Top quark Theory

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Top-quark physics is the most interesting known unknown physics to be studied at the LHC

Contents

- Intro
- Cross sections
- Forward Backward Charge Asymmetry
- Measuring the running top-quark mass
- Recent developments
- Conclusions

The top quark in the Standard Model

1. family	2. family	3. family	T_3 $(SU(2)_L)$	Y $(U(1)_Y)$	$Q=T_3+Y$
$\begin{pmatrix} u_L, u_L, u_L \\ d_L, d_L, d_L \end{pmatrix}$	$\begin{pmatrix} c_L, c_L, c_L \\ s_L, s_L, s_L \end{pmatrix}$	$\begin{pmatrix} t_L, t_L, t_L \\ b_L, b_L, b_L \end{pmatrix}$	1/2 -1/2	1/6 1/6	2/3 -1/3
u_R, u_R, u_R	c_R, c_R, c_R	t_R, t_R, t_R	0	2/3	2/3
d_R, d_R, d_R	s_R, s_R, s_R	b_R, b_R, b_R	0	-1/3	-1/3

Top quark interactions completely determined by the gauge structure of the SM

Only “2” free parameters:

- Top quark mass/Yukawa coupling
- CKM matrix

→ Top quark properties can be precisely predicted in the SM

Cabibbo-Kobayashi-Maskawa (CKM) Matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

eigenstates of the weak interaction

mass eigenstates

Global fit (SM, unitarity)

$$|V_{td}| = (8.14^{+0.32}_{-0.64}) \times 10^{-3},$$
$$|V_{ts}| = (41.61^{+0.12}_{-0.78}) \times 10^{-3},$$
$$|V_{tb}| = 0.999100^{+0.000034}_{-0.000004}$$

[PDG 06]

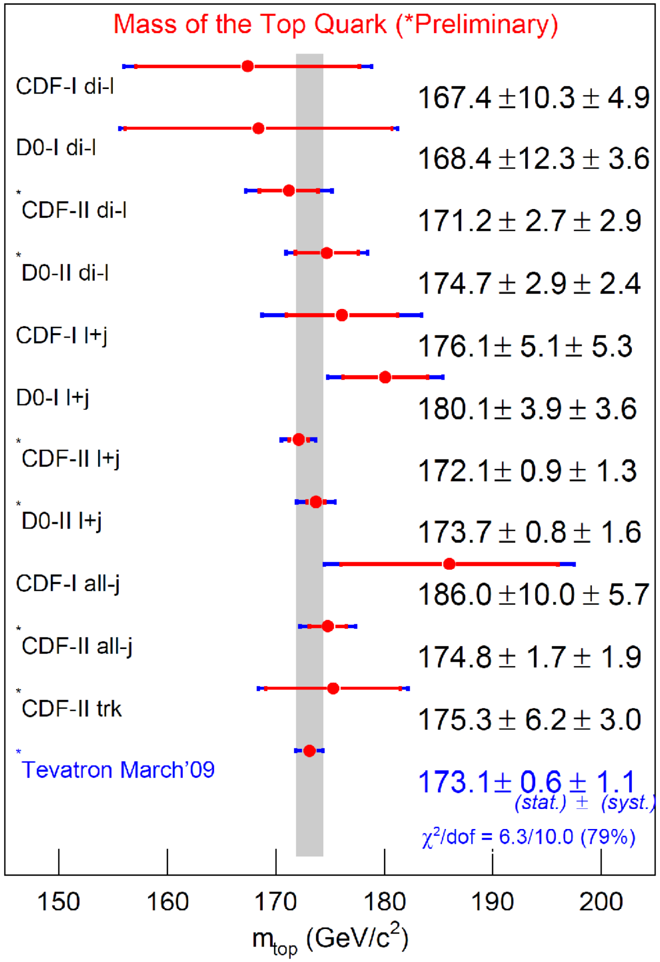
Almost no lower bound if additional families exist

!

→direct measurement important

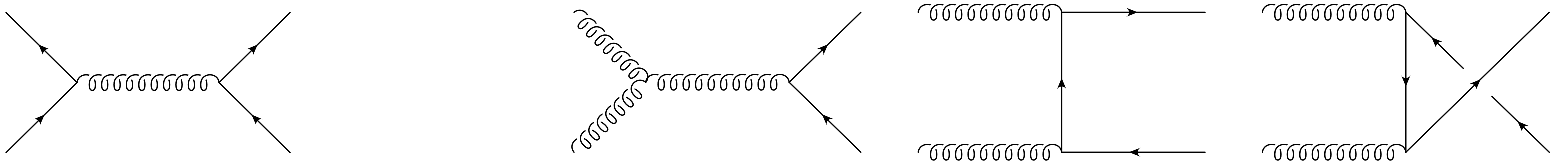
Experimental status – Top quark mass

From direct measurements at the Tevatron:



[arXiv0903.2503]

$$173.1 \pm 0.6 \pm 1.1 \text{GeV}$$



Partonic cross sections

$$\beta = \sqrt{1-\rho}$$
$$\rho = 4m_t^2/\hat{s}$$

$$\hat{\sigma}_{q\bar{q}} = \frac{8\pi\alpha_s^2}{27\hat{s}}\beta(1+\frac{\rho}{2})$$

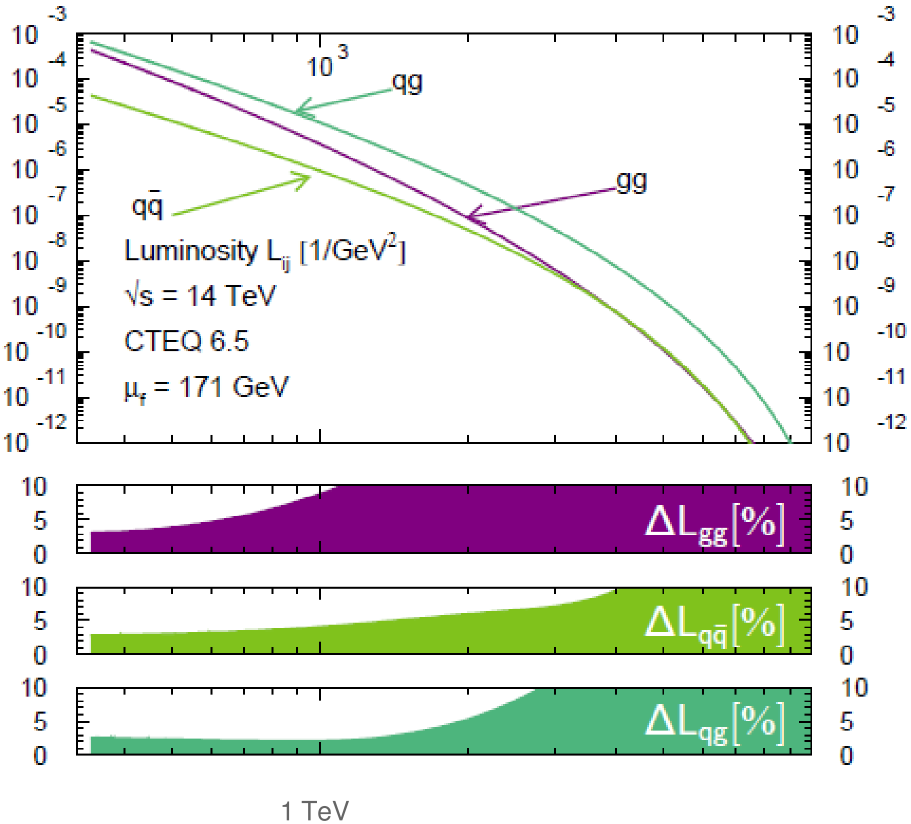
$$\hat{\sigma}_{gg} = \frac{4\pi\alpha_s^2}{12\hat{s}}\left[\left(1+\rho+\frac{\rho^2}{16}\right)\ln\left(\frac{1+\beta}{1-\beta}\right)-\beta\left(\frac{7}{4}+\frac{31}{16}\rho\right)\right]$$

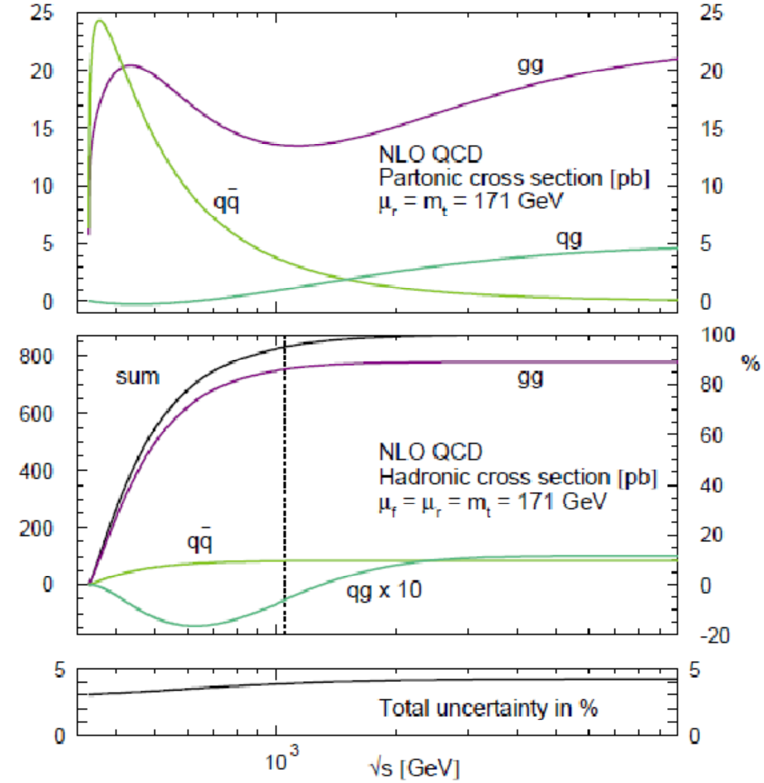
$$\sigma_{\mathrm{Had}} = \sum_{i,j}\int_0^1\int_0^1 F_{i/H_1}(x_1,\mu)F_{j/H_2}(x_2,\mu)\hat{\sigma}_{ij}(\hat{s}=x_1x_2s,\mu)$$

NLO corrections also known

[Dawson, Ellis, Nason ’89, Beenakker et al ’89,’91,Bernreuther, Brandenburg, Si, P.U. ’04, Czakon,Mitov 08]

Top quark pair production — NLO





$$L_{ij}(\hat{s}, s_{\text{had}}, \mu_f^2) = \frac{1}{s_{\text{had}}} \int_{\hat{s}}^{s_{\text{had}}} \frac{ds}{s} f_{i/p}\left(\mu_f^2, \frac{s}{s_{\text{had}}}\right) f_{j/p}\left(\mu_f^2, \frac{\hat{s}}{s}\right)$$

$$\sigma(s_{\text{had}}, m_t{}^2; s_{\max}) = \sum_{i,j=q,\bar{q},g} \int_{4m_t{}^2} d\hat{s}\, L_{ij}(\hat{s}, s_{\text{had}}, \mu_f^2)\, \hat{\sigma}_{ij\to t\bar{t}}(\hat{s}, m_t{}^2, \mu_f^2, \mu_r^2)$$

Coulomb singularity

Close to threshold NLO corrections are given by:

$$f_{q\bar{q}}^{(1)}(\rho) + \overline{f}_{q\bar{q}}^{(1)}(\rho)\ln\frac{\mu^2}{m^2} = \frac{1}{4\pi^2} f_{q\bar{q}}^{(0)}(\rho)\left\{\left(C_F - \frac{1}{2}C_A\right)\frac{\pi^2}{2\beta} + 2C_F\ln^2(8\beta^2) - (8C_F + C_A)\ln(8\beta^2) - 2C_F\ln(4\beta^2)\ln\frac{\mu^2}{m^2} + \overline{C}_2\left(\frac{\mu^2}{m^2}\right) - \mathcal{O}(1-\rho)\right\}, \quad (12)$$

$$f_{gg}^{(1)}(\rho) + \overline{f}_{gg}^{(1)}(\rho)\ln\frac{\mu^2}{m^2} = \frac{1}{4\pi^2} f_{gg}^{(0)}(\rho)\left\{\frac{N_c^2-2}{N_c(N_c^2-2)}\frac{\pi^2}{4\beta} + 2C_A\ln^2(8\beta^2) - \frac{(9N_c^2-20)C_A}{N_c^2-2}\ln(8\beta^2) - 2C_A\ln(4\beta^2)\ln\frac{\mu^2}{m^2} + \overline{C}_3\left(\frac{\mu^2}{m^2}\right) + \mathcal{O}(1-\rho)\right\}, \quad (13)$$

