CERN Summer lectures, August, 2008

POSSIBLE PHYSICS BEYOND THE STANDARD MODEL

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Quote

"Understanding nature is one of the noblest endeavors the human race has ever undertaken"

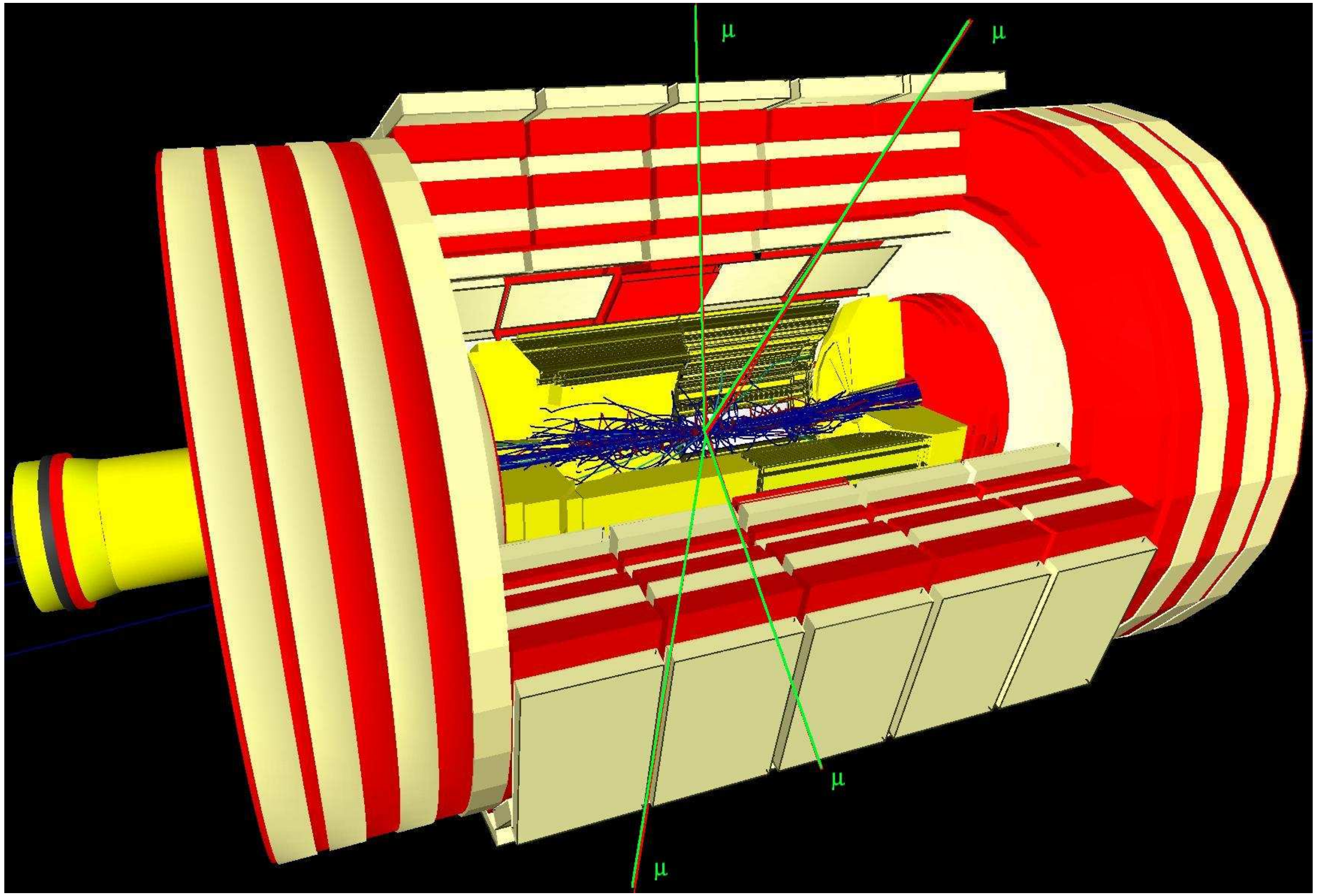
Steven Weinberg

Preview

- What do we expect to see at LHC?

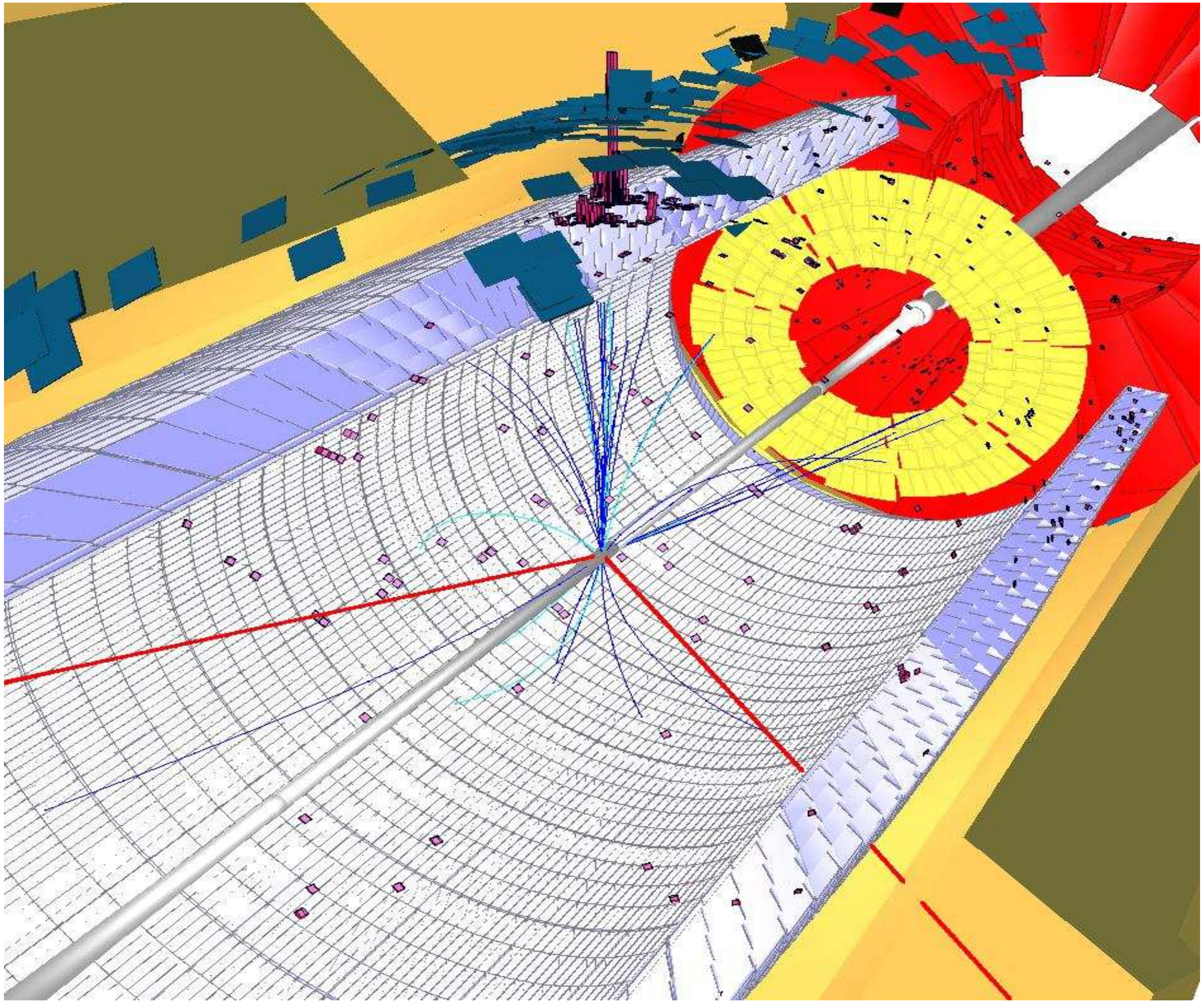
Preview: Higgs

Possible exciting physics we are preparing to search for: Higgs particle



Requested by the SM, might tell us a lot about the hierarchy puzzle.

Preview: Supersymmetry



SUSY event: A decay of a neutralino into Z + LSP, the Z decays into two muons.

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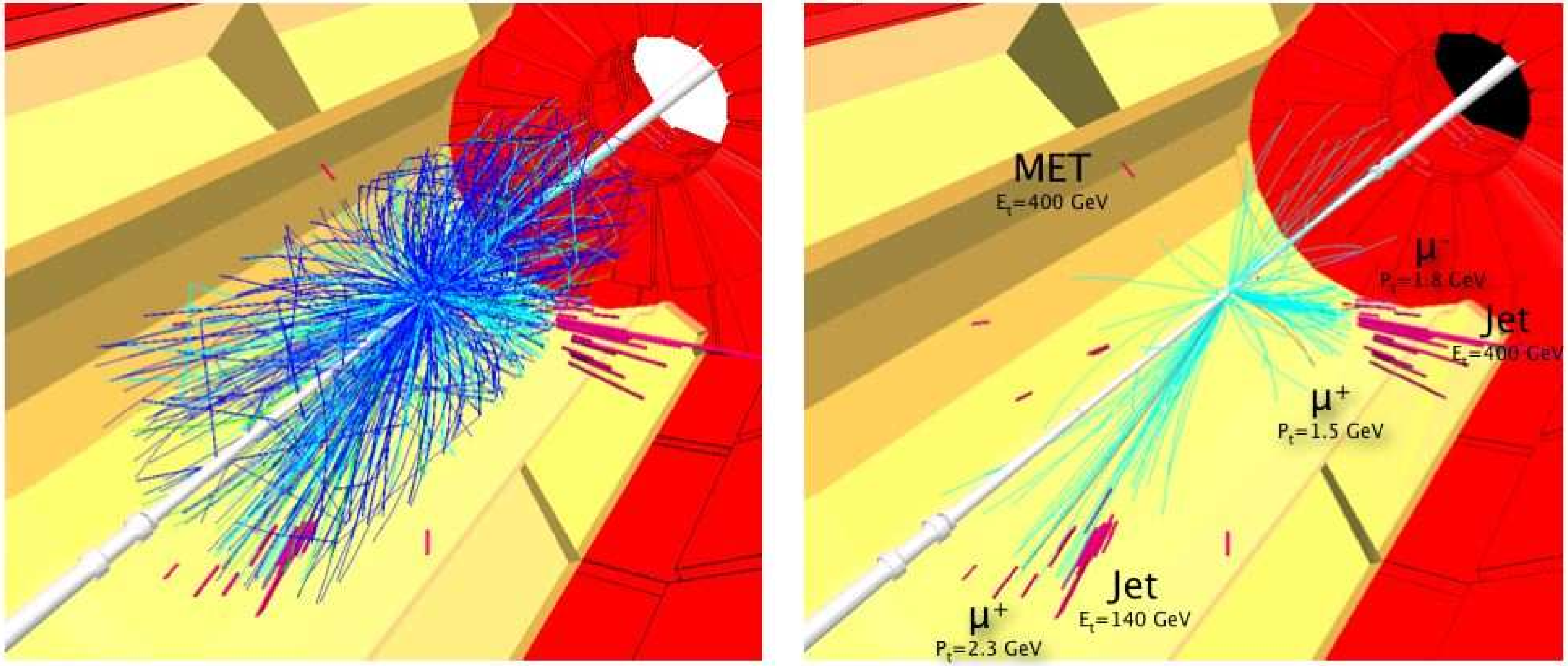
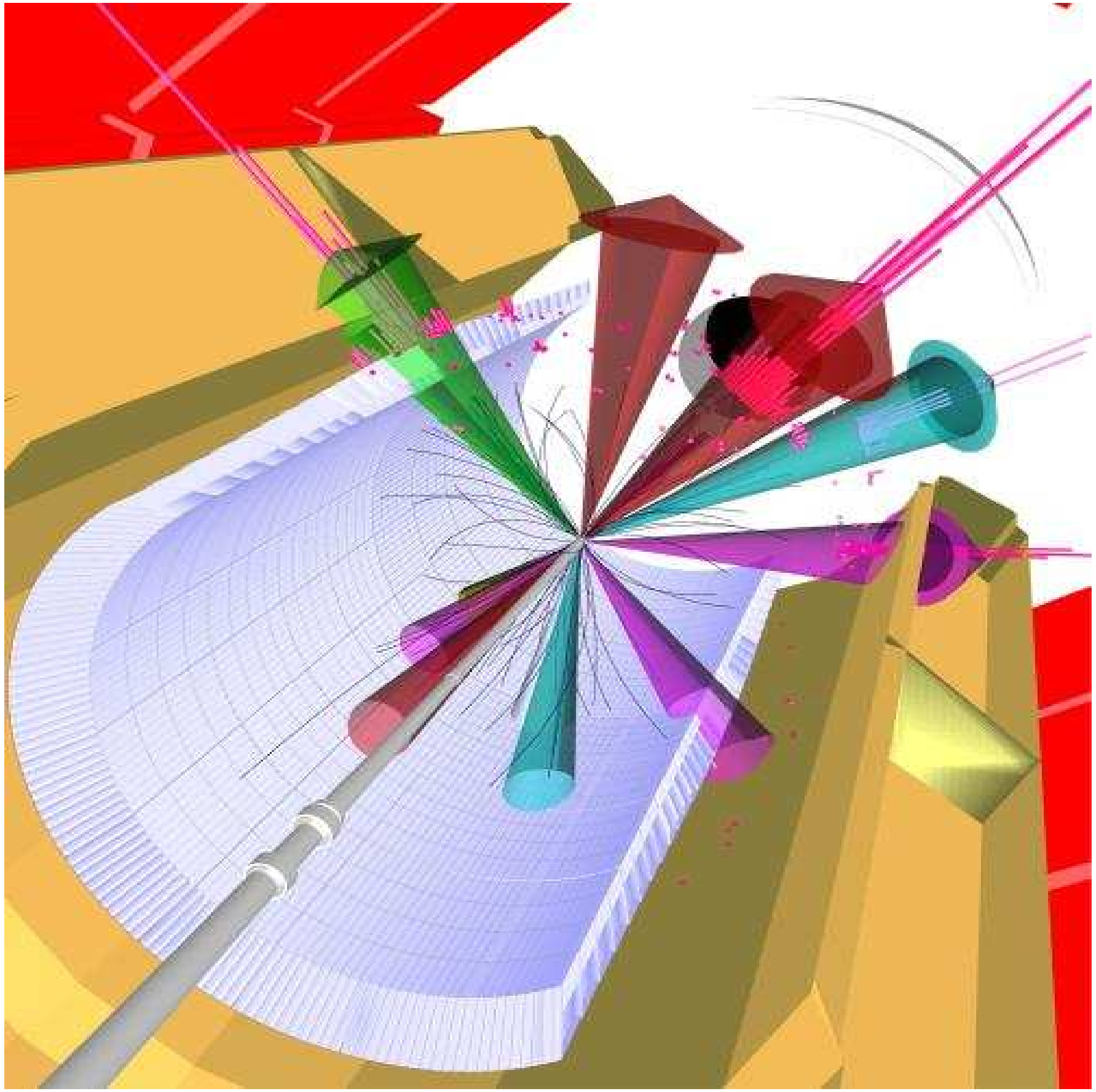
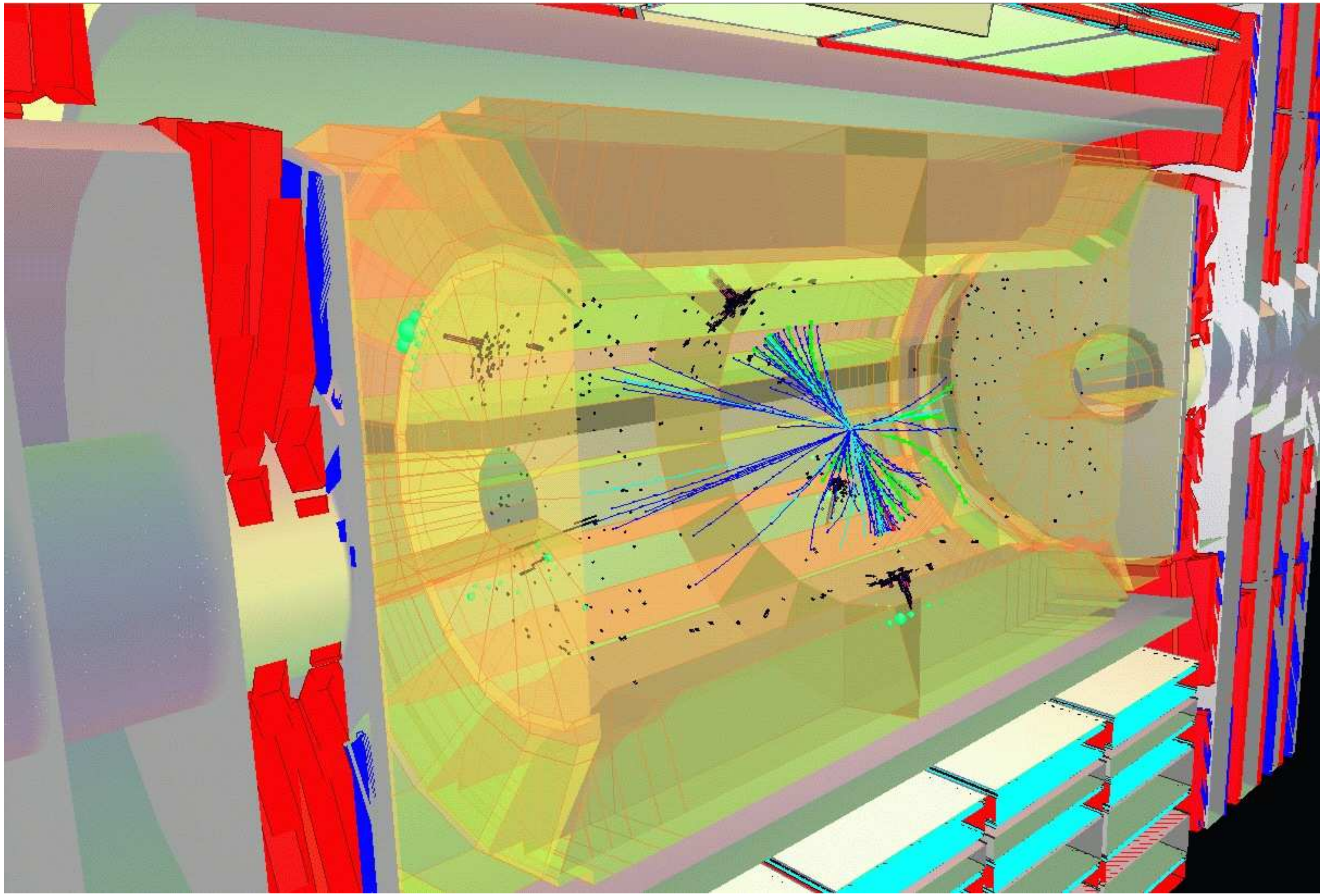


Missing transverse energy susy event at high luminosity

Preview: Small black-Hole production

Small black-hole may be produced and decay via Hawking radiation at LHC, if the scale of (quantum) gravity is low.





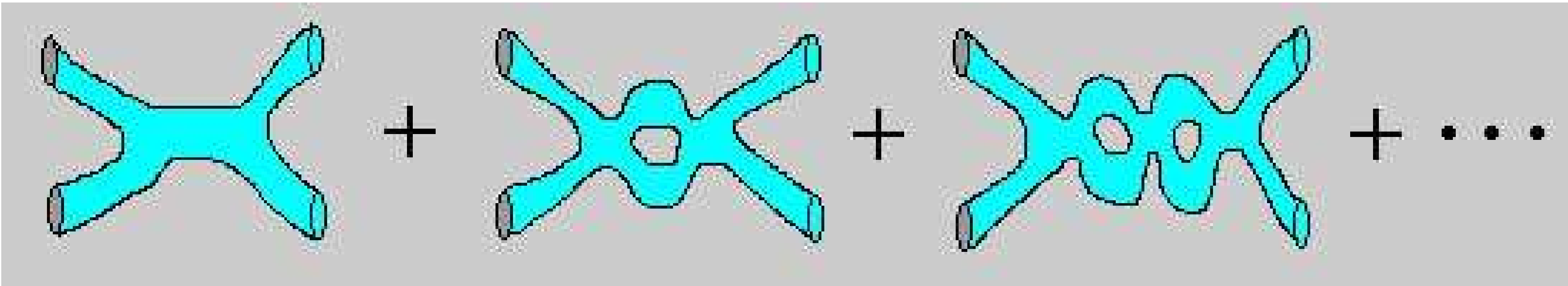
Event of BH production with $M_P = 1$ TeV and two extra dimensions.

See the Charybdis site

http://www.ippp.dur.ac.uk/montecarlo/leshouches/generators/charybdis/manual.html

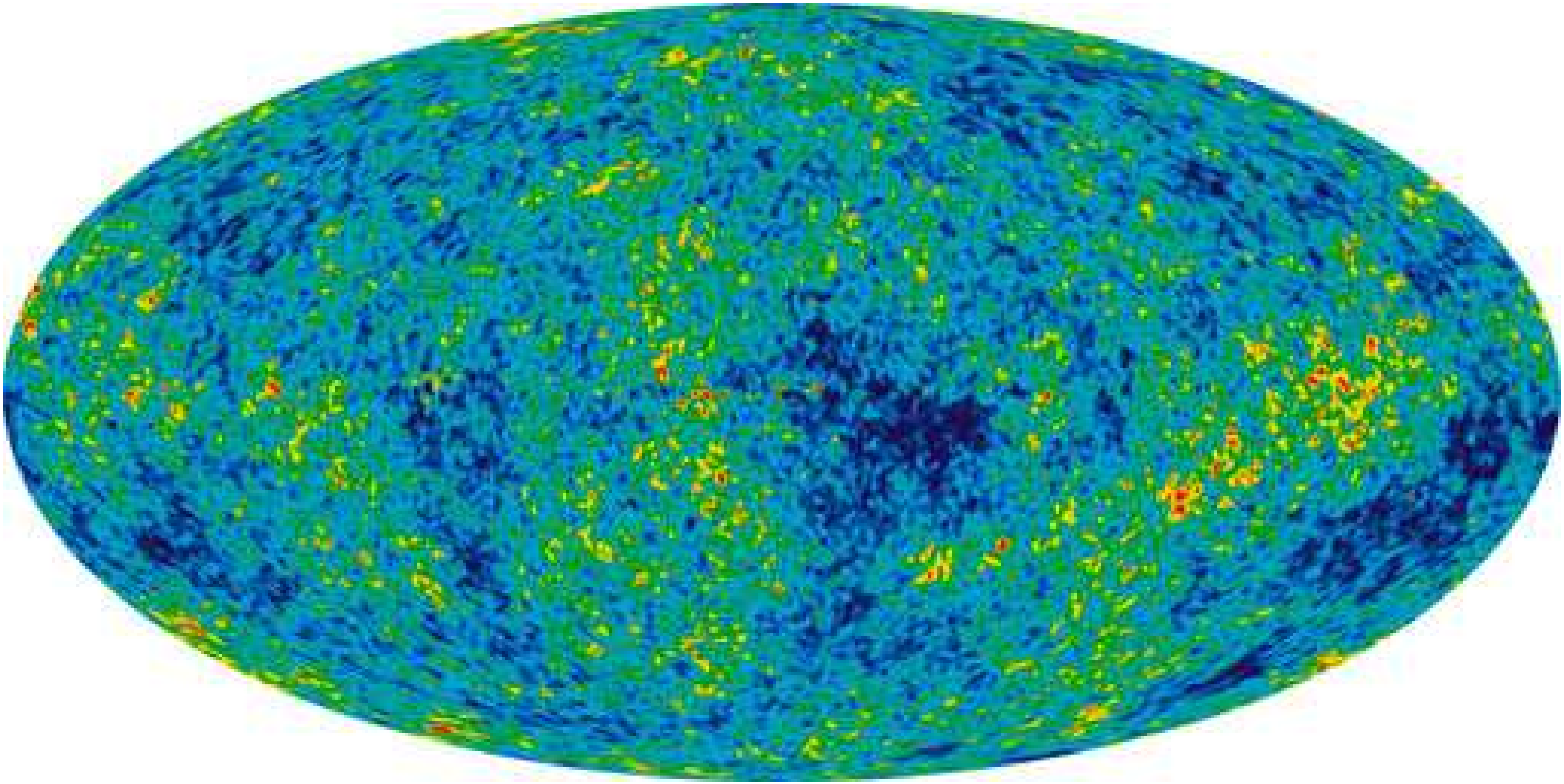
Preview: Strings?

String excited modes can also be produced if the string scale is sufficiently low.



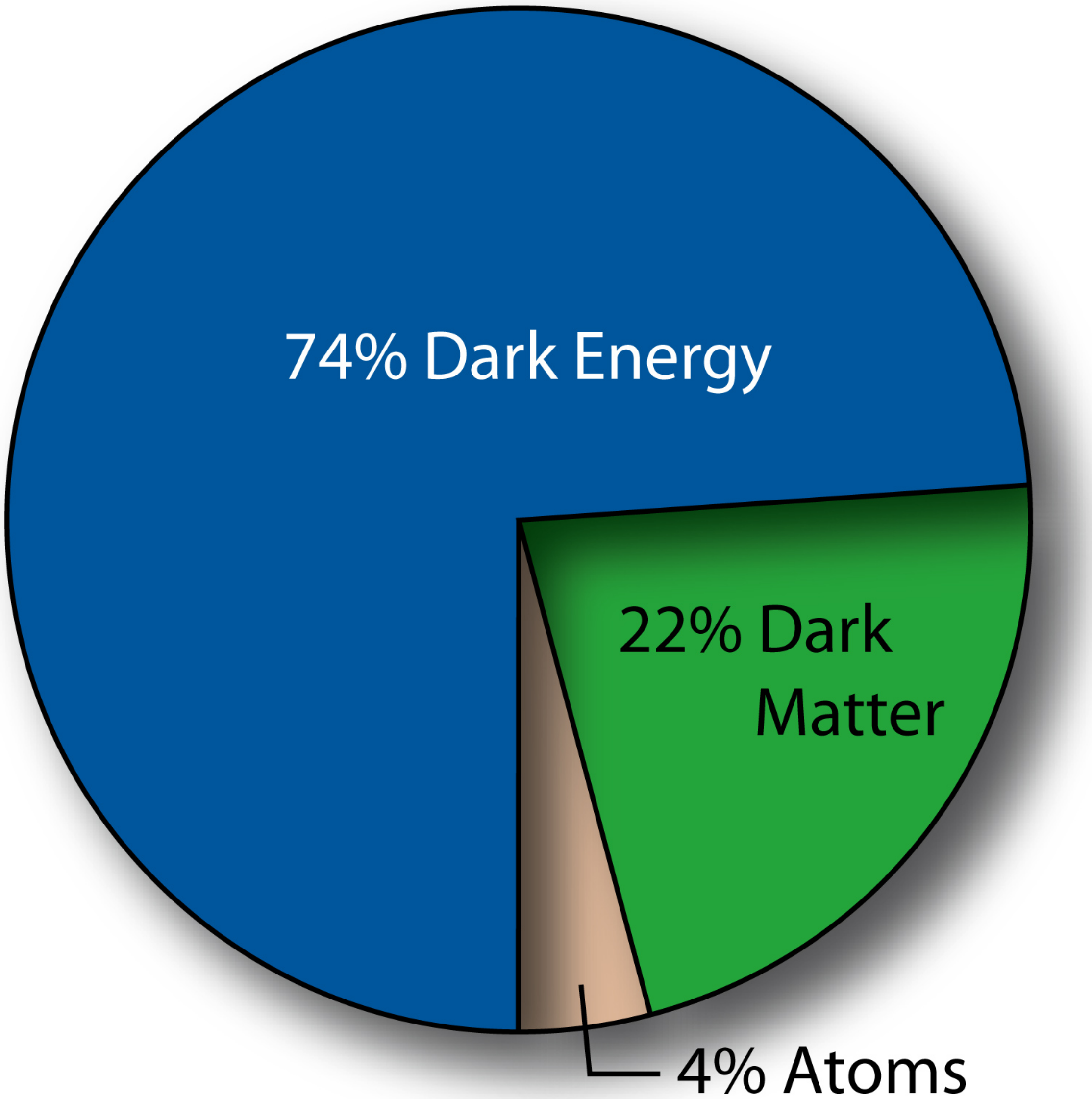
Preview: Dark Matter+Dark Energy

Can particle physics provide a candidate for the dark matter of the universe?



Can particle physics provide explanations for the primordial spectrum of cosmological fluctuations?

The cosmic Pie



What is the dark energy?

The purpose of these lectures

- Why we believe the Standard Model is not the final (fundamental) theory of the world?

- Why do we believe that there is new physics around the TeV range?

- What types of new physics at shorter distances theorists have guessed during the past twenty years and why?

♡ This is an exciting period because it is some of this new physics that we are going to test at LHC

♠ We are also living in an era where similar, revolutionary data are coming from cosmology and they also probe the nature of the fundamental theory

♣ Most probably it will be some of you that will solve the puzzles and nail down the fundamental theory that extends and completes the SM!

Suggested reading

- “Effective Field Theories” by Aneesh Manohar,
[arXiv:hep-ph/9606222]
A detailed discussion of the the concept of effective field theories with applications.

- “Beyond the Standard Model” by Fabio Zwirner,
http://doc.cern.ch/cernrep/1998/98-03/98-03.html
Hierarchy, technicolor, unification, supersymmetry, supergravity, supersymmetry breaking.

- “Supersymmetry and duality in field theory and string theory” by Elias Kiritsis
[arXiv:hep-ph/9911525]
Duality and monopoles in supersymmetric theories and string theory.

- “Supersymmetry Phenomenology” by Hitoshi Murayama
[arXiv:hep-ph/0002232]
Supersymmetric phenomenology.

- "Technicolor", by Ken Lane,
[arXiv:hep-ph/0007304]
Technicolor review.

- "Physics beyond the Standard Model" by Gian Giudice,
Lect. Notes Phys. **591**:294-327,2002

- "Beyond the Standard Model" by John Iliopoulos
http://preprints.cern.ch/cernrep/2004/2004-001/2004-001.html
Unification, supersymmetry, monopoles and electric-magnetic duality, supergravity, string theory.

