

Flavour as a Window to New Physics at the LHC

June 12, 2008

Muon  $g-2$   
and  
Electric Dipole Moments

Arkady Vainshtein

William Fine Theoretical Physics Institute  
University of Minnesota

# Outline

- ✪ Experimental data on  $g$  and EDM
- ✪ CPV scales and EDM calculations
- ✪ Anatomy of muon  $g-2$ , electroweak corrections
- ✪ Hadronic light-by-light scattering
- ✪ Do we see NP in the muon  $g-2$ ?

# Lepton magnetic moments

The present experimental values

**Electron:** Hanneke, Fogwell, and Gabrielse '08

$$g/2 = 1.001\,159\,652\,180\,73\,(28) \\ 0.28 \times 10^{(-12)} [0.28 \text{ ppt}]$$

New value of  $\alpha$  follows

$$1/\alpha = 137.035\,999\,084\,(51) \quad [0.37 \text{ ppb}]$$

**Muon:** BNL E821 '06

$$g/2 = 1.001\,165\,920\,80\,(63) \quad [630 \text{ ppt}]$$

**Tau:** Delphi at LEP2 '04

$$g/2 = 0.982(17)$$

# Electric dipole moments

Neutron: Baker et al '06

$$|d| < 2.9 \times 10^{-26} \text{ e} \cdot \text{cm}$$

Electron: Regan et al '06

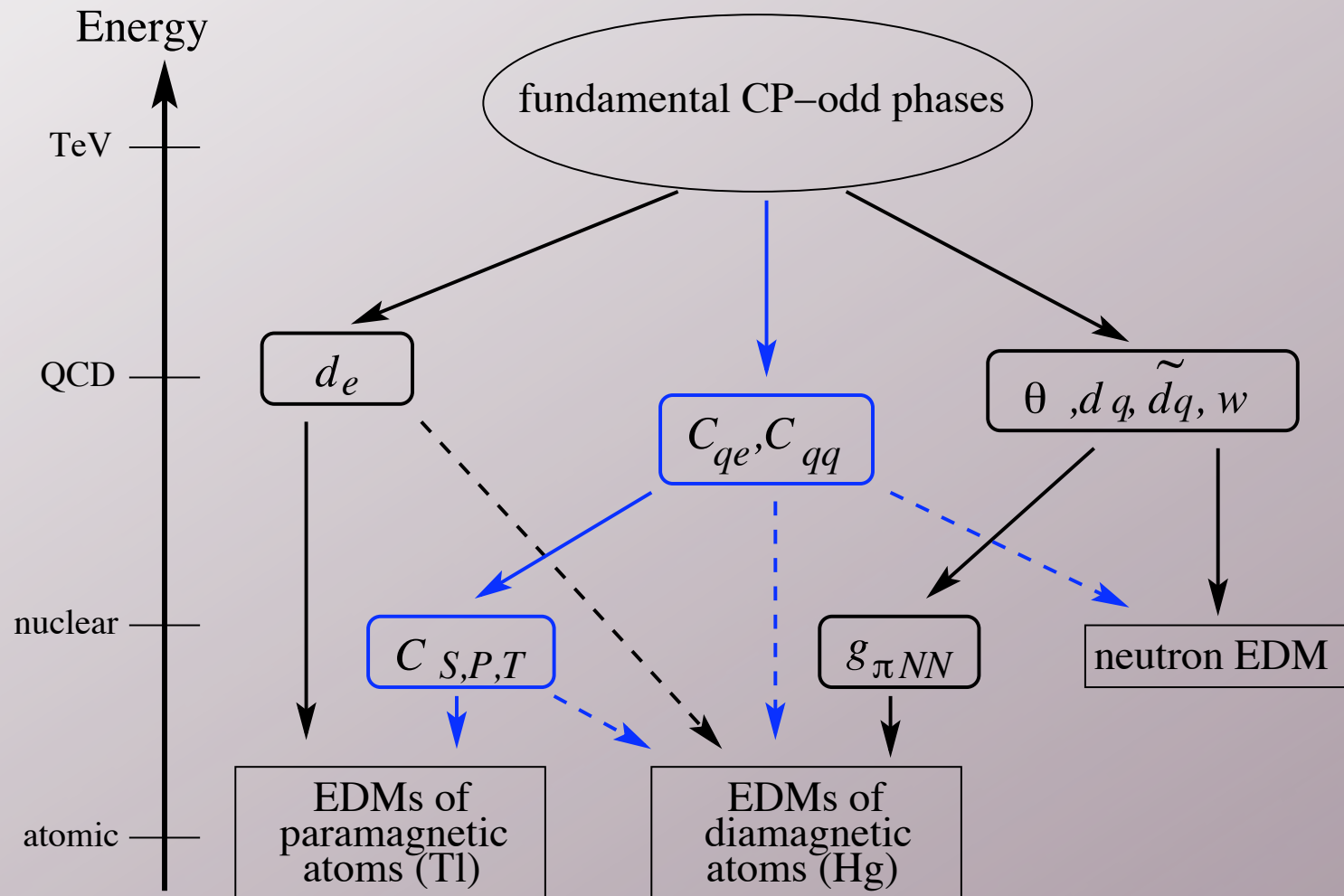
$$|d| < 1.6 \times 10^{-27} \text{ e} \cdot \text{cm}$$

Muon: Muon g-2 Collab '04

$$|d| < 2.8 \times 10^{-19} \text{ e} \cdot \text{cm}$$

# Scales hierarchy

Pospelov, Ritz '05



# Observables

$$\mathcal{L}_{eff}^{\text{nuclear}} = \mathcal{L}_{\text{edm}} + \mathcal{L}_{\pi NN} + \mathcal{L}_{eN},$$

$$\mathcal{L}_{\text{edm}} = -\frac{i}{2} \sum_{i=e,p,n} d_i \bar{\psi}_i (F\sigma) \gamma_5 \psi,$$

$$\begin{aligned} \mathcal{L}_{\pi NN} = & \bar{g}_{\pi NN}^{(0)} \bar{N} \tau^a N \pi^a + \bar{g}_{\pi NN}^{(1)} \bar{N} N \pi^0 \\ & + \bar{g}_{\pi NN}^{(2)} (\bar{N} \tau^a N \pi^a - 3 \bar{N} \tau^3 N \pi^0), \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{eN} = & C_S^{(0)} \bar{e} i \gamma_5 e \bar{N} N + C_P^{(0)} \bar{e} e \bar{N} i \gamma_5 N + C_T^{(0)} \epsilon_{\mu\nu\alpha\beta} \bar{e} \sigma^{\mu\nu} e \bar{N} \sigma^{\alpha\beta} N \\ & + C_S^{(1)} \bar{e} i \gamma_5 e \bar{N} \tau^3 N + C_P^{(1)} \bar{e} e \bar{N} i \gamma_5 \tau^3 N + C_T^{(1)} \epsilon_{\mu\nu\alpha\beta} \bar{e} \sigma^{\mu\nu} e \bar{N} \sigma^{\alpha\beta} \tau^3 N. \end{aligned}$$

