

---

---

# Exceptional field theory

Olaf Hohm, H.S.

[1307.0509], [1308.1673], [1312.0614], [1312.4542], [1406.3348]

O. Hohm, H. Godazgar, M. Godazgar, H. Nicolai, H.S. [1406.3235]

CERN Theory Division 10/2014



Henning Samtleben



# motivation

---

## exceptional symmetries in supergravity

show up in dimensional reduction of D=11 supergravity and beyond ..?

D=11 supergravity

[Cremmer, Julia, Scherk]

$T^7$      $GL(7) \subset E_{7(7)}$

- general scheme: toroidal compactification on  $T^d$  yields maximal supergravity with **global  $E_{d(d)}$**
- only part of it has a geometrical D=11 interpretation
- glimpses of the **exceptional symmetries  $E_{d(d)}$**  appear beyond toroidal compactification ...

D=4 supergravity

[Cremmer, Julia]

maximal supersymmetry

global  $E_{7(7)}$

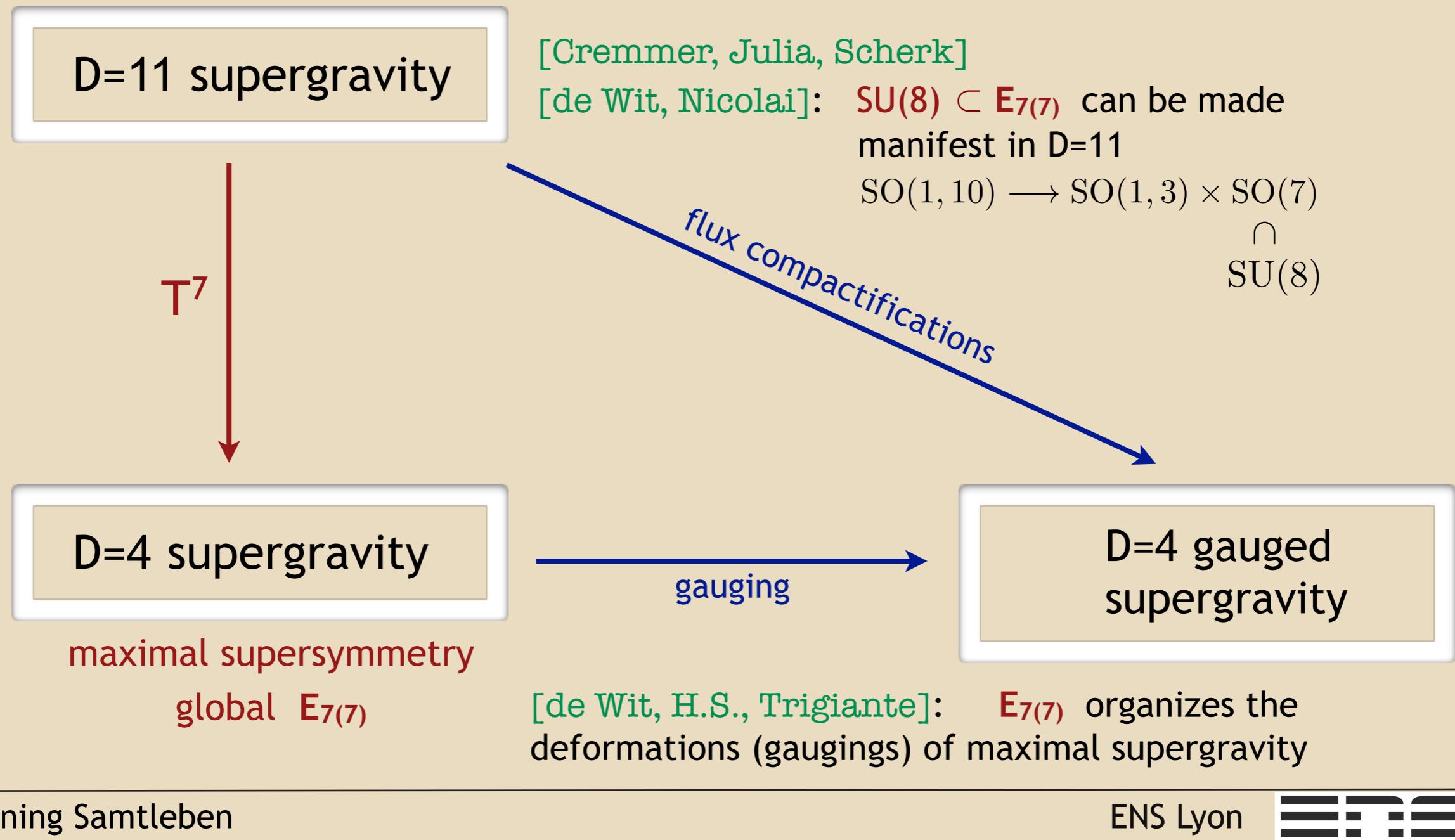
organises the couplings, Ward identities, ...

# motivation

---

## exceptional symmetries in supergravity

show up in dimensional reduction of D=11 supergravity and beyond ..?



