universal Extra Dimension Scenario and $\mathcal{B}(\bar{B} \rightarrow X_s \gamma)$

Haisch,Weiler [hep-ph/0703064]

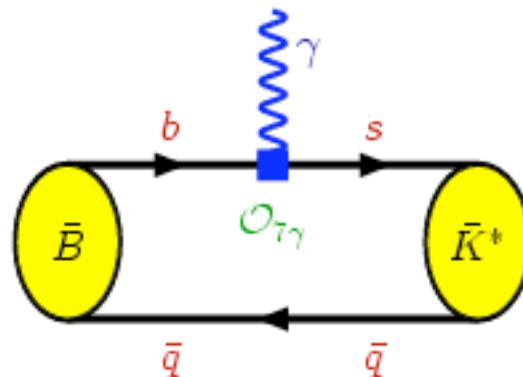


- LO KK modes & NNLO SM contributions and $\mathcal{B}(\bar{B} \rightarrow X_s \gamma)_{\text{exp}}$
 $\implies 1/R > 600 \text{ GeV}$

$B \rightarrow K^* \gamma$ Decays

$B \rightarrow K^* \gamma$ Branching Fraction in LO

- In LO, only the electromagnetic penguin operator $\mathcal{O}_{7\gamma}$ contributes to the $B \rightarrow K^* \gamma$ amplitude; involves the form factor $T_1^{(K^*)}(0)$



$$\mathcal{M}^{\text{LO}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* C_7^{(0)\text{eff}} \frac{e \bar{m}_b}{4\pi^2} T_1^{(K^*)}(0) [(Pq)(\epsilon^* \epsilon^*) - (e^* P)(\epsilon^* q) + i \text{eps}(\epsilon^*, \epsilon^*, P, q)]$$

Here, $P^\mu = p_B^\mu + p_K^\mu$; $q^\mu = p_B^\mu - p_K^\mu$ is the photon four-momentum; ϵ^μ is its polarization vector; ϵ^{μ} is the K^* -meson polarization vector

- Branching ratio:

$$\mathcal{B}^{\text{LO}}(B \rightarrow K^* \gamma) = \tau_B \frac{G_F^2 |V_{tb} V_{ts}^*|^2 \alpha M^3}{32\pi^4} \bar{m}_b^2(\mu_b) |C_7^{(0)\text{eff}}(\mu_b)|^2 |T_1^{(K^*)}(0, \mu_b)|^2$$

$B \rightarrow K^* \gamma$ decay rates in NLO

- Perturbative improvements undertaken in three approaches (QCD-F; PQCD; SCET)

Factorization Ansatz (QCDF):

[Beneke, Buchalla, Neubert, Sachrajda; Beneke & Feldmann]

$$\langle V\gamma|Q_i|\bar{B}\rangle = t_i^I \zeta_{V_\perp} + t_i^{II} \otimes \phi_+^B \otimes \phi_\perp^V + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)$$

- ζ_{V_\perp} (form factor) and $\phi^{B,V}$ (LCDAs) are non-perturbative functions
- t^I and t^{II} are perturbative hard-scattering kernels

$$t^I = \mathcal{O}(1) + \mathcal{O}(\alpha_s) + \dots, \quad t^{II} = \mathcal{O}(\alpha_s) + \dots$$

- The kernels t^I and t^{II} are known at $\mathcal{O}(\alpha_s)$ for some time;
include Hard-scattering and Vertex corrections
[Parkhomenko, AA; Bosch, Buchalla; Beneke, Feldmann, Seidel 2001]

SCET factorization formula for $B \rightarrow K^* \gamma$

[Chay, Kim '03; Grinstein, Grossman, Ligeti '04; Becher, Hill, Neubert '05]

$$\langle V\gamma | Q_i | \bar{B} \rangle = \Delta_i C^A \zeta_{V\perp} + (\Delta_i C^{B1} \otimes j_\perp) \otimes \phi_\perp^V \otimes \phi_+^B$$

- $\zeta_{V\perp}, \phi_\perp^V, \phi_+^B$ are matrix elements of SCET operators
- Hard-scattering kernels t^I, t^{II} = SCET matching coefficients

$$t_i^I = \Delta_i C^A(m_b); \quad t_i^{II} = \Delta_i C^{B1}(m_b) \otimes j_\perp(\sqrt{m_b \Lambda}) \quad (\text{subfactorization})$$

- Derivation of factorization in SCET
 - 1) QCD \rightarrow SCET_I: Integrate out m_b ; Defines vertex corrections $\Delta_i C^A = t_i^I$

$$Q_i \rightarrow \Delta_i C^A(m_b) J^A + \Delta_i C^{B1}(m_b) \otimes J^{B1} + \dots$$

- 2) SCET_I \rightarrow SCET_{II}: Integrate out $\sqrt{m_b \Lambda_{\text{QCD}}}$; Defines spectator corrections

$$J^{B1} \rightarrow j_\perp(\sqrt{m_b \Lambda_{\text{QCD}}}) \otimes O^{B1,\text{SCET}_{II}}(\Lambda_{\text{QCD}})$$

- 3) Large logs in t_i^{II} resummed by solving RG equations

$$[\Delta_i C^{B1} \otimes j_\perp] \rightarrow [\Delta_i C^{B1}(\mu_h) \otimes U(\mu_h, \mu_{hc}) \otimes j_\perp(\mu_{hc})]$$

$B \rightarrow K^* \gamma$ in SCET at NNLO

[Pecjak, Greub, AA '07]

Vertex Corrections

$$\Delta_i C^A = \Delta_7 C^{A(0)} \left[\Delta_{i7} + \frac{\alpha_s(\mu)}{4\pi} \Delta_i C^{A(1)} + \frac{\alpha_s^2(\mu)}{(4\pi)^2} \Delta_i C^{A(2)} \right]$$

- Contributions from O_7 and O_8 exact to NNLO $O(\alpha_s^2)$
- Contribution from O_2 exact at NLO $O(\alpha_s)$ but only large- β_0 limit at $O(\alpha_s^2)$

Spectator Corrections at $O(\alpha_s^2)$

$$t_i^{II(1)}(u, \omega) = \Delta_i C^{B1(1)} \otimes j_\perp^{(0)} + \Delta_i C^{B1(0)} \otimes j_\perp^{(1)}$$

- Status of $O(\alpha_s^2)$ Calculations
 - The one-loop jet-function $j_\perp^{(1)}$ known
[Becher and Hill '04; Beneke and Yang '05]
 - The one-loop hard coefficient $\Delta_7 C^{B1(1)}$ known
[Beneke, Kiyo, Yang '04; Becher and Hill '04]
 - The one-loop hard coefficient $\Delta_8 C^{B1(1)}$ known
[Pecjak, Greub, AA '07]
 - $\Delta_i C^{B1(1)}$ ($i = 1, \dots, 6$) remain unknown (require two loops)

Estimates of $\text{BR}(B \rightarrow K^* \gamma)$ in SCET at NNLO

[Pecjak, Greub, AA '07]

Estimates at NNLO in units of 10^{-5}

$$\mathcal{B}(B^+ \rightarrow K^{*+} \gamma) = 4.6 \pm 1.2[\zeta_{K^*}] \pm 0.4[m_c] \pm 0.2[\lambda_B] \pm 0.1[\mu]$$

[Expt. 4.03 ± 0.26 (HFAG 2008)]

$$\mathcal{B}(B^0 \rightarrow K^{*0} \gamma) = 4.3 \pm 1.1[\zeta_{K^*}] \pm 0.4[m_c] \pm 0.2[\lambda_B] \pm 0.1[\mu]$$

[Expt.: 4.01 ± 0.20 (HFAG 2008)]

$$\mathcal{B}(B_s \rightarrow \phi \gamma) = 4.3 \pm 1.1[\zeta_\phi] \pm 0.3[m_c] \pm 0.3[\lambda_B] \pm 0.1[\mu]$$

[Expt.: $5.7^{+2.1}_{-1.8}$ (BELLE)]

Comparison with current experiments

- $\frac{\mathcal{B}(B^+ \rightarrow K^{*+} \gamma)_{\text{NNLO}}}{\mathcal{B}(B^+ \rightarrow K^{*+} \gamma)_{\text{exp}}} = 1.14 \pm 0.35[\text{theory}] \pm 0.07[\text{exp}]$

- $\frac{\mathcal{B}(B_s \rightarrow \phi \gamma)_{\text{NNLO}}}{\mathcal{B}(B_s \rightarrow \phi \gamma)_{\text{exp}}} = 0.8 \pm 0.2[\text{theory}] \pm 0.3[\text{exp}]$

