

Parton distributions: HERA–Tevatron–LHC

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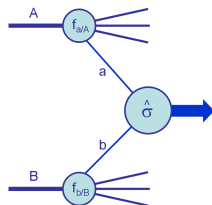
Introduction

- Protons are not elementary particles: made of **partons**.
 \Rightarrow **P**arton **D**istribution **F**unctions (**PDFs**) essential to relate theory to experiment at the LHC (and Tevatron, HERA, ...).
- $f_{a/A}(x, Q^2)$ gives *number density* of partons a in hadron A with momentum fraction x at a hard scale $Q^2 \gg \Lambda_{\text{QCD}}^2$.

$$\sigma_{AB} = \sum_{a,b=q,g} \int_0^1 dx_a \int_0^1 dx_b f_{a/A}(x_a, Q^2) f_{b/B}(x_b, Q^2) \hat{\sigma}_{ab}$$

Outline of talk:

- Global PDF analyses.
- HERA structure functions: F_2 , F_2^c , F_2^b , F_L .
- Tevatron Z , W , jet data and implications.
- α_S and SM cross sections at LHC.



Fixed-order collinear factorisation at hadron colliders

- The “standard” pQCD framework: holds up to formally power-suppressed (“higher-twist”) terms $\mathcal{O}(\Lambda_{\text{QCD}}^2/Q^2)$.
- Expand $\hat{\sigma}_{ab}$, $P_{aa'}$ and β as perturbative series in α_S ($\mu_R = \mu_F = Q$).

$$\sigma_{AB} = \sum_{a,b=q,g} [\hat{\sigma}_{ab}^{\text{LO}} + \alpha_S(Q^2)\hat{\sigma}_{ab}^{\text{NLO}} + \dots] \otimes f_{a/A}(x_a, Q^2) \otimes f_{b/B}(x_b, Q^2)$$

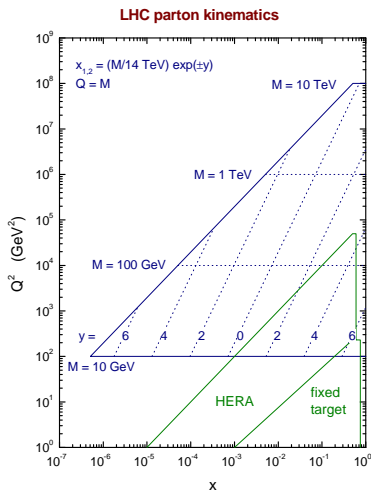
PDF evolution:
$$\frac{\partial f_{a/A}}{\partial \ln Q^2} = \frac{\alpha_S}{2\pi} \sum_{a'=q,g} [P_{aa'}^{\text{LO}} + \alpha_S P_{aa'}^{\text{NLO}} + \dots] \otimes f_{a'/A}$$

α_S evolution:
$$\frac{\partial \alpha_S}{\partial \ln Q^2} = -\beta^{\text{LO}} \alpha_S^2 - \beta^{\text{NLO}} \alpha_S^3 - \dots$$

- Need to extract input values $f_{a/A}(x, Q_0^2)$ and $\alpha_S(M_Z^2)$ from data.
- Structure functions in deep-inelastic scattering (DIS):

$$F_i(x_{\text{Bj}}, Q^2) = \sum_{a=q,g} C_{i,a} \otimes f_{a/A}, \quad C_{i,a} = C_{i,a}^{\text{LO}} + \alpha_S C_{i,a}^{\text{NLO}} + \dots$$

From HERA *et al.* to the LHC



- PDFs are **universal**.
- Fit existing data from **HERA** and **fixed-target** experiments, together with **Tevatron** data.
- **HERA** *ep* (H1, ZEUS).
- **Fixed-target** experiments:
lp, ld
 (BCDMS, NMC, E665, SLAC),
 νN
 (CCFR, NuTeV, CHORUS),
pp, pd (E866/NuSea).
- **Tevatron** *p \bar{p}* (CDF, DØ).
- DGLAP evolution gives PDFs at higher Q^2 for LHC.

Paradigm for PDF determination by “global analysis”

- 1 **Parameterise** the x dependence for each flavour $a = q, g$ at the input scale $Q_0^2 \sim 1 \text{ GeV}^2$ in some flexible form, e.g.

$$xf_{a/p}(x, Q_0^2) = A_a x^{\Delta_a} (1-x)^{\eta_a} (1 + \epsilon_a \sqrt{x} + \gamma_a x),$$

subject to number- and momentum-sum rule constraints.

- 2 **Evolve** the PDFs to higher scales $Q^2 > Q_0^2$ using the DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi) evolution equations.
- 3 **Convolute** the evolved PDFs with $C_{i,a}$ and $\hat{\sigma}_{ab}$ to calculate theory predictions corresponding to a wide variety of data.
- 4 **Vary** the input parameters $\{A_a, \Delta_a, \eta_a, \epsilon_a, \gamma_a, \dots\}$ to minimise

$$\chi^2 = \sum_{i=1}^{N_{\text{pts.}}} \left(\frac{\text{Data}_i - \text{Theory}_i}{\text{Error}_i} \right)^2$$

Determination of parton distributions by global analysis

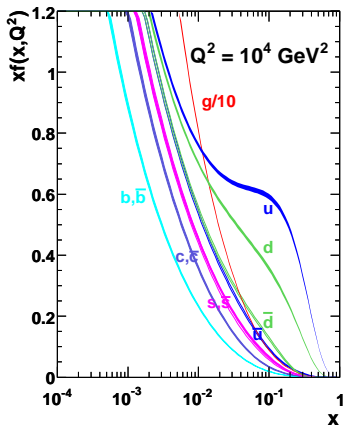
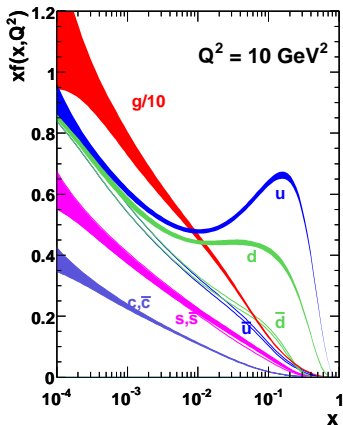
An “industry” for more than 20 years.

Regular updates as new data and theory become available.

- ① First NLO fit: **Martin+Roberts+Stirling** ('87) + **Thorne** ('98).
 Recently, “**MSTW**” = **MRST** – Roberts + **G.W.**
 {MRST 2001 LO, MRST 2004 NLO, MRST 2006 NNLO}
 → **MSTW 2008 LO, NLO, NNLO fits** [[arXiv:0901.0002](https://arxiv.org/abs/0901.0002)]
- ② Other major group: “**CTEQ**” = **C**oordinated
Theoretical–**E**xperimental Project on **Q**CD.
 - CTEQ6L1 LO [[hep-ph/0201195](https://arxiv.org/abs/hep-ph/0201195)]
 - CTEQ6.6 NLO [[arXiv:0802.0007](https://arxiv.org/abs/0802.0007)]
 - CTEQ NNLO?
- ③ Other groups fitting a restricted range of data with fewer free parameters: **S. Alekhin** *et al.*, **HERA** experiments (H1, ZEUS).
- ④ NNPDF Collaboration (see backup slides).