# motivation

---

## ■ to which extent are these symmetries present beyond D=4 ?

[de Wit, Nicolai] [Koepsell, Nicolai, H.S.] [West]

[Hull, Tseytlin, Duff, Siegel, Hillmann, Hohm, Zwiebach, Waldram, Pacheco, Coimbra, Strickland-Constable, Berman, Godazgar, Godazgar, Perry, West, Musaev, Kwak, Jeon, Lee, Park, Suh, Blair, Malek, Cederwall, Kleinschmidt, Thompson, Edlund, Karlsson, Aldazabal, Grana, Marques, Rosabal, Baron, Geissbühler, ... , ...]

## ■ exceptional field theory

double field theory,  
generalized geometry,  
exceptional geometry,  
gauged supergravity

## example : $E_{7(7)}$ formulation of D=11 supergravity

- ▶ D=4 :  $E_{7(7)}$  in dimensional reduction
- ▶ beyond :  $E_{7(7)}$  exceptional field theory (EXFT)
- ▶ supersymmetry : fermions and supersymmetric EXFT
- ▶ applications : generalized Scherk-Schwarz

---

---

# D = 4 : E<sub>7(7)</sub> in dimensional reduction

# $D = 4 : E_{7(7)}$ in dimensional reduction

## D=11 supergravity (bosonic sector)

$g_{\mathcal{M}\mathcal{N}}, A_{\mathcal{K}\mathcal{L}\mathcal{M}\mathcal{N}}$  metric and three-form

[Cremmer,Julia,Scherk]

$$\int d^{11}X \left( \sqrt{g}R - \frac{1}{12}\sqrt{g}F^{\mathcal{K}\mathcal{L}\mathcal{M}\mathcal{N}}F_{\mathcal{K}\mathcal{L}\mathcal{M}\mathcal{N}} + \frac{1}{2 \cdot 36^2} \epsilon^{\mathcal{K}\mathcal{L}\mathcal{M}\mathcal{N}\mathcal{P}\mathcal{Q}\mathcal{R}\mathcal{S}\mathcal{T}\mathcal{U}\mathcal{V}} F_{\mathcal{K}\mathcal{L}\mathcal{M}\mathcal{N}}F_{\mathcal{P}\mathcal{Q}\mathcal{R}\mathcal{S}}A_{\mathcal{T}\mathcal{U}\mathcal{V}} \right)$$

## dimensional reduction on $T^7$

$X^{\mathcal{M}} \rightarrow \{x^\mu, y^m\}$  split of coordinates

impose  $\partial_{\textcolor{brown}{m}} \Phi = 0$  for all fields, i.e.  $\Phi(x, \textcolor{brown}{y}) \rightarrow \Phi(x)$

the result is a four-dimensional supergravity with field content

$\{g_{\mu\nu}, A_\mu{}^{\textcolor{brown}{m}}, \phi^{mn}\}$

$\underbrace{\hspace{15em}}$

gravity coupled

GL(7)/SO(7) sigma model

$\{A_{\mu\nu\rho}, A_{\mu\nu\textcolor{brown}{m}}, A_{\mu\textcolor{brown}{m}\textcolor{brown}{n}}, A_{m\textcolor{brown}{n}\textcolor{brown}{k}}\}$

$\underbrace{\hspace{30em}}$

further coupled to 0-, 1-, 2-, 3-forms

# $D = 4 : E_{7(7)}$ in dimensional reduction

## The D=4 theory

scalars:  $\phi^{mn}$   $A_{m nk}$

1-forms:  $A_\mu{}^m$   $A_{\mu mn}$

2-forms:  $A_{\mu\nu m}$

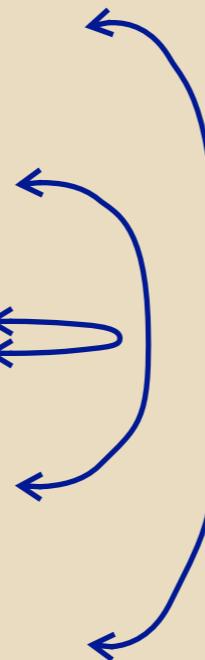
3-forms:  $A_{\mu\nu\rho}$

[28+35]

[7+21]

[7]

[1]



on-shell p-form dualities

# D = 4 : E<sub>7(7)</sub> in dimensional reduction

---

## The D=4 theory

scalars:  $\phi^{\textcolor{red}{mn}}$   $A_{\textcolor{red}{m}n k}$   $\tilde{A}^{\textcolor{red}{m}}$  [28+35+7]  $\mathcal{M}_{MN}$

1-forms:  $A_\mu{}^{\textcolor{red}{m}}$   $A_{\mu \textcolor{red}{m} n}$   $A_\mu{}^{mn}$   $A_{\mu m}$  [7+21+21+7]  $A_\mu{}^M$

---

scalar sector E<sub>7(7)</sub>/SU(8) sigma model

$$\mathcal{L} = \frac{1}{24} \partial_\mu \mathcal{M}_{MN} \partial^\mu \mathcal{M}^{MN} \quad \text{GL}(7) \subset E_{7(7)} \subset Sp(56)$$
$$\mathcal{M} = \mathcal{V} \mathcal{V}^T, \quad \mathcal{V} \in E_{7(7)}$$

