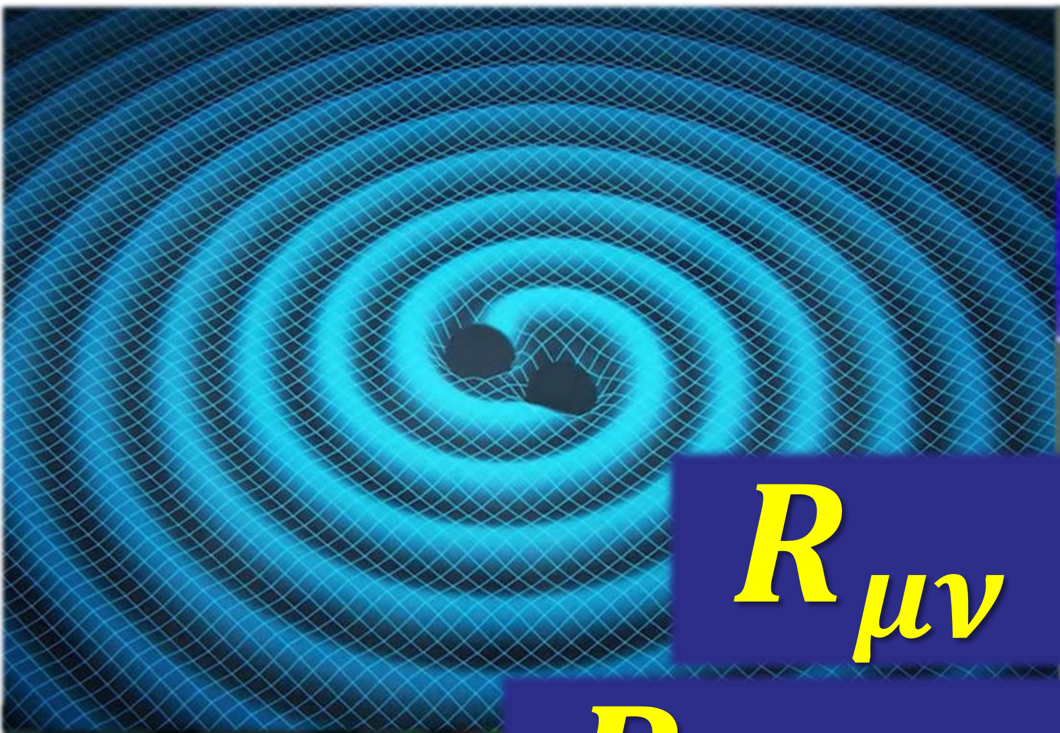
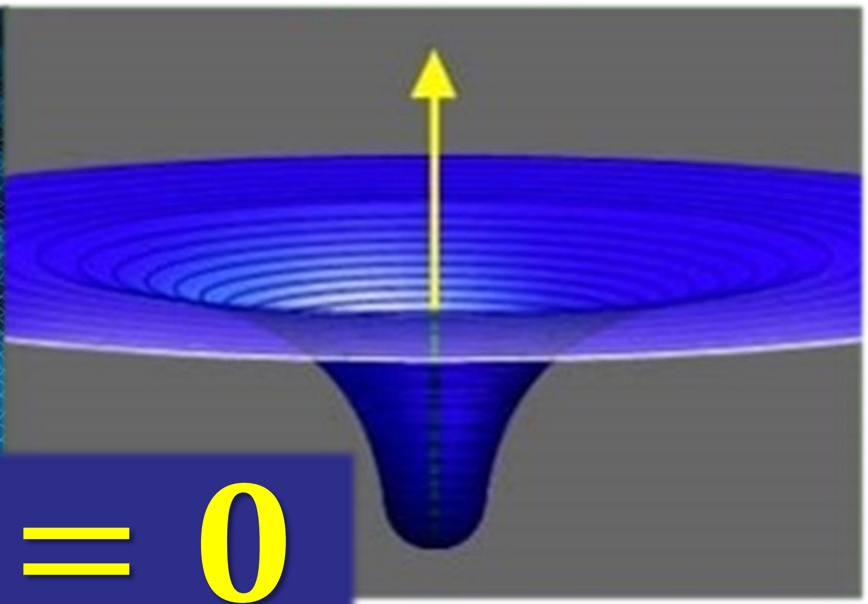
Black holes in the 1/D expansion

Roberto Emparan
ICREA & UBarcelona (& YITP Kyoto)

w/ Tetsuya Shiromizu, Ryotaku Suzuki,
Kentaro Tanabe, Takahiro Tanaka



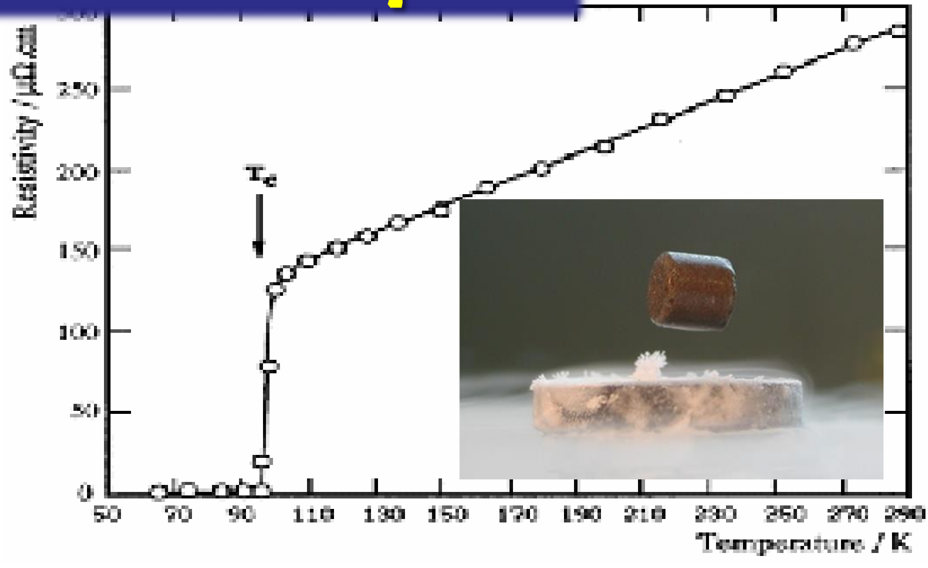


$$R_{\mu\nu} = 0$$

Nov 1915

$$R_{\mu\nu} = -\Lambda g_{\mu\nu}$$

Feb 1917



A dimensionless, adjustable parameter
is a good thing to have
for studying a theory

Quantum ElectroDynamics

Perturb around $e^2 = 0$

Quantum GluoDynamics

SU(3) Yang-Mills theory

No parameter?

Quantum GluoDynamics

SU(**N**) Yang-Mills theory

parameter!

What dimensionless parameter in

$R_{\mu\nu} = \Lambda\, g_{\mu\nu}$?

$$R_{\mu\nu} = \Lambda \, g_{\mu\nu}$$
$$\mu, \nu = 0, \ldots, \boldsymbol{D} - 1$$

YM

SU($N \to \infty$)

?

