QCD-like Approach (LσM) for Heavy-Ion Collisions

ABDEL NASSER TAWFIK

Egyptian Center for Theoretical Physics (ECTP),
World Laboratory for Cosmology And Particle Physics (WLCAPP)
Modern University for Technology and Information (MTI Univ.)

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http://atawfik.net/

Roots

Sigma-Model is a Physical system with the Lagrangian

$$\mathcal{L}(\phi_1, \phi_2, \ldots, \phi_n) = \sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}\, \mathrm{d}\phi_i \wedge *\mathrm{d}\phi_j$$

The fields φ_i represent **map** from a **base manifold** called worldsheet (spacetime) to a **target** (Riemannian) **manifold** of the scalars linked together by internal symmetries.

The scalars g_{ij} determines linear and non-linear properties.

It was introduced by **Gell-Mann** and **Levy** in **1960**. The name **σ-model** comes from a field in their model corresponding to a spinless meson ***σ***, a scalar introduced earlier by Schwinger.

Symmetries

- **Pisarski** and **Wilczek** discussed the order of the chiral transition using renormalization group arguments in the framework of **LσM**.
- **LσM** is the effective theory for the low-energy degrees of freedom of QCD and incorporates the global $SU(Nf)_r \times SU(Nf)_\ell \times$ **$U(1)_A$** symmetry, but not the local $SU(3)_c$ color symmetry.

- They found that, for Nf = 2 flavors of massless quarks, the transition can be of second order, if the **$U(1)_A$** symmetry is explicitly broken by instantons.
- It is driven first order by fluctuations, if the **$U(1)_A$** symmetry is restored at Tc.
- For Nf = 3 massless flavors, the transition is always first order. In this case, the term which breaks the **$U(1)_A$** symmetry explicitly is a cubic invariant, and consequently drives the transition first order.
- In the absence of explicit **$U(1)_A$** symmetry breaking, the transition is fluctuation-induced of first order.

R.D. Pisarski and F. Wilczek, Phys. Rev. D 29, 338 (1984).

Importance of LσM

- LσM is an alternative to lattice QCD.
- Various symmetry-breaking scenarios can be more easily investigated in LσM.
- But, finite-T LσM requires many-body resummation schemes, because infrared divergences cause naive perturbation theory to break down.
- Various properties of strongly interacting matter can be studied

Symmetries

- For Nf massless quark flavors, the QCD Lagrangian has a $SU(Nf)_r \times SU(Nf)_\ell \times \mathbf{U(1)_A}$ symmetry.
- In vacuum, a non-vanishing expectation value of the quark-antiquark condensate, spontaneously breaks this symmetry to the diagonal $SU(Nf)_V$ group of vector transformations, $V = r + \ell$.
- For Nf = 3, the effective, low-energy degrees of freedom of QCD are the **scalar** and **pseudoscalar mesons**. Since mesons are quark-antiquark states, they fall in singlet and octet representations of SU(3)V .

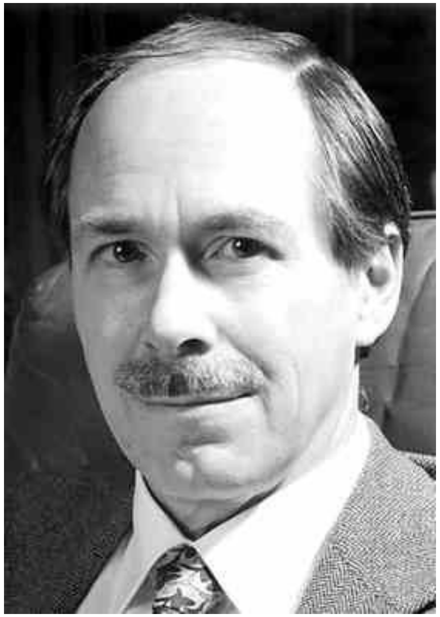
- The $SU(Nf)_r \times SU(Nf)_\ell \times \mathbf{U(1)_A}$ symmetry of QCD Lagrangian is explicitly broken by nonzero quark masses.
- For $M \leq Nf$ degenerate quark flavors, a $SU(M)_V$ symmetry is preserved.
- If $M < Nf$, the mass eigenstates are mixtures of singlet and octet states.

Jonathan T. Lenaghan,, Dirk H. Rischke, Jurgen Schaffner-Bielich, **Phys.Rev. D62 (2000) 085008**

Symmetries

According to 't Hooft , the instantons also break the $U(1)_A$ symmetry explicitly to $Z(Nf)_A$.

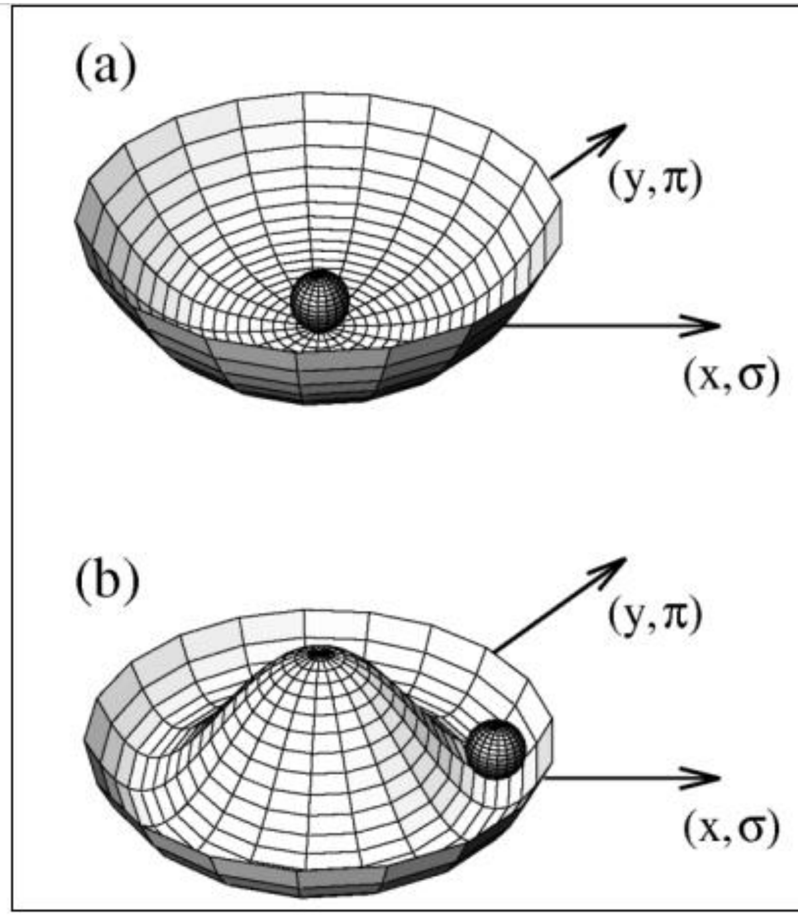
This discrete symmetry is irrelevant for the low-energy dynamics of QCD ➔ LσM has a $SU(Nf)r \times SU(Nf)\ell$ symmetry.



Chiral Symmetry

A great challenge trying to understand the processes, which led to the creation of the physical world around us.

Main goals of modern nuclear physics: investigation of hadron properties, (effective masses, decay widths, electromagnetic form factors etc.), inside nuclear matter under extreme conditions of high pressure and temperature.



di-lepton production in hot and dense medium ➔signals of the partial restoration of the chiral symmetry of QCD.

Chiral Symmetry

SYMMETRIES ALWAYS IMPLY CONSERVATION LAWS:

INVARIANCE OF LAGRANGIAN UNDER TRANSLATIONS IN SPACE AND TIME → MOMENTUM AND ENERGY CONSERVATION.

QCD LAGRANGIAN FOR MASSLESS QUARKS SHOWS A SYMMETRY UNDER VECTOR AND AXIAL TRANSFORMATION.

left- and right-handed parts treated EQUALLY (VECTOR)

DIFFERENTLY (AXIAL)

THIS IS THE CHIRAL SYMMETRY.

SYMMETRY OF VECTOR TRANSFORMATIONS LEADS TO ISOSPIN CONSERVATION.

Transformation

Chiral symmetry of vector field under unitary transformation $\vec{\Phi} \Longrightarrow e^{-i\ \theta^a\ T^a_{ij}}\ \vec{\Phi}$

θ^a corresponding the rotational angle, T^a_{ij} matrix generates the transformation and a index indicating several generators associated with the symmetry transformation.

Vector transformation Λ_V

$$\Psi \Longrightarrow e^{-i\ \frac{\vec{\tau}}{2}\vec{\theta}}\ \Psi \approx (1 - i\frac{\vec{\tau}}{2}\vec{\theta})\ \Psi$$

$$\bar{\Psi} \Longrightarrow e^{+i\ \frac{\vec{\tau}}{2}\vec{\theta}}\ \bar{\Psi} \approx (1 + i\frac{\vec{\tau}}{2}\vec{\theta})\ \bar{\Psi}$$

conjugate

Axil transformation Λ_A

$$\Psi \Longrightarrow e^{-i\gamma_5\ \frac{\vec{\tau}}{2}\vec{\theta}}\ \Psi \approx (1 - i\gamma_5\frac{\vec{\tau}}{2}\vec{\theta})\ \Psi$$

$$\bar{\Psi} \Longrightarrow e^{-i\ \gamma_5\frac{\vec{\tau}}{2}\vec{\theta}}\ \bar{\Psi} \approx (1 - i\gamma_5\frac{\vec{\tau}}{2}\vec{\theta})\ \bar{\Psi}$$

Fermions **Dirac Lagrangian which describes the free Fermion particle of mass m given by**

$$\mathcal{L}_D = \bar{\psi}(i\gamma_\mu\partial^\mu - m^2)\psi$$

Under vector transformation Λ_V is invariant BUT for Axial transformation Λ_A

$$\Lambda_A : \quad m\ \bar{\psi}\ \psi \Longrightarrow e^{-i\ \gamma^5\frac{\vec{\tau}}{2}\vec{\theta}}\ m\ \bar{\psi}\ \psi \approx (1 - i\gamma^5\frac{\vec{\tau}}{2}\vec{\theta})\ m\ \bar{\psi}\ \psi,$$