- "Phenomenological guide to physics beyond the standard model", by Stefan Pokorski,
[arXiv:hep-ph/0502132]
Hierarchy problem, unification, supersymmetry.

- "Phenomenology beyond the Standard Model" by Joe Lykken,
[arXiv:hep-ph/0503148]
Extra dimensions, little Higgs, Higgs-less models (orbifolds), Little Higgs.

Most obtainable from the archive http://xxx.arxiv.cornell.edu/

A tentative plan

- The Standard Model and its problems: Why do we expect new physics?

- Supersymmetry

- Grand Unification

- Gravity and String Theory

- The physics of extra dimensions.

High Energy Units

We use

$$h = 1 \quad , \quad c = 1$$

$$[Energy] \sim [Mass] \sim \frac{1}{[Length]} \sim \frac{1}{[Time]}$$

The Standard Model: principles

• The Standard Model of the Electroweak and Strong interactions has been a very successful theory.

• Effort started at the beginning of the twentieth century. Consolidated by the establishment of Quantum Field Theory.

QFT=Special Relativity+ Quantum Mechanics

• All interactions are based on the "gauge principle" (including gravity) $\Rrightarrow$ invariance under local (independent) symmetry transformations. (the first model for this was electromagnetism)

• Renormalizability was another principle at the time the SM was formulated. We understand today that it is not a necessity: Any extension of a fundamental theory (not including gravity) is necessarily renormalizable to begin with.

• Other important principles are: Locality, Unitarity.

Standard Model: Open problems

The standard model was constructed as a renormalizable theory
$\longrightarrow$ as such it can be extended in principle to very high energies.

Why do we believe that there is more to know beyond the Standard Model?

Three sets of experimental data that are not accounted for by the SM:

♣ Neutrinos have (VERY small) masses and they mix.

♠ There is a lot (22%) of dark (non-SM) matter in the universe.
Neutrinos are part of it but cannot account for most of it.

♠ There is another source of energy in the universe (74%) , known as "dark energy" (vacuum energy?) This translates to $|V_{vac}| \sim (10^{-3}\text{ eV})^4$.

In the SM $|V_{vac}| \gtrsim (10^{11})^4$ eV^4 $\gg$ $(10^{-3})^4$ eV^4

Off by 56 orders of magnitude

Standard Model: Open problems II

• (Quantum) Gravity is not part of the Standard Model. One of the deepest questions of modern theoretical physics is: why the characteristic scale of gravity

$$M_{\text{Planck}} = \frac{1}{\sqrt{G_N}} \simeq 10^{19} \quad \text{GeV}$$

is so much higher than the other scales of particle physics?

• The Standard model alone contains IR-free couplings $\Rrightarrow$ strongly-coupled UV physics.

• The SM has many unexplained parameters and patterns.

THEREFORE: SM is an Effective Field Theory (EFT) valid below 100 GeV. Must be replaced by a more fundamental theory at a higher scale Λ.

How big is Λ?

• Λ must be small: $\Lambda \sim$ a few TeV. Otherwise we suffer from a technical (fine-tuning) problem also known as the hierarchy problem (more later).

SM patterns and parameters

• The standard model group $SU(3) \times SU(2) \times U(1)$ is not "unified", the coupling constants

$$g_3^2 \simeq 1.5, \; g_2^2 \simeq 0.42, \; g_Y^2 \simeq 0.13$$

are independent parameters. This can be improved if the fundamental theory has a simple gauge group, like SU(5) that contains the SM gauge group.

• The matter content and representations seems not very "regular". Why not higher representations? Hypercharges seem also bizarre. (but up to normalizations they are determined by the absence of gauge anomalies (BIM)

• Why three families? ("Who ordered that?")

• What decides the scale of Electroweak symmetry breaking

$$v_F \simeq 174 \quad \text{GeV ?}$$

• What decides the mass of the Higgs?

The pattern of masses

- The pattern of SM masses is mysterious at least:

family \ type	ups	downs	leptons
3rd	$m_t = 175$	$m_b = 4.2$	$m_\tau = 1.7$
2nd	$m_c = 1.2$	$m_s = 0.1$	$m_\mu = 0.1$
1st	$m_u = 3 \times 10^{-3}$	$m_d = 5 \times 10^{-3}$	$m_e = 5 \times 10^{-4}$

- Neutrino masses seem to be in the $10^{-12} - 10^{-14}$ GeV range. SM masses span 16 orders of magnitude.

This is a question for the Yukawa couplings λ_i: $m_i = \lambda_i \; v_F$.
We want to explain their ratios and the absolute normalization, (as we can do it for the spectral lines of atoms.)

- There are other parameters of the Standard Model that we would like to understand.

How parameters affect us?

How academic is the issue of such parameters?

Most of them are crucial to the existence of our universe as we know it, and the existence of humans as we know them.

- $v_F \to 0$ then p is unstable to decay to neutrons $\to$ no Hydrogen.

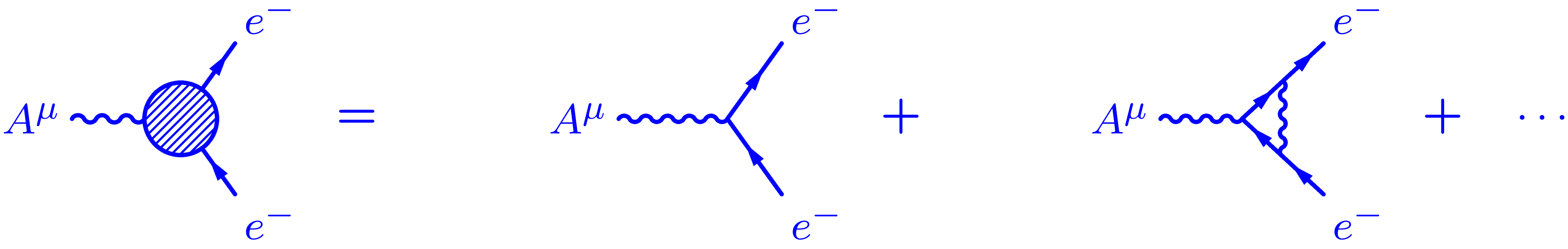
- $v_F >> 170GeV$ n-p mass difference is very large and the nuclear force becomes of shorter range $\to$ nuclei cannot be bound $\to$ nothing but hydrogen in the universe.

- changing the $\alpha_{em} \to$ no C^{12} resonance $\to$ no carbon in our universe.

Exercise : Justify by simple reasoning the first two statements. If unable look here.

Renormalization: "Integrating out" high-energy d.o.f

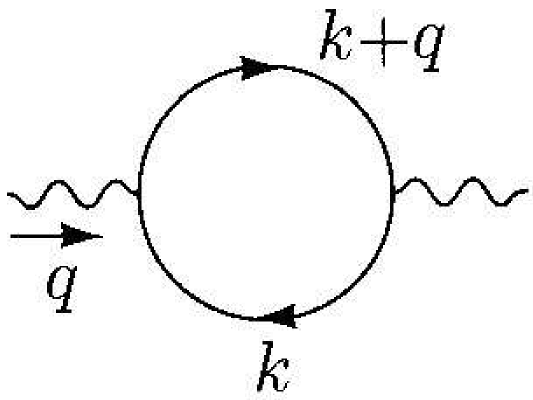
• In QFT physical processes have quantum corrections



• They involve "virtual" particles that do not satisfy the mass-shell condition $E^2 = \vec{p}^2 + m^2$. This is allowed because of the uncertainty principle.

• Therefore the energies of "virtual" particles are not constrained and can be arbitrarily high.

$$I_{vp} \sim \int_{-\infty}^{\infty} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + m_e^2} \frac{1}{(k+q)^2 + m_e^2}$$



• High energy degrees of freedom, unobservable directly in low energy experiments, make (indirect) quantum contributions to low-energy observables.

• We can therefore "integrate out" such degrees of freedom, and substitute their effects directly in the action of the low energy degrees of freedom (d.o.f).=Low Energy Effective Action

• This notion is more transparent in the path-integral formulation of Quantum Mechanics and QFT:

$$Z \equiv \int \mathcal{D}L\ \mathcal{D}H\ e^{iS(L,H)} \quad , \quad H \to \text{heavy}\ \ \text{d.o.f} \quad , \quad L \to \text{light}\ \ \text{d.o.f}$$

$$Z = \int \mathcal{D}L\ e^{iS_{\text{eff}}(L)} \quad , \quad e^{iS_{\text{eff}}(L)} \equiv \int \mathcal{D}H\ e^{iS(L,H)}$$

• It is also similar to what we do with the probabilities of unobservable events: P(x,y), with y unobservable gives a probability for x:

$$P_{eff}(x) = \int dy\ P(x,y)$$

Effective field theory

• Therefore, if we are interested in the low energy dynamics we can integrate-out the high energy d.o.f, and incorporate their effects in the action for the light particles. This we call the (low-energy) effective action.

• The heavy particles are unobservable from the low-energy point of view (cannot be produced) but they have "virtual" effects that affect the low energy dynamics.

• The effects of the high-energy d.o.f are summarized in the EFT by a **`few local interactions`** to a good degree of accuracy.

• The important concept that characterises interactions is their scaling dimension, Δ. In four dimensions, a scalar has $\Delta = 1$, a fermion $\Delta = 3/2$, a gauge field $\Delta = 1$ and a derivative $\Delta = 1$.

$$\int d^4x \, \left[(\partial\Phi)^2 + \bar{\psi}\partial\!\!\!/\psi - \frac{1}{4}(\partial A)^2\right]$$

For example the gauge interaction

$$\delta S \sim e \int d^4x \; A_\mu \bar{\psi} \gamma^\mu \psi$$

has scaling dimension $\Delta = 4$ and therefore e has scaling dimension zero.

- Effective interactions with scaling dimension Δ

$$\mathcal{S}_O = C_\Delta \int d^4x \; O_\Delta$$

have coefficients that by dimensional analysis are

$$C_\Delta \sim \frac{1}{\Lambda^{\Delta-4}} \quad , \quad C_{\Delta=4} \sim \log \Lambda$$

where Λ is the (large) scale of the high energy d.o.f.

- Corrections to interactions with $\Delta < 4$ are large.

This is what we call the **renormalizable interactions.** All the interactions of the Standard model are renormalizable interactions.
They are the most obvious avatar of the fundamental ($\Lambda = \infty$) Lagrangian.

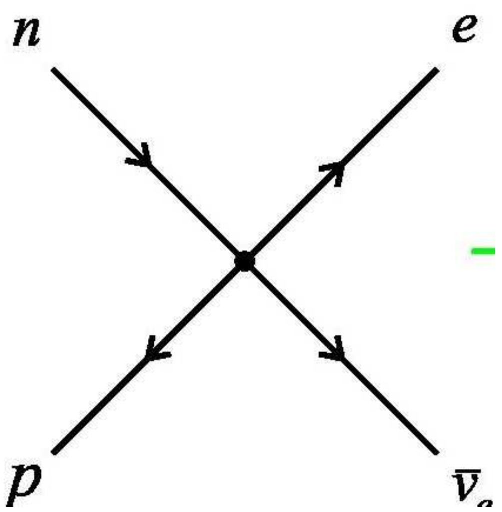
- Corrections to non-renormalizable interactions $\Delta > 4$ are small. This is where the new information of the high-energy theory is hiding (most of the time)

Effective couplings: the Fermi theory paradigm

The Fermi theory described the decay of neutrons:

$$n \to p + e + \bar{\nu}_e$$

via a four-fermion (dimension-6=non-renormalizable interaction)

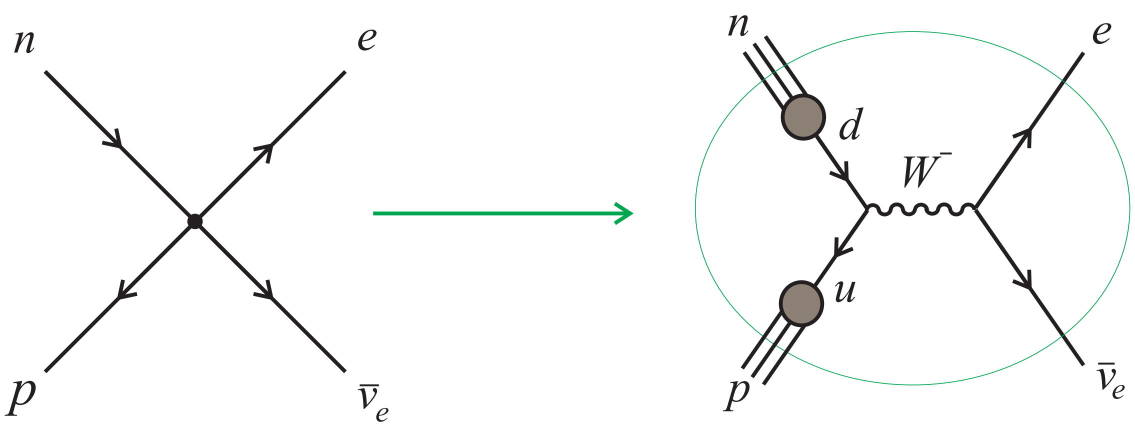


$$L_{\text{interaction}} = G_F \, (\bar{p} \, \gamma^\mu \, n)(\bar{\nu}_e \, \gamma_\mu \, e)$$

with $G_F \simeq \frac{1}{(300 \; GeV)^2} \quad \sim \quad \frac{1}{M^2}$

This descriptions is very accurate for energies $E << 100 GeV$.

However, with a better magnifying glass the four-fermi interaction originates from the Standard Model electroweak gauge interactions



Effective interaction :

$$p = (uud) \quad , \quad n = (udd) \quad , \quad d \to W^- + u \to (e^- + \bar{\nu}_e) + u$$

$$\frac{g_W^2}{p^2 + M_W^2} \simeq \frac{g_W^2}{M_W^2} - \frac{g_W^2}{M_W^2}\frac{p^2}{M_W^2} + \cdots = G_F + \cdots \quad , \quad p^2 << M_W^2$$

The effective interaction is dimension 6. It is the result of interactions with dimension 4 (renormalizable) interactions at higher energy.

- We can improve by including the next term:

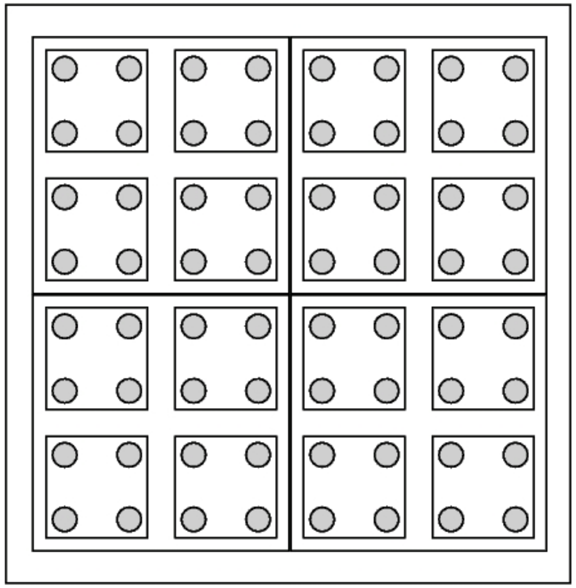
$$\frac{G_F}{M_W^2}(\bar{p}\ \gamma^\mu \partial_\mu\ n)(\bar{\nu}_e\ \gamma^\nu \partial_\nu\ e)$$

Irreversibility

- Integrating out high-energy d.o.f is an "irreversible" process.

- From a high energy theory, we calculate the low energy interactions. From a finite number of low-energy interactions we cannot reconstruct the high-energy theory.

- A simple example of integrating out: "renormalization": the block spin transformation.



- High energy $\sim$ Short distance

- An everyday analog: converting from RAW to jpeg format.

Original picture $\sim$ 2 Mb

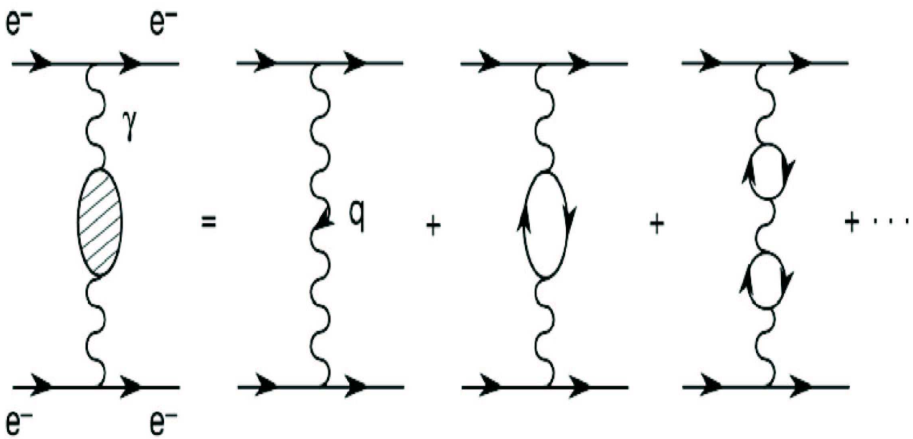
The original picture reduced 10 times $\sim$ 200 Kb

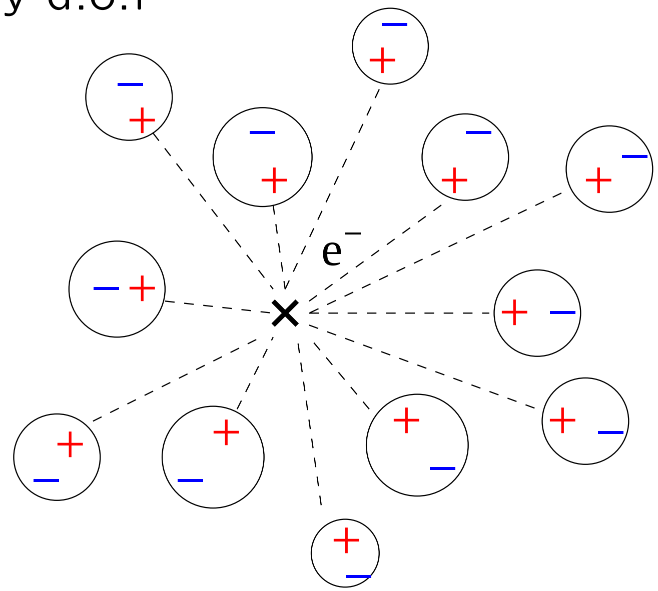
The original picture reduced 100 times $\sim$ 20 Kb

The original picture reduced 1000 times $\sim$ 2 Kb

Effective interactions: Running Couplings

You have learned that coupling constants "run" with energy. The reason is that they are the coefficients of the interaction terms in the effective action and therefore receive contributions from the quantum effects of the high-energy d.o.f





• In electromagnetism we have "screening":

• $e^+ - e^-$ pairs have the tendency to screen lone charges.

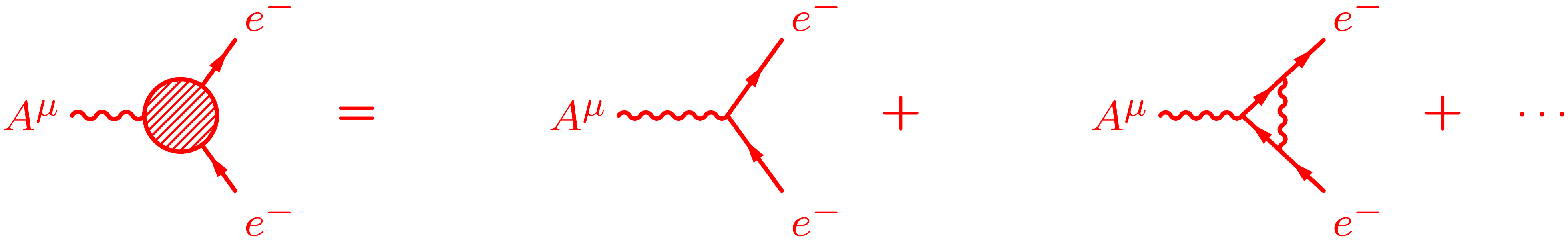
• The larger the distance $=$ more $e^+ - e^-$ pairs in-between $=$ more charge screening.

• Result: charge is a function of the energy$= \frac{1}{\text{distance}}$:

$$\alpha_{\rm em}(E) \simeq \frac{\alpha_{\rm em}(m_e)}{1-\frac{\alpha_{\rm em}(m_e)}{3\pi}\log\frac{E^2+m_e^2}{m_e^2}} \quad , \qquad \alpha_{\rm em} \equiv \frac{e^2}{2(\hbar c)}$$

• The charge becomes larger as we approach the electron closer.

• After taking into account these quantum effects on the coupling, we may replace the EM interaction by its corrected value:



$$\delta S(E) \quad \sim \quad e(E) \int A_\mu \, \bar{\psi}_e \, \gamma^\mu \, \psi_e$$

• This is the effective interaction valid at energy E.

Renormalization Summary

• A fundamental theory is defined at a high-energy scale $\Lambda \to \infty$.

• What we measure are effective interactions at low(er) energy (larger distance). They also contain the quantum effects of the higher energy modes.

• Knowledge of the high-energy (short-distance) theory defines completely the low energy theory. It does not work the other way around! (Universality!)

• At low energy , interactions of all possible dimensions (allowed by symmetries) are generated. Their effective couplings scale generically as

$$\lambda_{\Delta} \simeq \Lambda^{4-\Delta}\left[1+\mathcal{O}\left(\frac{E}{\Lambda}\right)\right] \quad , \quad \lambda_{\Delta=4} \sim \log \Lambda$$

Λ =the characteristic high energy scale, Δ = the (mass) dimension of the interaction.

• The (old) wisdom: a quantum theory must be renormalizable $\Rightarrow$ **Only renormalizable theories can be extended to high energy without modification** (They depend on a finite number of fundamental parameters)

• Suggestions or further reading are here

The hierarchy problem:introduction

- According to our previous discussion:

♠ Couplings of operators with $\Delta = 4$ depend logarithmically on the high energy scale Λ

♠ Operators of dimension two and three (mass terms for bosons and fermions) should have at low energy their coefficients scale as

$$m_i^2 \sim \Lambda^2 \quad \Leftrightarrow \quad m_i \sim \Lambda$$

If we want the SM to make sense up to $\Lambda \simeq 10^{18}$ GeV, then either:

- All masses are generically enormous (excluded from experiment)

- If there are light particles there are two possibilities:

♠ The masses are light due to a symmetry.

♠ The masses are accidentally light (fine-tuning).

♠ "No free lunch": we must have new physics at $\Lambda \sim 1 - 10$ TeV to avoid fine-tuning.

Fermion masses

Consider the electron Lagrangian written in terms of the left- and right-handed components of the electron,

$$e_{L,R} = \frac{1 \pm \gamma^5}{2}\; e$$

$$S = i\,[\bar{e}_R(\not\partial + \not A)e_R + \bar{e}_L(\not\partial + \not A)e_L] + m_e\;(\bar{e}_L e_R + \bar{e}_R e_L)$$

The theory has the usual vector U(1) symmetry (conservation of electric charge):

$$e_{L,R} \to e^{i\epsilon}\; e_{L,R} \quad , \quad \bar{e}_{L,R} \to e^{-i\epsilon}\; \bar{e}_{L,R}$$

When $m_e = 0$ there is more symmetry: chiral symmetry,

$$e_L \to e^{i\epsilon} e_L \quad , \quad e_L \to e^{-i\epsilon} e_L$$

Inversely: chiral symmetry forbids a mass.

The quantum corrections to the fermion mass coming from the diagrams

- We would expect that $\delta m \sim \Lambda \left[c_1 + c_2 \frac{m_e}{\Lambda} \log \frac{E}{\Lambda} + \mathcal{O}(\Lambda^{-2})\right]$.

- The result of the calculation gives though (to leading order in $\alpha_{\tt em}$)

$$m_{eff}(E) = m_e + \frac{3\alpha_{em}}{4\pi}\; m_e\; \log \frac{E}{\Lambda} = m_e \left[1 + \frac{3\alpha_{em}}{4\pi} \log \frac{E}{\Lambda}\right]$$

- There is no linear dependence on Λ!

- Therefore, **it is very insensitive to the high-energy scale Λ.** ($\sim 4\%$ for $E = 1$ GeV and $\Lambda = 10^{19}$ GeV).

Gauge boson masses

Unbroken gauge symmetry forbids gauge bosons to have a mass. Upon spontaneous breaking of the gauge symmetry gauge bosons acquire masses.

$$M_{Z,W^\pm} \sim g\ v_F \qquad v_F \sim \frac{\mu}{\sqrt{\lambda}} \quad , \quad V = -\frac{\mu^2}{2}H^2 + \lambda H^4$$

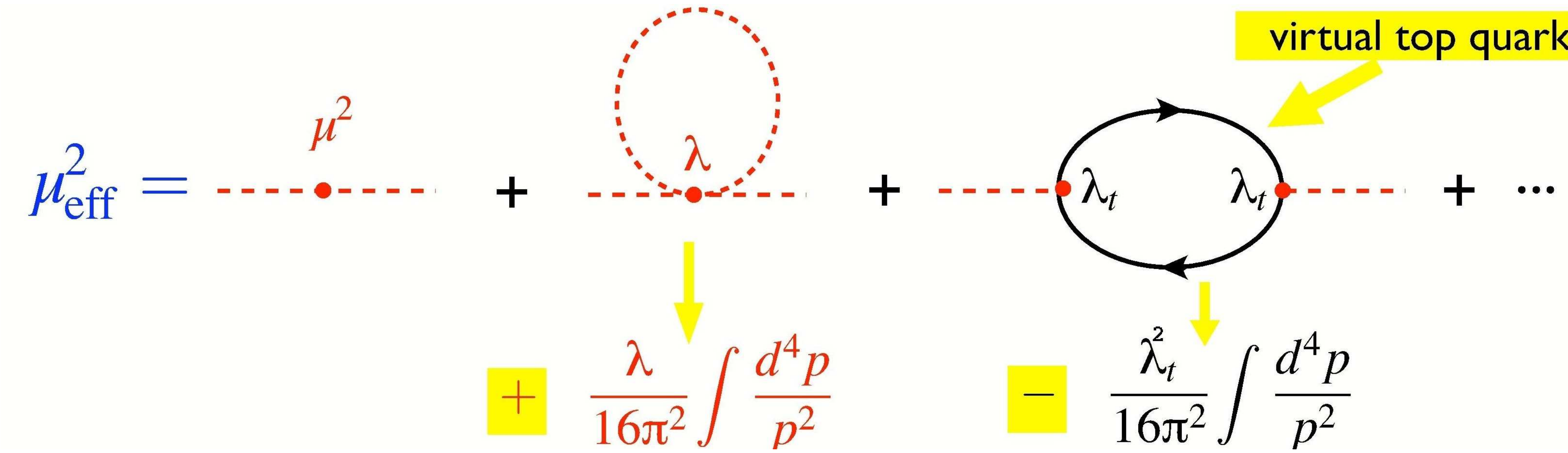
Dimensionless couplings run logarithmically $\sim \log\frac{E}{\Lambda}$ and therefore are not very sensitive to Λ.

• The important sensitivity comes from the renormalization of the mass-term of the Higgs, μ.

• In the SM this is also the case for the fermions as $m_f \sim \lambda_{\rm Yukawa}\ v_F$

The Higgs mass term

We have seen that the sensitivity of SM masses depends on the behavior of a single parameter: the mass term μ of the Higgs scalar.



$$\mu^2_{eff}(E) = \mu^2 + \frac{\lambda - \lambda_t^2}{4\pi^2}(\Lambda^2 - E^2)$$

The Higgs mass, and therefore many other SM masses depend quadratically on the UV scale Λ.

The hierarchy problem

We found that:

• All dimensionless couplings of the SM run logarithmically and are therefore not very sensitive to the UV scale of the theory.

• The Higgs quadratic term $\mu \Rightarrow$ the Higgs expectation value $v_F \Rightarrow$ Fermion and gauge-boson masses is linearly sensitive to Λ.

• The SM physics at high energy is therefore technically hard to calculate as fine-tuning is required.

This is the hierarchy problem:**It is very difficult in a theory where parameters run polynomially with the cutoff Λ to extend it to hierarchically higher energies.**

End of first act

Evading the hierarchy problem

Very SPECIAL theories may avoid the hierarchy problem.

- "Technicolor"

- "Supersymmetry"

- Large dimensions

- Pseudo-Goldstone particles (aka Little Higgs)

See [arXiv:hep-ph/0512128] and [arXiv:hep-ph/0502182]

Technicolor

• The idea, known under the name of "technicolor", is to assume that all particles in the fundamental theory except the gauge bosons are fermions.

• And the Higgs? It could be a bound state of two fermions (like meson scalars are bound states of quarks and anti-quarks, or the Cooper pair in superconductors).

• This needs a new gauge interaction (technicolor) that becomes strong at an energy $\Lambda_T > v_F$.

• For $\infty >> E >> \Lambda_T$ the theory is a theory of fermions and all masses run logarithmically.

• For $E << \Lambda_T$ the theory looks like the SM.

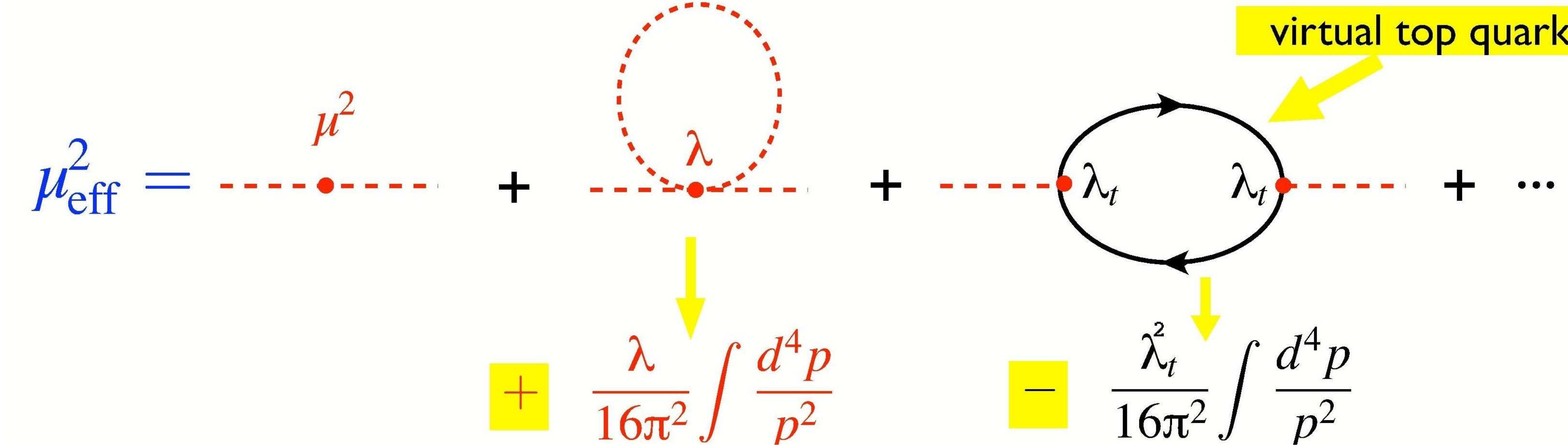
• Therefore $\Lambda_T \sim 1TeV$.