$\leftrightsquigarrow$

Quantum GR

SO ($D \to \infty$,1)

Quantum GR: SO(**D**-1,1) local Lorentz group

graviton polarizations grows with D

BUT:

No topological expansion of Feynman diagrams

Strominger 1981

Bjerrum-Bohr 2004

Even worse: UV behavior infinitely bad

YM
SU($N \to \infty$)

$\leftrightsquigarrow$

Quantum GR
SO ($D \to \infty$,1)

Classical General Relativity

D-diml Einstein's theory

Well-defined for all D

Many problems can be formulated keeping D arbitrary

→ D = continuous parameter

→ expand in 1/D

Kol et al

RE+Tanabe+Suzuki

Classical General Relativity

D-diml Einstein's theory

Large D:

Keeps essential physics of D=4

∃ black holes

∃ gravitational waves

Simplifies the theory

reformulation in terms of other variables?

The large D expansion can be useful

even if additional dimensions don't exist

It may (or may not) be accurate enough

in D=4 (or 10, 11...)

BH in *D* dimensions

$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right) dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

Localization of interactions

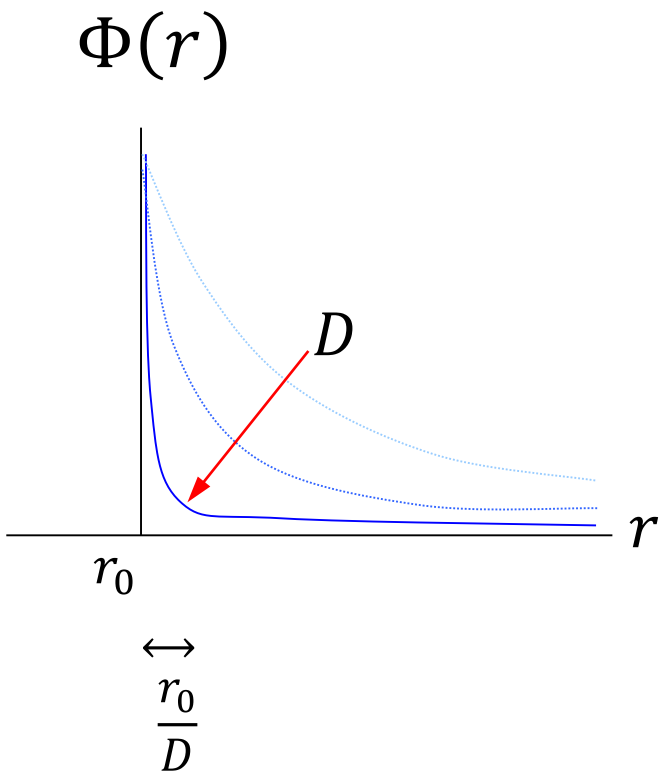
Large potential gradient:

$$\Phi(r) \sim \left(\frac{r_0}{r}\right)^{D-3}$$

$$\nabla\Phi\Big|_{r_0} \sim D/r_0$$

$\Longrightarrow$ Hierarchy of scales

$$\boxed{\frac{r_0}{D} \ll r_0}$$



Fixed $r > r_0$ $D \to \infty$

$$1 - \left(\frac{r_0}{r}\right)^{D-3} \to 1$$

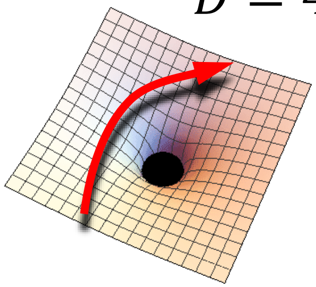
$$ds^2 \to -dt^2 + dr^2 + r^2 d\Omega_{D-2}$$