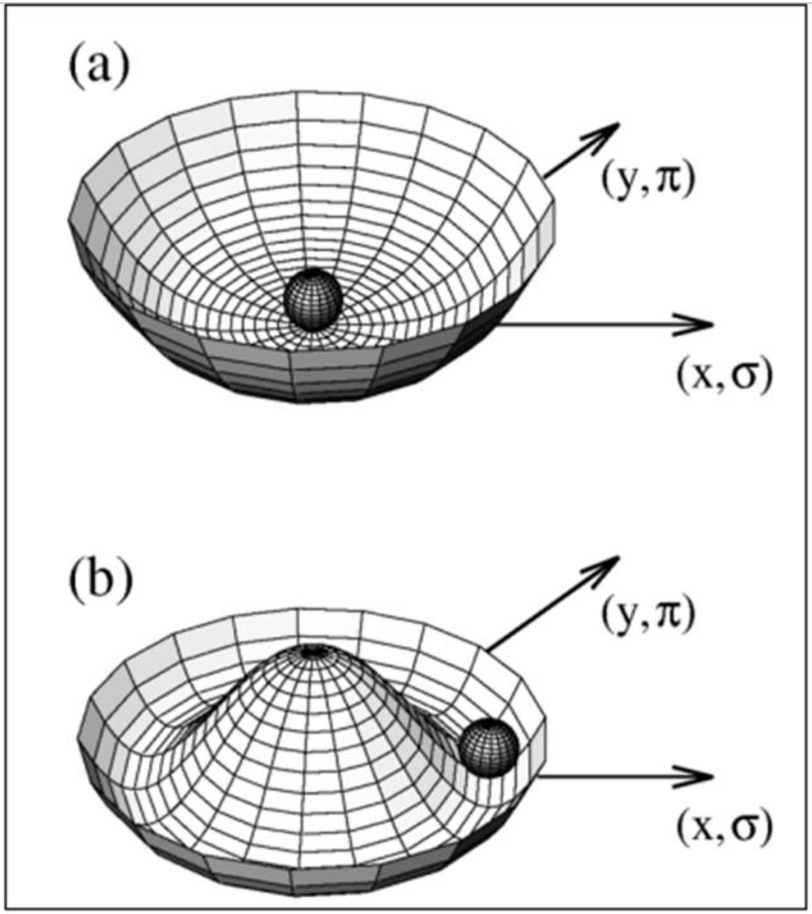
$$= m\ \bar{\psi}\ \psi - 2im\bar{\theta}(\bar{\psi}\gamma_5\frac{\vec{\tau}}{2}\psi)$$

ϕ are component fields such as π's

Symmetry Breaking

Spontaneous symmetry breaking

If a symmetry of the Lagrangian is not realized in the ground state.



The ground state is right in the middle (0,0) and the potential plus ground state are still invariant under rotations

The ground state is at a finite distance away from the center. The point at the center is a local maximum of the potential and thus unstable

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Sigma fields

Combination of quarks (q# of mesons), a meson-like state

(scalar Meson)
Sigma like state $J^p = 0^+$

(pseudoscalar Meson)
Pion like state $J^p = 0^-$

$$\sigma = \bar{\psi}\psi$$

$$\pi = i\bar{\psi}\tau\gamma_5\psi$$

Gell-Mann & Levy **obtained an invariant form if squares of the two states are summed**

(Vector Meson)
Sigma like state $J^p = 0^+$

(Axial-Vector Meson)
Pion like state $J^p = 0^-$

$$\Lambda_V: \quad \pi^2 \to \pi^2, \quad \sigma^2 \to \sigma^2$$

$$\Lambda_A: \quad \pi^2 \to \pi^2 + 2\sigma\theta\pi, \quad \sigma^2 \to \sigma^2 - 2\sigma\theta\pi$$

$$(\pi^2 + \sigma^2) \xleftrightarrow{\Lambda_V, \Lambda_A} (\pi^2 + \sigma^2)$$

Vector and Axial-Vector transformation

Vector transformation

$$\pi_i: \quad i\bar{\psi}\bar{\tau}\gamma_5\psi \longrightarrow i\bar{\psi}\bar{\tau}\gamma_5\psi + \theta_j\left[\bar{\psi}\tau_i\gamma_5\frac{\tau_j}{2}\psi - \bar{\psi}\frac{\tau_j}{2}\tau_i\gamma_5\psi\right]$$
$$= \quad i\bar{\psi}\bar{\tau}\gamma_5\psi + i\theta\epsilon_{ijk}\bar{\psi}\gamma_5\tau_k\psi,$$

Vector transformation

$$\left[\tau_i, \tau_j\right] = 2i\epsilon_{ijk}\tau_k$$

Levi-Civta Symbols

$$\epsilon_{ijk} = \begin{cases} +1 & for\ even\ permutation\ 1\,2\,3\,, \\ -1 & for\ odd\ permutation\ 1\,2\,3\,, \\ 0 & Otherwise \end{cases}$$

$$\bar{\pi} \longrightarrow \bar{\pi} + \epsilon_{ijk}\bar{\theta}\bar{\pi}_k$$

Axial-Vector transformation

$$\pi_i : \qquad i\bar{\psi}\bar{\tau}\gamma_5\psi \longrightarrow i\bar{\psi}\bar{\tau}\gamma_5\psi + \theta_j \left[\bar{\psi}\tau_i\gamma_5\gamma_5\frac{\tau_j}{2}\psi - \bar{\psi}\gamma_5\frac{\tau_j}{2}\gamma_5\tau_i\psi\right]$$
$$= \quad i\bar{\psi}\bar{\tau}\gamma_5\psi + \theta_j\bar{\psi}\psi\delta_{ij},$$

$\gamma_5\gamma_5 = 1$ and the commutation relation between matrices

$$[\tau_i, \tau_j] = 2\delta_{ij}$$

$$\delta_{ij} = \begin{cases} +1 & for\; i = j, \\ 0 & for\; i \neq j \end{cases}$$

$$\bar{\pi} \longrightarrow \bar{\pi} + \theta\bar{\pi}$$

Interactions

Pion-Nucleon Interaction involves pseudo-scalar combination of nucleon field multiplied by pion field

$$g_\pi (i\bar{\psi}\gamma_5\bar{\tau})\bar{\pi}$$

Where gπ is pion-nucleon coupling constant. The chiral invariance requires another term transforming sigma

$$g_\pi (i\bar{\psi}\psi)\sigma$$

Therefore, the interaction between nucleon and meson

$$\delta L = -g_\pi \left[(i\bar{\psi}\gamma_5\bar{\tau})\bar{\pi} + (i\bar{\psi}\psi)\sigma\right]$$

Nucleon Mass

- The nucleon mass breaks the chiral symmetry, explicitly.
- The simplest way to include nucleon mass and keeping chiral symmetry unbroken is to explode the coupling of the nucleon $g\pi$,

$$<\sigma> = \sigma_0 = f_\pi$$

We have to introduce a potential for sigma field with min. at $\sigma = f\pi$

$$V = V(\bar{\pi}^2, \sigma^2) = \frac{\lambda}{4}\left((\bar{\pi}^2 + \sigma^2) - f_\pi^2\right)$$

Lagrangian

The kinetic energy term for

nucleons **and** **mesons**

$$i\bar{\psi}\gamma_\mu\partial_\mu\psi$$

$$\frac{1}{2}\left(\partial_\mu\pi\partial^\mu\pi + \partial_\mu\sigma\partial^\mu\sigma\right)$$

The *LσM* Lagrangian

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial_\mu\psi + \frac{1}{2}(\partial_\mu\pi\partial^\mu\pi + \partial_\mu\sigma\partial^\mu\sigma) - g_\pi\left[(i\bar{\psi}\gamma_5\bar{\tau}\psi)\bar{\pi} + (i\bar{\psi}\psi)\sigma\right] - \frac{\lambda}{4}\left((\bar{\pi}^2 + \sigma^2) - f_\pi^2\right)$$

K. E Of nucleons

K. E Of Mesons

interaction term between nucleons and the mesons

Pion nucleon Potential

Nucleon mass term

The chiral part of LσM-Lagrangian has $SU(3)_R \times SU(3)_L$ symmetry

$$\mathcal{L}_{chiral} = \mathcal{L}_q + \mathcal{L}_m$$

where fermionic part

$$\mathcal{L}_q = \sum_f \overline{\psi}_f (i\gamma^\mu D_\mu - g T_a(\sigma_a + i\gamma_5 \pi_a))\psi_f$$

and mesonic part

$$\begin{aligned}\mathcal{L}_m = & \mathrm{Tr}(\partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2 \Phi^\dagger \Phi) - \lambda_1 [\mathrm{Tr}(\Phi^\dagger \Phi)]^2 \\ & - \lambda_2 \mathrm{Tr}(\Phi^\dagger \Phi)^2 + c[\mathrm{Det}(\Phi) + \mathrm{Det}(\Phi^\dagger)] \\ & + \mathrm{Tr}[H(\Phi + \Phi^\dagger)],\end{aligned}$$

- m^2 is tree-level mass of the fields in the absence of symmetry breaking
- λ_1 *and* λ_2 are the two possible quartic coupling constants,
- c is the cubic coupling constant,
- g flavor-blind Yukawa coupling of quarks to mesons and of quarks to background gauge field $A_\mu = \delta_{\mu 0} A_0$

$c = 4.80;$
$g = 6.5;$
$\lambda_1 = 5.90;$
$\lambda_2 = 46.48;$
$m^2 = (0.495)$^2;

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ϕ is a complex 3×3 matrix and parameterizing scalar σ_a and pseudoscalar π_a (nonets) mesons

$$\Phi = T_a \, \phi_a = T_a \, (\sigma_a + i\pi_a)$$

where σ_a are the scalar fields and π_a are the pseudoscalar fields. The *3 × 3* matrix *H* breaks the symmetry explicitly and is chosen as

$$H = T_a \, h_a$$

where h_a are nine external fields and $T_a = \hat{\lambda}_a /2$ are generators of U(3) with $\hat{\lambda}_a$ Are Gell-Mann matrices $\hat{\lambda}_0 = \sqrt{\frac{2}{3}}\,\mathbf{1}$

The T_a are normalized such that $\mathrm{Tr}(T_a T_b) = \delta_{ab}/2$ and obey the $U(3)$

$$[T_a, T_b] = i\, f_{abc} T_c \; ,$$
$$\{T_a, T_b\} = d_{abc} \, T_c \; ,$$

where f_{abc} and d_{abc} for $a, b, c = 1, \ldots, 8$ are the standard antisymmetric and symmetric structure constants of $SU(3)$ and

$$f_{ab0} \equiv 0 \quad , \quad d_{ab0} \equiv \sqrt{\frac{2}{3}}\, \delta_{ab}$$

m^2 is the tree-level mass of fields in absence of symmetry breaking, λ_1 and λ_2 are quadratic couplings while c is the cubic coupling. In 4D, these couplings are only relevant $SU(3)_r \times SU(3)_\ell$ invariant operators.

Terms in 1st line of mesonic part are invariant under $SU(3)_r \times SU(3)_\ell$ symmetry transformations

$$\Phi \longrightarrow U_r \, \Phi \, U_\ell^\dagger \;\; , \quad U_{r,\ell} \equiv \exp\left(i \, \omega_{r,\ell}^a \, T^a\right)$$

Introducing $\omega^a_{V,A} \equiv (\omega^a_r \pm \omega^a_\ell)/2$, the right- and left-handed symmetry transformations can be alternatively written as vector, $V = r + \ell$, and axial vector, $A = r - \ell$, transformations.

Φ is a singlet under $U(1)_V$ transformations $\exp(i\,\omega_V^0 T^0)$

where $U(1)_V$ is the U(1) of baryon # conservation and thus always respected.

The terms in last line of mesonic part are therefore invariant under

$$SU(3)_r \times SU(3)_\ell \times U(1)_A \cong SU(3)_V \times U(3)_A$$

and break the axial and possibly the $SU(3)_V$ vector symmetries, explicitly.