♣ Unfortunately detailed models that satisfy known experimental constraints are very difficult to construct.

(See here for the simplest model and hep-ph/0007304 for a comprehensive review.)

Supersymmetry

Another SPECIAL class of theories:



If $\lambda = \lambda_t^2$ then the quadratic divergence will cancel. Fermion and boson loops cancel each other.

The symmetry that imposes such relations is known as

supersymmetry (SUSY) .

$$\delta_{\text{SUSY}}\,(\text{Boson}) = \epsilon \cdot (\text{Fermion})$$

$$\delta_{\text{SUSY}}\,(\text{Fermion}) = \epsilon \cdot \partial\,(\text{Boson})$$

Therefore

$$\delta_{\text{SUSY}} \cdot \delta_{\text{SUSY}} \sim \partial$$

and in this sense SUSY is a "square root" of a translation.

Supersymmetry pairs a particle with spin j will another with spin $j \pm \frac{1}{2}$

Then the Higgs will have a fermionic partner (the Higgsino) whose effect will be to cancel the quadratic terms in the running of the mass.

The Supersymmetric Multiplets (representations)

There are two supersymmetric multiplets with spin at most 1:

• The chiral (or scalar) multiplet: (ϕ, ψ) $\rightarrow$ a complex scalar and a Weyl fermion.
• The vector multiplet (A_μ^a, λ^a) $\rightarrow$ a vector and Weyl fermion (gaugino).

• In the SSM we must promote all gauge bosons to vector multiplets containing a vector (gauge boson) and a Majorana fermion (gaugino) in the adjoint of the SM gauge group $\rightarrow$ (A_μ^a, λ^a).

• We also promote all fermions to chiral multiplets containing a complex scalar and a Weyl fermion, in the appropriate representation of the gauge group $\Phi_i \equiv (\phi^i, \psi^i)$.

♠ We must promote the Higgs doublet to chiral multiplet H_1^{-1} but we also need to introduce a second conjugate Higgs multiplet H_2^{+1}.

♣ This is to avoid $U(1)_Y$ anomalies and give masses to both up and down quarks (see later).

Exercise : Show that no particle of the SM can be a susy partner: a full doubling of the spectrum is necessary.

The quantum numbers

chiral supermultiplet	SU(3)	SU(2)	$U(1)_Y$
Q	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$
U^c	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$
D^c	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$
L	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
E^c	$\mathbf{1}$	$\mathbf{1}$	1
H_1	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
H_2	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$

Note that L and H_1 are indistinguishable in terms of gauge quantum numbers.

The Supersymmetric Standard Model

particles				Sparticles			
quarks	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	u_R	d_R	squarks	$\begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}$	$\tilde{u}_R$	$\tilde{d}_R$
leptons	$\begin{pmatrix} e_L \\ \nu_L \end{pmatrix}$	e_R		sleptons	$\begin{pmatrix} \tilde{e}_L \\ \tilde{\nu}_L \end{pmatrix}$	$\tilde{e}_R$	
Higgs doublets	H_1 (hypercharge $= -1$)			Higgsinos	$\tilde{H}_1$		
	H_2 (hypercharge $= +1$)				$\tilde{H}_2$		
	W^+, H^+			charginos	χ_1^+, χ_2^+		
	Z, γ, H_i^0	$(i = 1, 2, 3)$		neutralinos	χ_a^0	$(a = 1, 2, 3, 4)$	
	g			gluino	$\tilde{g}$		

The supersymmetric interactions

• The renormalizable interactions of a gauge theory are encoded in gauge couplings, Yukawa couplings and the potential for the scalars.

• In a supersymmetric renormalizable theory, the interactions are encoded into the gauge couplings and the super-potential W.

• W is a gauge-invariant function of the chiral fields, (but not of their complex conjugates). A renormalizable W is at most cubic.

• The kinetic terms of the fields and their couplings to the gauge bosons are standard and determined by the representations/charges and the gauge couplings.

.

- The Yukawa couplings are as follows:

$$L_Y = [i\sqrt{2}\ g\ \bar{\psi}_i\ \lambda^a (T^a\phi)^i + h.c.] - \frac{1}{2}\left[\frac{\partial^2 W}{\partial\phi^i\partial\phi^j}\ \bar{\psi}^i\psi^j + h.c.\right]$$

- Finally there is the scalar potential

$$V = \sum_i \left|F_i(\phi)\right|^2 + \frac{g^2}{2}\sum_a [D^a(\phi)]^2 \geq 0$$

with

$$F_i = \frac{\partial W}{\partial\phi^i} \quad , \quad D^a = \phi_i^*(T^a)^i{}_j\phi^j$$

• The most general cubic, gauge invariant and holomorphic superpotential:

$$W = \mu H_1 H_2 + \zeta^U \; Q U^c H_2 + \zeta^D \; Q D^c H_1 + \zeta^E \; L E^c H_1 +$$

$$+ \lambda Q D^c L + \lambda' L L E^c + \mu' L H_2 + \lambda'' U^c D^c D^c$$

Exercise Show this!

• μ' has one family index $(\zeta^U, \zeta^D, \zeta^E)$ have two such indices and $(\lambda, \lambda', \lambda'')$ have three.

• The last four terms violate baryon and lepton number.

$$\lambda, \lambda', \mu' \neq 0 \quad \to \quad \Delta B = 0, |\Delta L| = 1 \quad , \quad \lambda'' \neq 0 \quad \to \quad \Delta B = 1, |\Delta L| = 0$$

A discrete symmetry (R-parity) must be imposed to forbid them.

Exercise Why such offending terms are absent in the SM?

Exercise Why we do not set the coefficients of the offending (baryon+lepton violating) terms to zero in the superpotential?

Supersymmetric Renormalization

• The parameters (coupling constants) of the canonical supersymmetric quantum field theory include:

(1) The gauge coupling constant g (simple group).

(2) The coefficients of the monomials in the superpotential:

$$W = W_0 + W_1^i \Phi_i + W_2^{ij} \Phi_i \Phi_j + W_3^{ijk} \Phi_i \Phi_j \Phi_k + \cdots$$

- W_0 is a trivial constant that does not affect physics (in the absence of gravity).

- W_1^i contributes constants to the potential (vacuum energy).

- W_2^{ij} contribute to masses

- W_3^{ijk} contribute to Yukawa's and quartic scalar couplings.

If supersymmetry is unbroken, then :

- The superpotential is not renormalized in perturbation theory.
- The gauge coupling runs logarithmically with energy.

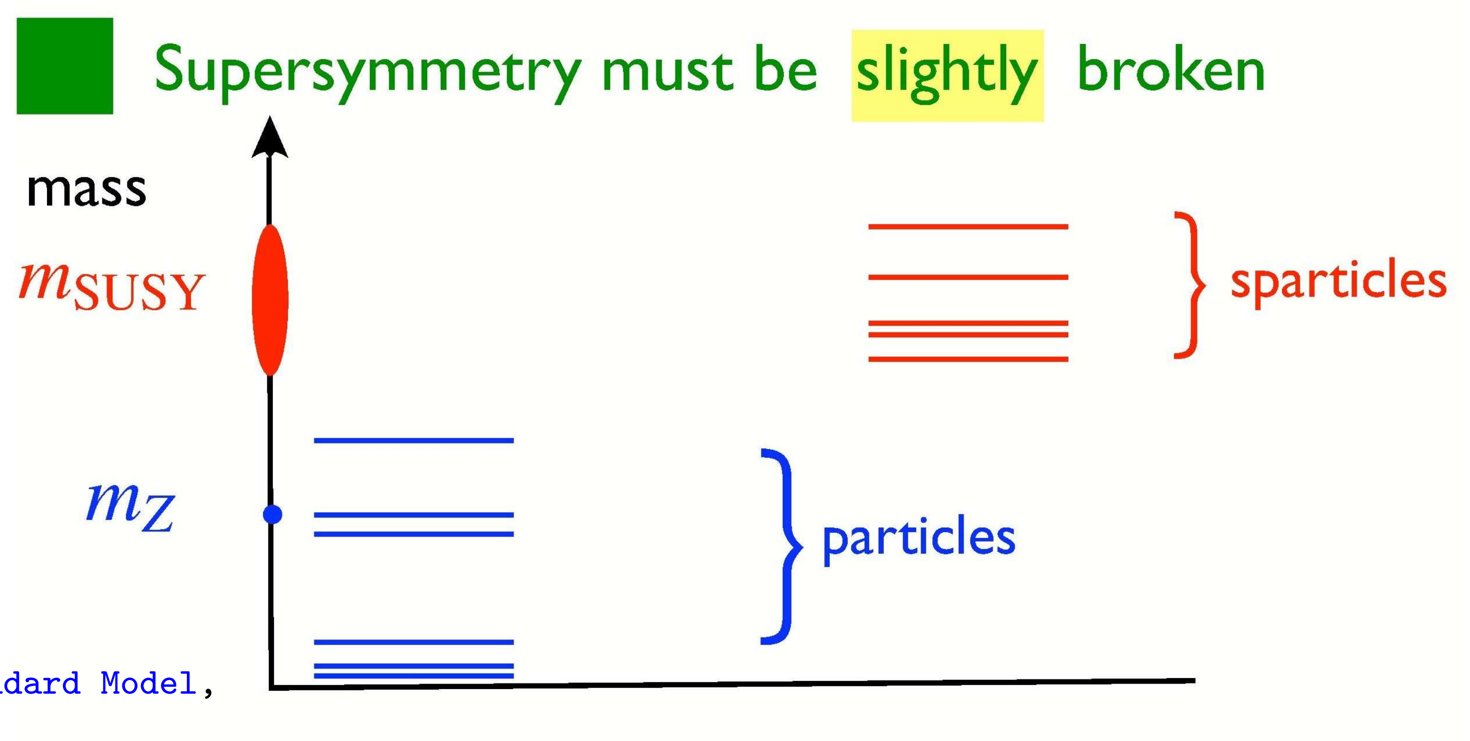
Therefore, all couplings including physical masses run at most logarithmically: there is no hierarchy problem in a supersymmetric QFT.

Supersymmetry breaking

So far we have neglected the fact that exact supersymmetry forces the superpartners to have the same mass as the SM particles, e.g.

$$m_e = m_{\tilde{e}} \quad , \quad \text{etc.}$$

It is unavoidable to conclude that:



Supersymmetry breaking, II

We must ensure that SUSY breaking does not destroy the good properties of SUSY:

• There are good reasons to believe that like gauge symmetry breaking, supersymmetry breaking must be spontaneous.

• It is characterized by a SUSY-breaking scale M_{SUSY}, that sets the scale for the masses of superpartners.

• Above M_{SUSY} the Higgs mass runs logarithmically like that of the fermions!

• M_{SUSY} must not be very far from v_F. It should be $1-10$ TeV.

• If $M_{\text{SUSY}} >> v_F$ the hierarchy problem resurfaces.

• Therefore, naturalness tells us that the superpartners must be in the TeV range.

• If this idea is correct, most probably the superpartners will be found at LHC.

R-parity

To avoid problems with fast proton decay and lepton number violation: assume the existence of an extra Z_2 symmetry

$$\text{R}-\text{parity} \quad = \quad (-1)^{2S+3(B-L)} \quad = \quad (-1)^{\text{number of Sparticles}}$$

- Sparticles can only be produced or annihilated in pairs (harder to produce).

- The lightest Sparticle (LSP) is absolutely stable.

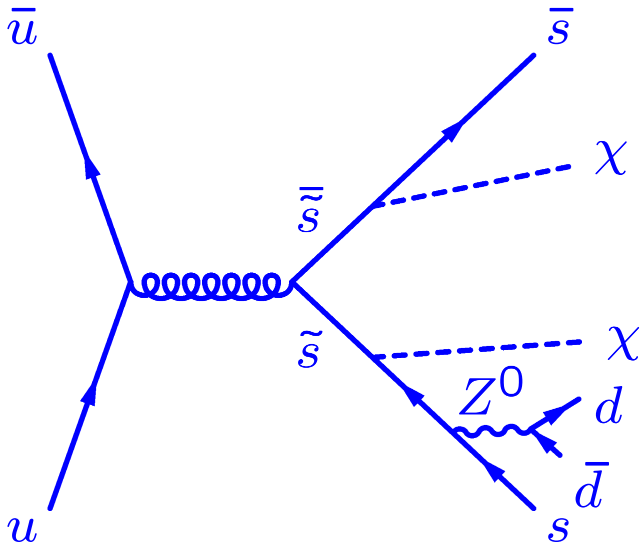
- It is almost always a neutralino $\to$ it has only weak interactions $\to$ it is not directly visible in experiments $\to$ missing energy.

This is a characteristic SUSY signal at LHC.

- When supersymmetry breaks, R-parity must remain intact!

Missing Energy

This is an example of a possible event that can be seen at LHC:

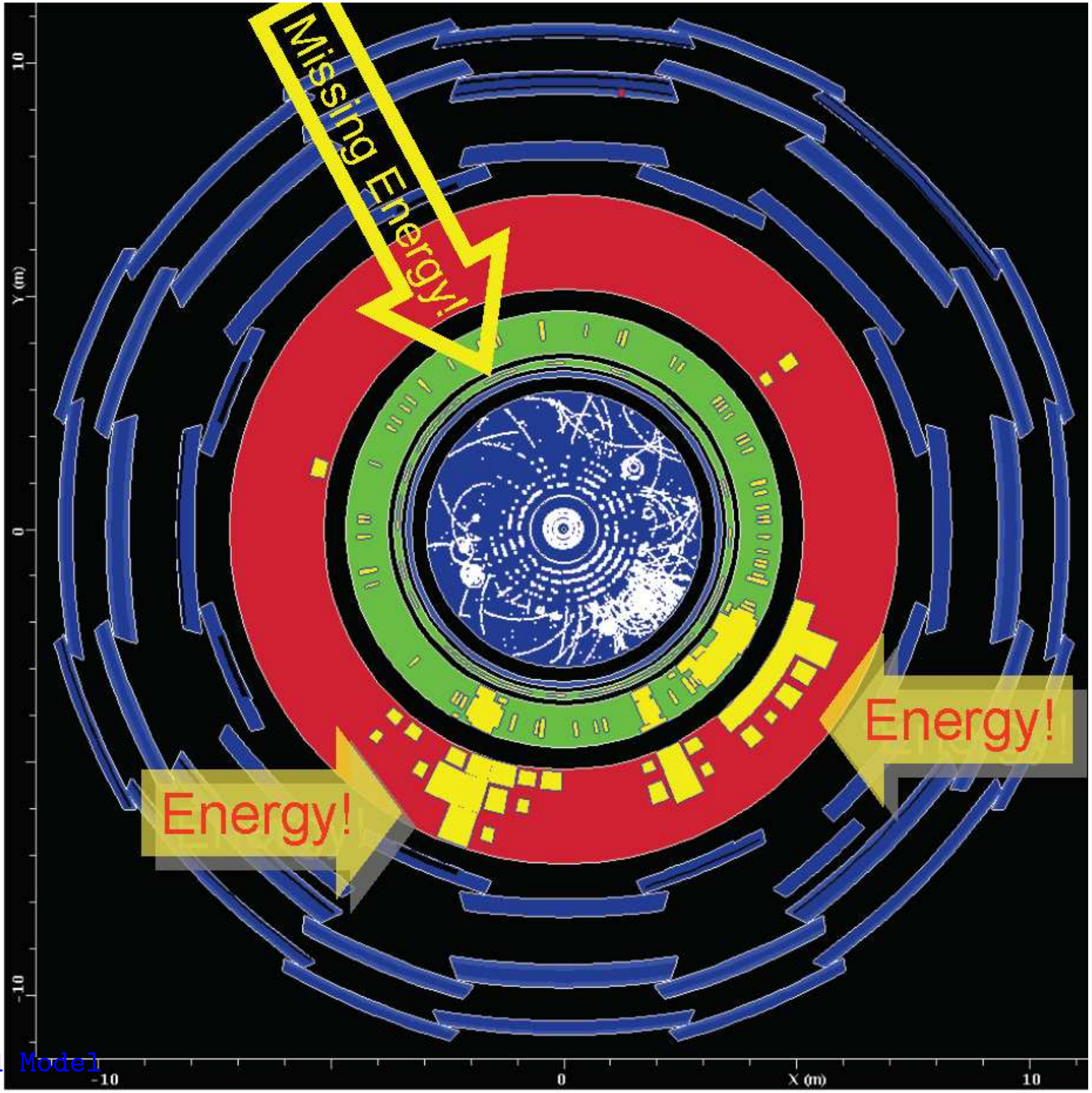


where:

$$\tilde{q} \quad = \quad squark$$

$$\chi \quad = \quad \text{LSP}$$

Missing Energy (Atlas simulation)



A link to the dark matter of the Universe

• The universe contains an important fraction (22%) of non-relativistic, non-SM matter. This is known as Dark Matter.

• There is solid evidence for it at the galactic level (rotation curves)

• Its presence is crucial for structure formation in the universe which more-over requires that it is NON-BARYONIC.

• It is mostly composed of Weakly Interacting (very) Massive Particles: WIMPS.

• The supersymmetric LSP, is an excellent candidate for forming the dark matter of our universe.

The soft supersymmetry breaking terms

• What is the effective theory of a spontaneously broken supersymmetric theory?

• Spontaneous supersymmetry breaking invalidates the supersymmetric non-renormalization theorems, but their violation is "soft".

• The (non-supersymmetric) effective action differs from the supersymmetric one by what are known as "soft terms".

• They have the property, that when added to a supersymmetric action, they do not affect the UV behavior of the theory. (super-renormalizable, $\Delta < 4$)

• They have dimension $\Delta = 2, 3$ and their coefficients are "masses", M_i.

• Such mass scales characterize the mass-splitting of supersymmetric multiplets.

• For $E \gg M_i$ the theory behaves as a supersymmetric quantum theory.

• The soft terms are:

(a)masses for the scalar field of chiral multiplets.

(b)masses for gaugini.

(c)Cubic scalar interactions (if allowed by the superpotential)

MSSM

- We parametrize the general supersymmetric+spontaneosuly broken effective theory.
- The most general soft terms for the SSM (preserving R-parity) are:

$$L_{\rm soft}=\sum_i \tilde m_i^2|\phi_i|^2+\frac{1}{2}\sum_A M_A\bar\lambda^A\lambda^A+$$

$$+\left(\zeta^U A^U\ \tilde q\tilde u^c h_2+\zeta^D A^D\ \tilde q\tilde d^c h_1+\zeta^E A^E\ \tilde\lambda\tilde e^c h_1+m_3^2\ h_1h_2+h.c.\right)$$

where $\tilde q,\tilde u^c,\tilde d^c$ are the respective squarks and $h_{1,2}$ the two Higgs doublets. This gives rise to the Minimal Supersymmetric Standard Model (MSSM).

- Since A^i are matrices in flavor space, we have a large number of parameters. For generic values of such parameters there are phenomenological problems (like flavor changing neutral currents).

The tree-level MSSM potential that involves the Higgses (There is also the potential for squarks and sleptons) is:

$$V=m_1^2|h_1|^2+m_2^2|h_2|^2+m_3^2(h_1h_2+h.c.)+\frac{g_2^2}{8}\left(h_2^\dagger\vec\sigma h_2+h_1^\dagger\vec\sigma h_1\right)^2+\frac{g_Y^2}{8}\left(|h_2|^2-|h_1|^2\right)^2$$

$$m_1^2=\mu^2+\tilde m_{h_1}^2\quad ,\quad m_2^2=\mu^2+\tilde m_{h_2}^2$$

• Without any extra input, there are no UV constraints on the MSSM parameters.

• A simple ansatz (compatible with data so far, and which can arise from supergravity/string theory) Imposed at some UV scale Λ:

Gaugino masses and soft scalar masses are universal

$$M_3 = M_2 = M_Y \equiv m_{1/2}$$

$$\tilde{m}_Q = \tilde{m}_{U^c} = \tilde{m}_{D^c} = \tilde{m}_L = \tilde{m}_{E^c} = \tilde{m}_{H_1} = \tilde{m}_{H_2} \equiv m_0$$

So are the soft scalar couplings

$$A^U = A^D = A^E \equiv A_0$$

If we now include the μ-term coefficient, μ and the soft breaking term m_3 we end up with 5 extra parameters on top of the SM ones:

$$\mu \quad , \quad m_{1/2} \quad , \quad m_0 \quad , \quad A_0 \quad , \quad m_3$$

• After minimization of the Higgs potential with $\langle H_1 \rangle = \binom{v_1}{0}$, $\langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$ we can trade μ and m_3 with sign(μ) and $\tan\beta \equiv \frac{v_1}{v_2}$.

$$\text{sign}(\mu) \quad , \quad m_{1/2} \quad , \quad m_0 \quad , \quad A_0 \quad , \quad \tan\beta$$

This is known as the mSUGRA parametrization of the MSSM.

• The parameters, $m_{1/2}, m_0, A_0$, must be evolved to low energy using the RGE equations and eventually compared to data. $\tan\beta$ is already a low energy parameter.

You are now guided to this page where a simple example is analyzed in order to illustrate the derivation of the softly broken supersymmetric action

SUSY outlook

• Supersymmetry provides a way out of the hierarchy problem if superpartners are in the TeV range

• We will see that this fits well with the Unification of SM coupling constants

• Supersymmetry can also be used to solve the hierarchy problems of unified theories.

♠ Supersymmetric extensions of the standard model are not however free of problems :

• Care is needed to avoid FCNC

• Dynamical Spontaneous supersymmetry breaking is a very tricky issue.

• No perfect or nearly perfect model

• Supersymmetry cannot really help with the cosmological constant problem.

Grand Unification: The idea

The Standard Model gauge group is not "fully unified". At higher energy, the symmetry becomes larger. At lower energies it breaks spontaneously to the standard model group: $SU(3)\times SU(2)\times U(1)_Y$

$$SU(3) \Longrightarrow U_3U_3^\dagger = \mathbf{1} \quad , \quad det(U_3) = 1$$

$$SU(2) \Longrightarrow U_2U_2^\dagger = \mathbf{1} \quad , \quad det(U_2) = 1$$

We can include $SU(3)\times SU(2)\times U(1)_Y$ inside SU(5)

$$SU(5) \Longrightarrow U_5U_5^\dagger = 1 \quad , \quad det(U_5) = 1$$

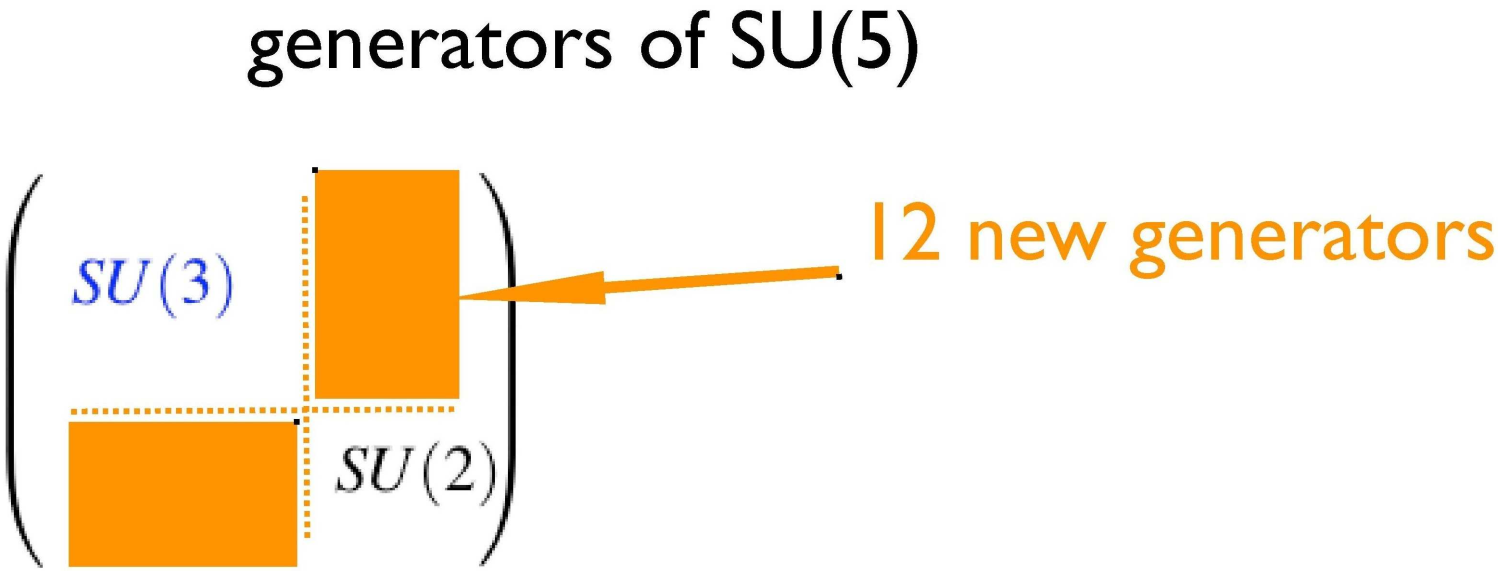
subgroups of SU(5)

$$SU(3) \Rightarrow \begin{pmatrix} U_3 & & & 0 & 0 \\ & & & 0 & 0 \\ & & & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} U_3 & 0 \\ 0 & \mathbf{1}_{2\times 2} \end{pmatrix}$$

$$SU(2) \Rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & & \\ 0 & 0 & 0 & & U_2 \end{pmatrix} = \begin{pmatrix} \mathbf{1}_{3\times 3} & 0 \\ 0 & U_2 \end{pmatrix}$$

$$U(1)_Y \sim \begin{pmatrix} e^{i\frac{\theta}{3}} & 0 & 0 & 0 & 0 \\ 0 & e^{i\frac{\theta}{3}} & 0 & 0 & 0 \\ 0 & 0 & e^{i\frac{\theta}{3}} & 0 & 0 \\ 0 & 0 & 0 & e^{-i\frac{\theta}{2}} & 0 \\ 0 & 0 & 0 & 0 & e^{-i\frac{\theta}{2}} \end{pmatrix}$$

Since dim($SU(5)$)=24, there are 12 extra gauge bosons apart from the SM ones.



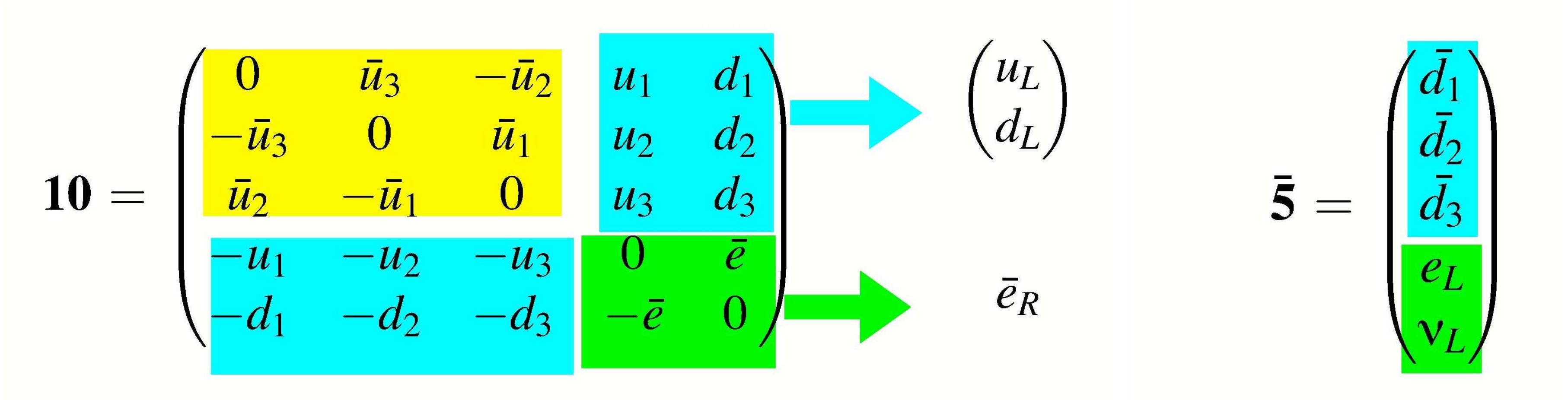
MORE

$$U(x)U(x)^\dagger = 1 \quad , \quad Det[U(x)] = 1$$

The infinitesimal generators of SU(5) are T^a, $a = 1, 2, \cdots, 24$

$$U(x) = e^{iT^a\theta^a(x)} \quad , \quad (T^a)^\dagger = T^a \quad , \quad Tr[T^a] = 0$$

SU(5): the matter



There should also be a singlet to accommodate ν_R.

• We have three copies of such representations to generate the three families.

