Estimating U-spin breaking in $B_{d,s} \rightarrow D_{d,s}(\pi, K)$

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Informal discussion

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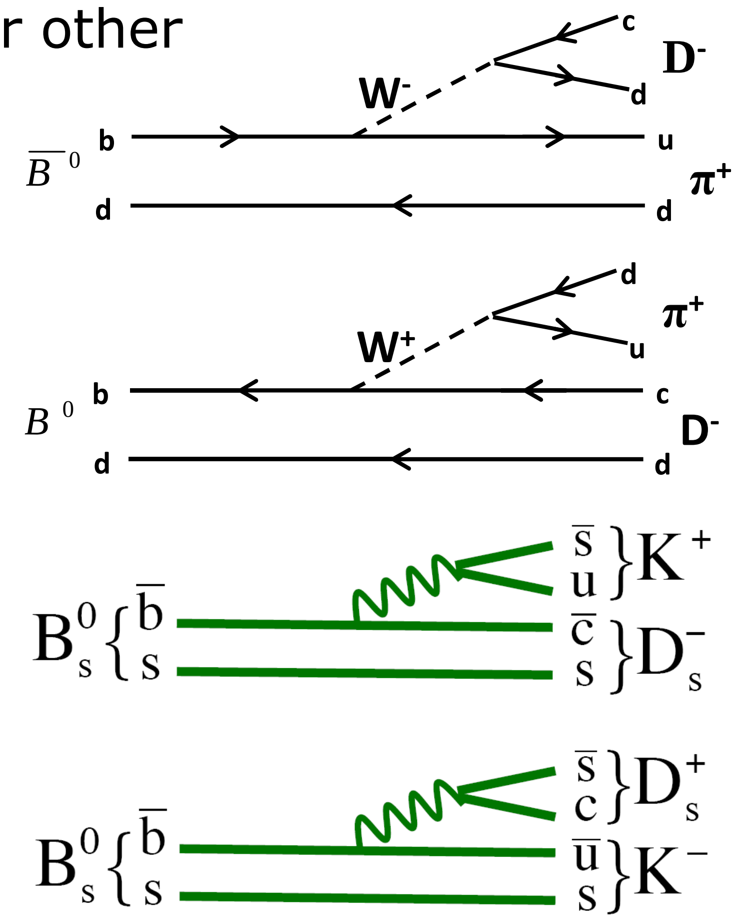
OVERVIEW

- Motivation for looking at $B_{d,s} \rightarrow D_{d,s}(\pi,K)$
- Constraining SU(3) uncertainties in $B \rightarrow D\pi$
- Expected precision from separate analyses of $B_s \rightarrow D_s K$ and $B \rightarrow D\pi$
- Combined analysis with U-spin symmetry
- Further theoretical inputs required

MOTIVATION

MEASURING γ WITH $B_q \rightarrow D_q u_q$

- The $B_q^0 \rightarrow D_q u_q$ family are tree level decays
  - Not sensitive to New Physics
  - Provide a SM baseline of γ for other measurements



Current SM values of CKM angles:

$$\alpha = \left(87.8^{+5.8}_{-5.4}\right)^\circ$$

$$\beta = \left(21.5^{+1.0}_{-1.0}\right)^\circ$$

$$\gamma = \left(72^{+34}_{-30}\right)^\circ$$

Ref : CKMFitter Moriond 2008

LHCb performance preview

What is the expected LHCb precision on γ?

- 10° with 1 year of data taking ($2fb^{-1}$) in $B_s \rightarrow D_s K$

- $\sim 20^\circ$ with 1 year of data taking ($2fb^{-1}$) possible in $B_d \rightarrow D\pi$

Can also use $B_d \rightarrow D^*\pi$, $B_s \rightarrow D_s^* K$

Will discuss in more detail later in the talk... just whetting your appetite for now.

DEPENDENCE ON γ

The dependence on γ comes from time dependent rate asymmetries:

$$A\left(B \to D_q \bar{u}_q\right) = \frac{C\cos(\Delta m\tau) + S\sin(\Delta m\tau)}{\cosh\left(\Delta\Gamma_q t/2\right) - A_{\Delta\Gamma}\sinh\left(\Delta\Gamma_q t/2\right)}$$

Since there are two possible final states, one obtains two asymmetries, and hence two (to first order) independent constraints on γ

ASYMMETRIES IN MORE DETAIL

C, S, $A_{\Delta\Gamma}$ are the observable parameters, from which γ is extracted (from now on "CP observables"

- **C** can only be resolved for large x_q
- **$A_{\Delta\Gamma}$** can only be resolved for large $\Delta\Gamma$

$$S = \frac{2x_q \sin(\delta_q + \phi_q - \gamma)}{(x_q^2 + 1)} \qquad C_q = -\frac{1 - x_q^2}{1 + x_q^2}$$

$$\left|A_{\Delta\Gamma}(B_q^0 \to D_q \bar{u}_q)\right|^2 + \left|C(B_q^0 \to D_q \bar{u}_q)\right|^2 + \left|S(B_q^0 \to D_q \bar{u}_q)\right|^2 = 1$$

And there are of course three analogous parameters for the "other" asymmetry

GOING DEEPER INTO THE TERMINOLOGY

The dependence on γ is contained in the CP-observable S

$$S = \frac{2x_q \sin\left(\delta_q + \phi_q - \gamma\right)}{\left(x_q^2 + 1\right)}$$

$\mathbf{x_q}$ is the ratio of the interfering tree-level diagrams; the bigger $\mathbf{x_q}$, the more sensitive the decay is to γ

$$\boxed{x_d = -\left(\frac{\lambda^2 R_b}{1-\lambda^2}\right) a_d} \qquad \boxed{x_s = R_b a_s}$$

ONE FINAL STEP...

The formulas for $x_{d,s}$ come from the decay amplitudes

$$R_b = \left(1 - \frac{\lambda^2}{2}\right)\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right| \approx 0.4$$

a_d, a_s are hadronic parameters of order 1, λ is of course 0.22 (the Cabbibo angle)

$$\boxed{x_s = R_b a_s \approx 0.4} \quad \boxed{x_d = -\left(\frac{\lambda^2 R_b}{1-\lambda^2}\right) a_d \approx 0.02}$$

x_s is large enough to fit from data

BUT

x_d must be externally constrained!

WHERE DOES THIS LEAVE US?

$B_s \rightarrow D_s K$ and $B_d \rightarrow D\pi$ decays are sensitive to γ

We measure γ from time dependent CP asymmetries

The observables which carry the dependence on γ also depend on the ratio of the interfering tree level diagrams

- This interference is big enough to fit from the data for the B_s case, but too small for B_d

In order to extract from $B_d \rightarrow D\pi$, we need an external constraint on x_d!

CONSTRAINING X_d

**Taken from
Max Baak's talk
to CKM 2006**

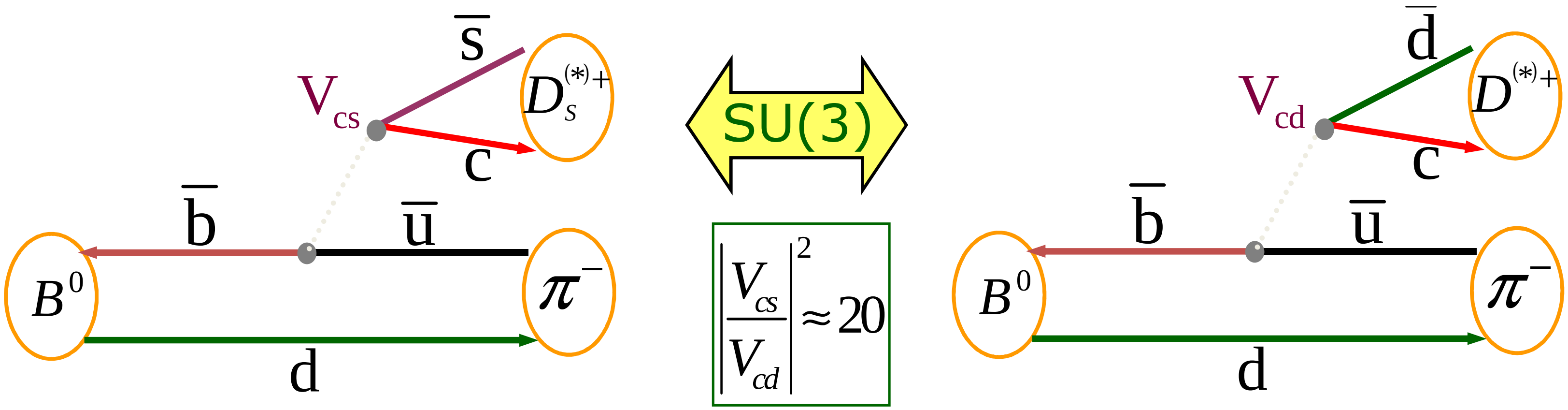
BEFORE WE PROCEED

In all following slides

$$\mathbf{r}^{\mathbf{D(*)h}} \equiv \mathbf{x_d}$$

THE STARTING POINT

Estimate $r^{D(*)h}$ from $B^0 \rightarrow D_s^{(*)+}\pi^-/\rho^-$ using SU(3) symmetry [1]



$$r^{D^{(*)}h} = \sqrt{\frac{\mathcal{B}(B^0 \rightarrow D_s^{(*)+}h^-)}{\mathcal{B}(B^0 \rightarrow D^{(*)-}h^+)}}\left|\frac{V_{cd}}{V_{cs}}\right|\frac{f_{D^{(*)}}}{f_{D_s^{(*)}}}$$

[1] I. Dunietz, Phys. Lett. B 427, 179 (1998)

SOURCES OF SU(3) BREAKING

Amplitude relation assumes factorization

- Not (yet) been proven to work for wrong-charm $b \to u$ transitions
- i.e. No theoretical handle on size of non-factorizable contributions involved

Three potential sources of SU(3) breaking between $D^{(*)}h$ and $D_s^{(*)}h$:

1. Unknown SU(3) breaking uncertainty from non-factorizable contributions
2. Final state interactions: different rescattering diagrams
3. Missing W-exchange diagrams in calculation

Accounted for by introducing theoretical uncertainty on **amplitude ratio $r^{D(*)h}$**

- Size of uncertainty not well understood
- Typically guestimated to be **30% of size of amplitude ratio**.