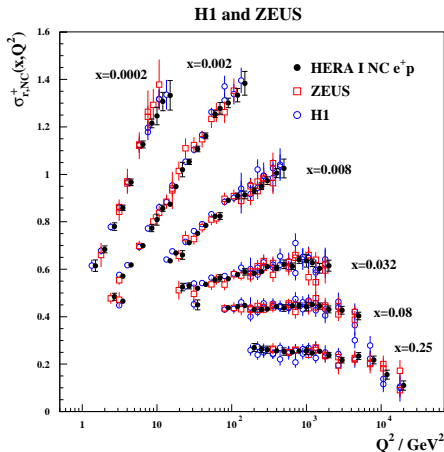
Example of PDFs obtained from global analysis

MSTW 2008 NLO PDFs (68% C.L.)



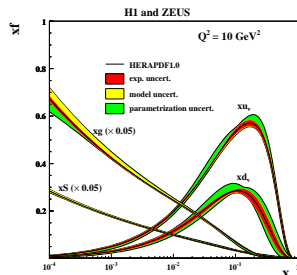
- **Error bands** shown are obtained from propagation of **experimental** uncertainties on the fitted data points.

Combination of H1 and ZEUS data [\[arXiv:0911.0884\]](https://arxiv.org/abs/0911.0884)



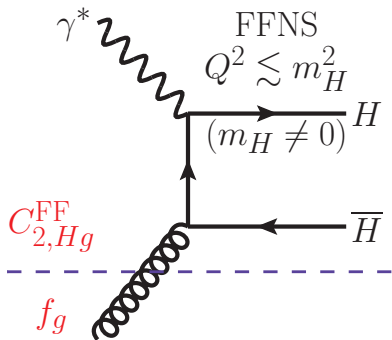
- H1 and ZEUS NC and CC measurements **combined** to improve accuracy: will be used in next generation of global fits.

Fit only to HERA data:



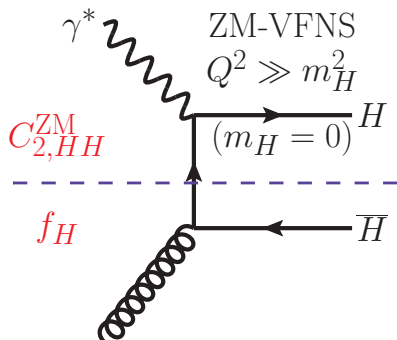
- 10 free input PDF parameters (cf. 28 for **MSTW 2008**).
- Experimental uncertainties using $\Delta\chi^2 = 1$.

Heavy quark contribution to DIS structure function F_2



Fixed flavour number scheme

- No heavy quark PDF.
- Includes $\mathcal{O}(m_H^2/Q^2)$ terms.
- No resummation of $\alpha_S \ln(Q^2/m_H^2)$ terms.



Zero-mass variable flavour number scheme

- Use heavy quark PDF.
- Mass dependence neglected.
- Resums $\alpha_S \ln(Q^2/m_H^2)$ terms similar to light quarks.

General-mass variable flavour number scheme (GM-VFNS)

- Interpolate between two well-defined regions.
- FFNS for $Q^2 \leq m_H^2$, ZM-VFNS for $Q^2 \gg m_H^2$.

CTEQ6.1 NLO (ZM-VFNS) → CTEQ6.5 NLO (GM-VFNS)

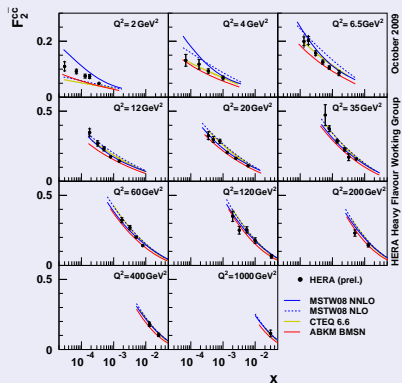
- **8% increase** in W and Z cross sections at LHC.

MRST 2004 → MRST 2006 [[arXiv:0706.0459](https://arxiv.org/abs/0706.0459)]

- The MRST group have used a GM-VFNS since 1998.
- At NNLO, PDFs are discontinuous at $Q^2 = m_H^2$, but neglected in MRST NNLO fits prior to 2006.
- 2004 NNLO → 2006 NNLO: **6% increase** in $\sigma_{W,Z}$ at LHC.
- Pre-2006 MRST *NNLO* PDF sets should be considered **obsolete** due to **incomplete heavy flavour treatment**.
- NNPDF fits (including future NNPDF2.0) still use ZM-VFNS.

Heavy flavour structure function data

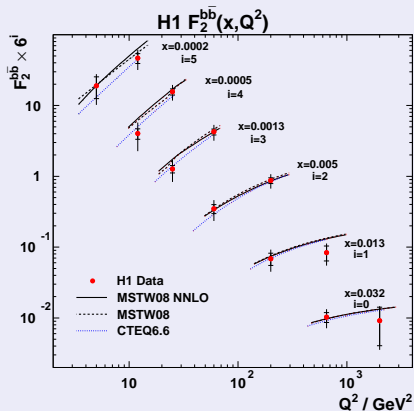
Charm structure function $F_2^{c\bar{c}}$



H1 and ZEUS combination (prel.)

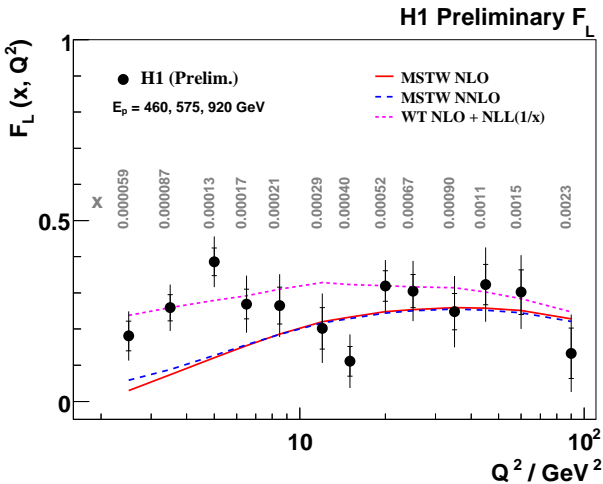
- Good agreement with theoretical predictions using GM-VFNS.

Beauty structure function $F_2^{b\bar{b}}$



[arXiv:0907.2643]

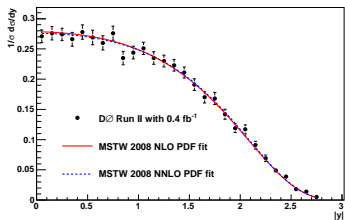
Longitudinal proton structure function at HERA



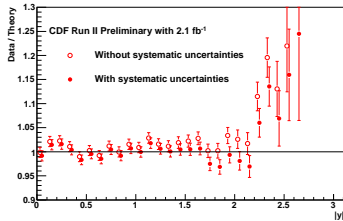
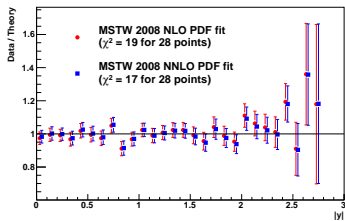
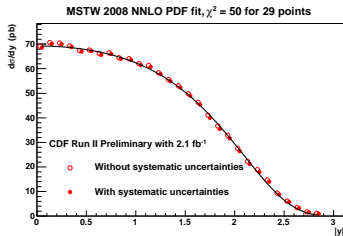
- NLO and NNLO calculations lower than data at low Q^2 .
- Small- x resummation helps [White, Thorne, [hep-ph/0611204](https://arxiv.org/abs/hep-ph/0611204)].