---

vector sector vectors combine in the fundamental 56 representation:  $A_\mu{}^M$

$$\mathcal{L}_{\text{vec}} = -\frac{1}{4} \mathcal{M}_{MN} \mathcal{F}_{\mu\nu}{}^M \mathcal{F}^{\mu\nu}{}^N$$

and twisted self-duality equation

$$\mathcal{F}_{\mu\nu}{}^M + \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \Omega^{MN} \mathcal{M}_{NK} \mathcal{F}^{\rho\sigma}{}^K = 0$$

---

separately gauge and E<sub>7(7)</sub> invariant, supersymmetry connects the different terms

# D = 4 : E<sub>7(7)</sub> in dimensional reduction – and beyond ..?

## The D=4 theory

$$\mathcal{L} = \frac{1}{24} \partial_\mu \mathcal{M}_{MN} \partial^\mu \mathcal{M}^{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}_{\mu\nu}^{\phantom{\mu\nu}M} \mathcal{F}^{\mu\nu N}$$

and       $\mathcal{F}_{\mu\nu}^{\phantom{\mu\nu}M} + \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \Omega^{MN} \mathcal{M}_{NK} \mathcal{F}^{\rho\sigma K} = 0$

- E<sub>7(7)</sub> appears in the D=4 theory (after dualization), what happens beyond ..?
- keeping all 11 coordinates: the fields appear ‘gauged’ under internal 7d diffeomorphisms with the KK vector

$$D_\mu \phi = \partial_\mu - A_\mu^{\phantom{\mu}\textcolor{red}{m}} \partial_{\textcolor{red}{m}} \phi$$

lift to E<sub>7(7)</sub> :       $A_\mu^{\phantom{\mu}\textcolor{red}{m}} \rightarrow A_\mu^{\phantom{\mu}M}$     56 vector fields       $\partial_{\textcolor{red}{m}} \rightarrow \partial_M$     56 coordinates

■ section condition       $(t_\alpha)^{MN} \partial_M \otimes \partial_N = 0$        $(t_\alpha)^{MN} \partial_M \partial_N f = 0$

[Berman, Perry, Coimbra, Strickland-Constable, Waldram,  
Cederwall, Kleinschmidt, Thompson]       $(t_\alpha)^{MN} \partial_M f \partial_N g = 0$

- covariant restriction to 7 coordinates
- analogue to the strong constraint of double field theory      [Hohm, Hull, Zwiebach]
- inequivalent solution! to 6 coordinates (IIB)



---

---

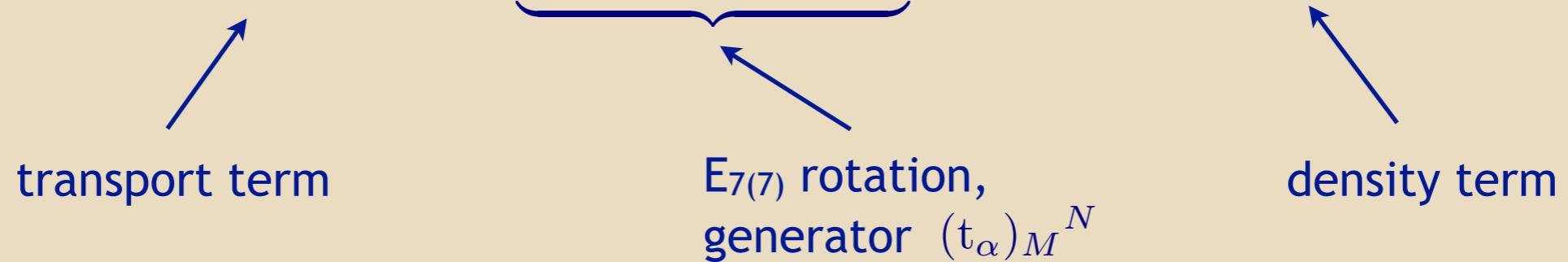
# $E_{7(7)}$ exceptional field theory

## E-bracket and tensor hierarchy

# $E_{7(7)}$ : exceptional field theory

- generalized diffeomorphisms : [Coimbra, Strickland-Constable, Waldram]

$$\mathcal{L}_\Lambda V^M = \Lambda^N \partial_N V^M - \kappa (\partial_L \Lambda^K) (t^\alpha)_K^L (t_\alpha)_N^M V^N - \lambda (\partial_K \Lambda^K) V^M$$



- closure of the algebra : E-bracket

$$[\Lambda_1, \Lambda_2]_E^M = 2\Lambda_{[1}^K \partial_K \Lambda_{2]}^M + 12 (t_\alpha)^{MN} (t^\alpha)_{KL} \Lambda_{[1}^K \partial_N \Lambda_{2]}^L - \frac{1}{4} \Omega^{MN} \Omega_{KL} \partial_N (\Lambda_1^K \Lambda_2^L)$$

(modulo section constraint. not associative ! Jacobiator ...)

