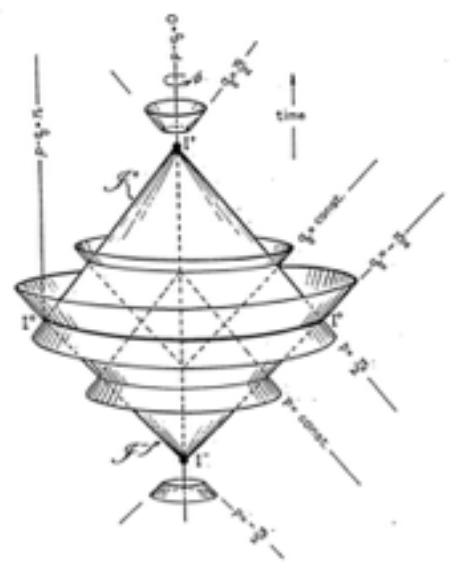
CERN TH String Theory Seminar
November 11, 2014



Penrose, Les Houches 1963

Holographic aspects of gravity in 4 and 3 dimensions

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Overview

Symmetry groups of asymptotically AdS and flat spacetimes

with C. Troessaert, P.H. Lambert

Current algebra of 4d flat case

with C. Troessaert

New results in 3 dimensions: 3d gravity as group theory

with A. Gomberoff, H.A. Gonzalez, B. Oblak

Asymptotic symmetries   Gauge fixation

Main idea : asymptotic symmetries = residual gauge symmetries

BMS ansatz

$$g^{\mu\nu} = \begin{pmatrix} 0 & -e^{-2\beta} & 0 \\ -e^{-2\beta} & -\frac{V}{r}e^{-2\beta} & -U^B e^{-2\beta} \\ 0 & -U^A e^{-2\beta} & g^{AB} \end{pmatrix}$$

$$u \qquad r \qquad x^A = \phi, \theta, \chi, \dots$$

null coordinate

d-1 gauge conditions $g^{uu} = 0 = g^{uA}$

determinant condition $\det g_{AB} = r^{2(d-2)} \det \bar{\gamma}_{AB}$ $\quad \bar{\gamma}_{AB} dx^A dx^B = e^{2\varphi} d^{d-2}\Omega$

conformal to metric on unit d-2 sphere

➡ fix diffeomorphism invariance in d dimensions

Minkowski and AdS backgrounds

backgrounds $\beta = 0 = U^A = \varphi, \quad \frac{V}{r} = -\frac{r^2}{l^2} - 1$

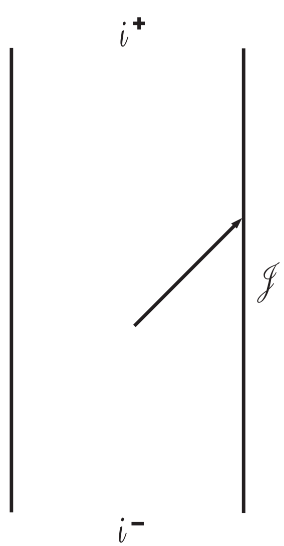
Minkowski d $l^{-1} = 0 \qquad t = u + r \qquad ds^2 = -dt^2 + dr^2 + r^2 d^{d-2}\Omega$

AdS d $t = u + l \arctan \frac{r}{l} \qquad ds^2 = -(\frac{r^2}{l^2} + 1)dt^2 + (\frac{r^2}{l^2} + 1)^{-1} dr^2 + r^2 d^{d-2}\Omega$

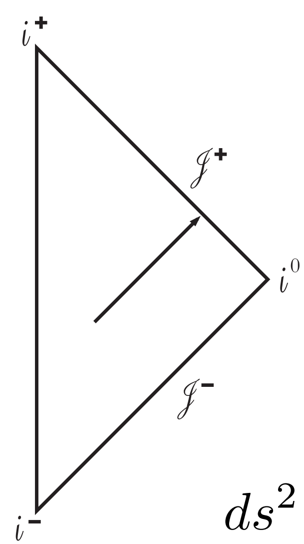
asymptotics $r \to \infty, \quad u, x^A \text{fixed}$

conf. rescaled
induced metric

$ds^2 = -l^{-2} du^2 + d^{d-2}\Omega$



spatial infinity



$ds^2 = 0 du^2 + d^{d-2}\Omega$

null infinity

Asymptotic symmetries   Residual gauge transformations

fall-off conditions

$$\begin{cases} \beta = o(1) \\ U^A = o(1) \\ g_{AB}dx^A dx^B = r^2 \bar{\gamma}_{AB} dx^A dx^B + o(r^2) \\ \frac{V}{r} = -\frac{r^2}{l^2} + o(r^2) \end{cases}$$

leave class of spacetimes invariant

exact conditions

$$\begin{cases} \mathcal{L}_\xi g_{rr} = 0 \\ \mathcal{L}_\xi g_{rA} = 0 \\ g^{AB}\mathcal{L}_\xi g_{AB} = 0 \end{cases} \Rightarrow \begin{cases} \xi^u = F \\ \xi^A = Y^A - \partial_B F \int_r^\infty dr' e^{2\beta} g^{AB} \\ \xi^r = -\frac{r}{d-2}(\bar{D}_B \xi^B - \partial_B \xi^u U^B) \end{cases}$$