[Nason, Dawson, Ellis 88,
Beenakker, kuijf, vNeerven,Smith '89]

Large logarithmic corrections in threshold region:

$$f_{ij}^{(n)}(\rho;\mu^2/m^2) \sim f_{ij}^{(0)}(\rho)\ln^{2n}\beta^2$$

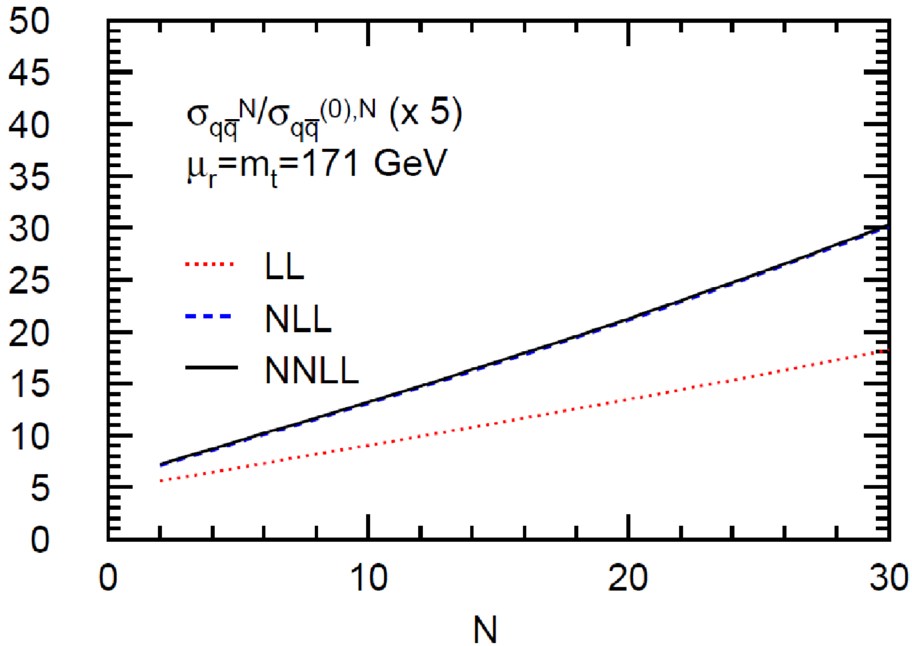
→Sum large logarithmic corrections to improve perturbation theory

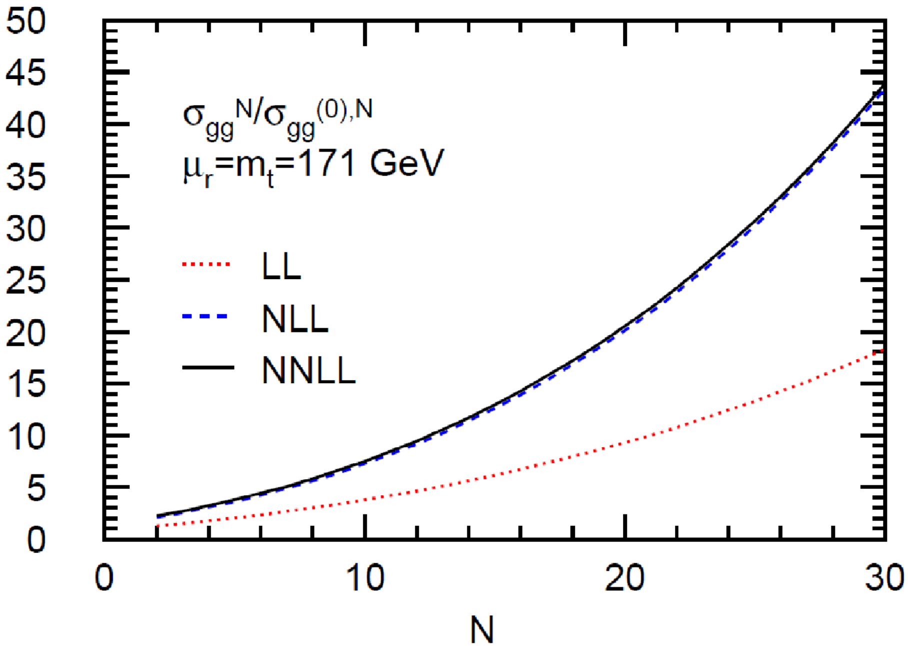
[Bonciani, Cacciari, Catani, Kidonakis, Laenen, Mangano, Moch, Nason, Ridolfi, Sterman, P.U.…]

Next-to-Next-to-Leading Log

NEW !

[Moch,PU '08]



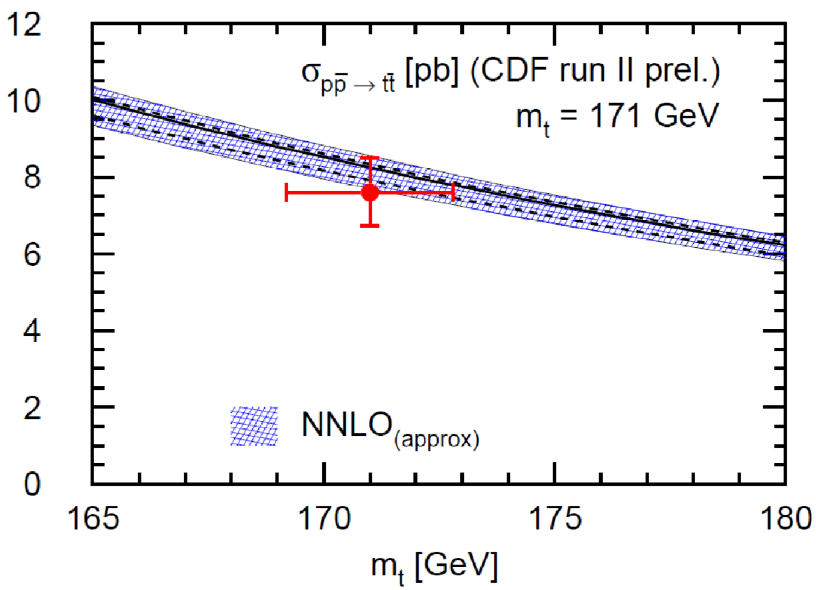


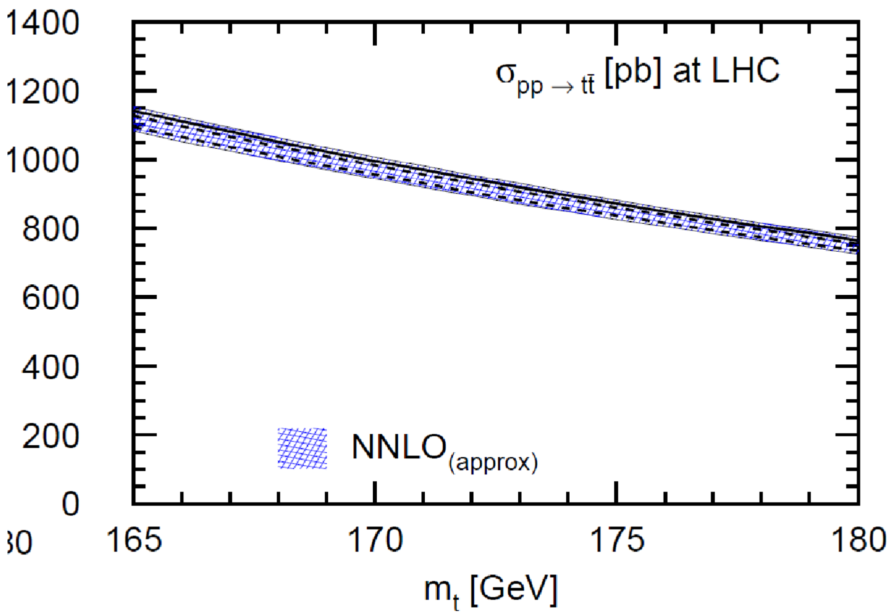
→ very small effect

→ use knowledge to construct Ansatz for NNLO

NNLO Ansatz

[Moch,PU '08]





Ansatz based on:

NEW !

- NNLL resummation (log(beta))
- 2-loop Coulomb singularity
- exact scale dependence at NNLO

→ approximation to full NNLO result

Theoretical predictions

NLL

[Cacciari,Frixione,Mangano,Nason,Ridolfi 08]

$$\sigma_{t\bar{t}}^{\rm NLO+NLL}({\rm LHC}, m_t = 171\ {\rm GeV}, {\rm MRST2006nnlo}) = 961\ {}^{+89(9.2\%)}_{-91(9.4\%)}\ ({\rm scales})\ {}^{+11(1.1\%)}_{-12(1.2\%)}\ ({\rm PDFs})\ \ {\rm pb}$$

$$\sigma_{t\bar{t}}^{\rm NLO+NLL}({\rm Tev}, m_t = 171\ {\rm GeV}, {\rm MRST2006nnlo}) = 7.93\ {}^{+0.34(4.3\%)}_{-0.56(7.1\%)}\ ({\rm scales})\ {}^{+0.24(3.1\%)}_{-0.20(2.5\%)}\ ({\rm PDFs})\ \ {\rm pb}\,.$$

$$\sigma_{t\bar{t}}^{\rm NLO+NLL}({\rm LHC}, m_t = 171\ {\rm GeV}, {\rm CTEQ6.5}) = 908\ {}^{+82(9.0\%)}_{-85(9.3\%)}\ ({\rm scales})\ {}^{+30(3.3\%)}_{-29(3.2\%)}\ ({\rm PDFs})\ \ {\rm pb}$$

$\sigma_{t\bar{t}}^{\rm NLO}$ **Assessment of scale uncertainty needs to be clarified** 's) pb

NNLO Approx.

[Langenfeld,Moch,PU,08,09]

$$\sigma_{\rm LHC} = 887\ {\rm pb}\ {}^{+9}_{-33}\ {\rm pb\ (scale)}\ {}^{+15}_{-15}\ {\rm pb\ (MSTW2008)},$$

$$\sigma_{\rm Tev} = 7.04\ {\rm pb}\ {}^{+0.24}_{-0.36}\ {\rm pb\ (scale)}\ {}^{+0.14}_{-0.14}\ {\rm pb\ (MSTW2008)},$$

$$\sigma_{\rm LHC} = 874\ {\rm pb}\ {}^{+9}_{-33}\ {\rm pb\ (scale)}\ {}^{+28}_{-28}\ {\rm pb\ (CTEQ6.6)},$$

$$\sigma_{\rm Tev} = 7.34\ {\rm pb}\ {}^{+0.24}_{-0.38}\ {\rm pb\ (scale)}\ {}^{+0.41}_{-0.41}\ {\rm pb\ (CTEQ6.6)}.$$

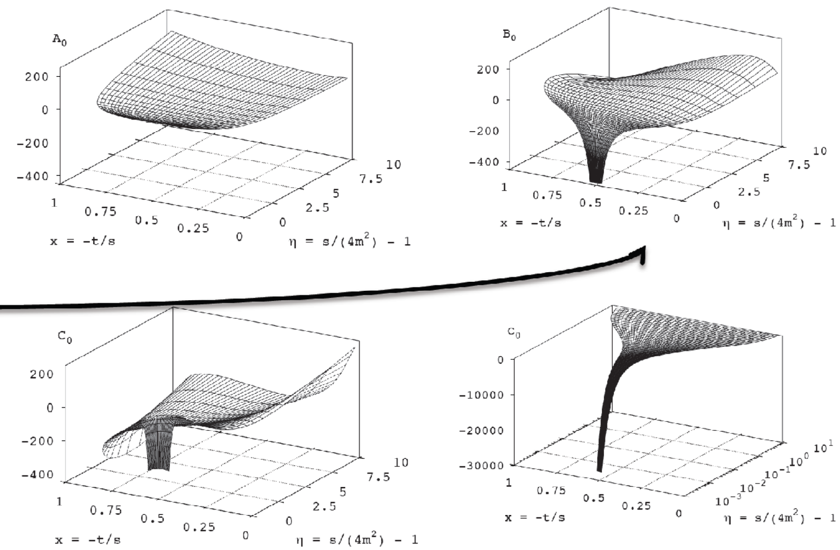
$m_t = 173$GeV

Similar results by Kidonakis

[Czakon QCD conf. Berlin]