- should become a ‘local’ ( $x^\mu$ - dependent) symmetry

$$\text{covariant derivatives } \mathcal{D}_\mu = \partial_\mu - \mathcal{L}_{A_\mu}$$

non-associativity leads to non-covariant field strengths

# $E_{7(7)}$ : exceptional field theory – tensor hierarchy

■ should become a ‘local’ ( $x^\mu$ - dependent) symmetry

$$\text{covariant derivatives } \mathcal{D}_\mu = \partial_\mu - \mathcal{L}_{A_\mu}$$

non-associativity leads to non-covariant field strengths

■ covariant field strength : coupling to two-forms  $\longrightarrow$  tensor hierarchy

$$\mathcal{F}_{\mu\nu}^M \equiv 2\partial_{[\mu} A_{\nu]}^M - 2[A_\mu, A_\nu]_E^M - 12(t^\alpha)^{MN} \partial_N B_{\mu\nu\alpha} - \frac{1}{2}\Omega^{MN} B_{\mu\nu N}$$

with 133 two-forms  $B_{\mu\nu\alpha}$  dual to the scalars

and a constrained compensator field  $(t_\alpha)^{MN} B_M B_N = 0$

covariant under gauge transformations

$$\delta_\Lambda A_\mu^M = \mathcal{D}_\mu \Lambda^M + 12(t^\alpha)^{MN} \partial_N \Xi_{\mu\alpha} + \frac{1}{2}\Omega^{MN} \Xi_{\mu N}$$

$$\delta_\Lambda B_{\mu\nu\alpha} = 2\mathcal{D}_{[\mu} \Xi_{\nu]\alpha} + (t_\alpha)_{KL} \Lambda^K \mathcal{F}_{\mu\nu}^L - (t_\alpha)_{KL} A_{[\mu}^K \delta A_{\nu]}^L$$

---

---

# $E_{7(7)}$ exceptional field theory

## The dynamics

# $E_{7(7)}$ : exceptional field theory – the action

■ the full action  $S = \int d^4x d^{56}Y \sqrt{g} \mathcal{L}$  [O. Hohm, H.S.]

$$\begin{aligned} \mathcal{L} = & \widehat{R} + \frac{1}{24} \mathcal{D}_\mu \mathcal{M}_{MN} \mathcal{D}^\mu \mathcal{M}^{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}_{\mu\nu}^{\phantom{\mu\nu}M} \mathcal{F}^{\mu\nu N} \\ & + \mathcal{L}_{\text{top}} - V_{\text{pot}}(\mathcal{M}_{MN}, g_{\mu\nu}) \end{aligned}$$

# $E_{7(7)}$ : exceptional field theory – the action

the full action  $S = \int d^4x d^{56}Y \sqrt{g} \mathcal{L}$  [O. Hohm, H.S.]

$$\mathcal{L} = \widehat{\bar{R}} + \frac{1}{24} \mathcal{D}_\mu \mathcal{M}_{MN} \mathcal{D}^\mu \mathcal{M}^{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}_{\mu\nu}{}^M \mathcal{F}^{\mu\nu N}$$

$$+ \mathcal{L}_{\text{top}} - V_{\text{pot}}(\mathcal{M}_{MN}, g_{\mu\nu})$$

covariantized Einstein-Hilbert term

$$\mathcal{D}_\mu e_\nu{}^a \equiv \partial_\mu e_\nu{}^a - A_\mu{}^M \partial_M e_\nu{}^a - \frac{1}{2} \partial_M A_\mu{}^M e_\nu{}^a$$

the vierbein carries weight  $\lambda = \frac{1}{2}$

with the ‘improved’ Riemann tensor  $\widehat{\bar{R}}_{\mu\nu}{}^{ab} = R_{\mu\nu}{}^{ab} + \mathcal{F}_{\mu\nu}{}^M e^{a\rho} \partial_M e_\rho{}^b$

weights:

$\mathcal{M}_{MN}$	$e_\mu{}^a$	$A_\mu{}^M$	$B_{\mu\nu\alpha}$	$\frac{1}{2} \times$ Weyl weight
0	$\frac{1}{2}$	$\frac{1}{2}$	1	

# $E_{7(7)}$ : exceptional field theory – the action

the full action  $S = \int d^4x d^{56}Y \sqrt{g} \mathcal{L}$  [O. Hohm, H.S.]

$$\mathcal{L} = \widehat{R} + \frac{1}{24} \mathcal{D}_\mu \mathcal{M}_{MN} \mathcal{D}^\mu \mathcal{M}^{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}_{\mu\nu}^M \mathcal{F}^{\mu\nu N}$$
$$+ \mathcal{L}_{\text{top}} - V_{\text{pot}}(\mathcal{M}_{MN}, g_{\mu\nu})$$

covariantized sigma model  $\mathcal{D}_\mu = \partial_\mu - \mathcal{L}_{A_\mu}$

the matrices  $\mathcal{M}_{MN}$  transform with weight  $\lambda = 0$

consistent with  $\det \mathcal{M} = 1$  and the total weight of the integrand

# $E_{7(7)}$ : exceptional field theory – the action

the full action  $S = \int d^4x d^{56}Y \sqrt{g} \mathcal{L}$  [O. Hohm, H.S.]

