Salam: Electroweak Unification

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Renormalizability

Chiral Fermions

Local gauge invariance

Edifice of Particle Physics is mainly built on these ingredients. **Abdus Salam** has made significant and far reaching contributions in all fields.

QED: First order perturbation theory gives results in good agreement with experiment. Beyond leading order yield infinite answers.

Renormalization: (Schwinger, Tomonaga, Feynman and Dyson) showed: How infinities can be subtracted by renormalizing mass and charge to give finite answers

QED: One of the most accurately tested theory: Successful predictions of radiative corrections to the magnetic moment of the electron and the Lamb shift in the energy levels of hydrogen atom.

Salam: solved the outstanding difficulty in the renormalization theory: That of "Overlapping divergences" to complete the Dyson's proof of renormalization of QED.

Renormalization is regarded as a basic principle: A successful theory must be renormalizable.

Chiral fermion:

$$\psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\psi$$

- for left handed

+ for right handed

1957: **Salam** introduced γ_5 invariance
If ψ is a solution of the Dirac equation for zero mass fermion (spin $\frac{1}{2}$): $\gamma_5\psi$ is also a solution of Dirac equation.
Neutrino : massless
γ_5 invariance holds for neutrino.
In chiral representation of γ matrices, γ_5 is diagonal
Dirac field

$$\psi = \begin{pmatrix} \zeta \\ \eta \end{pmatrix} \qquad \zeta, \eta = \frac{1}{2}(1 \mp \gamma_5)\psi$$

Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\psi = 0 \qquad \text{For } m = 0$$

$$(i\vec{\sigma} \cdot \vec{\nabla} - i\frac{\partial}{\partial t})\zeta = 0 \tag{1}$$

$$(i\vec{\sigma} \cdot \vec{\nabla} + i\frac{\partial}{\partial t})\eta = 0. \tag{2}$$

Equations. (1) and (2) are not disconnected. Take complex conjugate of Eq. (1)

$$(-i\vec{\sigma}^* \cdot \vec{\nabla} + i\frac{\partial}{\partial t})\zeta^*$$

$$\sigma^2(-i\vec{\sigma}^* \cdot \vec{\nabla} + i\frac{\partial}{\partial t})\sigma^2\sigma^2\zeta^* = 0$$

$$\sigma^2\vec{\sigma}^*\sigma^2 = -\vec{\sigma}$$

or

$$(i\vec{\sigma}^* \cdot \vec{\nabla} + i\frac{\partial}{\partial t})i\sigma^2\zeta^* = 0 \quad (3)$$

which is Eq. (2).
Under change conjugation

$$\zeta \to \zeta^c = -i\sigma^2\eta^*$$
$$\eta \to \eta^c = i\sigma^2\zeta^*$$

Thus if Eq. (1) is satisfied by left-handed particle field ζ, then it follows from Eq. (1) that right handed antiparticle field η^c satisfies Eq. (2).
$\zeta = \nu_L$: Left handed neutrino
$\eta^c = \nu_R^c$: Right handed anti-neutrino.

Write

$$\zeta(\vec{p}) = \omega(\vec{p})e^{-ip\cdot x} = \omega(\vec{p})e^{i(\vec{p}\cdot\vec{x}-Et)} \quad (4)$$

From Eq. (1)

$$\begin{aligned} (\vec{\sigma}\cdot\vec{p})\omega(\vec{p}) &= -E\omega(\vec{p}) \\ E^2 = \vec{p} \quad &, \quad E = \pm|\vec{p}| \end{aligned}$$

+tive energy spinor $u(\vec{p})$

$$\frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}u(\vec{p}) = -u(\vec{p})$$

-tive energy $(E = -|\vec{p}|)$ spinor v$(\vec{p})$

$$\frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}v(\vec{p}) = v(\vec{p})$$

Important result: left handed neutrino ν_L has helicity -1 and right handed anti-neutrino ν_R^c, has helicity +1.