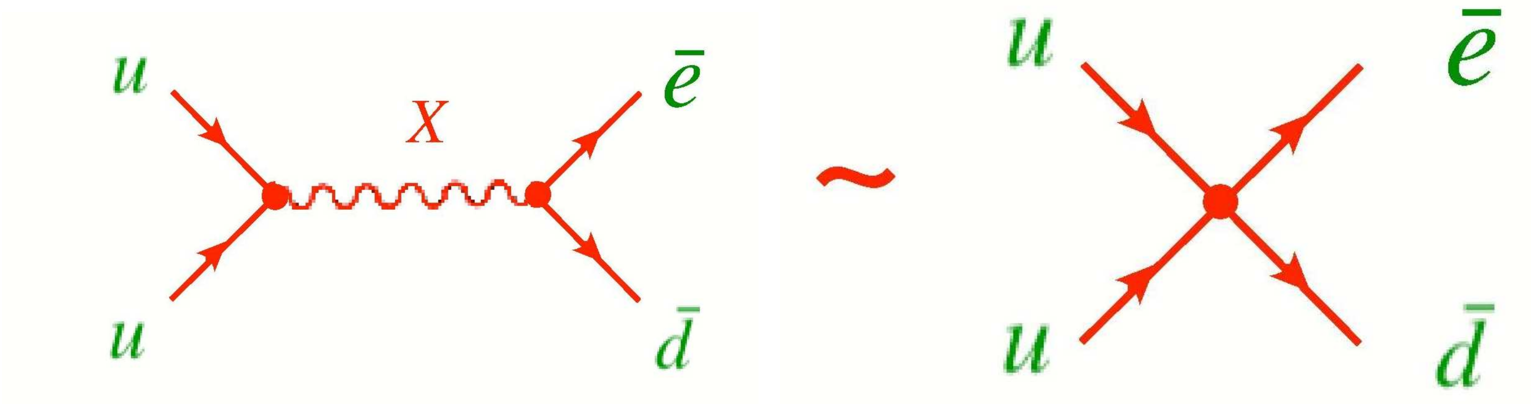
• The new, larger symmetry mixes quarks and leptons:We expect baryon and lepton number to be violated by the new gauge interactions.

• We also expect relations between masses of particles

• There are other grand unified groups like SO(10) that give also successful predictions in the neutrino sector.

Proton decay

The SU(5) symmetry should break spontaneously at some high energy scale Λ_{GUT} to $SU(3) \times SU(2) \times U(1)_Y$ (via a new Higgs effect). The 12 extra gauge bosons X will acquire masses $M_X \sim \Lambda_{GUT}$.



As with the Fermi example this four-fermion effective interaction has a coupling $\sim \frac{g_5^2}{M_X^2}$

From experiment we obtain that $\tau_p > 2.6 \times 10^{33}$ years. This implies

$$M_X > 10^{15} \text{ GeV}$$

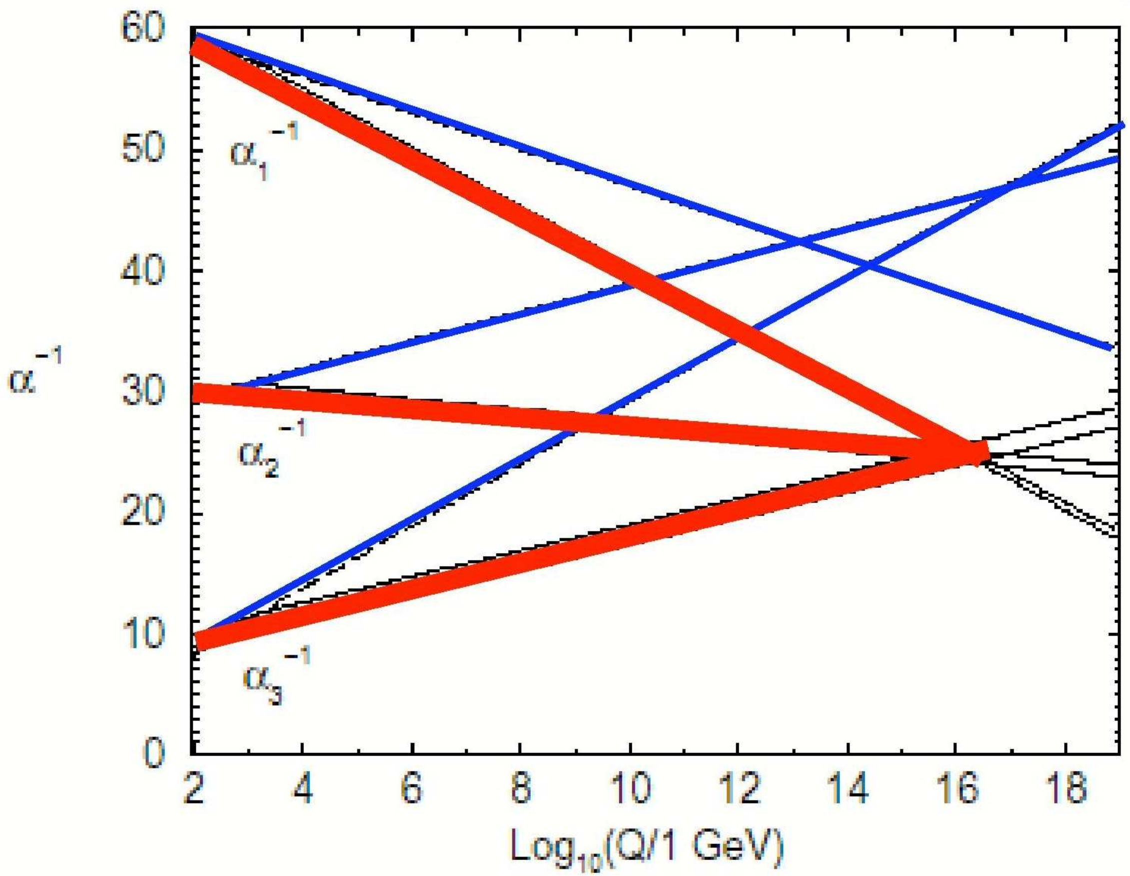
MORE

Coupling unification

We have coupling unification at the scale $\Lambda = M_X$

$$g_3 = g_2 = \sqrt{\frac{5}{3}}\, g_Y = g_5 \equiv g_{GUT}$$

This seems in good agreement with the data if we allow for the renormalization group running



The gravitational coupling

The coupling of gravity, Newton's constant G_N has dimensions M^{-2}. This is how we define the Planck Mass :

$$G_N = \frac{1}{M^2_{\text{Planck}}}$$

Gravitational force:

$$F = G_N \; \frac{M_1 \; M_2}{R^2} \sim G_N \; \frac{E_1 \; E_2}{R^2}$$

The dimensionless gravitational coupling runs fast with energy:

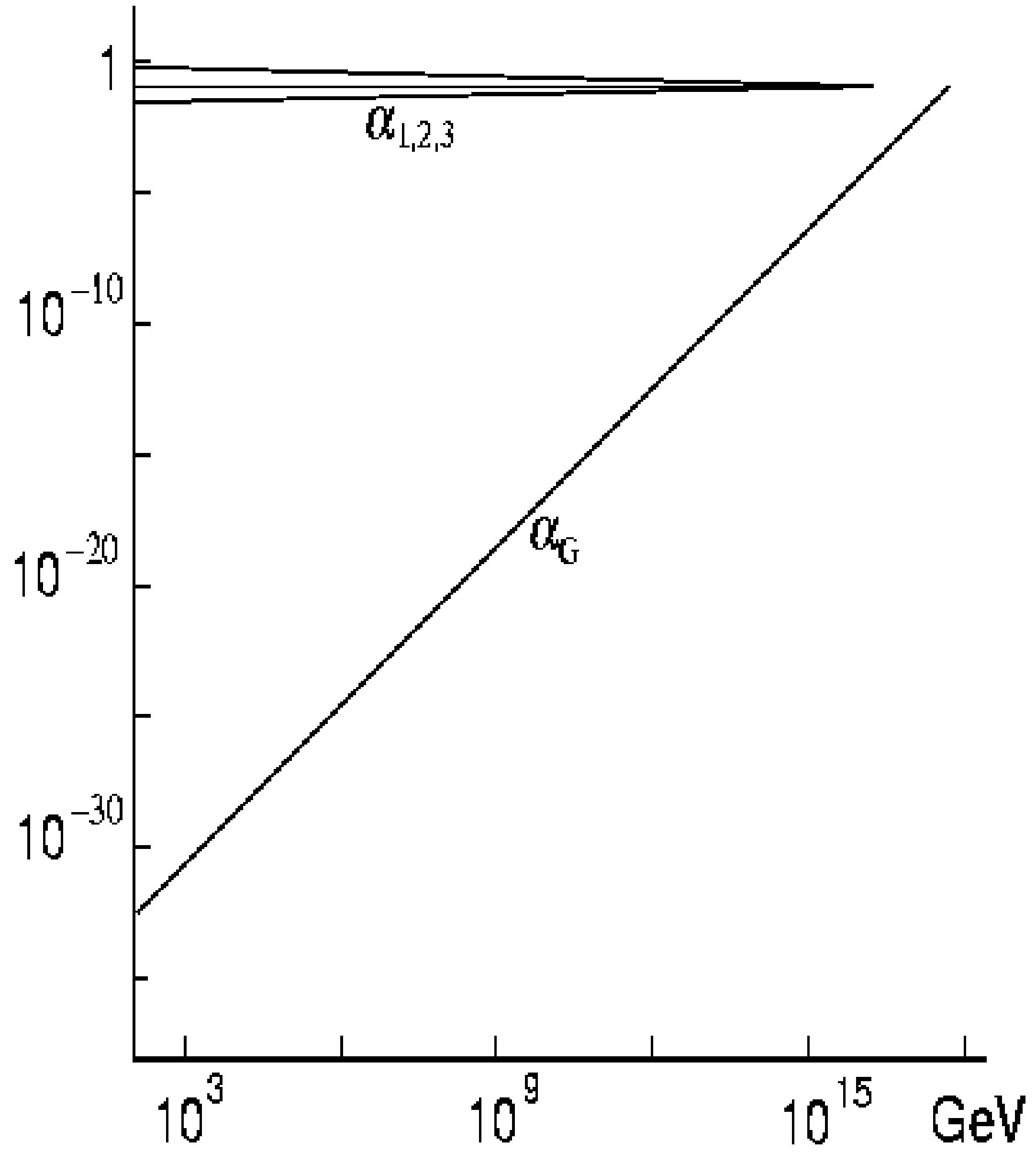
$$\alpha_{grav} \equiv G_N \; E^2 = \frac{E^2}{M^2_{\text{Planck}}}$$

Gravity versus other interactions

interaction	dimensionless coupling	strength
Strong	$\alpha_s = \frac{g_s^2}{4\pi\hbar c}$	~ 1
Electromagnetic	$\alpha_{\texttt{em}} = \frac{e^2}{4\pi\hbar c}$	$\sim \frac{1}{137}$
Weak	$G_F \ m_p^2$	$\sim 10^{-5}$
Gravity	$G_N \ \frac{m_p^2}{\hbar c}$	$\sim 10^{-36}$

Therefore until now gravity has been safely neglected in particle physics.

The running of all couplings



Gravity and the SM

The existence of gravity is one of the most solid arguments that the SM is not the final theory.

• Gravity interacts with SM fields.

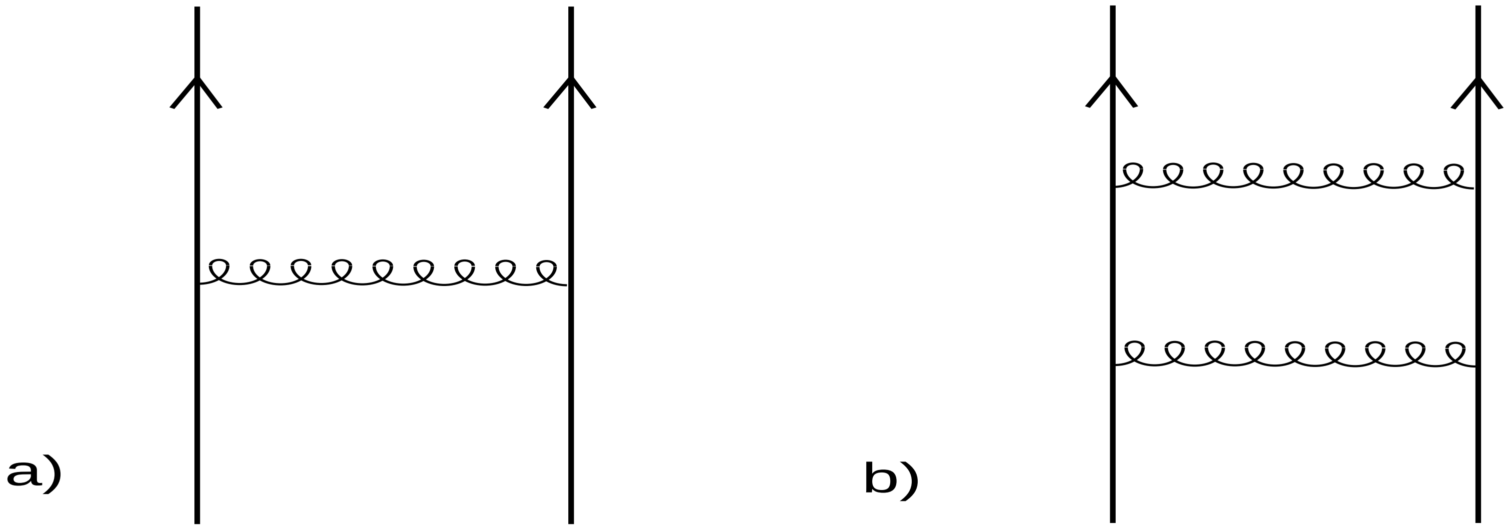
• At some high energy scale, Λ_P gravity will become strong, and quantum effects must be incorporated. This scale could be $M_P \sim 10^{19}$ GeV but (as we will see later) it could also be much lower.

• This fundamental theory, would look like classical gravity plus the SM at energies $E << \Lambda_P$.

• In this sense the SM is an effective theory, valid (at most) up to Λ_P.

• Things look bad, since classical gravity (general relativity) is a non-renormalizable theory.

Gravity at short distances?



- The classical gravitational theory is non-renormalizable

$$(b) \quad \sim \quad \frac{E^2}{M^4_{\mathsf{Planck}}} \int_0^{\Lambda} dp\ p \quad \sim \quad \frac{\Lambda^2\ E^2}{M^4_{\mathsf{Planck}}},$$

- At higher orders it gets worse and worse.

- No clue as to what the short distance theory is.

- This has been an open problem for more than 50 years.

Gravity and String Theory

• String theory is a different framework for describing and unifying all interactions.

• It always includes quantum gravity, without UV problems (UV divergences)

• It also includes the other ingredients of the SM: Gauge interactions, chiral matter (fermions) and if needed, supersymmetry.

• It offers some conceptual features that are appealing to (many) physicists:

(a) String theory ALWAYS contains gravity

(b) The existence of fermions implies supersymmetry at high energy.

(c) It has a priori no fundamental parameters but only one dimensionfull scale: the size of the strings. All dimensionless parameters of a given ground state of the theory are "dynamical" (expectation values of scalar fields). This allows for a multitude of different vacuum states ("the landscape").

(d) It contains solitonic extended objects (known as branes) that provide an incredible richness to the theory as well as a deep link between gauge theories and gravity.

What is String Theory?

Shift in paradigm: from point particle to a closed string.

• In QFT fields are “point-like”. In string theory, they depend not on a point of space-time but a loop in space-time (the position of a closed string).

What is the difference between a closed “fundamental” string and a loop of wire?

(A) The fundamental string is much smaller: its size is definitely smaller than 10^{-18} m. This would explain why we have not seen one so far.

(B) Apart from the usual degrees of freedom (their coordinates in space-time), fundamental strings have also fermionic degrees of freedom. There a kind of supersymmetry relating the coordinates to such fermionic degrees of freedom.

Since the smallest length we can see today (with accelerators) is approximately 10^{-18} m strings would appear in experiments so far as point-like objects.

• Fundamental strings, like the analogous classical objects, can vibrate in an infinite possible number of harmonics.

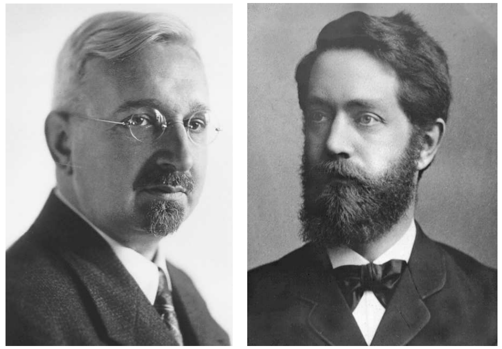
• Upon quantization, these harmonics behave like different particles in space-time.

A single string upon quantization $\Longrightarrow$ an infinite number of particles with ever increasing mass.

• Infinity of particles is responsible for the unusual properties of string theory (and its complicated structure).

• Strings live in diverse dimensions. Lorentz invariance/weak curvature $\Leftrightarrow$ 9+1 dimensions. Although this seems to contradict common experience it can be compatible under certain circumstances. How do we see the extra dimensions?

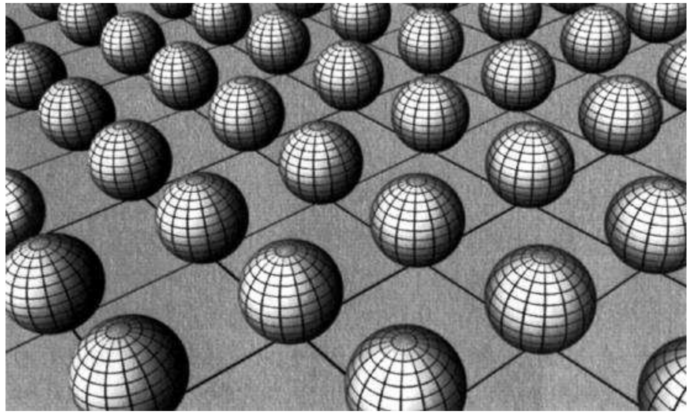
Extra space dimensions



- The idea that space has extra, hitherto unobservable dimensions goes back to the beginning of the twentieth century, by Kaluza (1925) and Klein (1926).

- It comes naturally in string theory.

How come they are not visible today?



(A) Because they compact and sufficiently small.

(B) Because we are "stuck" on the 4D world.

(C) Because they are of a more bizarre kind (for example, they are discretized appropriately)

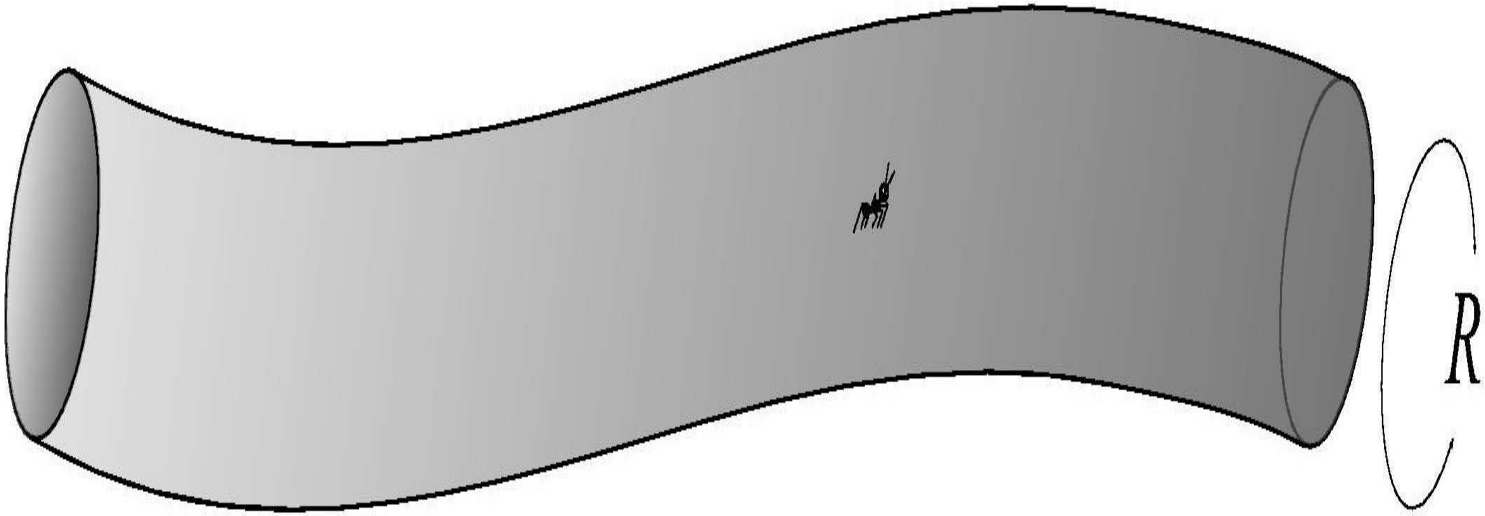
“Small” compact dimensions

A compact, sufficiently small extra dimension is not visible !

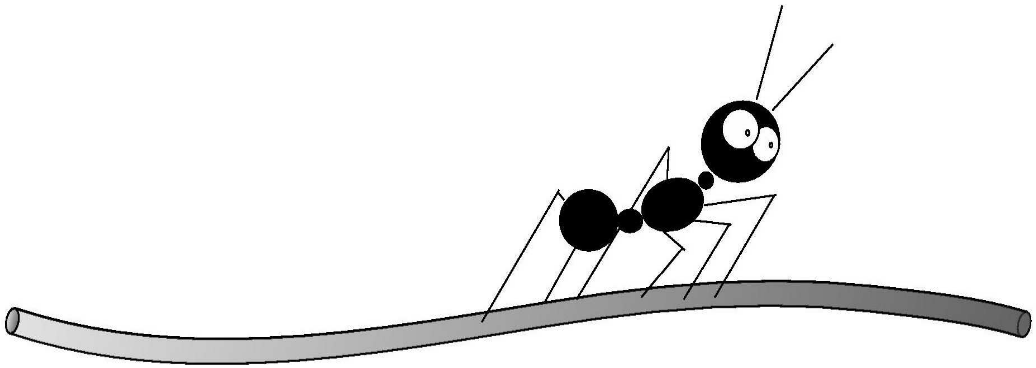
A simple example of a space with one compact (circle) and one non-compact (real line) dimension: a hose of infinite length and radius R.

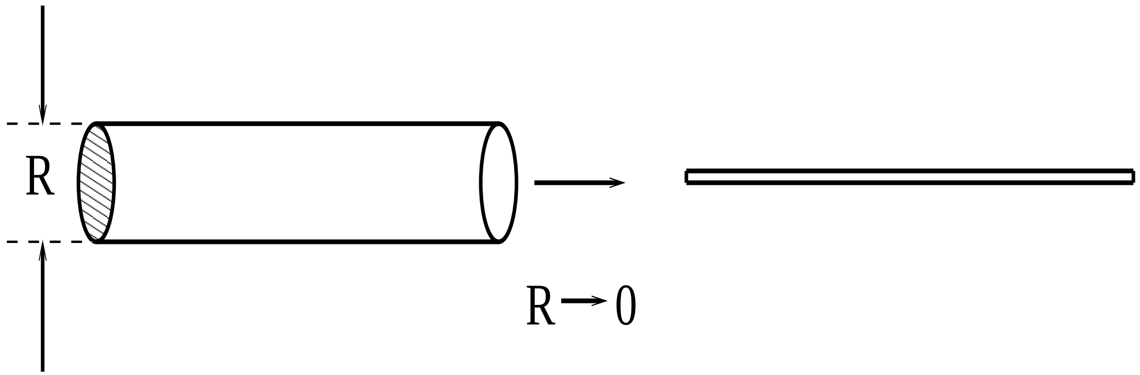
There are two regimes:

(A) At distance $<<$ R the space looks like an (infinite) two-dimensional plane.



(B) At distance $>> R$ the compact direction of the hose is invisible. The hose looks one-dimensional.





We will now make this intuition more precise.

Kaluza-Klein states

Consider the usual 3+1 dimensional space-time and a fifth dimension that is a circle of radius R. Consider also a free massless scalar field in this 5d space-time.

- From QM: the momentum on a circle is quantized.

$$\exp\left[ip_4\ (x^4+2\pi R)\right] = \exp\left[\ i\ p_4\ x^4\ \right] \quad \Rightarrow \quad e^{2\pi i\ p_4 R} = 1 \quad \longrightarrow \quad p_4 = \frac{n}{R}$$

From the mass-less condition in 5 dimensions:

$$E^2 - \vec{p}^2 = 0 \to E^2 - p_1^2 - p_2^2 - p_3^2 - p_4^2 = 0 \to E^2 - p_1^2 - p_2^2 - p_3^2 = \frac{n^2}{R^2}$$

Compare with four-dimensional relation for massive particles:

$$E^2 - p_1^2 - p_2^2 - p_3^2 = M^2$$

This is equivalent to an infinite tower of four-dimensional particles (KK states) with masses

$$M_n = \frac{|n|}{R} \quad , \quad n \in Z$$

- A single massless scalar in 5d is equivalent to an infinite collection of (mostly) massive scalar in 4d.

This result is generic and applies also to massive fields or fields with spin. (**Exercise** Derive the KK masses for a massive 5d scalar. Derive the analogous result for a 5-dimensional gauge field. What is the spin of 4-dimensional fields that are obtained and what are their masses?)

♣ If at low energy, our available energy in accelerators is

$$E \lesssim \frac{1}{R}$$

none of the massive KK-states can be produced ("seen").

The extra dimension is invisible!

♠ When $E >> \frac{1}{R}$ several KK states can be produced and studied. When many have been seen the extra compact dimension can be reconstructed.

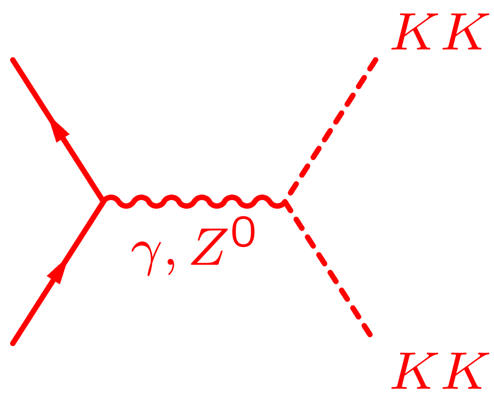
♢ The fact that till today in colliders we have not seen such states (with SM charges) gives a limit on R:

$$\frac{1}{R} > 300 \quad GeV$$

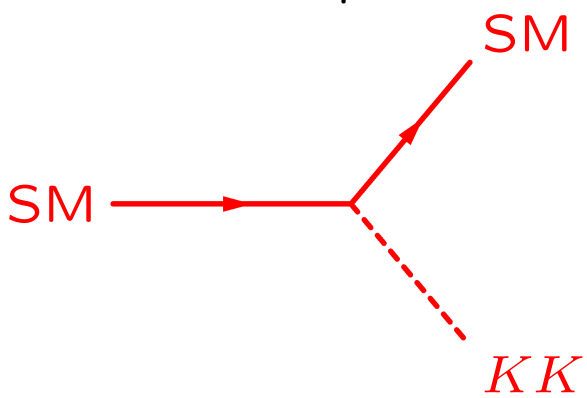
In LHC, there will be searches for KK states.

• Since a circle is translationally invariant, p_4 is conserved. n is therefore like a conserved KK U(1) charge.

• Therefore KK-states must be pair produced, so the threshold for their production is $\frac{2}{R}$.



• There are cases where the extra dimension is not translationally invariant. (e.g. a finite interval) Then KK-charge is not conserved, KK states can be singly produced and the threshold for production is $\frac{1}{R}$.



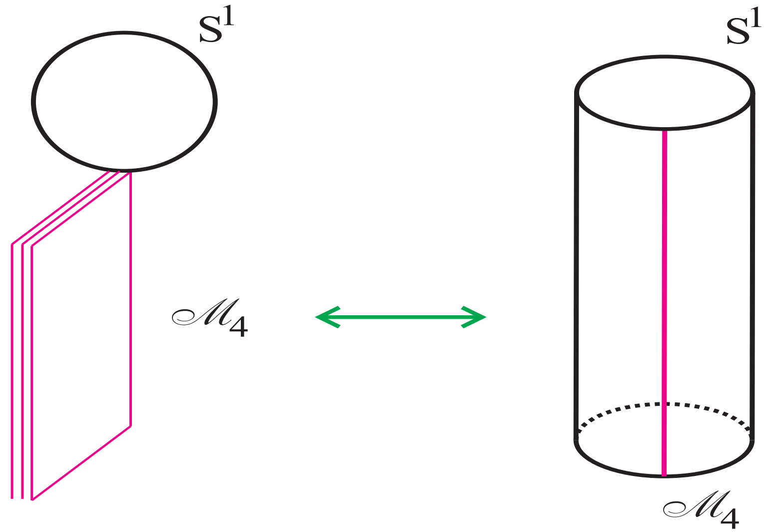
Branes and large extra dimensions

• The collider bound on R: $1/R > 300$ GeV can be simply evaded if the KK states carry no SM charges.
In the simplest case they couple gravitationally.

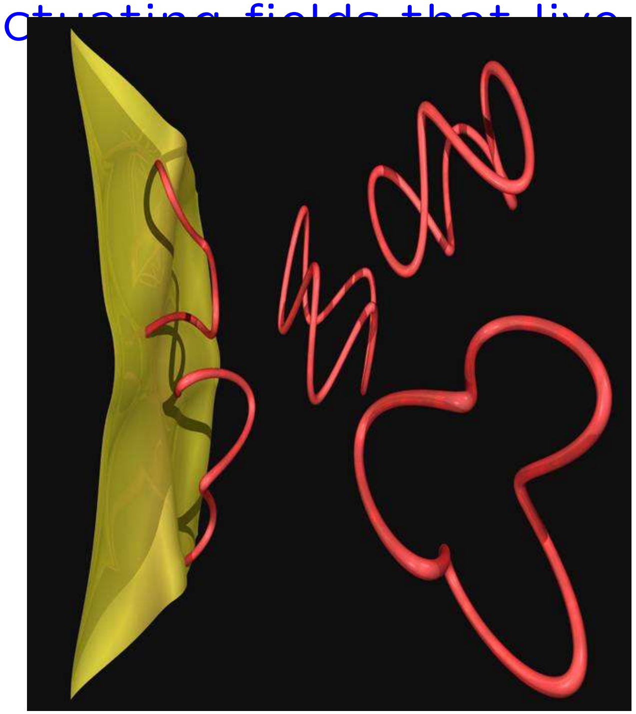
• This setup is possible using the idea of branes.

♠ Consider $M_4 \times S^1$ as an example , with the circle of radius R.

• A 3-brane is a (hyper)-membrane with 3 spacial dimensions. We can imagine such a 3-brane embedded inside our (4+1)-dimensional space.



• Branes are part of string theory. They have fluctuating fields that live on them.



• Such localized fields are typically gauge fields, fermions and scalars.

• We may therefore arrange that the SM fields live on such a 3-brane and cannot propagate in the rest n dimensions (the "bulk")

• The gravitational field on the other hand can propagate in all (4+1) directions.