Z/γ^* rapidity distributions from Tevatron Run II

Z/γ^* rapidity shape distribution from $D\bar{D}$



Z/γ^* rapidity distribution from CDF



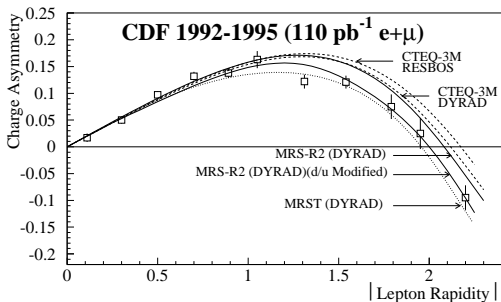
[Data: hep-ex/0702025]

[Data: [arXiv:0908.3914](https://arxiv.org/abs/0908.3914)]

$W \rightarrow \ell\nu$ charge asymmetry from Tevatron Run I

$$A_W(y_W) = \frac{d\sigma(W^+)/dy_W - d\sigma(W^-)/dy_W}{d\sigma(W^+)/dy_W + d\sigma(W^-)/dy_W} \approx \frac{u(x_1)d(x_2) - d(x_1)u(x_2)}{u(x_1)d(x_2) + d(x_1)u(x_2)}$$

But measure $A_\ell(\eta_\ell) = \frac{d\sigma(\ell^+)/d\eta_\ell - d\sigma(\ell^-)/d\eta_\ell}{d\sigma(\ell^+)/d\eta_\ell + d\sigma(\ell^-)/d\eta_\ell}$

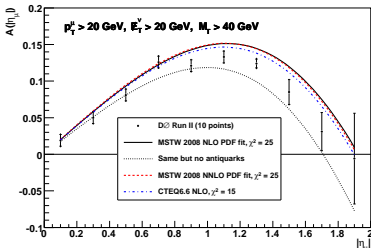


[[hep-ex/9809001](https://arxiv.org/abs/hep-ex/9809001)]

$W \rightarrow \ell\nu$ charge asymmetry from Tevatron Run II

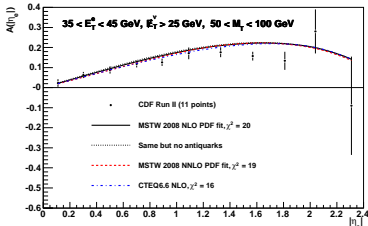
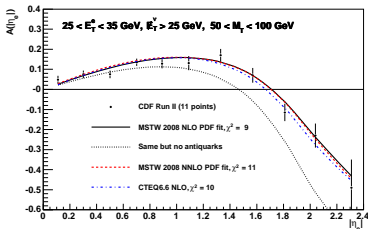
- Run II data in **MSTW 2008** fit.
- Mainly constraint on **down** quark.
- Antiquarks important at low p_T^ℓ .

D \bar{D} data on lepton charge asymmetry from $W \rightarrow \mu\nu$ decays



[Data: [arXiv:0709.4254](https://arxiv.org/abs/0709.4254)]

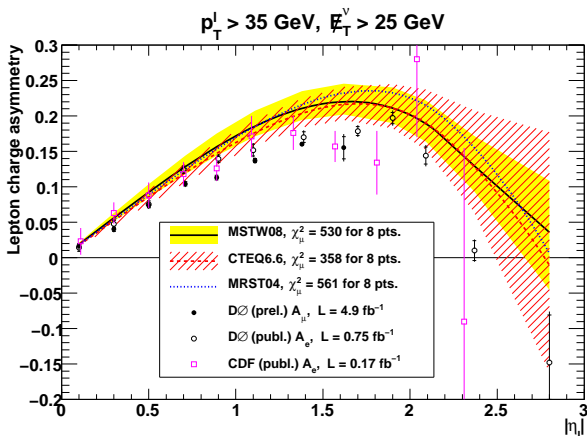
CDF data on lepton charge asymmetry from $W \rightarrow e\nu$ decays



[Data: [hep-ex/0501023](https://hep-ex.org/2005/10/23/)]

Latest DØ data on $W \rightarrow \ell\nu$ charge asymmetry

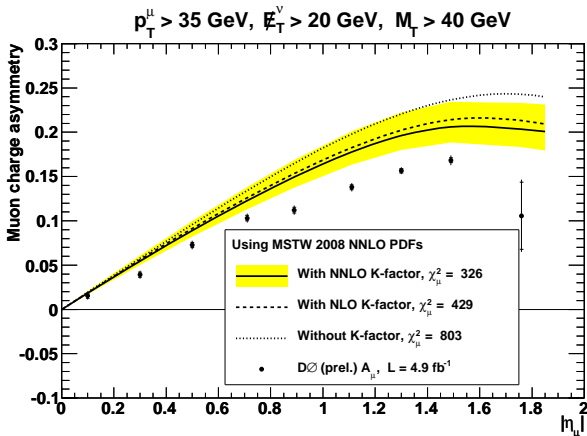
New DØ data: A_e [arXiv:0807.3367] and A_μ [DØ Note 5976-CONF]



- Problems describing new p_T^ℓ data at NLO, especially for $p_T^\ell > 35 \text{ GeV}$.

Latest $D\bar{D}$ data on $W \rightarrow \ell\nu$ charge asymmetry

- Effect of NNLO (or p_T^W -resummation, RESBOS) is small.

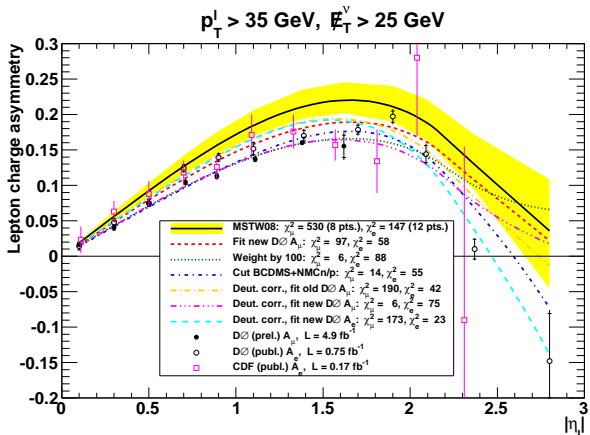


NNLO: Catani, Cieri, Ferrera, de Florian, Grazzini, arXiv:0903.2120

(Previous calculation: Melnikov, Petriello, hep-ph/0609070, FEWZ)

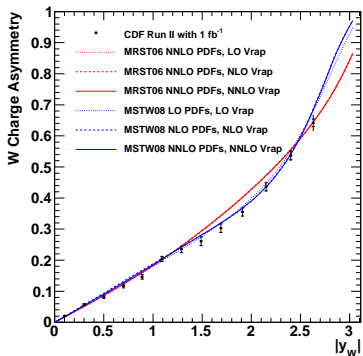
Latest $D\bar{D}$ data on $W \rightarrow \ell\nu$ charge asymmetry

- Can the PDFs be refitted to describe the new data?