$$\mathcal{L} = \widehat{R} + \frac{1}{24} \mathcal{D}_\mu \mathcal{M}_{MN} \mathcal{D}^\mu \mathcal{M}^{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}_{\mu\nu}^M \mathcal{F}^{\mu\nu N} + \mathcal{L}_{\text{top}} - V_{\text{pot}}(\mathcal{M}_{MN}, g_{\mu\nu})$$

non-abelian YM-term

with covariant field strength

$$\mathcal{F}_{\mu\nu}^M \equiv 2\partial_{[\mu} A_{\nu]}^M - 2[A_\mu, A_\nu]_E^M - 12(t^\alpha)^{MN} \partial_N B_{\mu\nu\alpha} - \frac{1}{2} \Omega^{MN} B_{\mu\nu N}$$

two-form field equations compensated by the topological term

and twisted self-duality equation

$$\mathcal{F}_{\mu\nu}^M + \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \Omega^{MN} \mathcal{M}_{NK} \mathcal{F}^{\rho\sigma K} = 0$$

# $E_{7(7)}$ : exceptional field theory – the action

the full action  $S = \int d^4x d^{56}Y \sqrt{g} \mathcal{L}$  [O. Hohm, H.S.]

$$\mathcal{L} = \widehat{R} + \frac{1}{24} \mathcal{D}_\mu \mathcal{M}_{MN} \mathcal{D}^\mu \mathcal{M}^{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}_{\mu\nu}{}^M \mathcal{F}^{\mu\nu}{}^N$$

$$+ \mathcal{L}_{\text{top}} - V_{\text{pot}}(\mathcal{M}_{MN}, g_{\mu\nu})$$

topological term

$$S_{\text{top}} \equiv -\frac{1}{24} \int_{\Sigma_5} d^5x \int d^{56}Y \varepsilon^{\mu\nu\rho\sigma\tau} \mathcal{F}_{\mu\nu}{}^M \mathcal{D}_\rho \mathcal{F}_{\sigma\tau M}$$

boundary term of five-dimensional bulk

separately  $\Lambda^M$  gauge invariant (weight of  $\mathcal{F}^M$  is  $\lambda = \frac{1}{2}$ )

duality equations  $-\frac{1}{2} (t_\alpha)_K{}^L (e \mathcal{D}^\mu \mathcal{M}^{KP} \mathcal{M}_{LP}) = \varepsilon^{\mu\nu\rho\sigma} \mathcal{H}_{\nu\rho\sigma\alpha}$

# $E_{7(7)}$ : exceptional field theory – the action

the full action  $S = \int d^4x d^{56}Y \sqrt{g} \mathcal{L}$  [O. Hohm, H.S.]

$$\mathcal{L} = \widehat{R} + \frac{1}{24} \mathcal{D}_\mu \mathcal{M}_{MN} \mathcal{D}^\mu \mathcal{M}^{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}_{\mu\nu}^M \mathcal{F}^{\mu\nu N}$$

$$+ \mathcal{L}_{\text{top}} - V_{\text{pot}}(\mathcal{M}_{MN}, g_{\mu\nu})$$

“potential”

$$\begin{aligned} V = & -\frac{1}{48} \mathcal{M}^{MN} \partial_M \mathcal{M}^{KL} \partial_N \mathcal{M}_{KL} + \frac{1}{2} \mathcal{M}^{MN} \partial_M \mathcal{M}^{KL} \partial_L \mathcal{M}_{NK} \\ & - \frac{1}{2} g^{-1} \partial_M g \partial_N \mathcal{M}^{MN} - \frac{1}{4} \mathcal{M}^{MN} g^{-1} \partial_M g g^{-1} \partial_N g - \frac{1}{4} \mathcal{M}^{MN} \partial_M g^{\mu\nu} \partial_N g_{\mu\nu}. \end{aligned}$$

invariance under  $\Lambda$  transformations is not manifest

transforms with weight  $\lambda = -1$ , great (!) :  $\delta(\sqrt{g} V_{\text{pot}}) = \partial_M (\Lambda^M \sqrt{g} V_{\text{pot}})$

# $E_{7(7)}$ : exceptional field theory – the action

- the full action  $S = \int d^4x d^{56}Y \sqrt{g} \mathcal{L}$  [O. Hohm, H.S.]

$$\begin{aligned}\mathcal{L} = & \widehat{R} + \frac{1}{24} \mathcal{D}_\mu \mathcal{M}_{MN} \mathcal{D}^\mu \mathcal{M}^{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}_{\mu\nu}{}^M \mathcal{F}^{\mu\nu N} \\ & + \mathcal{L}_{\text{top}} - V_{\text{pot}}(\mathcal{M}_{MN}, g_{\mu\nu})\end{aligned}$$

- all 5 terms are separately invariant under “generalized diffeomorphisms”  $\Lambda^M(x, Y)$
  - also manifestly invariant under “D=4 diffeomorphisms”  $\xi^\mu(x)$
  - last missing symmetry : “skew-diffeomorphisms”  $\xi^\mu(x, Y)$
- can be realized on the action ! fixes all relative coefficients !  
“plays the role of supersymmetry ...”

