Determination of V_{cb} and V_{ub}

N. Uraltsev

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Rate $\propto |V_{qb}|^2 \Longrightarrow$ measure a $b \to c$ ($b \to u$) decay rate. Need the coefficient accurately

hence *semileptonic* decays

- V_{cb} at zero recoil
    - $B \to D^* \ell\nu$
    - $B \to D\, \ell\nu$

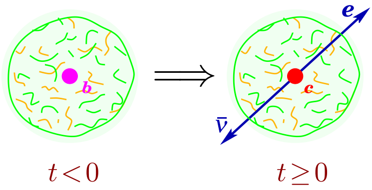
- V_{cb} from $\Gamma_{\rm sl}(B)$
    - extracting heavy quark parameters and V_{cb}
    - recent theoretical advances

- V_{ub} from inclusive $b \to u\, \ell\nu$ decays

V_{cb} at zero recoil

$$\mathrm{d}w\,(B \to D^* + \ell\bar{\nu}) \;\sim\; G_F^2 \cdot |V_{cb}|^2 \cdot |\vec{p}\,| \cdot |F_{B\to D^*}(\vec{p}\,)|^2$$

$|V_{cb}|$ requires $F_{B\to D^*}(\vec{p}\,)$ – it is shaped by bound-state physics



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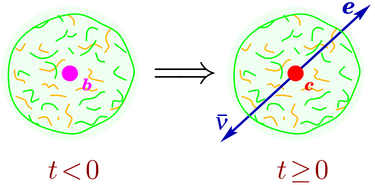

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Without *isotopic* effects (in the heavy quark limit) $F(\vec{p}=0)=1$:

$$F_{n/p}(0) \;=\; 1 + \frac{0}{m_{c,b}} + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{m^2_{c,b}}\right) + \mathcal{O}\left(\frac{\Lambda^3_{\rm QCD}}{m^3_{c,b}}\right) + \ldots$$

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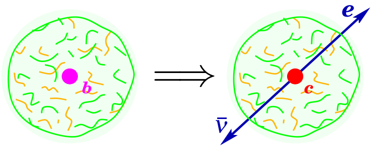

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No $1/m_{b,c}$-corrections

(cf. Ademollo-Gatto)

1986 Voloshin, Shifman
1990 Luke

Experimental issue: extrapolation to the zero-recoil point

$$\frac{1}{\sqrt{(M_B - M_{D^*})^2 - q^2}} \, \frac{\mathrm{d}\Gamma(B \to D^* \ell\nu)}{\mathrm{d}q^2}\Bigg|_{q^2=(M_B - M_{D^*})^2}$$

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In fact, *considerably larger*

Sum rules for heavy flavor transitions (can be paralleled in the nonrelativistic QM expansion):

Bigi, Shifman, N.U., Vainshtein 1994

$$F_{D^*}^2 + \sum_{f \neq D^*} |F_{B\to f}|^2 = \xi_A^{\text{pert}} - \frac{\mu_G^2}{3m_c^2} - \frac{\mu_\pi^2 - \mu_G^2}{4}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b}\right) - \Delta_{\frac{1}{m_Q^3}} + \Delta_{\frac{1}{m_Q^4}} + \ldots$$

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Guess: $\chi = 0.5 \pm 0.5$ $\quad$ In models typically get between 0.5 and 1.3

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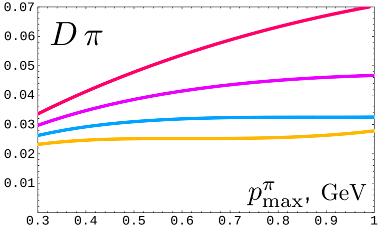
The $D\pi$ intermediate state contribution appears enhanced:



$g_{D^*D\pi} = 4.9 \quad (\Gamma_D = 96\,\mathrm{KeV})$
$g_{B^*B\pi}/g_{D^*D\pi} = 1,\ 0.8,\ 0.6$ and 0.4

$\delta_{D\pi} \simeq -(2.5\%$ to $3\%)$ corresponds alone to $\chi \gtrsim 0.4$

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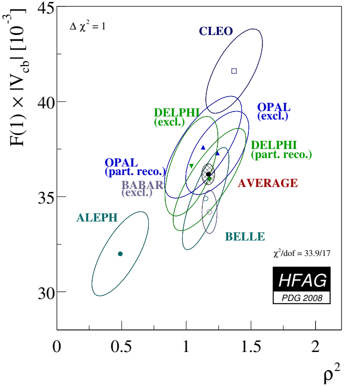

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c quark really is not sufficiently heavy...

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Is the literal disagreement too surprising? I do not think it is

Remains only if the error intervals are truly "$\pm$", not if many are '$-$'
Usually the sign is unknown, but sometimes there are physics arguments for a definite sign

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Is this what the lattice skeptics used to say?

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Can be understood?