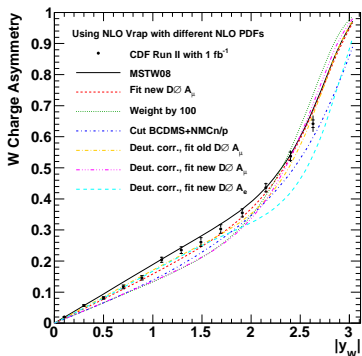


- ... Not both $D\bar{D}$ A_{μ} and A_e simultaneously.

W charge asymmetry from Tevatron Run II

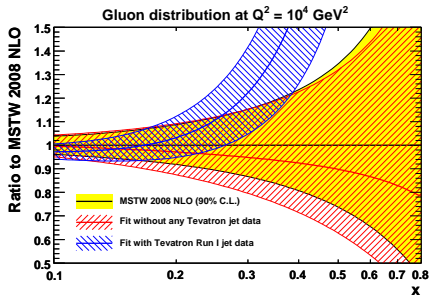
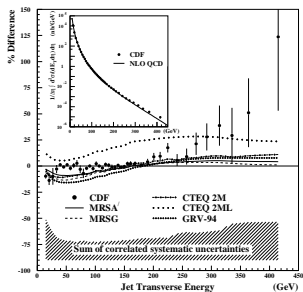


- Data: [arXiv:0901.2169](https://arxiv.org/abs/0901.2169)
- Vrap: Anastasiou, Dixon, Melnikov, Petriello, [hep-ph/0312266](https://arxiv.org/abs/hep-ph/0312266)



- **MSTW08** good description (better than MRST06).
- Modified fits to new $D \otimes A_{\ell}$ tend to **undershoot** CDF A_W .

Impact of Tevatron Run II inclusive jet production data



- Initial Tevatron Run I jet data showed an **excess** at high E_T , later **accommodated** by refitting gluon distribution.
- Run I data included in recent PDF fits up to **MRST 2006** (and current **CTEQ6.6**).
- **MSTW 2008** is first PDF fit to include Run II jet data: preference for **smaller** gluon distribution at high x .
- Similar findings by CTEQ [[arXiv:0904.2424](https://arxiv.org/abs/0904.2424)].

Tension between Run I and Run II inclusive jet data

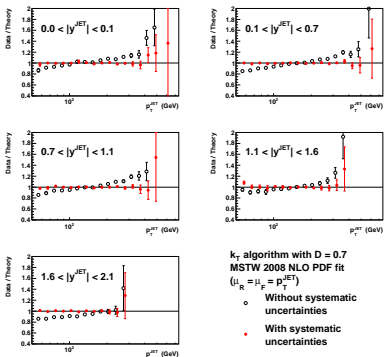
Highlighted numbers indicate χ^2 values for data sets explicitly included in various NLO global fits:

CDFI (33 pts.)	DØI (90 pts.)	CDFII(k_T) (76 pts.)	DØII (110 pts.)	$\Delta\chi^2_{\text{non-jet}}$ (2513 pts.)	$\alpha_S(M_Z^2)$
53	119	64	117	0	0.1197
51	48	132	180	9	0.1214
56	110	56	114	2	0.1202
53	85	68	117	1	0.1204

- Fit to Run I jets \Rightarrow description of Run II jets bad.
- Fit to Run II jets \Rightarrow description of Run I jets bad.
- Fit neither \Rightarrow similar description as fitting Run II only.
- **Summary:** Some inconsistency between Run I and Run II jets. Run II jets slightly more consistent with rest of data.

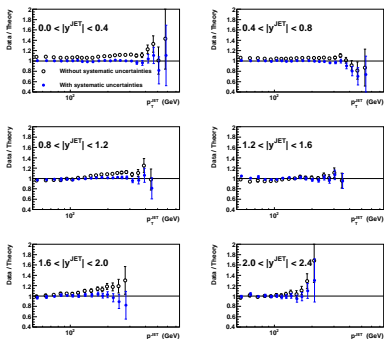
Description of Tevatron Run II inclusive jet data

CDF Run II inclusive jet data, $\chi^2 = 56$ for 76 pts.



[Data: [hep-ex/0701051](https://arxiv.org/abs/hep-ex/0701051)]

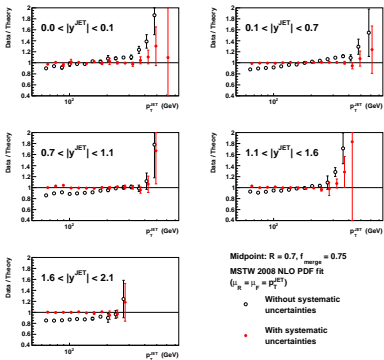
$\text{D}\emptyset$ Run II inclusive jet data (cone, $R = 0.7$)
 MSTW 2008 NLO PDF fit ($\mu_R = \mu_F = p_T^{\text{JET}}$), $\chi^2 = 114$ for 110 pts.



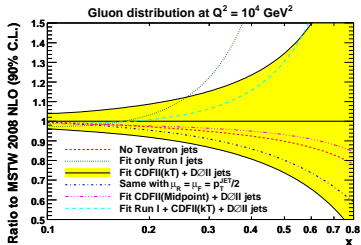
[Data: [arXiv:0802.2400](https://arxiv.org/abs/0802.2400)]

Effect of CDF Run II jet data using Midpoint algorithm

CDF Run II inclusive jet data, $\chi^2 = 108$ for 72 pts.

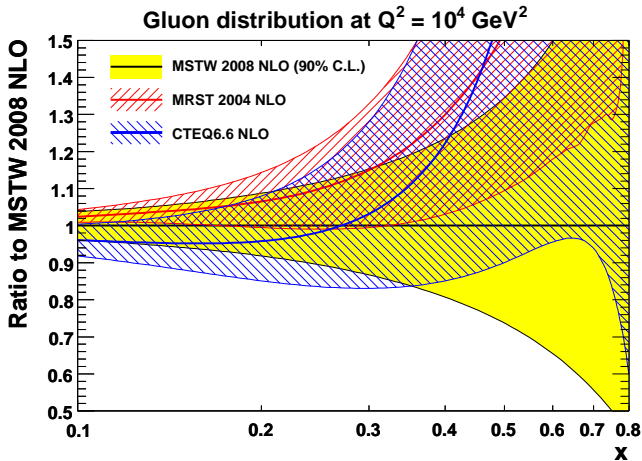


[Data: [arXiv:0807.2204](https://arxiv.org/abs/0807.2204)]



- Only slight change in gluon if replace CDF(k_T) by CDF(**Midpoint**) data.
- Scale choice
 $\mu_R = \mu_F = p_T/2$ gives smaller gluon, but within uncertainties.