Amplitudes for Quark Annihilation

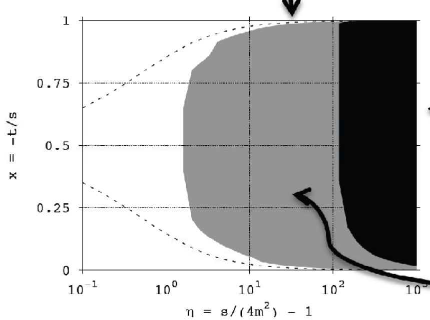
- complexity
  - 190 diagrams
  - 2812 integrals
  - 145 masters



High precision numerics

Convergence region for
A small mass expansion

MC '08



1% accuracy
MC, Mitov, Moch '07

0.1% from the
expansion

MC '08

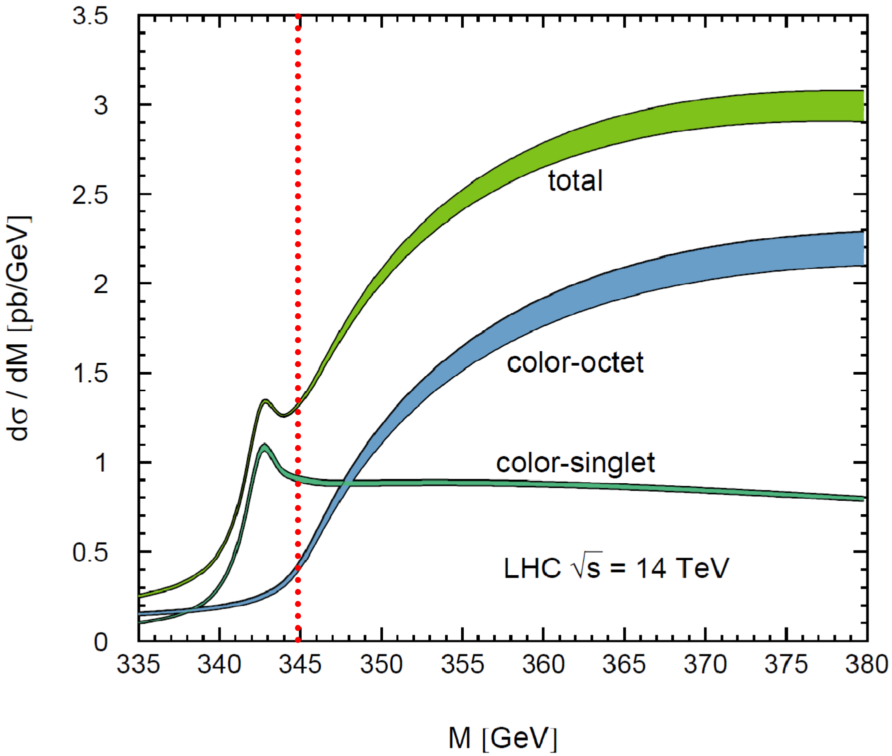
$$m^2 = 0.2s,\ t = -0.45s$$

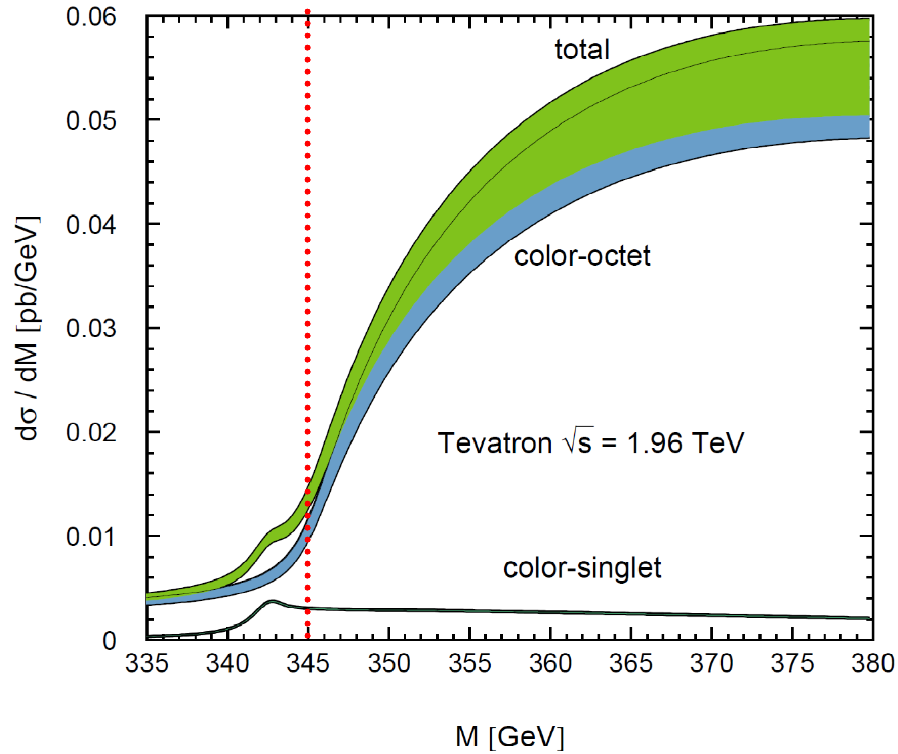
	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
A	0.22625	1.391733154	-2.298174307	-4.145752449	17.37136599
B	-0.4525	-1.323646320	8.507455541	6.035611156	-35.12861106
C	0.22625	-0.06808683395	-18.00716652	6.302454931	3.524044913
D_l		-0.22625	0.2605057339	-0.7250180282	-1.935417247
D_h			0.5623350684	0.1045606449	-1.704747998
E_l		0.22625	-0.3323207300	7.904121951	2.848697837
E_h			-0.5623350684	4.528240788	12.73232424
F_l					-1.984228442
F_{lh}					-2.442562819
F_h					-0.07924540546

gg case underway

Bound state effects at threshold

[Hagiwara, Suimino, Yokoya 08]
[Kiyo,Kühn,Moch,Steinhauser,P.U. 08]





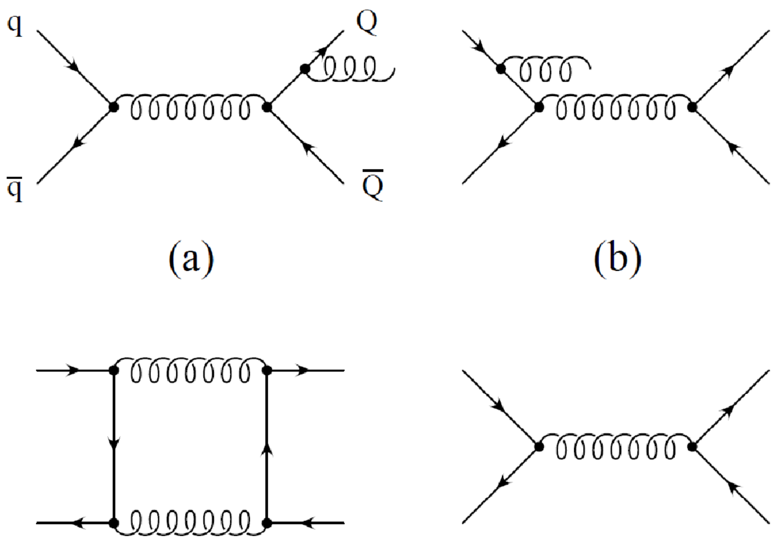
"Resonance structure" from would be boundstate

~10 pb shift of total cross section at LHC

Important measurements

- ✓ tt cross section — Precise determination of top mass, consistency checks with theo. predictions, search for new physics in the tt invariant mass spectrum
- ✓ W-Polarization in top decay — Test of the V-A structure in top decay
- ✓ ttH cross section — Measurement of the Yukawa coupling
- ✓ ttZ cross section — Measurement of the Z couplings
- ✓ Single top production — Direct measurement of the CKM matrix element Vtb, top polarization, search for anomalous Wtb couplings
- ✓ Spin correlations — Weak decay of a `free' quark, bound on the top width and V_{tb}, search for anomalous couplings
- ✓ tt+Jet(s) production — Search for anomalous couplings, important background
- ✓ ttγ cross section — Measurement of the electric charge
- ✓ b-quark distribution in decay — Sensitive to new physics t→bH^+
- ✓ top polarization — Sensitive to new physics

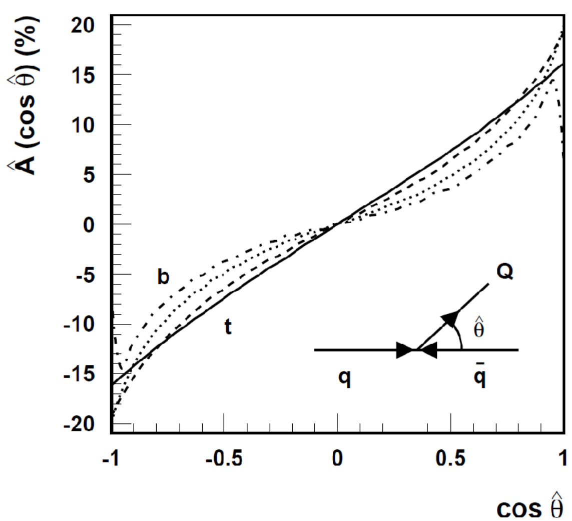
[Kühn,Rodrigo 98]

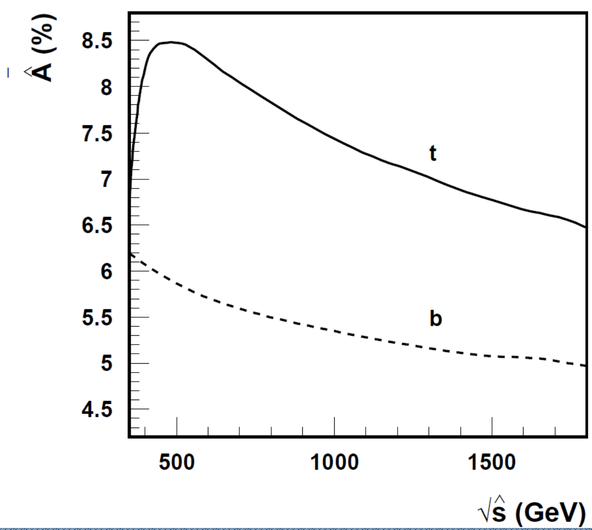


→ Interference produces charge asymmetry similar to QED

Appears first in NLO!