fix r dependence up to integration functions $\quad F = F(u, x^B), \quad Y^A = Y^A(u, x^B)$

asymptotic conditions

$$\begin{cases} \mathcal{L}_\xi g_{ur} = o(1) \\ \mathcal{L}_\xi g_{uA} = o(r^2) \\ \mathcal{L}_\xi g_{uu} = o(r^2) \\ \mathcal{L}_\xi g_{AB} = o(r^2) \end{cases} \Rightarrow \begin{cases} F = f + \int_0^u du' \bar{D}_B Y^B \\ Y^A = y^A + l^{-2} \int_0^u du' \bar{\gamma}^{AB} \partial_B F \end{cases}$$

fix u dependence up to integration functions $\quad f = f(x^B), \quad y^A = y^A(x^B)$

conformal Killing equation d-2 sphere $\quad \mathcal{L}_Y \bar{\gamma}_{AB} = \frac{2}{d-2} \bar{D}_B Y^B \bar{\gamma}_{AB}$

Results

AdS $d \geq 4$ $\mathfrak{so}(d-1,2)$ only exact Killing vectors of background

flat $d \geq 5$ $\mathfrak{so}(d-1,1) \ltimes ST$

cKv no constraint on f, angle dependent supertranslations

stronger fall-off's ➔ $iso(d-1,1)$ Poincaré algebra

AdS $d=3$ no constraint from cKe 2d conformal algebra

flat $d=3$ $\mathrm{Vect}(S^1) \ltimes_{\mathrm{ad}} \mathrm{Vect}(S^1)_{\mathrm{ab}}$ contraction of 2d conformal algebra

flat $d=4$ $\mathfrak{so}(3,1) \ltimes ST = \mathfrak{bms}_4^{\mathrm{glob}}$ globally well-defined BMS algebra

$$\zeta = e^{i\phi}\cot\frac{\theta}{2} \qquad d\theta^2 + \sin^2\theta d\phi^2 = P^{-2}d\zeta d\bar{\zeta}, \quad P(\zeta,\bar{\zeta}) = \frac{1}{2}(1+\zeta\bar{\zeta}),$$

standard GR choice: restrict to globally well-defined transformations

CFT choice : allow for meromorphic functions on the Riemann sphere

solution to conformal Killing equation $Y^\zeta = Y^\zeta(\zeta), \quad Y^{\bar\zeta} = Y^{\bar\zeta}(\bar\zeta)$

generators

$$l_n = -\zeta^{n+1}\frac{\partial}{\partial\zeta}, \quad \bar l_n = -\bar\zeta^{n+1}\frac{\partial}{\partial\bar\zeta}, \quad n \in \mathbb{Z}$$

superrotations

$$T_{m,n} = \zeta^m \bar\zeta^n, \quad m, n \in \mathbb{Z}$$

supertranslations

commutation relations

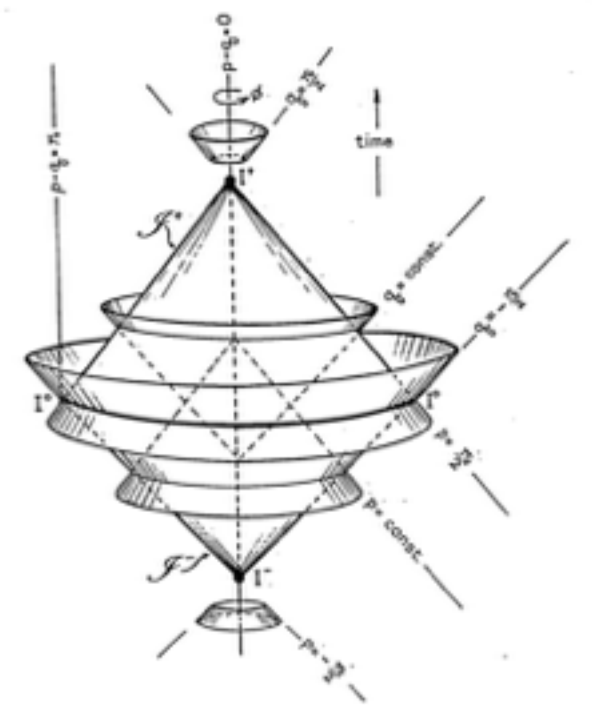
$$[l_m, l_n] = (m-n) l_{m+n}, \quad [\bar l_m, \bar l_n] = (m-n)\bar l_{m+n}, \quad [l_m, \bar l_n] = 0,$$
$$[l_l, T_{m,n}] = (\frac{l+1}{2} - m) T_{m+l,n}, \quad [\bar l_l, T_{m,n}] = (\frac{l+1}{2} - n) T_{m,n+l}.$$

Poincaré subalgebra $l_{-1},\, l_0,\, l_1, \quad \bar l_{-1},\, \bar l_0,\, \bar l_1, \quad T_{0,0},\, T_{1,0},\, T_{0,1},\, T_{1,1},$