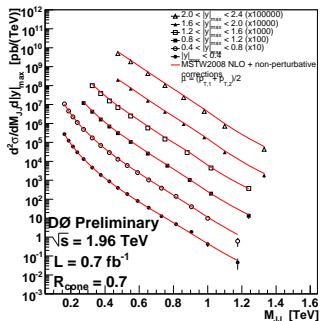
New high- x gluon distribution compared to previous sets



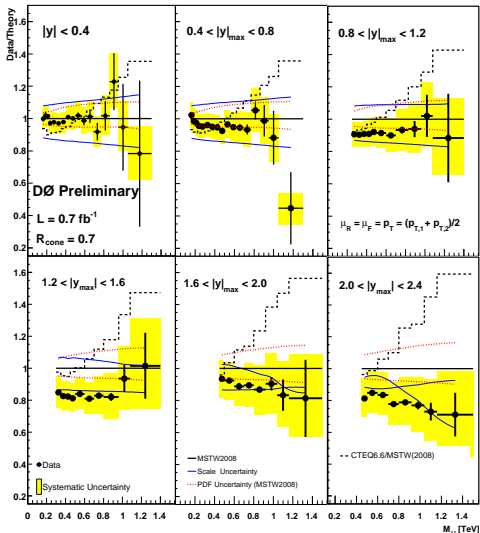
- Smaller high- x gluon than previous MRST and CTEQ fits.

Description of $D\bar{D}$ dijet mass spectrum

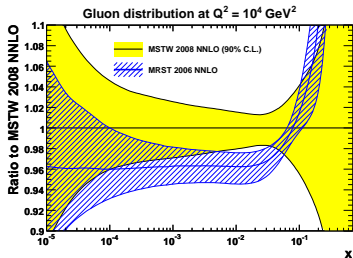
[$D\bar{D}$ Note 5919-CONF]



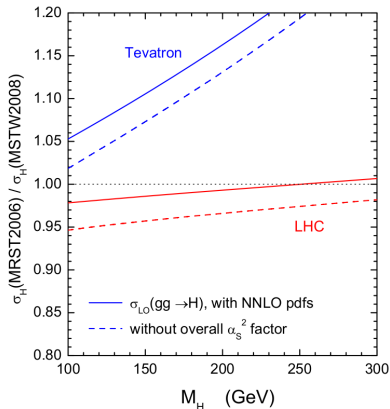
- Data favour less gluon at high x (MSTW 2008 over CTEQ6.6).



Implications of new PDFs for Higgs cross sections

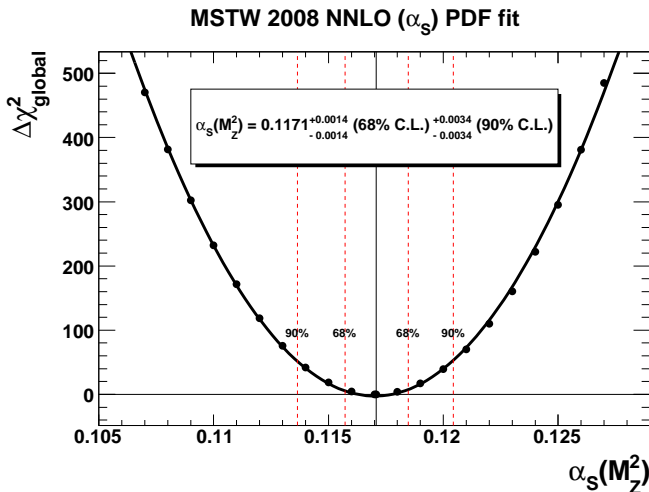


- NNLO trend similar to NLO (N.B. exact NNLO jet cross section unavailable, use threshold corrections).
- $\alpha_S(M_Z^2) = 0.1191$ (2006)
→ 0.1171 (MSTW 2008)



- Higgs cross sections **smaller** at Tevatron with **2008** PDFs.
- Used in Tevatron exclusion results (March 2009).

Determination of $\alpha_S(M_Z^2)$ from NNLO global PDF analysis

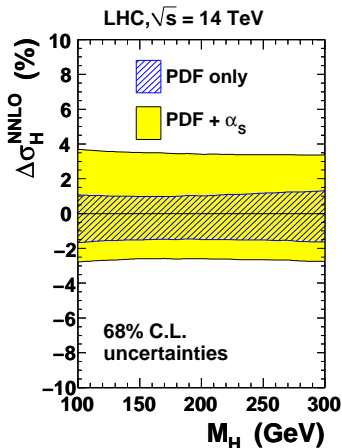
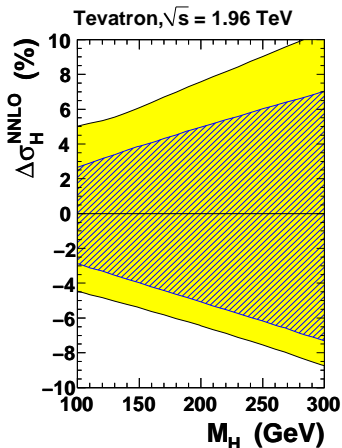


- Additional theory uncertainty ($\lesssim |\text{NNLO} - \text{NLO}| = 0.003$).
- cf. PDG world average value of $\alpha_S(M_Z^2) = 0.1176 \pm 0.002$.

Impact of α_S on SM Higgs uncertainty versus M_H

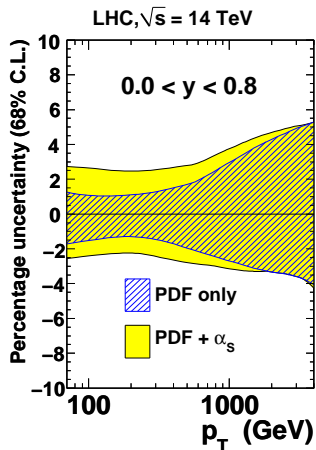
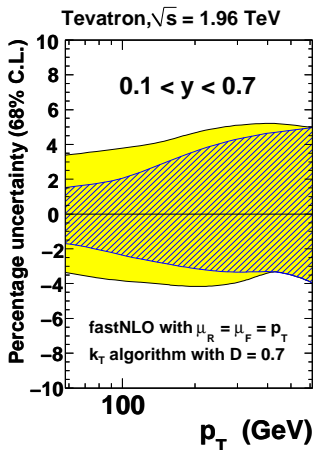
- **Correlation** between PDF and α_S uncertainties in cross section calculations [MSTW, arXiv:0905.3531].

Higgs cross sections with MSTW 2008 NNLO PDFs



Impact of α_S on inclusive jet uncertainty versus p_T

Inclusive jet cross sections with MSTW 2008 NLO PDFs



- Mostly gluon-initiated at low $p_T \Rightarrow$ correlated with α_S .
- Mostly quark-initiated at high $p_T \Rightarrow$ anticorrelated with α_S .