♠ Consider the Newton constant and Planck mass in a (4+N)-dimensional theory:

$$\mathcal{L} = \frac{1}{G_{4+N}} \int d^{4+N}x \, \sqrt{g} \, R_{N+4} \quad , \quad G_{4+N} \sim \frac{1}{M_*^{(N+2)}}$$

• At distances $l << R$ gravity is effectively (4+N)-dimensional.

$$V(r) \sim \frac{1}{r^{1+N}}$$

• At large distances $l >> R$ gravity is four-dimensional. The effective 4D Newton constant $G_4 = M_P^{-2}$ can be calculated from

$$M_*^{N+2} \int d^{4+N}x \sqrt{g}\, R_{4+N} \quad \sim \quad M_*^{N+2} R^N \int d^4x \sqrt{g}\, R_4 + \cdots$$

$$\frac{1}{G_4} \sim M_P^2 \sim M_*^{(N+2)} R^n$$

• By choosing appropriately the size of extra dimensions

$$R \quad \sim \quad 10^{\frac{32}{N}} \quad \text{TeV}^{-1} \quad \sim \quad 10^{\frac{32}{N}-12} \ \text{ eV}^{-1} \sim 10^{\frac{32}{N}-16} \ \text{ mm}$$

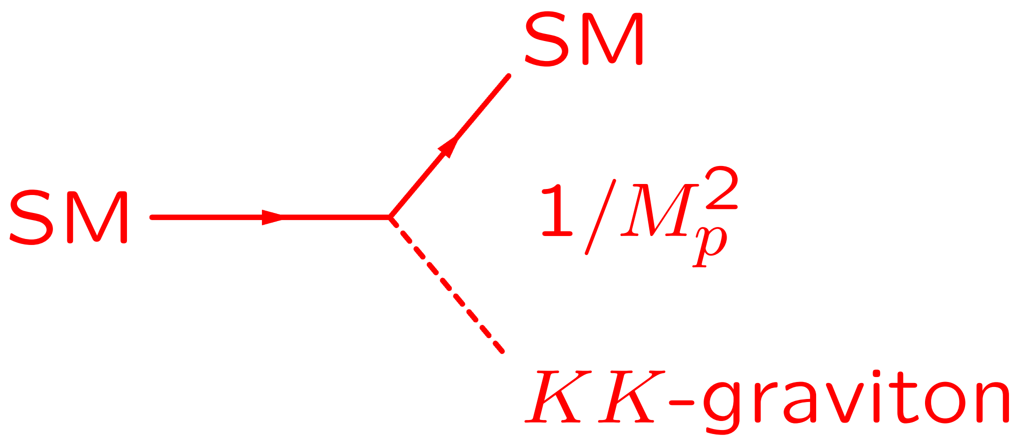
we can arrange that the quantum gravity scale of the full theory

M_* is as low as 1 TeV while $M_P = 10^{19}$ GeV.

• SM particles have no KK descendants (no bulk propagation). They do not directly feel the extra dimensions. The collider bound on R is not relevant here.

• The graviton has KK descendants, with the usual masses $\frac{|n|}{R}$. They couple to SM matter gravitationally.

• Each KK graviton couples with strength M_P^{-2} which is very weak.



• However, the existence of many KK-gravitons enhances this coupling (more later).

• For two extra dimensions their size can be 0.1 mm !!! How come we have not seen such a "large dimension"?

• It cannot be seen at accelerators because of the weak coupling of KK gravitons. (It becomes substantial at 1 TeV or more).

• For distances smaller than 0.1 mm gravity becomes higher-dimensional :

$$F \sim \frac{1}{r^{2+N}} \qquad or \qquad V \sim \frac{1}{r^{1+N}}$$

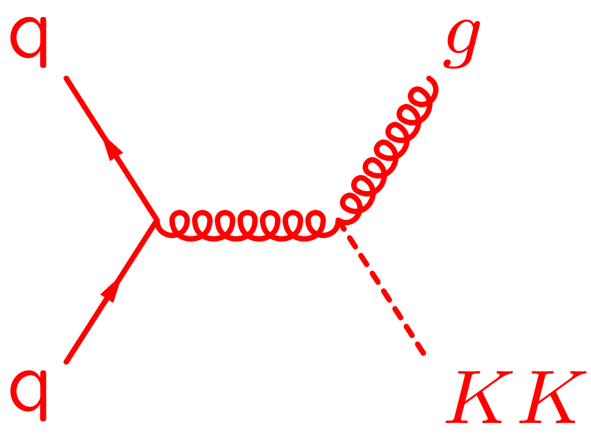
(Exercise : The compact Newton's law Show that the Newtonian force with N compact dimensions all of radius R is $F = G_* M^2 \sum_{\vec{n}\in Z^N} \frac{1}{|\vec{r}+2\pi R\vec{n}|^{(N+2)}}$ where $\vec{n}$ is a vector of integers)

♠ Surprisingly, until recently the gravitational law has been measured only up to distances of 1 mm! Today, the limiting distance has gone down to 10 μm.

Where can we see signals for all this?

(A) From tabletop short distance experiments

(B) At LHC. The signal is missing energy due to brehmstralung into KK gravitons that escape undetected in the bulk.



For $E >> \frac{1}{R}$

$$\sigma \sim \frac{1}{M_P^2}(\#\ of\ KK\ gravitons) \sim \frac{1}{M_P^2}(ER)^N \sim \frac{1}{M_*^2}(EM_*)^N$$

Exercise : calculate the number $(ER)^N$ of KK states that can contribute to this process.)

Here you may find some cross sections for KK-graviton production at LHC.

Further reading:[arXiv:hep-ph/0503148]

Black holes at colliders?

Black holes are very special (and singular) solutions of GR

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta \; d\phi^2)$$

$$f(r) = 1 - \frac{2M}{M_P^2 \; r} \quad , \quad f(R) = 0 \quad \to \quad R = 2\frac{M}{M_P^2} = 2\frac{M}{M_P} \; \ell_P$$

Far away, $r \to \infty$ the space is flat

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta \; d\phi^2) = -dt^2 + dx^i dx^i$$

- $r = R$ is the horizon, $r = 0$ is the singularity.
- Black holes are classically stable (and "hungry").
- In the quantum theory they decay via Hawking radiation.
- This fact is correlated with many of the theoretical puzzles posed by black-holes (black-hole thermodynamics and "the black-hole information paradox").

The black-hole information saga

• Particles with masses $M << M_P$ have an invisible horizon size: $R \ll \ell_P$
But very massive particles $M >> M_P$ have a macroscopic horizon: $R \gg \ell_P$
They should be treated as black holes

• In the case of large extra dimensions, the higher-dimensional Planck scale M_* is much smaller than the four-dimensional one M_P.

$$\frac{M_P^2}{M_*^2} \sim (M_* R)^n \sim \left(\frac{R}{\ell_*}\right)^N \gg 1$$

• If $M_* \sim 1$ TeV then multi-TeV particles will behave as (higher-dimensional*) black holes.

• They will be created during a collision, and they will decay (democratically) via Hawking radiation.

• Although we do not yet control the details of such processes at LHC energies, we may be faced with such events at LHC

Further reading: start from hep-ph/0111230

Exercise*: Derive the higher-dimensional black-hole solution, by thinking simply about its asymptotic properties. In particular it must satisfy the (higher-dimensional) Poisson equation.

Conclusions

We have seen that we already have experimental data that cannot be explained in the context of the Standard Model,

- Neutrino masses and mixings.
- Dark matter.
- Dark Energy.

We have also seen many ideas that attempt to unify the forces, make a UV stable theory, incorporate gravity, and try to explain the Standard Model patterns and the data above.

No theory so far can successfully accommodate all three data above.

♠ We need input from experiments!

Happily, data are still flowing in from cosmological observations, and accelerators like LHC are expected to provide complementary views of the fundamental physical theory.

◇ **We do count on your help!**

The (old) quest for understanding nature is still on!

Thank you!

The Standard Model: ingredients

A review of the ingredients

Gauge groups

♣ Strong force: SU(3)$_{\text{color}}$ $\rightarrow$ three colors.

Carriers: gluons are spin-one octets

$\rightarrow$ (color/anti-color) combinations. (SU(N) $\rightarrow$ N^2-1 gauge bosons)

They are confined inside hadrons $\Rrightarrow$ "glue".

♠ The electroweak force:SU(2)×U(1)$_Y$, it is spontaneously broken to U(1)$_{EM}$ by the Higgs effect.

Carriers: $W^{\pm}$, Z^0 (massive), γ (massless)

Standard Model: the quarks

Left-handed: $\begin{pmatrix} U_L \\ D_L \end{pmatrix}^a_{\frac{1}{6}}, \quad \begin{pmatrix} C_L \\ S_L \end{pmatrix}^a_{\frac{1}{6}}, \quad \begin{pmatrix} T_L \\ B_L \end{pmatrix}^a_{\frac{1}{6}},$

a=red, blue, green

Right-handed: $\begin{pmatrix} U_R \end{pmatrix}^a_{\frac{2}{3}}, \begin{pmatrix} D_R \end{pmatrix}^a_{-\frac{1}{3}}, \begin{pmatrix} C_R \end{pmatrix}^a_{\frac{2}{3}}, \begin{pmatrix} S_R \end{pmatrix}^a_{-\frac{1}{3}}, \begin{pmatrix} T_R \end{pmatrix}^a_{\frac{2}{3}},$
$\begin{pmatrix} B_R \end{pmatrix}^a_{-\frac{1}{3}}$

The SM is a chiral theory.

Standard Model: the leptons

Left-handed: $\begin{pmatrix} \nu_e \\ e_L \end{pmatrix}_{-\frac{1}{2}}$, $\begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix}_{-\frac{1}{2}}$, $\begin{pmatrix} \nu_\tau \\ \tau_L \end{pmatrix}_{-\frac{1}{2}}$

Right-handed $\left(\; e_R \; \right)_{-1}$, $\left(\; \mu_R \; \right)_{-1}$, $\left(\; \tau_R \; \right)_{-1}$

and

$$\left(\; \nu_e^R \; \right)_0 \; , \; \left(\; \nu_\mu^R \; \right)_0 \; , \; \left(\; \nu_\tau^R \; \right)_0$$

For the neutrino sector you will learn the recent developments from the forthcoming lectures of J. Gomez-Cadenas

All fermions come in three copies called families.

Standard Model: the Higgs

• The SM has interaction carriers with spin-one and "matter" with spin-$\frac{1}{2}$

• There is a spin-0 player as well: the Higgs. It is a (complex)-scalar SU(2) doublet with hypercharge $\frac{1}{2}$. Its "raison d'être" : break the electroweak symmetry spontaneously. As a result it gives masses to matter particles.

• Three of its components

$$H^{\pm} \quad , \quad Im(H^0)$$

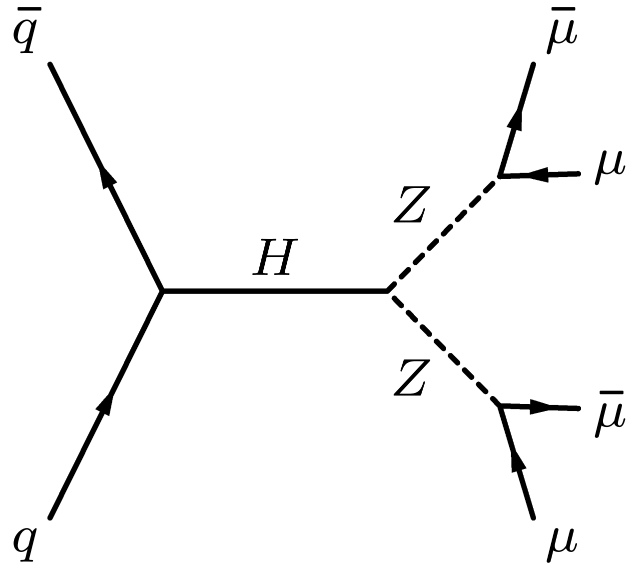
become the third components of the massive gauge bosons

$$W^{\pm} \quad , \quad Z^0$$

after electro-weak symmetry breaking.

• The fourth, Re(H^0) $\Rightarrow$ physical neutral scalar that we expect to see at LHC.

Higgs Event



Higgs→ZZ→ $\mu\mu\mu\mu$ event

How parameters affect us: some answers

- $v_F \to 0$ then p is unstable to decay to neutrons $\to$ no Hydrogen. The reason:

$$m_n - m_p \simeq m_d - m_u + E_n^{\text{EM}} - E_p^{\text{EM}}$$

The EM mass difference $E_n^{\text{EM}} - E_p^{\text{EM}} \sim -1.7$ MeV is independent of v_F but comes from quantum effects of electromagnetism. By lowering v_F we can make $m_d - m_u \simeq (\lambda_d - \lambda_u) v_F > 0$ very small. Then the EM Mass difference dominates and $m_p > m_n$.

- $v_F >> 170 GeV$ n-p mass difference is very large and the nuclear force becomes of shorter range $\to$ nothing but hydrogen in the universe.

From the GellMann-Oakes-Reines relation $m_\pi^2 = (m_u + m_d)\frac{\sigma}{f_\pi^2}$. σ is the vev of the chiral condensate for zero masses and depends only on QCD physics. The same applied to f_π that controls the pion self-interactions. This formula indicates that even for very small quark masses, the pion mass is determined by the quark masses.

If now the masses are much larger than Λ_{QCD} then we expect $\mu_\pi \sim m_u + m_d$. The range of the nuclear force (roughly due to the exchange of pions) is about $1/m_\pi$. The deuteron becomes unbound.

Eventually, the neutron becomes heavier than the proton plus its nuclear binding energy: bound neutrons would decay to
protons and complex nuclei cease to exist.
Much later, (for very large v_F) only $\Delta^{++} = uuu$ would become the only stable particle
See [arXiv:hep-ph/9801253] for more information

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Other parameters

There are others parameters, measured in the SM, whose values are not explained:

• The elements of the Kobayashi-Maskawa matrix: three mixing angles and a phase that controls CP violation. There is a similar matrix for the Neutrino sector.

• A non-perturbative parameter: the θ-angle of QCD: $\sim \theta \int d^4x \, \epsilon^{\mu\nu\rho\sigma} Tr[F_{\mu\nu} F_{\rho\sigma}]$

A non-zero value breaks CP in the strong interactions (contrary to observations.) This is the "strong CP-problem"

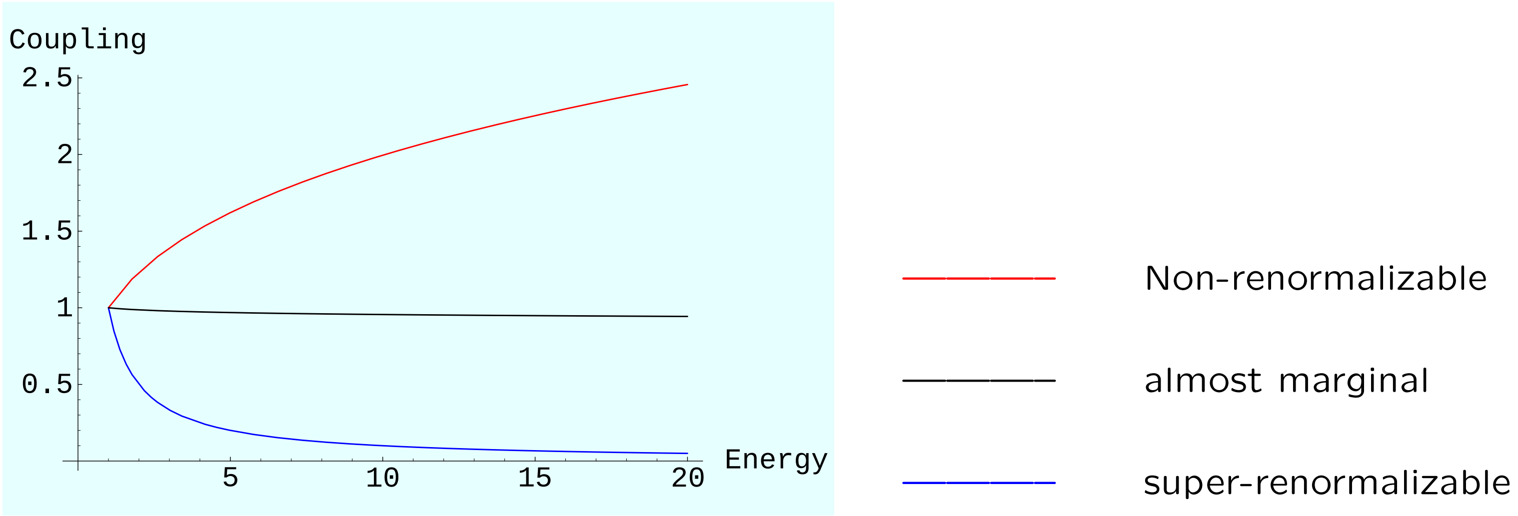
Experimentally

$$d_n \lesssim 10^{-25} \;\; e \text{ cm} \quad \rightarrow \quad \theta \lesssim 2 \times 10^{-10}$$

RETURN

Renormalization: the coupling constants

• Couplings λ_i depend on energy, E: $\lambda_i(E)$. This dependence is the result of including virtual effects at higher energies than E (shorter distances).



• Couplings can be irrelevant (non-renormalizable),

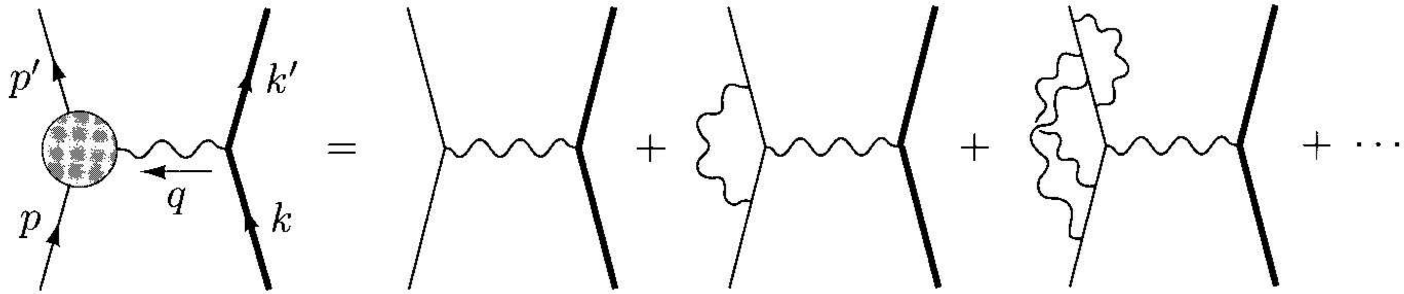
$$\lambda(E) = \lambda(E_0) \left(\frac{E}{E_0} \right)^{\Delta - 4} \quad , \quad \Delta > 4$$

relevant (super-renormalizable), or almost marginal (renormalizable, log running).

$$\lambda(E) = \frac{1}{\frac{1}{\lambda(E_0)} + b_0 \log \frac{E}{E_0}}$$

• Depending on the sign of b_0 this coupling marginally-relevant or marginally-irrelevant.

• We define a theory (via its couplings) at $E = \infty$ and uniquely determine (by calculating the quantum effects) the couplings at any lower energy (renormalization group flow).



• The reverse is not possible: we cannot guess the high-energy theory from the low-energy couplings. Two distinct theories can have the same low energy theory (universality).

• The couplings $\lambda_i(E)$ define the EFFECTIVE theory at energy E. It captures the low energy physics.

• When we make measurements at accelerators , we measure the effective couplings $\lambda_i(E_{\text{experiment}})$.

• Our goal is to find the UV couplings (complete specification of the theory).

Renormalization: the old view

In the traditional approach:

- $\Lambda = \infty$. The theory is defined to make sense at all possible energies.

- $\lambda_{n>0}(\infty) = 0$

Since

$$\tilde{\lambda}_n(E) \sim \Lambda^n$$

- Effective dimension>4 interactions are insensitive to high energy physics.

- Effective dimension≤ 4 couplings are infinite. We must choose carefully the $\lambda_n(\infty)$ so that this infinity cancels.

$$\tilde{\lambda}_2(E) = \lambda_2 + a\Lambda^2 + b\lambda_2 \log\frac{E^2}{\Lambda^2} + \text{finite} \quad \text{as} \quad \Lambda \to \infty$$

Choose

$$\lambda_2 = -a\Lambda^2 - b\lambda_2 \log\frac{E^2}{\Lambda^2}$$

Renormalized couplings: a concrete example

Consider that there exists at low energy a single scalar ϕ and we write the basic interactions at a high scale Λ. (this is the definition scale. At higher scales the theory may change):

$$S = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{m_0^2}{2}\phi^2 + \lambda_0\phi^4$$

We now calculate various low energy parameters, at a given scale $E_0 << \Lambda$.

$$m^2(E_0) = m_0^2 - \xi_1\Lambda^2 + \cdots \quad , \quad \frac{1}{\lambda(E_0)} = \frac{1}{\lambda_0} - b_0 \log\frac{E_0}{\Lambda} + \cdots$$

These are obtained from the two and four-point functions or equivalently from $\sigma_{2\to 2}$. If we now compute a $2 \to 4$ scattering cross section

$$\sigma_{2\to 4}(m(E_0), \lambda(E_0), \Lambda) \simeq \frac{\lambda_0^2}{m_0^2} \sim \frac{\lambda(E_0)^2}{m(E_0)^2 + \xi_1\Lambda^2}$$

From this we can "measure" $\Lambda = \Lambda_*$. If $\Lambda_* \neq \infty$ then the theory must change at $E \sim \Lambda_*$.

If instead we look at a theory like EM with a massive fermion, then both the gauge coupling constant and mass run logarithmically. Smaller sensitivity at the Λ scale.

Renormalization: Further reading

The following are introductory texts requiring mostly undergraduate knowledge.

• G. P. Lepage, "What is renormalization", [arXiv:hep-ph/0506330]

• J. Alexandre, "Concepts of renormalization in physics", [arXiv:physics/0508179]

• B. Delamotte, "A hint of renormalization", [arXiv:hep-th/0212049]

For the renormalization group in statistical mechanics:

• H. Maris and L. Kadanoff, " Teaching the renormalization group", Am. J. of Phys. 46 (1978) 652.

RETURN

Technicolor (extended discussion)

♠ Imagine a new SU(N)$_T$ interaction and new (massless) fermions

$$\begin{pmatrix} U_L \\ D_L \end{pmatrix} \sim (N,1,2,0) \quad , \quad U_R \sim (N,1,1,1/2) \quad , \quad D_R \sim (N,1,1,-1/2)$$

under $SU(N)_T \times SU(3) \times SU(2) \times U(1)_Y$.

• In the absence of SM interactions there a global "chiral symmetry" $SU(2)_L \times SU(2)_R$ with $(U_L, D_L) \sim (2,1)$, $(U_R, D_R) \sim (1,2)$.

• Note that $SU(2)_L$ is the same as the electroweak SU(2).

• Like in QCD, this $SU(2)_L \times SU(2)_R$ chiral symmetry will break spontaneously to $SU(2)_{\text{diagonal}}$ because of the strong IR dynamics of the technicolor gauge theory.

• A vev is generated

$$\begin{pmatrix} \langle U_L^\dagger U_R \rangle & \langle U_L^\dagger D_R \rangle \\ \langle D_L^\dagger U_R \rangle & \langle D_L^\dagger D_R \rangle \end{pmatrix} = \Lambda_T^3 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

• We may define the dimensionless (composite) doublet fields

$$Z_1 = \frac{1}{\Lambda_T^3}\begin{pmatrix} U_L^\dagger U_R \\ D_L^\dagger U_R \end{pmatrix} \quad , \quad Z_2 = \frac{1}{\Lambda_T^3}\begin{pmatrix} U_L^\dagger D_R \\ D_L^\dagger D_R \end{pmatrix}$$

They transform under $SU(2)_{EW} \times U(1)_Y$ as $Z_1 \sim (2,1/2)$, $Z_2 \sim (2,-1/2)$.

• They have kinetic terms

$$\mathcal{L}_{\text{kinetic}} = \frac{F_T^2}{2}\left[\partial_\mu Z_1 \cdot \partial^\mu Z_1^\dagger + \partial_\mu Z_2 \cdot \partial^\mu Z_2^\dagger\right]$$

• We may now identify them with a properly normalized pair of Higgs doublets

$$H_1 = F_T \; Z_1 \quad , \quad H_2^\dagger = F_T \; Z_2 \quad , \quad \langle H_1 \rangle = \langle H_2 \rangle = F_T \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

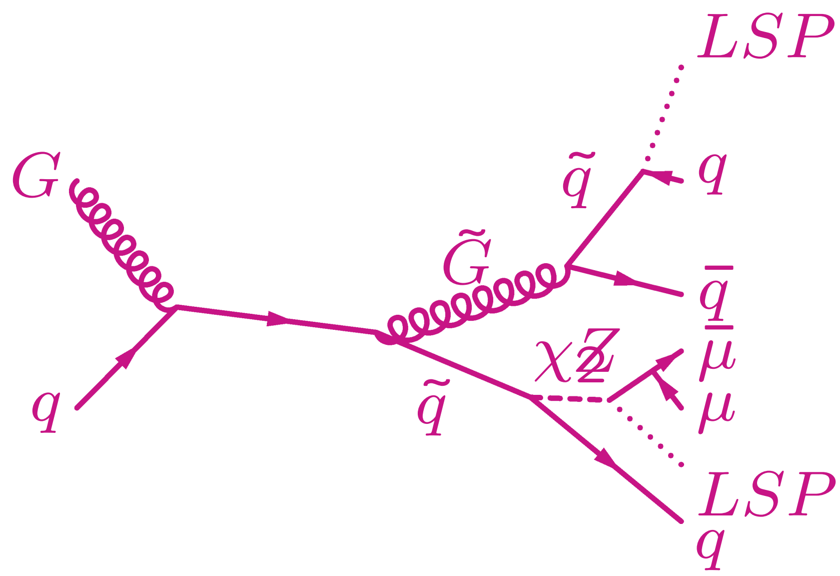
the break the EW symmetry with $v_F = F_T$.

• This simplest model needs improvement as al families are treated alike and the pattern of SM masses and mixings cannot be reproduced.

• This starts a series of complications that keeps expanding.

♣ Unfortunately detailed models that satisfy known experimental constraints are very difficult to construct. (See hep-ph/0007304 for a review.)

First SUSY event



The events were generated by Maria Spiropulu for the following SUSY mSUGRA parameters:

$$\tan\beta = 10 \quad , \quad m_{\frac{1}{2}} = 285 \quad \text{GeV} \quad , \quad m_0 = 210 \quad \text{GeV} \quad , \quad A = 0, \text{sign}(\mu) = +$$

This is known as the LM4 mSUGRA Point.

For these parameters the squark (gluino) masses are about 600 (700) GeV and the lightest neutralino, which escapes direct detection, has a mass of 114 GeV.

See http://iguanacms.web.cern.ch/iguanacms/gallery-page4.html

Weyl spinors

• Two component spinors: ψ_α and $\psi_{\dot\alpha}$ with $\alpha, \dot\alpha = 1, 2$. They transform in complex conjugate representations of SU(2).

• Complex conjugation intertwines the two types of spinors

$$(\psi_\alpha)^* = \bar\psi_{\dot\alpha} \quad , \quad (\psi_{\dot\alpha})^* = \bar\psi_\alpha$$

• We raise and lower indices by $\epsilon^{\alpha\beta}$ and $\epsilon^{\dot\alpha\dot\beta}$ with ϵ^{12}=1 and $\epsilon_{\alpha\beta} = -\epsilon^{\alpha\beta}$.

$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta \quad , \quad \psi^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\psi_{\dot\beta}$$

• The Pauli matrices intertwine the two chiralities

$$(\sigma_\mu)_{\alpha\dot\alpha} \equiv (\mathbf{1}, \vec\sigma)_{\alpha\dot\alpha} \quad , \quad (\bar\sigma_\mu)^{\dot\alpha\alpha} \equiv (\mathbf{1}, -\vec\sigma)^{\dot\alpha\alpha}$$

• In the Weyl representation the γ-matrices are

$$\gamma_\mu = \begin{pmatrix} 0 & i\sigma^\mu \\ i\bar\sigma^\mu & 0 \end{pmatrix} \quad , \quad \gamma^5 = i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

• In this representation a Dirac spinor ψ_D and a Majorana (real) spinor ψ_M can be written as

$$\psi_D = \begin{pmatrix} \psi_\alpha \\ \chi_{\dot\alpha} \end{pmatrix} \quad , \quad \psi_M = \begin{pmatrix} \psi_\alpha \\ \bar\psi^{\dot\alpha} \end{pmatrix}$$

Useful Identities

$$\sigma_\mu \bar\sigma^\mu = -4 \quad , \quad \sigma_\nu \sigma^\mu \bar\sigma^\nu = 2\sigma^\mu \quad , \quad (\sigma^\mu_{\alpha\dot\beta})^* = \bar\sigma^\mu_{\dot\alpha\beta}$$

$$\theta^\alpha \psi_\alpha = \epsilon^{\alpha\beta}\theta_\beta\psi_\alpha = -\epsilon^{\beta\alpha}\theta_\beta\psi_\alpha = -\theta_\beta\psi^\beta = \psi^\beta\theta_\beta$$

$$\theta^\alpha \sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\theta^\beta\sigma^\nu_{\beta\dot\beta}\bar\theta^{\dot\beta} = \frac{1}{2}\theta^\alpha\epsilon_{\alpha\beta}\theta^\beta \; \bar\theta^{\dot\alpha}\epsilon_{\dot\alpha\dot\beta}\bar\theta^{\dot\beta} \; \eta^{\mu\nu}$$

$$\theta^\alpha\chi_\alpha\theta^\beta\psi_\beta = -\frac{1}{2}\theta^\alpha\theta_\alpha\chi^\beta\psi_\beta$$

$$\theta^\alpha\psi_\alpha\lambda^\beta\chi_\beta + \theta^\alpha\lambda_\alpha\psi^\beta\chi_\beta + \theta^\alpha\chi_\alpha\lambda^\beta\psi_\beta = 0$$

Exercise: Prove the identities above

Further reading: "Supersymmetry and supergravity" by Bagger and Wess

The Supersymmetry algebra

• Supersymmetry generators are represented by fermionic operators, $Q^I_\alpha, \bar{Q}^I_{\dot{\alpha}}$, $I = 1, 2, \cdots, \mathcal{N}$.