Flat, empty space at $r > r_0$

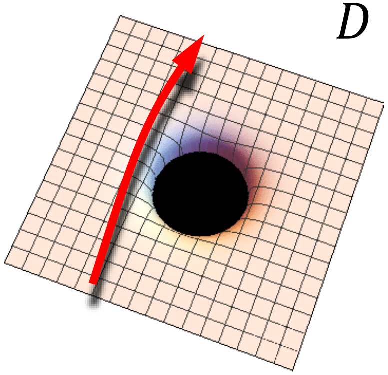
“**Far-zone**” limit

Black Hole scattering:
no deflection

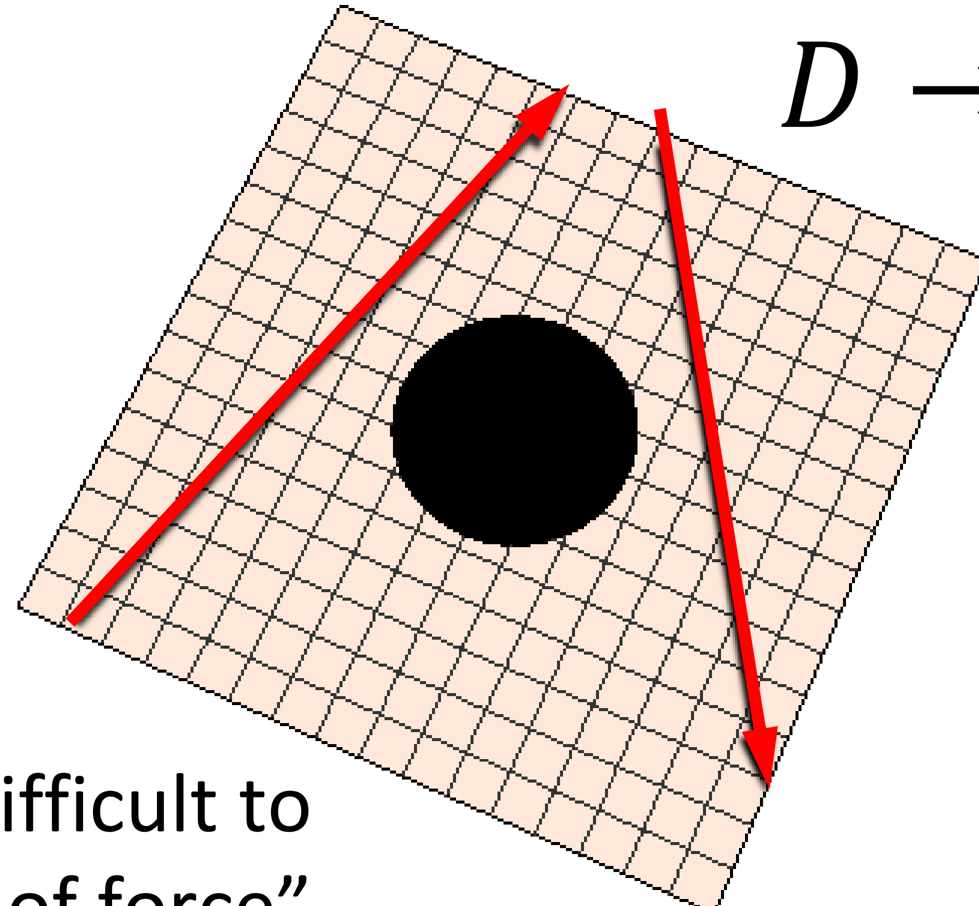
$D = 4$



$D \gg 4$

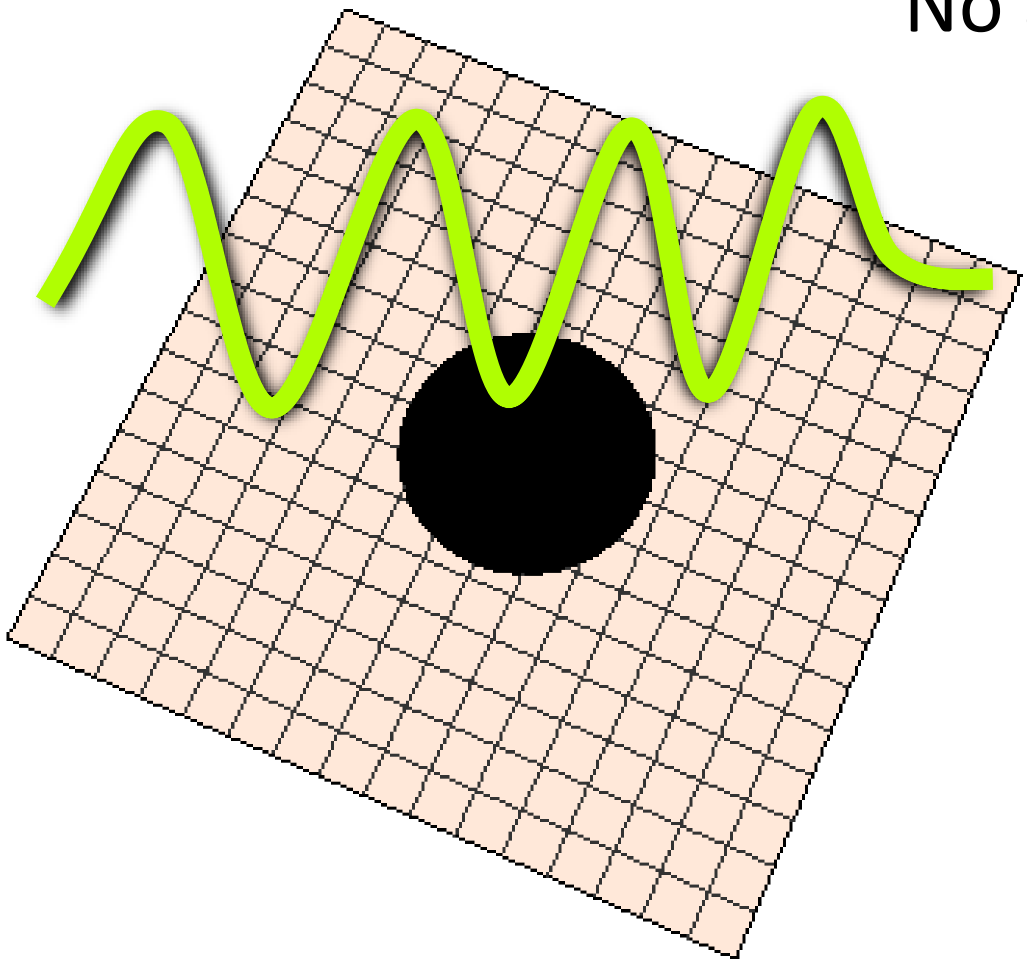


$D \to \infty$



"infinitely difficult to catch a line of force"