Lagrangian of Linear Sigma Model

generators of the U(3) symmetry are $T_a = \lambda_a/2$ where λ_a are Gell-Mann matrices with $\lambda_0 = \sqrt{\frac{2}{3}}\mathbf{I}$

$$\hat{\lambda}_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{\lambda}_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{\lambda}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{\lambda}_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\hat{\lambda}_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \hat{\lambda}_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \hat{\lambda}_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \hat{\lambda}_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

As required λ_a span all traceless Hermitian matrices. They follow

$$[T_a, T_b] = i\sum_{c=1}^{8} f_{abc}T_c \{T_a, T_b\} = \frac{1}{3}\delta_{ab} + \sum_{c=1}^{8} d_{abc}T_c$$

where f are structure constant given by

$$f_{123} = 1 f_{147} = -f_{156} = f_{246} = f_{257} = f_{345} = -f_{367} = \frac{1}{2} f_{458} = f_{678} = \frac{\sqrt{3}}{2},$$

$$d_{118} = d_{228} = d_{338} = -d_{888} = \frac{1}{\sqrt{3}} d_{448} = d_{558} = d_{668} = d_{778} = -\frac{1}{2\sqrt{3}}$$

$$H=\begin{pmatrix}\sqrt{\frac{2}{3}}h0+h3+\frac{h8}{\sqrt{3}} & h1-ih2 & h4-ih5\\ h1+ih2 & \sqrt{\frac{2}{3}}h0-h3+\frac{h8}{\sqrt{3}} & h6-ih7\\ h4+ih5 & h6+ih7 & \sqrt{\frac{2}{3}}h0-2\frac{h8}{\sqrt{3}}\end{pmatrix};$$

$$T_a\sigma_a = \frac{1}{\sqrt{2}}\begin{pmatrix}\frac{1}{\sqrt{2}}a_0^0+\frac{1}{\sqrt{6}}\sigma_8+\frac{1}{\sqrt{3}}\sigma_0 & a_0^- & \kappa^-\\ a_0^+ & -\frac{1}{\sqrt{2}}a_0^0+\frac{1}{\sqrt{6}}\sigma_8+\frac{1}{\sqrt{3}}\sigma_0 & \bar{\kappa}^0\\ \kappa^+ & \kappa^0 & -\frac{2}{\sqrt{3}}\sigma_8+\frac{1}{\sqrt{3}}\sigma_0\end{pmatrix},$$

$$T_a\pi_a = \frac{1}{\sqrt{2}}\begin{pmatrix}\frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\pi_8+\frac{1}{\sqrt{3}}\pi_0 & \pi^- & K^-\\ \pi^+ & -\frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\pi_8+\frac{1}{\sqrt{3}}\pi_0 & \bar{K}^0\\ K^+ & K^0 & -\frac{2}{\sqrt{3}}\pi_8+\frac{1}{\sqrt{3}}\pi_0\end{pmatrix}.$$

$\pi^{\pm} \equiv (\pi_1 \pm i\,\pi_2)/\sqrt{2}$ and $\pi^0 \equiv \pi_3$ charged and neutral pions, respectively

$$K^{\pm} \equiv (\pi_4 \pm i\,\pi_5)/\sqrt{2},$$
$$K^{0} \equiv (\pi_6 + i\,\pi_7)/\sqrt{2}$$
$$\bar{K}^{0} \equiv (\pi_6 - i\,\pi_7)/\sqrt{2}$$

are Kaons

π_0 and the π_8 are η and the η' meson

Other nonets can be generated, for instance, the parity partner of pions

$$a_0^{\pm} \equiv (\sigma_1 \pm i\,\sigma_2)/\sqrt{2} \text{ and } a_0^0 \equiv \sigma_3.$$

Symmetry breaking gives the Φ field a vacuum expectation value

$$\langle \Phi \rangle \equiv T_a\,\bar{\sigma}_a$$

Shifting the Φ field by this vacuum expectation value, the Lagrangian reads

$$\mathcal{L} = \frac{1}{2}\left[\partial_\mu \sigma_a \partial^\mu \sigma_a + \partial_\mu \pi_a \partial^\mu \pi_a - \sigma_a (m_S^2)_{ab} \sigma_b - \pi_a (m_P^2)_{ab} \pi_b\right] + \left(\mathcal{G}_{abc} - \frac{4}{3}\mathcal{F}_{abcd}\,\bar{\sigma}_d\right)\sigma_a\sigma_b\sigma_c - 3\left(\mathcal{G}_{abc} + \frac{4}{3}\mathcal{H}_{abcd}\,\bar{\sigma}_d\right)\pi_a\pi_b\sigma_c - 2\,\mathcal{H}_{abcd}\,\sigma_a\sigma_b\pi_c\pi_d - \frac{1}{3}\mathcal{F}_{abcd}\left(\sigma_a\sigma_b\sigma_c\sigma_d + \pi_a\pi_b\pi_c\pi_d\right) - U(\bar{\sigma})\ ,$$

where the tree-level potential is

$$U(\bar{\sigma}) = \frac{m^2}{2}\,\bar{\sigma}_a^2 - \mathcal{G}_{abc}\bar{\sigma}_a\bar{\sigma}_b\bar{\sigma}_c + \frac{1}{3}\mathcal{F}_{abcd}\bar{\sigma}_a\bar{\sigma}_b\bar{\sigma}_c\bar{\sigma}_d - h_a\bar{\sigma}_a$$

$\bar{\sigma}_a$ is determined from

$$\frac{\partial U(\bar{\sigma})}{\partial \bar{\sigma}_a} = m^2\,\bar{\sigma}_a - 3\,\mathcal{G}_{abc}\bar{\sigma}_b\bar{\sigma}_c + \frac{4}{3}\mathcal{F}_{abcd}\bar{\sigma}_b\bar{\sigma}_c\bar{\sigma}_d - h_a = 0$$

coefficients $\mathcal{G}_{abc}$, $\mathcal{F}_{abcd}$, and $\mathcal{H}_{abcd}$ are given by

$$\mathcal{G}_{abc} = \frac{c}{6}\left[d_{abc} - \frac{3}{2}\left(\delta_{a0}d_{0bc} + \delta_{b0}d_{a0c} + \delta_{c0}d_{ab0}\right) + \frac{9}{2}d_{000}\delta_{a0}\delta_{b0}\delta_{c0}\right] ,$$

$$\mathcal{F}_{abcd} = \frac{\lambda_1}{4}\left(\delta_{ab}\delta_{cd} + \delta_{ad}\delta_{bc} + \delta_{ac}\delta_{bd}\right) + \frac{\lambda_2}{8}\left(d_{abn}d_{ncd} + d_{adn}d_{nbc} + d_{acn}d_{nbd}\right)$$

$$\mathcal{H}_{abcd} = \frac{\lambda_1}{4}\delta_{ab}\delta_{cd} + \frac{\lambda_2}{8}\left(d_{abn}d_{ncd} + f_{acn}f_{nbd} + f_{bcn}f_{nad}\right) .$$

where

tree-level masses, $(m_S^2)_{ab}$ and $(m_P^2)_{ab}$ are given by

$$(m_S^2)_{ab} = m^2\,\delta_{ab} - 6\,\mathcal{G}_{abc}\,\bar{\sigma}_c + 4\,\mathcal{F}_{abcd}\,\bar{\sigma}_c\bar{\sigma}_d$$
$$(m_P^2)_{ab} = m^2\,\delta_{ab} + 6\,\mathcal{G}_{abc}\,\bar{\sigma}_c + 4\,\mathcal{H}_{abcd}\,\bar{\sigma}_c\bar{\sigma}_d$$

The masses are not diagonal, thus σ_a and π_a fields are not mass generators in standard basis of SU(3). As, the mass matrices are symmetric and real, diagonalization is achieved by an orthogonal transformation

$$\tilde{\sigma}_i = O_{ia}^{(S)}\,\sigma_a ,$$
$$\tilde{\pi}_i = O_{ia}^{(P)}\,\pi_a ,$$
$$\left(\tilde{m}_{S,P}^2\right)_i = O_{ai}^{(S,P)}\,\left(m_{S,P}^2\right)_{ab}\,O_{bi}^{(S,P)}$$

The expectation values $\langle \Phi \rangle = T_0 \, \bar{\sigma}_0 + T_8 \, \bar{\sigma}_8$

where

$$h_0 = \left[m^2 - \frac{c}{\sqrt{6}} \, \bar{\sigma}_0 + \left(\lambda_1 + \frac{\lambda_2}{3} \right) \bar{\sigma}_0^2 \right] \bar{\sigma}_0 + \left[\frac{c}{2\sqrt{6}} + (\lambda_1 + \lambda_2) \, \bar{\sigma}_0 - \frac{\lambda_2}{3\sqrt{2}} \, \bar{\sigma}_8 \right] \bar{\sigma}_8^2$$

$$h_8 = \left[m^2 + \frac{c}{\sqrt{6}} \, \bar{\sigma}_0 + \frac{c}{2\sqrt{3}} \, \bar{\sigma}_8 + (\lambda_1 + \lambda_2) \, \bar{\sigma}_0^2 - \frac{\lambda_2}{\sqrt{2}} \, \bar{\sigma}_0 \, \bar{\sigma}_8 + \left(\lambda_1 + \frac{\lambda_2}{2} \right) \bar{\sigma}_8^2 \right] \bar{\sigma}_8$$

From PCAC relations

$$\bar{\sigma}_0 = \frac{f_\pi + 2 \, f_K}{\sqrt{6}} \, , \qquad f_\pi = 92.4 \text{ MeV}, \; f_K = 113 \text{ MeV}$$

$$\bar{\sigma}_8 = \frac{2}{\sqrt{3}} \, (f_\pi - f_K)$$

The Scalar masses

$$
\begin{aligned}
\left(m_S^2\right)_{00} &= m^2 - \sqrt{\frac{2}{3}}\, c\,\bar{\sigma}_0 + (3\lambda_1 + \lambda_2)\,\bar{\sigma}_0^2 + (\lambda_1 + \lambda_2)\,\bar{\sigma}_8^2 \;, \\
\left(m_S^2\right)_{11} &= \left(m_S^2\right)_{22} = \left(m_S^2\right)_{33} \\
&= m^2 + \frac{c}{\sqrt{6}}\,\bar{\sigma}_0 - \frac{c}{\sqrt{3}}\,\bar{\sigma}_8 + (\lambda_1 + \lambda_2)\,\bar{\sigma}_0^2 + \sqrt{2}\,\lambda_2\,\bar{\sigma}_0\,\bar{\sigma}_8 + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\bar{\sigma}_8^2 \\
\left(m_S^2\right)_{44} &= \left(m_S^2\right)_{55} = \left(m_S^2\right)_{66} = \left(m_S^2\right)_{77} \\
&= m^2 + \frac{c}{\sqrt{6}}\,\bar{\sigma}_0 + \frac{c}{2\sqrt{3}}\,\bar{\sigma}_8 + (\lambda_1 + \lambda_2)\,\bar{\sigma}_0^2 - \frac{\lambda_2}{\sqrt{2}}\,\bar{\sigma}_0\,\bar{\sigma}_8 + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\bar{\sigma}_8^2 \\
\left(m_S^2\right)_{88} &= m^2 + \frac{c}{\sqrt{6}}\,\bar{\sigma}_0 + \frac{c}{\sqrt{3}}\,\bar{\sigma}_8 + (\lambda_1 + \lambda_2)\,\bar{\sigma}_0^2 - \sqrt{2}\,\lambda_2\,\bar{\sigma}_0\,\bar{\sigma}_8 + 3\left(\lambda_1 + \frac{\lambda_2}{2}\right)\bar{\sigma}_8^2 \\
\left(m_S^2\right)_{08} &= \left(m_S^2\right)_{80} = \left[\frac{c}{\sqrt{6}} + 2\,(\lambda_1 + \lambda_2)\,\bar{\sigma}_0 - \frac{\lambda_2}{\sqrt{2}}\,\bar{\sigma}_8\right]\bar{\sigma}_8 \;.
\end{aligned}
$$