W and Z total cross sections at the LHC

- Potential “**standard candle**” for luminosity determination.
- NNLO total cross sections with “**PDF+ α_s** ” uncertainties using MSTW 2008 NNLO PDFs:

LHC	$B_{\ell\nu} \cdot \sigma_W$ (nb)	$B_{\ell+\ell-} \cdot \sigma_Z$ (nb)	R_{WZ}
$\sqrt{s} = 7$ TeV	$10.47^{+0.27}_{-0.20}$ $\left(\begin{smallmatrix} +2.5\% \\ -1.9\% \end{smallmatrix} \right)$	$0.958^{+0.024}_{-0.018}$ $\left(\begin{smallmatrix} +2.5\% \\ -1.9\% \end{smallmatrix} \right)$	$10.92^{+0.03}_{-0.02}$ $\left(\begin{smallmatrix} +0.3\% \\ -0.2\% \end{smallmatrix} \right)$
$\sqrt{s} = 10$ TeV	$15.35^{+0.39}_{-0.31}$ $\left(\begin{smallmatrix} +2.6\% \\ -2.0\% \end{smallmatrix} \right)$	$1.429^{+0.037}_{-0.027}$ $\left(\begin{smallmatrix} +2.6\% \\ -1.9\% \end{smallmatrix} \right)$	$10.74^{+0.03}_{-0.03}$ $\left(\begin{smallmatrix} +0.3\% \\ -0.3\% \end{smallmatrix} \right)$
$\sqrt{s} = 14$ TeV	$21.72^{+0.56}_{-0.48}$ $\left(\begin{smallmatrix} +2.6\% \\ -2.2\% \end{smallmatrix} \right)$	$2.051^{+0.053}_{-0.043}$ $\left(\begin{smallmatrix} +2.6\% \\ -2.1\% \end{smallmatrix} \right)$	$10.59^{+0.03}_{-0.03}$ $\left(\begin{smallmatrix} +0.3\% \\ -0.3\% \end{smallmatrix} \right)$

LHC	$B_{\ell\nu} \cdot \sigma_{W^+}$ (nb)	$B_{\ell\nu} \cdot \sigma_{W^-}$ (nb)	R_{\pm}
$\sqrt{s} = 7$ TeV	$6.16^{+0.16}_{-0.12}$ $\left(\begin{smallmatrix} +2.6\% \\ -2.0\% \end{smallmatrix} \right)$	$4.31^{+0.11}_{-0.08}$ $\left(\begin{smallmatrix} +2.5\% \\ -2.0\% \end{smallmatrix} \right)$	$1.429^{+0.015}_{-0.012}$ $\left(\begin{smallmatrix} +1.1\% \\ -0.8\% \end{smallmatrix} \right)$
$\sqrt{s} = 10$ TeV	$8.88^{+0.23}_{-0.19}$ $\left(\begin{smallmatrix} +2.6\% \\ -2.1\% \end{smallmatrix} \right)$	$6.47^{+0.16}_{-0.13}$ $\left(\begin{smallmatrix} +2.5\% \\ -2.0\% \end{smallmatrix} \right)$	$1.373^{+0.013}_{-0.010}$ $\left(\begin{smallmatrix} +0.9\% \\ -0.7\% \end{smallmatrix} \right)$
$\sqrt{s} = 14$ TeV	$12.39^{+0.32}_{-0.28}$ $\left(\begin{smallmatrix} +2.6\% \\ -2.3\% \end{smallmatrix} \right)$	$9.33^{+0.24}_{-0.20}$ $\left(\begin{smallmatrix} +2.6\% \\ -2.1\% \end{smallmatrix} \right)$	$1.328^{+0.011}_{-0.009}$ $\left(\begin{smallmatrix} +0.9\% \\ -0.7\% \end{smallmatrix} \right)$

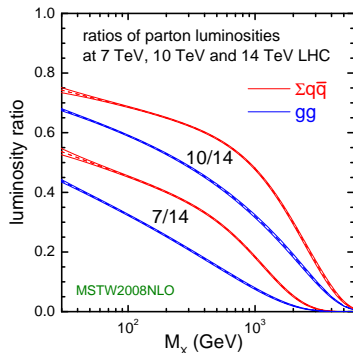
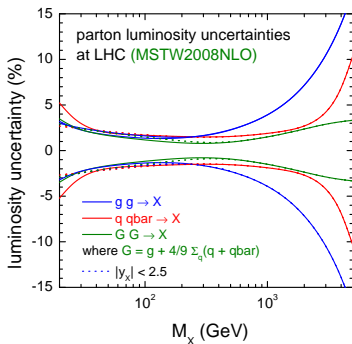
- Additional uncertainty from scale variation less than 1%.

Parton luminosity functions at the LHC

If $\hat{\sigma}_{ab} = C_{ab} \delta(\hat{s} - M_X^2)$, with $\hat{s} = x_a x_b s$, then

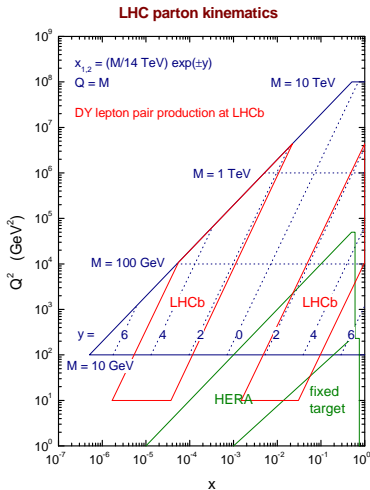
$$\sigma_{AB} = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b f_{a/A}(x_a, M_X^2) f_{b/B}(x_b, M_X^2) \hat{\sigma}_{ab} = \sum_{a,b} C_{ab} \frac{\partial \mathcal{L}_{ab}}{\partial M_X^2}$$

$$\frac{\partial \mathcal{L}_{ab}}{\partial M_X^2} = \int_{\tau}^1 \frac{dx}{x} f_{a/A}(x, M_X^2) f_{b/B}(\tau/x, M_X^2), \quad \tau = \frac{M_X^2}{s}$$

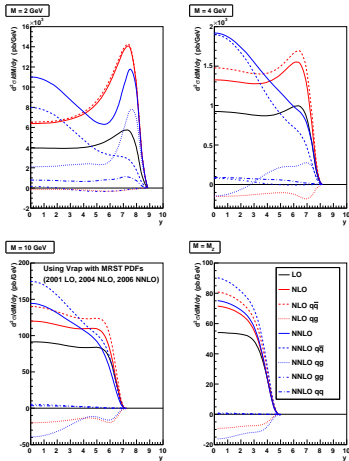


Precision measurements at high rapidity from LHCb

[MSTW, arXiv:0808.1847; R. McNulty, arXiv:0810.2550]



γ^*/Z rapidity distributions at LHC



Summary

- **Parton Distribution Functions (PDFs)** are a non-negotiable input to all theory predictions at hadron colliders.
- **HERA averaged** cross sections reduce uncertainties and will be an important input to future global PDF analyses.
- **Tevatron Run II W, Z** data provide PDF constraints, but **problems** describing new $W \rightarrow \ell\nu$ charge asymmetry data.
- **Tevatron Run II jets** prefer **smaller high- x gluon** than Run I: impact on Higgs cross sections at Tevatron.
- Now possible to consistently calculate combined “**PDF+ α_S** ” uncertainty on hadronic cross sections.