4d gravity is dual to an extended conformal field theory

Proposal : should be relevant for relevant for gravitational S-matrix

scattering theory between $\mathscr{I}^-$ and $\mathscr{I}^+$



Penrose, Les Houches 1963

action on gravitational phase space

particles as UIRREPS for $\mathrm{BMS4}$

McCarthy 1972: no continous spin representations for $\mathrm{BMS4}^{\mathrm{glob}}$

Strominger et al. : Ward identities for soft photon and graviton theorems

Asymptotic symmetries Digression: Asymptotics and soft behaviour

Gervais & Zwanziger 1980

$$A_\mu^{\mathrm{as}}(x) \equiv \lim_{\lambda\to\infty} \lambda A_\mu(\lambda x)\,,$$

$$\mathcal{A}_\mu(x) = (2\pi)^{-3/2} \int [\exp(\mathrm{i}k\cdot x)\,\alpha(\omega,\hat{\boldsymbol{k}}) + \mathrm{h.c.}]\,(2\omega)^{-1}\,\mathrm{d}^3k\,, \qquad \omega = |\boldsymbol{k}|\,, \qquad \hat{\boldsymbol{k}} = \boldsymbol{k}/\omega\,.$$

After rewriting $\mathcal{A}_\mu$ as

$$\mathcal{A}_\mu(x) = \lim_{\epsilon\to 0} \tfrac{1}{2}(2\pi)^{-3/2} \int \mathrm{d}\hat{\boldsymbol{k}} \int_0^\infty \mathrm{d}\omega\,[\omega\alpha(\omega,\hat{\boldsymbol{k}})\exp[\mathrm{i}\omega(\hat{\boldsymbol{k}}\cdot \boldsymbol{x} - t) - \epsilon t] + \mathrm{h.c.}]\,,$$

one obtains by standard arguments

$$\lim_{\lambda\to\infty} \lambda\,\mathcal{A}_\mu(\lambda x) = \frac{1}{2}\int \frac{\mathrm{d}\hat{\boldsymbol{k}}}{(2\pi)^{3/2}}\left[\frac{\beta(\hat{\boldsymbol{k}})}{\epsilon + \mathrm{i}(t - \hat{\boldsymbol{k}}\cdot \boldsymbol{x})} + \mathrm{h.c.}\right],$$

$$\beta(\hat{\boldsymbol{k}}) = \lim_{\omega\to 0} \omega\alpha(\omega,\hat{\boldsymbol{k}})\,.$$

In practice, new dynamical variables at infinity come into play

Current algebra 4d flat   Asymptotic solution space

conformally flat metric $$d\bar{s}^2 = \bar{\gamma}_{AB} dx^A dx^B = 2P^{-2} d\zeta d\bar{\zeta}$$

covariant derivative $$\eth \eta^s = P^{1-s} \bar{\partial}(P^s \eta^s), \qquad \bar{\eth} \eta^s = P^{1+s} \partial (P^{-s} \eta^s)\,,$$

$$[\bar{\eth}, \eth] \eta^s = \frac{s}{2} R\, \eta^s\,, \qquad R = 4P^2 \partial \bar{\partial} \ln P, \quad R_S = 2$$

conformal Killing vectors $$\mathcal{Y} = P^{-1} \bar{Y}(\bar{\zeta}), \quad \bar{\mathcal{Y}} = P^{-1} Y(\zeta)$$

spin and conformal weights $$-\delta_{\mathcal{Y},\bar{\mathcal{Y}}} \eta = \left[\mathcal{Y}\eth + \bar{\mathcal{Y}}\bar{\eth} + \frac{s-w}{2}\, \eth\mathcal{Y} - \frac{s+w}{2}\, \bar{\eth}\bar{\mathcal{Y}}\right]\eta$$

asymptotic solution space $$\chi(u, \zeta, \bar{\zeta}) = \{\sigma^0, \Psi_2^0, \Psi_1^0, + \text{c.c.}\}$$

parametrisation of leading part on-shell behaviour of $\frac{V}{r},\ U^A,\ \gamma_{AB}$

evolution equations $$\dot{\Psi}_2^0 = -\eth^2 \dot{\bar{\sigma}}^0 - \sigma^0 \ddot{\bar{\sigma}}^0 \qquad \dot{\Psi}_1^0 = \eth \Psi_2^0 - 2\sigma^0 \eth \dot{\bar{\sigma}}^0$$

free u dependence $\sigma^0(u, \zeta, \bar{\zeta})$ news tensor $\dot{\sigma}^0(u, \zeta, \bar{\zeta})$

on-shell constraints $$\Psi_2^0 - \bar{\Psi}_2^0 = \bar{\eth}^2 \sigma^0 - \eth^2 \bar{\sigma}^0 + \dot{\sigma}^0 \bar{\sigma}^0 - \sigma^0 \dot{\bar{\sigma}}^0\,,$$