1956: Lee and Yang pointed out that there is no experimental evidence, that in weak interactions left-right symmetry holds unlike electromagnetic and strong interactions. Lee and Yang proposed experiments to check this symmetry. Under space reflection (Parity)P : $\vec{x} \to -\vec{x}$; under charge conjugation C : particle $\to$ antiparticle. Both these discrete symmetries are violated in weak interaction but are preserved by the strong and electromagnetic interaction. First indication of parity violation was revealed in the decay of a particle with spin parity $J^P = 0^-$ called K meson into two modes $K^0 \to \pi^+\pi^-$ (parity violating) and $K^0 \to \pi^+\pi^-\pi^0$ (parity conserving).

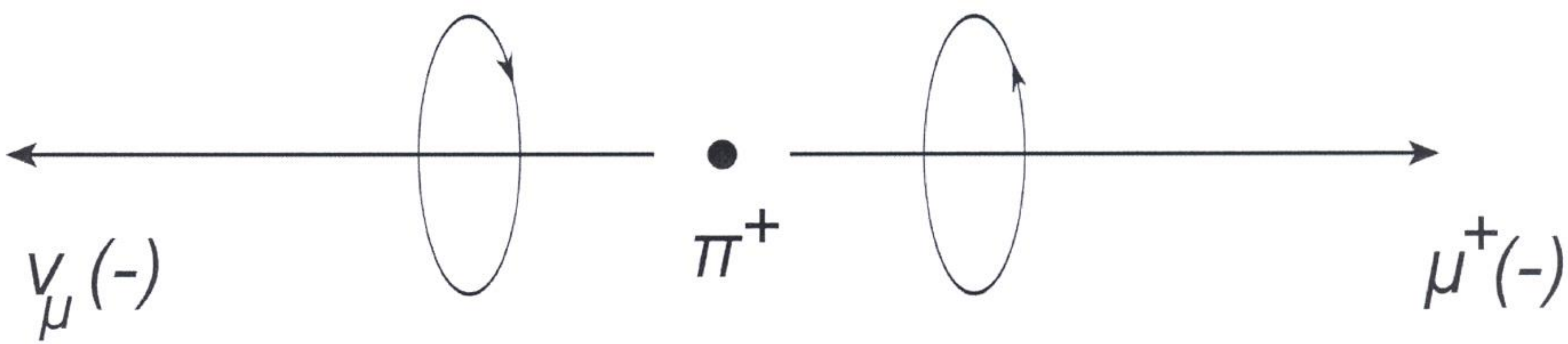

Figure: Parity Violation in Weak Decay

Both C and P are violated in the decay

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu$$

The helicity of μ^+ comes out to be negative, showing violation of parity or space reflection invariance.
Due to Conservation of spin, the neutrino also comes out with negative helicity $\nu_\mu(-)$

$$\text{Helicity } \mathcal{H} = \frac{\overrightarrow{\sigma} \, . \, \overrightarrow{p}}{|\overrightarrow{p}|}$$

$$\mathcal{H} \xrightarrow{P} -\mathcal{H}$$
$$\mathcal{H} \xrightarrow{C} \mathcal{H}$$

C-invariance implies

$$\begin{aligned} \Gamma_{\pi^- \to \mu^-(-)\overline{\nu}_\mu} &= \Gamma_{\pi^+ \to \mu^+(-)\nu_\mu} \\ \text{Exp. } \Gamma_{\pi^- \to \mu^-(-)\overline{\nu}_\mu} &\ll \Gamma_{\pi^+ \to \mu^+(-)\nu_\mu} \end{aligned}$$

Shows C-violation.