RESCATTERING CORRECTION

1. Rescattering is parametarized as a multiplicative correction to the amplitude ratio:

2. Rescattering is independent of formation process, so can be calculated from CKM-favoured modes

3. Fit to the strong-interaction rescattering matrix using experimental inputs to obtain correction factors

4. Can check validity of method by comparing predicted rescattering branching ratios to measured ones

BR (x10^{-4})	Factorized $\mathcal{B}$	Rescattered $\mathcal{B}$	Measured $\mathcal{B}$	χ
$B^+ \to \bar{D}^0\pi^+$	48.6	48.6	49.2 ± 2.0	+0.29
$B^0 \to D^-\pi^+$	32.7	28.0	28.3 ± 1.7	+0.20
$B^0 \to \bar{D}^0\pi^0$	0.50	2.39	2.61 ± 0.24	+0.90
$B^0 \to D_s^-K^+$	0.00	0.25	0.27 ± 0.06	+0.41
$B^0 \to \bar{D}^0\eta$	0.00	1.36	2.02 ± 0.35	+1.89
$B^0 \to \bar{D}^0\eta'$	0.08	1.25	1.25 ± 0.23	+0.02
$B^+ \to \bar{D}^0K^+$	3.90	3.90	4.08 ± 0.24	+0.75
$B^0 \to D^-K^+$	2.6	2.2	2.0 ± 0.6	−0.27
$B^0 \to \bar{D}^0K^0$	0.08	0.53	0.52 ± 0.07	−0.12
$B^+ \to \bar{D}^{*0}\pi^+$	50.3	50.3	46 ± 4	−1.08
$B^0 \to D^{*-}\pi^+$	33.0	28.3	27.6 ± 2.1	−0.34
$B^0 \to \bar{D}^{*0}\pi^0$	0.60	2.51	1.73 ± 0.42	−1.86
$B^0 \to D_s^{*-}K^+$	0.00	0.23	0.18 ± 0.06	−0.87
$B^0 \to \bar{D}^{*0}\eta$	0.07	1.34	1.78 ± 0.56	+0.79
$B^0 \to \bar{D}^{*0}\eta'$	0.10	1.24	1.23 ± 0.35	−0.03
$B^+ \to \bar{D}^{*0}K^+$	3.88	3.88	3.7 ± 0.4	−0.44
$B^0 \to D^{*-}K^+$	2.53	2.10	2.1 ± 0.2	+0.21
$B^0 \to \bar{D}^{*0}K^0$	0.09	0.53	0.36 ± 0.12	−1.35
$B^+ \to \bar{D}^0\rho^+$	101	101	134 ± 18	+1.86
$B^0 \to D^-\rho^+$	76.3	71.1	75 ± 12	+0.32
$B^0 \to \bar{D}^0\rho^0$	0.4	3.1	2.9 ± 1.1	−0.19
$B^0 \to D_s^-K^{*+}$	0.0	0.0	0.0 ± 6.6	0.00
$B^0 \to \bar{D}^0\omega$	0.2	2.6	2.6 ± 0.6	−0.10
$B^0 \to \bar{D}^0\phi$	0.0	0.0	–	–
$B^+ \to \bar{D}^0K^{*+}$	5.9	5.9	6.3 ± 0.8	+0.49
$B^0 \to D^-K^{*+}$	4.2	3.9	4.5 ± 0.7	+0.79
$B^0 \to \bar{D}^0K^{*0}$	0.08	0.36	0.40 ± 0.08	+0.48

Max Baak

CKM Workshop 2006, Nagoya

SU(3) correction from rescattering

➢ Naive amplitude ratio :

$$r^{D^{(*)}h} = \sqrt{\frac{\mathcal{B}(B^0 \to D_s^{(*)+} h^-)}{\mathcal{B}(B^0 \to D^{(*)-} h^+)}} \left|\frac{V_{cd}}{V_{cs}}\right| \frac{f_{D^{(*)}}}{f_{D_s^{(*)}}}$$

➢ SU(3) rescattering correction factor R_i to amplitude ratio $r^{D(*)h}$:

$$R_i \approx \left(\frac{p_{D^{(*)}}}{p_{D_s^{(*)}}}\right)^{\frac{2L+1}{2}} \left|\frac{1 + i\bar{r}'_0 + i\bar{r}'_a}{1 + i\bar{r}'_0}\right|$$

1. No annihilation rescattering for $B^0 \to h^- D_s^{(*)+}$

2. Kinematic factor

➢ Values of R_i :

Final state	R_i
$D^+\pi^-$	1.045 ± 0.005
$D^{*+}\pi^-$	1.072 ± 0.005
$D^+\rho^-$	1.036

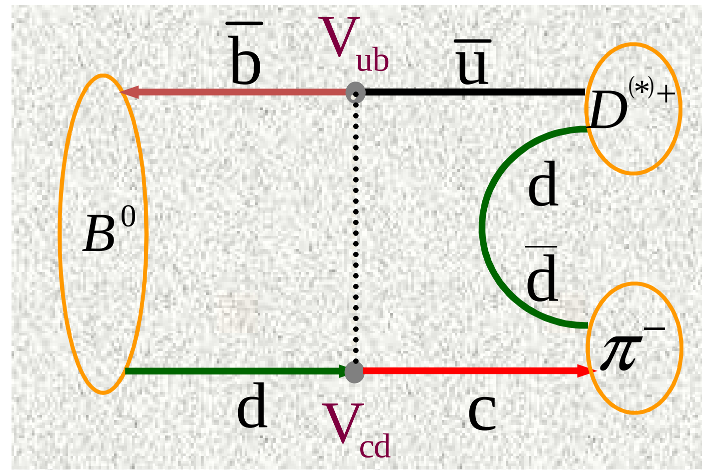
W EXCHANGE CORRECTION

1. Estimate from effective hamiltonians for the two processes (tree-level and exchange) using naive factorization

2. However, factorization is not reliable for colour-suppressed decays

3. Add a large systematic error to account for this:

No exchange diagram for final state $D_s\pi/\rho$



$$\boxed{|E/T| < 5.0\%}$$

Max Baak CKM Workshop 2006, Nagoya

NON-FACTORIZABLE SU(3) CORRECTIONS

1. Estimate residual SU(3) breaking from non-factorizable contributions using $B \rightarrow D_s^*\pi$

2. Relate the measured branching ratio to the rescattering-corrected factorization prediction

3. Precise estimate from factorization is possible by relating $B \rightarrow D_s^*\pi$ to semileptonic B decays

➢ Assuming up to 3 times typical SU(3) breaking scale for $B^0 \to \pi^- D_{(s)}^{*+}$:

$$\boxed{\begin{aligned} \left|\bar{a}^{c}\,\frac{2m_s}{\Lambda_\chi}\right| &< 2\left(\frac{2m_s}{\Lambda_\chi}\right)\left|\frac{\mathcal{R}e(\tilde{a}_s^{\mathrm{corr}}) + \frac{1}{2}|\tilde{a}_s^{\mathrm{corr}}|^2}{|1+\tilde{a}_s^{\mathrm{corr}}|^2}\right| \\ &< 0.085\ (0.120)\ @\ 68.3\%\ (90\%)\ \mathrm{C.L.} \end{aligned}}$$

THE FINAL ERROR BUDGET

- Amplitude ratios after rescattering correction:

$$r^{D^{(*)}h} = \sqrt{\frac{\mathcal{B}(B^0 \to D_s^{(*)+} h^-)}{\mathcal{B}(B^0 \to D^{(*)-} h^+)}} \left|\frac{V_{cd}}{V_{cs}}\right| \frac{f_{D^{(*)}}}{f_{D_s^{(*)}}} R_i$$

Decay	Predicted $r^{D^{(*)}h}$ $(\times 10^{-2})$
$B^0 \to D^\mp \pi^\pm$	$1.54 \pm 0.18\,(\mathcal{B}) \pm 0.09\,(r.f_{D_{(s)}}) \pm 0.17\,(V_{cq}) \pm 0.01\,(\text{rsc.})$
$B^0 \to D^{*\mp} \pi^\pm$	$2.15 \pm 0.30\,(\mathcal{B}) \pm 0.12\,(r.f_{D_{(s)}}) \pm 0.24\,(V_{cq}) \pm 0.01\,(\text{rsc.})$
$B^0 \to D^\mp \rho^\pm$	$0.33 \pm 0.59\,(\mathcal{B}) \pm 0.02\,(r.f_{D_{(s)}}) \pm 0.04\,(V_{cq})$

- New since PDG '06: large uncertaintainty from V_{cs}
- We add 9% Gaussian errors for SU(3) from non-factorizable contributions and 5% flat errors for SU(3) breaking from W-exchange diagrams.