- Theory error is about 30%; dominantly from ζ_{V_L} , m_c and λ_B

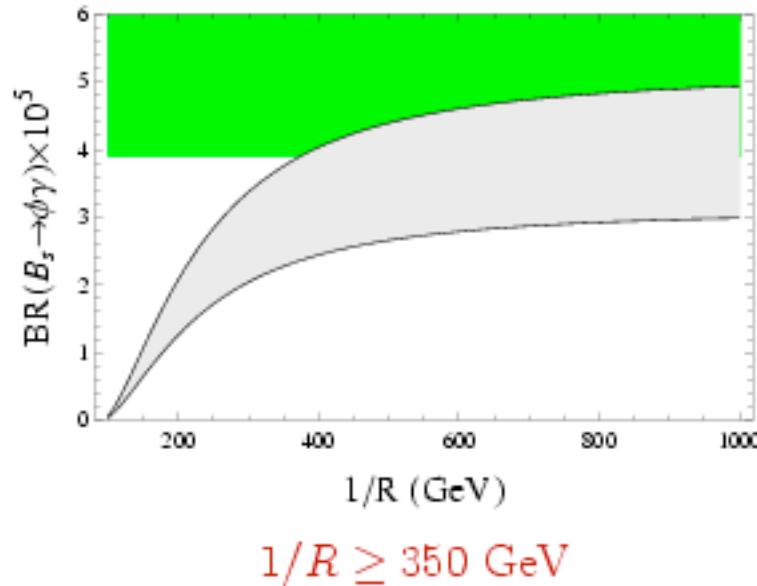
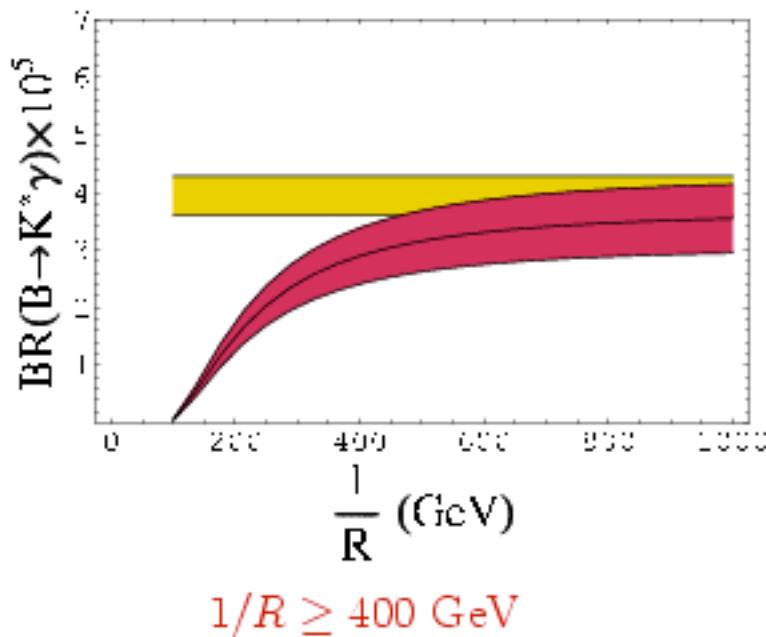
$B \rightarrow K^*\gamma$ & $B_s \rightarrow \phi\gamma$ decays in the mUED scenario

P. Colangelo et al., arXiv:hep-ph/0604029 & arXiv:0709.2817 [hep-ph]

$$\mathcal{B}(B \rightarrow K^*\gamma) = \tau_B \frac{\alpha G_F^2}{8\pi^4} |V_{tb} V_{ts}^*|^2 m_b^2 |C_7^{(0)\text{eff}}(\mu_b, x_t, 1/R)|^2 |T_1^{B \rightarrow K^*}(0)|^2 M_B^3 [1 - m_{K^*}^2/M_B^2]^3$$

$$\mathcal{B}(B_s \rightarrow \phi\gamma) = \left[\frac{T_1^{B_s \rightarrow \phi}(0)}{T_1^{B \rightarrow K^*}(0)} \right]^2 \left[\frac{M_{B_d}}{M_{B_s}} \right]^3 \left[\frac{M_{B_s}^2 - m_\phi^2}{M_{B_d}^2 - m_{K^{*0}}^2} \right]^3 \frac{\tau_{B_s}}{\tau_{B_d}} \mathcal{B}(B_d \rightarrow K^{*0}\gamma)$$

Transition form factors are estimated by the LCSR method



Time-Dependent CP-Asymmetry in $B_{(s)}^0 \rightarrow V\gamma$

- Time-dependent decay width

$$\Gamma[B_q^0(\bar{B}_q^0) \rightarrow f^{\text{CP}}\gamma] \propto e^{-\Gamma_q t} \left[\cosh \frac{\Delta\Gamma_q t}{2} - \mathcal{A}^\Delta \sinh \frac{\Delta\Gamma_q t}{2} \pm \mathcal{C} \cos(\Delta M_q t) \mp \mathcal{S} \sin(\Delta M_q t) \right]$$

- Time-dependent CP-asymmetry in the $B_q^0 \rightarrow V\gamma$ decays

$$\mathcal{A}_{\text{CP}}(t) = \frac{\Gamma(\bar{B}_q^0 \rightarrow V\gamma) - \Gamma(B_q^0 \rightarrow V\gamma)}{\Gamma(\bar{B}_q^0 \rightarrow V\gamma) + \Gamma(B_q^0 \rightarrow V\gamma)} = \frac{\mathcal{S} \sin(\Delta M_q t) - \mathcal{C} \cos(\Delta M_q t)}{\cosh(\Delta\Gamma_q t/2) - \mathcal{A}^\Delta \sinh(\Delta\Gamma_q t/2)}$$

- In the SM (with ψ being the fraction of "wrong" polarization)

$$\mathcal{C} = 0 \quad (\text{direct CPV}) \quad \mathcal{S} = \sin(2\psi) \sin \phi \quad \mathcal{A}^\Delta = \sin(2\psi) \cos \phi$$

- B-factories measured CP-asymmetry $\mathcal{A}_{\text{CP}}(t)$ in $B_d^0 \rightarrow K^{*0}(K_s\pi^0)\gamma$

$$\mathcal{C} = -0.03 \pm 0.14 \quad \mathcal{S} = -0.19 \pm 0.23 \quad [\text{HFAG, 2008}]$$

- As $\Delta\Gamma_s \neq 0$, decay $B_s^0 \rightarrow \phi\gamma$ probes \mathcal{A}^Δ as well as \mathcal{C} and \mathcal{S}
- LHCb projected sensitivity in $B_s^0 \rightarrow \phi\gamma$: $\sigma(A^\Delta) = 0.22$; $\sigma(S, C) = 0.11$
- SM (pQCD): Wang et al. (arxiv:0711.0432)
 $C_{\phi\gamma} = (0.3 \pm 0.1)\%$; $S_{\phi\gamma} = (0.2 \pm 0.1)\%$; $\mathcal{A}_{\phi\gamma}^\Delta = -(5.5 \pm 0.4)\%$

$\bar{B} \rightarrow X_s l^+ l^-$

- The NNLO calculation of $\bar{B} \rightarrow X_s l^+ l^-$ corresponds to the NLO calculation of $\bar{B} \rightarrow X_s \gamma$, as far as the number of loops in the diagrams is concerned.
- Wilson Coefficients of the two additional operators

$$O_i = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L) (\bar{l} \gamma^\mu \gamma_5 l), \quad i = 9, 10$$

have the following perturbative expansion:

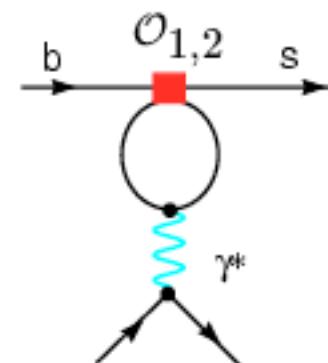
$$\begin{aligned} C_9(\mu) &= \frac{4\pi}{\alpha_s(\mu)} C_9^{(-1)}(\mu) + C_9^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_9^{(1)}(\mu) + \dots \\ C_{10} &= C_{10}^{(0)} + \frac{\alpha_s(M_W)}{4\pi} C_{10}^{(1)} + \dots \end{aligned}$$

- After an expansion in α_s , the term $C_9^{(-1)}(\mu)$ reproduces (the dominant part of) the electroweak logarithm that originates from photonic penguins with charm quark loops:

$$\frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) = \frac{4}{9} \ln \frac{M_W^2}{m_b^2} + \mathcal{O}(\alpha_s)$$

$$C_9^{(-1)}(m_b) \simeq 0.033 \ll 1 \quad \Rightarrow \quad \frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) \simeq 2$$

On the other hand: $C_9^{(0)}(m_b) \simeq 2.2$; need to calculate NNLO



Comparison of $B \rightarrow X_s \ell^+ \ell^-$ with Data

- $B \rightarrow X_s \ell^+ \ell^-$ decay rate

$$\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)(M_{\ell\ell} > 0.2 \text{ GeV}) = (4.50^{+1.03}_{-1.02}) \times 10^{-6} \quad [\text{HFAG'07}]$$

$$SM : (4.2 \pm 0.7) \times 10^{-6} \quad [\text{AGHL'01}]; \quad (4.6 \pm 0.8) \times 10^{-6} \quad [\text{GHIY'04}]$$

- Differential distributions in $B \rightarrow X_s \ell^+ \ell^-$

- $M(X_s)$ -distribution: tests $s \rightarrow X_s$ fragmentation model; FMs provide reasonable fit to data; improved theoretical distributions calculated in HQET more recently [Lee, Ligeti, Stewart, Tackmann 2006]

- $q^2 = M_{\ell^+\ell^-}^2$ -distribution away from the $J/\psi, \psi', \dots$ resonances is sensitive to short-distance physics; current data in agreement with the SM estimates but the precision is not better than 25%