PDFs for use in LO Monte Carlo event generators

- Which PDFs (f^X , $X = \text{LO}, \text{NLO}, \dots$) to use if only a LO $\hat{\sigma}$ is available? Define the “truth” to be $f^{\text{NLO}} \otimes \hat{\sigma}^{\text{NLO}}$, and

$$K(X) = \frac{f^{\text{NLO}} \otimes \hat{\sigma}^{\text{NLO}}}{f^X \otimes \hat{\sigma}^{\text{LO}}}$$

- [A. Sherstnev](#) and [R. Thorne](#) have studied modified LO PDFs (LO* and LO**) which give $K(\text{LO}^*)$ or $K(\text{LO}^{**})$ much closer to 1 than either $K(\text{LO})$ or $K(\text{NLO})$ for a variety of processes.
 - LO* [[arXiv:0711.2473](#)]: LO PDF fit with violation of momentum-sum rule and NLO α_S .
 - LO** [[arXiv:0807.2132](#)]: same but also modified α_S scale in PDF evolution, similar to in parton shower.

Based on MRST 2006 analysis: will be updated soon.

- [CTEQ](#) [[arXiv:0910.4183](#)]: similar idea with NLO “pseudodata”.

Criteria for choice of tolerance $T = \sqrt{\Delta\chi^2_{\text{global}}}$

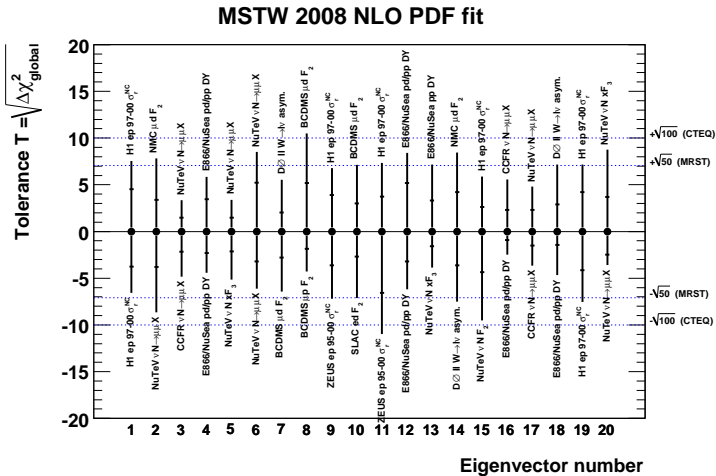
Parameter-fitting criterion

- $T^2 = 1$ for 68% ($1-\sigma$) C.L., $T^2 = 2.71$ for 90% C.L.
- **In practice:** minor inconsistencies between fitted data sets, and unknown experimental and theoretical uncertainties, so **not appropriate for global PDF analysis.**

Hypothesis-testing criterion (proposed by CTEQ)

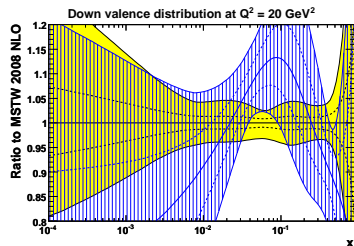
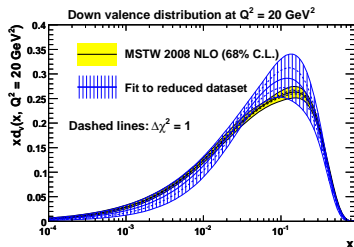
- Much weaker: treat PDF sets obtained from eigenvectors of covariance matrix as **alternative hypotheses.**
- Determine T^2 from the criterion that **each data set should be described within its 90% C.L. limit.** Very roughly, a “good” fit has $\chi^2 \simeq N_{\text{pts.}} \pm \sqrt{2N_{\text{pts.}}}$ for each data set.
- **CTEQ:** $T^2 = 100$ for 90% C.L. limit, **MRST:** $T^2 = 50$.

Dynamic tolerance: different for each eigenvector



- Outer (inner) error bars give tolerance for 90% (68%) C.L.

Test of dynamic tolerance: fit to reduced dataset



- Fit to **reduced dataset** comprising **589** DIS data points, cf. **2699** data points in **global** fit.
- Errors given by $T^2 = 1$ don't overlap \Rightarrow inconsistent data sets included in global fit.
- **Dynamic tolerance** $T^2 > 1$ **accommodates** mildly inconsistent data sets.
- **J. Pumplin** [[arXiv:0909.0268](https://arxiv.org/abs/0909.0268)]: significant tension from BCDMS/NMC supports $T^2 \approx 10$ for 90% C.L. uncertainties.
- **Issues:**
 - ① $T^2 > 1$ not rigorous?
 - ② Dependence on input parameterisation?

Alternative approach: NNPDF Collaboration

NNPDF Collaboration: R. Ball, L. Del Debbio, S. Forte,
A. Guffanti, J. Latorre, A. Piccione, J. Rojo, M. Ubiali

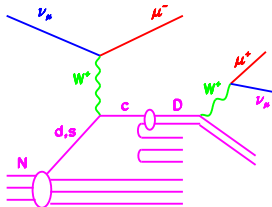
MSTW approach [[arXiv:0901.0002](https://arxiv.org/abs/0901.0002)] (CTEQ similar)

Parameterisation	$xf_{a/p} \sim A_a x^{\Delta_a} (1-x)^{\eta_a} (1 + \epsilon_a \sqrt{x} + \gamma_a x)$
Minimisation	Non-linear least-squares (Marquardt method)
Error propagation	Hessian method with dynamical tolerance
Application	Use best-fit and 40 eigenvector PDF sets

NNPDF approach [[arXiv:0808.1231](https://arxiv.org/abs/0808.1231), [arXiv:0906.1958](https://arxiv.org/abs/0906.1958)]

Parameterisation	Neural network (37 free parameters per PDF)
Minimisation	Genetic algorithm (stop before overlearning)
Error propagation	Generate $N_{\text{rep}} \sim \mathcal{O}(1000)$ MC data replicas
Application	Calculate average and s.d. over N_{rep} PDF sets

Illustration: NuTeV/CCFR dimuon cross sections



$$\frac{d\sigma}{dx dy}(\nu_\mu N \rightarrow \mu^+ \mu^- X) \propto \frac{d\sigma}{dx dy}(\nu_\mu N \rightarrow \mu^- c X)$$

- ν_μ and $\bar{\nu}_\mu$ cross sections constrain s and \bar{s} , respectively, for $0.01 \lesssim x \lesssim 0.2$.