Leads to Forward-Backward charge asymmetry at the Tevatron





$$\bar{\hat{A}} = \frac{N_t(\cos\hat\theta \geq 0) - N_{\bar t}(\cos\hat\theta \geq 0)}{N_t(\cos\hat\theta \geq 0) + N_{\bar t}(\cos\hat\theta \geq 0)}$$

The Forward-Backward charge asymmetry

Integrated charge asymmety at the Tevatron (1.8TeV):

$$\bar{A} = \frac{N_t(\cos\theta \geq 0) - N_{\bar{t}}(\cos\theta \geq 0)}{N_t(\cos\theta \geq 0) + N_{\bar{t}}(\cos\theta \ldots}$$;

[Kühn,Rodrigo 98]

Theor. prediction only leading order

...ev)

Recently ...

$$A_{fb} = 0.05 \pm 0.015$$

[Antunano,Kühn,Rodrigo 08]

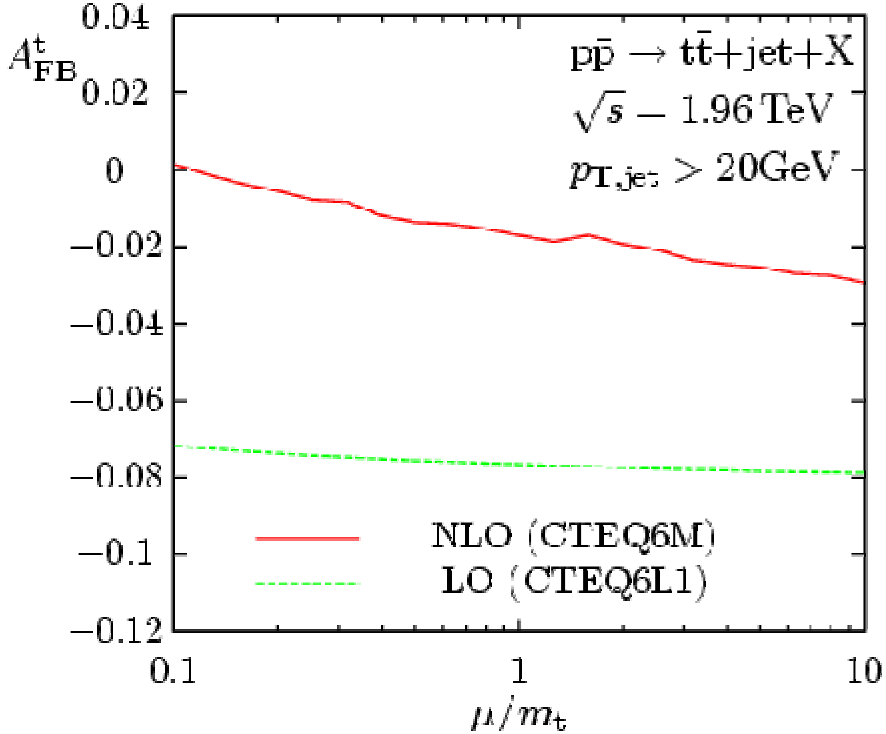
[CDF/ANAL/TOP/PUBLIC/9724, March 2009] (3.2 1/fb)

$$A_{fb} = 0.193 \pm 0.065^{stat} \pm 0.024^{syst}$$

Forward-backward charge asymmetry in tt+1Jet

[Dittmaier, PU, Weinzierl PRL 98:262002, '07]

$$A_{FB}^{t} = \frac{\sigma(y_t > 0) - \sigma(y_t < 0)}{\sigma(y_t > 0) + \sigma(y_t < 0)}$$



centr.val.(num.int.err.)$^{\text{shift } \mu=2m}_{\text{shift } \mu=m/2}$

	$\sigma_{t\bar{t}jet}$[pb]		A^t_{FB}[%]	
$p_{T,jet,cut}$ [GeV]	LO	NLO	LO	NLO
20	$1.583(2)^{+0.96}_{-0.55}$	$1.791(1)^{+0.16}_{-0.31}$	$-7.69(4)^{+0.10}_{-0.085}$	$-1.77(5)^{+0.58}_{-0.30}$
30	$0.984(1)^{+0.60}_{-0.34}$	$1.1194(8)^{+0.11}_{-0.20}$	$-8.29(5)^{+0.12}_{-0.085}$	$-2.27(4)^{+0.31}_{-0.51}$
40	$0.6632(8)^{+0.41}_{-0.23}$	$0.7504(5)^{+0.072}_{-0.14}$	$-8.72(5)^{+0.13}_{-0.10}$	$-2.73(4)^{+0.35}_{-0.49}$
50	$0.4670(6)^{+0.29}_{-0.17}$	$0.5244(4)^{+0.049}_{-0.096}$	$-8.96(5)^{+0.14}_{-0.11}$	$-3.05(4)^{+0.49}_{-0.39}$

What do we mean by the mass of a confined quark ?

Mass is just a **parameter** of the theory like for example α_s

→ Mass parameter depends on the renormalization scheme

Common schemes:

- On-shell / pole-mass scheme
- $\overline{\text{MS}}$ mass, *running mass*

Other definitions:

potential subtracted mass, 1S mass, kinetic mass,…

(used in e^+e^- annihilation, also useful for hadron collider ?)

How do we measure a quark mass ?

By comparing the theoretical prediction (in a specific scheme) for a measurable quantity with the experimentally measured value

Examples:

- Measurement of the b-quark mass at LEP from 3-jet cross sections, $\rightarrow \overline{m}_b(\mu = m_Z)$
- Determination of m_t from electroweak fits.

Note:

Need to go to NLO in pert. theory to give a meaningful definition of the parameters, i.e. masses,couplings

In LO the renormalization scheme is not determined!

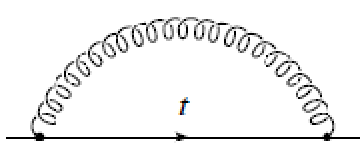
(… this is what pure theory tells us…)

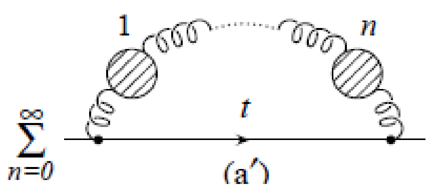
Pole mass versus $\overline{\mathrm{MS}}$ mass

$$m_{\text{pole}} = 173\text{GeV} \rightarrow \overline{m}(\overline{m}) = 163\text{GeV}$$

Difference of 10 GeV → make sure that
we know which mass we measure !

Due to infrared renormalon in pole-mass,
pole mass has intrinsic uncertainty of order Λ_{QCD}

$\Sigma^{(1)} =$ 



$$\frac{16 m_R}{3\beta_0}\sum_{n=0}^{\infty} c_n a^{n+1}$$

$$c_n \overset{n\to\infty}{\longrightarrow} e^{-C/2} 2^n n!$$

Pole mass not well defined beyond pert. theory,
Limitation for reachable precision!

[Smith, Willenbrock 97]

There is no pole in full QCD !

Jetmass in e+e-

[Hoang 07,08]

Double differential invariant mass distribution:

$$Q = 5 \times 172\ \text{GeV}$$

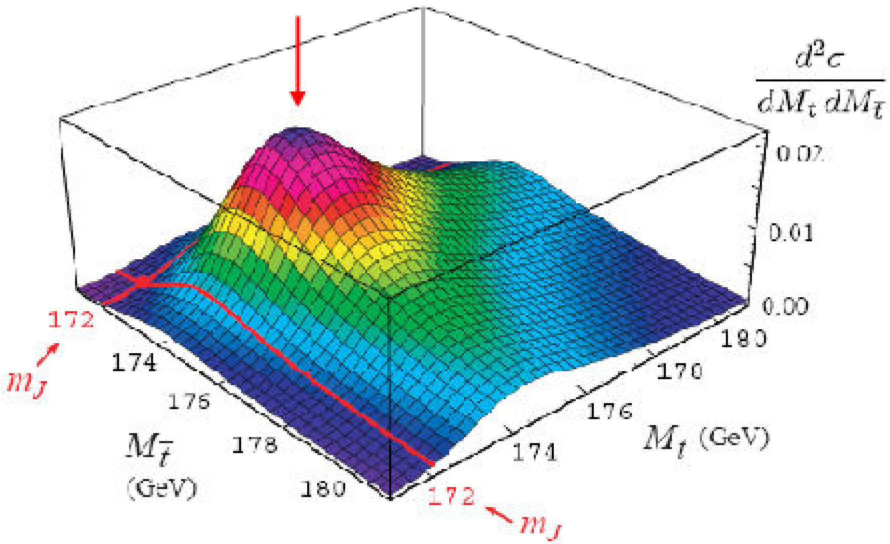
$$\Gamma = 1.43\ \text{GeV}$$

$$m_J(2\,\text{GeV}) = 172\ \text{GeV}$$

$$\mu_\Gamma = 5\ \text{GeV}$$

$$\mu_\Lambda = 1\ \text{GeV}$$

$$a = 2.5\,, \quad b = -0.4$$
$$\Lambda = 0.55\ \text{GeV}$$

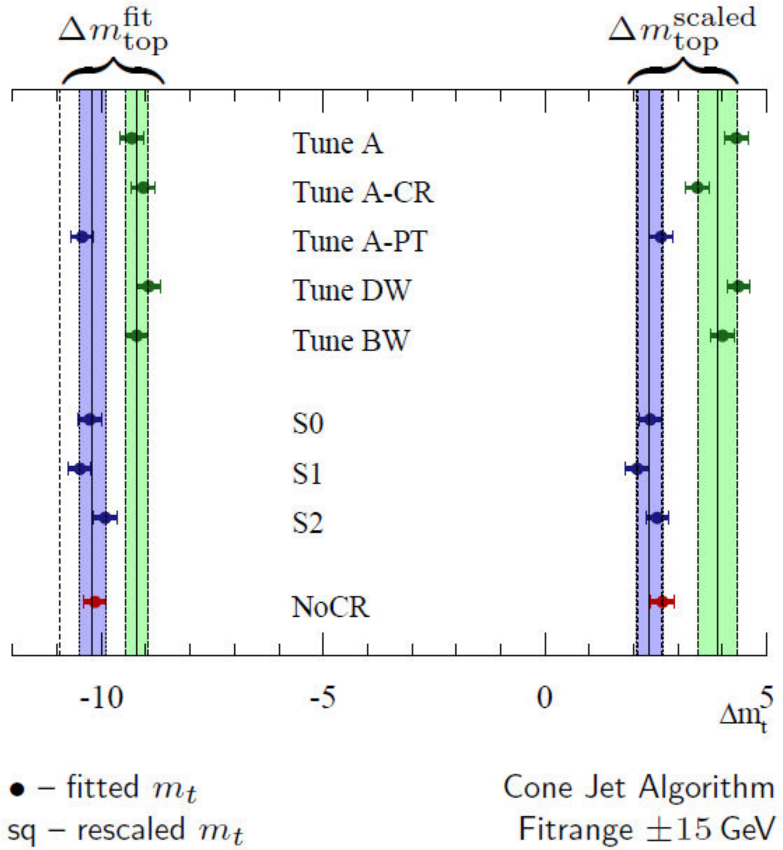


Non-perturbative corrections shift peak by ~2.4 GeV and broaden the distribution

Non-perturbative effects at the LHC

[Skands,Wicke 07,08]

Simulate top mass measurement using different models/tunes for non-perturbative physics