• Haag, Lopusanski and Sonius have shown that the most general symmetry of a Lorentz-invariant QFT S-matrix is a (semi)direct product of the $\mathcal{N}$-extended Super-Poincaré algebra, and all internal symmetries.

$$\{Q^I_\alpha, Q^J_\beta\} = \epsilon_{\alpha\beta} Z^{IJ} \quad , \quad \{\bar{Q}^I_{\dot{\alpha}}, \bar{Q}^J_{\dot{\beta}}\} = \epsilon_{\dot{\alpha}\dot{\beta}} \bar{Z}^{IJ} \quad , \quad \{Q^I_\alpha, \bar{Q}^J_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}}\, P_\mu$$

$Z^{IJ}, \bar{Z}^{IJ}$ are antisymmetric "central charges" (relevant for $\mathcal{N} \geq 1$).

• Extended ($\mathcal{N} > 1$) supersymmetry algebras do not have chiral representations. They have very special and interesting properties though.

• We will focus on $\mathcal{N} = 1$ supersymmetry from now on:

$$\{Q_\alpha, Q_\beta\} = 0 \quad , \quad \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0 \quad , \quad \{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}}\, P_\mu$$

The Supersymmetric representations

• Consider first massive one-particle states with mass M. We go to the rest frame where $P_\mu = (M, \vec{0})$. We obtain

$$\{Q_\alpha, Q_\beta\} = 0 \quad , \quad \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0 \quad , \quad \{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2M\ \delta_{\alpha\dot\beta} \quad \leftarrow \quad 2\sigma^\mu_{\alpha\dot\beta}\ P_\mu$$

• The operators $A_\alpha = \frac{Q_\alpha}{\sqrt{2M}}$, $A^\dagger_\alpha = \frac{\bar{Q}_{\dot\alpha}}{\sqrt{2M}}$ satisfy

$$\{A_\alpha, A_\beta\} = 0 \quad , \quad \{A^\dagger_\alpha, A^\dagger_\beta\} = 0 \quad , \quad \{A_\alpha, A^\dagger_\beta\} = \delta_{\alpha\beta}$$

and are therefore two fermionic creation and annihilation operators.

• All representations can be constructed out of a "ground state" with spin S, S_z satisfying

$$A_\alpha|S, S_z\rangle = 0 \quad , \quad \alpha = 1, 2$$

by acting with creation operators $A^\dagger_\alpha$ taking into account the Pauli principle: $(A^\dagger_1)^2 = (A^\dagger_2)^2 = 0$. There then 4 states in each irreducible representation :

$$|S, S_z, 0, 0\rangle \equiv |S, S_z\rangle \ , \ |S, S_z, 1, 0\rangle \equiv A^\dagger_1|S, S_z\rangle \ , \ |S, S_z, 0, 1\rangle \equiv A^\dagger_2|S, S_z\rangle \ , \ |S, S_z, 1, 1\rangle \equiv A^\dagger_1 A^\dagger_2|S, S_z\rangle$$

• $|S, S_z, 0, 0\rangle$ and $|S, S_z, 1, 1\rangle$ have spin which is equal to that of the ground state.

• $|S, S_z, 1, 0\rangle$ and $|S, S_z, 0, 1\rangle$ have spin which is equal to the $S \otimes \frac{1}{2}$ representation of the rotation group.

• In total the spin content of a massive rep is $S \otimes \left[2(0) + \left(\frac{1}{2}\right)\right]$.

• Parity acts as $A^\dagger_1 \leftrightarrow A^\dagger_2$ so the two singlets have opposite parity.

The massless representations

• In this case we choose a frame where $P_\mu = (E, 0, 0, -E)$, $E > 0$.

• The supersymmetric anticommutator now becomes

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2E(1+\sigma^3) = 4E\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

• Note that the second component satisfies $Q_2^2 = (\bar{Q}_2)^2 = \{Q_2, \bar{Q}_2\} = 0$.
In a unitary theory, this implies $Q_2 = \bar{Q}_2 = 0$ in the Hilbert space.

• The operators $A = \frac{Q_1}{\sqrt{4E}}$, $A^\dagger = \frac{\bar{Q}_1}{\sqrt{4E}}$ are fermionic oscillator operators

$$\{A, A\} = \{A^\dagger, A^\dagger\} = 0 \quad , \quad \{A, A^\dagger\} = 1$$

and the representation is now two dimensional: starting with a ground state of helicity λ

$$A|\lambda\rangle = 0$$

we construct a single "excited" state with helicity $\lambda + \frac{1}{2}$

$$\left|\lambda + \frac{1}{2}\right\rangle = A^\dagger|\lambda\rangle$$

• Similarly from a helicity $-\lambda$ ground state we obtain a helicity $-\left(\lambda + \frac{1}{2}\right)$ excited state.

• Interesting examples: $\lambda = \frac{1}{2} \to$, a massless vector and a massless spinor (gaugino)
$\lambda = \frac{3}{2} \to$, a massless spin-2 (graviton) and a massless spin-3/2 (gravitino)

The Supersymmetry transformations

• For the chiral multiplet, (ϕ, ψ)

$$\delta_\xi \phi = \xi^\alpha \psi_\alpha \quad , \quad \delta_\xi \psi_\alpha = \sigma^\mu_{\alpha\dot\beta} \bar\xi^{\dot\beta} \partial_\mu \phi \tag{1}$$

• The supersymmetry algebra closes only on-shell (imposing the free equations of motion).

• The algebra will be unconstrained if we introduce an "auxiliary" scalar field F:

$$\delta_\xi \phi = \xi^\alpha \psi_\alpha \quad , \quad \delta_\xi \psi_\alpha = \sigma^\mu_{\alpha\dot\beta} \bar\xi^{\dot\beta} \partial_\mu \phi + 2\xi_\alpha \; F \quad , \quad \delta_\xi F = i \partial_\mu \psi_\alpha \sigma^\mu_{\alpha\dot\beta} \bar\xi^{\dot\beta} \tag{2}$$

• The auxiliary field is a non-propagating field (Free-field equations of motion imply $F = 0$)

• For the vector multiplet $(A^a_\mu, \lambda^a_\alpha, D^a)$:

$$\delta A^a_\mu = -i \bar\lambda^a \bar\sigma^\mu \xi + i \bar\xi \bar\sigma^\mu \lambda^a \; , \; \delta\lambda^a = \sigma^{\mu\nu} \xi F^a_{\mu\nu} + i\xi \; D^a \; , \; \delta D^a = -\xi \sigma^\mu (\mathcal{D}_\mu \bar\lambda)^a - (\mathcal{D}_\mu \lambda)^a \sigma^\mu \bar\xi$$

$$(\mathcal{D}_\mu \lambda)^a \equiv \partial_\mu \lambda^a + i g f^a{}_{bc} A^b_\mu \lambda^c \quad , \quad [T^a, T^b] = i f^{ab}{}_c \; T^c$$

Exercise: Compute the supersymmetry commutator $[\delta_{\xi_1}, \delta_{\xi_2}]$ and verify that (1) closes on shell while (2) closes off-shell.

The simplest scalar action

• We consider a chiral multiplet $\Phi \equiv (\phi, \psi_\alpha, F)$

• The most general (real) supersymmetric Lagrangian with a general scalar potential depends on an arbitrary (holomorphic) function: The superpotential $W(\Phi)$

• It is given by

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi^* + \frac{i}{2}\bar{\psi}\partial\!\!\!/\psi + FF^* + \frac{\partial W(\phi)}{\partial \phi}F + \frac{\partial W(\phi^*)}{\partial \phi^*}F^* -$$

$$-\frac{1}{2}\frac{\partial^2 W(\phi)}{\partial \phi^2}\psi^\alpha \psi_\alpha - \frac{1}{2}\frac{\partial^2 W^*(\phi^*)}{\partial (\phi^*)^2}\psi^{\dot{\alpha}}\psi_{\dot{\alpha}}$$

• We may "integrate out" the non-propagating auxiliary field, by solving its equations of motion: $F = -\frac{\partial W(\phi)}{\partial \phi}$. Substituting back in the action we obtain

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi^* + \frac{i}{2}\bar{\psi}\partial\!\!\!/\psi - \frac{1}{2}\frac{\partial^2 W(\phi)}{\partial \phi^2}\psi\psi + c.c. - V(\phi, \phi^*)$$

$$V(\phi, \phi^*) = |F|^2 = \left|\frac{\partial W(\phi)}{\partial \phi}\right|^2$$

Exercise: Supersymmetry implies the conservation of the spin-3/2 supercurrent, $\partial_\mu G^\mu_a = 0$. Derive the supercurrent for this simple theory using the supersymmetry transformations and the Noether procedure. The conserved charges of the susy algebra are given as usual by $Q_\alpha = \int d^3x\ G^0_\alpha$

The supersymmetric gauge theory

• The general supersymmetric gauge theory contains a vector multiplet in the adjoint $\to$ $(A^a_\mu, \lambda^a, D^a)$ of a gauge group G

• "Matter" is composed of chiral multiplets $\Phi^i = (\phi^i, \psi^i, F^i)$ transforming in a reducible representation R of G.

• The general supersymmetric action with canonical (quadratic) kinetic terms is completely determined by the superpotential: a gauge-invariant and holomorphic function of the chiral superfields, $W(\Phi^i)$

$$\mathcal{L}_{\text{SUSY}} = -\frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} + \frac{i}{2}\bar{\lambda}^a\gamma^\mu(\mathcal{D}_\mu\lambda)^a + (\mathcal{D}_\mu\phi)^\dagger_i(\mathcal{D}^\mu\phi)^i + \frac{i}{2}\bar{\psi}_i\gamma^\mu(\mathcal{D}_\mu\psi)^i +$$

$$+\left[i\sqrt{2}g\ (\bar{\psi}_i\lambda^a)(T^a\phi)^i + h.c.\right] - \left[\frac{1}{2}\frac{\partial^2 W}{\partial\phi^i\partial\phi^j}\ \bar{\psi}^i\psi^j + h.c.\right] - V(\phi,\phi^\dagger)$$

$$(\mathcal{D}_\mu\lambda)^a \equiv \partial_\mu\lambda^a + igf^a{}_{bc}A^b_\mu\lambda^c \quad , \quad (\mathcal{D}_\mu\phi)^i \equiv \partial_\mu\phi^i + ig(T^a)^i{}_jA^a_\mu\phi^j$$

$$V(\phi,\phi^\dagger) = F^*_iF_i + \frac{g^2}{2}D^aD^a = \sum_i\left|\frac{\partial W}{\partial\phi^i}\right|^2 + \frac{g^2}{2}\sum_a\left[\phi^*_i(T^a)^i{}_j\phi^j\right]^2$$

• Couplings are unified. For a renormalizable theory, the superpotential must be at most cubic. The kinetic terms of the fields and their couplings to the gauge bosons are standard and determined by the representations/charges and the gauge couplings.

The Fayet-Iliopoulos term

• There is one extra addition to the supersymmetric gauge theory action that is allowed by supersymmetry when there is a U(1) gauge group factor: the addition of

$$\delta\mathcal{L}_{FI} = \xi \; D$$

to the supersymmetric Lagrangian:

$$\mathcal{L}_D = \frac{1}{2g^2}D^2 + \left(\xi + \sum_i Q_i|\phi_i|^2\right) D$$

• Integrating out D, its only effect is to modify the D-term potential

$$V_{U(1)} = \frac{g^2}{2}\left(\xi + \sum_i Q_i|\phi_i|^2\right)^2$$

• If $Tr[U(1)] \neq 0$, then a non-zero $\xi \sim \Lambda^2$ is generated at one loop and upsets the nice structure of supersymmetric perturbation theory.

The supersymmetric vacuum

Taking the trace of the susy algebra

$$\delta^{a\dot{\beta}}\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2Tr[\sigma^\mu]\ P_\mu = 4P_0 = 4H$$

In the quantum theory $\bar{Q}_{\dot{\alpha}} = Q^\dagger_\alpha$ we obtain

$$H = \frac{1}{2}\left[Q_1 Q_1^\dagger + Q_2 Q_2^\dagger\right]$$

- The Hamiltonian of a supersymmetric theory is a positive definite operator:

$$\langle\psi|\ H\ |\psi\rangle \geq 0$$

- The vacuum preserves supersymmetry if it is annihilated by all conserved susy charges:

$$Q_\alpha|0\rangle = 0 \quad , \quad Q^\dagger_\alpha|0\rangle = 0$$

- Therefore if susy is unbroken, then $H|0\rangle = 0$. This is an exact equation in supersymmetric QFT.

- The converse is also true: susy is spontaneously broken iff $H|0\rangle \neq 0$ and positive.

Exercise: Show that in a supersymmetric theory $[P_\mu, Q_\alpha] = 0$. In particular $[H, Q_\alpha] = 0$ as it should for conserved charges.

Spontaneous supersymmetry breaking

Spontaneous supersymmetry breaking is an important problem. There are many different classes of models.

• Global supersymmetry breaks spontaneously, when $\langle V \rangle > 0$. Since $V \sim |F_i|^2 + |D^a|^2$, this implies that if some $\langle F_i \rangle$ or $\langle D^a \rangle$ are non-zero susy is broken.

• Like standard global symmetries, there is a massless fermion, the Goldstino, $\tilde{G} = \langle F_i \rangle \psi^i + \langle D_a \rangle \lambda^a$, associated with spontaneous global supersymmetry breaking.

• Supersymmetry can be promoted into a local symmetry. The appropriate theory then contains also gravity and is known as supergravity.

• In particular, the "gauge-field" associated to local supersymmetry is a spin-3/2 fermion known as the gravitino. It is the supersymmetric partner of the graviton. Like the graviton it is massless when supersymmetry is unbroken.

• When supersymmetry breaks spontaneously, the gravitino acquires a non-zero mass $m_{3/2}$. It becomes massive by combining with the Goldstino field. This is the super-Higgs mechanism.

• The supersymmetry breaking scale Λ_S is related to the gravitino mass in a universal fashion:

$$\Lambda_S = \sqrt{3\ m_{3/2}\ M_P}$$

• The superpartner mass splittings depend on the sector I of the theory as:

$$(\Delta m^2)_I \quad \sim \quad \lambda_I\ \Lambda_S^2$$

where λ_I is the (renormalized) Goldstino/gravitino coupling to sector I.

There are two rough avenues to arrange for $\Delta m \sim$ TeV:

(A) Heavy gravitino mass $\to$ large Λ_S, but very small λ_I.

(B) Light gravitino mass, and $\lambda_I \sim 1$.

The O'Raifertaigh example

• Consider a theory with chiral multiplets X, Y_1, Y_2 and a (renormalizable) superpotential

$$W = X^2 Y_1 + X Y_2 - a Y_2 \quad , \quad a \neq 0$$

• The potential is $V = |F_1|^2 + |F_2|^2 + |F_X|^2$ with

$$F_1 = \frac{\partial W}{\partial Y_1} = X^2 \quad , \quad F_2 = \frac{\partial W}{\partial Y_2} = X - a \quad , \quad F_X = \frac{\partial W}{\partial X} = 2XY_1 + Y_2$$

• There is no supersymmetric vacuum (solution to $F_1 = F_2 = F_X = 0$).

• The minimum of the potential is at $X_0 = X_0^*$, which minimizes

$$|F_1|^2 + |F_2|^2 = |X|^4 + |X - a|^2$$

• Moreover, at the minimum $Y_2 + 2X_0 Y_1 = 0$. There is a flat direction: one of the vevs (for example Y_1) is arbitrary.

minimal Gauge Mediated Susy Breaking

• There is a source of supersymmetry breaking due to a vev $\langle X\rangle = M+\theta^2\ F$ in a hidden (secluded) sector.

• There are messenger superfields in complete SU(5) reps (not to upset gauge coupling unification) Φ_i that couple as $\lambda_{ij}\ \bar\Phi_i\ X\ \Phi_j$ both to the secluded and the SSM sector. They modify the GUT scale coupling as

$$\delta\alpha^{-1}_{GUT} = -\frac{N}{2\pi}\log\frac{M_{GUT}}{M}\quad,\quad N=\sum_i\ n_i$$

• Diagonalize and absorb λ's into (M, F) $\to (M_i, F_i)$. Then the gaugino and scalar masses are given by

$$M_a = k_a\frac{\alpha_a}{4\pi}\Lambda_G\quad,\quad \Lambda_G=\sum_i n_i\frac{F_i}{M_i}\quad,\quad k_Y=\frac{5}{3}\ ,\ k_2=k_3=1$$

$$m_i^2(t) = 2\sum_{a=1}^{3}C_a^i\ k_a\frac{\alpha_a^2(0)}{(4\pi)^2}\left[\Lambda_S^2+h_a\Lambda_G^2\right]\quad,\quad h_a=\frac{k_a}{b_a}\left[1-\frac{\alpha_a^2(t)}{\alpha_a^2(0)}\right]\quad,\quad \Lambda_S^2=N\frac{F^2}{M^2}$$

• The MSSM soft parameters are here parameterized by $(M, N, \Lambda_G, \tan\beta, \text{sign}(\mu))$

Further reading: [arXiv:hep-ph/9801271]

minimal Anomaly Mediated Susy Breaking

• The idea of anomaly mediated supersymmetry breaking comes from brane realizations of the SM.

• The "hidden" sector where supersymmetry breaks spontaneously is localized on a brane different from the SSM-brane.

• The breaking of supersymmetry is communicated to the SSM via the Weyl anomaly.

• The form of the gaugino and scalar soft masses is of the form

$$M_a = \beta_a\ M \quad , \quad m_i^2 = m_0^2 - C_i^a\ \beta_a\ M^2$$

where M is a characteristic energy scale and m_0 a phenomenological parameter and β_a the gauge β-functions.

• This mechanism is still in its infancy and has many obscure points. It is known as mAMSB and characterized by the parameters $(m_0, M, \tan\beta, \text{sign}(\mu))$

Further reading:[arXiv:hep-th/9810155], [arXiv:hep-ph/9810442]

A simple example of a softly-broken supersymmetric theory

We consider a theory that contains the supersymmetric chiral multiplets, Φ_i, $i = 1, 2, 3$. Each contains a complex scalar ϕ_i and a Weyl Fermion, ψ_i.

$$\Phi_i \quad \equiv \quad (\phi_i, \psi_i)$$

We will not include gauge interactions. In this case the only interactions will come from the superpotential. It must be written in terms of the chiral multiplets Φ_i but not of their conjugates, $\bar{\Phi}_i \equiv (\phi^*, \psi_c^i)$. We will choose it to be simple:

$$W = h\ \Phi_1\ \Phi_2\ \Phi_3$$

We fist compute the Yukawa couplings using $\sum_{i,j} \frac{\partial^2 W}{\partial\phi_i \partial\phi_j} \bar{\psi}_i \psi_j$ to find

$$\mathcal{L}_{Yukawa} = h(\phi_2\ \bar{\psi}_3 \psi_3 + \phi_2\ \bar{\psi}_1 \psi_3 + \phi_3\ \bar{\psi}_1 \psi_2) + c.c.$$

We then compute the potential from $V = \sum_i |F_i|^2 = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2$ (there are no D-terms as there is no gauge group and gauge interactions)

$$V = |h|^2 \left[|\phi_1 \phi_2|^2 + |\phi_1 \phi_3|^2 + |\phi_2 \phi_3|^2 \right]$$

It contains only quartic couplings of the scalars. The complete supersymmetric Lagrangian is therefore

$$\mathcal{L} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{Yukawa}} - V$$

where $\mathcal{L}_{\text{kinetic}}$ contains the standard kinetic terms

$$\mathcal{L}_{\text{kinetic}} = \frac{1}{2}\sum_{i=1}^{3}\partial_\mu\phi_i\partial^\mu\phi_i^* + \sum_{i=1}^{3}\bar{\psi}_i\partial\!\!\!/\psi_i$$

We will now add all allowed soft terms that would be present id supersymmetry is broken:

- Masses $\tilde{m}_i$ for the scalars of the chiral multiplets.

$$\mathcal{L}^{\text{soft}-\text{sb}}_{\text{mass}} = \frac{1}{2}\sum_{i=1}^{3}\ \tilde{m}_i^2\ \phi_i\phi_i^*$$

- Cubic couplings for the scalars proportional to the superpotential couplings: The only superpotential non-zero coupling is $\Phi_1\Phi_2\Phi_3$ so

$$\mathcal{L}^{\text{soft}-\text{sb}}_{\text{cubic}} = A(\phi_1^*\phi_2\phi_3 + \phi_1\phi_2^*\phi_3 + \phi_1\phi_2\phi_3^*) + c.c.$$

- Gaugino masses are also soft, but there are no gaugini in this simple theory.

Exercise so that you see if you understood the above:

Exercise 1: Consider now the same theory with the following superpotential $W = \sum_{i=1}^{3}\left[h_i\ \Phi_i^3 + \mu_i\Phi_i^2 + \zeta_i\Phi_i\right]$
Repeat the procedure above to produce the softly broken supersymmetric action

Exercise 2: Go back now to the MSSM superpotential derive the Yukawa couplings and the potential and the soft breaking terms.

RETURN

SSB: Heavy gravitino mass

• Here the supersymmetry breakings happens in a "hidden sector".

• It is communicated to the observable sector by the gravitational interaction

$$\lambda_I \sim \frac{\Lambda_S^2}{M_P^2} \quad , \quad \Lambda_S \sim \sqrt{(\Delta m) M_P} \sim 10^{10} - 10^{11} \ \text{GeV} \quad , \quad m_{3/2} \sim 1 \ \text{TeV}$$

Taking the limit $M_P \to \infty$ to recover the EFT, we obtain the MSSM with typically universal soft terms.

• Such breaking can be realized in supergravity and in superstring vacua where susy is broken by hidden gaugino condensation.

• The EFT is MSSM and is valid up to close the Planck scale.

• There is another "mechanism" in this class: Anomaly Mediated Susy Breaking.

Further reading: http://doc.cern.ch/cernrep/1998/98-03/98-03.html

SSB: Light gravitino mass

This may be realized when supersymmetry is broken in a hidden sector, and is communicated to the observable sector by gauge or Yukawa interactions. Here $\lambda_I \sim \mathcal{O}(1)$.

• To obtain the desired mass splittings, $\Lambda_S \sim$ TeV and therefore $m_{3/2} \sim 10^3 - 10^{-5}$ eV.

• A class of models realizing this supersymmetry breaking pattern are known as messenger or gauge mediated supersymmetry breaking models. They contain apart from the observable sector, the "messenger" sector and the "hidden" sector.

• Here the gravitino is part of the low energy spectrum and its Goldstino component couples to the low energy fields with strength that ranges from order the gauge couplings to several orders smaller.

• Such theories have new physics well below the Planck scale.

• The LSP is the gravitino.

Further reading: http://doc.cern.ch/cernrep/1998/98-03/98-03.html

SU(5) generators: continued

- SU(3) generators

$$\begin{pmatrix} * & * & * & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- SU(2) generators

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & * & * \end{pmatrix}$$

- $U(1)_Y$ generator.

$$\begin{pmatrix} \mathbf{2} & 0 & 0 & 0 & 0 \\ 0 & \mathbf{2} & 0 & 0 & 0 \\ 0 & 0 & \mathbf{2} & 0 & 0 \\ 0 & 0 & 0 & -\mathbf{3} & 0 \\ 0 & 0 & 0 & 0 & -\mathbf{3} \end{pmatrix}$$

$$T_{13} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad T_{14} = \begin{pmatrix} 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \ldots \qquad T_{24}$$

12 extra gauge bosons ~ 6 complex fields

$$SU(2) \quad \begin{pmatrix} X_1 & X_2 & X_3 \\ Y_1 & Y_2 & Y_3 \end{pmatrix} = \left(\bar{3}, 2, \frac{5}{6}\right)$$

$SU(3)$

Decomposition of the 10 of SU(5) in SM reps

Exercise: Use the decomposition under $SU(5) \ni SU(3) \times SU(2)$

$$5 \to (3,1) + (1,2)$$

$$10 = (5 \otimes 5)_{\text{antisymmetric}}$$

and

$$(3 \otimes 3)_{\text{antisymmetric}} = \bar{3}$$

to show that

$$10 \to (\bar{3},1) + (3,2) + (1,1)$$

SU(5) symmetry breaking

- At a high scale M_{GUT} SU(5) must break to $SU(3) \times SU(2) \times U(1)_Y$.

- The simplest way to do this is via Higgs scalar Φ in the adjoint of SU(5) (a 5x5 hermitian traceless matrix):

$$\Phi(x)' = U(x)\ \Phi(x)\ U(x)^\dagger \quad , \quad U(x)U(x)^\dagger = 1 \quad , \quad Det[U(x)] = 1$$

or in terms of the infinitesimal generators of SU(5), T^a, $a = 1, 2, \cdots, 24$

$$U(x) = e^{iT^a\theta^a(x)} \quad , \quad (T^a)^\dagger = T^a \quad , \quad Tr[T^a] = 1$$

$$\delta\Phi(x) = i\theta^a(x)\ [T^a, \Phi(x)]$$

- The vev that does the required symmetry breaking is proportional to the 5×5 traceless matrix

$$\hat{\lambda} = \frac{1}{\sqrt{15}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix}$$

Exercise: Show that such a vev does not break $SU(3)\times SU(2)\times U(1)_Y$. You will have to identify the generators of $SU(3)\times SU(2)\times U(1)_Y$ inside the T^a, and show that they commute with $\hat{\lambda}$.

- The most general renormalizable potential is

$$V(\Phi) = -\frac{m^2}{2}\,Tr[\Phi^2] + \frac{h_1}{4}\,(Tr[\Phi^2])^2 + \frac{h_2}{2}\,Tr[\Phi^4]$$

- When both h_1 and h_2 are positive, the global minimum of the potential is at

$$\Phi = A\ \hat{\lambda} \quad , \quad A^2 = \frac{15m^2}{15h_1 + 7h_2}$$

and breaks $SU(5) \to S(3)\times SU(2)\times U(1)_Y$.

• At around 200 GeV we need an extra symmetry breaking: $SU(2)\times U(1)_Y \to U(1)_{em}$. This needs an extra Higgs scalar.

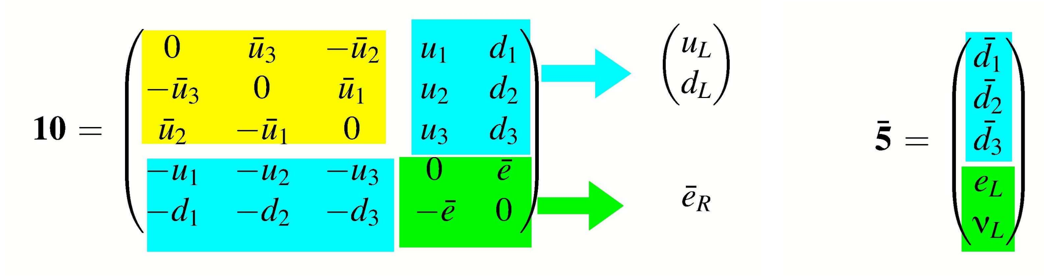
To see this decompose the adjoint of SU(5) under $SU(3)\times SU(2)$

$$24 \to (8,1)\otimes(1,3)\otimes 2(3,2)\otimes(1,1)$$

This can be done using the decomposition

$$5\to(3,1)+(1,2)\quad,\quad \bar{5}=(\bar{3},1)+(1,\bar{2})\quad,\quad 5\otimes\bar{5}=24+1$$

No (1,2) piece!



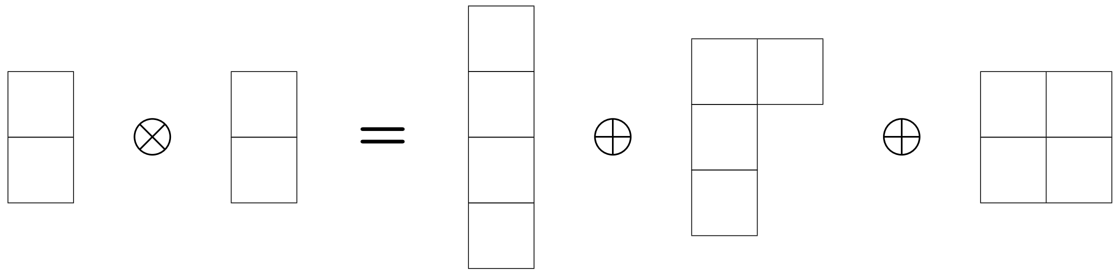
• We also need to give masses to quarks and leptons:

$\psi_{10}\psi_{10}$ gives masses to up quarks

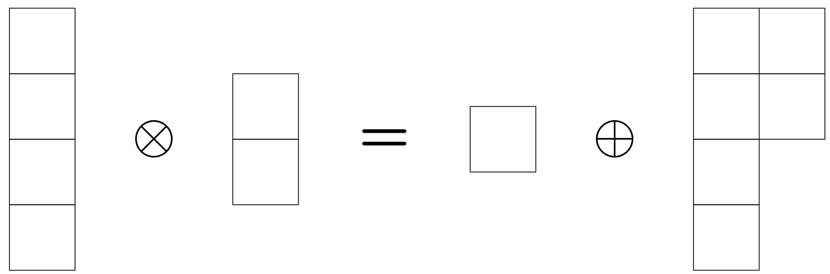
$\psi_{10}\psi_{\bar{5}}$ gives masses to down quarks and charged leptons.

We therefore need Yukawas of the form $\psi_{10}\psi_{10}Z$ and $\psi_{10}\psi_{\overline{5}}Z$ for some scalar rep Z of SU(5)

- from group theory:



$$10 \otimes 10 = \overline{5} \oplus \overline{45} \oplus \overline{50}$$



$$\overline{5} \otimes 10 = 5 \oplus 45$$

• Therefore a Higgs multiplet in $5 \to H$ can give masses to both quarks and leptons.

• It transforms under SU(5) as

$$H'(x) = U(x)\ H(x) \quad , \quad U(x) \in SU(5)$$

• It also contains a (1,2) to break the EW symmetry.