$$
\begin{aligned}
m_\sigma^2 &\equiv (\tilde{m}_S^2)_0 = (m_S^2)_{00}\,\cos^2\theta_S + (m_S^2)_{88}\,\sin^2\theta_S + 2\,(m_S^2)_{08}\,\cos\theta_S\,\sin\theta_S \\
m_{f_0}^2 &\equiv (\tilde{m}_S^2)_8 = (m_S^2)_{00}\,\sin^2\theta_S + (m_S^2)_{88}\,\cos^2\theta_S - 2\,(m_S^2)_{08}\,\cos\theta_S\,\sin\theta_S
\end{aligned}
$$

where

$$
\tan 2\theta_S = \frac{2\,(m_S^2)_{08}}{(m_S^2)_{00} - (m_S^2)_{88}}
$$

The Pseudo-Scalar masses

$$
\begin{aligned}
(m_P^2)_{00} &= m^2 + \sqrt{\frac{2}{3}}\, c\,\bar{\sigma}_0 + \left(\lambda_1 + \frac{\lambda_2}{3}\right)\left(\bar{\sigma}_0^2 + \bar{\sigma}_8^2\right) \;, \\
(m_P^2)_{11} &= (m_P^2)_{22} = (m_P^2)_{33} \\
&= m^2 - \frac{c}{\sqrt{6}}\,\bar{\sigma}_0 + \frac{c}{\sqrt{3}}\,\bar{\sigma}_8 + \left(\lambda_1 + \frac{\lambda_2}{3}\right)\bar{\sigma}_0^2 + \frac{\sqrt{2}}{3}\,\lambda_2\,\bar{\sigma}_0\,\bar{\sigma}_8 + \left(\lambda_1 + \frac{\lambda_2}{6}\right)\bar{\sigma}_8^2 \;, \\
(m_P^2)_{44} &= (m_P^2)_{55} = (m_P^2)_{66} = (m_P^2)_{77} \\
&= m^2 - \frac{c}{\sqrt{6}}\,\bar{\sigma}_0 - \frac{c}{2\sqrt{3}}\,\bar{\sigma}_8 + \left(\lambda_1 + \frac{\lambda_2}{3}\right)\bar{\sigma}_0^2 - \frac{\lambda_2}{3\sqrt{2}}\,\bar{\sigma}_0\,\bar{\sigma}_8 + \left(\lambda_1 + \frac{7}{6}\,\lambda_2\right)\bar{\sigma}_8^2 \;, \\
(m_P^2)_{88} &= m^2 - \frac{c}{\sqrt{6}}\,\bar{\sigma}_0 - \frac{c}{\sqrt{3}}\,\bar{\sigma}_8 + \left(\lambda_1 + \frac{\lambda_2}{3}\right)\bar{\sigma}_0^2 - \frac{\sqrt{2}}{3}\,\lambda_2\,\bar{\sigma}_0\,\bar{\sigma}_8 + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\bar{\sigma}_8^2 \;, \\
(m_P^2)_{08} &= (m_P^2)_{80} = \left[-\frac{c}{\sqrt{6}} + \frac{2}{3}\,\lambda_2\,\bar{\sigma}_0 - \frac{\lambda_2}{3\sqrt{2}}\,\bar{\sigma}_8\right]\bar{\sigma}_8 \;.
\end{aligned}
$$

$$
\begin{aligned}
m_{\eta'}^2 \equiv (\tilde{m}_P^2)_0 &= (m_P^2)_{00}\,\cos^2\theta_P + (m_P^2)_{88}\,\sin^2\theta_P + 2\,(m_P^2)_{08}\,\cos\theta_P\,\sin\theta_P \;, \\
m_\eta^2 \equiv (\tilde{m}_P^2)_8 &= (m_P^2)_{00}\,\sin^2\theta_P + (m_P^2)_{88}\,\cos^2\theta_P - 2\,(m_P^2)_{08}\,\cos\theta_P\,\sin\theta_P \;,
\end{aligned}
$$

$$
\tan 2\theta_P = \frac{2\,(m_P^2)_{08}}{(m_P^2)_{00} - (m_P^2)_{88}}
$$

- The chiral effective models is not able to describe the effects of gluonic degrees of freedom in QCD.
- The lack of confinement in these models results in a non-zero quark number density even in the confined phase.
- The Polyakov loop can incorporate these effects in the coupling of these models, explicitly.
- The functional form of the potential is motivated by the QCD symmetries of in the pure gauge limit.
- Polyakov loop potential produces a first-order transition in the pure gauge limit with *Nc* = 3 colors.

$$\frac{\mathcal{U}(\phi,\phi^*,T)}{T^4} = -\frac{b_2(T)}{2}|\phi|^2 - \frac{b_3}{6}(\phi^3+\phi^{*3}) + \frac{b_4}{4}(|\phi|^2)^2,$$

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3.$$

$a_0 = 6.75,$	$a_1 = -1.95,$	$a_2 = 2.625,$	$a_3 = -7.44$
	$b_3 = 0.75$	$b_4 = 7.5$	

The thermal expectation value of a color traced Wilson loop in the temporal direction determines the Polyakov-loop effective potential

$$\Phi(\vec{x}) = \frac{1}{N_c}\langle \mathcal{P}(\vec{x})\rangle,$$

Polyakov-loop potential and its conjugate

$$\phi = (\mathrm{Tr}_c \mathcal{P})/N_c,$$
$$\phi^* = (\mathrm{Tr}_c \mathcal{P}^\dagger)/N_c,$$

This can be represented by a matrix in the color space

$$\mathcal{P}(\vec{x}) = \mathcal{P}\exp\left[i\int_0^\beta d\tau A_4(\vec{x},\tau)\right],$$

$\beta = 1/T$ Temperature

$A_4 = iA^0$ Polyakov gauge

The coupling between Polyakov loop and quarks is given by the covariant derivative

$$D_\mu = \partial_\mu - iA_\mu \text{ in PLSM Lagrangian}$$

$$A_\mu = \delta_{\mu 0} A_0$$ in the chiral limit

$$\mathcal{L}_{PLSM} = \mathcal{L}_{meson} + \mathcal{L}_{quark} + \bar{q}\gamma_0 \mathcal{A}_0 - \mathcal{U}(\phi, \phi^*, T),$$

$$\mathcal{L}_{chiral} = \mathcal{L}_{meson} + \mathcal{L}_{quark} + \bar{q}\gamma_0 \mathcal{A}_0$$

invariant under the chiral flavor group (like QCD Lagrangian)

$U(\phi, \phi^*, T)$ is T-dependent Polyakov Potential

In case of no quarks, then $\phi = \phi^*$ and the Polyakov loop is considered as an order parameter for the deconfinement phase-transition

In thermal equilibrium, the grand partition function can be defined by using a path integral over the quark, antiquark and meson field.

$$\mathcal{Z} = \mathrm{Tr}\,\exp[-(\hat{\mathcal{H}} - \sum_{f=u,d,s} \mu_f \hat{\mathcal{N}}_f)/T]$$

$$= \int \prod_a \mathcal{D}\sigma_a \mathcal{D}\pi_a \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left[\int_x (\mathcal{L} + \sum_{f=u,d,s} \mu_f \bar{\psi}_f \gamma^0 \psi_f)\right],$$

where $\int_x \equiv i \int_0^{1/T} dt \int_V d^3x$ **and** μ_f **chemical potential**

Thermodynamic potential density

$$\Omega(T,\mu) = \frac{-T \ln \mathcal{Z}}{V} = U(\sigma_x, \sigma_y) + \mathcal{U}(\phi, \phi^*, T) + \Omega_{\bar{\psi}\psi}.$$

The quarks and antiquarks Potential contribution

$$\Omega_{\bar{\psi}\psi} = -2TN\int_0^\infty \frac{d^3\vec{p}}{(2\pi)^3}\left\{\ln\left[1+3(\phi+\phi^* e^{-(E-\mu)/T})\times e^{-(E-\mu)/T}+e^{-3(E-\mu)/T}\right]\right.$$
$$\left.+\ln\left[1+3(\phi^*+\phi e^{-(E+\mu)/T})\times e^{-(E+\mu)/T}+e^{-3(E+\mu)/T}\right]\right\},$$

where N gives the number of quark flavors, $E = \sqrt{\vec{p}^2+m^2}$

$$m_q = g\frac{\sigma_x}{2},$$
$$m_s = g\frac{\sigma_y}{\sqrt{2}}.$$

Mesonic potential $U(\sigma_x,\sigma_y) = \frac{m^2}{2}(\sigma_x^2+\sigma_y^2) - h_x\sigma_x - h_y\sigma_y - \frac{c}{2\sqrt{2}}\sigma_x^2\sigma_y$
$$+\frac{\lambda_1}{2}\sigma_x^2\sigma_y^2 + \frac{1}{8}(2\lambda_1+\lambda_2)\sigma_x^4 + \frac{1}{4}(\lambda_1+\lambda_2)\sigma_y^4.$$