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Not a drawback in the era of dynamics

$$F_+ = 1 + \left(\frac{\overline{\Lambda}}{2} - \overline{\Sigma}\right)\left(\frac{1}{m_c} - \frac{1}{m_b}\right)\frac{M_B - M_D}{M_B + M_D} - \mathcal{O}\left(\frac{1}{m_Q^2}\right)$$

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$$\boxed{\frac{M_B + M_D}{2\sqrt{M_B M_D}}\, f_+(0) \;= 1.04 \pm 0.01 \;\pm\; 0.01}$$

N.U. 2003

All orders in $1/m$ in 'BPS', to $1/m^2 \cdot 1/\mathrm{BPS}^2$, α_s^1

The bulk 3% is the perturbative factor

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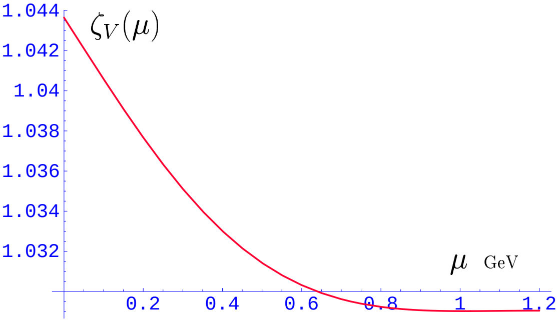
The bulk 3% is the perturbative factor, only a percent comes from power terms

Numerical evaluation of the formfactor requires accounting for perturbative renormalization:

Must be compatible with BPS in the nonperturbative domain

This can be done in the Wilsonian approach



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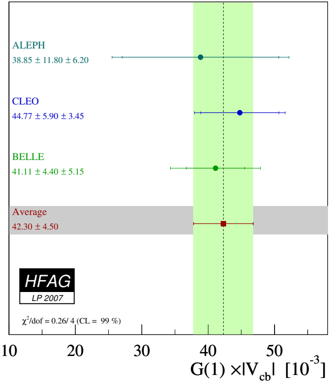

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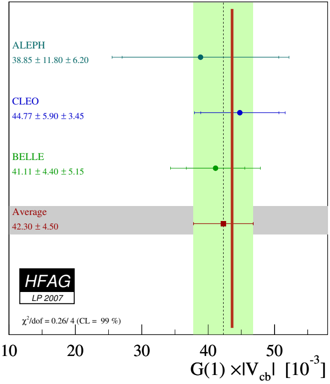

This leads to

$$|V_{cb}| = (40.7 \pm 4.4) \cdot 10^{-3}$$

Using $|V_{cb}|$ from $\Gamma_{\rm sl}(B)$ we predict

$$|V_{cb}|G(1) = 43.7 \cdot 10^{-3}$$

Extracting $|V_{cb}|$ from $\Gamma_{\rm sl}(B)$

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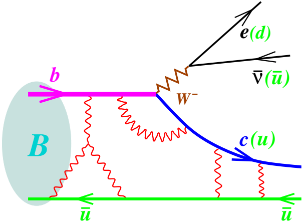

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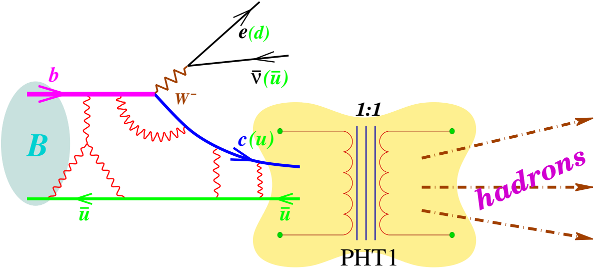

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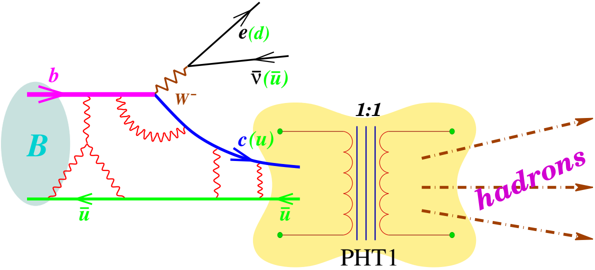

Folklore: A parton-hadron transformer, efficiency $\eta = 1$

Now we treat this scientifically and know that $\eta \neq 1$: calculate it in the $1/m_b$-expansion

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These QCD entities replace models and their attributes used early on

m_b, m_c, μ_π^2, ... (properly defined) can be determined from the semileptonic $(b \to s + \gamma)$ decay distributions themselves

BSUV, 1993-1994

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Use well-defined QCD parameters and enjoy numerically stable perturbation theory

Now adopted for analysis in all experiments

Experiment provides many observables, e.g.

$$\langle E_\ell \rangle, \;\; \langle E_\ell^2 \rangle, \;\; \langle E_\ell^3 \rangle; \qquad \langle M_X^2 \rangle, \;\; \langle M_X^4 \rangle, \;\; \langle M_X^6 \rangle \; \ldots$$

all as functions of the lower cut on charged lepton energy

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all as functions of the lower cut on charged lepton energy

The special role of the hadronic mass moments:

if m_c were large enough, the first would yield $\overline{\Lambda}$, the second μ_π^2, the third ρ_D^3 more or less directly

BSUV 1993-94

Precision data on the photon spectrum in $B \to X_s + \gamma$ are important!