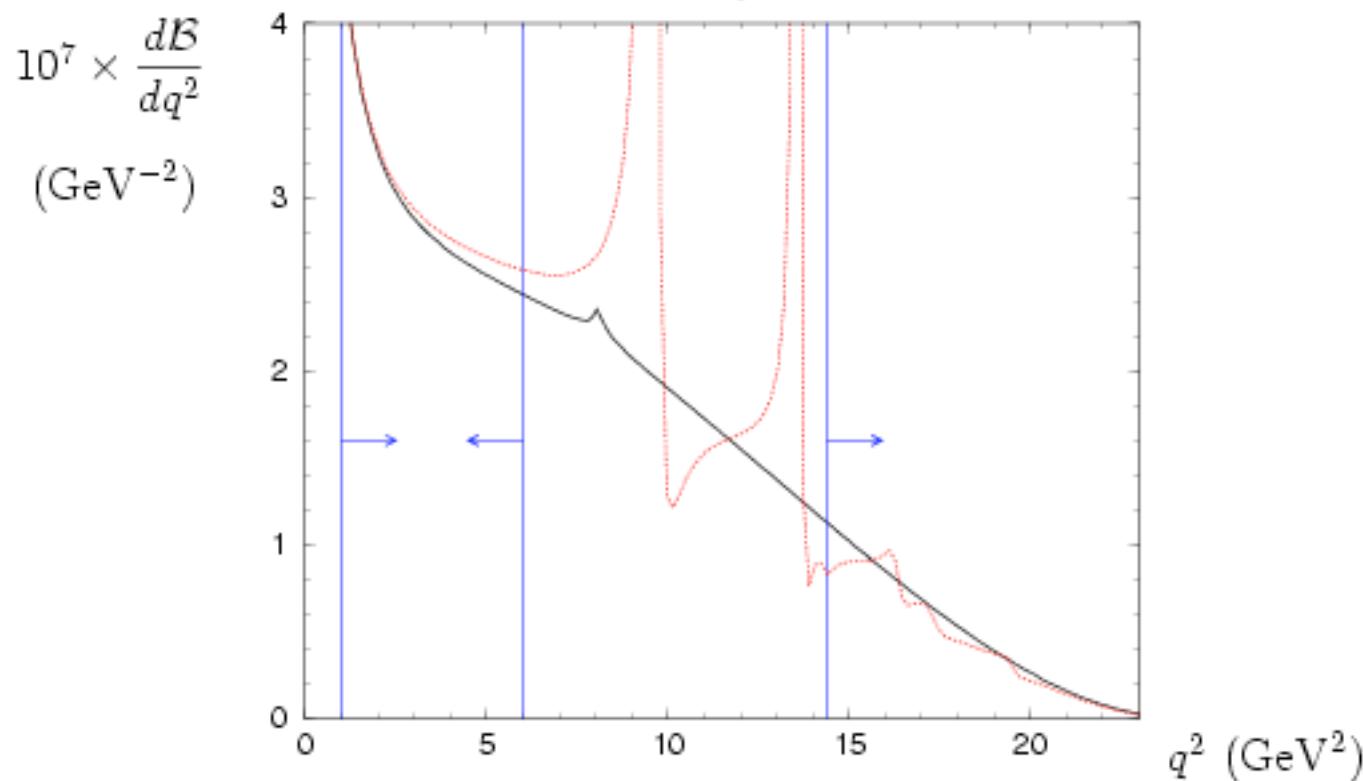
- Forward-Backward Asymmetry (FBA) is likewise sensitive to the SM and BSM effects, in particular encoded in the Wilson coefficients C_7, C_9 and C_{10}

$$A_{FB}(\hat{s}) \sim C_{10}(2C_7 + C_9(\hat{s})\hat{s}); \quad \hat{s} = q^2/M_B^2$$

- $A_{FB}(\hat{s})$ not yet measured; possible only in experiments at B factories

Dilepton invariant mass distribution in $\bar{B} \rightarrow X_s \ell^+ \ell^-$:

[Ghinculov, Hurth, Isidori, Yao 2004]



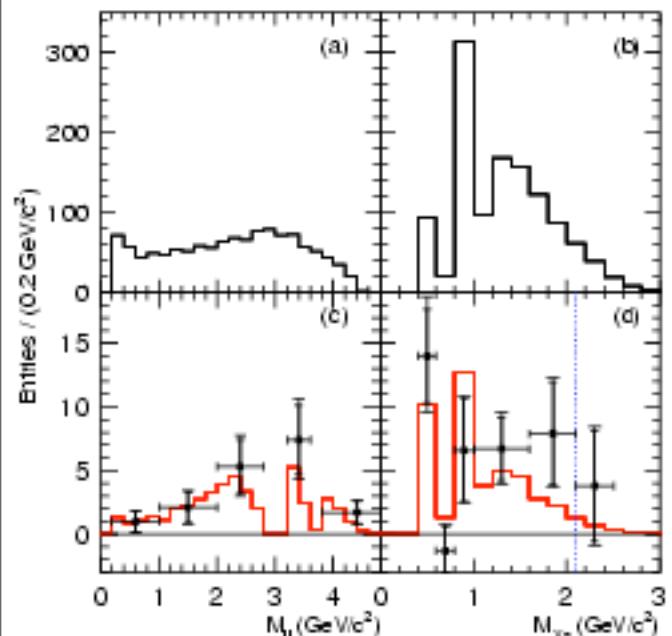
- $\text{BR}(\bar{B} \rightarrow X_s \ell^+ \ell^-); q^2 \in [1, 6] \text{ GeV}^2 = (1.63 \pm 0.20) \times 10^{-6}$
- $\text{BR}(\bar{B} \rightarrow X_s \ell^+ \ell^-); q^2 > 14 \text{ GeV}^2 = (4.04 \pm 0.78) \times 10^{-7}$
- $\text{BR}(\bar{B} \rightarrow X_s \ell^+ \ell^-); q^2 > 4m_\mu^2 = (4.6 \pm 0.8) \times 10^{-6}$,

in agreement with the other NNLO analysis [AA, Greub, Hiller, Lunghi 2001; Bobeth, Gambino, Gorbahn, Haisch, 2003; Huber, Lunghi, Misiak, Wyler 2005]

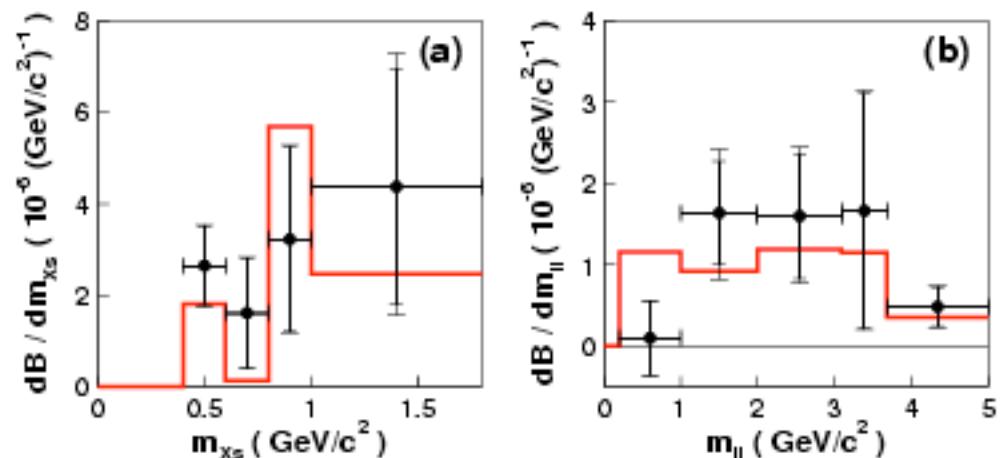
Decay distributions in $\bar{B} \rightarrow X_s \ell^+ \ell^-$

$M_{\ell\ell}$ and M_{X_s} Spectra

[BELLE]



[BABAR]



- In agreement with the NNLO SM calculations

NNLL Forward-Backward Asymmetry in $B \rightarrow X_s \ell^+ \ell^-$

[Asatrian, Bieri, Greub, Hovhannisyan; Ghinculov, Hurth, Isidori, Yao]

Normalized FB Asymmetry

$$\overline{A}_{\text{FB}}(\hat{s}) = \frac{\int_{-1}^1 \frac{d^2\Gamma(b \rightarrow X_s \ell^+ \ell^-)}{d\hat{s} dz} \text{sgn}(z) dz}{\int_{-1}^1 \frac{d^2\Gamma(b \rightarrow X_s \ell^+ \ell^-)}{d\hat{s} dz} dz}$$

Unnormalized FB Asymmetry

$$A_{\text{FB}}(\hat{s}) = \frac{\int_{-1}^1 \frac{d^2\Gamma(b \rightarrow X_s \ell^+ \ell^-)}{d\hat{s} dz} \text{sgn}(z) dz}{\Gamma(B \rightarrow X_c e \bar{\nu}_e)} \text{BR}_{s1}$$

$$\begin{aligned} \int_{-1}^1 \frac{d^2\Gamma(b \rightarrow X_s \ell^+ \ell^-)}{d\hat{s} dz} \text{sgn}(z) dz &= \left(\frac{\alpha_{\text{em}}}{4\pi} \right)^2 \frac{G_F^2 m_{b,\text{pole}}^5 |V_{ts}^* V_{tb}|^2}{48\pi^3} (1-\hat{s})^2 \\ &\times \left[-3\hat{s} \text{Re}(\tilde{C}_9^{\text{eff}} \tilde{C}_{10}^{\text{eff}*}) \left(1 + \frac{2\alpha_s}{\pi} f_{910}(\hat{s}) \right) - 6 \text{Re}(\tilde{C}_7^{\text{eff}} \tilde{C}_{10}^{\text{eff}*}) \left(1 + \frac{2\alpha_s}{\pi} f_{710}(\hat{s}) \right) \right] \end{aligned}$$

- NNLL Contributions stabilize the scale ($= \mu$) dependence of the FB Asymmetry