Current algebra 4d flat   Conformal transformations

bms4 transformations

$$-\delta_\xi \sigma^0 = [f\partial_u + \mathcal{Y}\eth + \overline{\mathcal{Y}\eth} + \frac{3}{2}\eth\mathcal{Y} - \frac{1}{2}\overline{\eth\mathcal{Y}}]\sigma^0 - \eth^2 f\,,$$

$$-\delta_\xi \dot{\sigma}^0 = [f\partial_u + \mathcal{Y}\eth + \overline{\mathcal{Y}\eth} + 2\eth\mathcal{Y}]\dot{\sigma}^0 - \frac{1}{2}\eth^2\psi\,,$$

$$-\delta_\xi \Psi_2^0 = [f\partial_u + \mathcal{Y}\eth + \overline{\mathcal{Y}\eth} + \frac{3}{2}\eth\mathcal{Y} + \frac{3}{2}\overline{\eth\mathcal{Y}}]\Psi_2^0 - 2\eth f \eth\dot{\bar{\sigma}}^0,$$

$$-\delta_\xi \Psi_1^0 = [f\partial_u + \mathcal{Y}\eth + \overline{\mathcal{Y}\eth} + 2\eth\mathcal{Y} + \overline{\eth\mathcal{Y}}]\Psi_1^0 + 3\eth f \Psi_2^0$$

$$f = T + \frac{1}{2}u\psi \qquad \psi = (\eth\mathcal{Y} + \overline{\eth\mathcal{Y}})$$

(field dependent) inhomogeneous pieces, Schwarzian derivatives

Strominger: soft gravitons = Goldstone modes for these transformations

Motivation for current algebra

interpretation requires charges, canonical generators for the transformations + Dirac bracket algebra

Problem: ADM type charges for superrotations diverge because of poles on the sphere

$$L_m \to \infty$$

Local non integrated version of Ward identities

$$\partial_a^x \langle J^a_{Q_1}(x) J^b_{Q_2}(y) X(z)\rangle = i\delta(x-y)\langle J^b_{[Q_1,Q_2]}(y)X(z)\rangle + i\delta(x-z)\langle J^b_{Q_2}(y)\delta_{Q_2}X(z)\rangle$$

classical version $\delta_{Q_1}:$

$$dJ_{Q_2} = Q_2^i \frac{\delta L}{\delta\phi^i} d^n x$$

$$\Rightarrow \quad \delta_{Q_1} J_{Q_2} = J_{[Q_1,Q_2]} + T + d(\cdot) + K_{Q_1,Q_2}$$

$$T + d(\cdot) \sim 0$$

Belinfante ambiguities

central extension highly constrained

$$[K_{Q_1,Q_2}] \in H^{n-1}(d)$$

may be field dependent

$$\delta_{Q_1} K_{Q_1,Q_2} - \frac{1}{2} K_{[Q_1,Q_2],Q_3} + \text{cyclic } (1,2,3) = 0$$

Current algebra 4d flat   Holographic version

Holography: understand gauge symmetries $\delta_f \phi^i = R^i_\alpha(f^\alpha) = R^i_\alpha f^\alpha + R^{i\mu}_\alpha \partial_\mu f^\alpha + \dots$

trivial Noether current $S_f = (R^{i\mu}_\alpha f^\alpha \frac{\delta L}{\delta \phi^i} + \dots)(d^{n-1}x)_\mu$

conserved n-2 form in linearised theory $k_f[\delta\phi] = (\frac{1}{2}\delta\phi^i \frac{\partial}{\partial \partial_\nu \phi^i} + \dots) \frac{\partial}{\partial dx^\nu} S_f$

$$dk_f[\delta\phi] = 0 \Leftarrow \begin{cases} \frac{\delta L}{\delta \phi} = 0 \\ \delta \frac{\delta L}{\delta \phi} = 0 \\ R^i_\alpha(f^\alpha) = 0 \end{cases}$$

in GR, 1-1 correspondence $[k_f] \longleftrightarrow$ Kvf of background, charges = ADM type charges

asymptotic case $x^\mu = (u, r, x^A) \quad r = \mathrm{cte} \to \infty$

$k_f = k_f^{[\mu\nu]}(d^{n-2})x_{\mu\nu} \longrightarrow$ current of lower dimensional theory $x^a = (u, x^A)$

integrability ? $k_f^{[ur]} \approx \delta J^u_f, \; k^{Ar}_f \approx \delta J^A_f$

No news ➡ no superrotations $\eth^3\mathcal{Y} = 0 = \bar{\eth}^3\bar{\mathcal{Y}}$

$$\mathcal{J}^u_\xi = -\frac{1}{8\pi G}\Big[\big(f(\Psi^0_2 + \sigma^0\dot{\bar{\sigma}}^0) + \mathcal{Y}(\Psi^0_1 + \sigma^0\eth\bar{\sigma}^0 + \frac{1}{2}\eth(\sigma^0\bar{\sigma}^0))\big) + \text{c.c.}\Big]$$