# $E_{7(7)}$ : exceptional field theory – the action

- the full action  $S = \int d^4x d^{56}Y \sqrt{g} \mathcal{L}$  [O. Hohm, H.S.]

$$\begin{aligned}\mathcal{L} = & \widehat{R} + \frac{1}{24} \mathcal{D}_\mu \mathcal{M}_{MN} \mathcal{D}^\mu \mathcal{M}^{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}_{\mu\nu}{}^M \mathcal{F}^{\mu\nu}{}^N \\ & + \mathcal{L}_{\text{top}} - V_{\text{pot}}(\mathcal{M}_{MN}, g_{\mu\nu})\end{aligned}$$

- unique action with generalized diffeomorphism invariance in all 4+56 coordinates (modulo section condition)

- upon explicit solution of the section condition  $(t_\alpha)^{MN} \partial_M \otimes \partial_N = 0$   
by splitting  $56 \rightarrow 7_{+2} + 21_{+1} + 21_{-1} + 7_{-2}$   
 $Y^M \rightarrow \{y^m, y_{mn}, y^{mn}, y_m\} \longrightarrow \Phi = \Phi(x^\mu, y^m, y_{mn}, y^{mn}, y_m)$

the theory **coincides with the full D=11 supergravity !**

# $E_{7(7)}$ : exceptional field theory – the action

■ the full action  $S = \int d^4x d^{56}Y \sqrt{g} \mathcal{L}$  [O. Hohm, H.S.]

$$\begin{aligned} \mathcal{L} = & \widehat{R} + \frac{1}{24} \mathcal{D}_\mu \mathcal{M}_{MN} \mathcal{D}^\mu \mathcal{M}^{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}_{\mu\nu}^{\phantom{\mu\nu}M} \mathcal{F}^{\mu\nu N} \\ & + \mathcal{L}_{\text{top}} - V_{\text{pot}}(\mathcal{M}_{MN}, g_{\mu\nu}) \end{aligned}$$

■ upon different (inequivalent) solution of the section condition

$$56 \rightarrow (6, 1)_{+2} + (6', 2)_{+1} + (20, 1)_0 + (6, 2)_{-1} + (6', 1)_{-2}$$

the theory coincides with the IIB theory !

---

---

# **supersymmetric exceptional field theory**

## **fermions and supersymmetry**

[Godazgar, Godazgar, Hohm, Nicolai, H.S.]

# $E_{7(7)}$ : exceptional field theory – supersymmetry

---

- supersymmetry is not needed to fix all relative couplings  
(that is done by the generalized diffeomorphisms)
  - but the full system can be supersymmetrized upon adding fermions
- 

fermions in the D=4 theory	spin	SO(1,3)	SU(8)
– gravitini $\psi^i$	$3/2$		<b>8</b>
– matter fermions	$1/2$		<b>56</b>

---

$$\mathcal{D}_\mu \epsilon^i \equiv \partial_\mu \epsilon^i + \frac{1}{4} \omega_\mu{}^{ab} \gamma_{ab} \epsilon^i + (Q_\mu)_j{}^i \epsilon^j$$

spin connection $\omega_\mu{}^{ab}$	composite SU(8) connection $(Q_\mu)_i{}^j$
function of the vierbein $\mathcal{E}$	function of the 56-bein $\mathcal{V}$
determined by vanishing torsion $\Gamma_{[\mu\nu]}{}^\lambda = 0$	$\mathcal{V}^{-1} \partial_\mu \mathcal{V} \equiv \underline{Q_\mu} + P_\mu$ SU(8)

---

Killing spinor equations	$\delta \psi_\mu^i = \mathcal{D}_\mu \epsilon^i + \frac{1}{4} \mathcal{V}_M{}^{ij} \mathcal{F}_{\rho\sigma}{}^M \gamma^{\rho\sigma} \gamma_\mu \epsilon_j$
--------------------------	--

# $E_{7(7)}$ : exceptional field theory – supersymmetry

spin connection  $\omega_\mu{}^{ab}$

function of the vierbein  $\mathcal{E}$

determined by vanishing torsion  $\Gamma_{[\mu\nu]}{}^\lambda = 0$

composite SU(8) connection  $(Q_\mu)_i{}^j$

function of the 56-bein  $\mathcal{V}$

$$\mathcal{V}^{-1}\partial_\mu\mathcal{V} \equiv \frac{Q_\mu + P_\mu}{\text{SU}(8)}$$

spin connection  $\omega_M{}^{ab}$

function of the vierbein  $\mathcal{E}$

$$\mathcal{E}^{-1}\partial_M\mathcal{E} \equiv \frac{\omega_M + p_M + \mathcal{M}_{MN}\mathcal{F}^N}{\text{SO}(1, 3)}$$

composite SU(8) connection  $(Q_M)_i{}^j$

function of the 56-bein  $\mathcal{V}$

determined by vanishing torsion  $\Gamma_{MN}{}^K \Big|_{\mathbf{912}} = 0$   
(up to “irrelevant” terms)