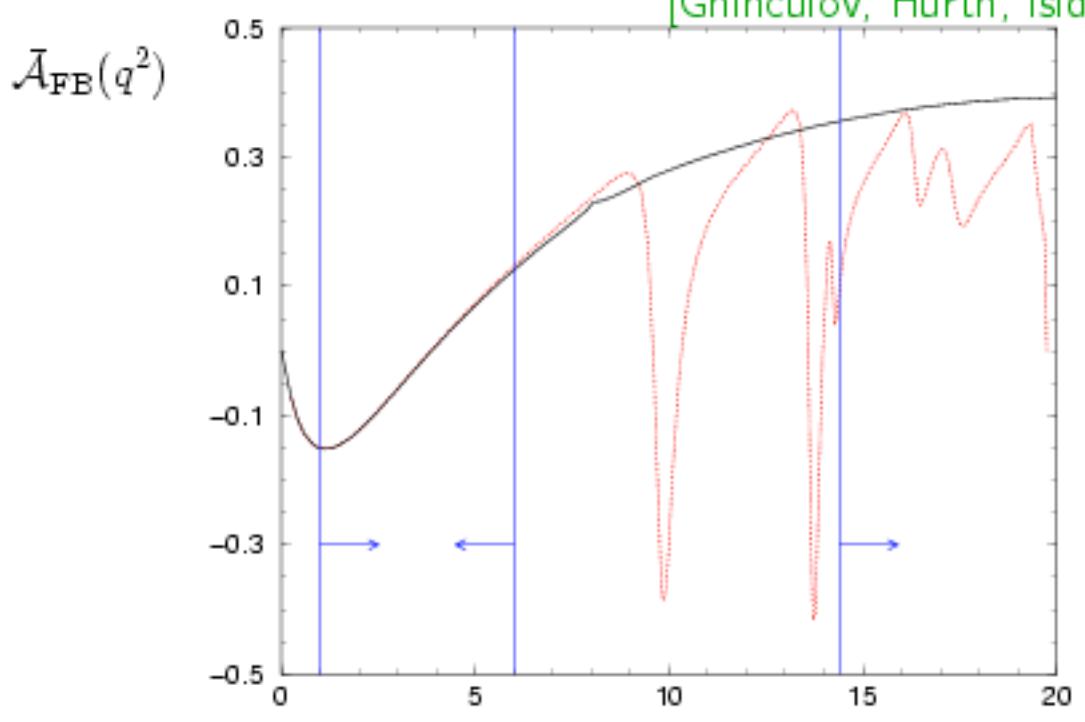
$$A_{\text{FB}}^{\text{NLL}}(0) = -(2.51 \pm 0.28) \times 10^{-6}; \quad A_{\text{FB}}^{\text{NNLL}}(0) = -(2.30 \pm 0.10) \times 10^{-6}$$

- Zero of the FB Asymmetry is a precise test of the SM, correlating \tilde{C}_7^{eff} and \tilde{C}_9^{eff}

$$\hat{s}_0^{\text{NLL}} = 0.144 \pm 0.020; \quad \hat{s}_0^{\text{NNLL}} = 0.162 \pm 0.008$$

Normalized FB-Asymmetry in $\bar{B} \rightarrow X_s \ell^+ \ell^-$:

[Ghinculov, Hurth, Isidori, Yao 2004]



$$\bar{A}_{FB}(q^2) = \frac{1}{d\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)/dq^2} \int_{-1}^1 d\cos\theta_\ell \frac{d^2\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)}{dq^2 d\cos\theta_\ell} \text{sgn}(\cos\theta_\ell)$$

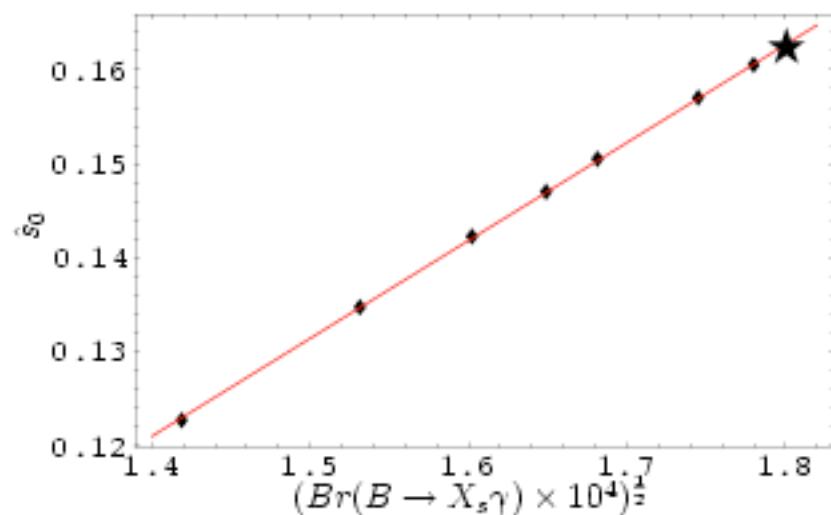
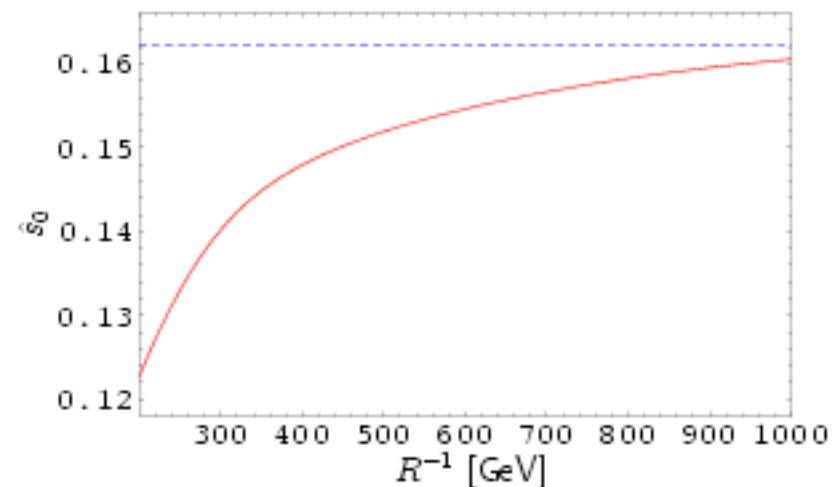
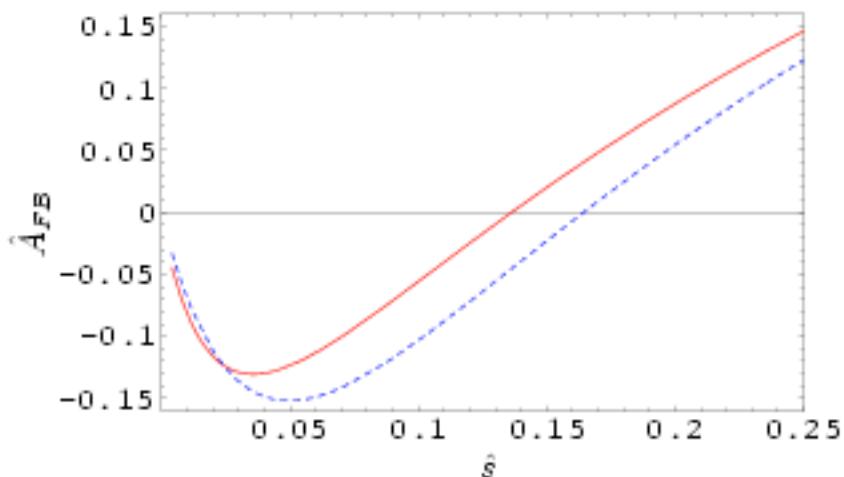
- Zero of the FB-Asymmetry is a precision test of the SM

$$q_0^2 = (3.90 \pm 0.25) \text{ GeV}^2 \quad [\text{Ghinculov, Hurth, Isidori, Yao 2004}]$$

$$q_0^2 = (3.76 \pm 0.22_{\text{theory}} \pm 0.24_{m_b}) \text{ GeV}^2 \quad [\text{Bobeth, Gambino, Gorbahn, Haisch 2003}]$$

Minimal Universal Extra Dimension and $\mathcal{B}(\bar{B} \rightarrow X_s \ell^+ \ell^-)$

Buras et al. [hep-ph/0306158]



Exclusive Decays $B \rightarrow (K, K^*)\ell^+\ell^-$

- $B \rightarrow K$ (pseudoscalar P); $B \rightarrow K^*$ (Vector V) Transitions involve the currents:

$$\Gamma_\mu^1 = \bar{s}\gamma_\mu(1 - \gamma_5)b, \quad \Gamma_\mu^2 = \bar{s}\sigma_{\mu\nu}q^\nu(1 + \gamma_5)b$$

$$\langle P|\Gamma_\mu^1|B\rangle \supset f_+(q^2), f_-(q^2)$$

$$\langle P|\Gamma_\mu^2|B\rangle \supset f_T(q^2)$$

$$\langle V|\Gamma_\mu^1|B\rangle \supset V(q^2), A_1(q^2), A_2(q^2), A_3(q^2)$$

$$\langle V|\Gamma_\mu^2|B\rangle \supset T_1(q^2), T_2(q^2), T_3(q^2)$$

- 10 non-perturbative q^2 -dependent functions (Form factors) \implies model-dependence
- Data on $B \rightarrow K^*\gamma$ provides normalization of $T_1(0) = T_2(0) \simeq 0.28$
- HQET/SCET-Approach allows to reduce the number of independent form factors from 10 to 3; perturbative symmetry-breaking corrections [Beneke, Feldmann, Seidel; Beneke, Feldmann]
- HQET & $SU(3)_F$ relate $B \rightarrow (\pi, \rho)\ell\nu_\ell$ and $B \rightarrow (K, K^*)\ell^+\ell^-$ to determine the remaining FF's

Experimental data vs. SM in $B \rightarrow (X_s, K, K^*)\ell^+\ell^-$ Decays

Branching ratios (in units of 10^{-6}) [HFAG: April 2008]

SM: [A.A., Greub, Hiller, Lunghi hep-ph/0112300]

Decay Mode	Expt. (BELLE & BABAR)	Theory (SM)
$B \rightarrow K\ell^+\ell^-$	0.39 ± 0.06	0.35 ± 0.12
$B \rightarrow K^*e^+e^-$	$1.13^{+0.28}_{-0.26}$	1.58 ± 0.49
$B \rightarrow K^*\mu^+\mu^-$	$1.03^{+0.26}_{-0.23}$	1.19 ± 0.39
$B \rightarrow X_s\mu^+\mu^-$	$4.3^{+1.3}_{-1.2}$	4.2 ± 0.7
$B \rightarrow X_se^+e^-$	4.7 ± 1.3	4.2 ± 0.7
$B \rightarrow X_s\ell^+\ell^-$	$4.50^{+1.03}_{-1.01}$	4.2 ± 0.7