However

$$\Gamma_{\pi^+ \to \mu^+ (-) \nu_\mu} \xrightarrow{CP} \Gamma_{\pi^- \to \mu^- (+) \overline{\nu}_\mu}$$

This is what is observed: CP-conservation hold.
Left-handed ν_μ and right-handed $\overline{\nu}_\mu$.
Both C and P are maximally violated $V - A$ theory.
Salam, Landau, Lee, Yang and Ohme independently pointed that not only parity is violated but charge conjugation is also violated which is the case experimentally.

Local Gauge invariance

Fundamental constituent of matter: spin 1/2

Dirac Equation

$$(i\gamma^\mu \partial_\mu - m)\psi(x) = 0$$

Dirac Lagrangian

$$\mathcal{L}_{\text{Dirac}} = \overline{\psi}(x)(i\gamma^\mu \partial_\mu - m)\psi(x)$$

Invariant

$$\psi(x) \longrightarrow e^{ie\Lambda}\psi(x) : \text{ global gauge transf}$$

Not invariant under local gauge transf

$$\psi(x) \longrightarrow e^{ie\Lambda(x)}\psi(x) \tag{5}$$

Replace

$$\begin{aligned} \partial_\mu &\longrightarrow D_\mu = \partial_\mu + ieA_\mu \\ D_\mu\psi(x) &\longrightarrow e^{ie\Lambda(x)} D_\mu\psi(x) \end{aligned}$$

Under Eq. (5) and $A^\mu \longrightarrow A^\mu - \partial^\mu \Lambda$

$$\begin{aligned} \mathcal{L} &= \overline{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \\ &= \overline{\psi}(x)(i\gamma^\mu(\partial_\mu + ieA_\mu) - m)\psi(x) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \end{aligned}$$

invariant under local gauge transformation.

$$
\begin{aligned}
\mathcal{L}_{int} &= -eJ^{\mu}_{em}A_{\mu} \\
J^{\mu}_{em} &= \overline{\psi}(x)\gamma^{\mu}\psi(x) \\
&= -\overline{e}\gamma^{\mu}e - \overline{\mu}\gamma^{\mu}\mu - \overline{\tau}\gamma^{\mu}\tau \\
&\quad +(\frac{2}{3}\overline{u}\gamma^{\mu}u - \frac{1}{3}\overline{d}\gamma^{\mu}d) + (\frac{2}{3}\overline{c}\gamma^{\mu}c - \frac{1}{3}\overline{s}\gamma^{\mu}s) + (\frac{2}{3}\overline{t}\gamma^{\mu}t - \frac{1}{3}\overline{b}\gamma^{\mu}b) \\
\partial_{\mu}J^{\mu}_{em} &= 0
\end{aligned}
$$

Universal: irrespective of nature of particle, the strength of electromagnetic int. is determined by thee electric charge.

J^{μ}_{em} is conserved.

Associated quantum of E.M. field photon has spin 1.

Photon is massless, mass term is not allowed by gauge invariance. Photon has only transverse polarization

$$k.\epsilon = 0, \overrightarrow{k}.\overrightarrow{\epsilon} = 0$$

Standard Model

Electromagnetic interaction is mediated by photon, the quantum of electromagnetic field whose strength is determined by the electric charge. Photon spin-1 massless particle with no longitudinal polarization. Underlying gauge symmetry group U(1).

Understanding of fundamental constituents of matter and their interactions lies in discovering a gauge symmetry group.

Fundamental representation of which gives the content of fundamental fermions and adjoint representation of the group gives the mediators. The strength of interaction is determined by the gauge coupling constant.