The thermodynamic potential

$$\Omega(T,\mu) = \frac{-T \ln \mathcal{Z}}{V} = U(\sigma_x,\sigma_y) + \mathcal{U}(\phi,\phi^*,T) + \Omega_{\bar{\psi}\psi}.$$

has the parameters

$m^2, h_x, h_y, \lambda_1, \lambda_2, c$ and g

σ_x and σ_y condensates

ϕ and ϕ^* order parameters

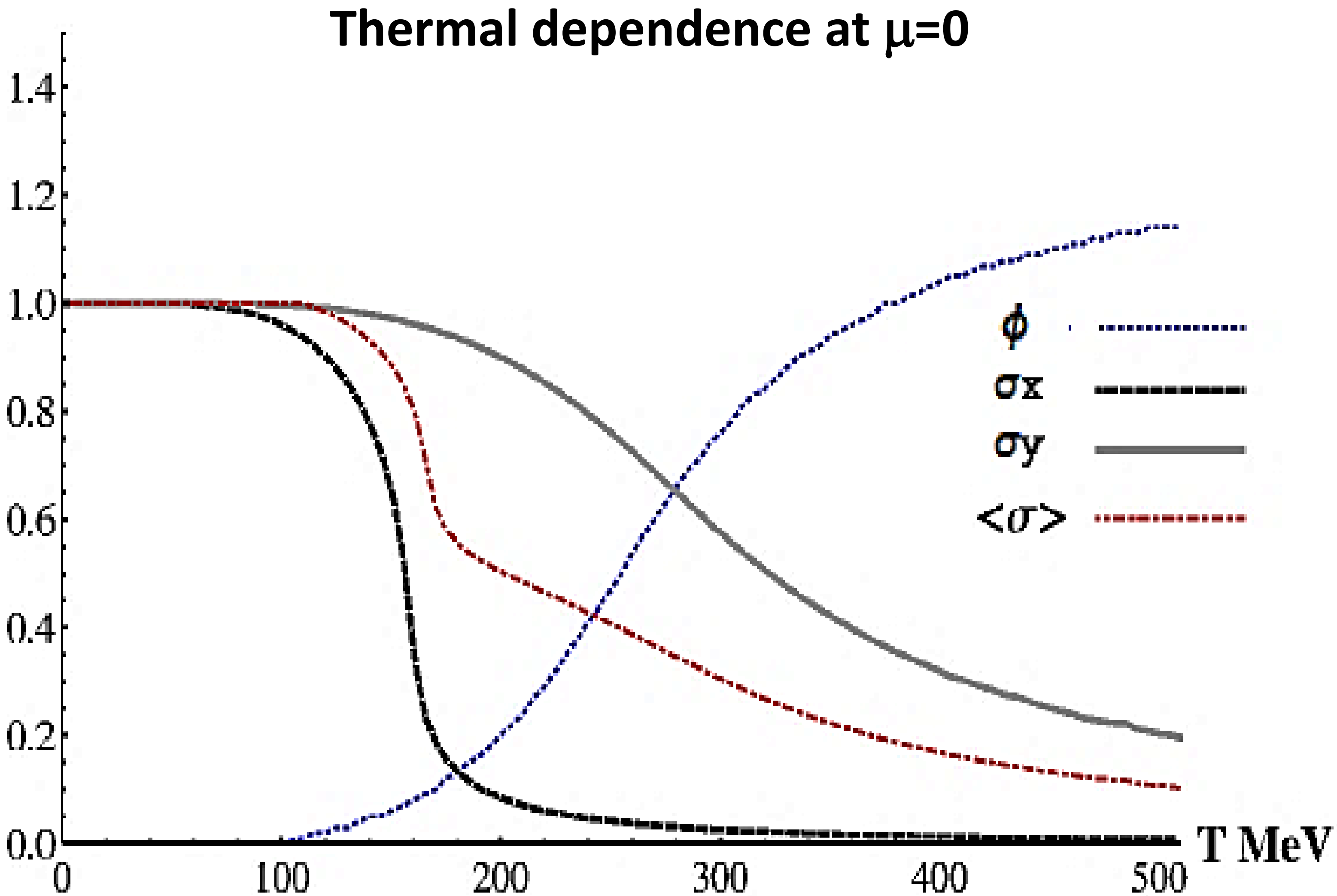
$m^2, h_x, h_y, \lambda_1, \lambda_2$ and c can be fixed experimentally

σ_x, σ_y, ϕ and ϕ^* minimizing the potential

$$\frac{\partial \Omega}{\partial \sigma_x} = \frac{\partial \Omega}{\partial \sigma_y} = \frac{\partial \Omega}{\partial \phi} = \left.\frac{\partial \Omega}{\partial \phi^*}\right|_{min} = 0,$$

$\sigma_x = \bar{\sigma_x}, \sigma_y = \bar{\sigma_y}, \phi = \bar{\phi}$ and $\phi^* = \bar{\phi}^*$ are the global minimum

Condensates and order parameter



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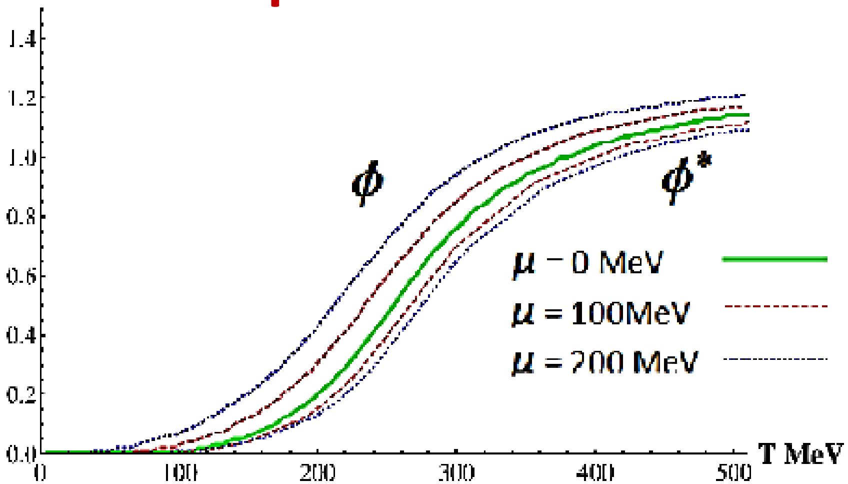
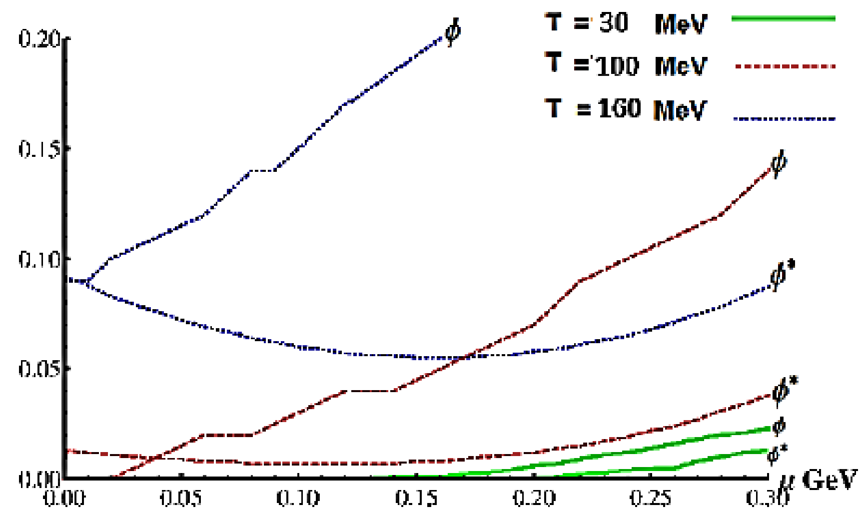
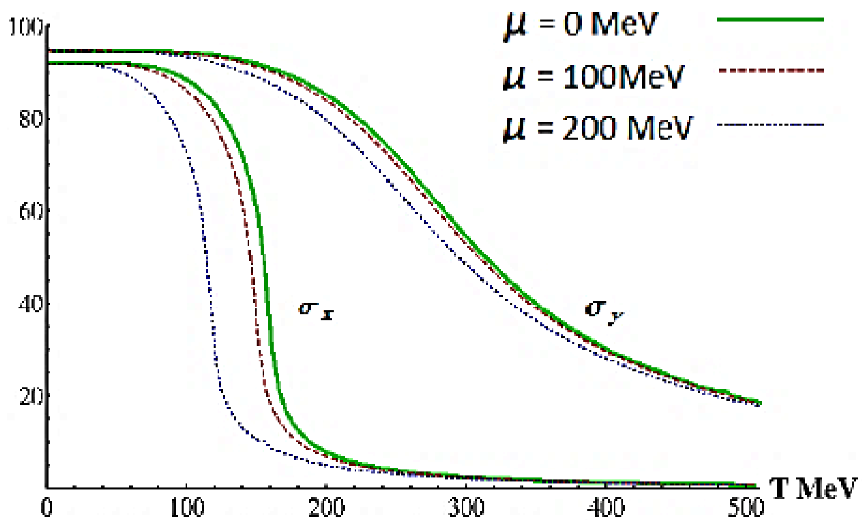
Condensates and order parameter

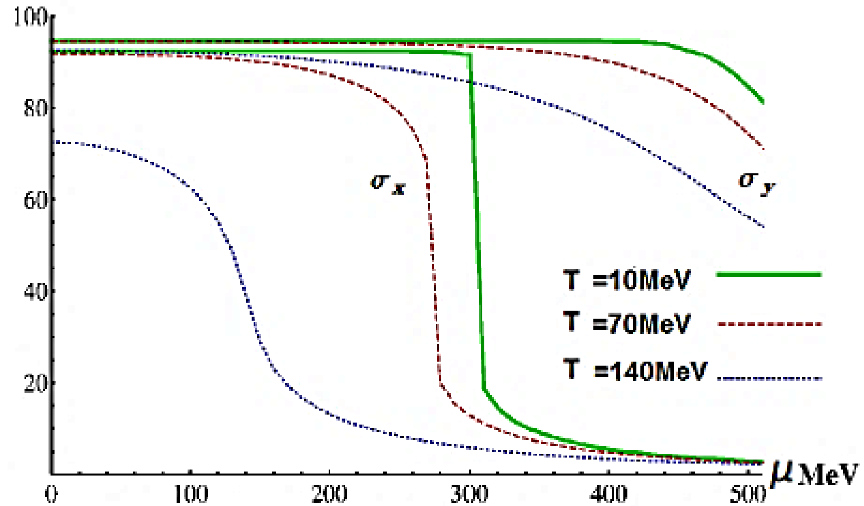
Thermal dependence





Density-dependence





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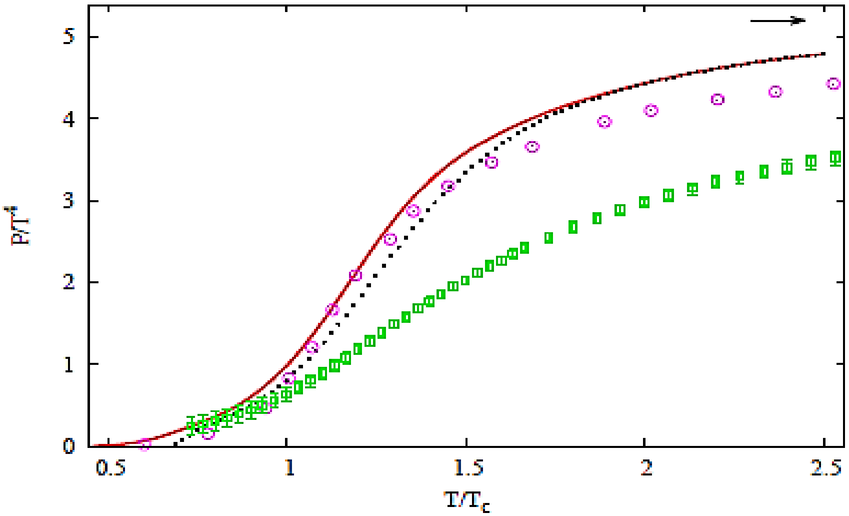
Thermodynamics

Pressure p/T^4 $P = -\Omega(T, \mu).$

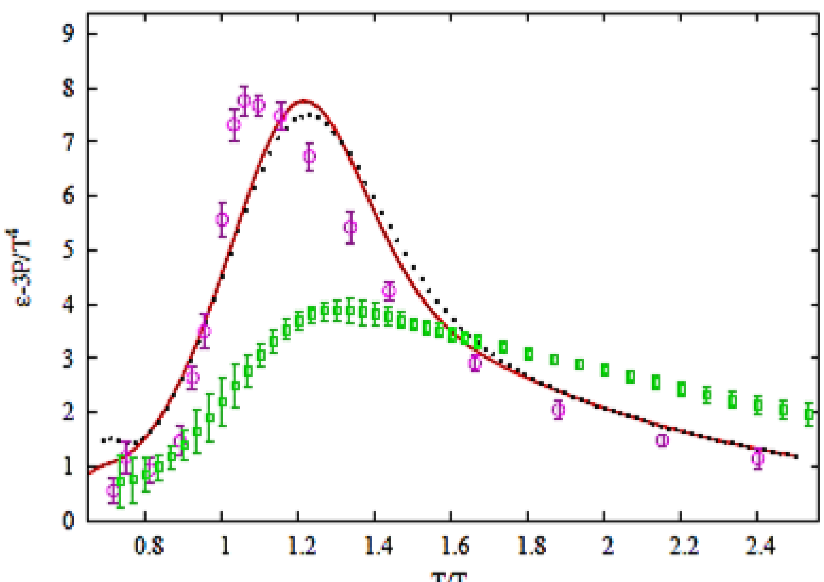


PLSM P is compared with the lattice QCD calculations (empty circles & rectangles) at $\mu = 0$.