No SU(3) uncertainties included

OVERALL ERROR NOW TAKEN AS 20%

EXPECTED PRECISIONS AT LHCB

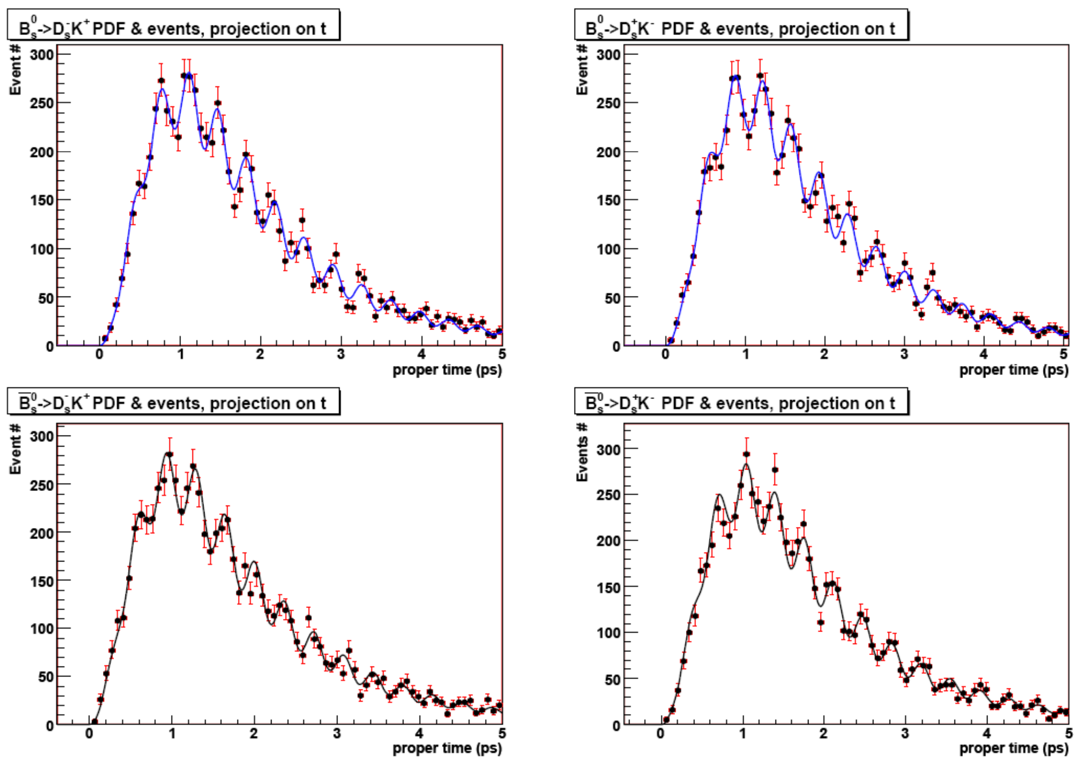
$$\gamma = 60^\circ$$

ASSUMED THROUGHOUT

$B_s \rightarrow D_s K$

- Use untagged $B_s \rightarrow D_s K$ events to resolve $\mathbf{A}_{\Delta\Gamma}$
- Use $B_s \rightarrow D_s \pi$ events to help constrain $\Delta\Gamma_s$ and $\Delta \mathbf{m}_s$
- Results in twofold ambiguity on γ

	Yield ($2fb^{-1}$)	B/S
$B_s \rightarrow D_s K$	6.2k	0.2
$B_s \rightarrow D_s \pi$	140k	0.7

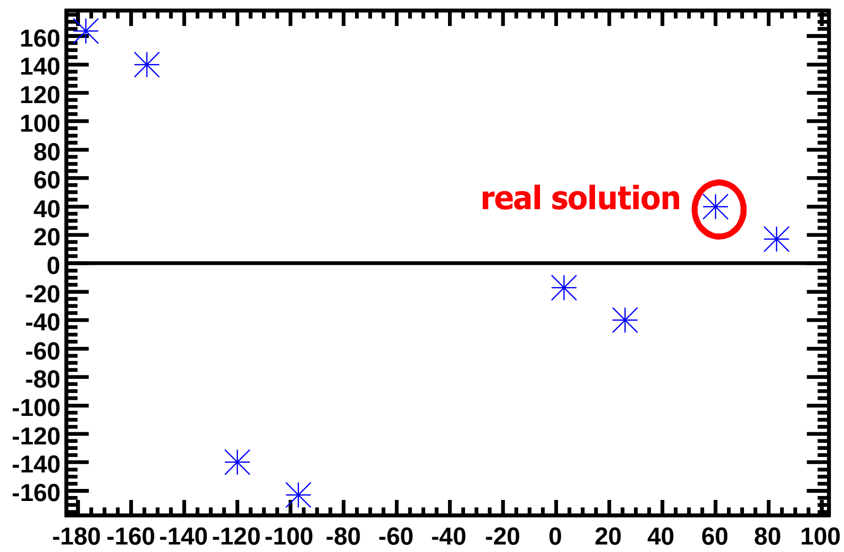


With $2fb^{-1}$ of data:

	Precision with tagged & untagged events
$\gamma+\phi_s$	10.3°
Δ_{ms}	0.007 ps^{-1}
x_s	0.06

Ref: CERN-LHCb-2005-036
CERN-LHCb-2007-017
CERN-LHCb-2007-041

$B_D \rightarrow D\pi$



Two problems:

1) The uncertainty on $\mathbf{x_d}$ introduces correlations between the two asymmetries.

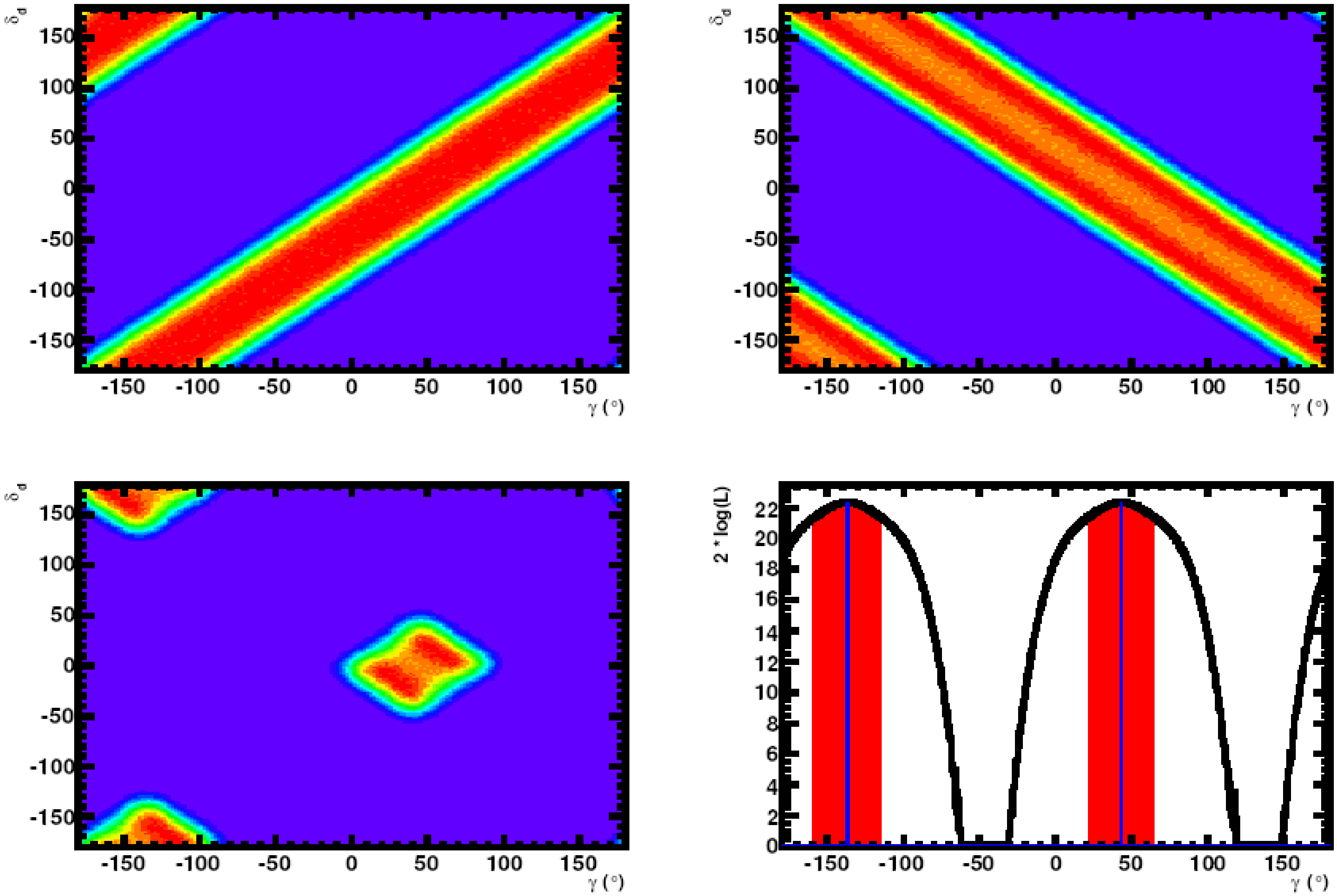
   - **The errors on each observable worsen, and after some time are saturated by the correlations.**

2) The negligible lifetime difference in the $\mathbf{B_d}$ system means $\mathbf{A_{\Delta\Gamma}}$ is not accesible