The full (renormalizable) Higgs potential:

$$V(H,\Phi) = V(\Phi) + V(H) + V(\Phi,H)$$

$$V(H) = -\frac{\mu^2}{2}H^\dagger H + \frac{\lambda}{4}(H^\dagger H)^2 \quad , \quad V(\Phi,H) = \alpha H^\dagger H\ Tr[\Phi^2] + \beta H^\dagger\ \Phi^2\ H$$

Exercise: Show that this the most general gauge-invariant and renormalizable potential

- For appropriate couplings there is a desired minimum:

$$\langle\Phi\rangle = A \begin{pmatrix} \mathbf{1} & 0 & 0 & 0 & 0 \\ 0 & \mathbf{1} & 0 & 0 & 0 \\ 0 & 0 & \mathbf{1} & 0 & 0 \\ 0 & 0 & 0 & -\frac{\mathbf{3}}{\mathbf{2}} - \frac{\epsilon}{\mathbf{2}} & 0 \\ 0 & 0 & 0 & 0 & -\frac{\mathbf{3}}{\mathbf{2}} + \frac{\epsilon}{\mathbf{2}} \end{pmatrix} \quad , \quad \langle H\rangle = v \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

- We must have $\epsilon \ll 1$ in order to have $M_X \gg M_W$

$$\epsilon = \frac{2\beta v^2}{20 h_2 A^2} + \mathcal{O}\left(\frac{v^4}{A^4}\right) \sim 10^{-28}$$

The SU(5) hierarchy problem

• We can arrange $V(H)$ and $V(\Phi)$ so that $m_H \sim v \sim 246$ GeV, and $m_\Phi \sim A \sim 10^{16}$ GeV, two very different scales.

• But the mixed potential $V(\Phi, H)$ induces a correction to $\delta m_H \sim A$

• Even if $V(\Phi, H) = 0$ at tree level it is expected to become non-zero because of quantum corrections.

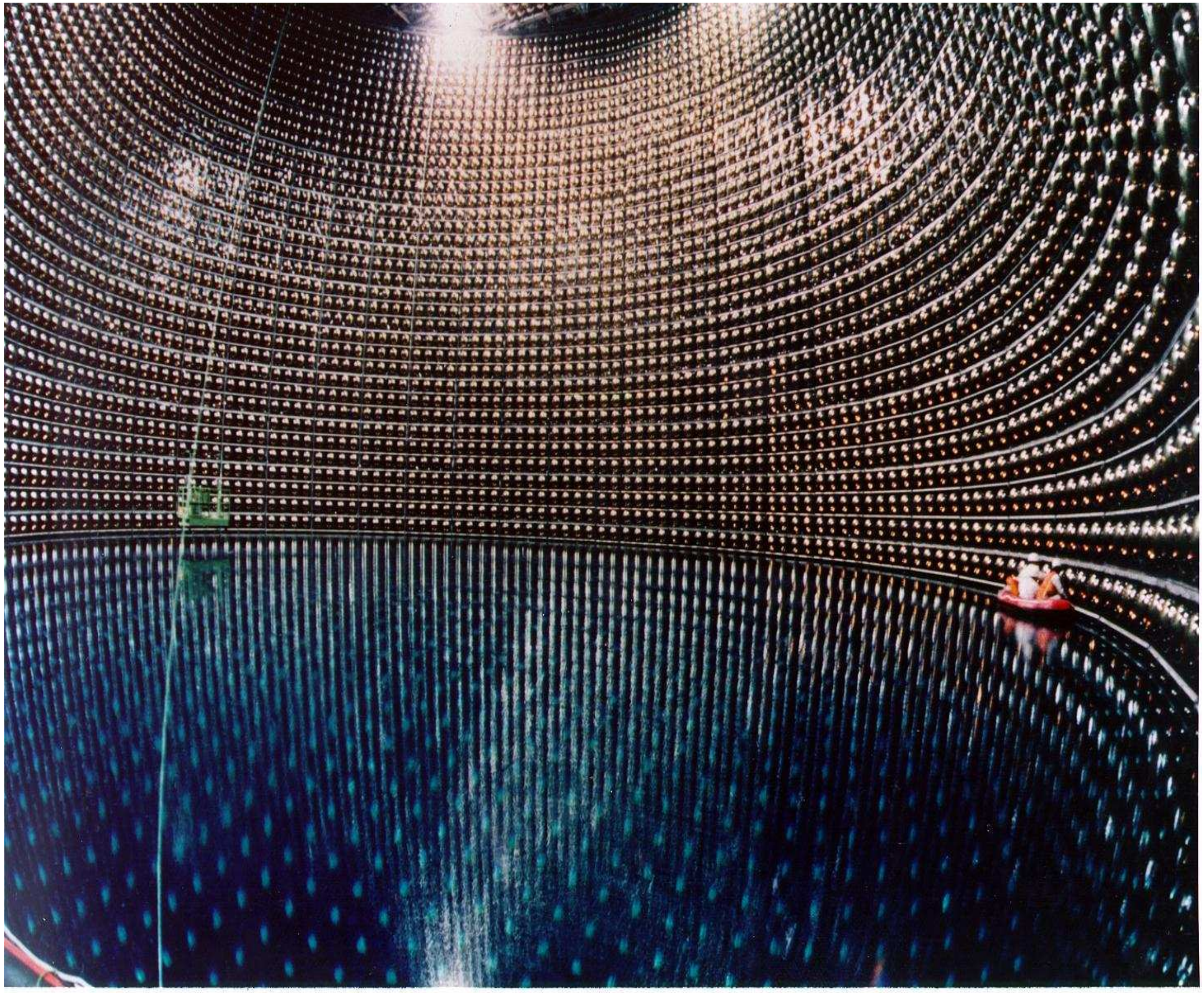
• Also $H \to (3,1) \oplus (1,2)$ under $SU(3) \times SU(2)$. The (2,1) is the standard Higgs doublet with small mass. but the $(3,1)$ must have a mass $\sim A$ because it mediates proton decay (see later).

• This is the "doublet-triplet" splitting problem.

• The SU(5) model with a high-unification scale needs "unatural" fine-tuning: it is practically uncalculable

Proton decay experiments

Large detectors (known today also as "neutrino telescopes") search for signals from the decay of protons.



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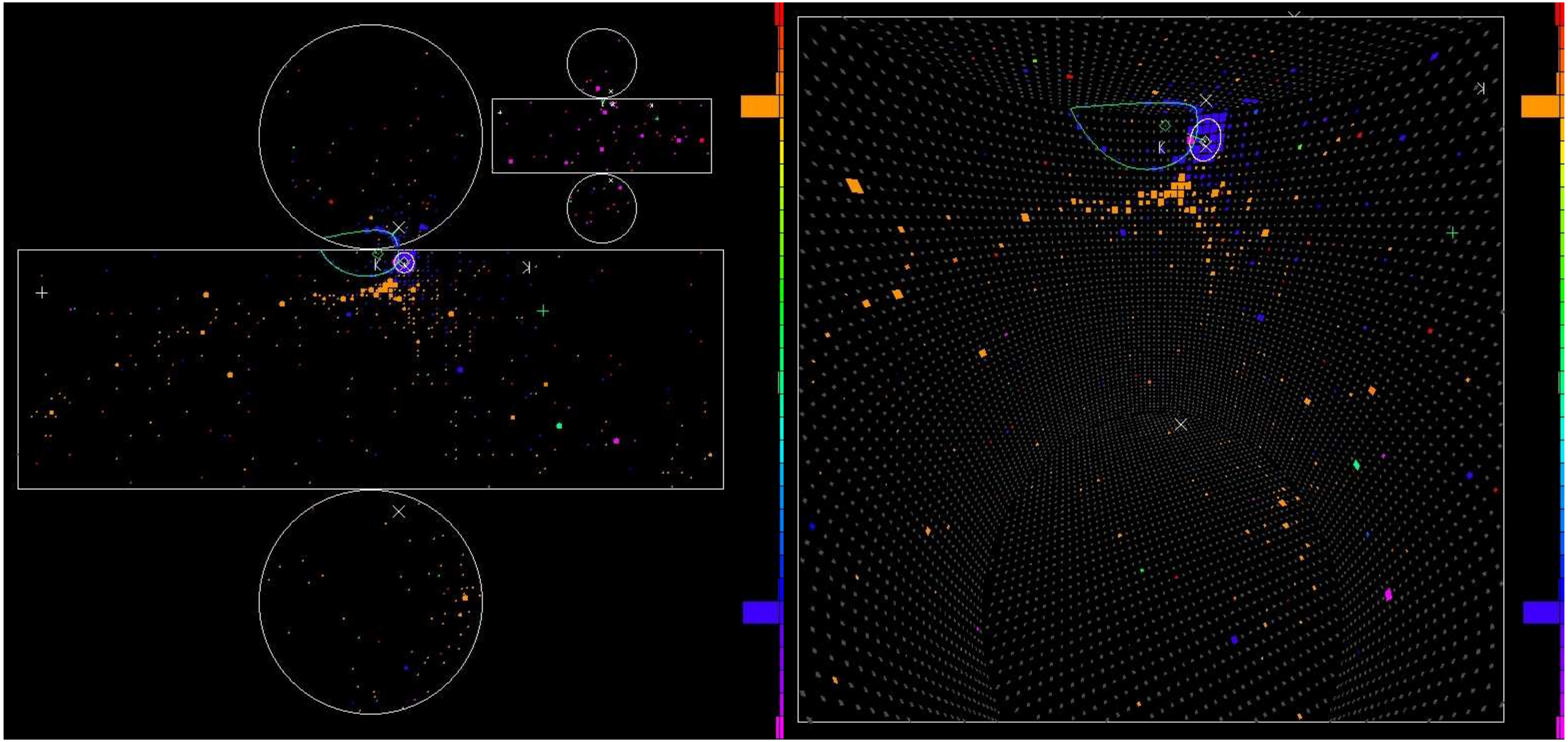

Neutrino event inside the SKM detector. It could come from one of the potential decay channels of the proton.

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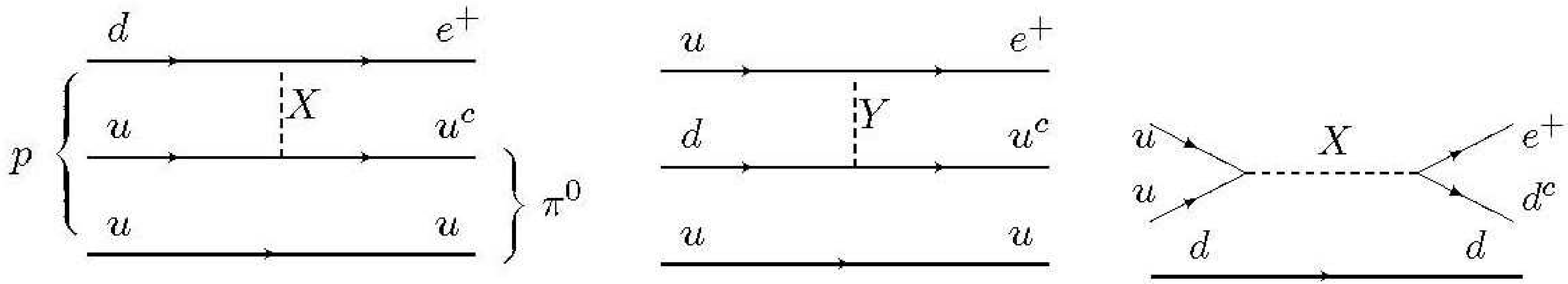
Proton decay channels

In standard GUTs the nucleon decay channels are as follows:

$$p \to \pi^0 + e^+ \quad \text{or} \quad p \to \pi^0 + \mu^+ \quad , \quad p \to K^+ + \bar{\nu}$$

$$p \to K^0 + e^+ \quad \text{or} \quad p \to K^0 + \mu^+ \quad , \quad n \to K^0 + \bar{\nu}$$



Exercise: Starting from the basic SU(5) baryon-violating reaction $uu \to e^+ \, \nu_e$ and all other obtained from this one by the SU(5) symmetry, derive the rest of the reactions above

For more details see http://arxiv.org/pdf/hep-ph/0211024

RETURN

Hypercharge normalization

We have seen that

$$Y = \frac{1}{6}\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix}$$

But we use matrices normalized to 1/2 to define the gauge theory

$$T_Y = \xi\ Y \quad , \quad Tr[T_Y T_Y] = \frac{1}{2} \quad , \quad \xi = \sqrt{\frac{3}{5}}$$

so that

$$A_\mu = T_Y\ B_\mu \quad , \quad \delta L = -\frac{1}{2}Tr[F_A]^2 + g_{GUT}Tr[A_\mu J^\mu] = -\frac{1}{4}F_B^2 + \xi\ g_{\text{GUT}}Tr[Y\ B_\mu J^\mu]$$

$$g_Y = \xi\ g_{\text{GUT}} = \sqrt{\frac{3}{5}}\ g_{\text{GUT}}$$

SU(5) Mass relations

- For each generation we have only two independent Yukawa couplings:

$$\lambda_{10105}\ \epsilon^{abcde}\ \psi_{ab}\psi_{cd}H_e \quad , \quad \lambda_{10\bar{5}5}\ \psi_{ab}\psi^{\bar{a}}(H^{\dagger})^{\bar{b}}$$

$$m_u = \lambda_{10105}\ v \quad , \quad m_d = \lambda_{10\bar{5}5}\ v \quad , \quad m_e = \lambda_{10\bar{5}5}\ v$$

- Therefore

$$m_d = m_e \quad , \quad m_s = m_\mu \quad , \quad m_b = m_\tau$$

- These relations are valid at $E = M_X$

- The "clean" one is the last and it is successful

SO(10) unification

• In SU(5) quarks and leptons are in three representations of the gauge group (10, $\bar{5}$ and 1)

• They may be combined in one representation of a higher group: this group is SO(10) and the relevant representation is the 16-dimensional MW-spinor of SO(10).

• $SU(5) \subset SO(10)$ and $16 \to 10 + \bar{5} + 1$.

• Gauge bosons are in the $45 \to 24 \oplus 10 \oplus \overline{10} + 1$ (the singlet is B-L)

• SO(10) has no gauge anomalies

• Neutrinos are unified with the rest of the fermions.

• There are several ways to break SO(10) to the SM group and several Higgs representations are needed.

Neutrino masses and the see-saw mechanism

- In SO(10), unlike SU(5), a neutrino singlet ν_R is "imposed".

- The Dirac mass term $\bar{\nu}_R\nu_L$ is definitely allowed. As ν_L is an SU(2) doublet the appropriate term is

$$(L^\dagger H)\nu_R \sim v_F\ \nu_L^\dagger \nu_R$$

- Since B-L is eventually broken, a Majorana mass term $\nu_R^c\nu_R$ is also allowed

$$M_D\nu_R^c\nu_R$$

where M_D is of the order of the unification scale

The neutrino mass matrix (one generation) is then of the form

$$L_{\nu-mass} = (\nu_L^\dagger, \nu_R^c)\begin{pmatrix} 0 & v_F \\ v_F & m_D \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$$

- There are two mass eigenstates with masses

$$M_{\text{light}} \simeq \frac{v_F^2}{m_D} \quad , \quad M_{\text{heavy}} \simeq m_D$$

- For $v_F \simeq 246$ GeV and $m_D \sim 10^{16}$ GeV, we obtain $M_{\text{light}} \sim 10^{-3}$ eV

Outlook on baryon number violation

• There are many models of unified gauge theories (most "popular" groups are SU(5), SO(10), E_6)

• Their generic prediction is that Baryon number is violated and the proton must decay

• The current limit on the lifetime is $\tau \succeq 10^{33}$ years.

• Baryon number is already violated in the SM, by electroweak instantons. However the rate at zero temperature is tiny (unobservable).

• The universe has an important baryon-asymmetry $\frac{n_B}{n_B+n_{\bar{B}}} \sim 10^{-9}$.

• The cosmological baryon asymmetry could have been generated during the EW phase transition, although detailed analyses are at best inconclusive and generically indicate that this may be impossible.

• In light of this, we need other sources of baryon number violation to generate the cosmological baryon asymmetry.

• According to Sakharov (1966) we need :

(1) CP violation

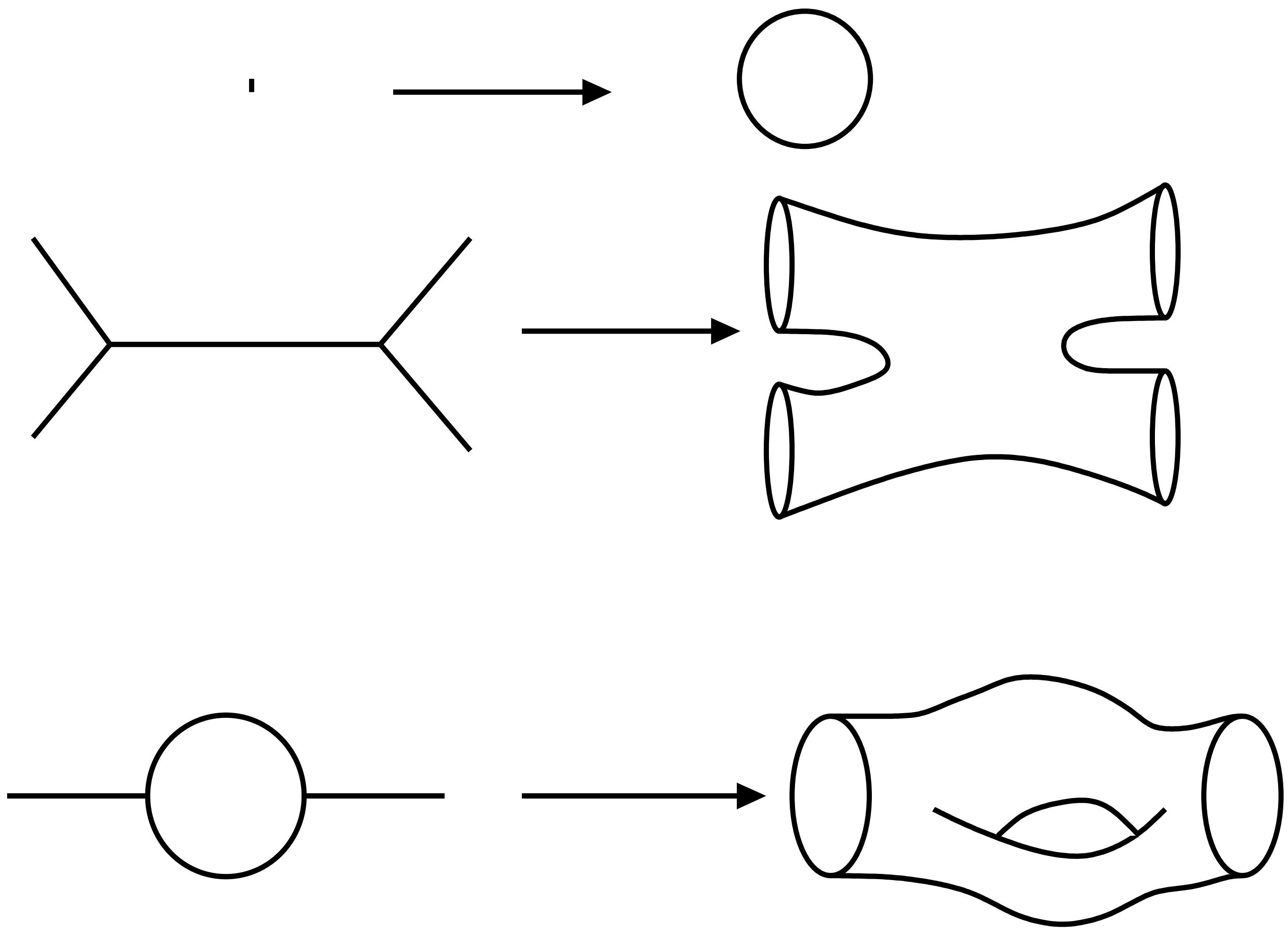
(2) Baryon number violation

(3) Out-of-equilibrium conditions

to generate the baryon asymmetry.

• There are several suggested solutions but the problem is still considered open. RETURN

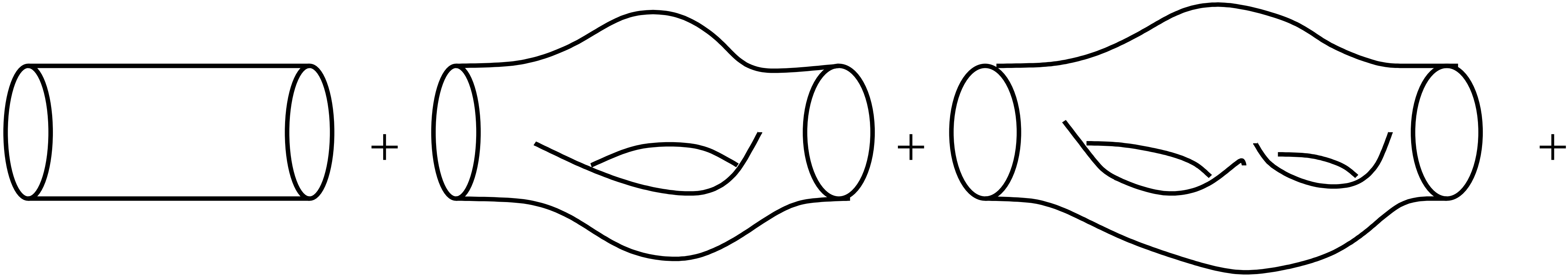
• In perturbation theory, standard QFT Feynman diagrams are replaced with string diagrams (two-dimensional surfaces)



String perturbation theory

♣ In QFT perturbation theory is formulated using Feynman diagrams.

♠ In string theory we have Riemann surfaces. For closed strings, each order contains a single diagram. At low energy, they reduce to the (many) QFT Feynman diagrams.



• String theory diagrams, when appropriately defined, give finite amplitudes in the UV. Quantum gravity, which is part of string theory is essentially finite.

The compact Newton's law

Assume 3+1 non-compact dimensions, and a single compact direction of radius R ($x^4 \to x^4 + 2\pi R$). The Newton's law, obtained by the method of images is

$$F = \frac{M_1 M_2}{M_*^3} \sum_{n\in Z} \frac{1}{[(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4 + 2\pi n R)^2]^{\frac{3}{2}}}$$

$$r = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2}$$

is the usual distance in 3+1 dimensions.

• When $r << R$, all other images $n \neq 0$ are far away and can be neglected. Therefore we have 5d gravity.

$$F(r << R) \simeq \frac{M_1 M_2}{M_*^3 \ [r^2 + x_4^2]^{\frac{3}{2}}}$$

• When $r >> R$ all images give equally important contributions. The result can be obtained by a Poisson resummation: $\sum_{n\in Z} f(2\pi n) = \sum_{n\in Z} \tilde{f}(n)$

$$\sum_{n\in Z} \frac{1}{[r^2 + (x^4 + 2\pi n R)^2]^{\frac{3}{2}}} \simeq \frac{1}{\pi R r^2}\left[1 + \mathcal{O}\left(\frac{R}{r}\right)\right]$$

$$F(r >> R) \simeq \frac{M_1 M_2}{\pi M_*^3 \ R} \frac{1}{r^2} \quad , \quad M_P^2 = \pi M_*^3 R$$

Kaluza-Klein states in string theory

In string theory the KK spectrum is more complex: beyond the usual KK states, the string can wind around the circle, m times. This gives an extra contribution to the energy:

$$\sim T\ (2\pi m\ R) \quad , \quad m \in Z \quad , \quad T = \frac{1}{2\pi\ \ell_s^2}$$

The spectrum of KK masses now becomes

$$M^2 = \frac{n^2}{R^2} + (2\pi T R)^2 m^2 = \frac{n^2}{R^2} + \ell_s^4 R^2 m^2 \quad , \quad m, n \in Z$$

- The spectrum of stringy KK states is invariant under T-duality

$$m \leftrightarrow n \quad , \quad R \leftrightarrow \frac{\ell_s^2}{R}$$

There is no circle with $R < \ell_s$ in string theory!

LHC cross-sections for KK-gravitons

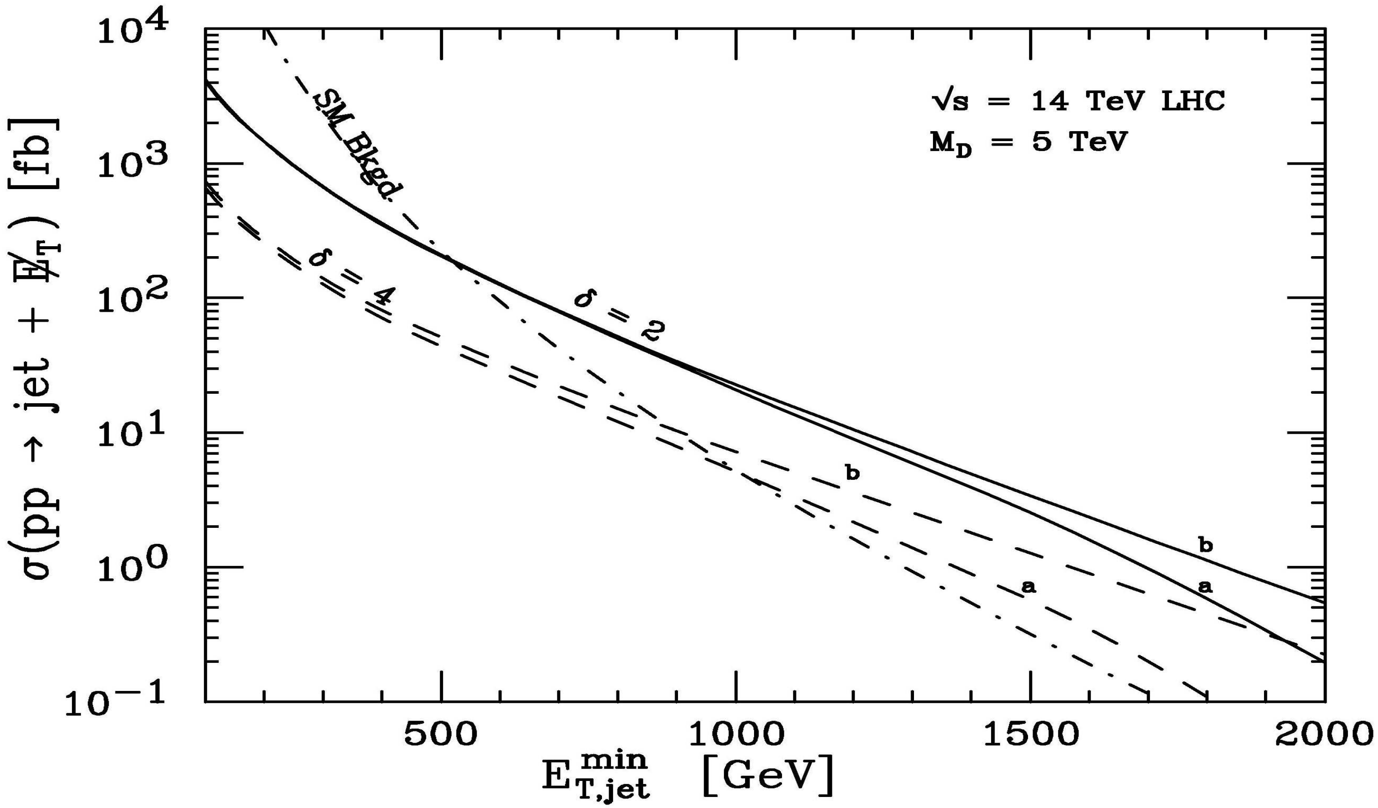


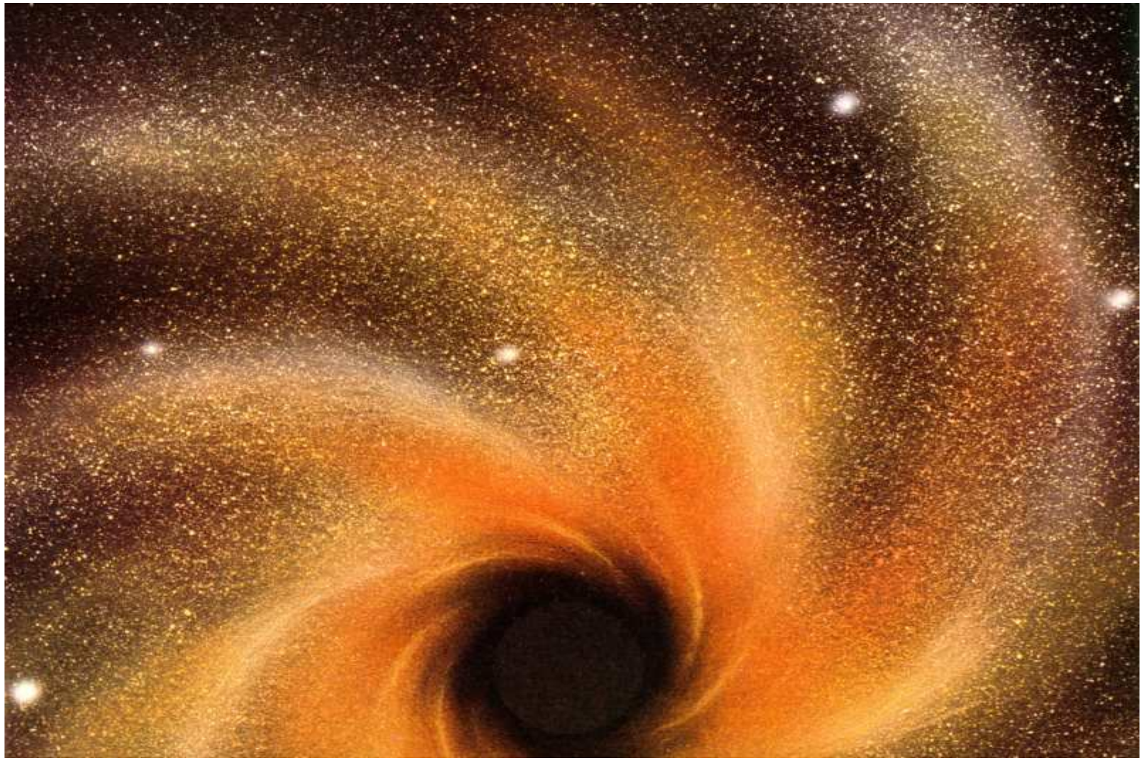
Figure 3: The total jet + nothing cross-section at the LHC integrated for all $E_{T,\rm jet} > E_{T,\rm jet}^{\rm min}$ with the requirement that $|\eta_{\rm jet}| < 3.0$. The Standard Model background is the dash-dotted line, and the signal is plotted as solid and dashed lines for fixed $M_D = 5\,\text{TeV}$ with $\delta = 2$ and 4 extra dimensions. The **a** (**b**) lines are constructed by integrating the cross-section over $\hat{s} < M_D^2$ (all $\hat{s}$).