Bondi mass & angular momentum

current algebra represents BMS4 (global) $-\delta_{\xi_2}\mathcal{J}^a_{\xi_1} = \mathcal{J}^a_{[\xi_1,\xi_2]} + \text{trivial}$

integrable, no problem with integration of charges $Q_\xi = \int_{S^2} d^2\Omega\ \mathcal{J}^u_\xi$

mass and angular momentum for Kerr

news ➡ non conserved & non integrable currents

$$-\delta_{\xi_2}\mathcal{J}^a_{\xi_1} + \Theta^a_{\xi_2}(-\delta_\xi\chi) = \mathcal{J}^a_{[\xi_1,\xi_2]} + K^a_{\xi,\xi_2} + \text{trivial}$$

$$\Theta^u_\xi(\delta\chi) = \frac{1}{8\pi G}\Big[f\dot{\bar{\sigma}}^0\delta\sigma^0 + \text{c.c.}\Big], \qquad \mathcal{K}^u_{\xi_1,\xi_2} = \frac{1}{8\pi G}\Big[\big(\frac{1}{2}\bar{\sigma}^0 f_1\eth^3\mathcal{Y} - (1\leftrightarrow 2)\big) + \text{c.c.}\Big]$$

field dependent central charge ➡ use in Cardy-type formula for Kerr BH ?

Symmetries and solutions

residual symmetries $l^{-1} \neq 0 \qquad \xi = Y^{+}(x^{+})\partial_{+} + Y^{-}(x^{-})\partial_{-} \qquad x^{\pm} = \frac{u}{l} \pm \phi$

$l^{-1} = 0 \qquad \xi = Y(\phi)\partial_{\phi} + (T + uY')\partial_{u}$

general solution to EOM $ds^2 = \left(-\frac{r^2}{l^2} + \mathcal{M}\right) du^2 - 2dudr + 2\mathcal{N} dud\phi + r^2 d\phi^2$

$$l^{-1} \neq 0 \qquad \mathcal{M}(u,\phi) = 2(\Xi_{++} + \Xi_{--}), \quad \mathcal{N}(u,\phi) = l(\Xi_{++} - \Xi_{--})$$

closed form

$$\Xi_{\pm\pm} = \Xi_{\pm\pm}(x^{\pm})$$

$$l^{-1} = 0 \qquad \mathcal{M} = \Theta(\phi),\ \mathcal{N} = \Xi(\phi) + \frac{u}{2}\partial_{\phi}\Theta$$

transformations $l^{-1} \neq 0 \quad -\delta_Y \Xi_{\pm\pm} = Y^{\pm}\partial_{\pm}\Xi_{\pm\pm} + 2\partial_{\pm}Y^{\pm}\Xi_{\pm\pm} - \frac{1}{2}\partial^3_{\pm}Y^{\pm}$

$$\widetilde{x}^{\pm} = F^{\pm}(x^{\pm}), \quad \widetilde{\Xi}_{\pm\pm} = (\partial_{\pm}F^{\pm})^{-2}(\Xi + \{F^{\pm}; x^{\pm}\})$$

Schwarzian derivative, EM tensor

$$l^{-1} = 0 \qquad -\delta\Theta = Y\Theta' + 2Y'\Theta - 2Y''' \qquad -\delta\Xi = Y\Xi' + 2Y'\Xi + \frac{1}{2}T\Theta' + T'\Theta - T'''$$

$$\widetilde{\Theta}(\widetilde{\phi}) = (\widetilde{\phi}')^{-2}\left[\Theta(\phi) + 2\{\widetilde{\phi};\phi\}\right] \qquad \widetilde{\Xi}(\widetilde{\phi}) = (\widetilde{\phi}')^{-2}\left[\Xi - \frac{\alpha}{2}\Theta' - \alpha'\Theta + \alpha'''\right]$$

3d AdS & flat

Charge algebra

no need for current algebras

$$Q_\xi = \frac{l}{8\pi G}\int_0^{2\pi} d\phi\,[Y^+\Xi_{++} + Y^-\Xi_{--}]$$

$$Q_\xi = \frac{1}{16\pi G}\int_0^{2\pi} d\phi\,[T\Theta + Y\Xi]$$

Fourier modes

$$i\{L^\pm_m, L^\pm_n\} = (m-n)L^\pm_{m+n} + \frac{c^\pm}{12}m(m^2-1)\delta^0_{m+n}, \quad \{L^\pm_m, L^\mp_n\} = 0$$

Dirac bracket algebra

$$c^\pm = \frac{3l}{2G}$$

BMS3 algebra

$$i\{J_m, J_n\} = (m-n)J_{m+n} + \frac{c_1}{12}m(m^2-1)\delta^0_{m+n},$$

$$i\{J_m, P_n\} = (m-n)P_{m+n} + \frac{c_2}{12}m(m^2-1)\delta^0_{m+n},$$

$$i\{P_m, P_n\} = 0,$$

$$c_1 = 0, \quad c_2 = \frac{3}{G}$$

3d flat — Charge algebra contraction

appropriate combination for the limit

$$P_m = \frac{1}{l}(L^+_m + L^-_{-m}), \quad J_m = L^+_m - L^-_{-m}$$

$$i\{J_m, J_n\} = (m-n)J_{m+n} + \frac{c^+ - c^-}{12} m(m^2-1)\delta^0_{m+n},$$

$$i\{J_m, P_n\} = (m-n)P_{m+n} + \frac{c^+ + c^-}{12\ell} m(m^2-1)\delta^0_{m+n},$$

$$i\{P_m, P_n\} = \frac{1}{l^2}\left((m-n)J_{m+n} + \frac{c^+ - c^-}{12} m(m^2-1)\delta^0_{m+n}\right)$$