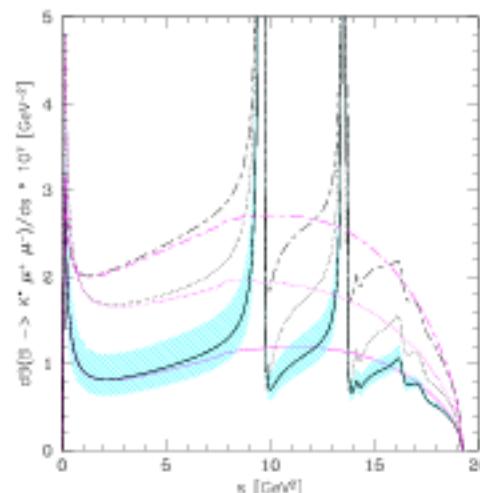
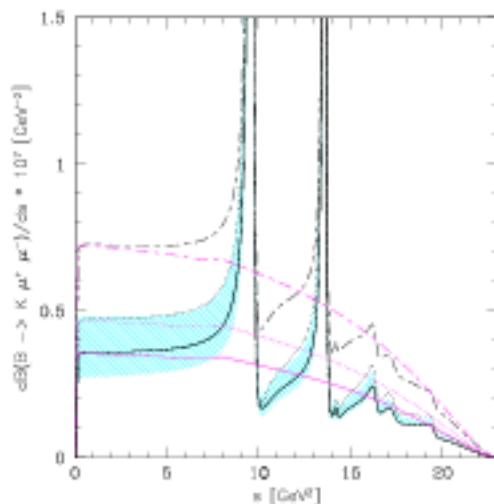
- Inclusive measurements and the SM rates include the cut $M_{\ell^+\ell^-} > 0.2$ GeV
- SM & Data agree within 25%

Dilepton mass-Spectrum in $\bar{B} \rightarrow (K, K^*)\ell^+\ell^-$ in SM and SUSY AA, Ball, Handoko, Hiller; hep-ph/9910221

- NP contributions coded in $R_i(\mu)$; $i = 7, 9, 10$

$$R_i(\mu) \equiv \frac{C_i^{\text{NP}} + C_i^{\text{SM}}}{C_i^{\text{SM}}}$$

- SM (solid); SUGRA [$R_7 = -1.2$] (dots);
- MIA [$R_7 = -0.83, R_9 = 0.92, R_{10} = 1.6$] (dashed)



Leading order in $1/m_b$ and all orders in α_s

[AA, Kramer, Zhou; hep-ph/0601034]

Introduction $B \rightarrow K^* e^+ e^-$ decay Summary

SCET formulae Phenomenological discussion

The factorization formula in SCET

$$\langle K_a^* \ell^+ \ell^- | H_{\text{eff}} | B \rangle = T_a^I(q^2) \zeta_a(q^2) + \\ \sum_{\pm} \int_0^{\infty} \frac{d\omega}{\omega} \phi_{\pm}^B(\omega) \int_0^1 du \phi_{K^*}^a(u) T_{a,\pm}^{II}(\omega, u, q^2)$$

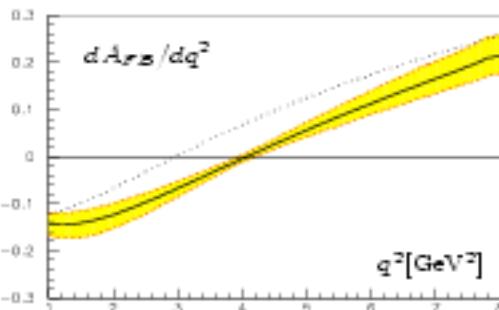
where $a = \parallel, \perp$ denotes the polarization of the K^* meson

- formally coincides with the formula in QCD Factorization [Beneke/Feldmann/Seidel 2001], but valid to all orders of α_s
- for T^{II} , the logarithms are summed from $\mu = m_b$ to $\sqrt{m_b \Lambda_h}$
- Compared with BFS, the definition of $\zeta_{\perp,\parallel}$ is also different here

Reduction of Scale Uncertainty in SCET

Introduction $B \rightarrow K^+ \ell^+ \ell^-$ decay Summary SCET formulae Phenomenological discussion

Forward-backward asymmetry



$A_{FB}(q_0^2) = 0$ free of hadronic uncertainties [Burdman 1998, Ali et al., 2000]

$q_0^2 = (4.07^{+0.16}_{-0.13}) \text{ GeV}^2$ with $\Delta(q_0^2)_{\text{scale}} = {}^{+0.08}_{-0.05} \text{ GeV}^2$

QCD-F [Beneke/Feldmann/Seidel 2001]

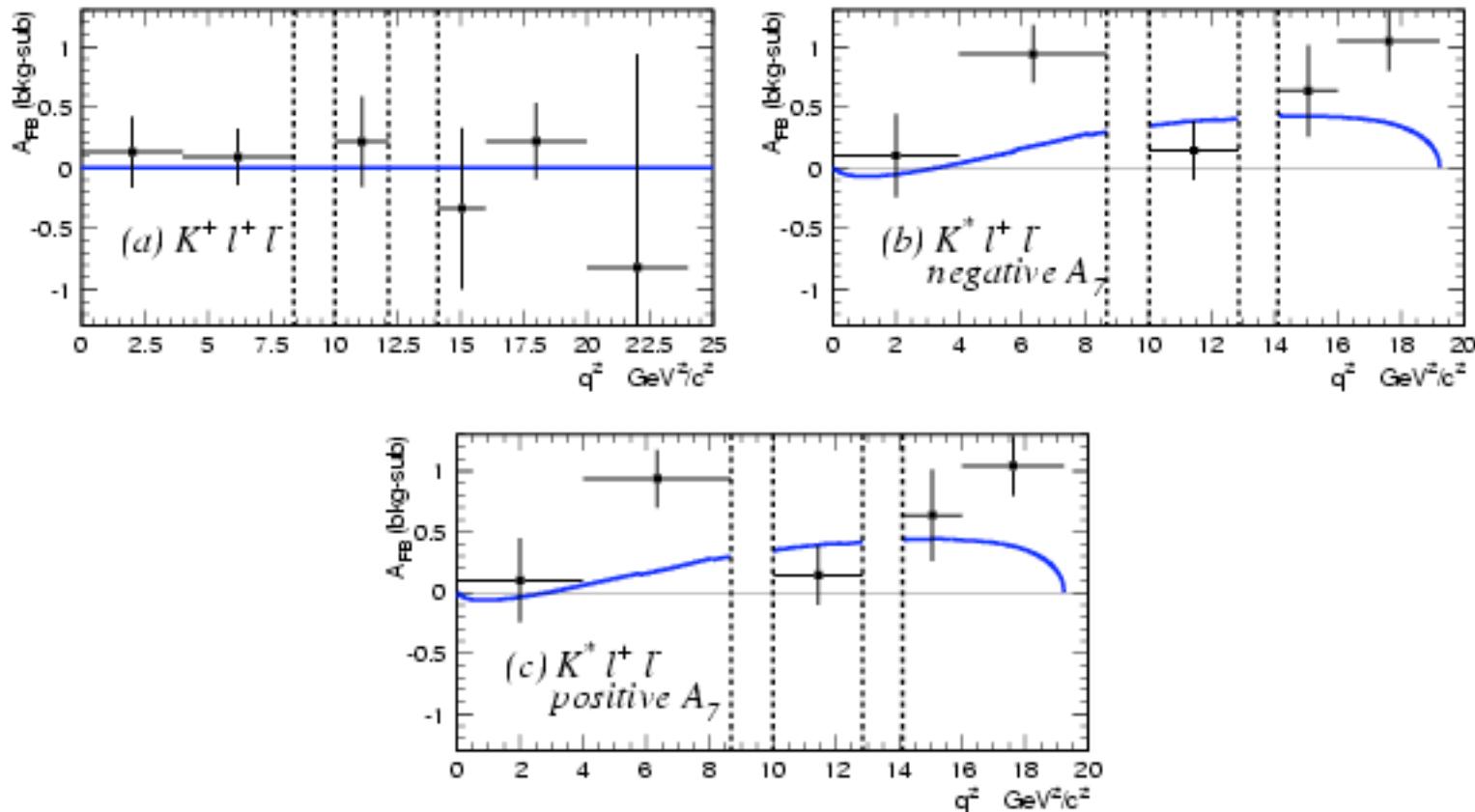
$q_0^2 = (4.39^{+0.38}_{-0.35}) \text{ GeV}^2$ with $\Delta(q_0^2)_{\text{scale}} = \pm 0.25 \text{ GeV}^2$



Ahmed Ali

$B \rightarrow K^+ \ell^+ \ell^-$ decay in soft-collinear effective theory

Belle FB Asymmetry Distributions (EPS 2005)