A technical detail: in higher hadronic moments should not include $M_B - m_b$ into counting rules in $\mu_{\rm hadr}$ (although $M_B - m_b \propto \mu_{\rm hadr}^1$), rather treat as an arbitrary scale parameter

For skeptics – study the modified hadronic moments $\langle \tilde{N}_X^k \rangle$ (Gambino, N.U.) more directly related to higher-dimensional expectation values

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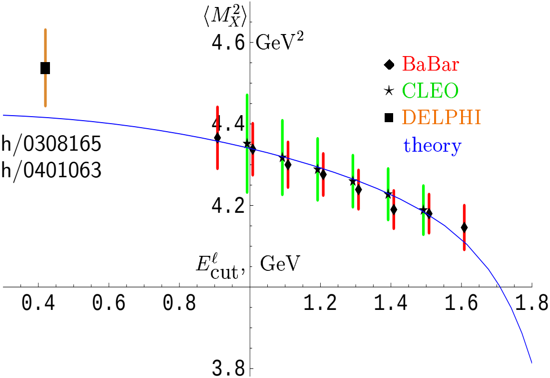
The first extensive data analysis along these lines was accomplished in 2004-2005 and turned out quite successful

- $\langle M_X^2 \rangle$ vs. $E^\ell_{\rm cut}$

Robust OPE approach à la Wilson, $\mu = 1\,\mathrm{GeV}$:

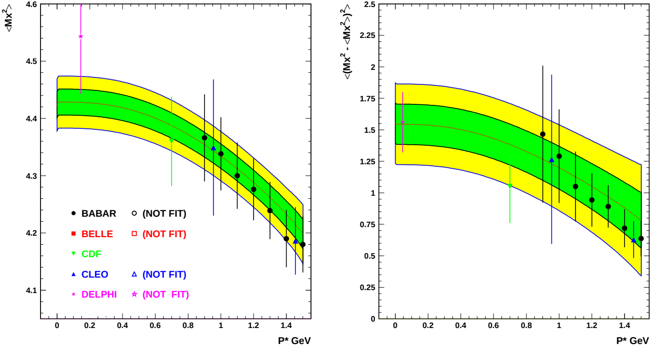
Data and expectations
as of July 2003

Bigi, N.U. hep-ph/0308165
Gambino, N.U. hep-ph/0401063



Second mass^2 moment $\langle [M_X^2 - \langle M_X^2 \rangle]^2 \rangle$:

hep-ph/0507253



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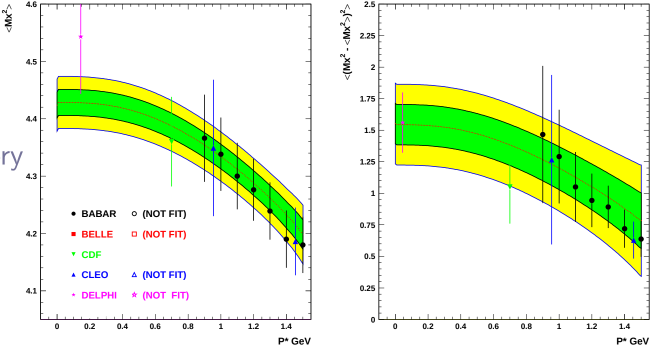
Good agreement

where the right theory is used right



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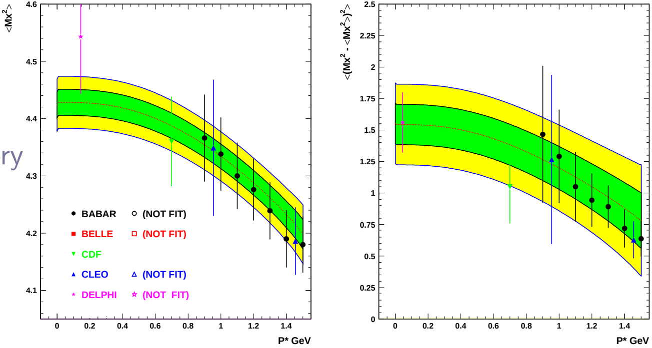

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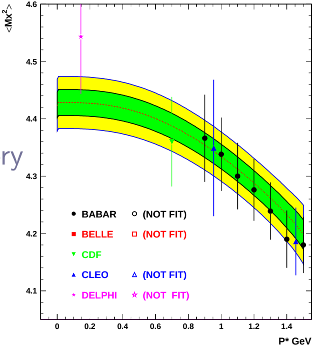