Virasoro algebra contracts to

$$\mathfrak{bms}_3$$

$$\cup$$

$$\mathfrak{iso}(2,1)$$

relation to AdS_3 similar to contraction between $\mathfrak{so}(2,2) \rightarrow \mathfrak{iso}(2,1)$

Virasoro factor: centrally non extended superrotations math summary

covariant phase space of 3d gravity

$l^{-1} \neq 0$ 2 copies of coadjoint representation $\mathfrak{vir}^*$ of $\widehat{\mathrm{Diff}^+}(S^1)$ at $c^{\pm} = \frac{3l}{2G}$

$l^{-1} = 0$ coadjoint representation $\mathfrak{bms}_3^*$ of $\widehat{\mathrm{Diff}^+}(S^1) \ltimes_{\mathrm{Ad}} \widehat{\mathrm{Vect}}(S^1)_{\mathrm{ab}}$ at $c_1 = 0, \quad c_2 = \frac{3}{G}$

Dirac bracket = Kirillov-Kostant bracket on $\mathfrak{g}^*$ $\{x_a, x_b\} = f^c_{ab} x_c$

useful ?

AdS3 & 3d flat — Zero mode solutions

zero mode solutions in both cases

BMS form

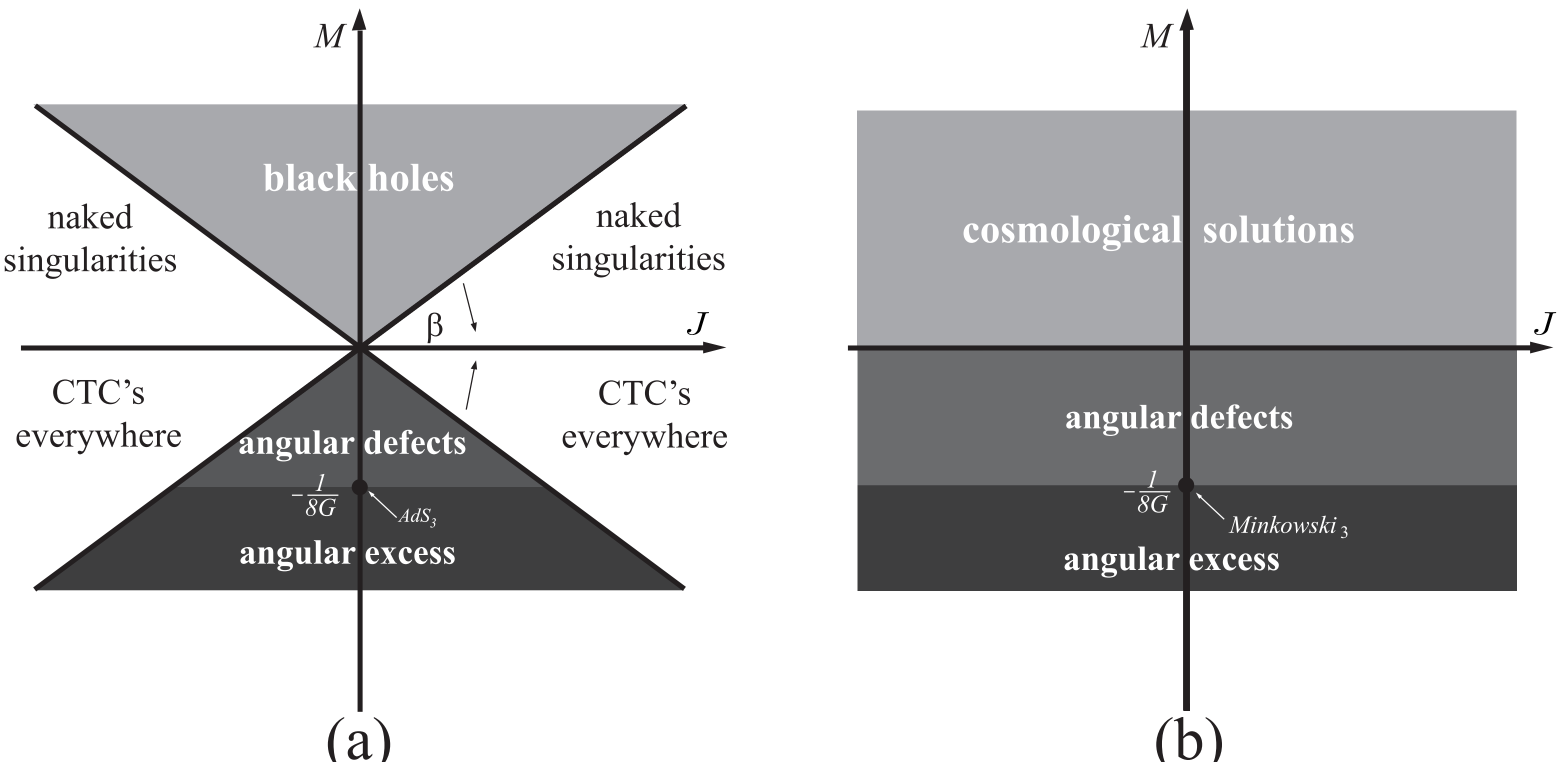
$$ds^2 = (-\frac{r^2}{l^2} + 8GM)du^2 - 2dudr + 8GJdud\phi + r^2 d\phi^2$$