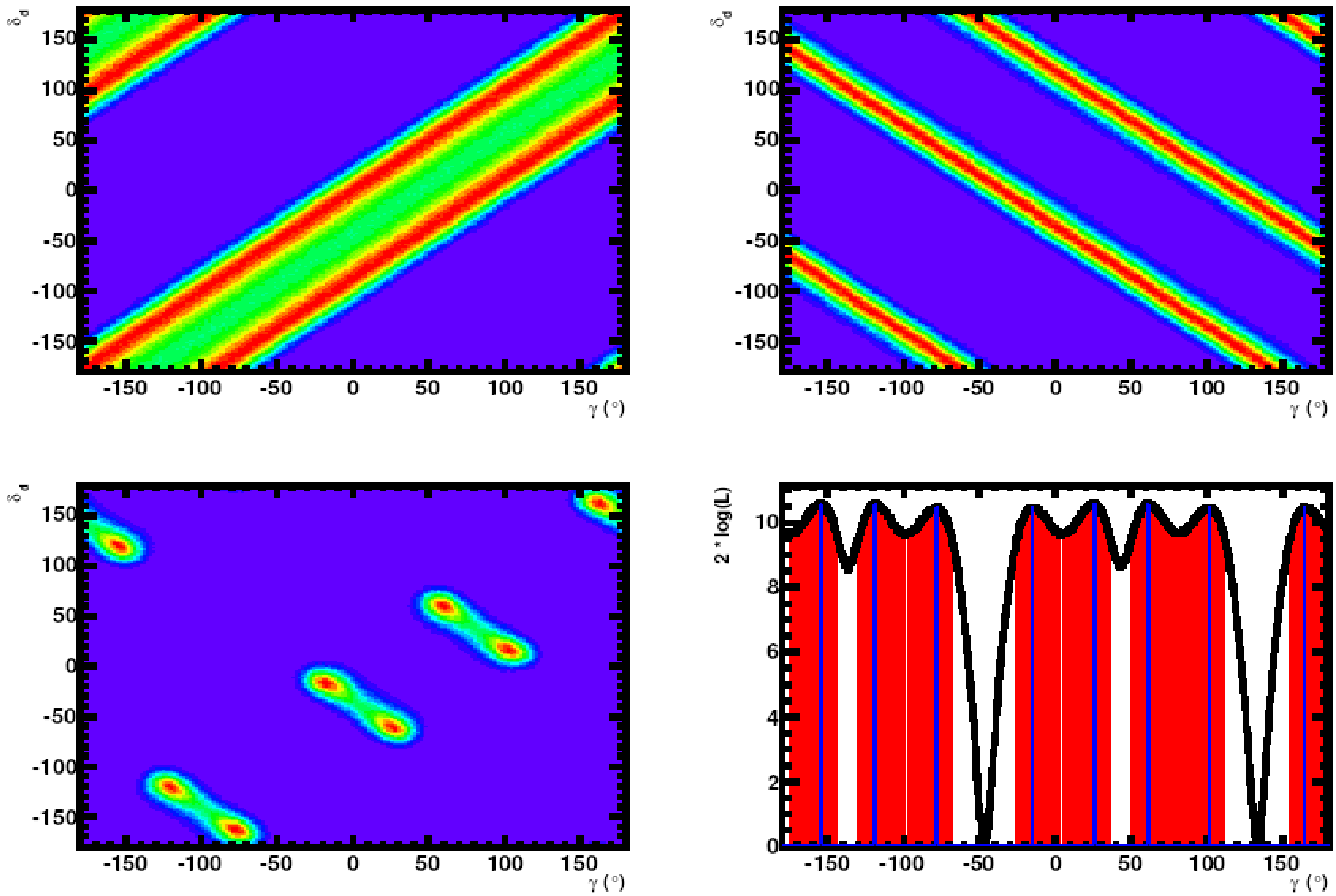
   - **The eight-fold ambiguity on γ remains. Also, the precisions vary with the value of the strong phases.**

Both will be resolved by using U-spin symmetry!

$B_D \rightarrow D\pi$: 5 years, factorization limit



$B_D \rightarrow D\pi$: 5 YEARS, LARGE STRONG PHASE



USING U-SPIN

U-SPIN OVERVIEW

U–spin is a subgroup of SU(3)

- QCD effects same if decays are related by interchange of **d** and **s** quarks

$$x_s = R_b a_s$$

$$x_d = -\left(\frac{\lambda^2 R_b}{1-\lambda^2}\right) a_d$$

QCD effects are parameterized by strong amplitudes ($\mathbf{a_{s,d}}$) and phases ($\delta_{\mathbf{s,d}}$)

$$S = \frac{2x_q \sin\left(\delta_q + \phi_q - \gamma\right)}{\left(x_q^2 + 1\right)}$$

Three different assumptions: equal phases and amplitudes, equal phases only, equal amplitudes only

Major advantage : no need to resolve $\mathbf{x_d}$

Ref: Fleischer, hep-ph/0304027

ASSUMING EQUAL STRONG PHASES

Can make a “minimal” U-spin assumption

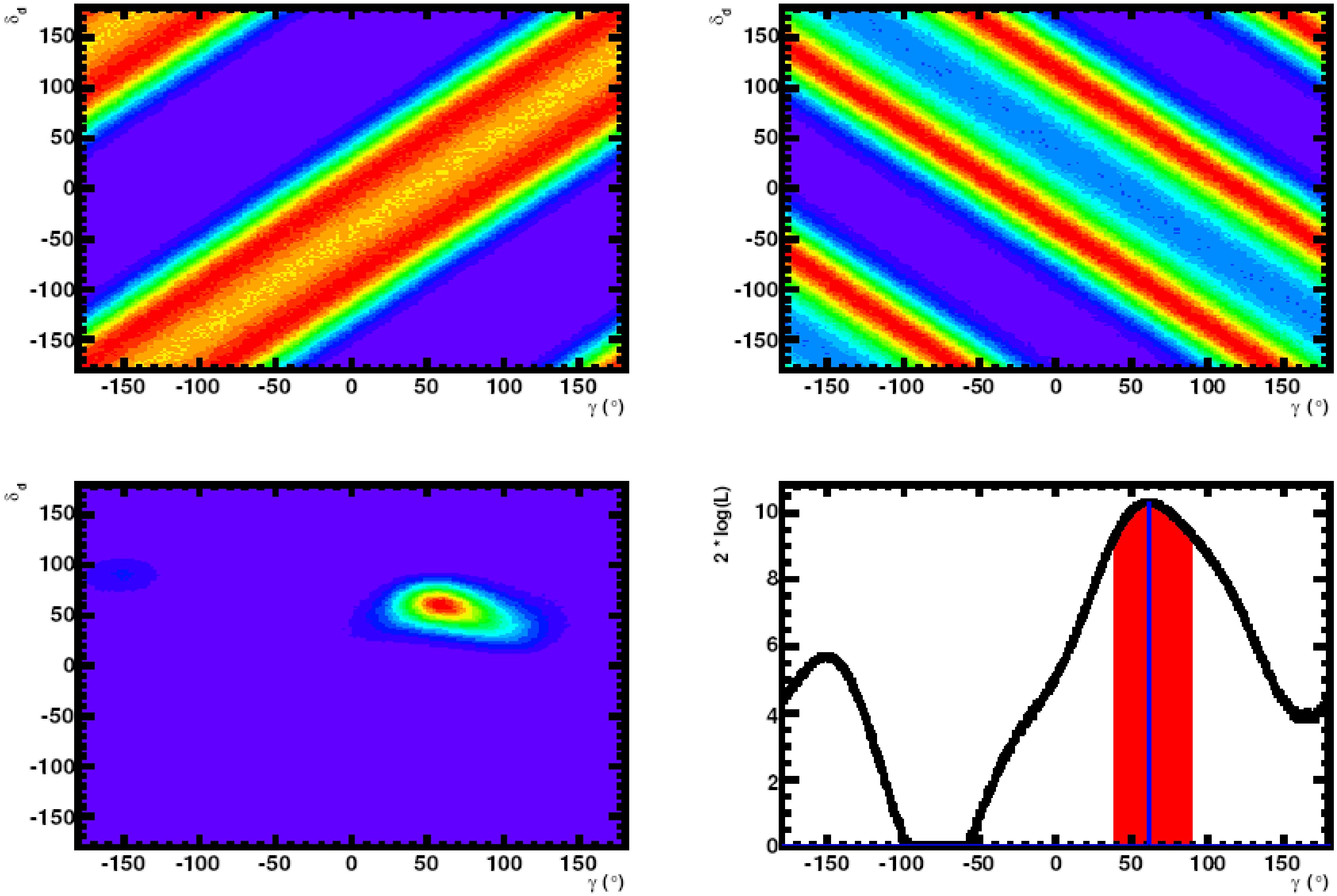
Strong phase in $B \rightarrow D\pi$ is the same as in $B_s \rightarrow D_s K$

Introduce this as a Gaussian constraint in the contour plots to resolve the ambiguities

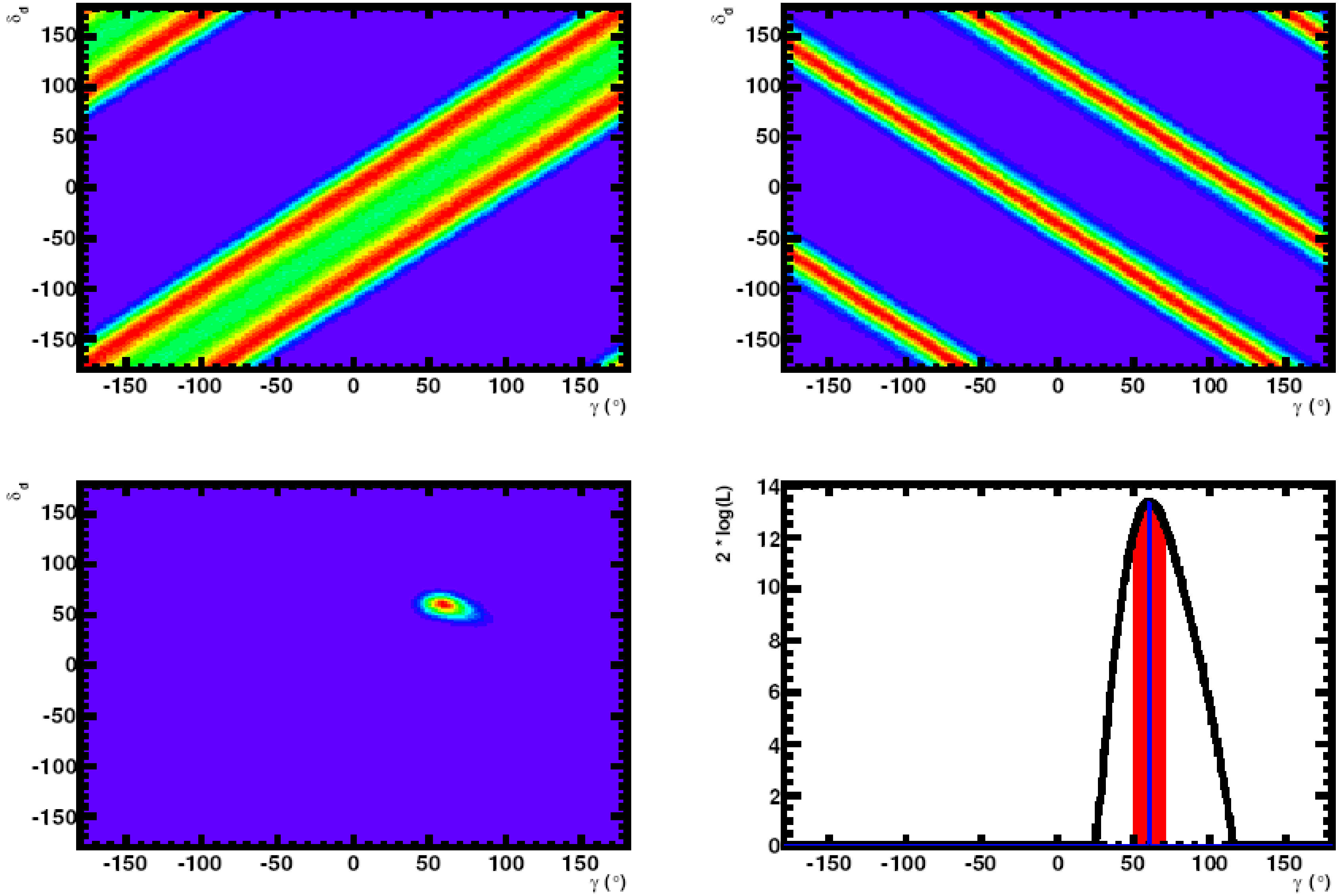
- Assume strong phase known to 20° (theoretical and experimental error) after 1 year
- And 10° after 5 years