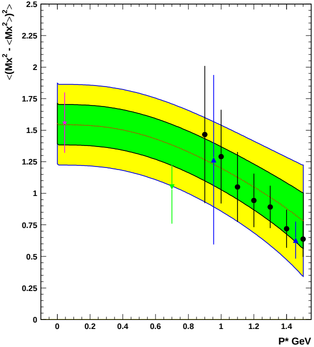

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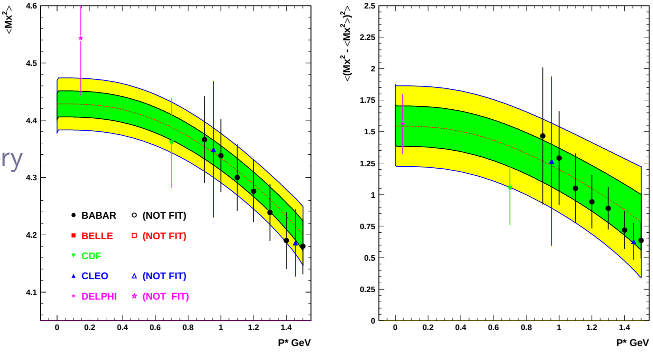
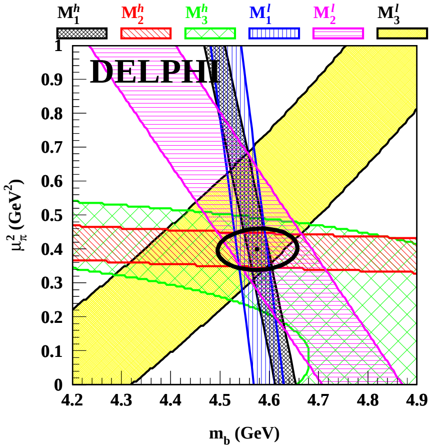


OPE works well even where it can be expected to break down

The Heavy Quark Expansion is based on the smart application of the Wilsonian OPE

It has nothing to do with integrating α_s over the Landau singularity or with summing non-summable perturbative series

IR domain *is excluded* from the perturbative calculations



M_1^h
M_2^h
M_3^h
M_1^l
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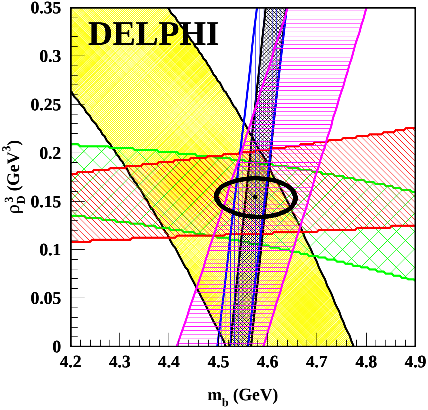
M_3^l
DELPHI
μ_π^2 (GeV^2)
m_b (GeV)



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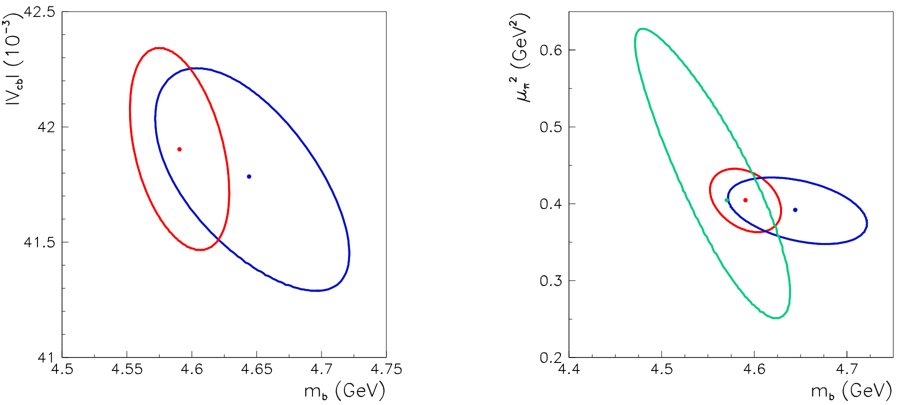
ρ_D^3 (GeV^3)
m_b (GeV)

A comprehensive fit including all moment measurements:

(by the professionals)



Important: HQ values emerged in accord with the theoretical expectations

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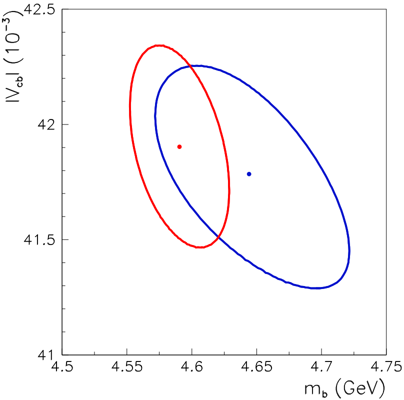