# Low Energy Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{dim}=4} + \mathcal{L}_{\text{dim}=5} + \mathcal{L}_{\text{dim}=6} + \cdots .$$

$$\mathcal{L}_{\text{dim}=4} = \frac{g_s^2}{32\pi^2} \bar{\theta} G_{\mu\nu}^a \tilde{G}^{\mu\nu,a},$$

$$\mathcal{L}_{\text{dim}=5} = -\frac{i}{2} \sum_{i=u,d,s,e,\mu} d_i \bar{\psi}_i (F\sigma) \gamma_5 \psi_i - \frac{i}{2} \sum_{i=u,d,s} \tilde{d}_i \bar{\psi}_i g_s (G\sigma) \gamma_5 \psi_i,$$

Effective dim=6. Thus, we need to add

$$\mathcal{L}_{\text{dim}=6} = \frac{1}{3} w f^{abc} G_{\mu\nu}^a \tilde{G}^{\nu\beta,b} G_{\beta}^{\mu,c} + \sum_{i,j} C_{ij} (\bar{\psi}_i \psi_i) (\bar{\psi}_j i \gamma_5 \psi_j) + \cdots$$

# Examples of calculation

$$d_n(\bar{\theta}) = (1 \pm 0.5) \frac{|\langle \bar{q}q \rangle|}{(225 \text{ MeV})^3} \bar{\theta} \times 2.5 \cdot 10^{-16} e \text{ cm},$$

$$d_n^{\text{PQ}}(d_q, \tilde{d}_q) = (1 \pm 0.5) \frac{|\langle \bar{q}q \rangle|}{(225 \text{ MeV})^3} \left[ 1.1e(\tilde{d}_d + 0.5\tilde{d}_u) + 1.4(d_d - 0.25d_u) \right]$$

In the Standard Model

Khriplovich '86

$$d_d = e \frac{m_d m_c^2 \alpha_s G_F^2 J_{CP}}{108 \pi^5} \ln^2(m_b^2/m_c^2) \ln(M_W^2/m_b^2).$$

$$d_d^{\text{KM}} \simeq 10^{-34} e \text{ cm}.$$

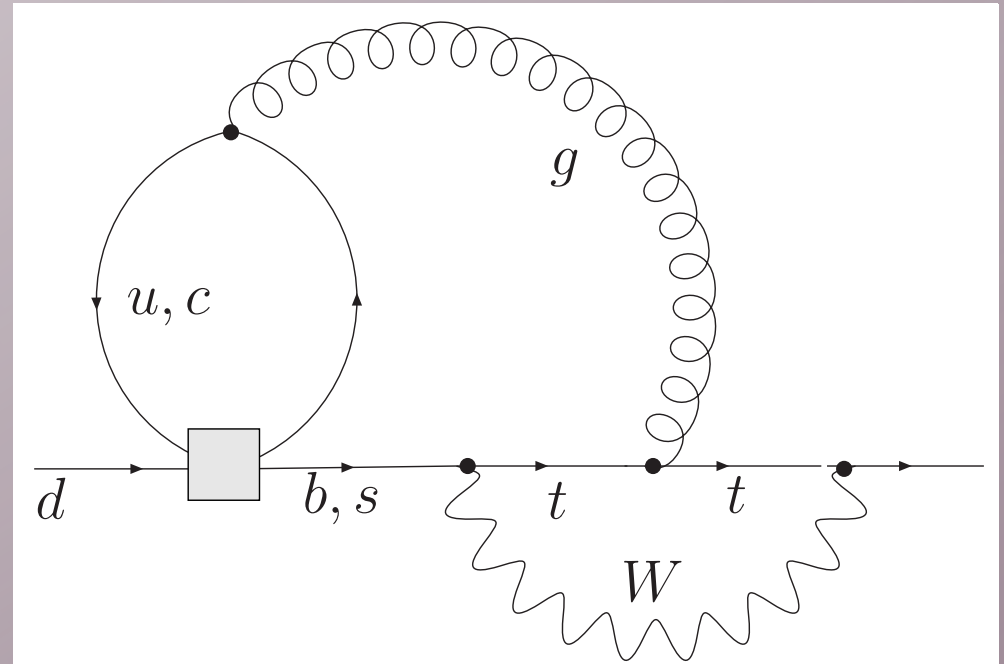
$$d_n^{\text{KM}} \simeq 10^{-32} e \text{ cm}.$$

Gavela; Khriplovich, Zhitnitsky '82

$$d_e^{\text{KM}} \leq 10^{-38} e \text{ cm}.$$

Khriplovich, Pospelov '91

Potential for NP to show up!