Black Hole scattering

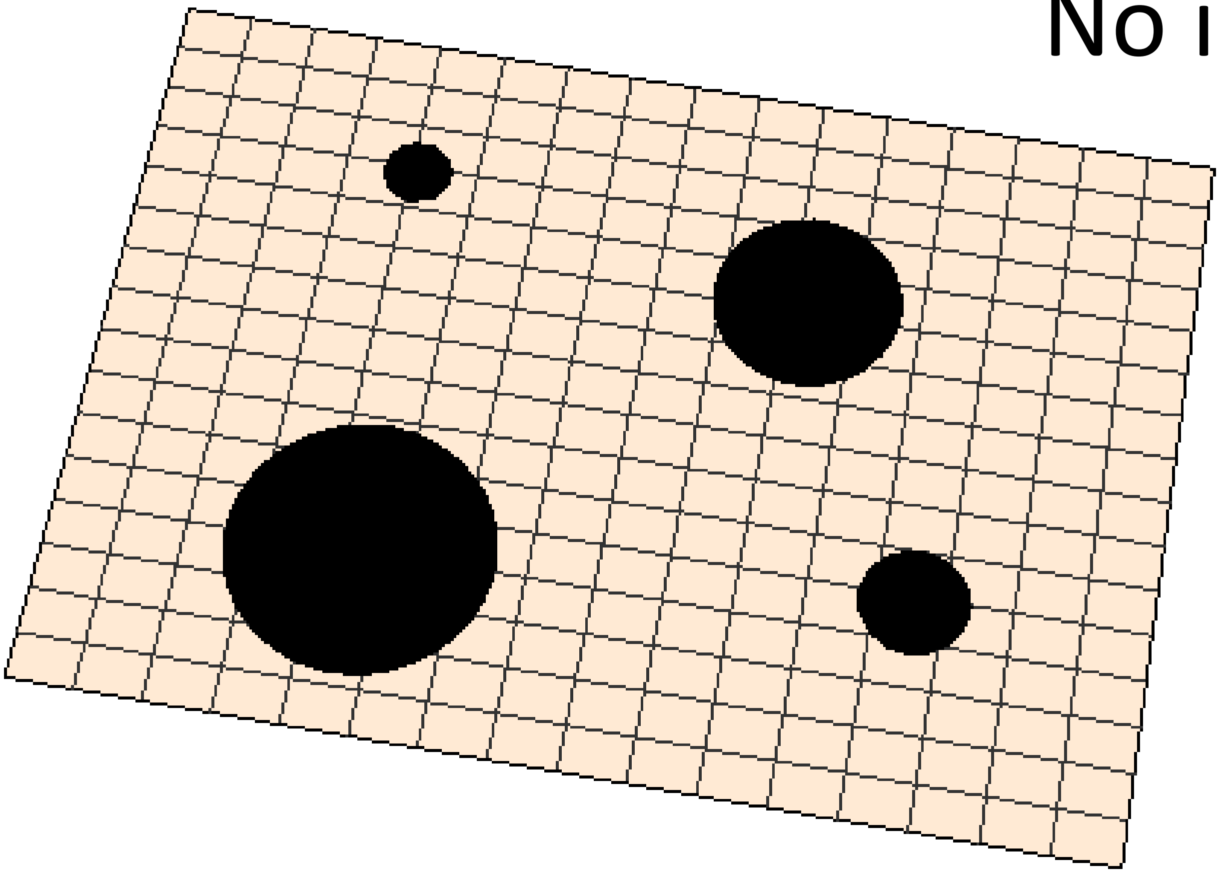


No absorption of waves

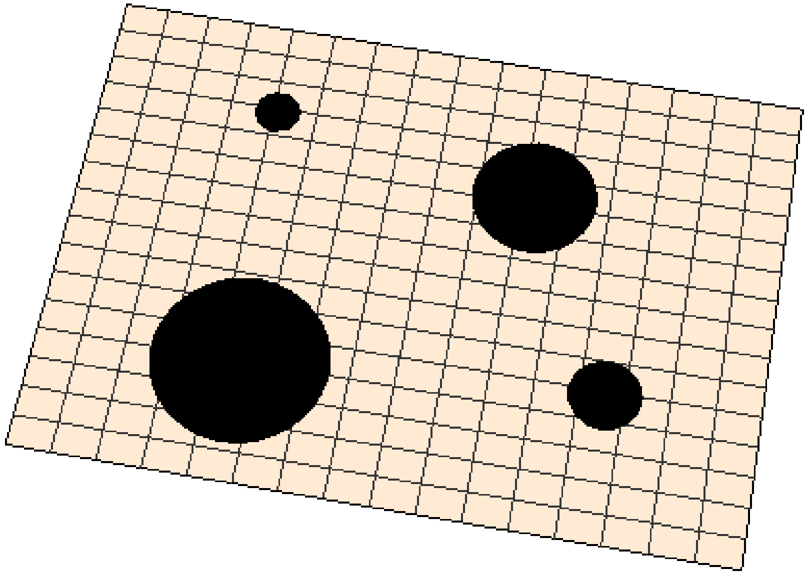
with wavelength

$\lambda \sim r_0$

No interaction



Holes cut out in Minkowski space



We are keeping length scales $\sim r_0$ finite as we send $D \to \infty$

“**Far-zone**” limit

Now take a limit that does *not trivialize* the gravitational field

$$\left(\frac{r_0}{r}\right)^{D-3} = \mathcal{O}(1) \iff r - r_0 < \frac{r_0}{D}$$

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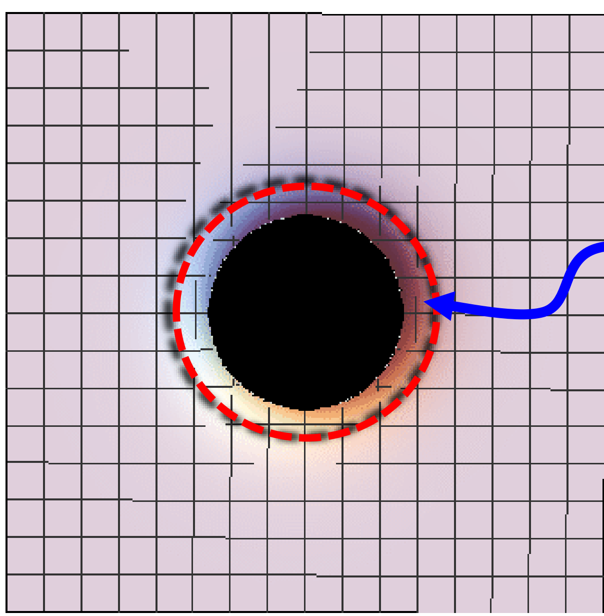


$$r - r_0 \sim \frac{r_0}{D}$$

"**Near-horizon**" limit

Near-horizon geometry

$$ds^2 = -\left(1-\left(\frac{r_0}{r}\right)^{D-3}\right)dt^2 + \frac{dr^2}{1-\left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

$$\left.\begin{aligned} \left(\frac{r}{r_0}\right)^{D-3} &= \cosh^2\rho \\ t_{near} &= \frac{D}{2r_0}t \end{aligned}\right\} \text{finite as } D \to \infty$$

Near-horizon geometry

2d string bh

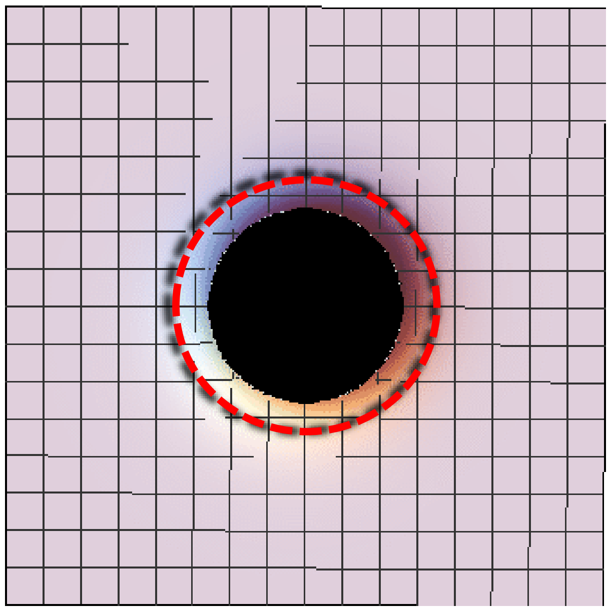
$$ds^2_{nh} \to \frac{4r_0^2}{D^2}\left(-\tanh^2\rho\ dt^2_{near} + d\rho^2\right)$$

$$+\, r_0^2\,(\cosh\rho)^{4/D}\, d\Omega^2_{D-2}$$

Soda 1993
Grumiller et al 2002
RE+Grumiller+Tanabe 2013

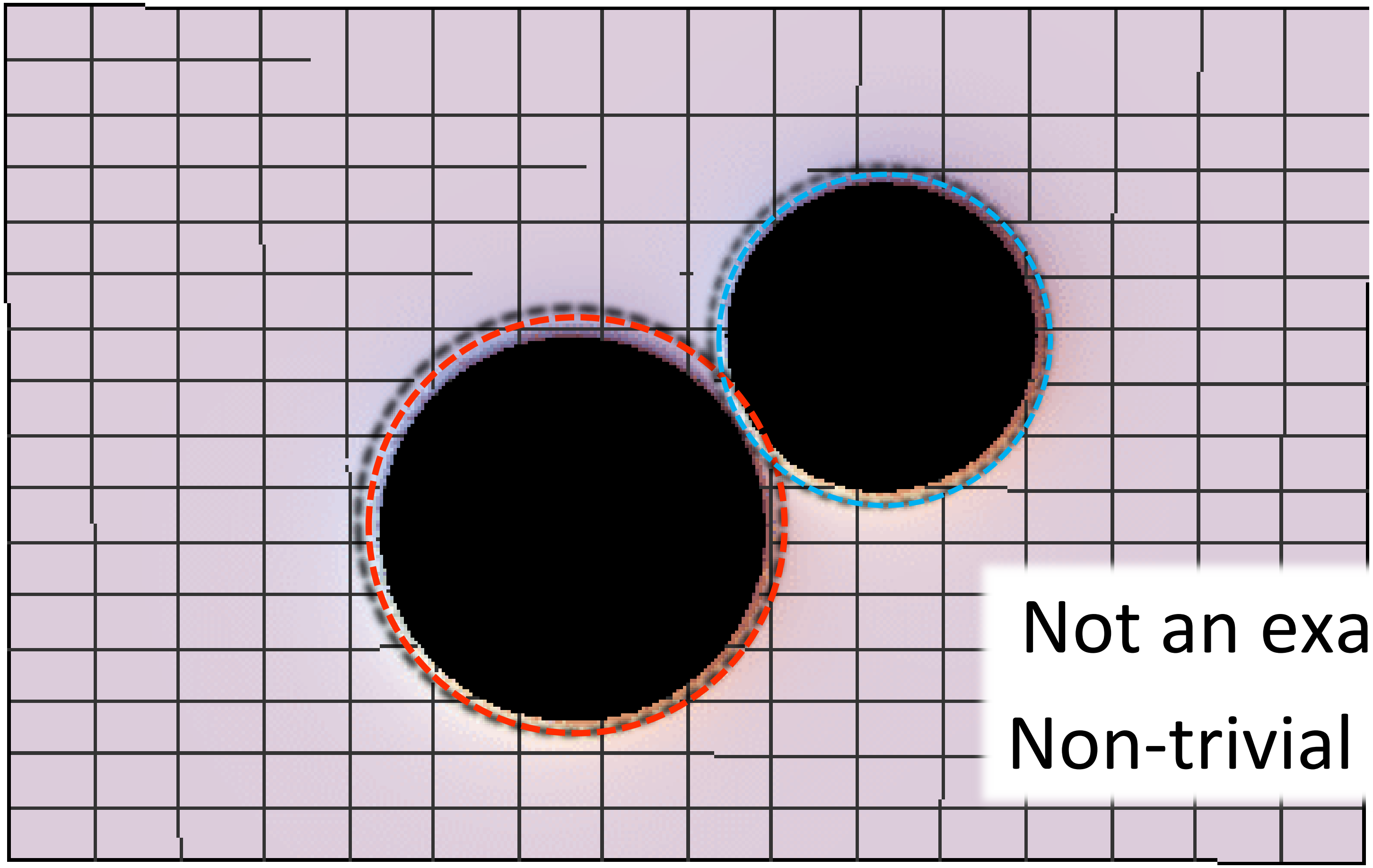
'string length' $\ell_s \sim \frac{r_0}{D}$