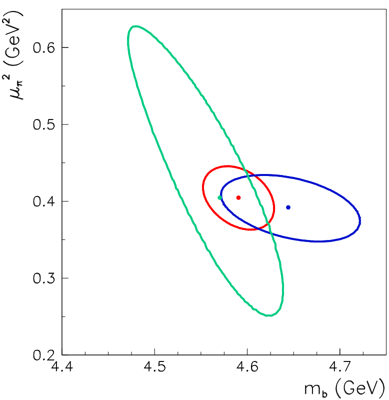

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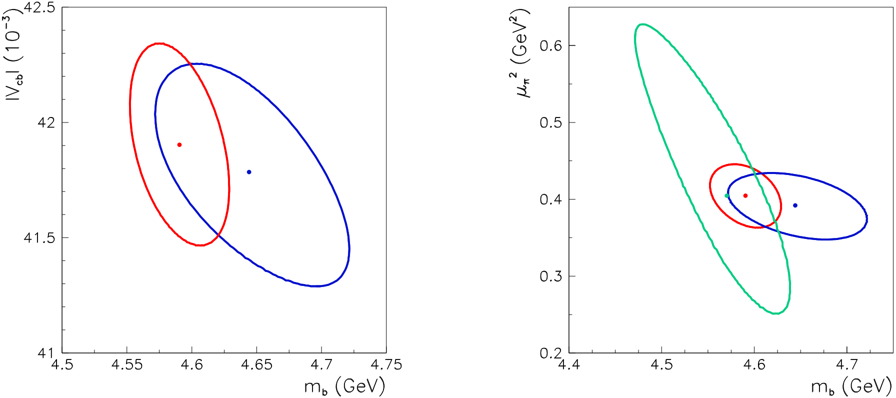

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'Theoretical correlations'

Status

Four years is quite a period   Some changes were inevitable

HFAG:

$$|V_{cb}| = (4.191 \pm 0.019 \pm 0.028 \pm 0.59) \cdot 10^{-3}$$

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$$m_b = 4.613 \pm 0.035\,\mathrm{GeV} \qquad m_b(m_b) \simeq 4.22\,\mathrm{GeV}$$

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Recent theory improvements

'Intrinsic charm' effects

Benson, Bigi, Mannel, N.U. 2003
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Required in the consistent OPE Benson et al., hep-ph/0302262

Generate enhanced effects $\frac{1}{m_b^3}\frac{1}{m_c^{2+k}}$ or even $\frac{1}{m_b^3}\frac{\alpha_s}{m_c^{1+k}}$ in the naive $1/m_Q$ expansion

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Experiment directly constrains the effect at 1 to 2% level

Expect improvement down to 0.5% where it would not affect precision of V_{cb}

The values of the q^2-moments are sensitive to these effects

Regular $1/m_b^4$ corrections

Dassinger, Mannel, Turczyk hep-ph/0611168

More expectation values appear. Expect small effect for $\Gamma_{\rm sl}(B)$, however noticeable for higher moments where so far both the experimental and theory accuracy have been limited

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Important to check their impact on $E_\ell^{\rm cut}$ dependence

Full α_s^2 corrections to decay distributions

Melnikov arXiv:0803.0951 [hep-ph]

So far incorporated α_s, $\beta_0\alpha_s^2$, all-order BLM. Complete α_s^2 had been evaluated only in $\Gamma_{\rm sl}(b)$

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N.U.; Czarnecki, Melnikov, N.U. PRL 1998

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N.U.; Czarnecki, Melnikov, N.U. PRL 1998

Using the more physical effective coupling is advantageous

In particular, in $b \to c\, \ell\nu$ the bulk of the QCD effects are encoded in the *dipole radiation coupling* $\alpha_s^{(d)}$:

$$\alpha_s^{(d)} = \bar{\alpha}_s - \frac{\alpha_s^2}{\pi}\, \underbrace{C_A\left(\frac{\pi^2}{6} - \frac{13}{12}\right)}_{1.67} + \ldots$$

Full α_s^2 corrections to decay distributions

Melnikov arXiv:0803.0951 [hep-ph]

So far incorporated α_s, $\beta_0\alpha_s^2$, all-order BLM. Complete α_s^2 had been evaluated only in $\Gamma_{\rm sl}(b)$

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NB: the 'dipole' coupling is an objective reality; -1.67 is an artifact of the $\overline{\rm MS}$ scheme

Table: Lepton energy moments

n	$E_\ell^{\rm cut}$, GeV	$L_n^{(0)}$	$L_n^{(1)}$	$L_n^{(2)}$
0	0	1	-1.77759	3.40
1	0	0.307202	−0.55126	1.11
2	0	0.10299	−0.1877	0.394
0	1	0.81483	-1.4394	2.63
1	1	0.27763	-0.49755	1.00
2	1	0.09793	-0.17846	0.382

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1	0	0.307202	-0.55126	1.11	-2.01
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n	$E_\ell^{\rm cut}$, GeV	$H_n^{(0)}$	$H_n^{(1)}$	$H_n^{(2)}$	$H_n^{(2)}/H_n^{(1)}$
1	1	0.334	-0.57728	1.02	-1.77
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The residual genuine non-BLM effects are suppressed!