In this case, still need external kowledge of x_d

$B_D \rightarrow D\pi$: 1 YEAR, LARGE STRONG PHASE, U-SPIN



$B_D \rightarrow D\pi$: 5 YEARS, LARGE STRONG PHASE, U-SPIN



γ known to 10 degrees – useful for a global constraint!

MORE SOPHISTICATED U-SPIN TREATMENT

Introduce new "orthogonal" CP-observables

$$\langle S_q \rangle_+ = \frac{S_q + \bar{S}_q}{2} = \frac{2x_q \cos\delta_q}{1+x_q^2}\sin(\varphi_q + \gamma)$$

$$\langle S_q \rangle_- = \frac{S_q - \bar{S}_q}{2} = \frac{2x_q \sin\delta_q}{1+x_q^2}\cos(\varphi_q + \gamma)$$

Will now use $B_s \rightarrow D_s K$ and $B \rightarrow D\pi$ information at the same time to get a combined constraint on γ

STRONG U-SPIN ASSUMPTION

Uses the relations

(1) $$\left[\frac{a_s \cos\delta_s}{a_d \cos\delta_d}\right] R = -\left[\frac{\sin(\phi_d+\gamma)}{\sin(\phi_s+\gamma)}\right]\left[\frac{\langle S_s\rangle_+}{\langle S_d\rangle_+}\right]$$

(2) $$\left[\frac{a_s \sin\delta_s}{a_d \sin\delta_d}\right] R = -\left[\frac{\cos(\phi_d+\gamma)}{\cos(\phi_s+\gamma)}\right]\left[\frac{\langle S_s\rangle_-}{\langle S_d\rangle_-}\right]$$

to extract γ under the assumptions $\boldsymbol{\delta_d=\delta_s}$ and $\boldsymbol{a_d=a_s}$,

The parameter **R** can be determined from $B_s \rightarrow D_s K$ ➡ $$R = \left(\frac{1-\lambda^2}{\lambda^2}\right)\left[\frac{1+x_d^2}{1+x_s^2}\right]$$

- $\boldsymbol{x_d}$ is a negligable second order correction.

PHASE U-SPIN ASSUMPTION

Uses the relation

$$\left[\frac{\tan(\phi_d+\gamma)}{\tan(\phi_s+\gamma)}\right]=\left[\frac{\tan\delta_s}{\tan\delta_d}\right]\left[\frac{\langle S_s\rangle_-}{\langle S_s\rangle_+}\right]\left[\frac{\langle S_d\rangle_+}{\langle S_d\rangle_-}\right]$$

to extract γ under the assumption $\delta_d=\delta_s$. It does not require any assumption about the value of a_d or a_s.

AMPLITUDE U-SPIN ASSUMPTION

Uses the relation

$$\left(\frac{a_s}{a_d}\right)R=\sigma\left|\frac{\sin(2\phi_d+2\gamma)}{\sin(2\phi_s+2\gamma)}\right|\sqrt{\frac{\langle S_s\rangle_+^2\cos^2(\phi_s+\gamma)+\langle S_s\rangle_-^2\sin^2(\phi_s+\gamma)}{\langle S_d\rangle_+^2\cos^2(\phi_d+\gamma)+\langle S_d\rangle_-^2\sin^2(\phi_d+\gamma)}}$$

to extract γ under the assumption $\mathbf{a_d=a_s}$. It does not require any assumption about the value of δ_d or δ_s, apart from an assumption about their relative signs

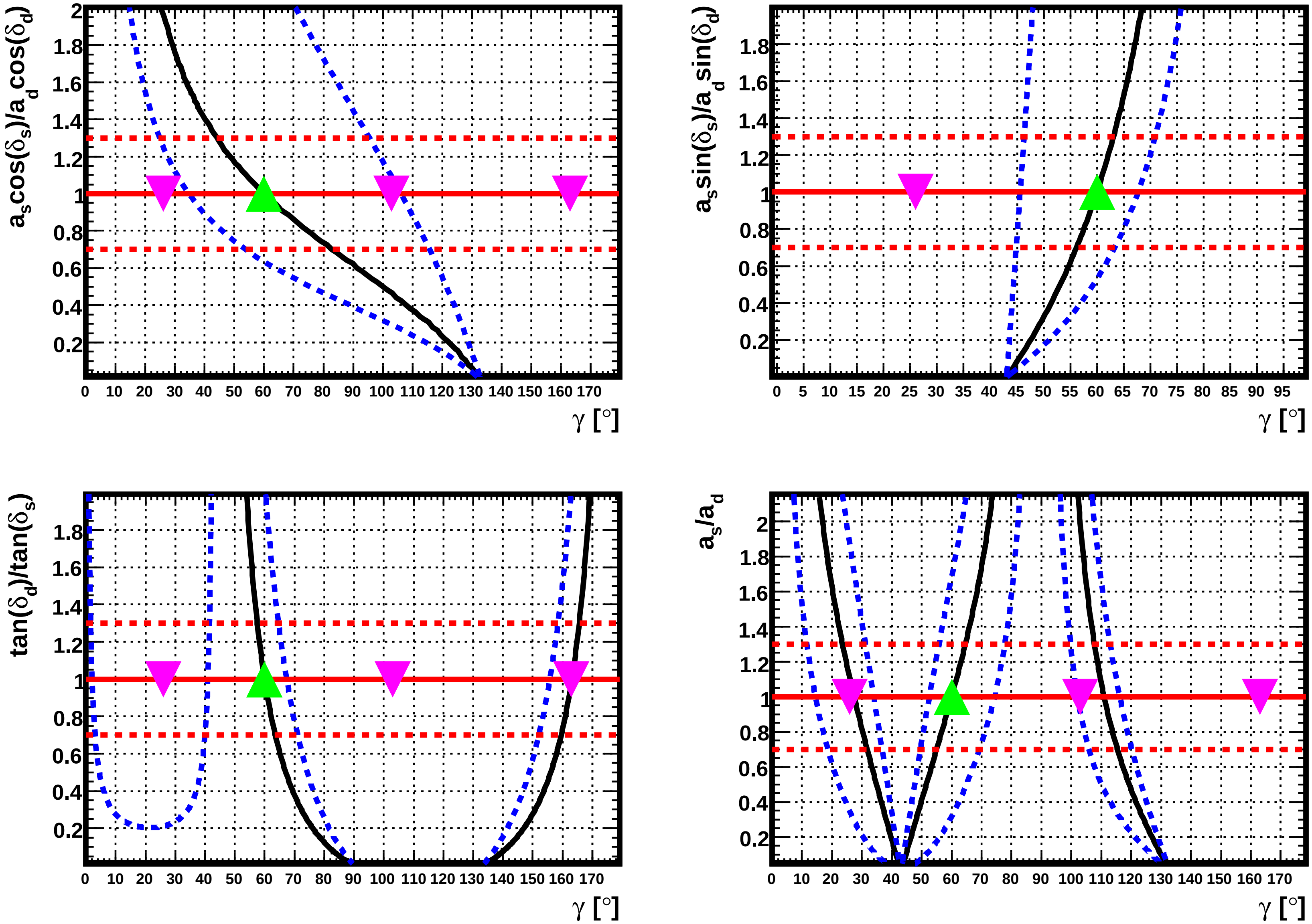
$$\text{if } \cos(\delta_d) \text{ has the same sign as } \cos(\delta_s),$$

$$\sigma=-\mathrm{sgn}\left[\langle S_s\rangle_+\langle S_d\rangle_+\sin(\phi_d+\gamma)\sin(\phi_s+\gamma)\right]$$

$$\text{if } \sin(\delta_d) \text{ has the same sign as } \sin(\delta_s),$$

$$\sigma=-\mathrm{sgn}\left[\langle S_s\rangle_-\langle S_d\rangle_-\cos(\phi_d+\gamma)\cos(\phi_s+\gamma)\right]$$

EXAMPLE RESULT: γ=60°, δ=60° (~1 YEAR)



ESTIMATING U-SPIN BREAKING

U-spin breaking is typically guesstimated at 30%

Has been argued to be a better symmetry than SU(3) in certain cases...

- Because U-spin does not depend on assumptions about relative sizes of different decay topologies, unlike SU(3)*

Would be nice to have a detailed error budget before we try to publish a measurement...