Physics at $\sim r_0/D$ close to the horizon is *not* trivial



Perfect absorption
of waves with
$\lambda \sim r_0/D$
$\omega \sim D/r_0$

"**Near-horizon**" dynamics



"**Near-horizon**" dynamics

Near-horizon universality

2d string bh = near-horizon geometry of **all neutral non-extremal bhs**

rotation = local boost

(along horizon)

cosmo const = 2d bh mass-shift

Physics by scales

When D is large, we have different physics at different length scales

Long-length physics: $\ell \gg \frac{r_0}{D}$

Short-length physics: $\ell \ll r_0$

Why Black Hole dynamics is difficult – *at fixed D*

In a typical situation, all scales are characterized by r_0

BH field not distinct from background

Why Large D

Large D allows us to introduce a generic small parameter

It gives us a new kind of **effective theory** for black holes

Large D Effective Theory

Solve near-horizon equations

integrate-out short-distance dynamics

→ Boundary conds for far-zone fields

Long-distance effective theory

Black hole perturbations ✓ *all analytic*

Scattering

Quasinormal modes

Ultraspinning instability

Holographic superconductors

Full non-linear GR ✓

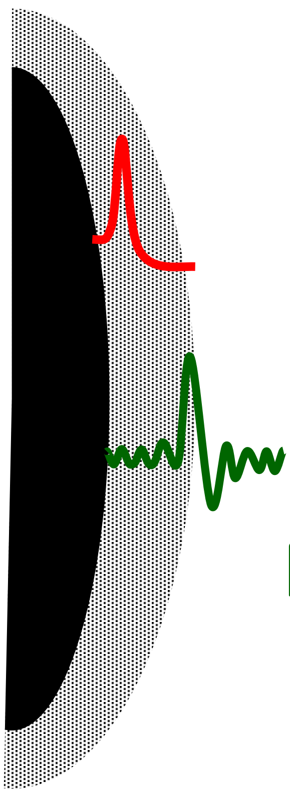
General theory of static black holes: ***Soap-film*** theory

Black droplets

simple ODE

Non-uniform black strings

BH excitations (quasinormal modes)



Decoupled normalizable states

very few modes: $\mathcal{O}(D^0)$

slow modes $\omega \sim D^0/r_0$

non-universal: hydro & instabilities

Non-decoupled non-normalizable states

most modes: $\mathcal{O}(D^2)$

fast modes $\omega \sim D/r_0$

universal: hole in space

BH perturbations: How accurate?

Small expansion parameter: $\frac{1}{D-3}$

not quite good for $D = 4$ ...

BH perturbations: How accurate?

Small expansion parameter: $\frac{1}{D-3}$

not quite good for $D = 4$ ...

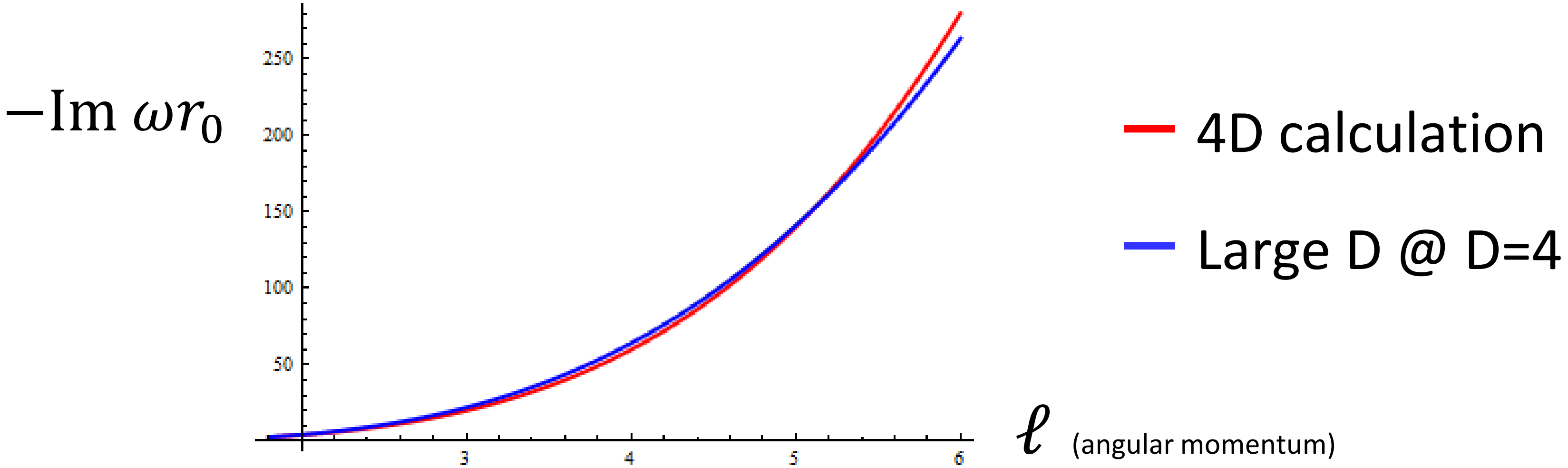
But it seems to be $\frac{1}{\mathbf{2}(D-3)}$

not *so* bad in $D = 4$, if we can compute higher orders

(in AdS: $\frac{1}{2(D-1)}$)