Running of $\alpha_s^{(d)}$ is given by the same β-function up to three loops, hence BLM resummation etc. remain literally valid

The change simply amounts to using a 10% smaller input value of α_s in all the expressions:

$$\alpha_s(4.6\,\mathrm{GeV}) = 0.22 \quad \text{vs.} \quad 0.25$$

That was actually applied in the fit par our suggestions

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The results are likely not to change when including full α_s^2

The corrections are significantly *smaller* than allowed for in our analysis of the moments

α_s-corrections to the power-suppressed Wilson coefficients:

for a long time the principal limiting factor

Remain largely unknown...

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α_s-corrections to c_π

Becher, Boos, Lunghi arXiv:0708.0855 [hep-ph]

$E_\ell^{\rm cut} = 1\,{\rm GeV}$

	1	$\frac{\alpha_s}{\pi}$	$\frac{\mu_\pi^2}{2m_b^2}$	$\frac{\alpha_s}{\pi}\,\frac{\mu_\pi^2}{2m_b^2}$	$(\frac{\alpha_s}{\pi}\mu_\pi^2)/\mu_\pi^2$	$(\frac{\alpha_s}{\pi})/1$
1	0.5149	-0.910	-0.5692	0.987	-1.73	-1.77
$\hat{E}_l$	0.1754	-0.314	0.0109	-0.024	-2.20	-1.79
$\hat{E}_l^2$	0.06189	-0.1128	0.1105	-0.202	-1.83	-1.82
$\hat{E}_l^3$	0.02251	-0.0418	0.09269	-0.1722	-1.86	-1.86
$\hat{E}_x$	0.2111	-0.365	-0.5694	1.010	-1.77	-1.73
$\hat{E}_x^2$	0.08917	-0.1482	-0.3378	0.576	-1.71	-1.66
$\hat{E}_x^3$	0.03867	-0.0606	$-0.16898(6)$	0.2639	-1.56	-1.57
$(\hat{p}_x^2-\rho)$	0	0.03618	-0.6855	1.213	-1.77	
$(\hat{p}_x^2-\rho)^2$	0	0.002808	0.15198	-0.4388	-2.89	
$(\hat{p}_x^2-\rho)^3$	0	0.0004053	0	0.020998		
$\hat{E}_x(\hat{p}_x^2-\rho)$	0	0.01801	-0.20707	0.2961	-1.43	
$\hat{E}_x(\hat{p}_x^2-\rho)^2$	0	0.0015307	0.06794	-0.1897	-2.79	
$\hat{E}_x^2(\hat{p}_x^2-\rho)$	0	0.009147	-0.05271	0.0304	-0.58	

Typically $\quad \mu_\pi^2 \implies \left(1 - (1.5 \text{ to } 2.2)\,\frac{\alpha_s}{\pi}\right)$

$E_\ell^{\rm cut} = 1\,\mathrm{GeV}$

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Increase in the extracted value of μ_π^2 by 10-15% ?

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Increase in the extracted value of μ_π^2 by 10-15% ?

Would be welcomed,

might account for certain difference between $b \to c\,\ell\nu$ and $b \to s+\gamma$

V_{cb}, possibly, is not affected: in $\Gamma_{\rm sl}$ this has been accounted for, it depends on nearly the same combination as does $\langle M_X^2 \rangle$

$\langle M_X^2 \rangle$ is dominated by $\langle E_X \rangle$:

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Have approached the level of nearly '1%' theoretical accuracy in V_{cb}
Accurate implementation of the recent improvements along with calculation of α_s-corrections to o_G and o_D would provide

the *real 1% accuracy*

$B \to (\pi, \rho, a_1, ...)\ \ell\nu$:   need formfactors

V_{ub}

$B \to (\pi, \rho, a_1, ...)\ \ell\nu$: need formfactors LCSR, lattices

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a) parametrization of the shape fitted to the data: P. Ball 2006

$$|V_{ub} f^+_{B\pi}(0)| = (0.91 \pm [0.06]_{\rm shape} \pm [0.03]_{\rm BR}) \times 10^{-3}$$

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b) LCSR calculation of $f^+_{B\pi}(0)$:

$f^+_{B\pi}(0) = 0.26^{+0.04}_{-0.03}$ Duplancić, Khodjamirian, Mannel, Melić, Offen 2008

with

$$|V_{ub}| = (3.5 \pm 0.4_{\rm th} \pm 0.2_{\rm shape} \pm 0.1_{\rm BR}) \times 10^{-3}$$

previous LCSR result (Ball, Zwicky 2004): $f^+_{B\pi}(0) = 0.258 \pm 0.031$

V_{ub} determinations from $B \to \pi \ell \nu$

	$f^+_{B\pi}(q^2)$ calculation	$V_{ub} \times 10^3$
Okamoto et al.	lattice ($n_f = 3$)	$3.78 \pm 0.25 \pm 0.52$
HPQCD	lattice ($n_f = 3$)	$3.55 \pm 0.25 \pm 0.50$
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Lattices: the first-principle approach to formulate a field theory

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V_{ub} determinations from $B \to \pi \ell \nu$

	$f^+_{B\pi}(q^2)$ calculation	$V_{ub} \times 10^3$
Okamoto et al.	lattice ($n_f = 3$)	$3.78 \pm 0.25 \pm 0.52$
HPQCD	lattice ($n_f = 3$)	$3.55 \pm 0.25 \pm 0.50$
Becher & Hill	-	$3.7 \pm 0.2 \pm 0.1$
Flynn et al.	-	$3.47 \pm 0.29 \pm 0.03$
Ball & Zwicky	LCSR	$3.5 \pm 0.4 \pm 0.1$
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In many instances the accuracy is being learned

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There is no reason to have a cut on a single variable, can introduce a domain in $\{q^2, q_0\} \Longleftrightarrow \{M_X, |\vec{q}|\}$

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More inclusive rates are better controlled theoretically

'Distance' in q_0 to the free-quark kinematics defines the OPE expansion parameter

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OPE suggests excluding large q^2 from the domain to calculate

Advantage of the cut over P_+?

Advantage of the cut over P_+? I doubt. The universality with $b \to s + \gamma$ holds to the same extent as the universality allowing to translate the distribution to arbitrary light-cone kinematics

Strategy:

- Deemphasize large q^2
- Impose cuts on $\{M_X, q^2\}$ to balance experimental selectivity and efficiency with the theory accuracy

Dealing with Fermi motion:

Earlier strategy from the 1990s: relate $b \to u$ distributions to $b \to s + \gamma$ relying on the FM universality

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The same idea drives the later approach by Lange et al.

Gambino *et al.*:

- $1/m^k$ corrections are included into Fermi Motion without additional model-dependence
- WA is allowed for
- All the known constraints provided by the OPE from $b \to c\, \ell\nu$ ($b \to s + \gamma$) are incorporated
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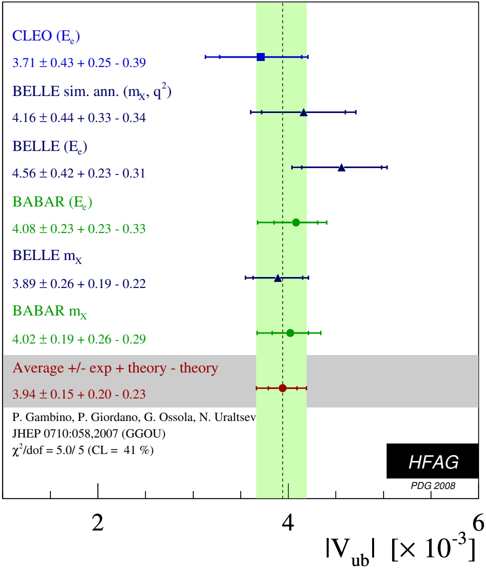
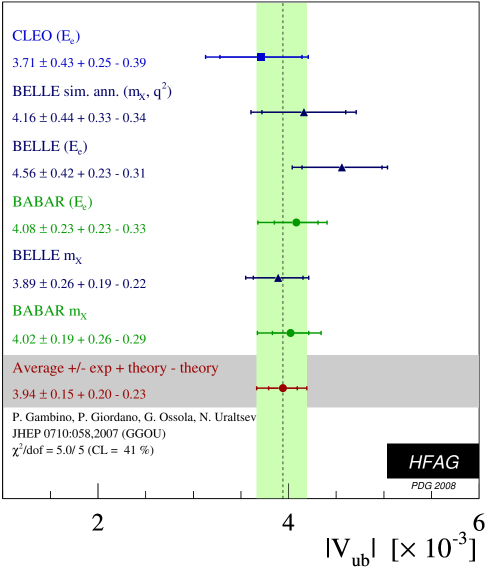
Generate rate/moments over arbitrary kinematic domain, however differential rates over certain regions are model-dependent and not to be taken literally