# Anatomy of muon g-2

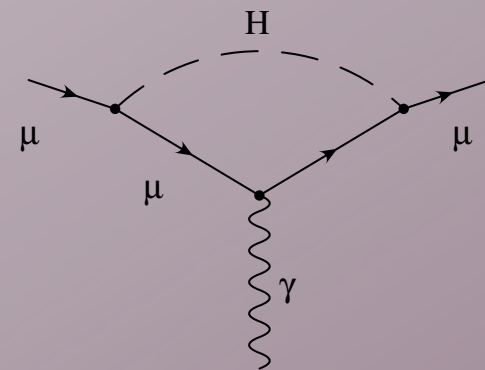
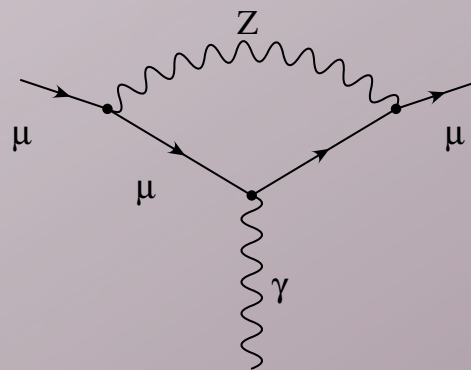
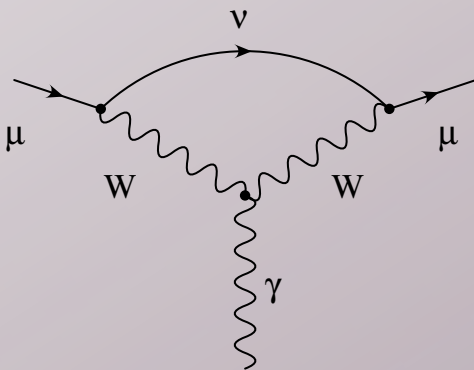
$$a_{\mu}^{\text{SM}} = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{had}} + a_{\mu}^{\text{EW}}$$

$$a_{\mu}^{\text{QED}} = 116\,584\,718.09(15) \times 10^{-11} \quad \text{Kinoshita et al}$$

$$a_{\mu}^{\text{EW}} = 154(1)(2) \times 10^{-11}$$

Czarnecki, Marciano, AV '02

$$a_{\mu}^{\text{EW}}(\text{1-loop}) = \frac{5 G_{\mu} m_{\mu}^2}{24 \sqrt{2} \pi^2} \left[ 1 + \frac{1}{5} (1 - 4 \sin^2 \theta_W)^2 + \mathcal{O} \left( \frac{m_{\mu}^2}{m_{W,H}^2} \right) \right] = 194.8 \times 10^{-11}$$



## Two-loop corrections are more involved

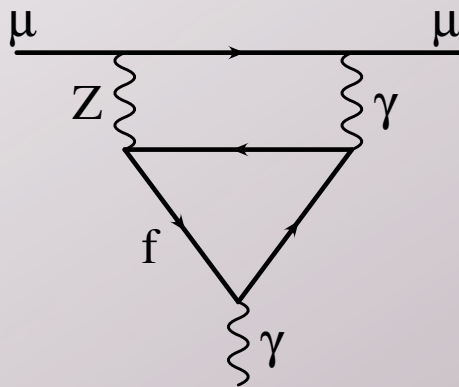
$$a_{\mu}^{\text{EW}}(2\text{-loop})_{LL} = \frac{5G_{\mu}m_{\mu}^2}{24\sqrt{2}\pi^2} \cdot \frac{\alpha}{\pi} \left\{ -\frac{43}{3} \ln \frac{m_Z}{m_{\mu}} + \frac{36}{5} \sum_{f \in F} N_f Q_f^2 I_f^3 \ln \frac{m_Z}{m_f} \right\}$$

$$\approx -37 \times 10^{-11} \quad F = \tau, u, d, s, c, b$$

Kukhto, Kuraev, Schiller, Silagadze '92

Peris, Perrottet, Rafael '95

Czarnecki, Krause, Marciano '95



$$m_{u,d} = 0.3 \text{ GeV}, m_s = 0.5 \text{ GeV}, \\ m_c = 1.5 \text{ GeV}, m_b = 4.5 \text{ GeV}$$

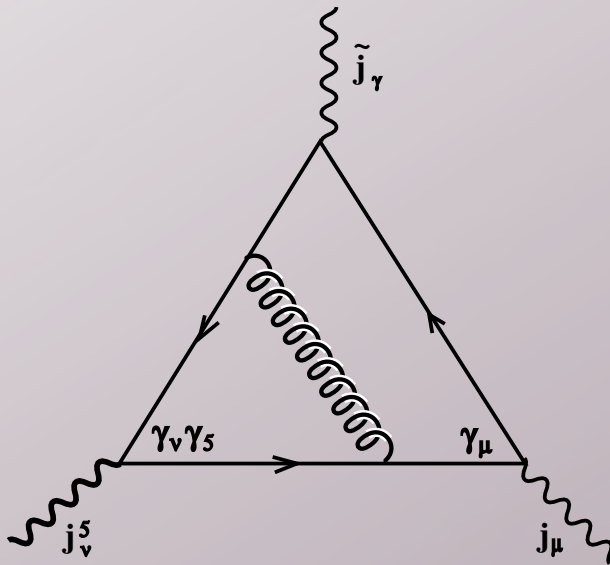
Fermion triangles (  $Z^* \gamma \gamma^*$  vertex)

$$\text{Total: } a_{\mu}^{\text{EW}} = 152(4) \times 10^{-11} \quad \text{Czarnecki, Marciano '01}$$

$$T_{\mu\nu} = -\frac{ie}{4\pi^2} \left[ w_T(q^2) \left( -q^2 \tilde{f}_{\mu\nu} + q_\mu q^\sigma \tilde{f}_{\sigma\nu} - q_\nu q^\sigma \tilde{f}_{\sigma\mu} \right) + w_L(q^2) q_\nu q^\sigma \tilde{f}_{\sigma\mu} \right],$$

$$\tilde{f}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\gamma\delta} f^{\gamma\delta}, \quad f_{\mu\nu} = k_\mu e_\nu - k_\nu e_\mu$$

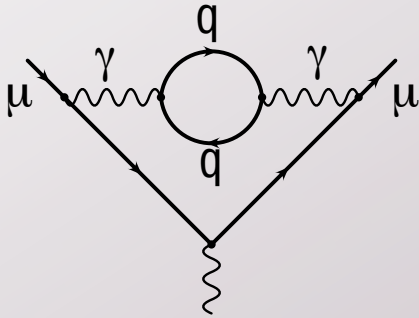
$$w_L^{1\text{-loop}}[f] = 2 w_T^{1\text{-loop}}[f] = 4 I_f^3 N_f Q_f^2 \frac{1}{Q^2}$$