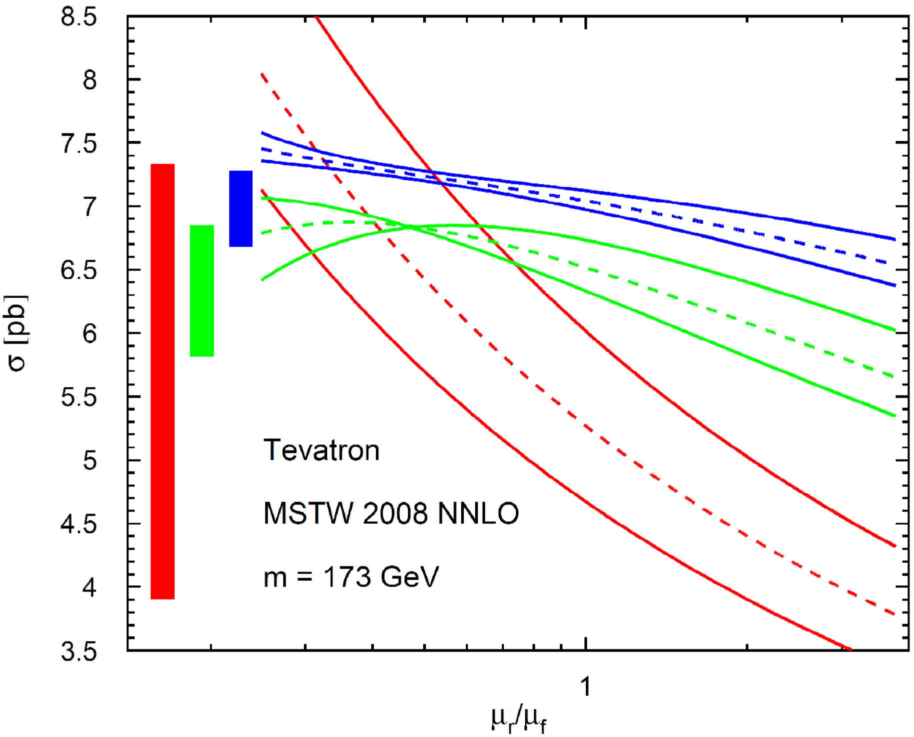


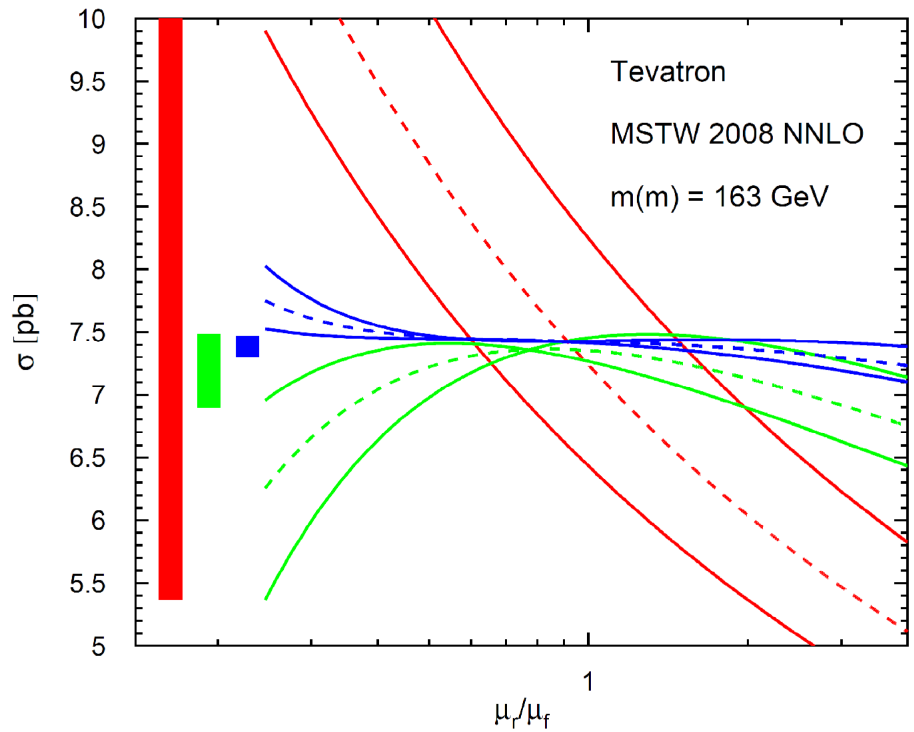
Non-perturbative effects result in shift of the order of 500 MeV

[Langenfeld, Moch, P.U. 09]

Idea:

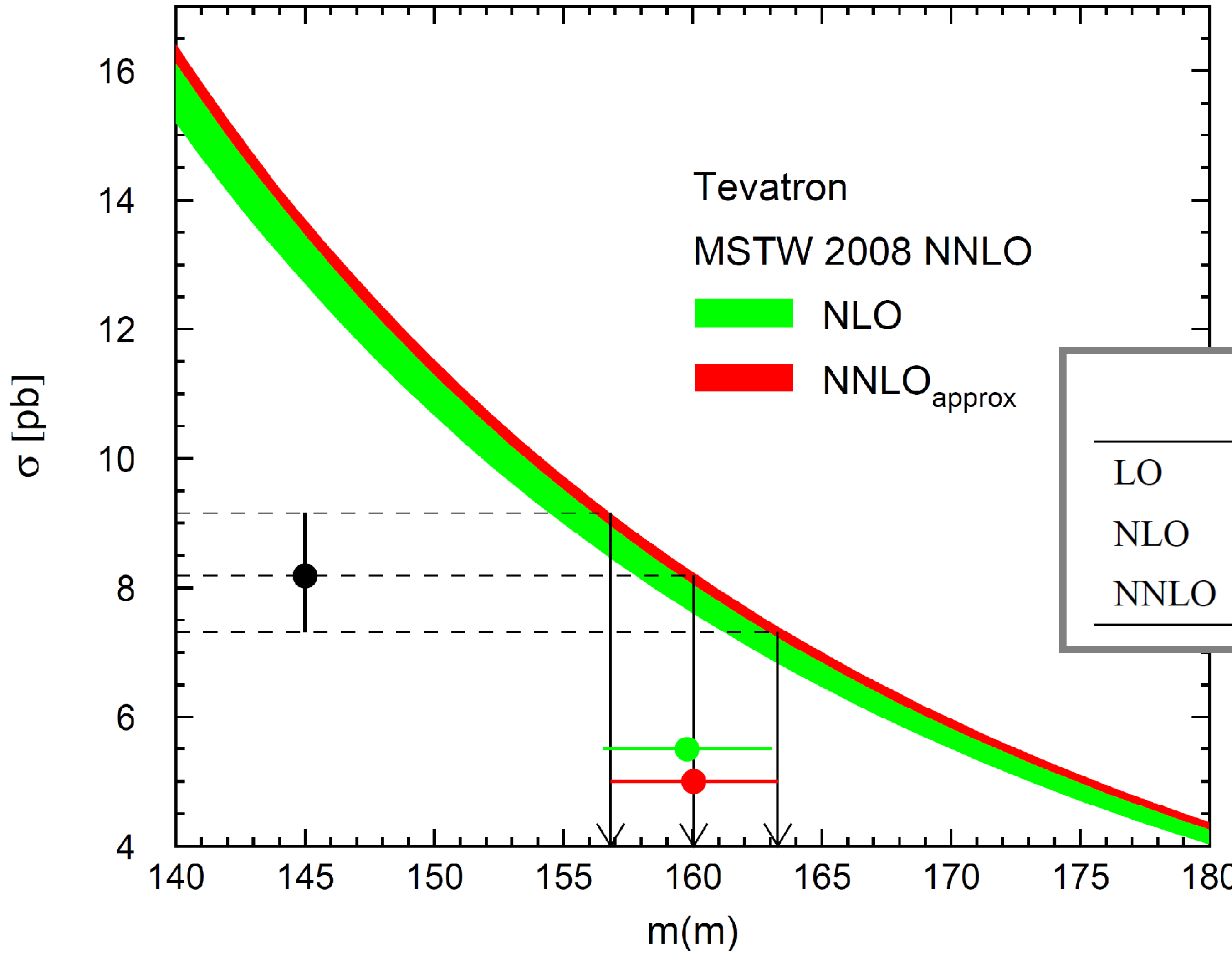
Use total cross section as function of the MS mass





First direct determination of the MS mass

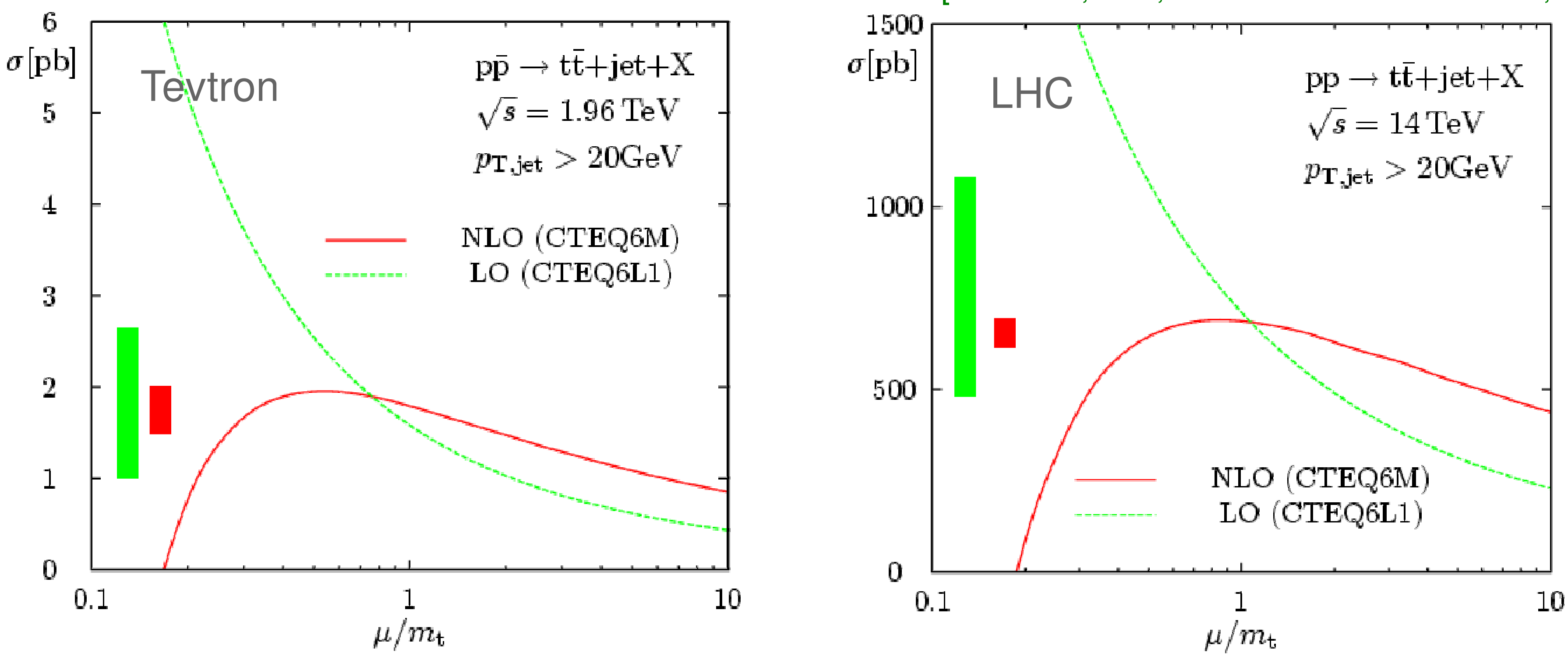
[Langenfeld, Moch, P.U. 09]



	$\overline{m}$ [GeV]	m_t [GeV]
LO	$159.2^{+3.5}_{-3.4}$	$159.2^{+3.5}_{-3.4}$
NLO	$159.8^{+3.3}_{-3.3}$	$165.8^{+3.5}_{-3.5}$
NNLO	$160.0^{+3.3}_{-3.2}$	$168.2^{+3.6}_{-3.5}$

$$m_t = 168.9^{+3.5}_{-3.4}$$

[Dittmaier, P.U., Weinzierl PRL 98:262002, '07]