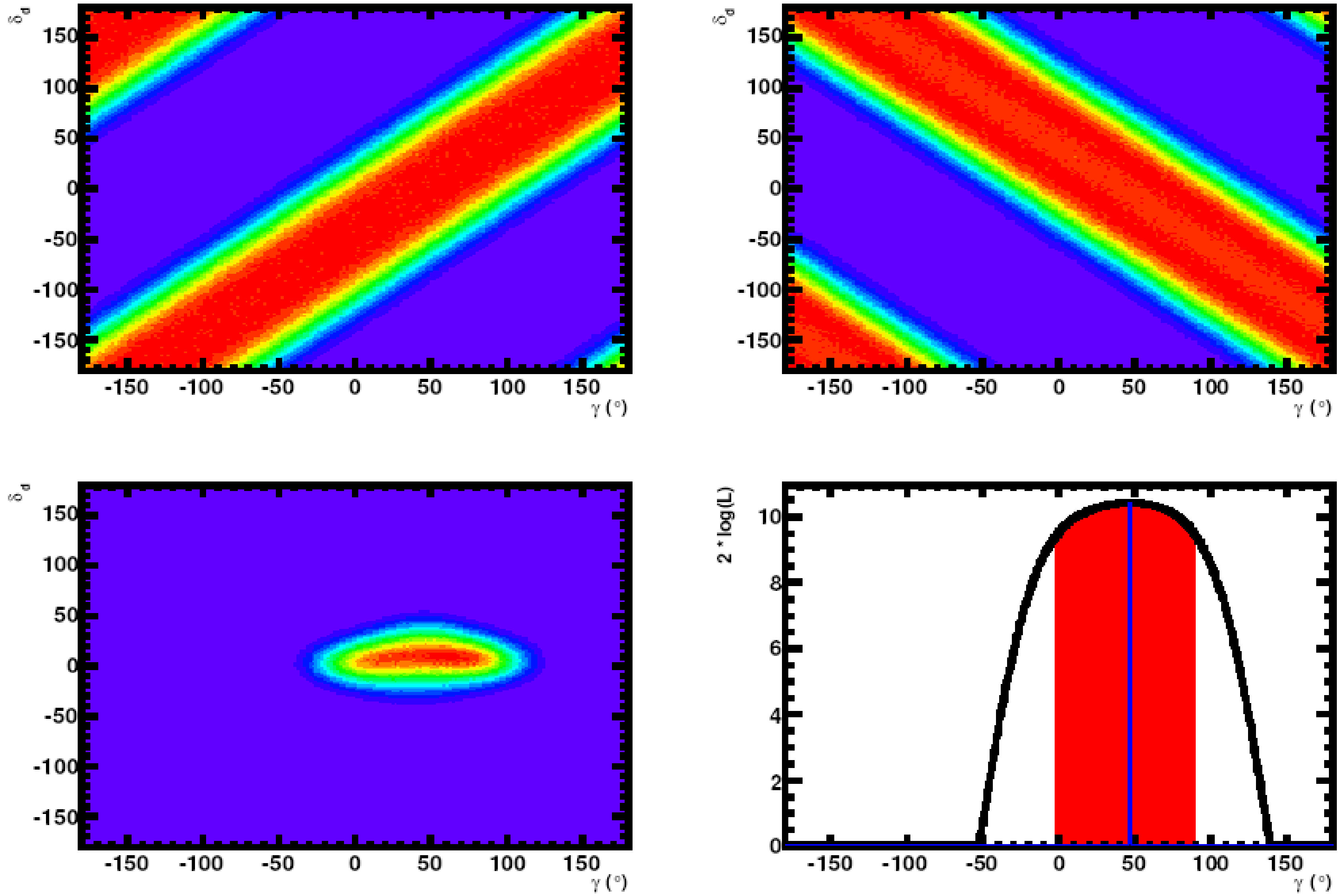
Ideally a list as produced by Max Baak for x_d:

- U-spin breaking effect **X** can be estimated at **Y%** from control channel(s) $\mathbf{Z_{1,2,3,...}}$

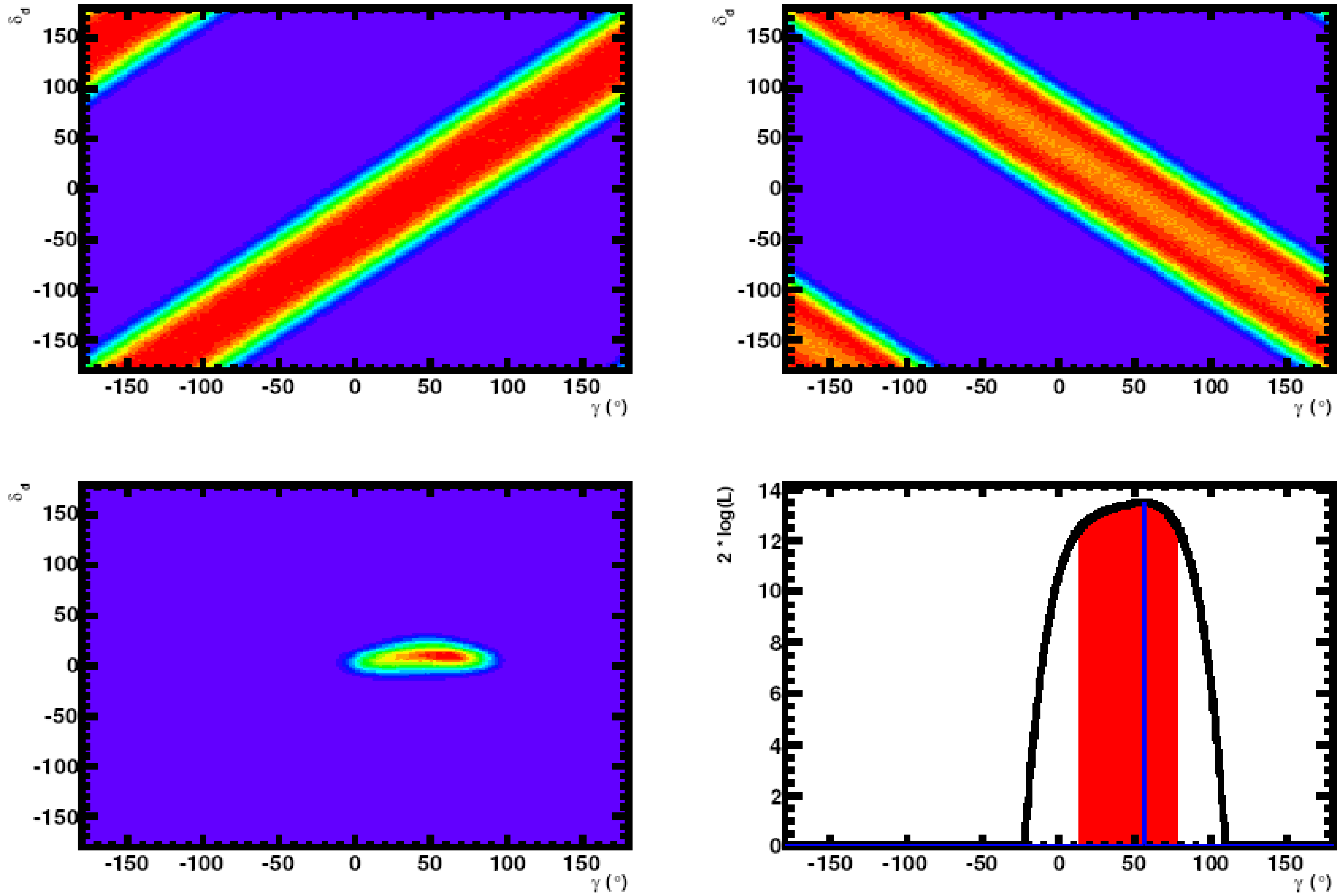
***Ref: Soni&Suprun, hep-ph/0609089**

BACKUP

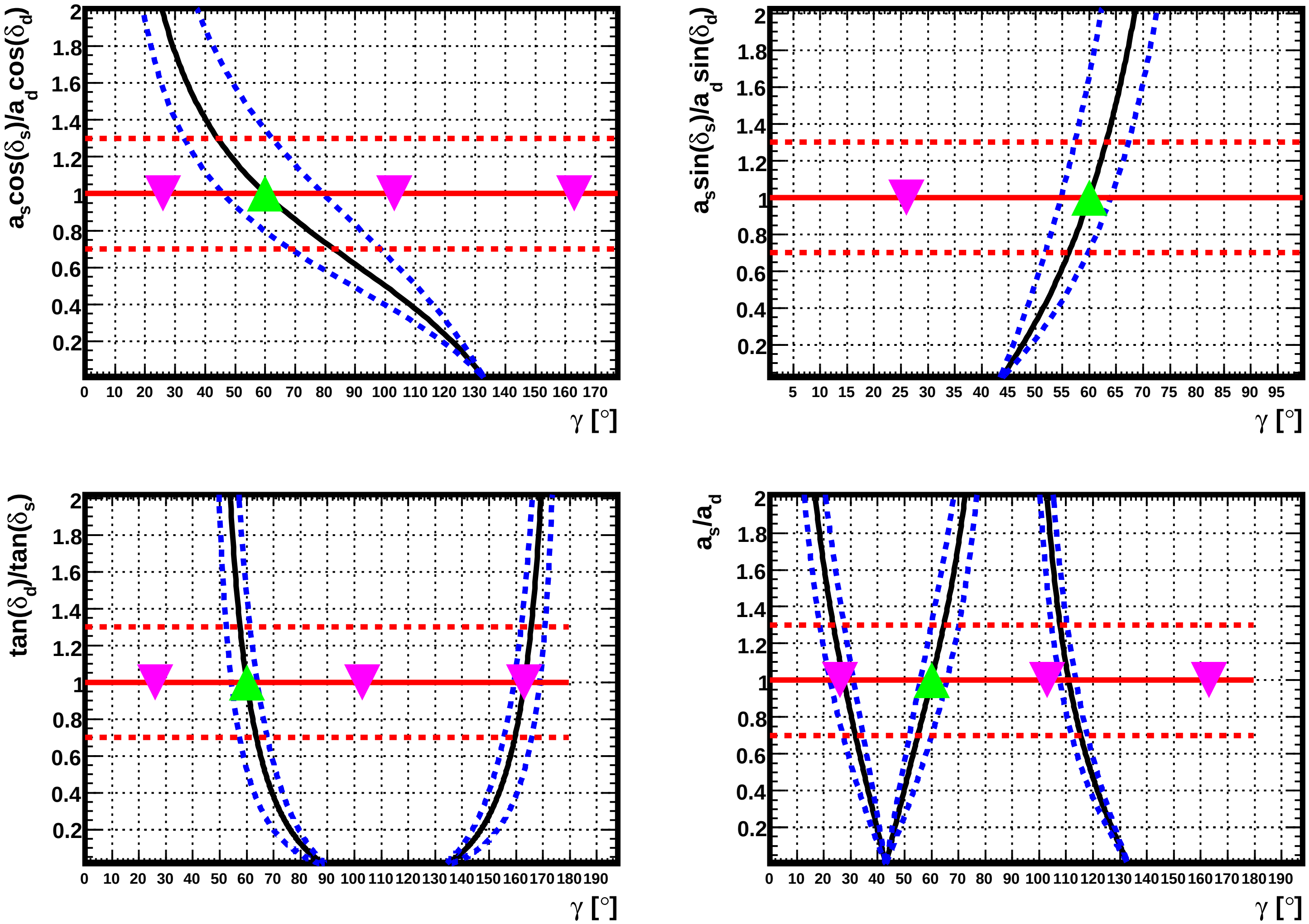
$B_D \rightarrow D\pi$: 1 year, factorization limit, U-spin