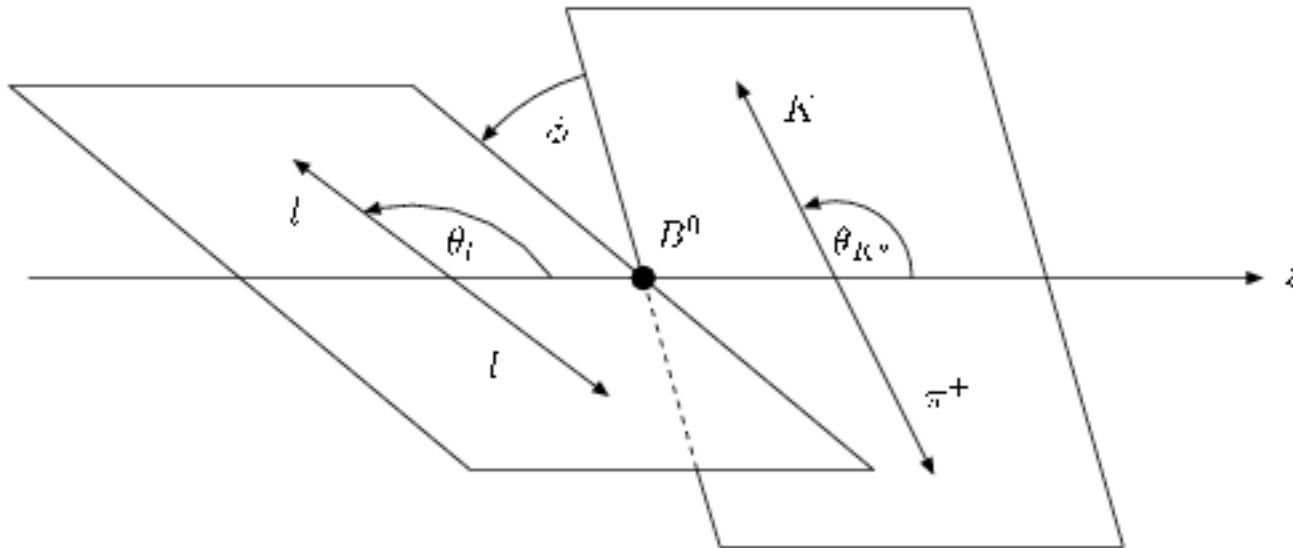


Best Fits

- $A_7 = -0.33$; $A_9/A_7 = -15.3^{+3.4}_{-4.8}$; $A_{10}/A_7 = 10.3^{+5.2}_{-3.5}$
- $A_7 = +0.33$; $A_9/A_7 = -16.3^{+3.7}_{-5.7}$; $A_{10}/A_7 = 11.1^{+6.0}_{-3.9}$
- SM: $A_7 = -0.33$; $A_9/A_7 = -12.3$; $A_{10}/A_7 = 12.8$

Angular Distributions in $B \rightarrow K^* \ell^+ \ell^-$

[Krüger, Sehgal, Sinha, Sinha; Fässler et al.; Krüger, Matias; Safir,AA]



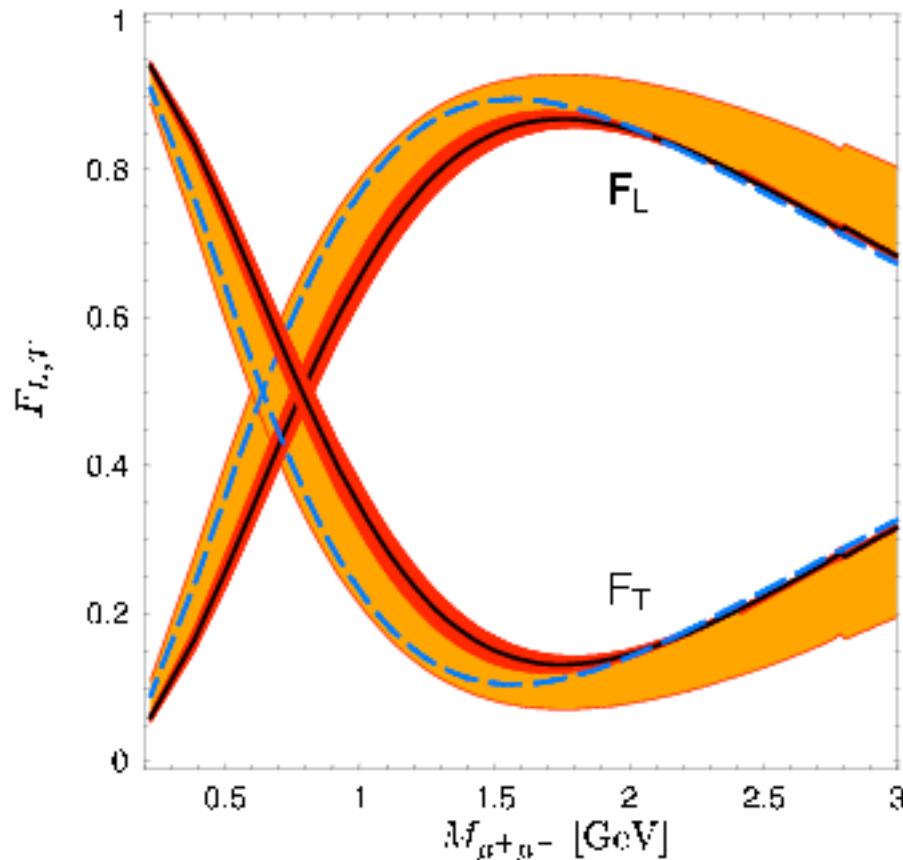
Angular Distributions

- θ_K : angle between the K - and B -momentum directions in the K^* rest frame
$$\frac{d\Gamma}{d \cos \theta_K} \propto \frac{3}{2} F_L \cos^2 \theta_K + \frac{3}{4} (1 - F_L)(1 - \cos^2 \theta_K)$$
 - Determines F_L , the K^* -longitudinal polarization
- θ_ℓ : angle between the $\ell^+(\ell^-)$ and $B(\bar{B})$ -momentum directions in the $\ell^+\ell^-$ c.o.m. frame
$$\frac{d\Gamma}{d \cos \theta_\ell} \propto \frac{3}{4} F_L (1 - \cos^2 \theta_\ell) + \frac{3}{8} (1 - F_L)(1 + \cos^2 \theta_\ell) + \mathcal{A}_{FB} \cos \theta_\ell$$
 - Determines \mathcal{A}_{FB}

SM Predictions for F_L & F_T in $B \rightarrow K^* \ell^+ \ell^-$

Krüger, Matias

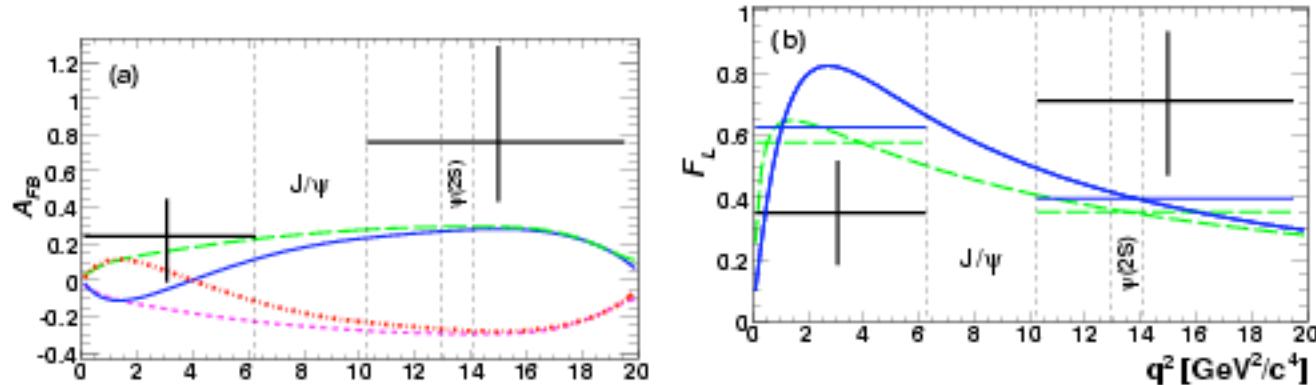
$$F_T = 1 - F_L$$



(Based on QCD-Factorization framework [Beneke, Feldmann, Seidel] with $\xi_\perp(0)$ varied between 0.24 and 0.35)

BABAR's measurement of the angular distributions in $B \rightarrow K^* \ell^+ \ell^-$

B. Aubert et al. SLAC-PUB-13133 [arxiv:0804.4412]

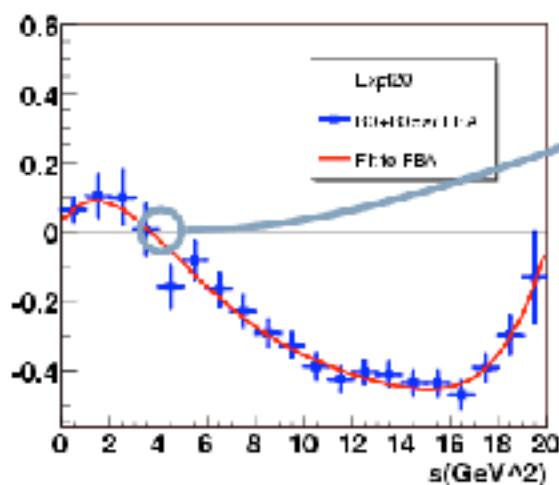
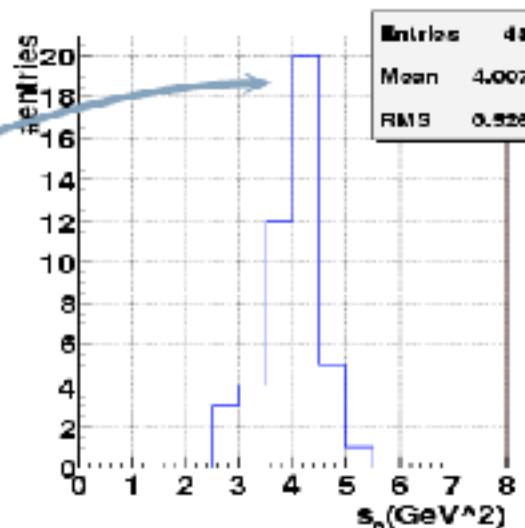


- Low- q^2 region ($m_{\ell\ell} < 2.5$ GeV)
 $\mathcal{A}_{FB} = 0.24^{+0.18}_{-0.23} \pm 0.05$; $F_L = 0.35 \pm 0.16 \pm 0.04$
[SM: $\mathcal{A}_{FB} = -0.03 \pm 0.01$ (Huber, Hurth, Lunghi); $F_L = 0.63 \pm 0.03$ (Krüger, Matias)]
- High- q^2 region ($m_{\ell\ell} > 3.2$ GeV)
 $\mathcal{A}_{FB} = 0.76^{+0.52}_{-0.32} \pm 0.07$; $F_L = 0.71^{+0.20}_{-0.22} \pm 0.04$
[SM: $\mathcal{A}_{FB} = 0.26 \pm 0.01^{+0.00}_{-0.05}$ (AA,Ball,Handoko,Hiller; Hovhannisyan, Hou, Mahajan); $F_L = 0.40 \pm 0.03$ (Krüger, Matias)]
- Measurements not yet quantitative