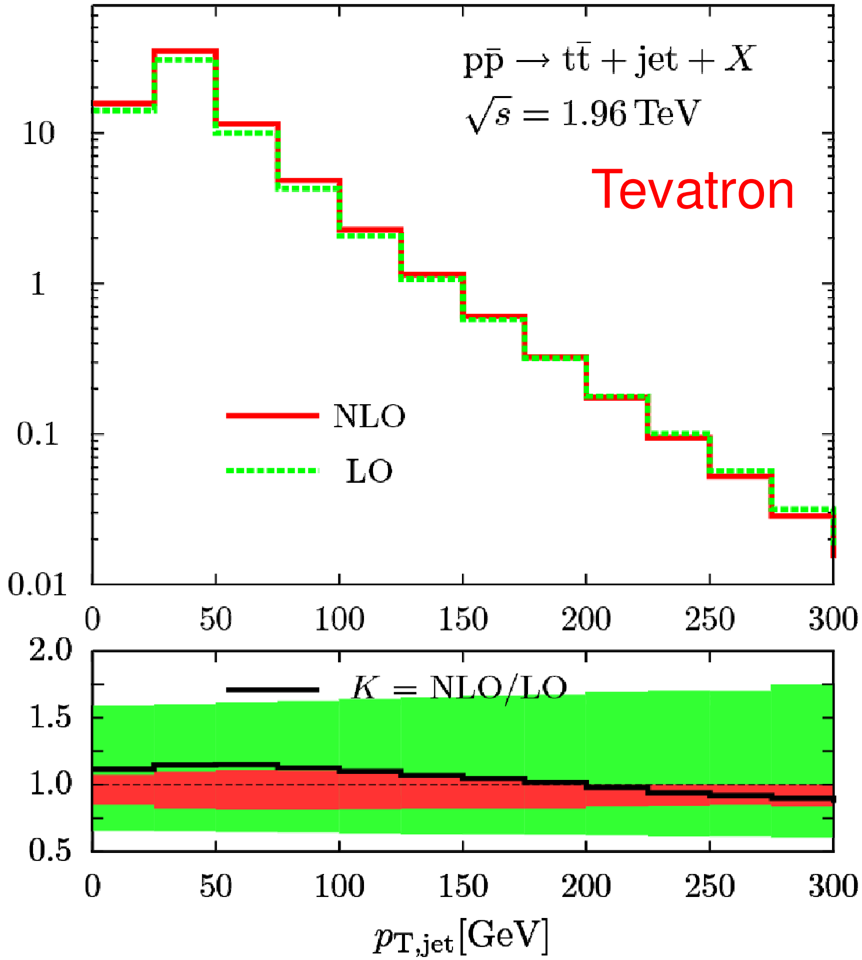


- Scale dependence is improved
- Corrections are moderate in size

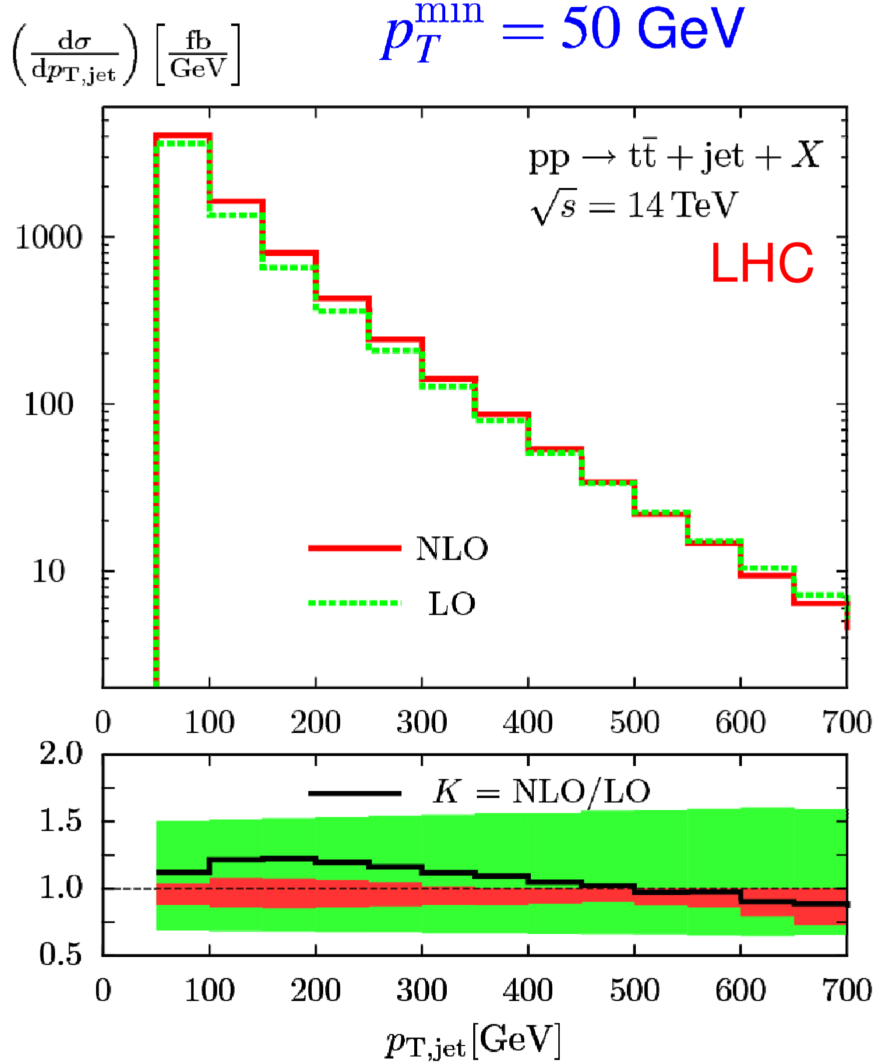
Recent Progress: tt + 1 Jet production at NLO

[Dittmaier, P.U., Weinzierl, '07,'08]

$p_T^{min} = 20$ GeV

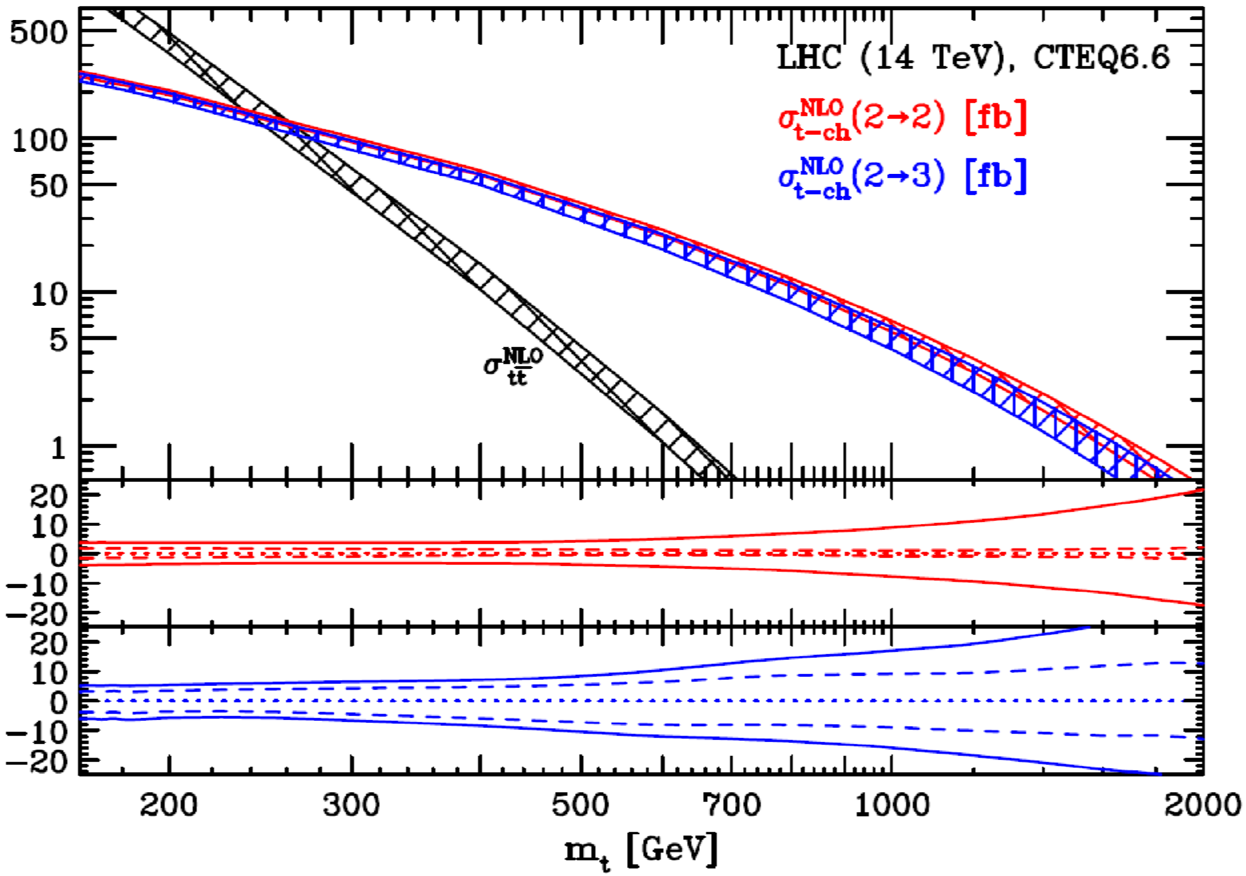


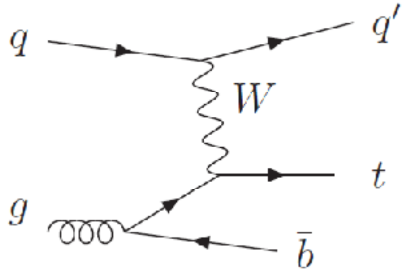
$p_T^{min} = 50$ GeV

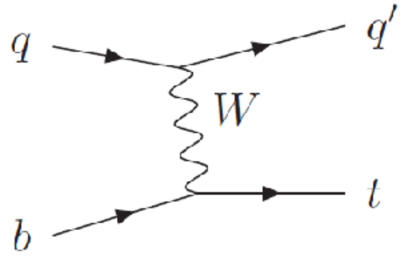


Corrections of the order of 10-20 %, again scale dependence is improved

[Campbell, Frederix, Maltoni, Tramontano 09]