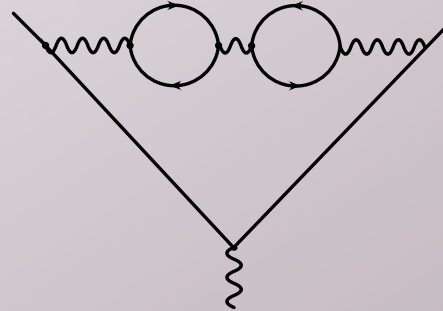
No perturbative corrections both in longitudinal and transversal parts in the chiral limit

# Hadronic contributions

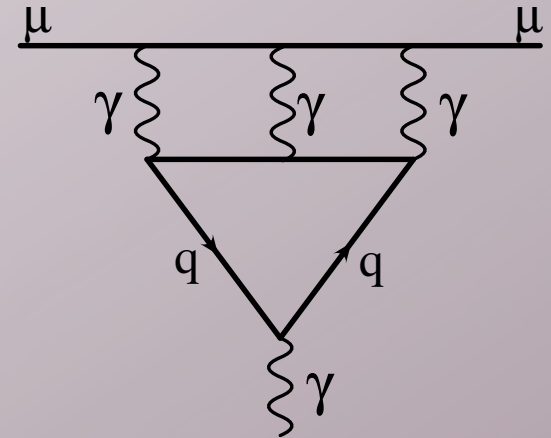
$$a_{\mu}^{\text{had}} = a_{\mu}^{\text{had,LO}} + a_{\mu}^{\text{had,HO}} + a_{\mu}^{\text{LBL}}$$



Lowest order hadronic contribution represented by a quark loop



An example of higher order hadronic contribution



Light-by-light scattering contribution

In theory

$$a_{\mu}^{\text{had,LO}} = \left( \frac{\alpha m_{\mu}}{3\pi} \right)^2 \int_{4m_{\pi}^2}^{\infty} \frac{ds}{s^2} K(s) R(s)$$

$K(s)$  is the known function,  $K(s) \rightarrow 1$ ,  $s \gg m_{\mu}^2$

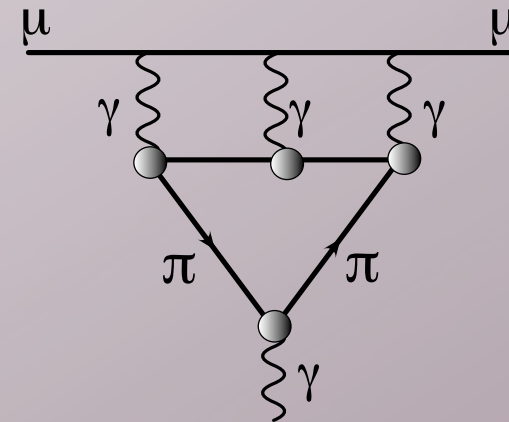
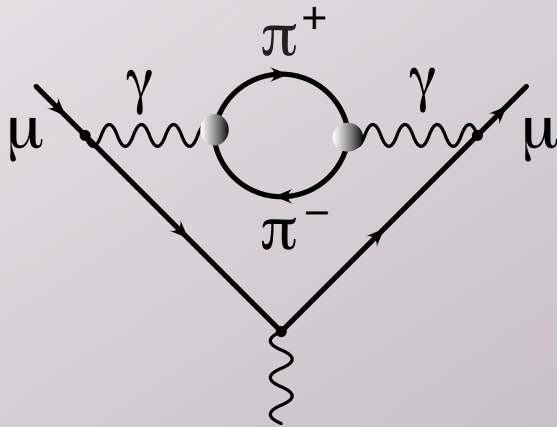
$R(s)$  is the cross section of  $e^+e^-$  annihilation into hadrons in units of  $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ . ■

In difference with  $a_{\mu}^{\text{had,LO}}$  there is no experimental input for the light-by-light contribution. What are possible theoretical parameters to exploit?

Smallness of chiral symmetry breaking,  $m_{\rho}^2/m_{\pi}^2 \gg 1$

$$a_{\mu}^{(n)} \sim c_1 \left( \frac{\alpha}{\pi} \right)^n \frac{m_{\mu}^2}{m_{\pi}^2},$$

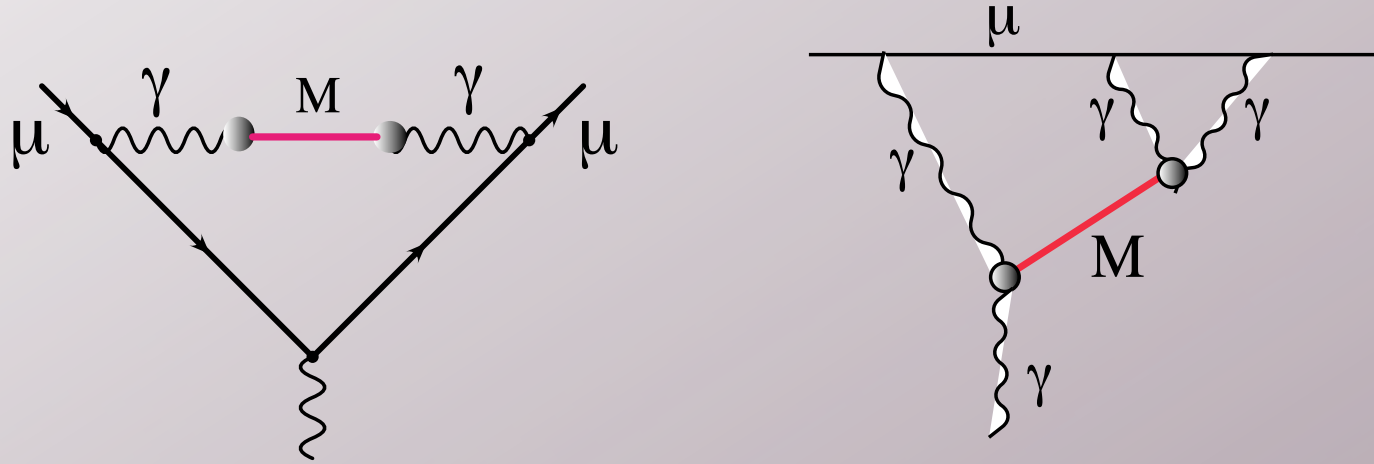
$$\text{LO} : n = 2, \quad \text{LbL} : n = 3$$



The Goldstone nature of pion implies  $m_{\pi}^2 \propto m_q$  much less than typical  $M_{\text{had}}^2 \sim m_{\rho}^2$ . Thus, the threshold range in pion loops produces the  $1/m_{\pi}^2$  enhancement.