From Giudice, Rattazzi and Wells [arXiv:hep-ph/9811291]

RETURN

Black holes and the information paradox

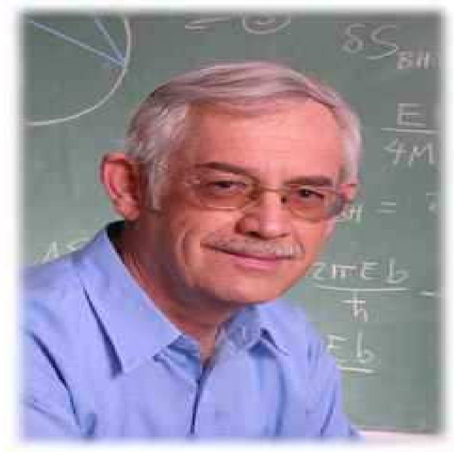
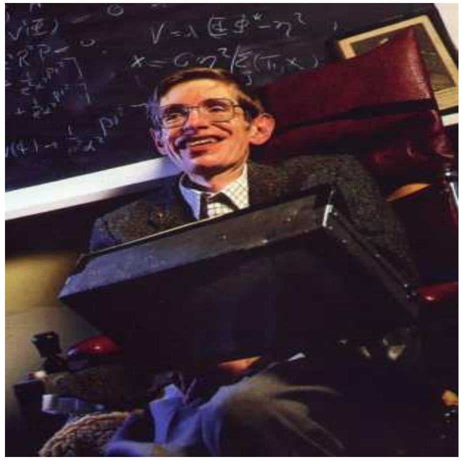
♠ SURGEON'S WARNING: The following descriptions are very qualitative and gloss over several important details. They are meant to convey the spirit of the current progress in understanding the black hole paradoxes in string theory



• General Relativity predicts black hole solutions. They are sinks of energy. They are surrounded by a horizon. Classically they are the ultimate vacuum cleaners.

- Black holes follow the laws of thermodynamics!!!

Christodoulou, Carter, Bekenstein, Hawking, 1970-72

- The simplest black hole solution (found by Schwarzschild) is characterized only by its mass M.

- We may define the Hawking temperature and the Bekenstein-Hawking entropy as :

$$T_H \equiv \frac{\hbar \ c^3}{8\pi \ k} G \ M$$

$$S_B \equiv 4\pi \frac{G}{\hbar \ c} M^2 = \frac{1}{4} \ \frac{\text{Area of the horizon}}{\ell_P^2}$$

Black hole thermodynamics

0-th law: gravitational equilibrium = constant temperature

1st law: $\Delta E = T\ \Delta S +$ WORK DONE

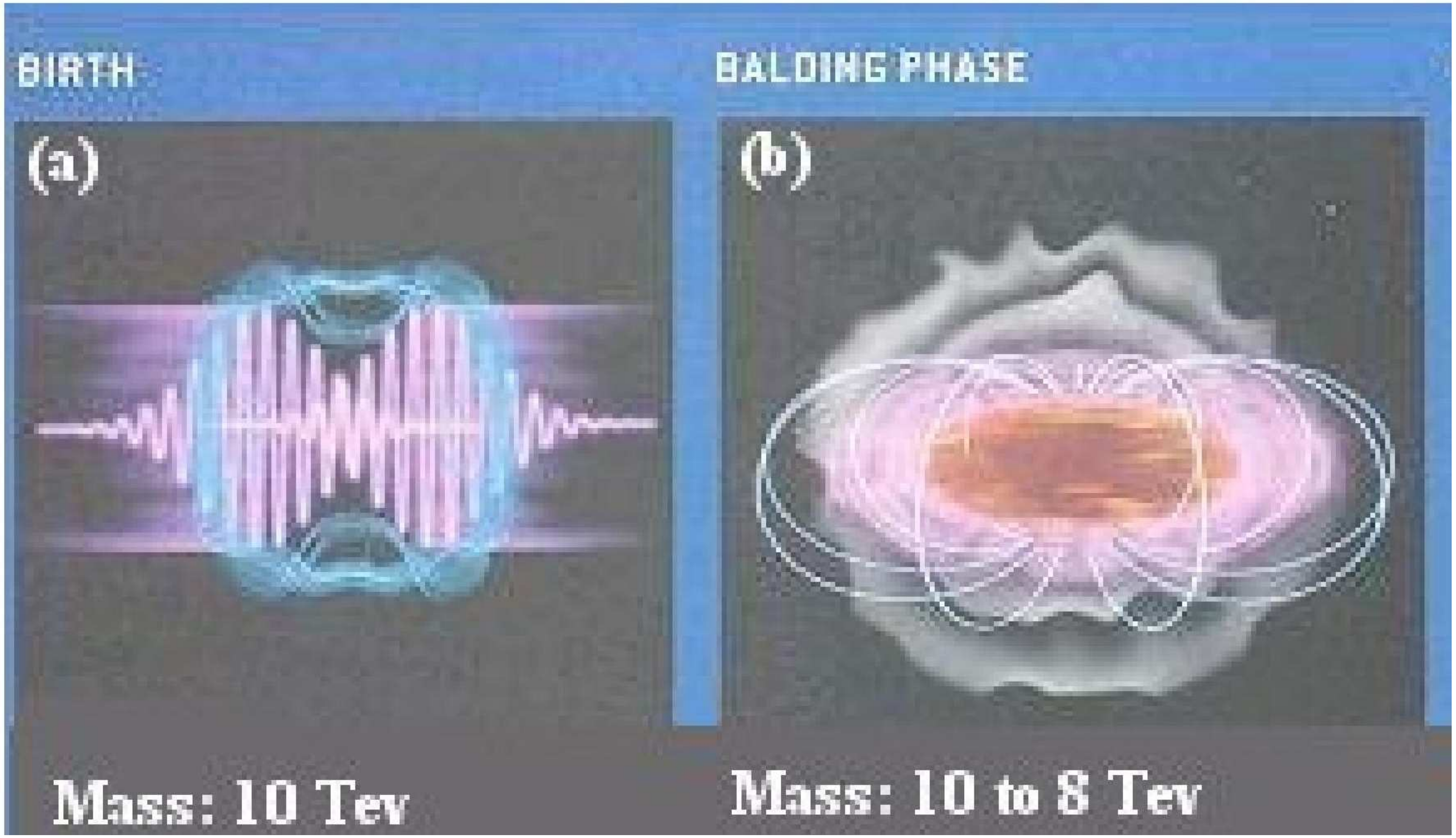
2nd law: $\Delta S \quad \geq \quad 0$

3rd law: No finite physical process can bring $T = 0$.

- The presence of such laws was a mystery

- In 1974 Hawking showed that if we treat matter around a black hole quantum mechanically (but gravity classically) , the black hole evaporates emitting particles with a black-body spectrum at temperature T_H (the Hawking temperature)

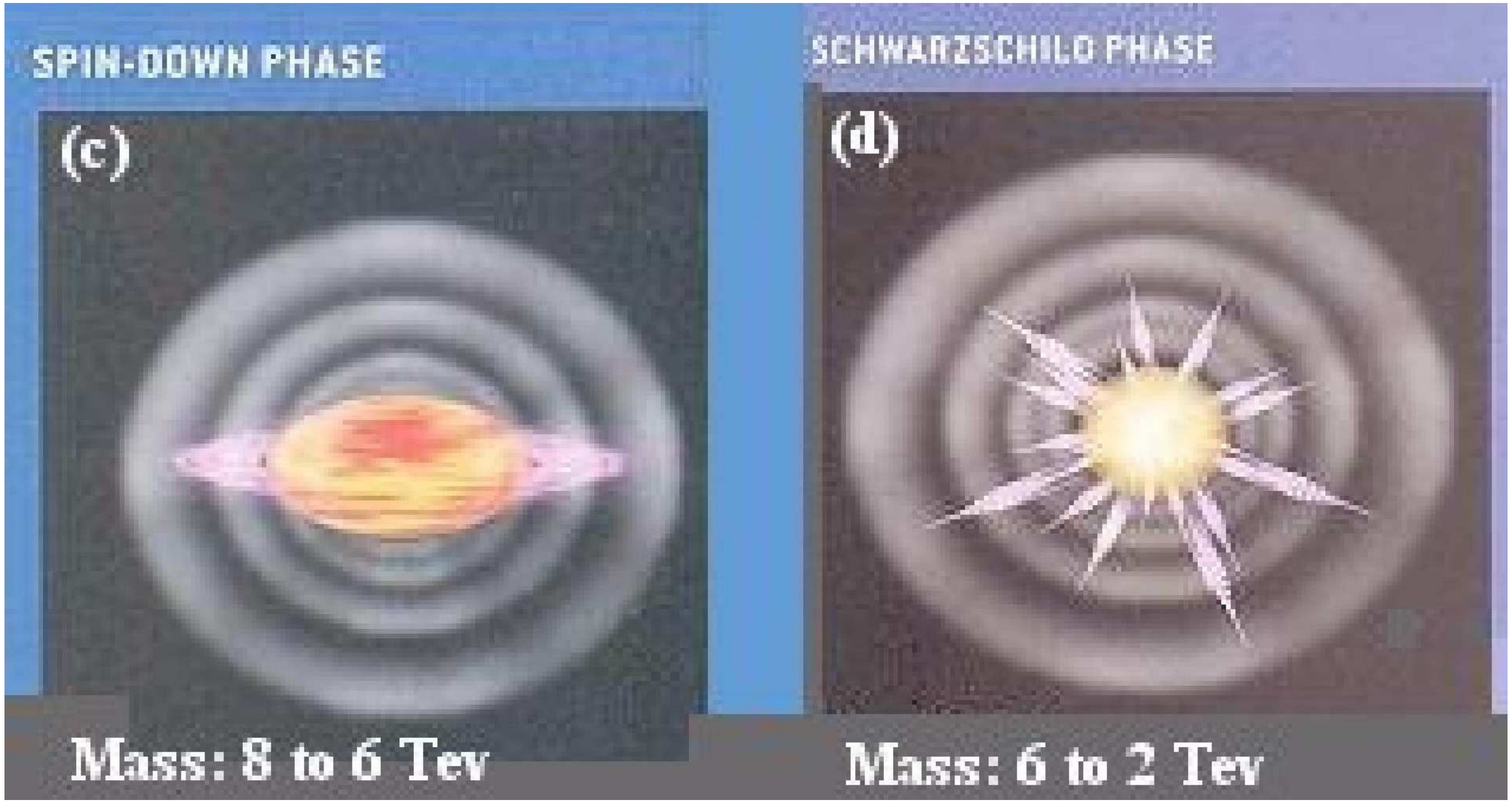
Black-hole evaporation



A simulation of black-hole evaporation:(a)+(b) Phase

The black hole is created by gravitationally collapsing particles (phase a). It first radiates aways possible conserved charges like electric charge (phase b)

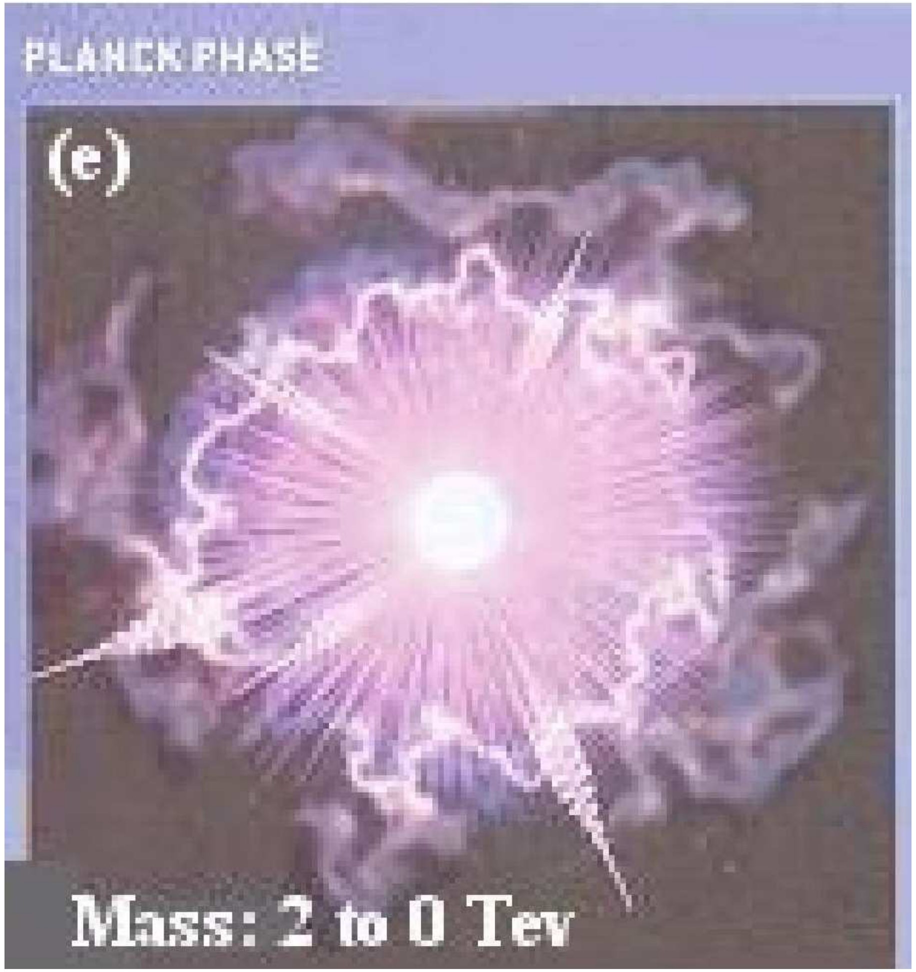
.



A simulation of black-hole evaporation:(c)+(d) Phase

It subsequently radiates away particles with angular momentum therefore spinning down (phase c) turning into a Schwarzschild black hole that continues to radiate (phase d)

.



A simulation of black-hole evaporation: Final phase

when the black hole is very small the radiation rate is very large ending in a final explosion

Where are the microstates?

• In quantum statistical mechanics entropy is a measure of the possible different microstates available to the macroscopic system, (N).

$$S \sim \log[N]$$

• What are the microstates that give rise to the Bekenstein-Hawking entropy?

• All reasonable arguments give:

$$\mathbf{Entropy} \sim \mathbf{volume}$$

and not $\sim$ surface as in the black-hole case.

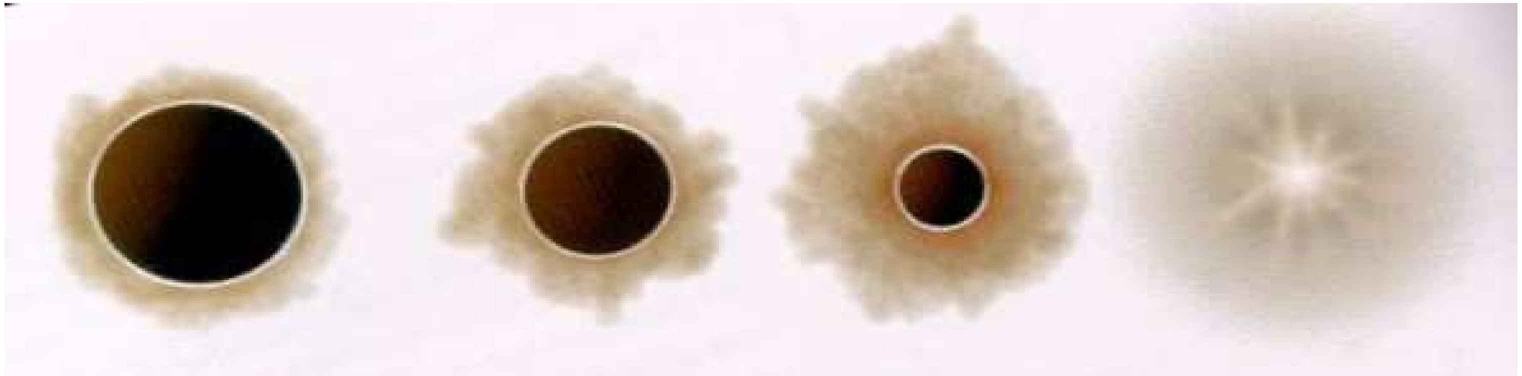
• What is the fundamental microscopic explanation of black-hole thermodynamics?

The information paradox

• According to quantum mechanics information is conserved: (pure states evolve to pure states).

But....

• A black hole that is created by matter+information (a pure initial state), radiates à la Hawking thermal radiation (that does not carry information), and evaporates leaving nothing behind.



Where did the original information go?

Interlude

• Hawking in 1974 conjectured that the information is lost permanently and therefore quantum mechanics must be modified in the presence of (strong) gravity.

• Last year he accepted defeat by embracing the solution given by the holographic description of string theory and paid his lost bet to John Preskill

Some string theory answers, I

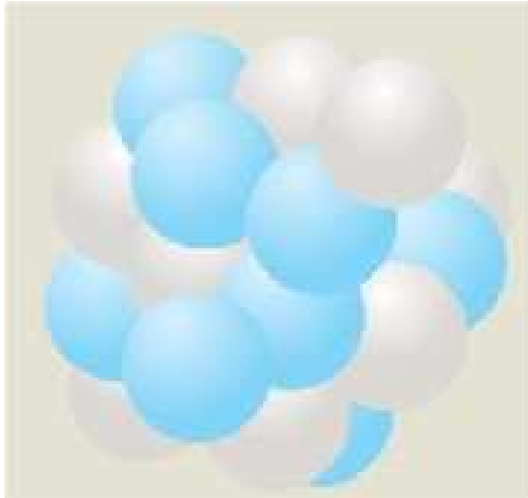
- We may theoretically construct microscopic black-holes from (many) D-brane bound states.
Strominger+Vafa, 1996

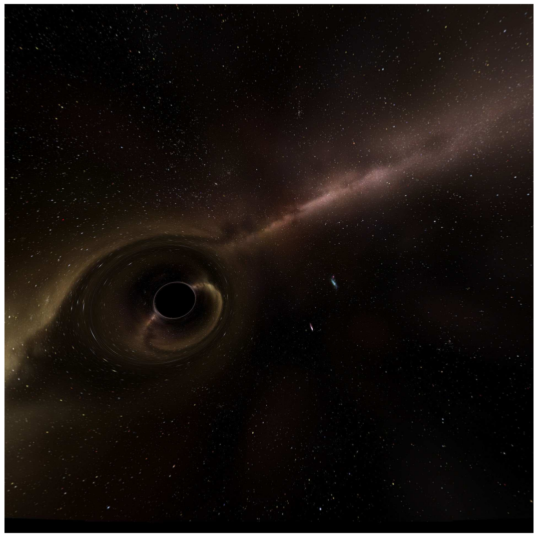
- When the string coupling constant is small $g_s << 1$, these bound states look like heavy composite particles without a horizon.

- In this case we can compute the possible microscopic quantum states and the answer agrees with the (gravitational) Bekenstein-Hawking entropy

- If we now extrapolate to $g_s >> 1$, the composite particle becomes a heavy black hole with a macroscopic horizon.

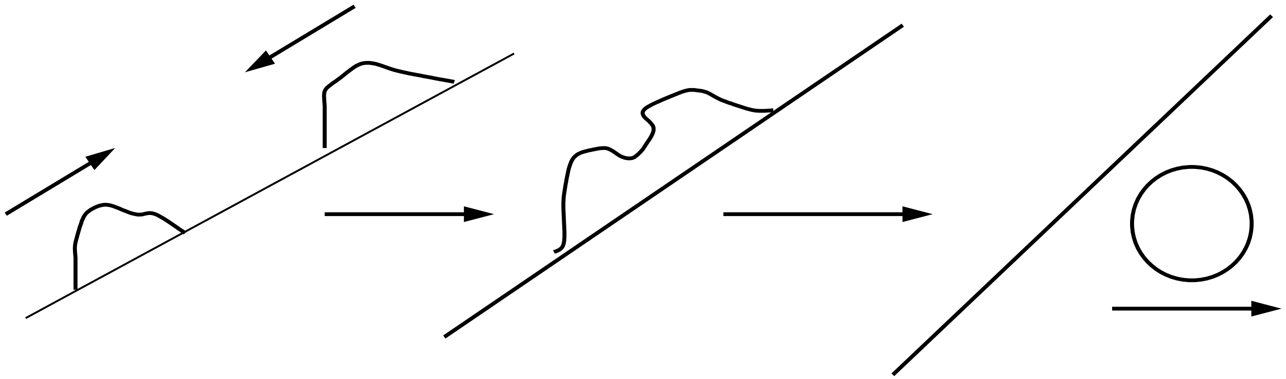
- Do we trust our calculations when $g_s >> 1$? Yes! (in special cases that are "protected" by supersymmetry





Some string theory answers, II

• We can also microscopically produce the Hawking radiation



• Open string degrees of freedom describe the fluctuations of D-branes and therefore those of their bound states. When such open string scatter on the surface of the bound-state they may combine to form a closed string and leave the bound state. This process can be modeled and computed from first principles n string theory (in some interesting limits).

• This detailed microscopic string theory calculation agrees with the semi-classical Hawking result in their common area of validity.

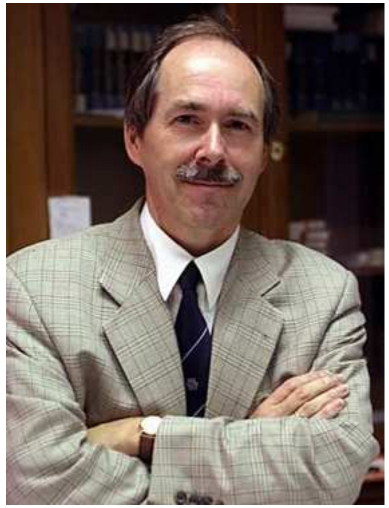
• This gives us confidence that this is the correct picture of black-hole microstates (at least for special black holes)!

• It also suggests that information is not lost.

Gravity and Holography

- In standard dynamical systems entropy is proportional to the volume.

- In black hole systems it is proportional to the (horizon) area

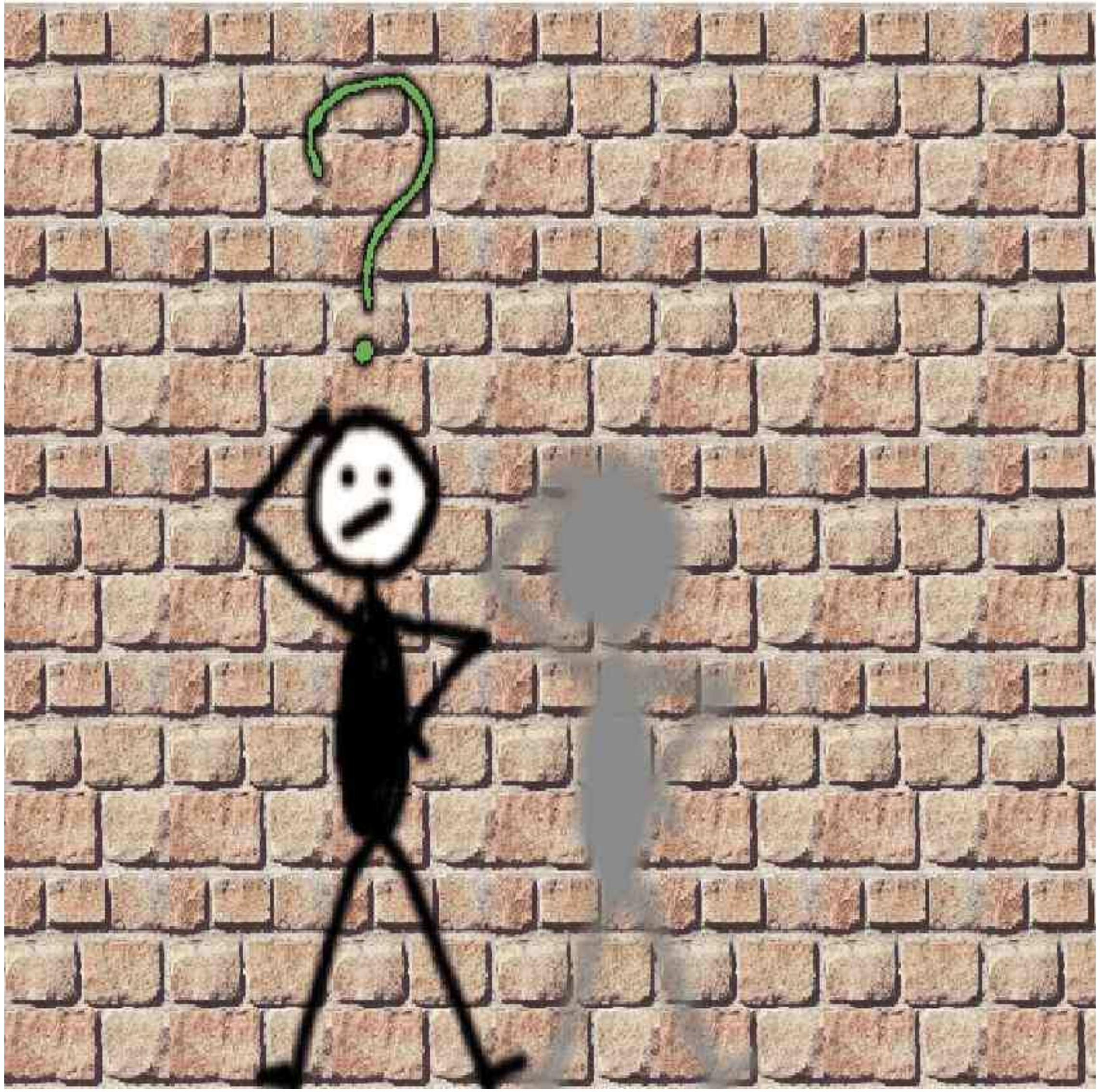


- This suggests that the gravitational degrees of freedom are much less in number that what we thought (based on QFT).

- The surface degrees of freedom seem to capture all the information about the volume they surround. (holographic property)

- 't Hooft (1992) : The holographic property is a general property of any consistent theory of quantum gravity.

This property reminds us of "Plato's cave"



Holographic gauge theory/string theory correspondence

• Recently concrete examples have been found where the holographic correspondence can be understood.

• A gauge theory living on the boundary of a space-time corresponds holographically to the string theory that lives in the bulk of the space-time.

Maldacena, 1997

• This opens the door for a deeper understanding of the puzzles of quantum gravity using gauge theory dynamics.

• It is also suggests that information is not lost from black-holes as the dual description in terms of the gauge theory is explicitly unitary.

• Recently holographic techniques start making an impact as a tool to understand strong coupling dynamics in QCD both zero temperature, as well as for the physics of the quark gluon plasma.

Further reading

• Introductory general relativity:
http://www.phys.uu.nl/~thooft/lectures/genrel.pdf

• Black holes: [arXiv:gr-qc/9707012]

• Introduction to black-hole thermodynamics:
http://www.glue.umd.edu/~tajac/BHTlectures/lectures.ps

• Introductory descriptions on the counting of black-hole microstates and the holographic (bulk-boundary) correspondence can be found in the following string theory books:

♠ C. Johnson, "D branes"

♠ K+M. Becker, John Schwarz, "String theory and M-theory

♠ E. Kiritsis, "String theory in a nutshell

RETURN

Plan and Links

- The Standard Model: principles 16 minutes
- Standard Model: Open Problems 18 minutes
- Standard Model: Open Problems II 20 minutes
- SM patterns and parameters 22 minutes
- The pattern of masses 24 minutes
- How parameters affect us 26 minutes
- Renormalization: Integrating out high-energy d.o.f 30 minutes
- The effective field theory 32 minutes
- Effective couplings: the Fermi Theory paradigm 36 minutes
- Irreversibility 39 minutes
- Running Couplings 42 minutes
- Renormalization Summary 46 minutes
- End of lecture 1

- Supersymmetry Breaking 101 minutes
- Supersymmetry Breaking, II 105 minutes
- R-parity 108 minutes
- Missing Energy 109 minutes
- Missing Energy (Atlas simulation) 110 minutes
- A link to dark matter 113 minutes
- The soft supersymmetry breaking terms 120 minutes
- MSSM 127 minutes
- SUSY Outlook 131 minutes
- Grand Unification: The idea 136 minutes
- SU(5): the matter 139 minutes
- Proton decay 142 minutes
- Coupling Unification 146 minutes
- End of lecture 3

Appendices

- The Standard Model: ingredients
- Standard Model: the quarks
- Standard Model: the leptons
- Standard Model: the Higgs
- Higgs event
- How parameters affect us: some answers
- Other parameters in the SM
- Renormalization:the coupling constants
- Renormalization: the old view
- renormalized couplings: an example
- Renormalization: further reading
- Technicolor (extended discussion)
- First SUSY event

- Weyl spinors
- The supersymmetry algebra
- The supersymmetric representations
- The massless representations
- The supersymmetry transformations
- The simplest scalar action
- The supersymmetric gauge theory
- The Fayet-Iliopoulos term
- The supersymmetric vacuum
- Spontaneous Supersymmetry Breaking
- The O'Raifertaigh example
- minimal Gauge Mediated Susy Breaking
- minimal Anomaly Mediated Susy Breaking
- A simple example of a softly-broken supersymmetric theory
- SSB: Heavy gravitino mass
- SSB: Light gravitino mass

- SU(5) generators: continued
- Decomposition of the 10 of SU(5) into SM reps
- SU(5) symmetry breaking 205 minutes
- The SU(5) hierarchy problem 209 minutes
- Proton Decay Experiments
- Proton decay channels
- Hypercharge normalization
- Mass relations 212 minutes
- SO(10) unification 215 minutes
- Neutrino masses and the see-saw mechanism 218 minutes
- Outlook on baryon number violation 223 minutes

- String Theory, Vol III
- String perturbation theory
- The compact Newton's law
- Kaluza-Klein states in string theory
- LHC cross-sections for Kaluza-Klein graviton production
- Black holes and the information paradox
- Black hole thermodynamics
- Black hole evaporation
- Where are the microstates?
- The information paradox
- Interlude
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- Some string theory answers, II
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