[Coimbra, Strickland-Constable, Waldram / Cederwall]



# $E_{7(7)}$ : exceptional field theory – supersymmetry

spin connection  $\widehat{\omega}_\mu{}^{ab}$

function of the vierbein  $\mathcal{E}$

determined by vanishing torsion  $\widehat{\Gamma}_{[\mu\nu]}{}^\lambda = 0$

composite SU(8) connection  $(Q_\mu)_i{}^j$

function of the 56-bein  $\mathcal{V}$

$$\mathcal{V}^{-1} \mathcal{D}_\mu \mathcal{V} \equiv \frac{\widehat{\mathcal{Q}}_\mu + \widehat{\mathcal{P}}_\mu}{\text{SU}(8)}$$

spin connection  $\omega_M{}^{ab}$

function of the vierbein  $\mathcal{E}$

$$\mathcal{E}^{-1} \partial_M \mathcal{E} \equiv \frac{\omega_M + p_M + \mathcal{M}_{MN} \mathcal{F}^N}{\text{SO}(1, 3)}$$

composite SU(8) connection  $(Q_M)_i{}^j$

function of the 56-bein  $\mathcal{V}$

determined by vanishing torsion  $\left. \Gamma_{MN}{}^K \right|_{\mathbf{912}} = 0$   
(up to “irrelevant” terms)

[Coimbra, Strickland-Constable, Waldram / Cederwall]

Killing spinor equations

combines IIA / IIB Killing spinor equations

$$\delta \psi_\mu^i = \mathcal{D}_\mu \epsilon^i - i \mathcal{V}^{Mij} \nabla_M (\gamma_\mu \epsilon_j)$$

---

---

## applications

### Scherk-Schwarz & consistent truncations

[Hohm, H.S., to appear]

# Scherk-Schwarz & consistent truncations

---

■ generalized Scherk-Schwarz ansatz

$$\mathcal{V}_M(x, Y) = U_M{}^N(Y) \mathcal{V}_N(x)$$

$$A_\mu{}^M(x, Y) = (U^{-1})_N{}^M(Y) \mathcal{A}_\mu{}^N(x) \quad \text{with E}_{7(7)}\text{-valued twist matrix } U_M{}^N(Y)$$

$$B_{\mu\nu\alpha}(x, Y) = U_\alpha{}^\beta(Y) \mathcal{B}_{\mu\nu\beta}(x)$$

consistency condition :  
for Y-dependence to factor out

$$\left[ (U^{-1})_K{}^P (U^{-1})_M{}^L \partial_P U_L{}^N \right] \Big|_{912} = \alpha(Y) X_{KM}{}^N$$

with constant  $X_{KM}{}^N$  :

‘structure constants’ of the gauged theory

■ standard Scherk-Schwarz:  $(U^{-1})_{[K}{}^P (U^{-1})_{M]}{}^L \partial_P U_L{}^N = f_{[KM]}{}^N$

Lie algebra, in EXFT replaced by Leibniz algebra

[Aldazabal, Grana, Marques, Rosabal]

[Musaev, Berman, Thompson]

[Lee, Strickland-Constable, Waldram]

[Hohm, H.S.]

# Scherk-Schwarz & consistent truncations

## ■ generalized Scherk-Schwarz ansatz

$$[(U^{-1})_K{}^P(U^{-1})_M{}^L \partial_P U_L{}^N] \Big|_{\mathbf{9}_{12}} = \alpha(Y) X_{KM}{}^N$$

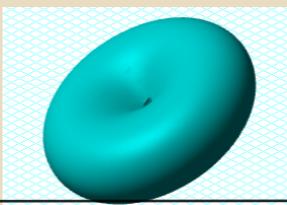
$$\mathcal{V}_M(x, Y) = U_M{}^N(Y) \mathcal{V}_N(x) \quad A_\mu{}^M(x, Y) = (U^{-1})_N{}^M(Y) A_\mu{}^N(x) \quad B_{\mu\nu\alpha}(x, Y) = U_\alpha{}^\beta(Y) B_{\mu\nu\beta}(x)$$

■ example: SL(8) subsector  
with twist matrix  $U \in \text{SL}(8)$

$$U_M{}^N = \begin{pmatrix} u_{mn}{}^{pq} & 0 \\ 0 & (u^{-1})_{pq}{}^{mn} \end{pmatrix}$$

explicit solutions to twist equations:  
gives rise to  $\text{SO}(p, q)$  gauged supergravity, automatically consistent truncation

■ linearisation of the highly non-linear reduction ansatz !  
(spheres, hyperboloids, etc.)  
plethora of new consistent truncations