Large number of colors,  $N_c$

Quark loops clearly give  $a_\mu \propto N_c$ . Dual not to pion loops but to meson exchanges.



No continuum in the large  $N_c$  limit.

$M = \rho^0, \omega, \phi, \rho', \dots$  for the polarization operator

$M = \pi^0, \eta, \eta', a_0, a_1, \dots$  (and any C-even meson) for the light-by-light

$$a_\mu^{(n)} \sim c_2 \left( \frac{\alpha}{\pi} \right)^n N_c \frac{m_\mu^2}{m_\rho^2}$$

We can check for  $a_\mu^{\text{had,LO}}$

Two regions. The threshold region  $s \sim 4m_\pi^2$  where

$$R(s) \approx \frac{1}{4} \left( 1 - \frac{4m_\pi^2}{s} \right)^{3/2}$$

and the resonance region  $s \sim m_\rho^2$  where by quark-hadron duality on average

$$R(s) \approx N_c \sum Q_q^2$$

The chirally enhanced threshold region gives numerically

$$a_\mu^{\text{had,LO}}(4m_\pi^2 \leq s \leq m_\rho^2/2) \approx 400 \times 10^{-11}$$

Compare with the  $N_c$  enhanced  $\rho$  peak,

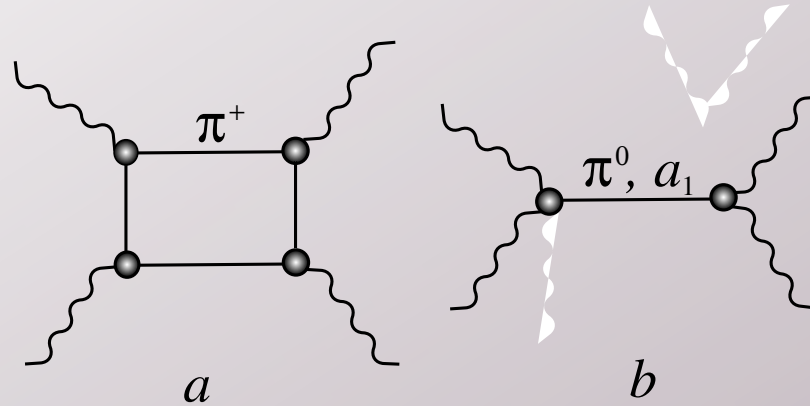
$$a_\mu^{\text{had,LO}}(\rho) = \frac{m_\mu^2 \Gamma(\rho \rightarrow e^+ e^-)}{\pi m_\rho^3} \approx 5000 \times 10^{-11}$$

This contribution is enhanced by  $N_c$ ,

$$a_\mu(\rho) \sim c_2 \left( \frac{\alpha}{\pi} \right)^2 N_c \frac{m_\mu^2}{m_\rho^2}$$

What is a lesson from this exercise? We see that the large  $N_c$  enhancement prevails over chiral one.

## In light-by-light



The chirally enhanced pion box contribution does not result in large number, it is actually rather small,

$$a_{\mu}^{\text{LbL}}(\text{pion box}) \approx -4 \times 10^{-11} \quad \text{Hayakawa, Kinoshita, Sanda; Melnikov}$$

similarly to the hadronic polarization case above.

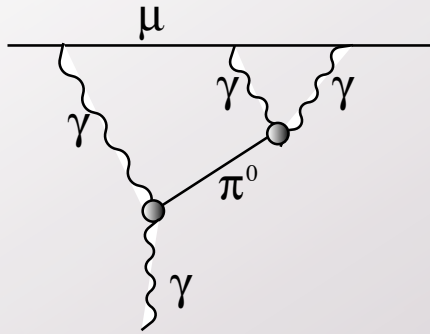
A larger value (-19) for the pion box was obtained by Bijmans, Pallante, Prades

Instability of the number is due to relatively large pion momenta in the loop, of order of  $4m_\pi$  as we estimated. Then details of the model becomes important and theoretical control is lost. In HSL model few first terms of  $m_\pi^2/m_\rho^2$  expansion are

$$a_\mu(\text{charged pion loop}) \times 10^{11} = -46.37 + 35.46 + 10.98 - 4.7 + \dots = -4.9$$

If momenta were small compared with  $m_\rho$  the result would be close to the leading term – free pion loop.

In case of polarization operator the suppression of the leading term in the chiral expansion (larger momenta) can be related to the  $p$ -wave  $p^3$  suppression. There is a suppression for  $s$ -wave in two-pion intermediate state near threshold in the case of LbL.



Hayakawa, Kinoshita, Sanda  
 Bijnens, Pallante, Prades  
 Barbieri, Remiddi  
 Pivovarov  
 Bartos, Dubničkova, Dubnička, Kuraev, Zemlyanaya  
 Knecht, Nyffeler  
 Knecht, Nyffeler, Perrottet, de Rafael  
 Ramsey-Musolf, Wise  
 Blokland, Czarnecki, Melnikov  
 Melnikov, A.V.

Different models: constituent quark loop, extended Nambu–Jano-Lasinio model (ENJL), hidden local symmetry (HLS) model ...