ADM form

$$ds^2 = -N^2 dt^2 + N^{-2} dr^2 + r^2(d\varphi + N^\varphi dt)^2,$$

$$N^2 = \frac{r^2}{l^2} - 8MG + \frac{16G^2J^2}{r^2}, \quad N^\varphi = \frac{4GJ}{r^2}$$

$$\Xi_{\pm\pm} = 2G(M \pm \frac{J}{l})$$



repeat derivation of entropy of cosmological solutions from Cardy type formula

Coadjoint orbits

classification of complete solution space through coadjoint orbits

$l^{-1} \neq 0$ answer to which of the general solution $\Xi_{\pm\pm} = \Xi_{\pm\pm}(x^{\pm})$

cannot be obtained from a zero mode solution through a large diffeomorphism ?

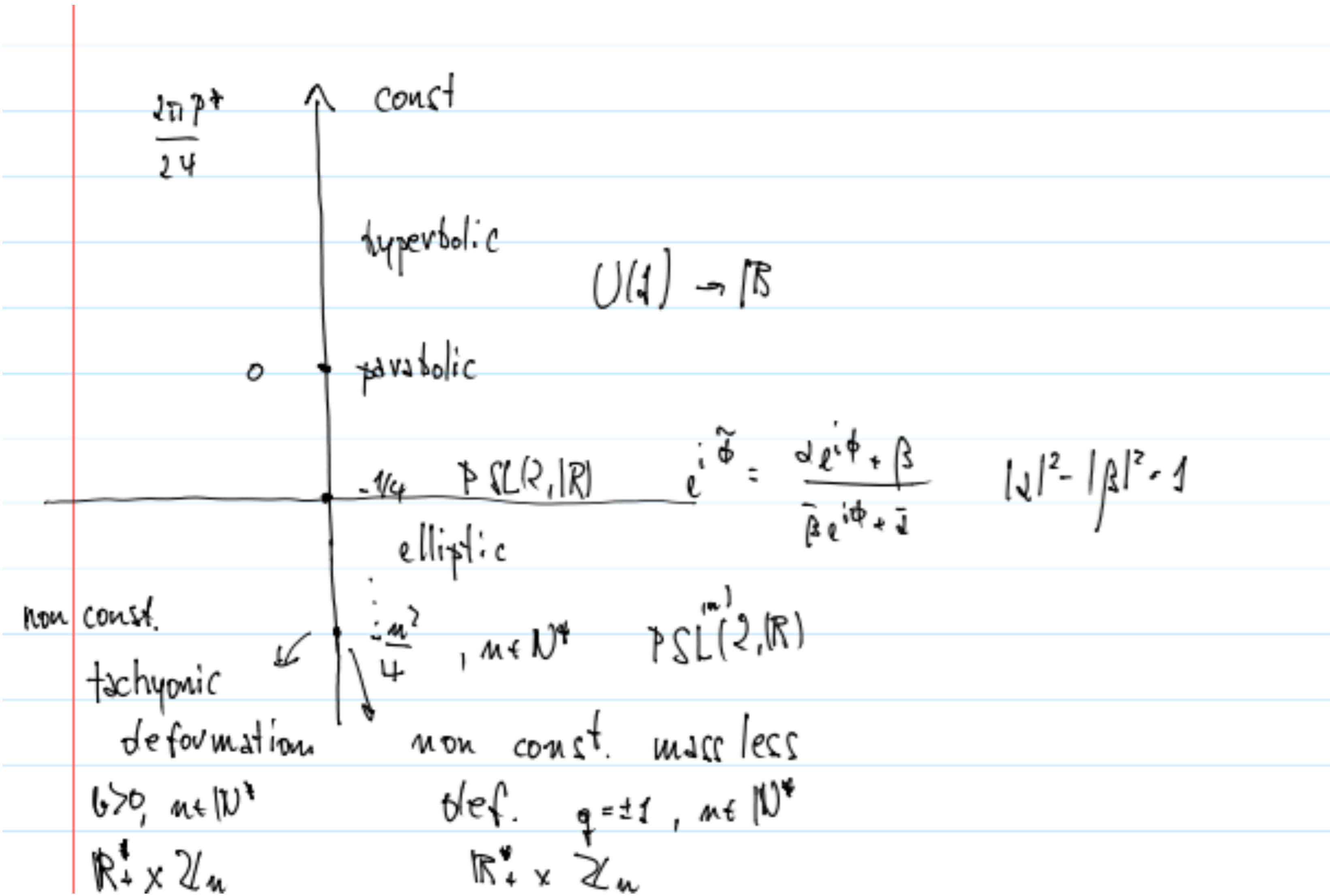
well-known math problem Witten, Coadjoint orbits of the Virasoro group, CMP '88