The pressure increases with T until it gets close to the value of massless gas (SB limit 5.2).

Solid curve at g=6.5
Dotted Curve at g=10.5

Trace anomaly (ε-3p)/T^4 $= T\frac{\partial}{\partial T}(p/T^4)$



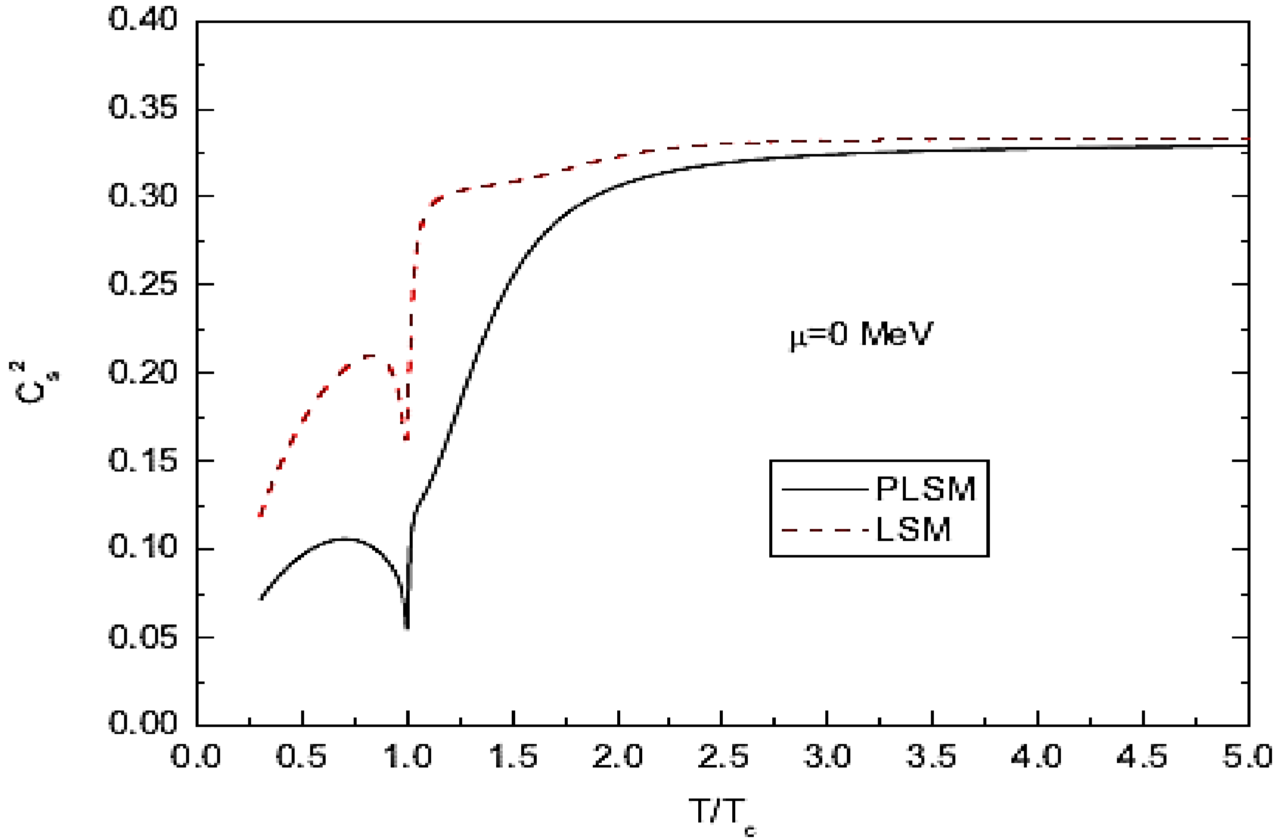
The values are small in hadronic phase and gradually increase in the region of phase transition (Cross over)

They are decreasing in the deconfined phase.

The trace anomaly shows a peak around the critical temperature

Equation of State

Squared speed of sound $C_s^2 = \frac{dp}{d\varepsilon} = \frac{s}{T ds/dT} = \frac{s}{C_v},$



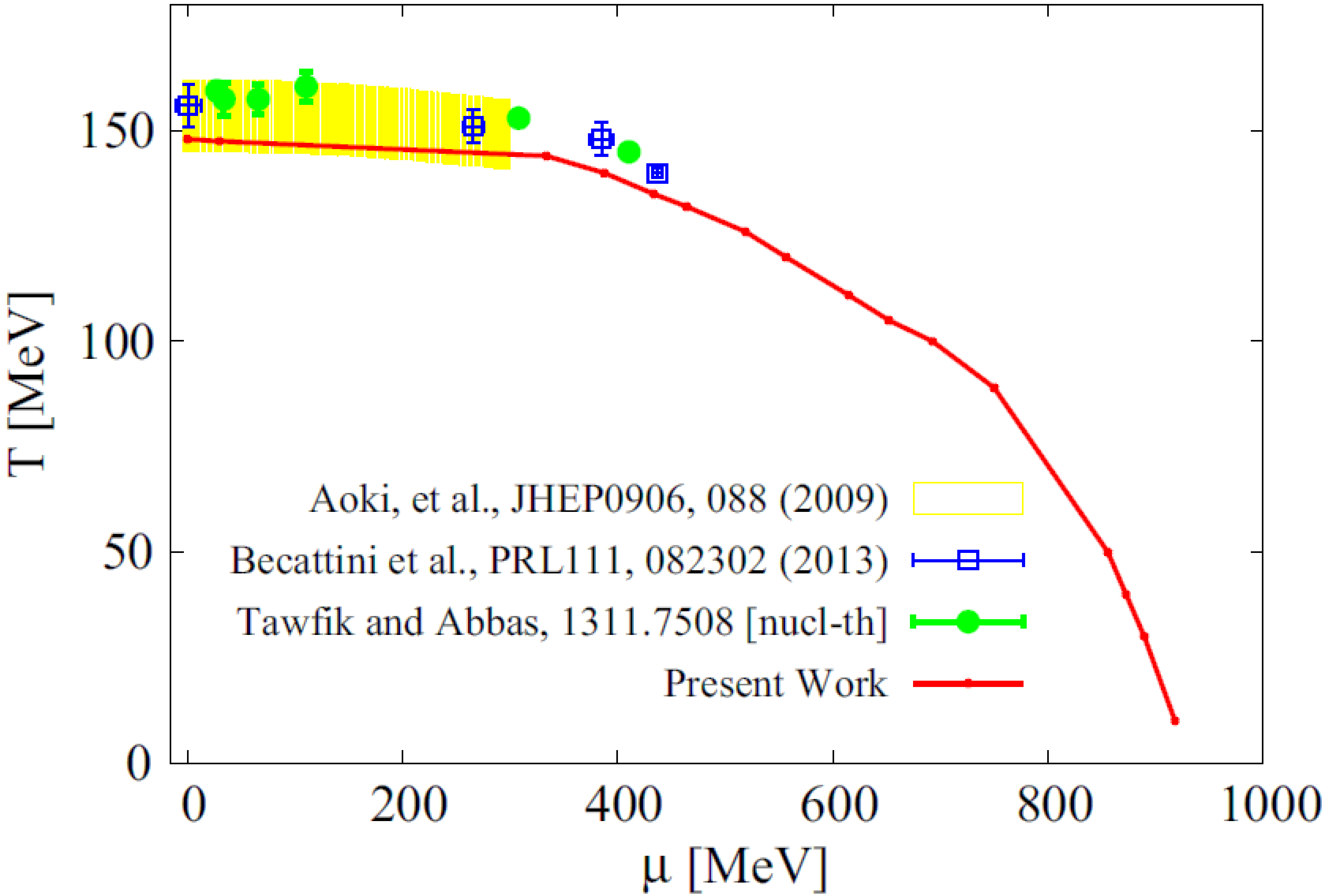
In conformal field theories including free field theory, the squared speed of sound is 1/3.

$Near\ T_c \rightarrow$ downward cusp.

At zero density, $\rightarrow$ minimum

WLCAPP World Laboratory Cosmology And Particle Physics

Chiral phase-diagram



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Thermal Evolution of Meson Masses

Masses are defined by the second derivative w.r.t. the corresponding field (scalar, pseudoscalar, etc.)

$$m_{i,ab}^{2} = \frac{\partial^2 \Omega(T,\mu_f)}{\partial \zeta_{i,a} \partial \zeta_{i,b}}\Big|_{\min}$$

where i stands for scalar, pseudoscalar, vector and axial-vector mesons

- **The mesonic part of the potential determines the mass matrix, entirely.**
- **The squared masses for scalar/pseudoscalar sector, are formulated in the nonstrange-strange basis**
- **The vacuum contribution vanishes.**

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Thermal Evolution of Meson Masses