The  $\pi^0$  pole part of LbL contains besides  $N_c$  the chiral enhancement in the logarithmic form, leading to the model-independent analytical expression

$$a_{\mu}^{\text{LbL}}(\pi^0) = \left(\frac{\alpha}{\pi}\right)^3 N_c \frac{m_{\mu}^2 N_c}{48\pi^2 F_{\pi}^2} \ln^2 \frac{m_{\rho}}{m_{\pi}} + \dots$$

However next, model dependent, terms are comparable with the the leading or  
 Numerically

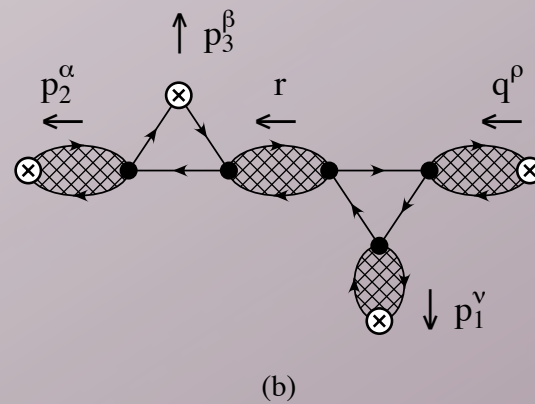
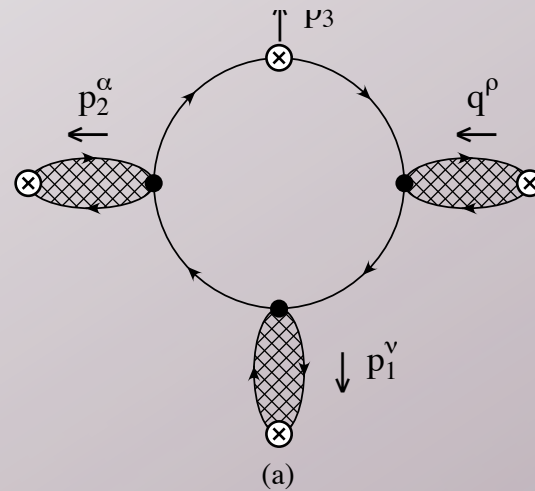
$$a_{\mu}^{\text{LbL}}(\pi^0) = 58(10) \times 10^{-11}$$

Knecht, Nyffeler

# Models

HLS model is a modification the Vector Meson Dominance model.

ENJL model is represented by the following graphs



## OPE constraints and hadronic model

$$\epsilon_i^\mu(q_i), \quad i = 1, 2, 3, 4, \quad \sum q_i = 0$$

$$\epsilon_4 \text{ represents the external magnetic field } f^{\gamma\delta} = q_4^\gamma \epsilon_4^\delta - q_4^\delta \epsilon_4^\gamma, \quad q_4 \rightarrow 0.$$

The LbL amplitude

$$\begin{aligned} \mathcal{M} &= \alpha^2 N_c \text{Tr} [\hat{Q}^4] \mathcal{A} = \alpha^2 N_c \text{Tr} [\hat{Q}^4] \mathcal{A}_{\mu_1 \mu_2 \mu_3 \gamma \delta} \epsilon_1^{\mu_1} \epsilon_2^{\mu_2} \epsilon_3^{\mu_3} f^{\gamma\delta} \\ &= -e^3 \int d^4x d^4y e^{-iq_1 x - iq_2 y} \epsilon_1^{\mu_1} \epsilon_2^{\mu_2} \epsilon_3^{\mu_3} \langle 0 | T \{ j_{\mu_1}(x) j_{\mu_2}(y) j_{\mu_3}(0) \} | \gamma \rangle \end{aligned}$$

The electromagnetic current  $j_\mu = \bar{q} \hat{Q} \gamma_\mu q$ ,  $q = \{u, d, s\}$

Three Lorentz invariants:  $q_1^2, q_2^2, q_3^2$

Consider the Euclidian range  $q_1^2 \approx q_2^2 \gg q_3^2 \gg \Lambda_{\text{QCD}}^2$

We can use OPE for the currents that carry large momenta  $q_1, q_2$

$$i \int d^4x d^4y e^{-iq_1x - iq_2y} T \{j_{\mu_1}(x), j_{\mu_2}(y)\} = \int d^4z e^{-i(q_1+q_2)z} \frac{2i}{\hat{q}^2} \epsilon_{\mu_1\mu_2\delta\rho} \hat{q}^\delta j_5^\rho(z) + \dots$$

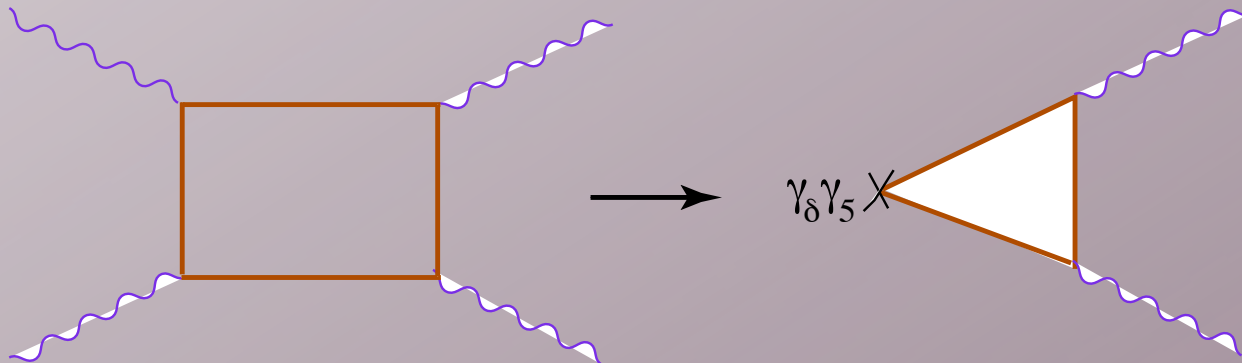
$\hat{q} = (q_1 - q_2)/2$ , the axial current  $j_5^\rho = \bar{q} \hat{Q}^2 \gamma^\rho \gamma_5 q$  is the linear combination of

$$j_{5\rho}^{(3)} = \bar{q} \lambda_3 \gamma^\rho \gamma_5 q \quad \text{isovector}$$

$$j_{5\rho}^{(3)} = \bar{q} \lambda_8 \gamma^\rho \gamma_5 q \quad \text{hypercharge}$$

$$j_{5\rho}^{(3)} = \bar{q} \gamma^\rho \gamma_5 q \quad \text{singlet}$$

$$j_{5\rho} = \sum_{a=3,8,0} \frac{\text{Tr} [\lambda_a \hat{Q}^2]}{\text{Tr} [\lambda_a^2]} j_{5\rho}^{(a)}$$