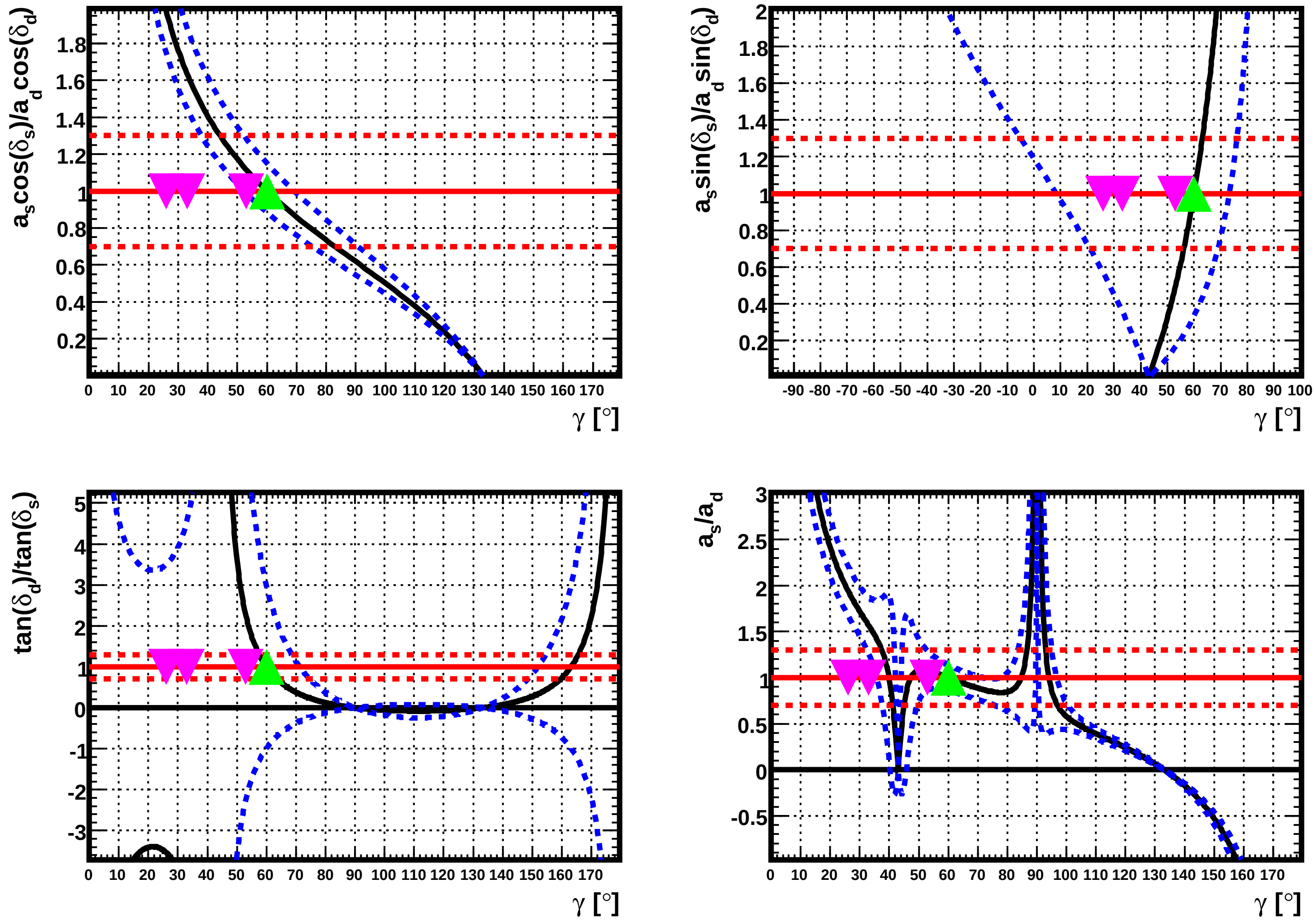
$B_D \rightarrow D\pi$: 5 YEARS, FACTORIZATION LIMIT, U-SPIN



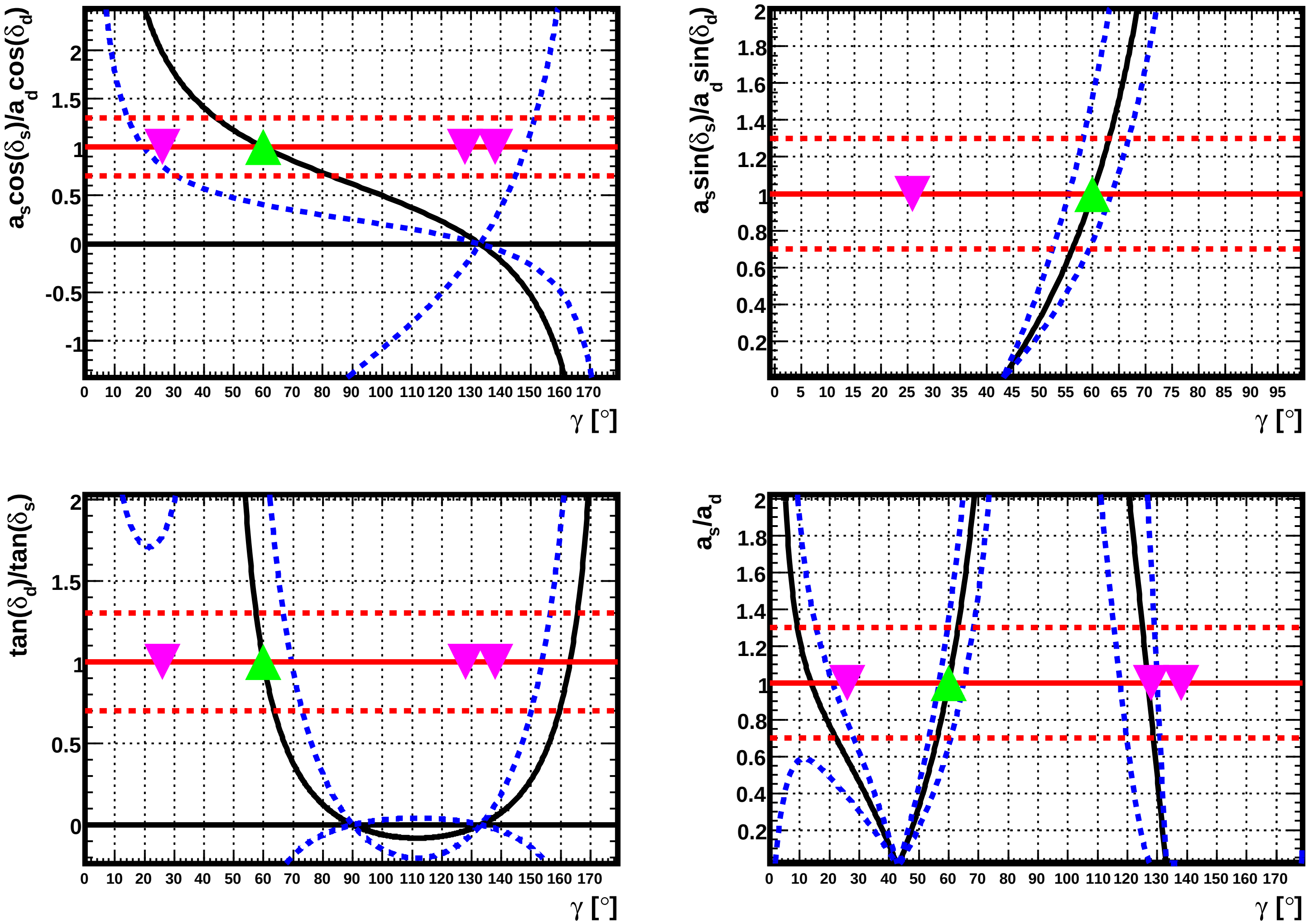
EXAMPLE RESULT: γ=60°, δ=60° (5 YEARS)