Scalar Masses

$$m_{a_0}^2 = m^2 + \lambda_1\left(\bar{\sigma}_x^2 + \bar{\sigma}_y^2\right) + \frac{3\lambda_2}{2}\bar{\sigma}_x^2 + \frac{\sqrt{2}c}{2}\bar{\sigma}_y,$$

$$m_{\kappa}^2 = m^2 + \lambda_1\left(\bar{\sigma}_x^2 + \bar{\sigma}_y^2\right) + \frac{\lambda_2}{2}\left(\bar{\sigma}_x^2 + \sqrt{2}\bar{\sigma}_x\bar{\sigma}_y + 2\bar{\sigma}_y^2\right) + \frac{c}{2}\bar{\sigma}_x,$$

$$m_{\sigma}^2 = m_{s,00}^2\cos^2\theta_s + m_{s,88}^2\sin^2\theta_s + 2m_{s,08}^2\sin\theta_s\cos\theta_s,$$

$$m_{f_0}^2 = m_{s,00}^2\sin^2\theta_s + m_{s,88}^2\cos^2\theta_s - 2m_{s,08}^2\sin\theta_s\cos\theta_s,$$

where

$$m_{s,00}^2 = m^2 + \frac{\lambda_1}{3}\left(7\bar{\sigma}_x^2 + 4\sqrt{2}\bar{\sigma}_x\bar{\sigma}_y + 5\bar{\sigma}_y^2\right) + \lambda_2\left(\bar{\sigma}_x^2 + \bar{\sigma}_y^2\right) - \frac{\sqrt{2}c}{3}\left(\sqrt{2}\bar{\sigma}_x + \bar{\sigma}_y\right),$$

$$m_{s,88}^2 = m^2 + \frac{\lambda_1}{3}\left(5\bar{\sigma}_x^2 - 4\sqrt{2}\bar{\sigma}_x\bar{\sigma}_y + 7\bar{\sigma}_y^2\right) + \lambda_2\left(\frac{\bar{\sigma}_x^2}{2} + 2\bar{\sigma}_y^2\right) + \frac{\sqrt{2}c}{3}\left(\sqrt{2}\bar{\sigma}_x - \frac{\bar{\sigma}_y}{2}\right),$$

$$m_{s,08}^2 = \frac{2\lambda_1}{3}\left(\sqrt{2}\bar{\sigma}_x^2 - \bar{\sigma}_x\bar{\sigma}_y - \sqrt{2}\bar{\sigma}_y^2\right) + \sqrt{2}\lambda_2\left(\frac{\bar{\sigma}_x^2}{2} - \bar{\sigma}_y^2\right) + \frac{c}{3\sqrt{2}}\left(\bar{\sigma}_x - \sqrt{2}\bar{\sigma}_y\right).$$

and $$\tan 2\theta_i = \frac{2m_{i,08}^2}{m_{i,00}^2 - m_{i,88}^2}$$

Thermal Evolution of Meson Masses

Pseudo-Scalar Masses

$$m_\pi^2 = m^2 + \lambda_1 \left(\bar{\sigma}_x^2 + \bar{\sigma}_y^2\right) + \frac{\lambda_2}{2}\bar{\sigma}_x^2 - \frac{\sqrt{2}c}{2}\bar{\sigma}_y,$$

$$m_K^2 = m^2 + \lambda_1 \left(\bar{\sigma}_x^2 + \bar{\sigma}_y^2\right) + \frac{\lambda_2}{2}\left(\bar{\sigma}_x^2 - \sqrt{2}\bar{\sigma}_x\bar{\sigma}_y + 2\bar{\sigma}_y^2\right) - \frac{c}{2}\bar{\sigma}_x,$$

$$m_{\eta'}^2 = m_{p,00}^2 \cos^2\theta_p + m_{p,88}^2 \sin^2\theta_p + 2m_{p,08}^2 \sin\theta_p \cos\theta_p,$$

$$m_\eta^2 = m_{p,00}^2 \sin^2\theta_p + m_{p,88}^2 \cos^2\theta_p - 2m_{p,08}^2 \sin\theta_p \cos\theta_p,$$

where

$$m_{p,00}^2 = m^2 + \lambda_1 \left(\bar{\sigma}_x^2 + \bar{\sigma}_y^2\right) + \frac{\lambda_2}{3}\left(\bar{\sigma}_x^2 + \bar{\sigma}_y^2\right) + \frac{c}{3}\left(2\bar{\sigma}_x + \sqrt{2}\bar{\sigma}_y\right),$$

$$m_{p,88}^2 = m^2 + \lambda_1 \left(\bar{\sigma}_x^2 + \bar{\sigma}_y^2\right) + \frac{\lambda_2}{6}\left(\bar{\sigma}_x^2 + 4\bar{\sigma}_y^2\right) - \frac{c}{6}\left(4\bar{\sigma}_x - \sqrt{2}\bar{\sigma}_y\right),$$

$$m_{p,08}^2 = \frac{\sqrt{2}\lambda_2}{6}\left(\bar{\sigma}_x^2 - 2\bar{\sigma}_y^2\right) - \frac{c}{6}\left(\sqrt{2}\bar{\sigma}_x - 2\bar{\sigma}_y\right),$$

Thermal Evolution of Meson Masses

Vector Masses

$$m_\rho^2 = m_1^2 + \frac{1}{2}(h_1+h_2+h_3)\,\bar{\sigma}_x^2 + \frac{h_1}{2}\bar{\sigma}_y^2 + 2\delta_x \,,$$

$$m_{K^\star}^2 = m_1^2 + \frac{\bar{\sigma}_x^2}{4}(g_1^2+2h_1+h_2) + \frac{\bar{\sigma}_x\bar{\sigma}_y}{\sqrt{2}}(h_3-g_1^2) + \frac{\bar{\sigma}_y^2}{2}(g_1^2+h_1+h_2) + \delta_x + \delta_y$$

$$m_{\omega_x}^2 = m_\rho^2 \,,$$

$$m_{\omega_y}^2 = m_1^2 + \frac{h_1}{2}\bar{\sigma}_x^2 + \left(\frac{h_1}{2}+h_2+h_3\right)\bar{\sigma}_y^2 + 2\delta_y \,,$$

Axial-Vector Masses

$$m_{a_1}^2 = m_1^2 + \frac{1}{2}(2g_1^2+h_1+h_2-h_3)\,\bar{\sigma}_x^2 + \frac{h_1}{2}\bar{\sigma}_y^2 + 2\delta_x,$$

$$m_{K_1}^2 = m_1^2 + \frac{1}{4}(g_1^2+2h_1+h_2)\,\bar{\sigma}_x^2 - \frac{1}{\sqrt{2}}\bar{\sigma}_x\bar{\sigma}_y\,(h_3-g_1^2) + \frac{1}{2}(g_1^2+h_1+h_2)\,\bar{\sigma}_y^2 + \delta_x + \delta_y,$$

$$m_{f_{1x}}^2 = m_{a_1}^2,$$

$$m_{f_{1y}}^2 = m_1^2 + \frac{\bar{\sigma}_x^2}{2}h_1 + \left(2g_1^2+\frac{h_1}{2}+h_2-h_3\right)\bar{\sigma}_y^2 + 2\delta_y.$$

Thermal Evolution of Meson Masses

In order to include the quark contribution in the grand potential, the mesonic masses should be modified due to the in-medium effects of finite temperature.

The explicit quark contribution to LSM-potential

$$\Omega_{\bar{q}q}(T,\mu_f) = \nu_c T \sum_{f=u,d,s} \int_0^\infty \frac{d^3k}{(2\pi)^3} \left\{ \ln(1 - n_{q,f}(T,\mu_f)) + \ln(1 - n_{\bar{q},f}(T,\mu_f)) \right\}$$

where $n_{q,f}\left(T,\mu_f\right) = \dfrac{1}{1+\exp\left[(E_f-\mu_f)/T\right]}$ **and** $E_f = \sqrt{k^2+m_f^2}$

Then

$$m_{i,ab}^2 = \frac{\partial^2 \Omega(T,\mu_f)}{\partial \zeta_{i,a} \partial \zeta_{i,b}}\bigg|_{\min} = \nu_c \sum_{f=l,s} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{q,f}} \left[(n_{q,f} + n_{\bar{q},f}) \left(m_{f,ab}^2 - \frac{m_{f,a}^2 m_{f,b}^2}{2E_{q,f}^2} \right) - (b_{q,f} + b_{\bar{q},f}) \left(\frac{m_{f,a}^2 m_{f,b}^2}{2E_{q,f} T} \right) \right].$$

$$b_{\bar{q},f}(T,\mu_f) = b_{q,f}(T,-\mu_f) \qquad b_{q,f}(T,\mu_f) = n_{q,f}(T,\mu_f)(1 - n_{q,f}(T,\mu_f))$$

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Thermal Evolution of Meson Masses

The meson-masses modification due to the in-medium effects of finite temperature in PLSM.

$$m^2_{i,ab} = \frac{\partial^2 \Omega(T,\mu_f)}{\partial \zeta_{i,a} \partial \zeta_{i,b}}\Big|_{\min} = \nu_c \sum_{f=l,s} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{q,f}} \Big[(N_{q,f} + N_{\bar{q},f}) \Big(m^2_{f,ab} - \frac{m^2_{f,a} m^2_{f,b}}{2E^2_{q,f}} \Big) + (B_{q,f} + B_{\bar{q},f}) \Big(\frac{m^2_{f,a} m^2_{f,b}}{2E_{q,f} T} \Big) \Big].$$

$$N_{q,f} = \frac{\Phi e^{-E_{q,f}/T} + 2\Phi^* e^{-2E_{q,f}/T} + e^{-3E_{q,f}/T}}{1 + 3(\phi + \phi^* e^{-E_{q,f}/T}) e^{-E_{q,f}/T} + e^{-3E_{\bar{q},f}/T}}$$

$$N_{\bar{q},f} = \frac{\Phi^* e^{-E_{\bar{q},f}/T} + 2\Phi e^{-2E_{\bar{q},f}/T} + e^{-3E_{\bar{q},f}/T}}{1 + 3(\phi^* + \phi e^{-E_{\bar{q},f}/T}) e^{-E_{\bar{q},f}/T} + e^{-3E_{\bar{q},f}/T}}$$

For quark, $B_{q,f} = 3(N_{q,f})^2 - C_{q,f}$ and for antiquark, $B_{\bar{q},f} = 3(N_{\bar{q},f})^2 - C_{\bar{q},f}$,

$$C_{q,f} = \frac{\Phi e^{-E_{q,f}/T} + 4\Phi^* e^{-2E_{q,f}/T} + 3e^{-3E_{q,f}/T}}{1 + 3(\phi + \phi^* e^{-E_{q,f}/T})\, e^{-E_{q,f}/T} + e^{-3E_{\bar{q},f}/T}},$$

$$C_{\bar{q},f} = \frac{\Phi^* e^{-E_{\bar{q},f}/T} + 4\Phi e^{-2E_{\bar{q},f}/T} + 3e^{-3E_{\bar{q},f}/T}}{1 + 3(\phi^* + \phi e^{-E_{\bar{q},f}/T})\, e^{-E_{\bar{q},f}/T} + e^{-3E_{\bar{q},f}/T}},$$