Underlying gauge group of standard model is semi-simple group

$$\begin{array}{ccccc} SU_C(3) & \times & SU_L(2) & \times & U(1) \\ g_s & & g & & g' \end{array}$$

$$\psi_q \quad : \quad \begin{pmatrix} u_i \\ d_i' \end{pmatrix}_L : (\underline{3}, \underline{2}, 1/3)$$

$$\psi_l \quad : \quad \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L : (\underline{1}, \underline{2}, -1)$$

$$u_{iR} \quad : \quad (\underline{3}, \underline{1}, -4/3), d_{iR} : (\underline{3}, \underline{1}, 2/3)$$

$$e_{iR} \quad : \quad (\underline{1}, \underline{1}, -2), \nu_i \text{ only left-handed}$$

i : generation index

$$i = 1 \quad \begin{pmatrix} u \\ d' \end{pmatrix}, \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$$

$$i = 2 \quad \begin{pmatrix} c \\ s' \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}$$

$$i = 3 \quad \begin{pmatrix} t \\ b' \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

$SU_C(3)$: Adjoint rep $\underline{8}$: Mediators 8 gluons: carry color massless. QCD quantum chromodynamics. Interquarks forces to form a hadrons are mediated by 8 gluons.

$$\alpha_s = \frac{g_s^2}{4\pi}$$
$$\alpha_s(1.7GeV) = 0.33, \alpha_s(34GeV) = 0.14, \alpha_s(80GeV) \simeq 0.12$$

Asymtotic freedom
$SU_L(2)$: adjoint rep $\underline{3}$: Mediators isospin triplet $W_\mu^+, W_\mu^-, W_\mu^0$.
$U_Y(1)$: Mediator B_μ Hypercharge.
$SU_C(3)$: is required to cancel the anomalies for renormalization.

The electric charge is given by

$$Q = I_{3L} + \frac{1}{2}Y \tag{6}$$

Since weak forces are short range, the gauge group $SU(2) \times U(1)$ is spontaneously broken:

$$SU(2) \times U(1) \rightarrow U_{em}(1)$$

From the Eq (6):

$$\begin{aligned} \frac{A_\mu}{e} &= \frac{W_{3\mu}}{g} + \frac{B_\mu}{g'} \\ A_\mu &= \frac{e}{g} W_{3\mu} + \frac{e}{g'} B_\mu \end{aligned}$$

and the orthogonal

$$Z_\mu = \frac{e}{g'} W_{3\mu} - \frac{e}{g} B_\mu \tag{7}$$

Put

$$\begin{aligned} \frac{e}{g} &= \sin\theta_W, \quad \frac{e}{g'} = \cos\theta_W \\ W_{3\mu} &= W_\mu^0 \end{aligned}$$

Thus the mass eigenstates are

$$\begin{aligned} A_\mu &= \cos\theta_W B_\mu + \sin\theta_W W_\mu^0 \\ Z_\mu &= -\sin\theta_W B_\mu + \cos\theta_W W_\mu^0 \end{aligned}$$

W^+, W^-, Z_μ are mediators of weak interactions. A_μ: Photon mediator of electromagnetic interaction.
Masses of vector bosons and $W^\pm$ and Z are generated by spontaneous breaking of gauge symmetry (SSB). For this purpose it is necessary to introduce a self-interacting complex scalar field

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

which is a doublet under $SU_L(2)$ and has $Y = 1$. This so-called Higgs field also interacts with the chiral fermions introduced earlier as well as with gauge vector bosons, $W^\pm, W^0$ and B. The scalar field ϕ develops a non-zero vacuum expectation value:

$$\langle\phi\rangle = \langle 0|\phi|0\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

thereby breaking the gauge symmetry of the ground state $|0>$.

This amounts to rewriting

$$\phi = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(v + H + i\eta) \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + H) \end{pmatrix}$$

the would be Goldstone bosons $H^\pm$ and η have been absorbed in $W^\pm$ and Z to give them longitudinal component and mass. In contrast to the gauge invariant vertices [which are not affected by SSB], one starts with manifestly gauge invariant vertices involving ϕ and other fields and then by spontaneous breaking of of local gauge symmetry, all gauge vector bosons except photon and fermions (except neutrinos: they left-handed and have no right handed component) acquire masses. However because of mixing between W^0 and B, the physical particles are γ and Z. This requires diagonalization of the mass matrix for $W_3 - B$ sectors, after diagonalization:

$$m_W = \frac{1}{2} g v, \, m_A = 0, \quad m_Z = \frac{m_W}{\cos\theta_W} = \frac{1}{2}\sqrt{g^2 + g'^2}\, v$$