HFAG preliminary:

$$|V_{ub}| = (3.94 \pm 0.15^{+0.20}_{-0.23}) \cdot 10^{-3}$$

Not fully explored yet



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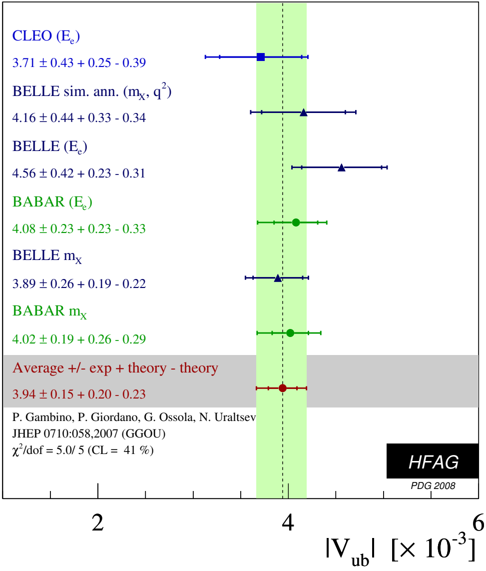

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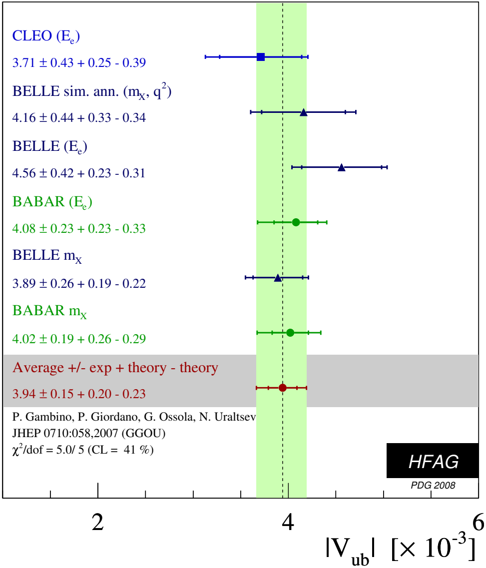

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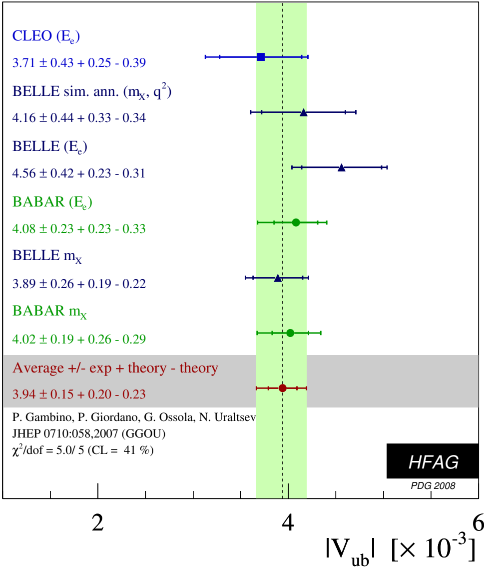

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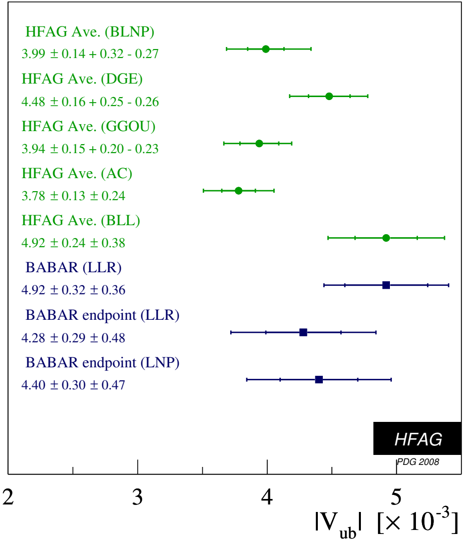
May lower V_{ub} by about 5%

All methods:



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BABAR (LLR)
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BABAR endpoint (LLR)
4.28 ± 0.29 ± 0.48
BABAR endpoint (LNP)
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HFAG
PDG 2008
2
3
4
5
|V_ub| [× 10^-3]

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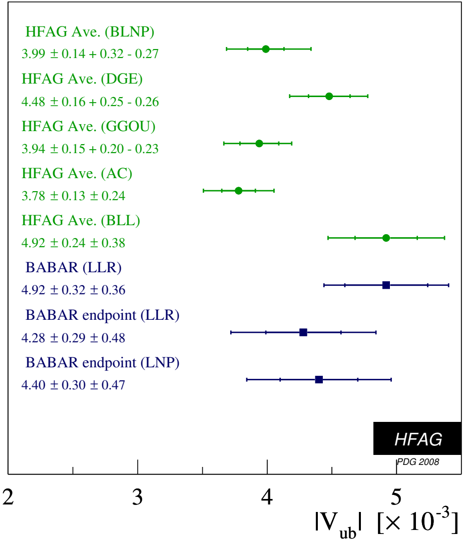

The more robust approaches with adequate theory descriptions seem to provide the stable result for V_{ub}