Balog et al., HEP-TH/9703045

coadjoint orbits are symplectic spaces, integrable systems ➡ geometric quantization/ UIRREPS

orbit $\mathcal{O} \cong \widehat{\mathrm{Diff}^{+}}(S^1)/G_0$ little group G_0 leaves given coadjoint vector invariant

Coadjoint orbits



Positive energy theorems

study behaviour of energy functional on orbit

$$E[P^*_{\widetilde{\phi}}] = \int_0^{2\pi} d\phi \Big(P^*(\widetilde{\phi})(\widetilde{\phi}')^2 - \frac{c}{24\pi}\{\widetilde{\phi}, \phi\}\Big)$$

energy of asymptotically flat spacetime $c = \frac{3}{G}$

chiral energy of asymptotically AdS spacetime $c^{\pm} = \frac{3l}{2G}$

Schwarzian derivative $$\int_0^{2\pi} d\phi \left[\{\widetilde{\phi}, \phi\} + \frac{1}{2}((\widetilde{\phi}')^2 - 1) \right] \leqslant 0 \quad \forall \widetilde{\phi}$$

$$= 0 \iff e^{i\widetilde{\phi}} = \frac{\alpha e^{i\phi} + \beta}{\bar{\beta} e^{i\phi} + \bar{\alpha}}, \quad |\alpha|^2 - |\beta|^2 = 1.$$

allows to show that energy is bounded from below iff orbit has a constant representative above Minkowski-space time (+1 class with non constant representatives)

similar results in AdS3

3d flat

BMS3 particles: Induced representations

scattering theory between $\mathscr{I}^-$ and $\mathscr{I}^+$

particle : UIRREP of BMS3

structure: $G \ltimes_{\mathrm{Ad}} \mathfrak{g}_{\mathrm{ab}}$

finite-dimensional Lie groups Wigner-Mackey

all UIRREPS :

1) determine characters of $\mathfrak{g}_{\mathrm{ab}}$ ➡ $\mathrm{Ad} \to \mathrm{Ad}^*$

2) determine orbits & little groups of Ad^*

3) induce UIRREPS of $G \ltimes_{\mathrm{Ad}} \mathfrak{g}_{\mathrm{ab}}$ out of UIRREPS of little group

cf. Poincaré group

again classified by coadjoint orbits of Virasoro group

3d AdS & flat — Dual 2d theories

start from CS formulation of 3d gravity

solve constraints with asymptotic condition $l^{-1} \neq 0$

$l^{-1} = 0$

2 copies of SL(2,R) cWZW model
= non chiral SL(2,R) WZW model

Hamiltonian reduction (Drinfeld-Sokolov)

Liouville theory with

$$c^{\pm} = \frac{3l}{2G}$$

iso(2,1) cWZW model

$$I[\lambda, \alpha] = \frac{k}{\pi} \int du d\phi \; \mathrm{Tr}\Big[\dot{\lambda}\lambda^{-1}\alpha' - \frac{1}{2}(\lambda'\lambda^{-1})^2\Big]$$

$$k = \frac{1}{4G}$$

$$\{P_a(\phi), P_b(\phi')\}^* = 0,$$

$$\{J_a(\phi), P_b(\phi')\}^* = \epsilon_{ab}{}^c P_c(\phi)\delta(\phi - \phi') - \frac{k}{2\pi}\eta_{ab}\partial_\phi\delta(\phi - \phi'),$$

$$\{J_a(\phi), J_b(\phi')\}^* = \epsilon_{ab}{}^c J_c(\phi)\delta(\phi - \phi')$$

iso(2,1) current algebra

3d AdS & flat　　　　Dual 2d theories

BMS Liouville with centrally extended global BMS3 symmetry algebra

Hamiltonian form of Liouville

$$l \to \infty$$

$$\mathcal{L}_H = \pi\dot{\varphi} - \frac{1}{2}\pi^2 - \frac{1}{2l^2}\varphi'^2 - \frac{\mu}{2\gamma^2}e^{\gamma\varphi} \quad \Longrightarrow \quad \mathcal{L}_H = \Pi\dot{\Phi} - \frac{1}{2}\Phi'^2 - \frac{\nu}{2\beta^2}e^{\beta\Phi}$$

$$\varphi = l\Phi, \quad \pi = \frac{\Pi}{l}$$

$$\beta = \gamma l, \nu = \mu l^2 \text{ fixed}$$

$$\{\widetilde{\mathcal{H}}(\phi), \widetilde{\mathcal{H}}(\phi')\}^* = 0,$$

$$\{\widetilde{\mathcal{H}}(\phi), \widetilde{\mathcal{P}}(\phi')\}^* = (\widetilde{\mathcal{H}}(\phi) + \widetilde{\mathcal{H}}(\phi'))\partial_\phi\delta(\phi - \phi') - \frac{k}{2\pi}\partial_\phi^3\delta(\phi - \phi'),$$

$$\{\widetilde{\mathcal{P}}(\phi), \widetilde{\mathcal{P}}(\phi')\}^* = (\widetilde{\mathcal{P}}(\phi) + \widetilde{\mathcal{P}}(\phi'))\partial_\phi\delta(\phi - \phi')$$

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