Quite accurate

Quasinormal frequency in $D = 4$ (vector-type)

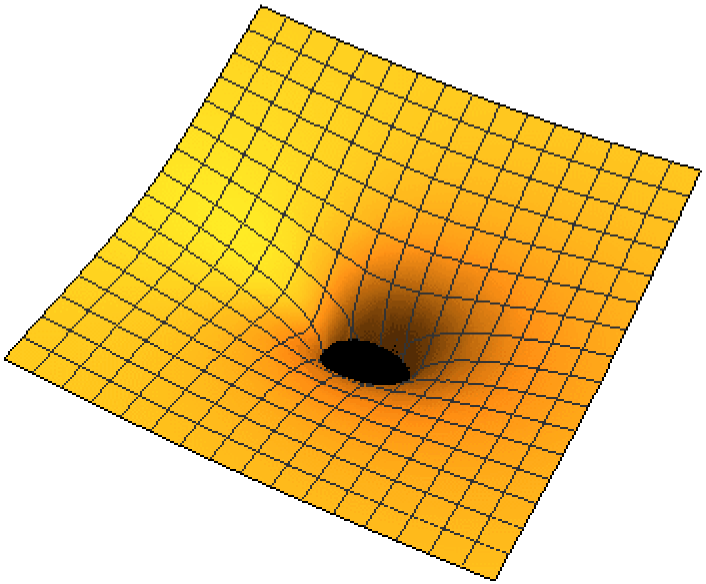


Calculation up to $\frac{1}{D^3}$ yields 6% accuracy in $D = 4$

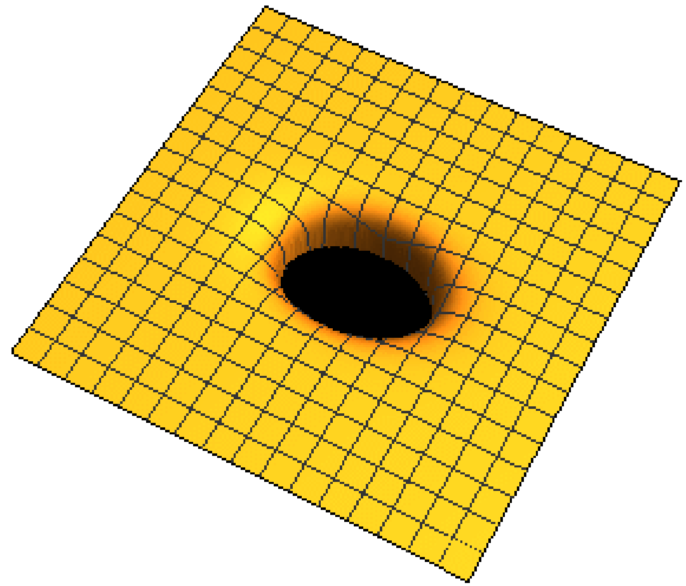
$$6\% = \left.\frac{1}{\left(2(D-3)\right)^4}\right|_{D=4}$$

Fully non-linear GR @ large D

Large-D ⇒ neat separation bh/background



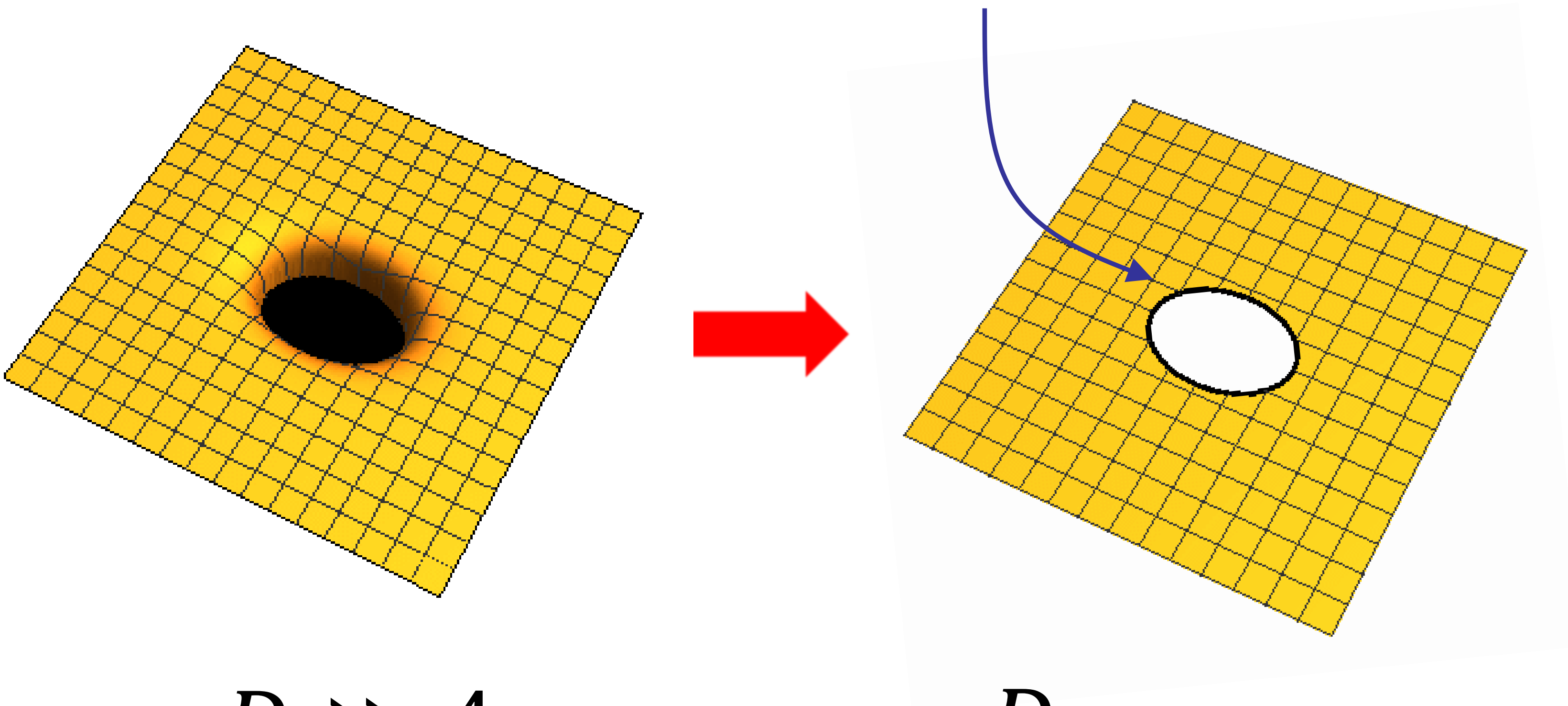
$D = 4$



$D \gg 4$

Replace bh ⟶ Surface in background

What eqs determine this surface?