Zero of $B \rightarrow \mu\mu K^*$ A_{FB} 

From Toy MC

- 2 fb^{-1} : $(4.0 \pm 1.2) \text{ GeV}^2$
- 10 fb^{-1} : $(4.0 \pm 0.5) \text{ GeV}^2 \Rightarrow 13\% \text{ error on } C_7/C_9$

Typical $A_{FB}(s)$ measurementSpread of s_0 

P Koppenburg

LHC — rare semi-leptonic and radiative B decays — Reach 2008 — p 18/21

$\Delta B = 1$ effective Hamiltonian with (NP) parity-flipped operators

Bobeth, Hiller, Piranishvili [arxiv:0805.2525]

Relevant for $b \rightarrow s\gamma$ and $b \rightarrow s\bar{l}l$ observables

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left(\mathcal{H}_{\text{eff}}^{(t)} + \hat{\lambda}_u \mathcal{H}_{\text{eff}}^{(u)} \right), \quad \hat{\lambda}_u = V_{ub} V_{us}^* / V_{tb} V_{ts}^*$$

$$\mathcal{H}_{\text{eff}}^{(t)} = C_1 \mathcal{O}_1^c + C_2 \mathcal{O}_2^c + \sum_{i=3}^{10} C_i \mathcal{O}_i + \sum_{i=7,9,10} C'_i \mathcal{O}'_i, \quad \mathcal{H}_{\text{eff}}^{(u)} = C_1 (\mathcal{O}_1^c - \mathcal{O}_1^u) + C_2 (\mathcal{O}_2^c - \mathcal{O}_2^u)$$

$$\mathcal{O}_7 = \frac{e}{(4\pi)^2} \overline{m}_b [\bar{s}\sigma^{\mu\nu} P_R b] F_{\mu\nu}, \quad \mathcal{O}'_7 = \frac{e}{(4\pi)^2} \overline{m}_b [\bar{s}\sigma^{\mu\nu} P_L b] F_{\mu\nu},$$

$$\mathcal{O}_9 = \frac{e^2}{(4\pi)^2} [\bar{s}\gamma_\mu P_L b][\bar{l}\gamma^\mu l], \quad \mathcal{O}'_9 = \frac{e^2}{(4\pi)^2} [\bar{s}\gamma_\mu P_R b][\bar{l}\gamma^\mu l],$$

$$\mathcal{O}_{10} = \frac{e^2}{(4\pi)^2} [\bar{s}\gamma_\mu P_L b][\bar{l}\gamma^\mu \gamma_5 l], \quad \mathcal{O}'_{10} = \frac{e^2}{(4\pi)^2} [\bar{s}\gamma_\mu P_R b][\bar{l}\gamma^\mu \gamma_5 l]$$

- $C_i = C_i^{\text{SM}} + C_i^{\text{NP}}$; $C'_i = C'_i{}^{\text{NP}}$; $i = 7, 9, 10$

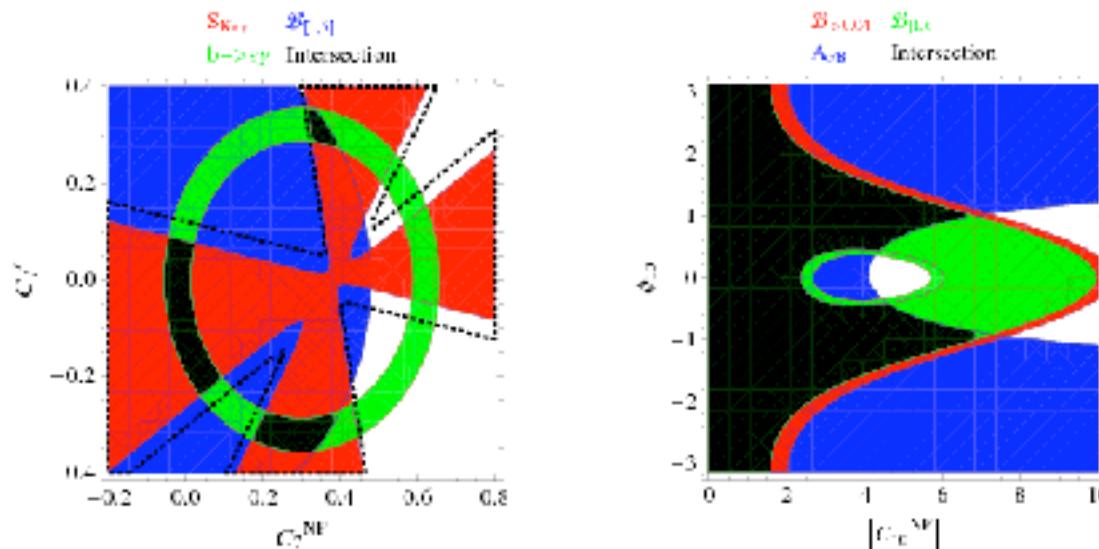
Constraints on Wilson coeffs. from $b \rightarrow s$ transitions in a NP scenario

Bobeth, Hiller, Piranishvili [arxiv:0805.2525]

Relevant $b \rightarrow s\gamma$ and $b \rightarrow s\bar{l}l$ observables

observable	SM	data
$\mathcal{B}(B \rightarrow X_s\gamma)^a$	$(3.15 \pm 0.23) \cdot 10^{-4}$	$(3.52 \pm 0.25) \cdot 10^{-4}$
$S_{K^*\gamma}^b$	$(-2.8^{+0.4}_{-0.5}) \cdot 10^{-2}$	-0.19 ± 0.23
$\mathcal{B}(B \rightarrow X_s\ell^+\ell^-)_{[1,6]}$	$(1.59 \pm 0.11) \cdot 10^{-6}$	$(1.60 \pm 0.51) \cdot 10^{-6}$
$\mathcal{B}(B \rightarrow X_s\ell^+\ell^-)_{(>0.04)}$	$(4.15 \pm 0.70) \cdot 10^{-6}$	$(4.5 \pm 1.0) \cdot 10^{-6}$
$\langle A_{FB} \rangle_{[high\,q^2]}$	< 0	$-(0.76^{+0.52}_{-0.32} \pm 0.07)$
$\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$	$\simeq 3 \cdot 10^{-9}$	$< 4.7 \cdot 10^{-8}$

^aWith photon energy cut $E_\gamma > 1.6$ GeV. ^bSM value obtained with $m_s = 0.12$ GeV.



$B_s \rightarrow \mu^+ \mu^-$ in the SM

- Effective Hamiltonian

$$\mathcal{H}_{eff} = -\frac{G_F \alpha}{\sqrt{2}\pi} V_{ts}^* V_{tb} \sum_i [C_i(\mu) \mathcal{O}_i(\mu) + C'_i(\mu) \mathcal{O}'_i(\mu)]$$

$$\begin{aligned} \mathcal{O}_{10} &= (\bar{s}_\alpha \gamma^\mu P_L b_\alpha) (\bar{l} \gamma_\mu \gamma_5 l), & \mathcal{O}'_{10} &= (\bar{s}_\alpha \gamma^\mu P_R b_\alpha) (\bar{l} \gamma_\mu \gamma_5 l) \\ \mathcal{O}_S &= m_b (\bar{s}_\alpha P_R b_\alpha) (\bar{l} l), & \mathcal{O}'_S &= m_s (\bar{s}_\alpha P_L b_\alpha) (\bar{l} l) \\ \mathcal{O}_P &= m_b (\bar{s}_\alpha P_R b_\alpha) (\bar{l} \gamma_5 l), & \mathcal{O}'_P &= m_s (\bar{s}_\alpha P_L b_\alpha) (\bar{l} \gamma_5 l) \end{aligned}$$

$$\begin{aligned} BR(\bar{B}_s \rightarrow \mu^+ \mu^-) &= \frac{G_F^2 \alpha^2 m_{B_s}^2 f_{B_s}^2 \tau_{B_s}}{64\pi^3} |V_{ts}^* V_{tb}|^2 \sqrt{1 - 4\hat{m}_\mu^2} \\ &\quad \times \left[(1 - 4\hat{m}_\mu^2) |F_S|^2 + |F_P + 2\hat{m}_\mu^2 F_{10}|^2 \right] \end{aligned}$$

where $\hat{m}_\mu = m_\mu / m_{B_s}$ and

$$F_{S,P} = m_{B_s} \left[\frac{C_{S,P} m_b - C'_{S,P} m_s}{m_b + m_s} \right], \quad F_{10} = C_{10} - C'_{10}$$

$$BR(\bar{B}_s \rightarrow \mu^+ \mu^-)_{SM} = (3.61 \pm 0.39) \times 10^{-9}$$

[Blanke et al., arxiv:0805.4393]

$$f_{B_s} = (245 \pm 25) \text{ MeV}$$

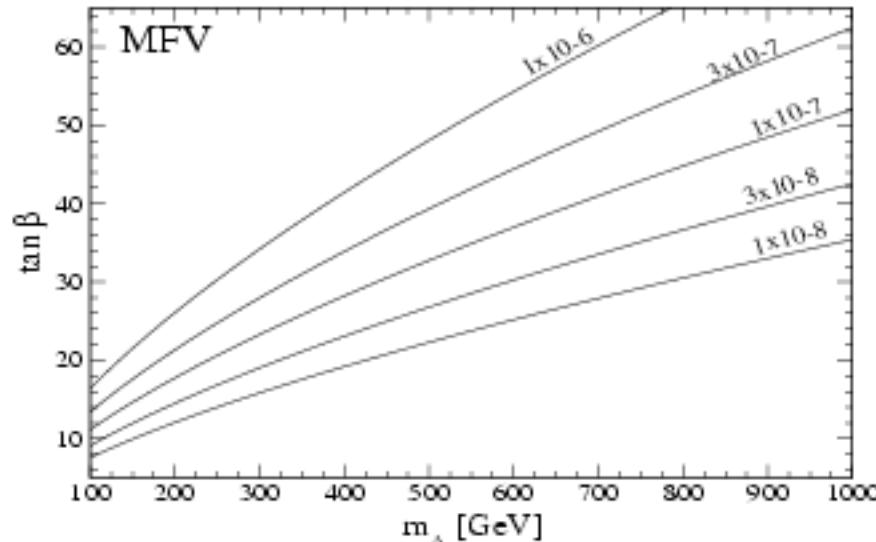
$B_s \rightarrow \mu^+ \mu^-$ in Minimal Flavor Violation SUSY Models