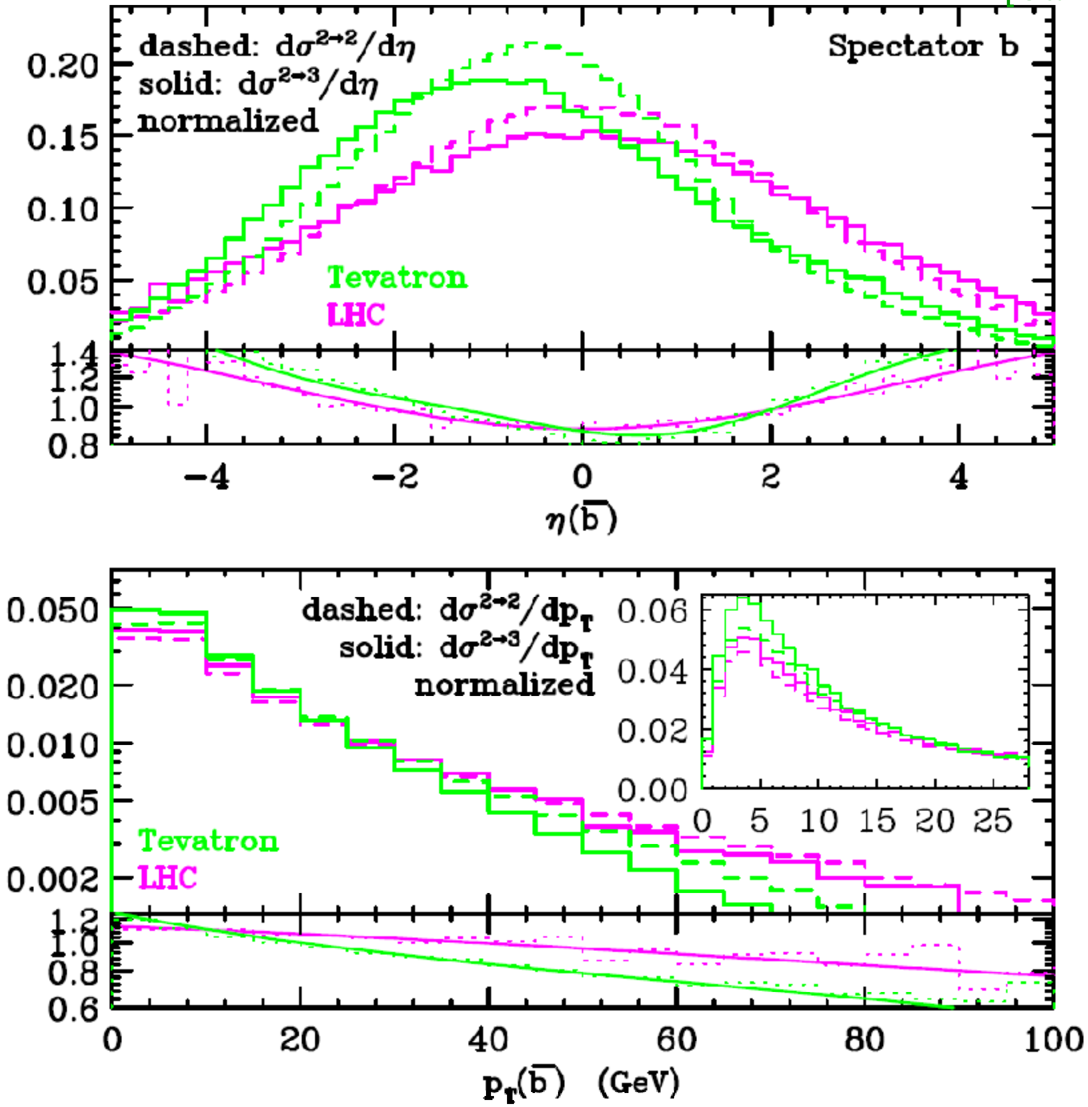






→ Good agreement between 4 and 5 flavour theory

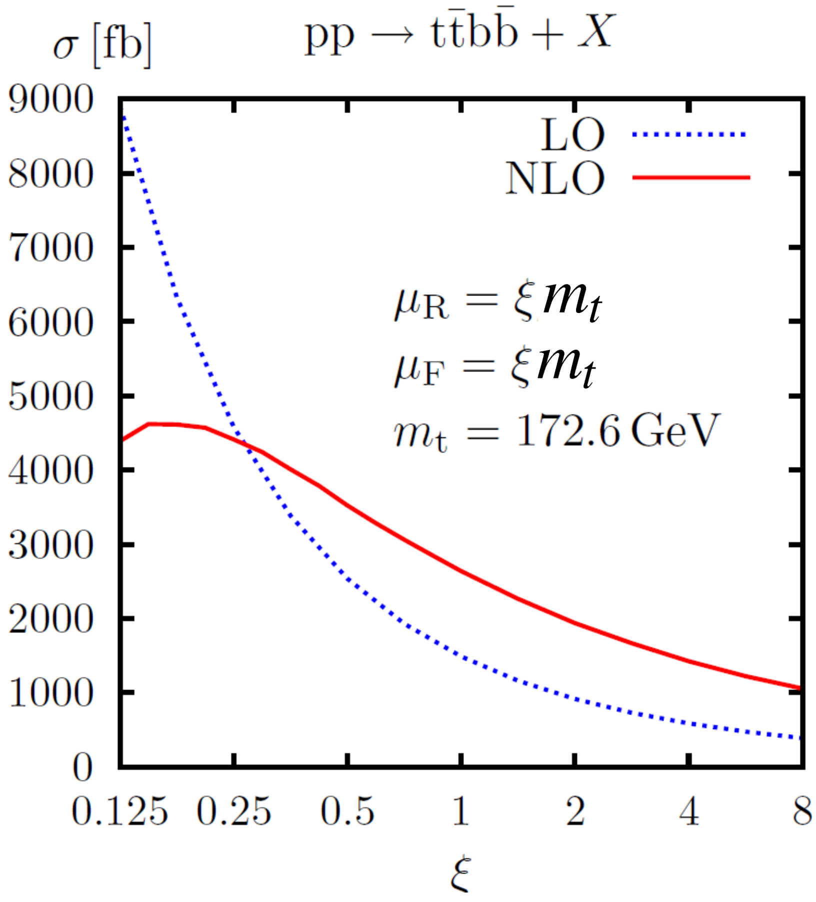
[Campbell, Frederix, Maltoni, Tramontano 09]

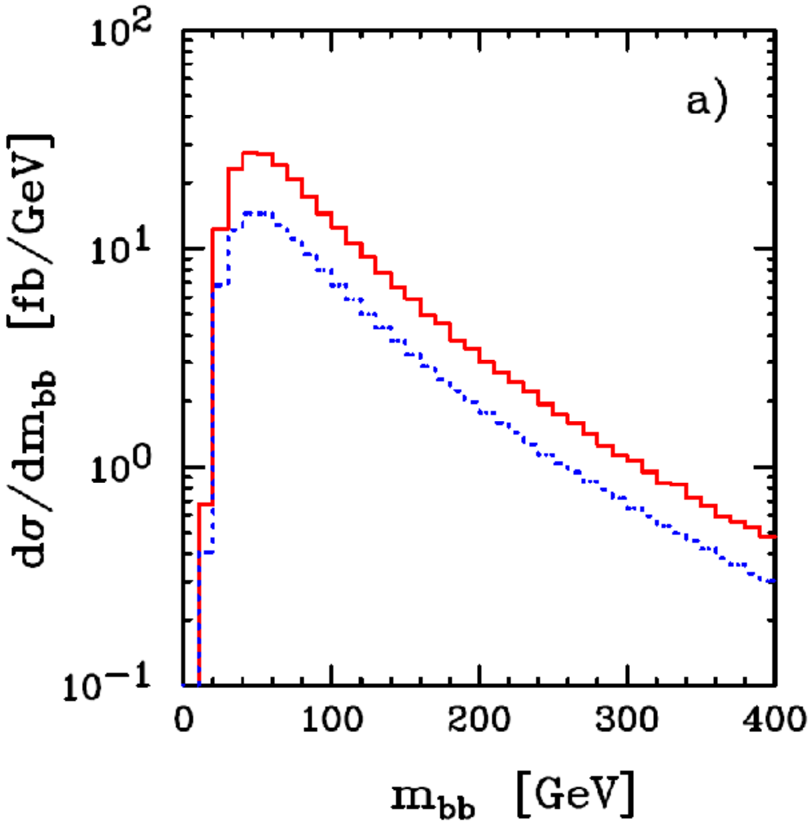


Differences if the observable is sensitive to specatator b !

[Bredenstein,Denner,Dittmaier,Pozzorini 08,09]

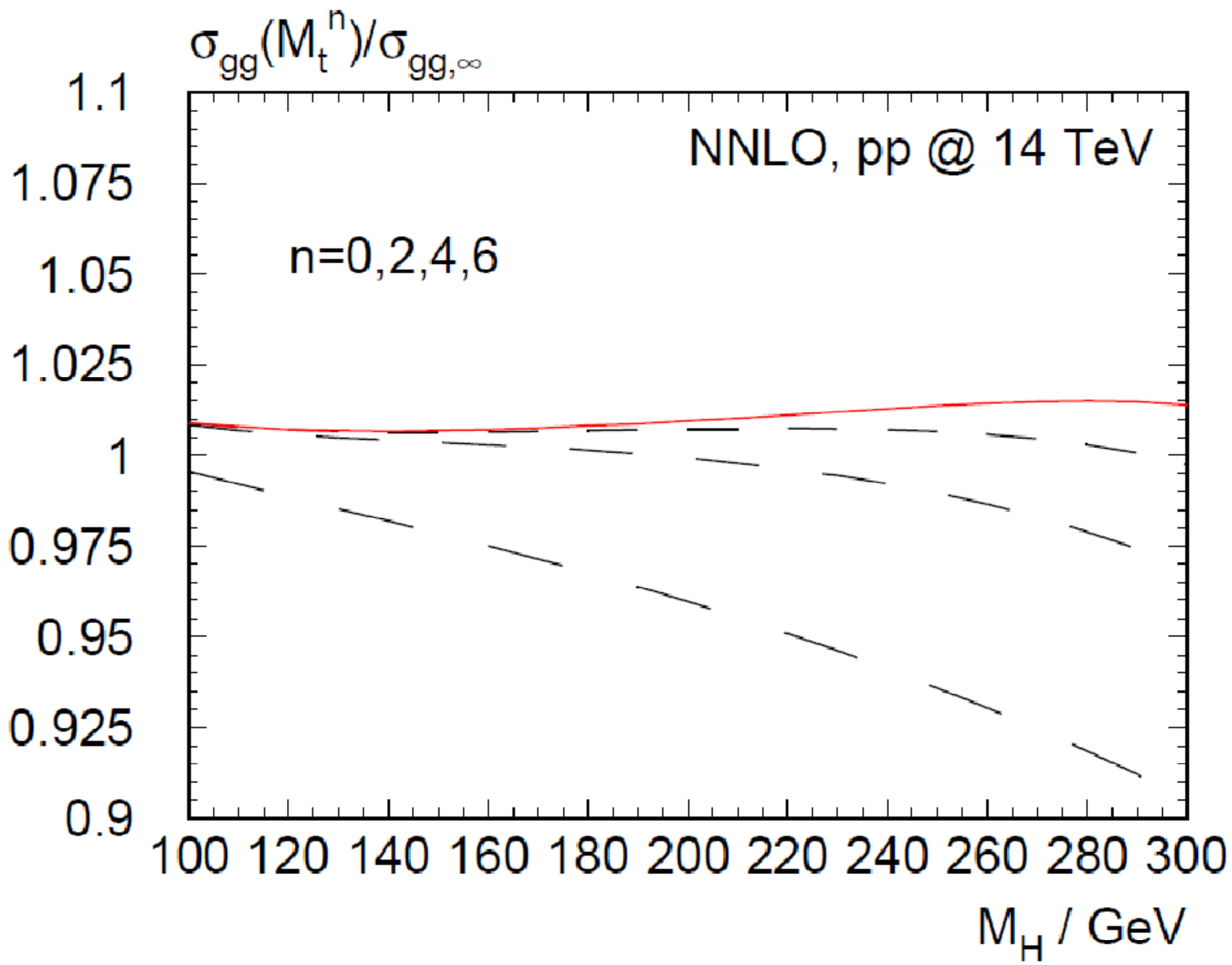
[Bevilacqua,Czakon,Papadopoulos,Pittau,Worek 09]

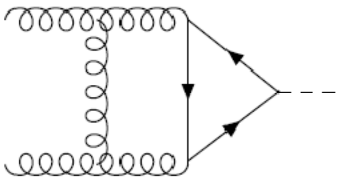




→ large corrections, scale dep. reduced

[Harlander, Ozeren 09]





→ Small deviations less 1%

Conclusions

Top-quark physics is the most interesting known unknown physics to be studied at the LHC

It is interesting as

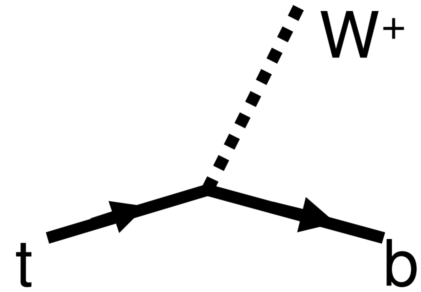
- theoretical laboratory for field theory
- such
- to search for new physics

Thank you for your attention

Top quark decay – Main decay in the SM

$$|V_{tb}| \approx 1 \quad \rightarrow \quad t \rightarrow Wb$$

$$W \hookrightarrow q\bar{q}', \bar{\ell}\nu_\ell$$



Width calculable in the SM:

$$\Gamma_t = \frac{G_F m_t^3}{8\pi\sqrt{2}}\left(1-\frac{m_W^2}{m_t^2}\right)^2\left(1+\frac{2m_W^2}{m_t^2}\right)\left(1-\frac{2\alpha_s}{3\pi}\left(\frac{2\pi^2}{3}-\frac{5}{2}\right)\right)$$

$$\approx 1.48 \quad GeV$$

Two-loop QCD and one-loop EW corrections also known!

→ Life time: $\Gamma \approx 1.4\text{GeV} \rightarrow \tau_t \approx 0.4 \times 10^{-25} s < \tau_{\text{QCD}} \approx 3\times 10^{-24} s$

"Top quark decays before it can hadronize"

[Bigi, Dokshitzer, Khoze, Kühn, Zerwas ´86]

→ No bound states, spin observables are "good" observables

Top quark pair production — NLO

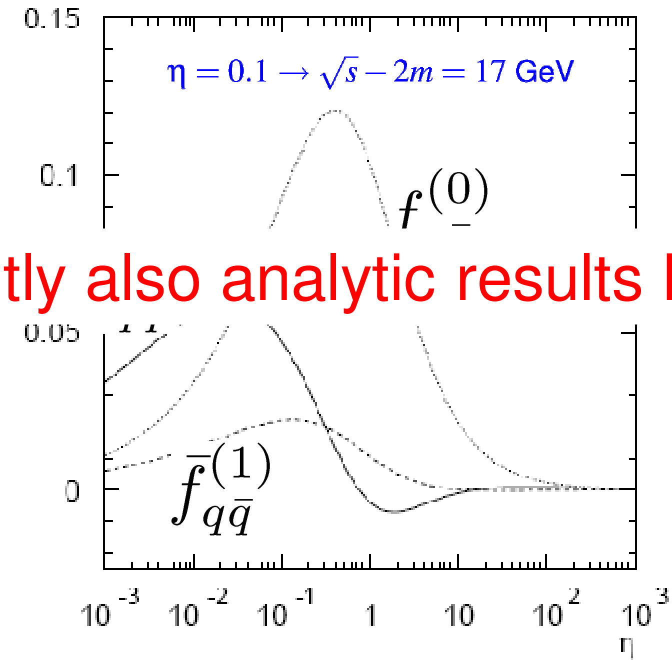
General form:

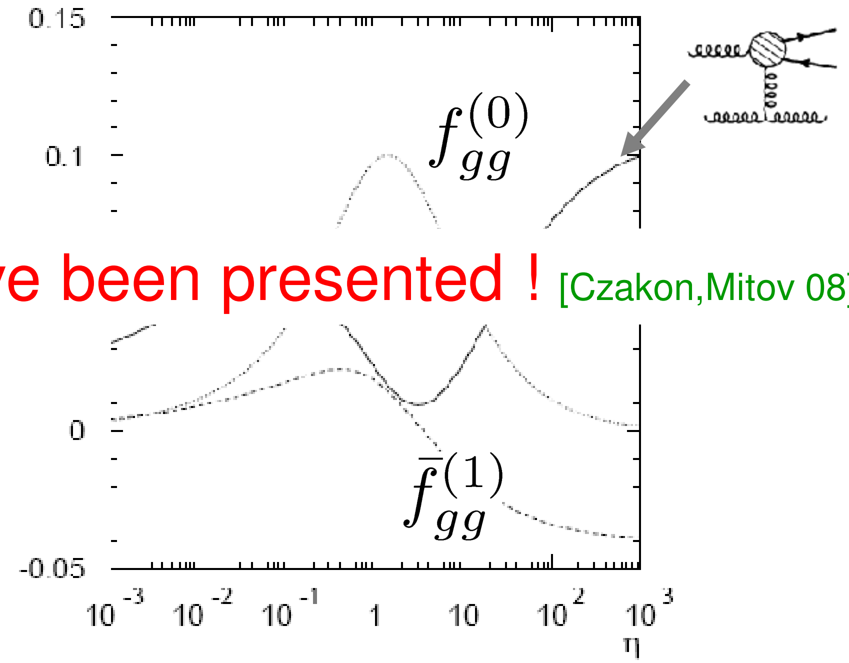
QCD corrections

$$\hat{\sigma}_{ij}(\hat{s}, m_t, \mu) = \frac{\alpha_s(\mu)}{m_t^2}\left(f_{ij}^{(0)}(\rho) + 4\pi\alpha_s(\mu)\left(f_{ij}^{(1)}(\rho) + \bar{f}_{ij}^{(1)}(\rho)\ln\left(\frac{\mu^2}{m_t^2}\right)\right)\right)$$

Functions $f_{ij}^{(1)}$ are obtained from complete NLO calculation

[Dawson et al 88, Beenakker et al 89, Bernreuther et a





Recently also analytic results have been presented ! [Czakon,Mitov 08]

$$\eta = \frac{1}{\rho} - 1$$

$$\rho = \frac{4m^2}{\hat{s}}$$

Top quark spin correlation

Due to parity invariance of QCD, top's produced in qq→tt and gg→ tt are essentially unpolarized

But: **Spins of top quark and antiquark are correlated**

[Bernreuther,Brandenburg 93, Mahlon, Parke 96, Stelzer,Willenbrock 96, Bernreuther, Brandenburg, Si, P.U. 04]

Quantum mechanics: $^{(2s+1)}L_J$

close to threshold:

$^3S_1$ $\quad q\bar{q} \to t\bar{t}$

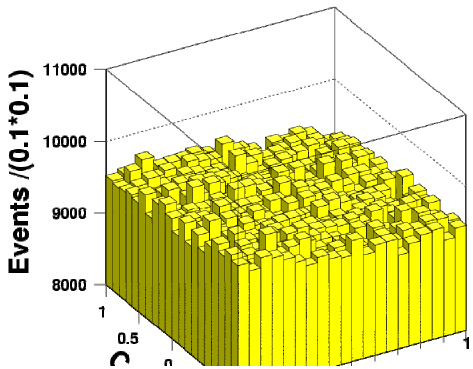
$^1S_0$ $\quad gg \to t\bar{t}$

→ Spins are parallel or anti-parallel close to threshold

$$C_{t\bar{t}} = \frac{\sigma(\uparrow\uparrow)+\sigma(\downarrow\downarrow)-\sigma(\uparrow\downarrow)-\sigma(\downarrow\uparrow)}{\sigma(\uparrow\uparrow)+\sigma(\downarrow\downarrow)+\sigma(\uparrow\downarrow)+\sigma(\downarrow\uparrow)}$$

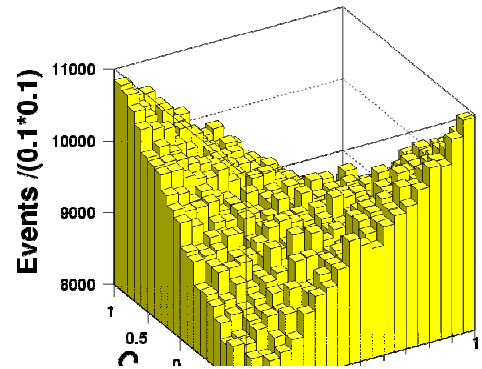
A realistic analysis…

no correlation

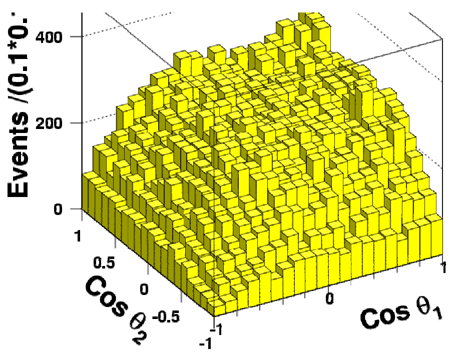


parton level

with correlation

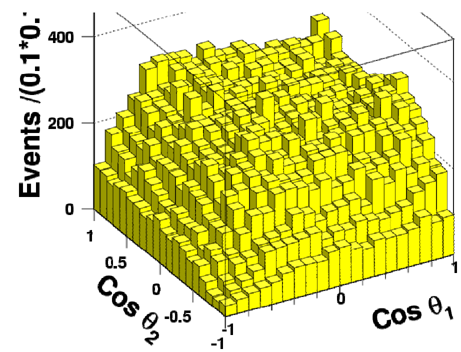


„In theory there is no difference between theory and practice. In practice there is.“ **[Yogi Ber(r)a]**



PYTHIA, reconstruction + cuts

after reconstruction

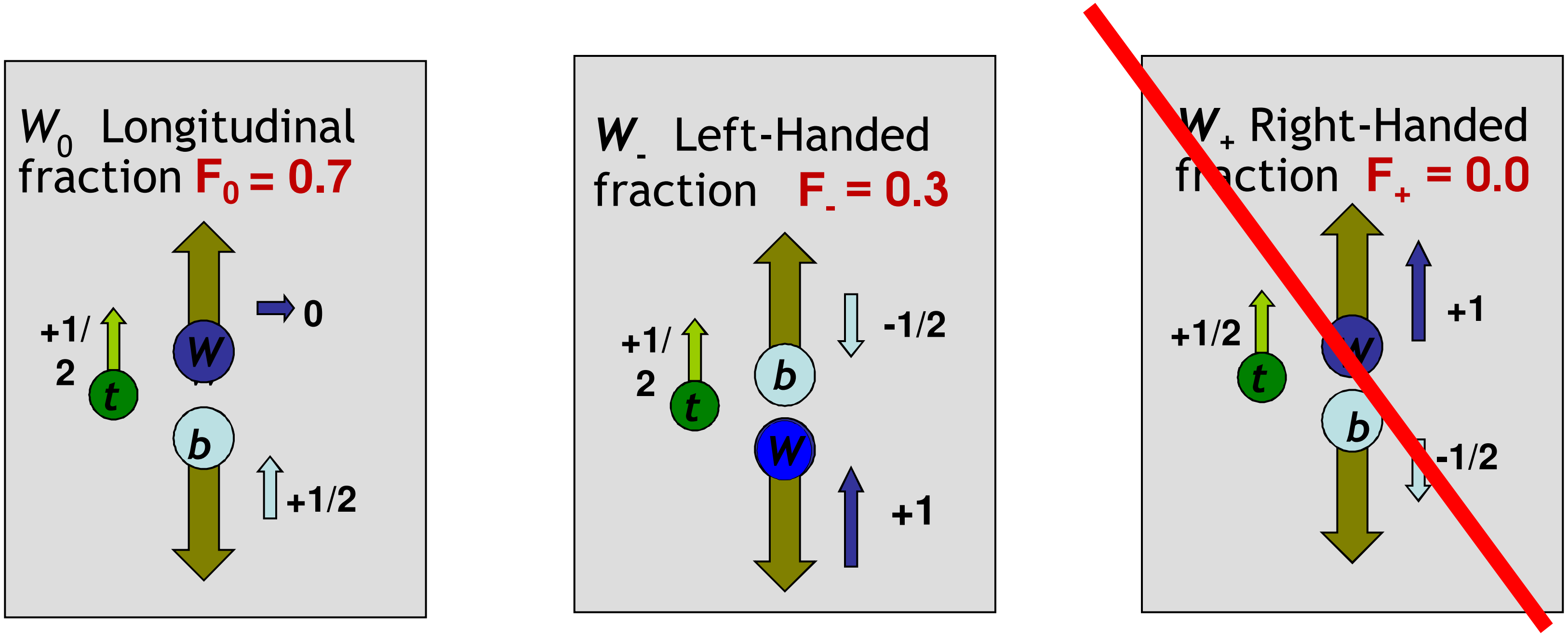


TopReX, reconstruction + cuts

Very difficult analysis, dominated by systematic uncertainties

[ATLAS collaboration, P. Pralavorio, F.Hubaut, E.Monnier]

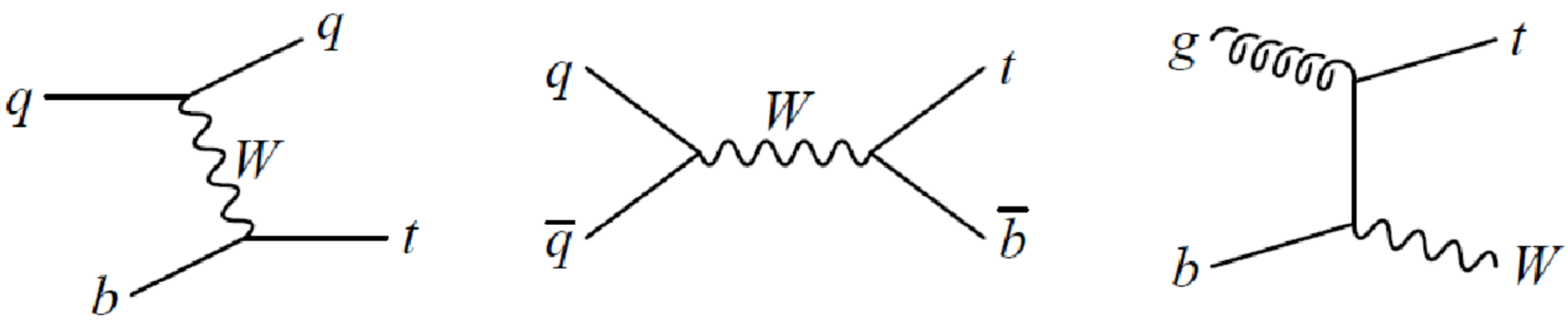
Test of the V-A structure of the Wtb vertex



can be measured at the LHC

➡ Unique source of longitudinal W-bosons at the LHC !

Single top quark production



t-channel *s*-channel *Wt* mode

Known at next-to-leading order QCD

[Smith,Willenbrock; Bordes,van Eijk; Stelzer,Sullivan,Willenbrock; Harris,Laenen,Phaf,Sullivan,Weinzierl; Sullivan]

→ Residual scale uncertainty about 5%

Top's are polarised !

Single top quark production

cross section	t channel	s channel	tW mode
$\sigma^t_{\text{Tevatron}}$	1.15 ± 0.07 pb	0.54 ± 0.04 pb	0.14 ± 0.03 pb
σ^t_{LHC}	150 ± 6 pb	7.8 ± 0.7 pb	44 ± 5 pb
$\sigma^{\bar{t}}_{\text{LHC}}$	92 ± 4 pb	4.3 ± 0.3 pb	44 ± 5 pb

err = PDF, scale, mt

New physics can affect s- and t-channel differently:

[Tait, Yuan 2001]