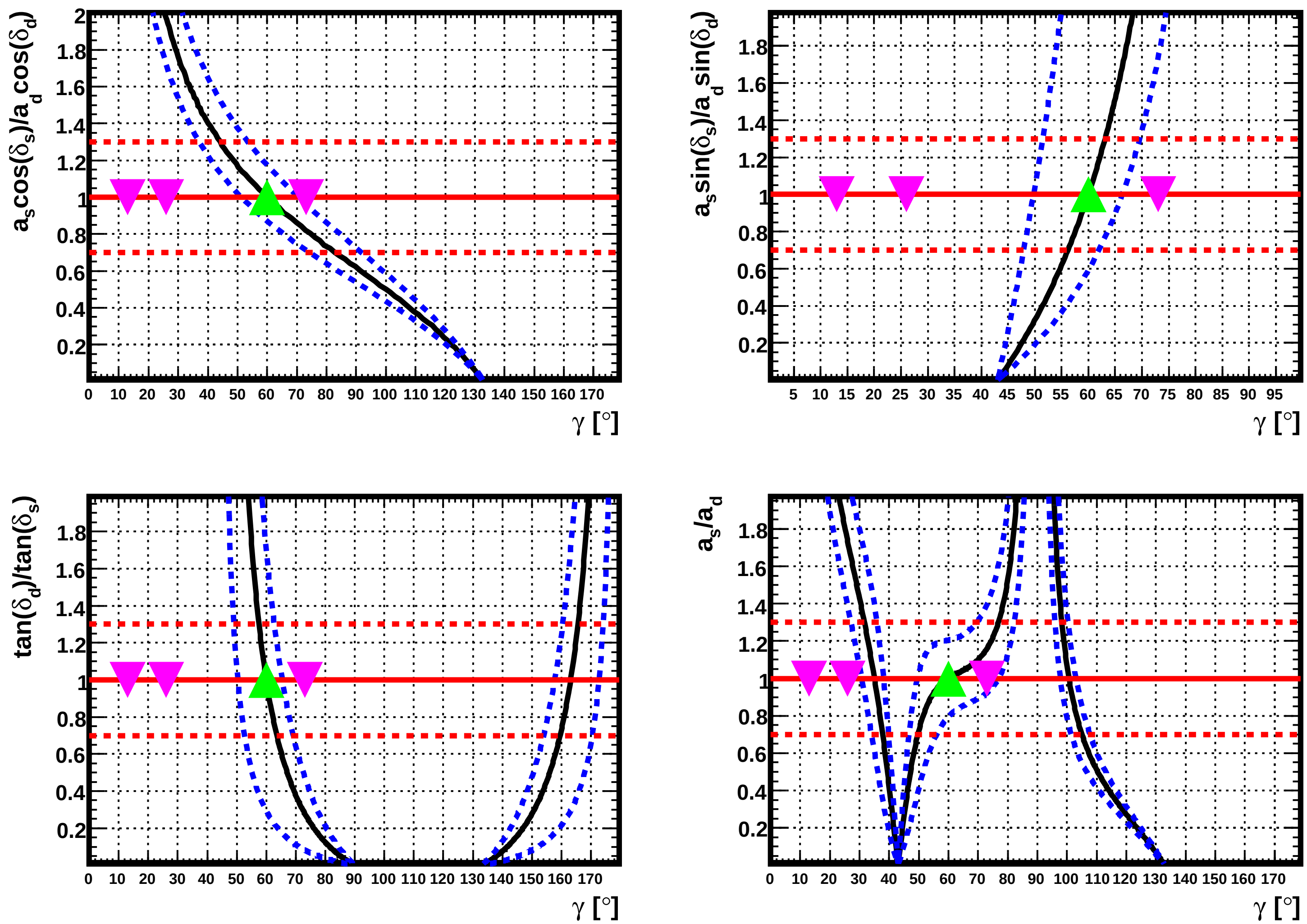
EXAMPLE RESULT: γ=60°, δ=10° (5 YEARS)



EXAMPLE RESULT: γ=60°, δ=85° (5 YEARS)



Example result: γ=60°, δ=30° (5 years)



W-exchange amplitudes

- SU(3) breaking error on r[D$^{(*)}$h] from missing exchange diagram:

$$\left|\frac{E}{T}\right| = \sqrt{\frac{BR\left(B^0 \to D_s^{(*)-}K^+\right)}{BR\left(B^0 \to D^{(*)-}\pi^+\right)}} \approx 10\%$$

- Ignores rescattering contribution to D$_s$K $\Rightarrow$ overestimation of E
- W-exchange amplitudes from rescattering fit consistent with naive factorization estimates!

$$D\pi \;:\; \left|\frac{E}{T}\right| < 0.029\ (0.058)\ @\ 68\%\ (95\%)$$

$$D^*\pi \;:\; \left|\frac{E}{T}\right| < 0.021\ (0.041)\ @\ 68\%\ (95\%)$$

$$D\rho \;:\; \left|\frac{E}{T}\right| < 0.033\ (0.066)\ @\ 68\%\ (95\%)$$

b $\to$ c transition

$$\left|\frac{E}{T}\right| = \frac{a_2}{a_1}\frac{f_B}{f_\pi}\left(\frac{m_D^2 - m_\pi^2}{m_B^2 - m_D^2}\right)\frac{F_0^{0\to D\pi}\left[m_B^2\right]}{F_0^{B\to D}\left[m_\pi^2\right]} \approx 0.7\%$$

- Large uncertainty on |E/T| estimate for b $\to$ u transition:

1. Factorization uncertainty for b $\to$ u
2. Value of Callan-Treiman prediction:

$$F_0^{0\to D\pi}\left[m_B^2\right] \approx \frac{m_D^2}{m_B^2}\frac{f_D}{f_\pi} = 0.21$$

b $\to$ u transition

$$\left|\frac{E}{T}\right| = \frac{a_2}{a_1}\frac{f_B}{f_D}\left(\frac{m_D^2 - m_\pi^2}{m_B^2 - m_\pi^2}\right)\frac{F_0^{0\to D\pi}\left[m_B^2\right]}{F_0^{B\to \pi}\left[m_D^2\right]} \approx 1.3\%$$

- Add 200% error on predicted ratio: $\rightarrow$ $\boxed{|E/T| < 5.0\%}$

Non-factorizable SU(3) breaking

- SU(3) breaking in amplitude ratio r from non-factorizable contributions:

$$
\begin{aligned}
|\Delta| &= \Delta_0 \left| \frac{1+\tilde{a}_d^{\rm corr}}{1+\tilde{a}_s^{\rm corr}} \right| \\
&= \Delta_0 \left| 1 + \left(\frac{\tilde{a}_d^{\rm corr} - \tilde{a}_s^{\rm corr}}{1+\tilde{a}_s^{\rm corr}} \right) \right| \\
&\approx \Delta_0 \left(1 + \frac{[\mathcal{R}e(\tilde{a}_d^{\rm corr}) + \frac{1}{2}|\tilde{a}_d^{\rm corr}|^2] - [\mathcal{R}e(\tilde{a}_s^{\rm corr}) + \frac{1}{2}|\tilde{a}_s^{\rm corr}|^2]}{|1+\tilde{a}_s^{\rm corr}|^2} \right) \\
&\equiv \Delta_0 \left(1 + \boxed{\bar{a}^c \frac{2m_s}{\Lambda_\chi}} \right)
\end{aligned}
$$

Non-factorizable amplitude $B^0 \to h^- D^{(*)+}$

Absorbs one unit of $2m_s/\Lambda_\chi$

< 0.17 @ 68.3% CL

- Additional SU(3) breaking proportional to non-factorizable contributions times perturbation parameter
- Assuming up to 3 times typical SU(3) breaking scale for $B^0 \to \pi^- D_{(s)}^{*+}$:

$$
\boxed{
\begin{aligned}
\left| \bar{a}^c \frac{2m_s}{\Lambda_\chi} \right| &< 2 \left(\frac{2m_s}{\Lambda_\chi} \right) \left| \frac{\mathcal{R}e(\tilde{a}_s^{\rm corr}) + \frac{1}{2}|\tilde{a}_s^{\rm corr}|^2}{|1+\tilde{a}_s^{\rm corr}|^2} \right| \\
&< 0.085\ (0.120)\ @\ 68.3\%\ (90\%)\ \text{C.L.}
\end{aligned}
}
$$

Non-factorizable b → u contributions

- Relative size of non-factorizable amplitude: $\tilde{a}_s^{corr}$

$$|1+\tilde{a}_s^{\rm corr}| \equiv \sqrt{\frac{\mathcal{B}_{\rm meas}(B^0\to\pi^-D_s^{*+})}{\mathcal{B}_{\rm resc}(B^0\to\pi^-D_s^{*+})}} \approx 1+\mathcal{R}e(\tilde{a}_s^{\rm corr})+\tfrac{1}{2}|\tilde{a}_s^{\rm corr}|^2$$
$$= 1.176\pm0.167\,({\rm exp.})\pm0.057\,(f_{D_s^*})\pm0.014\,({\rm rsc.})$$

$$\left|\frac{\mathcal{R}e(\tilde{a}_s^{\rm corr})+\frac{1}{2}|\tilde{a}_s^{\rm corr}|^2}{|1+\tilde{a}_s^{\rm corr}|^2}\right| \approx \left|\frac{1}{|1+\tilde{a}_s^{\rm corr}|}-\frac{1}{|1+\tilde{a}_s^{\rm corr}|^2}\right|$$
$$< 0.17\ (0.24)\ @\ 68.3\%\ (90\%)\ {\rm C.L.}$$

- Limit should improve with updates of: BR($B\to\pi l\nu$), BR($B^0\to\pi^- D_s^{*+}$), f_{Ds}.

- Two definitions to describe SU(3) breaking from non-factorizable corrections:

1.

$$2m_s/\Lambda_\chi \approx [0.16, 0.26]$$
$$f_{D_s}/f_D - 1 = 0.25\pm0.11 \approx 2m_s/\Lambda_\chi$$

typical SU(3) perturbation parameter

SU(3) breaking parameter in amplitude ratio $r^{D(*)h}$

(contains 1 unit of $2m_s/\Lambda_\chi$)

$$r^{D^{(*)}h} = (\tan\theta_c)\,\Delta_0\left|\frac{A(B^0\to D_s^{(*)+}h^-)}{A(B^0\to D^{(*)-}h^+)}\right|$$
$$\Delta_0 \equiv \frac{f_{D^{(*)}}F(m^2_{D^{(*)}})}{f_{D_s^{(*)}}F(m^2_{D_s^{(*)}})}R_i$$

2.