## The triangle amplitude

$$T_{\mu_3\rho}^{(a)} = i \langle 0 | \int d^4 z e^{iq_3 z} T \{ j_{5\rho}^{(a)}(z) j_{\mu_3}(0) \} | \gamma \rangle$$

kinematically is expressed via two scalar amplitudes

$$T_{\mu_3\rho}^{(a)} = -\frac{ie N_c \text{Tr} [\lambda_a \hat{Q}^2]}{4\pi^2} \left\{ w_L^{(a)}(q_3^2) q_{3\rho} q_3^\sigma \tilde{f}_{\sigma\mu_3} + \right. \\ \left. + w_T^{(a)}(q_3^2) \left( -q_3^2 \tilde{f}_{\mu_3\rho} + q_{3\mu_3} q_3^\sigma \tilde{f}_{\sigma\rho} - q_{3\rho} q_3^\sigma \tilde{f}_{\sigma\mu_3} \right) \right\}$$

Longitudinal  $w_L$ : pseudoscalar mesons exchange

Transversal  $w_T$ : pseudovector mesons exchange

In perturbation theory for massless quarks

$$w_L^{(a)}(q^2) = 2w_T^{(a)}(q^2) = -\frac{2}{q^2}$$

Nonvanishing  $w_L$  is the signature of the axial Adler–Bell–Jackiw anomaly.

Moreover, for nonsinglet  $w_L^{(3,8)}$  it is the *exact* QCD result, no perturbative as well as nonperturbative corrections. So the pole behavior is preserved all way down to small  $q^2$  where the pole is associated with Goldstone mesons  $\pi^0, \eta$ .

Comparing the pole residue we get the famous ABJ result

$$g_{\pi\gamma\gamma} = \frac{N_c \text{Tr} [\lambda_3 \hat{Q}^2]}{16\pi^2 F_\pi}$$

There exists the nonrenormalization theorem for  $w_T$  as well but only in respect to perturbative corrections. A.V. '02; Knecht, Peris, Perrottet, de Rafael '03

Higher terms in the OPE does not vanish in this case, they are responsible for shift of the pole  $1/q^2 \rightarrow 1/(q^2 - m_{V,PV}^2)$

Combining we get at  $q_1^2 \approx q_2^2 \gg q_3^2$

$$\begin{aligned} \mathcal{A}_{\mu_1\mu_2\mu_3\gamma\delta} f^{\gamma\delta} &= \frac{8}{\hat{q}^2} \epsilon_{\mu_1\mu_2\delta\rho} \hat{q}^\delta \sum_{a=3,8,0} W^{(a)} \left\{ w_L^{(a)}(q_3^2) q_3^\rho q_3^\sigma \tilde{f}_{\sigma\mu_3} \right. \\ &\quad \left. + w_T^{(a)}(q_3^2) \left( -q_3^2 \tilde{f}_{\mu_3}^\rho + q_{3\mu_3} q_3^\sigma \tilde{f}_\sigma^\rho - q_3^\rho q_3^\sigma \tilde{f}_{\sigma\mu_3} \right) \right\} + \dots \end{aligned}$$

where the weights  $W^{(3)} = 1/4, W^{(8)} = 1/12, W^{(0)} = 2/3$ .

# The model

Melnikov, AV '03

$$\mathcal{A} = \mathcal{A}_{\text{PS}} + \mathcal{A}_{\text{PV}} + \text{permutations},$$

$$\mathcal{A}_{\text{PS}} = \sum_{a=3,8,0} W^{(a)} \phi_L^{(a)}(q_1^2, q_2^2) w_L^{(a)}(q_3^2) \{f_2 \tilde{f}_1\} \{\tilde{f} f_3\},$$

$$\begin{aligned} \mathcal{A}_{\text{PV}} = & \sum_{a=3,8,0} W^{(a)} \phi_T^{(a)}(q_1^2, q_2^2) w_T^{(a)}(q_3^2) \left( \{q_2 f_2 \tilde{f}_1 \tilde{f} f_3 q_3\} \right. \\ & \left. + \{q_1 f_1 \tilde{f}_2 \tilde{f} f_3 q_3\} + \frac{q_1^2 + q_2^2}{4} \{f_2 \tilde{f}_1\} \{\tilde{f} f_3\} \right). \end{aligned}$$

For  $\pi^0$

$$w_L^{(3)}(q^2) = \frac{2}{q^2 + m_\pi^2},$$

$$\phi_L^3(q_1^2, q_2^2) = \frac{N_c}{4\pi^2 F_\pi^2} F_{\pi\gamma^*\gamma^*}(q_1^2, q_2^2)$$

Knecht, Nyffeler

$$= \frac{q_1^2 q_2^2 (q_1^2 + q_2^2) - h_2 q_1^2 q_2^2 + h_5 (q_1^2 + q_2^2) + (N_c M_1^4 M_2^4 / 4\pi^2 F_\pi^2)}{(q_1^2 + M_1^2)(q_1^2 + M_2^2)(q_2^2 + M_1^2)(q_2^2 + M_2^2)}$$

The model results in

$$a_{\mu}^{\pi^0} = 76.5 \times 10^{-11}, \quad a_{\mu}^{\text{PS}} = 114(10) \times 10^{-11}$$

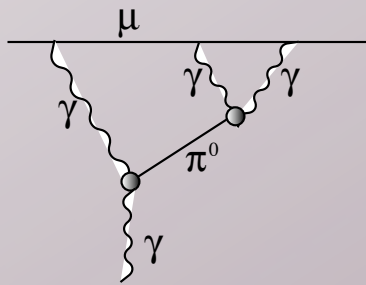
A similar analysis for pseudovector exchange gives

$$a_{\mu}^{\text{PV}} = 22(5) \times 10^{-11}$$

and finally

$$a_{\mu}^{\text{LbL}} = 136(25) \times 10^{-11}$$

## Comparison with other models



The difference with meson exchange models, like Knecht, Nyffeler et al, is due to absence of the form factor in the vertex with the soft photon (magnetic field),  $76.5 \times 10^{-11}$  versus  $58 \times 10^{-11}$  for  $\pi^0$  exchange. ■

ENJL model **Bijnens, Pallante, Prades** is conceptually not much different from our model. Indeed, we use meson exchange model which interpolates between the OPE at short distances and meson poles at large ones. It results in a less suppression at large momenta (no form factor in the vertex with magnetic field).