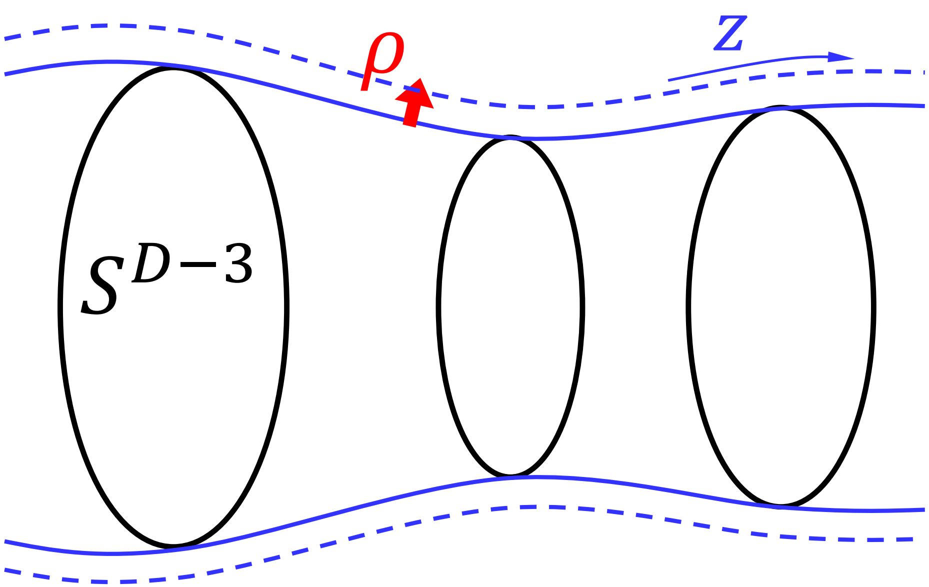


$D \gg 4$ $D \to \infty$

Gradient hierarchy

⊥ Horizon: $\partial_\rho \sim D$

∥ Horizon: $\partial_z \sim 1$

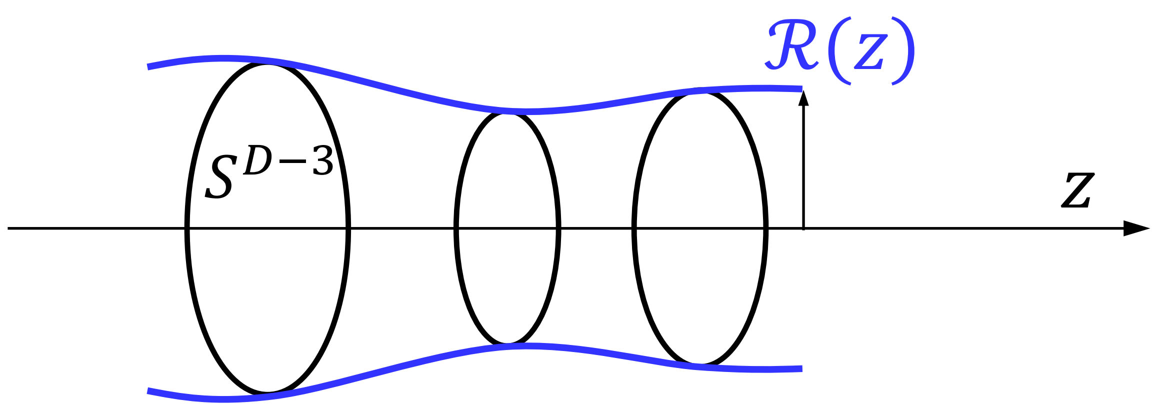


Einstein ‘momentum-constraint’ in ρ:

$$\sqrt{-g_{tt}}K = \text{const}$$

K = mean curvature of ‘horizon surface’

$$ds^2\Big|_h = g_{tt}(z)dt^2 + dz^2 + \mathcal{R}^2(z)d\Omega_{D-3}$$



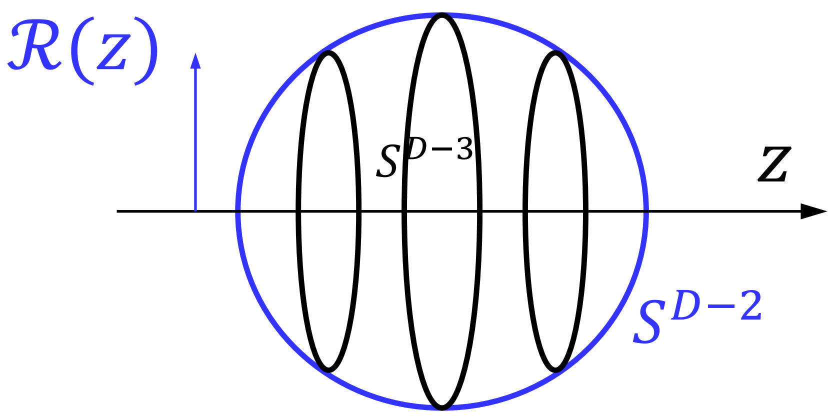
Soap-film equation (redshifted)

$$\sqrt{-g_{tt}}K = \text{const}$$

Some applications

Soap bubble in Minkowski = Schw BH

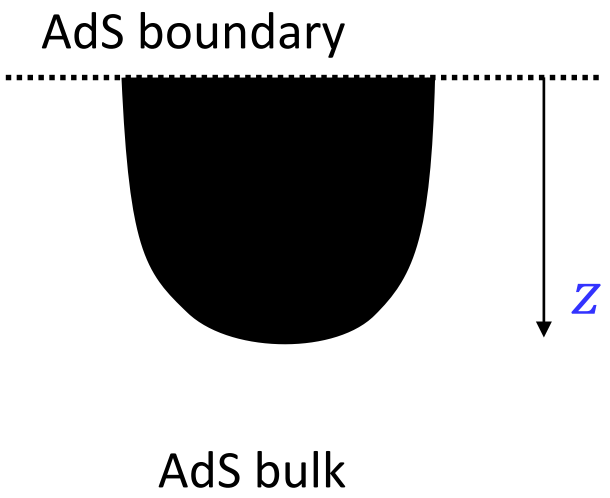
$$\sqrt{-g_{tt}}K = \text{const} \;\Rightarrow\; \mathcal{R}'^2 + \mathcal{R}^2 = 1$$



$$\Rightarrow \mathcal{R}(z) = \sin z$$

Black droplets

Black hole at boundary of AdS



dual to CFT in BH background

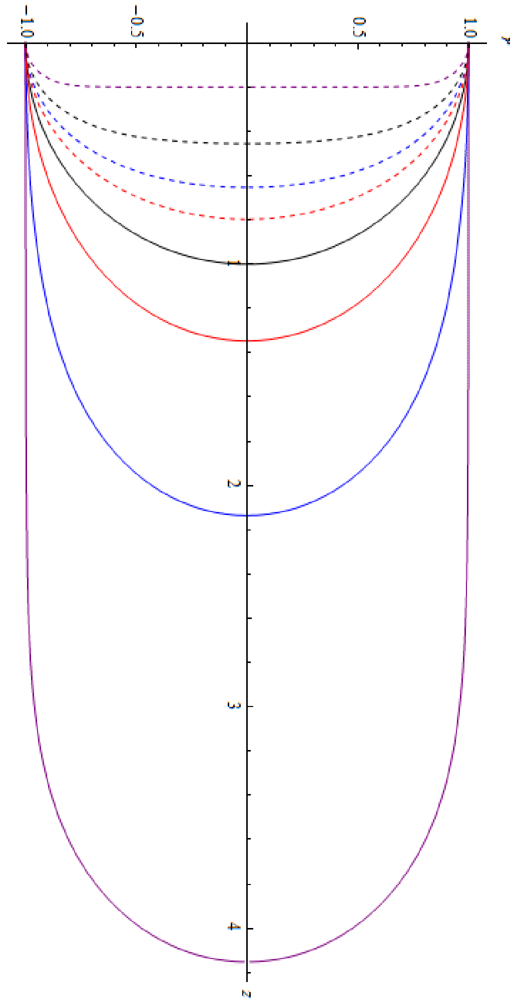
Numerical solution:

Figueras+Lucietti+Wiseman

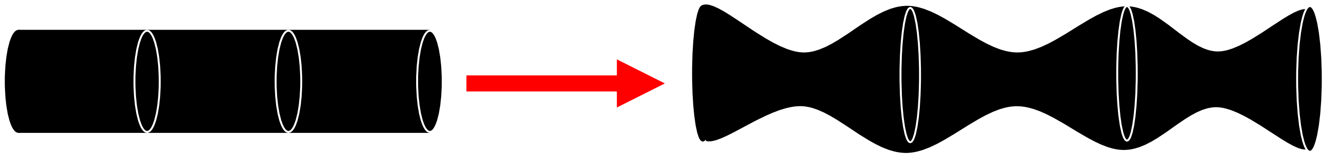
Numerical code

```
zmin = 0.000001;
zmax = 0.67;
r0 = .5;
NDSolve[{r'[z] == -z/r[z] (1 - Sqrt[r[z]^2 + z^2 (1 - r[z]^2)])/(1 - z^2), r[zmin] == r0}, r, {z, zmin, zmax}]
```

Black droplets



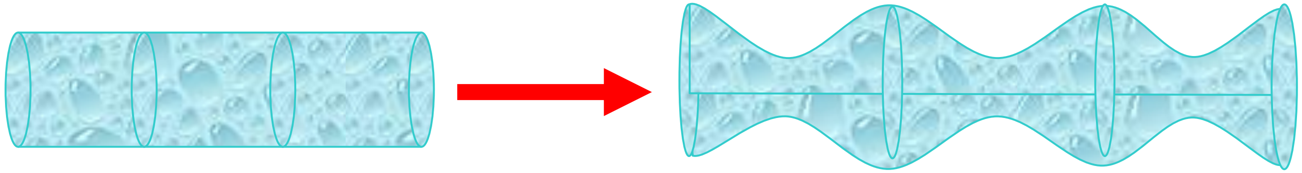
Non-uniform black strings



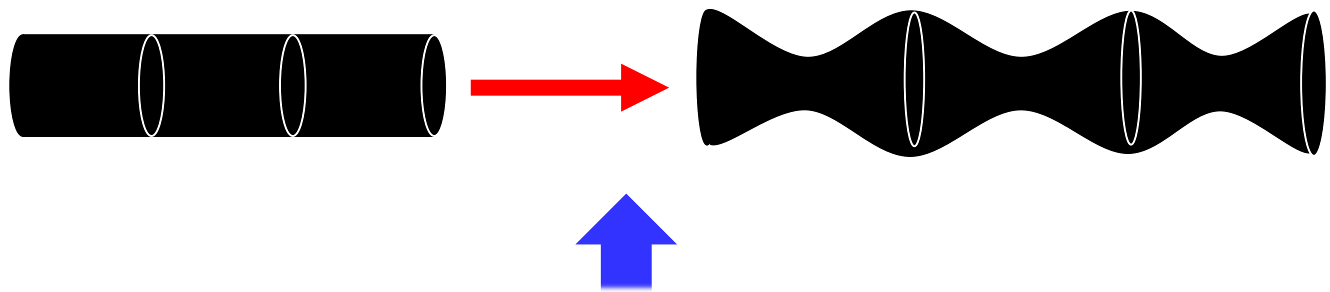
Gregory+Laflamme 1993

"Analogous" to fluid tubes (Rayleigh-Plateau)

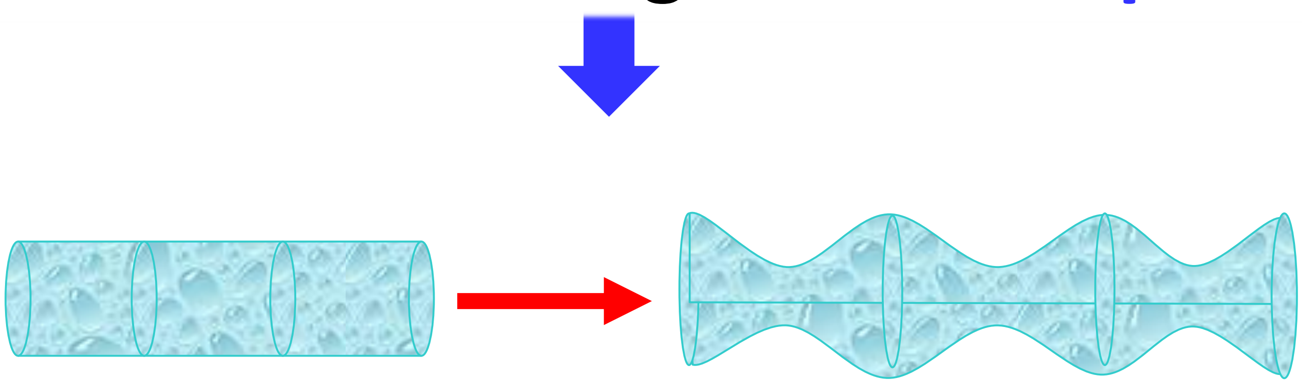
Cardoso+Dias 2006



Non-uniform black strings



$D \to \infty$: not "analogous" but **equivalent**



In progress

Extensions of $\sqrt{-g_{tt}}\,K = \text{const}$

Charged black holes

Rotating black holes

(Time-evolving black holes)

Conclusions

1/D: it works

(not obvious beforehand!)

Large N:

effective reformulation of YM

with strings as

basic (extended) objects

Can we reformulate GR

with black holes as

basic (extended) objects?

The large D limit may give us precisely this

Quantum effects?

Dimensionful scale:

$$L_{Planck} = (G\hbar)^{\frac{1}{D-2}}$$

Quantum effects governed by $\frac{r_0}{L_{Planck}}$

If $\frac{r_0}{L_{Planck}} \sim D^0$ the bh is fully quantum:

Entropy $\rightarrow 0$

Temperature $\rightarrow \infty$

Evaporation lifetime $\rightarrow 0$

But other scalings are possible

Scaling $\frac{r_0}{L_{Planck}}$ with D:

how large are the black holes,

which quantum effects are finite at large D

Finite entropy: $r_0/L_{Planck} \sim D^{1/2}$

Finite temperature: $r_0/L_{Planck} \sim D$

Finite energy of Hawking radn: $r_0/L_{Planck} \sim D^2$