## exceptional field theory

- **unique** theory with generalized diffeomorphism invariance in all coordinates (**modulo section condition**)
- upon an explicit solution of the section condition the theory **coincides** with the full D=11 supergravity
- upon different solution of the section condition the theory **coincides** with the full D=10 IIB supergravity
- **supersymmetric** upon adding fermions
- consistently includes the **dual graviton** degrees of freedom
- **generalized Scherk-Schwarz ansatz** describes sphere and other reductions

## exceptional field theory

### applications

- ▶ generalized Scherk-Schwarz ansatz :  
embedding and consistency of sphere compactifications (and more...)
- ▶ study of generalized BPS equations

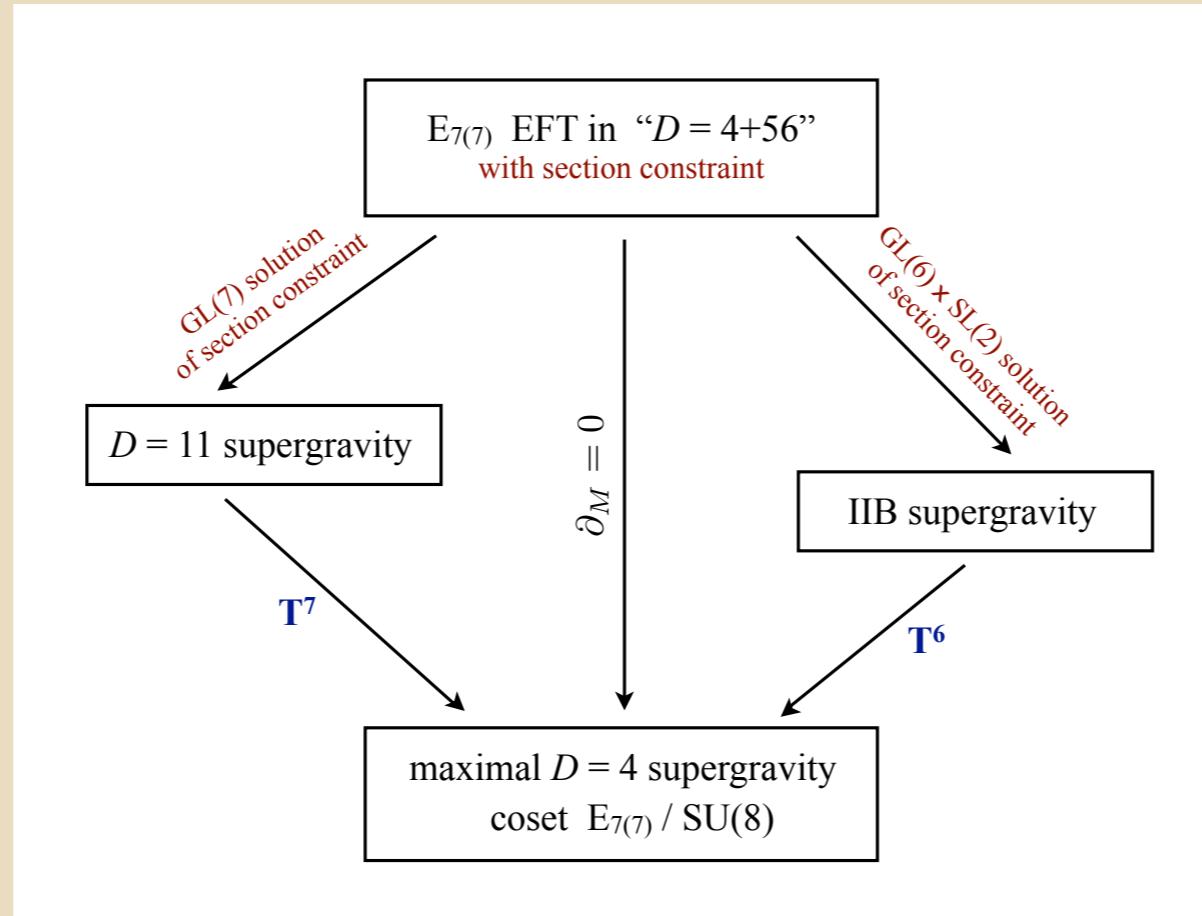
### towards a more fundamental formulation of supergravity

- ▶ extend to (infinite-dimensional)  $E_{9(9)}$ ,  $E_{10(10)}$ ,  $E_{11(11)}$  ...??
- ▶ better understand / consistently relax / the section condition ..??

$$\mathbb{P}_{KL}{}^{MN} \partial_M \otimes \partial_N \equiv 0$$

# conclusions

$$\begin{aligned}\mathcal{L} = & \widehat{R} + \frac{1}{24} \mathcal{D}_\mu \mathcal{M}_{MN} \mathcal{D}^\mu \mathcal{M}^{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}_{\mu\nu}^M \mathcal{F}^{\mu\nu N} \\ & + \mathcal{L}_{\text{top}} - V_{\text{pot}}(\mathcal{M}_{MN}, g_{\mu\nu})\end{aligned}$$



Killing spinor equations

$$\delta\psi_\mu^i = \mathcal{D}_\mu \epsilon^i - i \mathcal{V}^{M\,ij} \nabla_M (\gamma_\mu \epsilon_j)$$