In the ENJL model high momenta asymptotics are provided by adding up the quark loops. Thus, our asymptotics are the same and difference is mostly in details of interpolations between high and low momenta.

Bijnens and Prades demonstrated nicely, in particular, that the asymmetric configuration of momenta  $q_1 \approx q_2 \gg q_3$  plays a dominant role in both models.

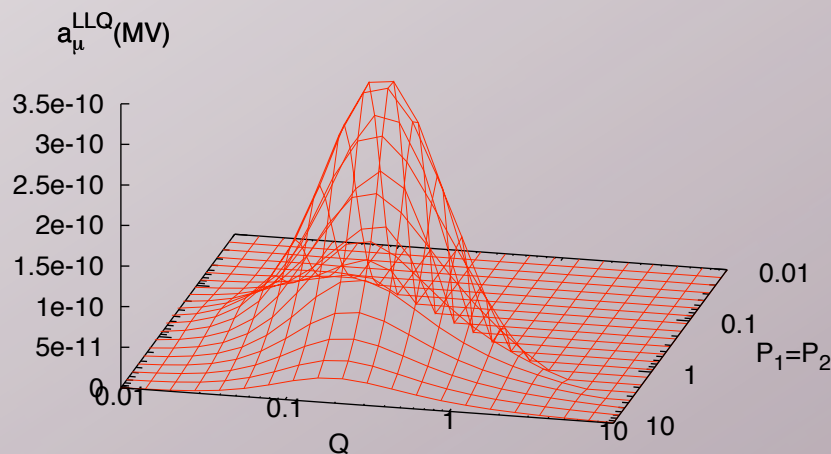


Fig. 8. The quantity  $a_\mu^{LLQ}$  of Eq. 10) as a function of  $Q$  and  $P_1 = P_2$  for the MV choice.  $a_\mu$  is directly related to the volume under the surface as plotted.

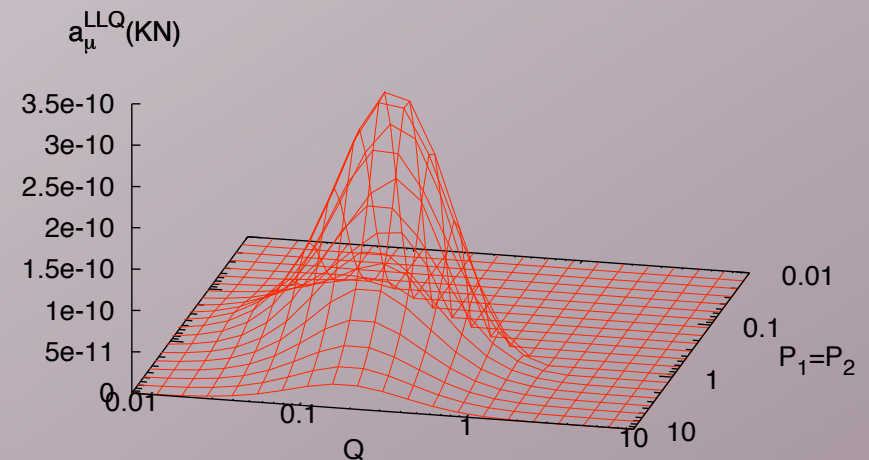


Fig. 9. The quantity  $a_\mu^{LLQ}$  of Eq. 10) as a function of  $Q$  and  $P_1 = P_2$  for the KN choice.  $a_\mu$  is directly related to the volume under the surface as plotted.

■ Let us compare the sum of pseudoscalar exchanges. We got it  $114 \times 10^{-11}$ , a 50% increase over the ENJL value  $85 \times 10^{-11}$ . However, adding up the ENJL result for the quark loop,  $22 \times 10^{-11}$ , we get  $109 \times 10^{-11}$ . Of course, we imply here that the bulk of the quark loop refers to the pseudoscalar exchange.

The difference in results come also from few other sources:

- (i) charge pion loop, zero versus  $(-19) \times 10^{-11}$  in ENJL,
- (ii) scalar exchange, zero versus  $(-7) \times 10^{-11}$  in ENJL,
- (iii) pseudovector exchange,  $22 \times 10^{-11}$  versus in  $2.5 \times 10^{-11}$  ENJL.■

The first point was discussed above, we do not see this contribution as distinguishable from other unaccounted contributions suppressed by  $1/N_c$ .■

The scalar exchange is not suppressed by  $1/N_c$ . We did not account it in our model because it does not show up at short distances. This means that the scalar exchange falls off at large momenta faster diminishing the integral. Indeed, numerically the scalar exchange is rather small contributions. Moreover, at this level other exchanges like spin two mesons are also relevant. It is not clear at all what would be a combined effect.■

The pseudovector exchange occurs to be very sensitive to interpolation between low and high momenta and to the model of mixing in the flavor SU(3).

# Summary for LbL

Our final result

$$a_{\mu}^{\text{LbL}} = 136(25) \times 10^{-11}$$

looks significantly larger than the ENJL one,  $83(32) \times 10^{-11}$ . However, without the charged pion loop and scalar exchange contribution, the ENJL number is  $109(32) \times 10^{-11}$ . ■

Recently Bijns and Prasad suggested  $110(40) \times 10^{-11}$  as an educated guess.

We see that the difference in results refers to rather subtle issues where it is not easy to find solid arguments for resolution.

So my conclusion is rather pessimistic in regards to perspective of diminishing of theoretical error in the hadronic light-by-light contribution.

# Do we see NP in the muon $g-2$ ?

QED	$116\,584\,718.09(.14)(.08)(.04) \times 10^{-11}$	
Electroweak	$154(2)(1) \times 10^{-11}$	
Hadronic LO	$6\,901(42)(19)(07) \times 10^{-11}$	
Hadronic HO	$-97.9(0.9)(0.3) \times 10^{-11}$	
Hadronic LbL	$110(40) \times 10^{-11}$	
Total SM	$11\,659\,1785(61) \times 10^{-11}$	
Experimental $a$	$11\,659\,2080(63) \times 10^{-11}$	
$\Delta a$	$295(88) \times 10^{-11}$	$3.4\sigma$

Both experimental and theoretical uncertainty should be reduced to be sure of NP.