Further we note from the above picture that the mass of a fermion of flavor f and that of Higgs particle H are respectively given by

$$m_f = \frac{h_f v}{\sqrt{2}}, \quad m_H = \sqrt{2v^2\lambda}$$

Thus the fermions masses and Higgs boson mass are free parameters in the standard model and

$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2}$$

For $q^2 << m_W^2$, the gauge coupling constant g in terms of G_F is given by

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} = \frac{e^2}{8m_W^2\sin^2\theta_W}$$

Thus

$$m_W = \left[\frac{\pi\alpha}{\sqrt{2}G_F\sin^2\theta_W}\right]^{1/2}$$

The mixing angle $sin^2\theta_W$ is a free parameter. From diverse experiments, the value of $sin^2\theta_W \approx 0.23$, showing the consistency of the unification model. With this value of $sin^2\theta_W$ and $\alpha = \frac{1}{137}$, one gets $m_W \approx 77.75$GeV, $m_Z \approx 8.860$ to be compared with the experimental values $m_W = 80.399 \pm 0.023$, $m_Z = 91.1876 \pm 0.0021 GeV$. However, since the measurements are made at m_Z, it is appropriate to use $\alpha(m_Z) \approx \frac{1}{128.28}$. With this value of α: $m_W \approx 80.35 GeV$, $m_Z \approx 91.56$GeV close to the experimental values for these bosons. After radiative corrections mass of $m_W = 80.39 GeV$, $m_Z = 91.8 GeV$, in remarkable agreement with the experimental values.

However, the missing link is the Higgs boson. The Higgs mass is arbitrary, the best limits on Higgs mass are

$$130 GeV < m_H < 180 GeV$$

Finally, the electroweak unification scale is given by
$v = \frac{2m_W}{g} = (\sqrt{2}G_F)^{-1/2} \approx 246 GeV$.
The year 1978 saw a remarkable set of experiments, confirming the existence of neutral weak interaction as predicted by the electroweak theory.
Since the weak and electromagnetic interactions are different manifestation of the same force and since weak interactions can distinguish between left and right; one should be able to see this minute effect in the scattering of polarized electrons on nuclei. This effect was detected experimentally; further confirming the theory. The crowning verification of electroweak theory was achieved in 1980's, when three spin 1 particles W, Z with mass 80-90 times proton mass were discovered at CERN collider experiments in Geneva. Recent discovery of a spin zero particle at CERN near 125 GeV is land mark discovery-providing the last missing link of electroweak theory.

Conclusion Remark: Weinberg

"Higgs is the one elementary particle whose mass does not arise from the breakdown of electorweak symmetry. As far as underlying principles of electroweak theory are concerned, the Higgs mass could have any value This is why neither Salam nor I could predict it. "

Except its mass all other properties of Higgs (for example its decays) can be calculated by electroweak theory.

Still there is a problem called "hierarchy problem" why Higgs mass that sets the scale of masses of all other known particles so small as compared to Planck mass which is the fundamental unit of mass in theroy of gravitation.

Gravitational force is as strong as electromagnetic force at Planck mass and weak force is as strong as electromagnetic force at electroweak unification scale. Planck mass:

$$M_P = \sqrt{\frac{\hbar c}{G_N}} = 10^{19}\, Gev/c^2$$

$$\alpha_{G_N} = \frac{1\, GeV^2}{M_P^2}\alpha \approx 10^{-38}\alpha \approx 10^{-40}$$

$$\sqrt{\frac{(\hbar c)^3}{G_F}} \approx 300 \quad GeV$$

$$\alpha_W = \frac{(1\, GeV)^2}{(300\, GeV)^2} \approx 10^{-7}.$$