- Higgsino contribution to $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ [Babu, Kolda; Kane, Kolda, Lennon; ...]

$$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) \simeq \frac{G_F^2}{8\pi} \eta_{\text{QCD}}^2 m_{B_s}^3 f_{B_s}^2 \tau_{B_s} m_b^2 m_\mu^2 \left(\frac{\tan^2 \beta}{\cos^4 \beta} \right) \left(\frac{\kappa_{\widetilde{H}}^2}{m_A^4} \right).$$

- $\eta_{\text{QCD}} \simeq 1.5$ is the QCD correction due to the RG between the SUSY and B_s scales

$$\kappa_{\widetilde{H}} = -\frac{G_F m_t^2 V_{ts} V_{tb}}{4\sqrt{2}\pi^2 \sin^2 \beta} \mu A_t f(\mu^2, m_{\tilde{t}_L}^2, m_{\tilde{t}_R}^2)$$



B Physics Observables & MSSM

Ellis et al. [arxiv:0709.00982]

WMAP-Compliant Benchmark Surfaces for MSSM

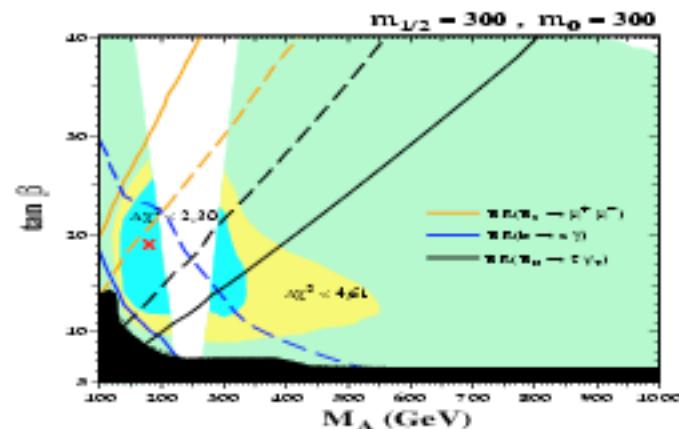
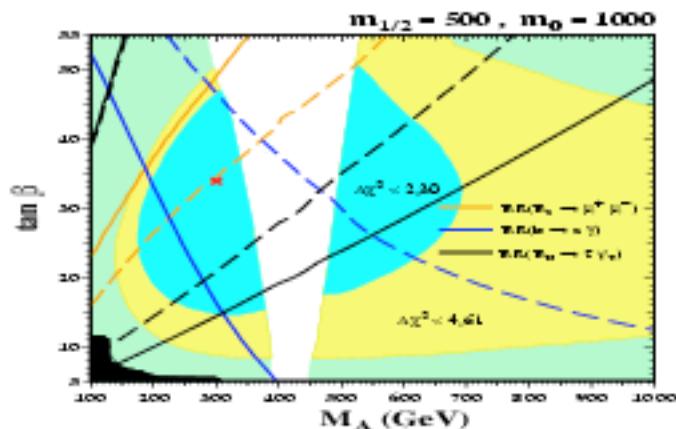
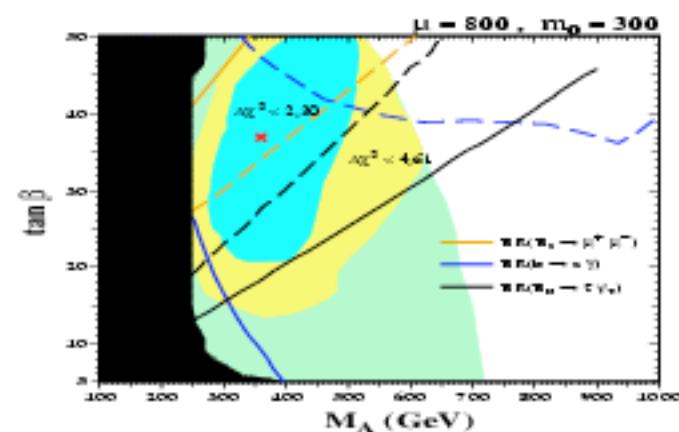
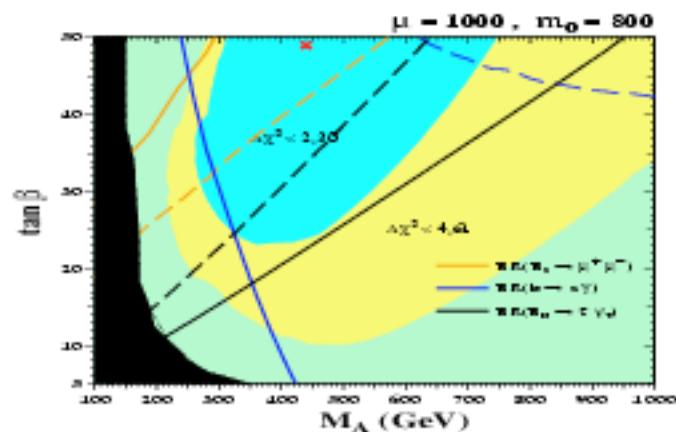
- The four NUHM benchmark surfaces obtained by fixing some parameters and allowing M_A and $\tan\beta$ to vary freely.

	$m_{1/2}$	m_0	A_0	μ	χ^2_{\min}
P1	$\sim \frac{9}{8} M_A$	800	0	1000	7.1
P2	$\sim 1.2 M_A$	300	0	800	3.1
P3	500	1000	0	250 ... 400	7.4
P4	300	300	0	200 ... 350	5.6

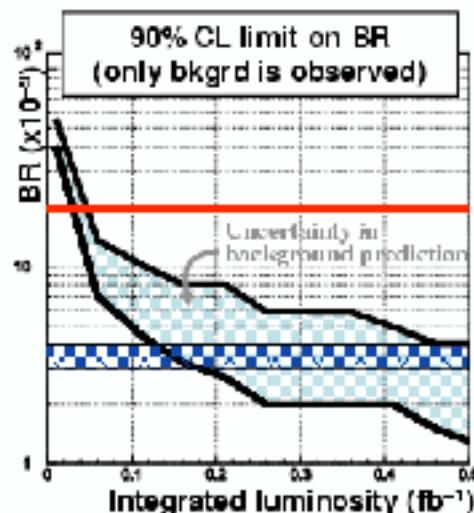
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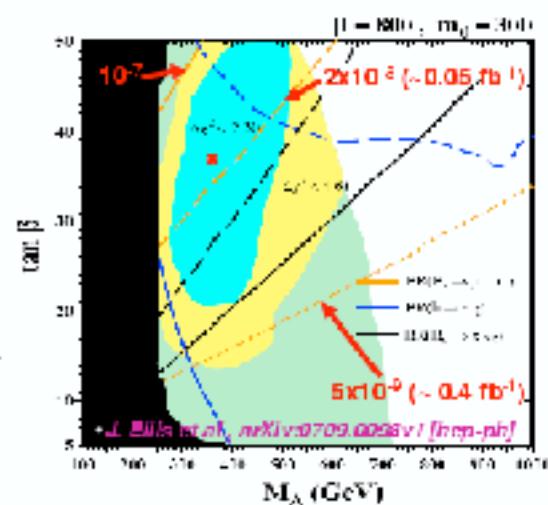


$B_s \rightarrow \mu\mu$ at LHCb



Exclusion:

$0.1 \text{ fb}^{-1} \Rightarrow \text{BR} < 10^{-8}$
 $0.5 \text{ fb}^{-1} \Rightarrow < \text{SM}$



With SM Branching ratio

$2 \text{ fb}^{-1} \Rightarrow 3\sigma$ evidence
 $6 \text{ fb}^{-1} \Rightarrow 5\sigma$ observation

With 0.1 fb^{-1} can measure BR $9(15) \times 10^{-6}$ at $3(5)\sigma$

With 0.5 fb^{-1} can measure BR $5(9) \times 10^{-6}$ at $3(5)\sigma$

9

Summary

- All current measurements involving CC and FCNC processes (decay rates and distributions, Mixings, CP Violation) of the B^\pm and B_d^0 mesons are in agreement with the SM. However, still plenty of room for NP contributions
- Tevatron (& Belle) have provided first measurements for the B_s^0 , including the first measurements of $S_{J/\psi\phi}$ not in accord with the SM (a Harbinger of NP or just an experimental Hickup?)
- Experiments at the LHC will test this precisely, together with other benchmark measurements: ΔM_s , γ , $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$; $B_s \rightarrow \phi \gamma$ and various CP asymmetries
- A lot of theoretical interest remains in precision studies of Rare B -decays: $\mathcal{B}(B \rightarrow (X_s, K^*)\gamma)$; $\mathcal{B}(B \rightarrow (X_d, \rho, \omega, \phi)\gamma)$; $\mathcal{B}(B \rightarrow (X_s, K^*)\ell^+\ell^-)$, in particular the dilepton invariant mass spectra and the Forward-Backward Asymmetries of the leptons. LHC experiments will measure the exclusive decays precisely
- Hope that the synergy of high energy frontier and low energy precision physics, which worked so well in piecing together the SM yielding precise knowledge of the CKM matrix, will continue to hold sway in the LHC-era, providing valuable information on the flavour aspects of the next Paradigm!