- Can **relax assumption** made in previous fits that

$$s(x, Q_0^2) = \bar{s}(x, Q_0^2) = \frac{\kappa}{2} [\bar{u}(x, Q_0^2) + \bar{d}(x, Q_0^2)], \text{ with } \kappa \approx 0.5.$$

- MSTW **parameterise** at input scale of $Q_0^2 = 1 \text{ GeV}^2$ in the form:

$$xS^+(x, Q_0^2) \equiv xS(x, Q_0^2) + x\bar{s}(x, Q_0^2) = A_+ (1-x)^{\eta_+} xS(x, Q_0^2),$$

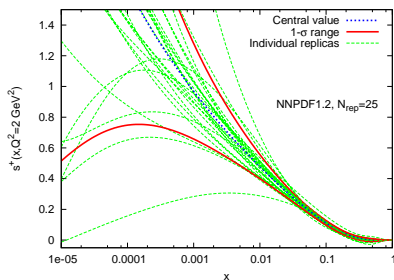
$$xS^-(x, Q_0^2) \equiv xS(x, Q_0^2) - x\bar{s}(x, Q_0^2) = A_- x^{0.2} (1-x)^{\eta_-} (1-x/x_0).$$

- x_0 fixed by zero strangeness: $\int_0^1 dx [s(x, Q_0^2) - \bar{s}(x, Q_0^2)] = 0$.

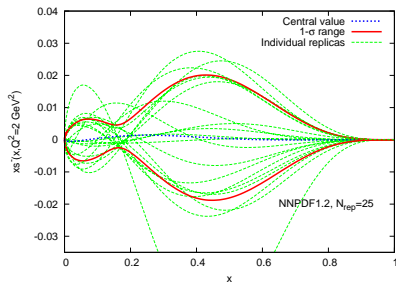
First fits from NNPDF Collaboration

- NNPDF1.0: fit only DIS structure function data.
Fix $s = \bar{s} = (\bar{u} + \bar{d})/4$ at $Q_0^2 = 2 \text{ GeV}^2$.
- NNPDF1.1: free strangeness but no νN dimuon data.
- NNPDF1.2: free strangeness and add νN dimuon data.

$s^+ \equiv s + \bar{s}$ at $Q^2 = 2 \text{ GeV}^2$:



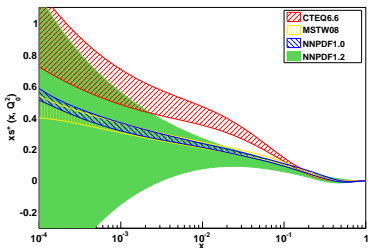
$xs^- \equiv xs - x\bar{s}$ at $Q^2 = 2 \text{ GeV}^2$:



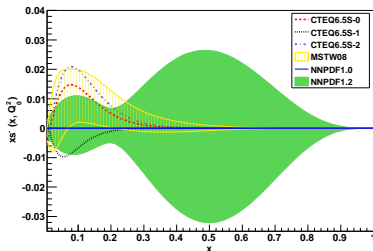
- Data only constrain $0.01 \lesssim x \lesssim 0.2$.

NNPDFs compared to “standard” PDFs

$$x s^+ \equiv x s + x \bar{s} \text{ at } Q^2 = 2 \text{ GeV}^2:$$



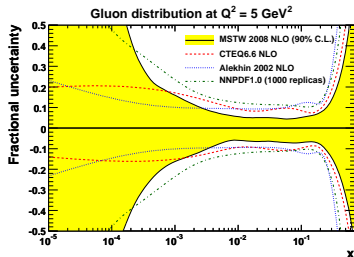
$$x s^- \equiv x s - x \bar{s} \text{ at } Q^2 = 2 \text{ GeV}^2:$$



- NNPDF uncertainties much larger in regions of no data.
- MSTW use a relatively restrictive input parameterisation:
 - ① Restrict small- x s^+ to be (mass-suppressed) fraction of \bar{u}, \bar{d} .
 - ② No reason to expect very large asymmetry s^- at large x .
- CTEQ s^+ disagrees even in region of data: not understood.

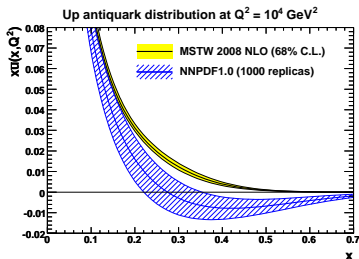
NNPDFs compared to “standard” PDFs

Gluon fractional uncertainty



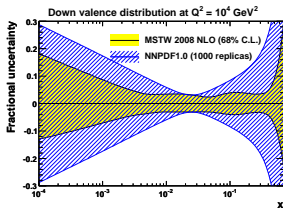
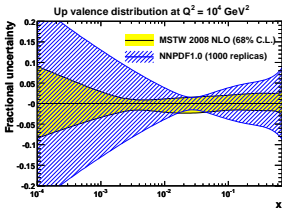
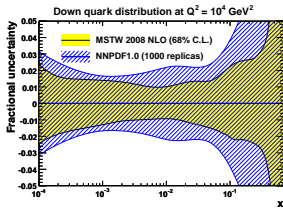
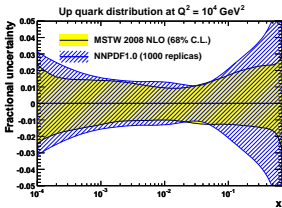
- MSTW small- x gluon:
 $xg(x, Q_0^2) \sim A_g x^{\delta_g} + A_{g'} x^{\delta_{g'}}$
- CTEQ, Alekhin:
 $xg(x, Q_0^2) \sim A_g x^{\delta_g}$.

Up antiquark at large x



- NNPDF1.0 negative by $\sim 2\text{-}\sigma$ at large $x \sim 0.5$.
- No E866/NuSea Drell–Yan data to constrain.

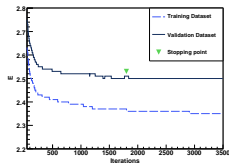
Uncertainties for MSTW 2008 and NNPDF1.0



- NNPDF1.0 uncertainties generally larger, but also less data.
- Fully global fit with Tevatron data (NNPDF2.0) in progress.

Potential issues with stopping criterion in NNPDF fits

- Partition each data set into “**training**” and “**validation**” to avoid overlearning. Stop fit when χ^2_{val} increases. Complications due to each data set having a different **training length**.



Talk by S. Forte, PDF4LHC, DESY, 23rd October 2009:

REMOVE STOPPING: OVERLEARNING FIT

PERFORM A FIT WITH A FIXED, VERY LARGE NUMBER OF GA GENERATIONS:

25000 gens. (AVERAGE 1000 gens. FOR STANDARD FIT)

	STANDARD STOPPING			FIXED LONG	
	REPLICAS	CENTRAL VALUE	FIXED PARTITION	REPLICAS	CENTRAL VALUE
χ^2	1.32	1.32	~ 1.3	1.18	1.19
$\langle \chi^2_{\text{rep}} \rangle$	2.79 ± 0.24	1.65 ± 0.20	$\sim 1.6 \pm 0.2$	2.43 ± 0.13	1.29 ± 0.06
$\langle \chi^2_{\text{tr}} \rangle_{\text{rep}}$	2.76	1.59	~ 1.6	2.40	1.27
$\langle \chi^2_{\text{val}} \rangle_{\text{rep}}$	2.80	1.61	~ 1.6	2.47	1.30
$\langle \sigma^{\text{dat}} \rangle$	0.039	0.035	~ 0.03	0.032	0.019

χ^2 OF THE GLOBAL FIT DECREASES A LOT!
IS IT REALLY OVERLEARNING?

- $\langle \chi^2_{\text{val}} \rangle_{\text{rep}}$ continues decreasing after standard stopping.
- Stopping point has significant influence on PDF uncertainties.