Thermal Evolution of Meson Masses

The first and second derivative of the quark mass matrix given by

	$m^2_{l,a} m^2_{q,b}/g^4$	$m^2_{l,ab}/g^2$	$m^2_{s,a} m^2_{s,b}/g^4$	$m^2_{s,ab}/g^2$
$\sigma_0 \;\; \sigma_0$	$\frac{1}{3}\sigma_x^2$	$\frac{2}{3}$	$\frac{1}{3}\sigma_y^2$	$\frac{1}{3}$
$\sigma_1 \;\; \sigma_1$	$\frac{1}{2}\sigma_x^2$	1	0	0
$\sigma_4 \;\; \sigma_4$	0	$\sigma_x \frac{\sigma_x+\sqrt{2}\sigma_y}{\sigma_x^2-2\sigma_y^2}$	0	$\sigma_y \frac{\sqrt{2}\sigma_x+2\sigma_y}{2\sigma_y^2-\sigma_x^2}$
$\sigma_8 \;\; \sigma_8$	$\frac{1}{6}\sigma_x^2$	$\frac{1}{3}$	$\frac{2}{3}\sigma_y^2$	$\frac{2}{3}$
$\sigma_0 \;\; \sigma_8$	$\frac{\sqrt{2}}{6}\sigma_x^2$	$\frac{\sqrt{2}}{3}$	$-\frac{\sqrt{2}}{3}\sigma_y^2$	$-\frac{\sqrt{2}}{3}$
$\pi_0 \;\; \pi_0$	0	$\frac{2}{3}$	0	$\frac{1}{3}$
$\pi_1 \;\; \pi_1$	0	1	0	0
$\pi_4 \;\; \pi_4$	0	$\sigma_x \frac{\sigma_x-\sqrt{2}\sigma_y}{\sigma_x^2-2\sigma_y^2}$	0	$\sigma_y \frac{\sqrt{2}\sigma_x-2\sigma_y}{\sigma_x^2-2\sigma_y^2}$
$\pi_8 \;\; \pi_8$	0	$\frac{1}{3}$	0	$\frac{2}{3}$
$\pi_0 \;\; \pi_8$	0	$\frac{\sqrt{2}}{3}$	0	$-\frac{\sqrt{2}}{3}$

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Sector	Symbol	PDG [28]	PLSM	PNJL [24, 25]	Lattice QCD	
					Hot QCD[26]	PACS-CS [27]
Scalar $J^{PC} = 0^{++}$	a_0	$a_0(980^{\pm 20})$	1026	837		
	κ	$K_0^*(1425^{\pm 50})$	1115	1013		
	σ	$\sigma(400-1200)$	800	700		
	f_0	$f_0(1200-1500)$	1284	1169		
Pseudoscalar $J^{PC} = 0^{-+}$	π	$\pi^0(134.97^{\pm 6.9})$	120	126	$134^{\pm 6}$	$135.4^{\pm 6.2}$
	K	$K^0\ (497.614^{\pm 24.8})$	509	490	$422.6^{\pm 11.3}$	$498^{\pm 22}$
	η	$\eta(547.853^{\pm 27.4})$	553	505	$579^{\pm 7.3}$	$688^{\pm 32}$
	η'	$\eta'(957.78^{\pm 60})$	965	949	– –	– –
Vector $J^{PC} = 1^{--}$	ρ	$\rho(775.49^{\pm 38.8})$	745	– –	$756.2^{\pm 36}$	$597^{\pm 86}$
	ω_X	$\omega(782.65^{\pm 44.7})$	745	– –	$884^{\pm 18}$	$861^{\pm 23}$
	K^*	$K^*(891.66^{\pm 26})$	894	– –	$1005^{\pm 93}$	$1010.2^{\pm 77}$
	ω_y	$\phi(1019.455^{\pm 51})$	1005	– –	– –	– –
Axial-Vector $J^{PC} = 1^{++}$	a_1	$a_1(1030-1260)$	980	– –		
	f_{1x}	$f_1(1281^{\pm 60})$	980	– –		
	K_1^*	$K_1^*(1270^{\pm 7})$	1135	– –		
	f_{1y}	$f_1(1420^{\pm 71.3})$	1315	– –		

Comparison between pseudoscalar and axialvector masses in PLSM, PNJL, PDG and LQCD

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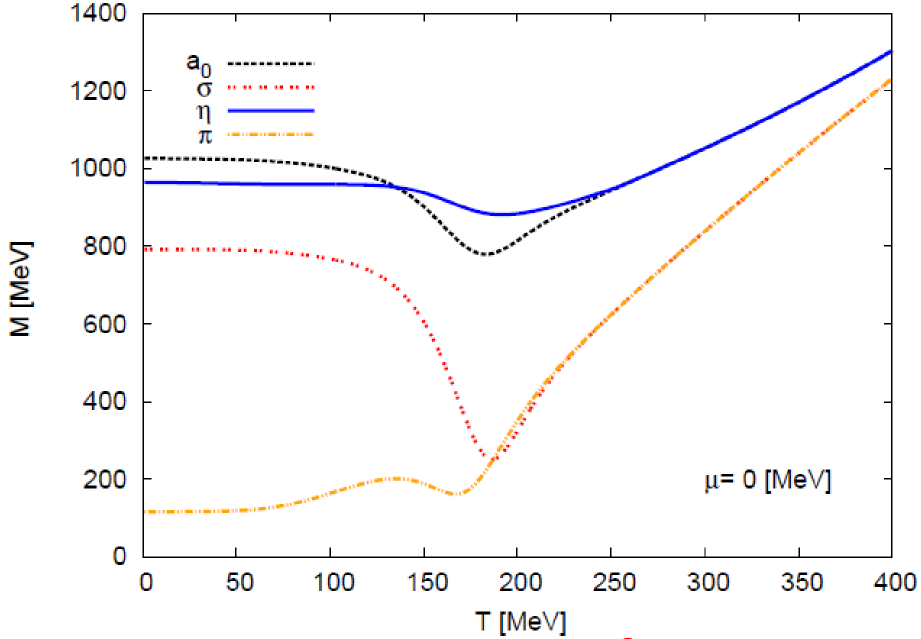
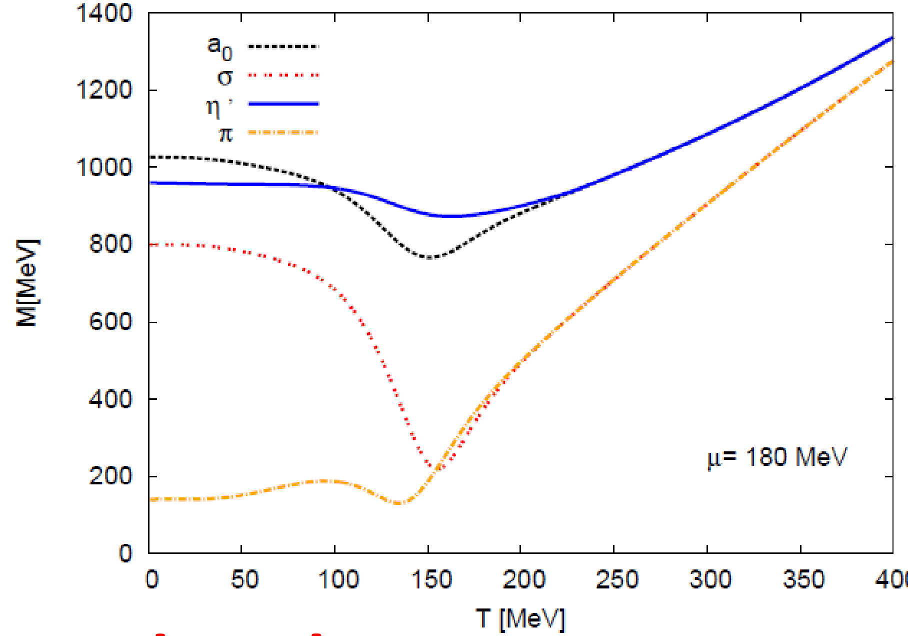
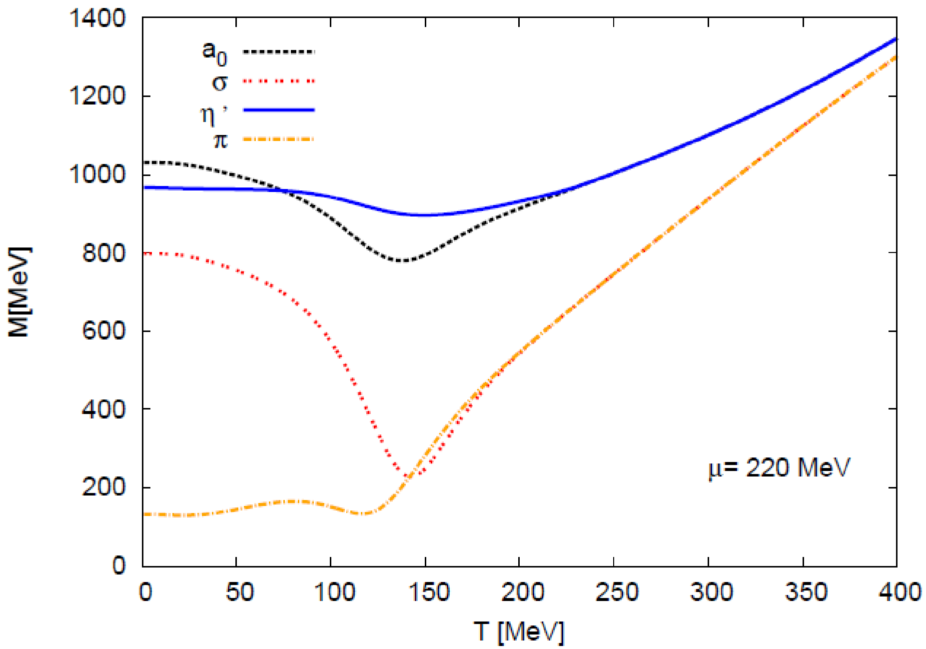
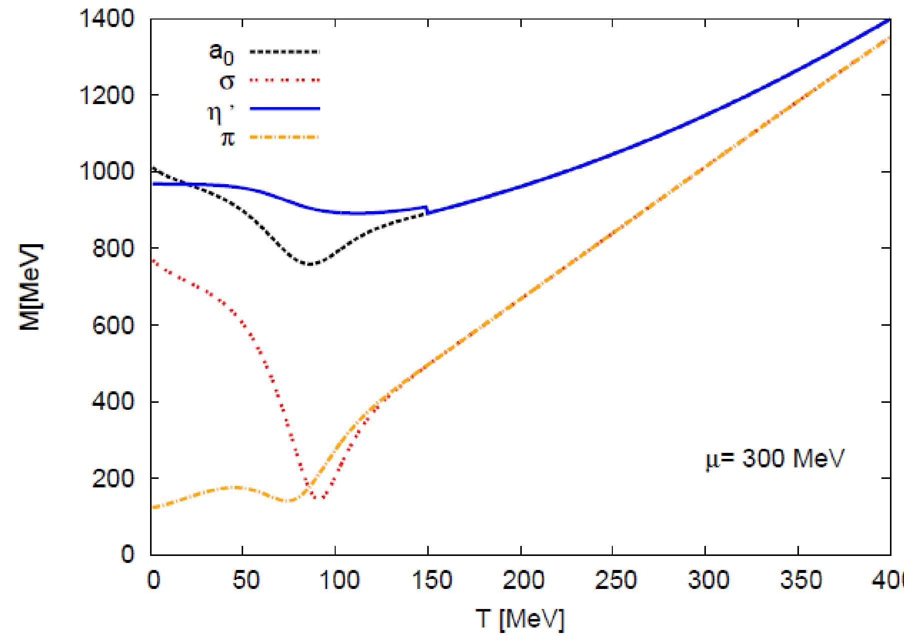
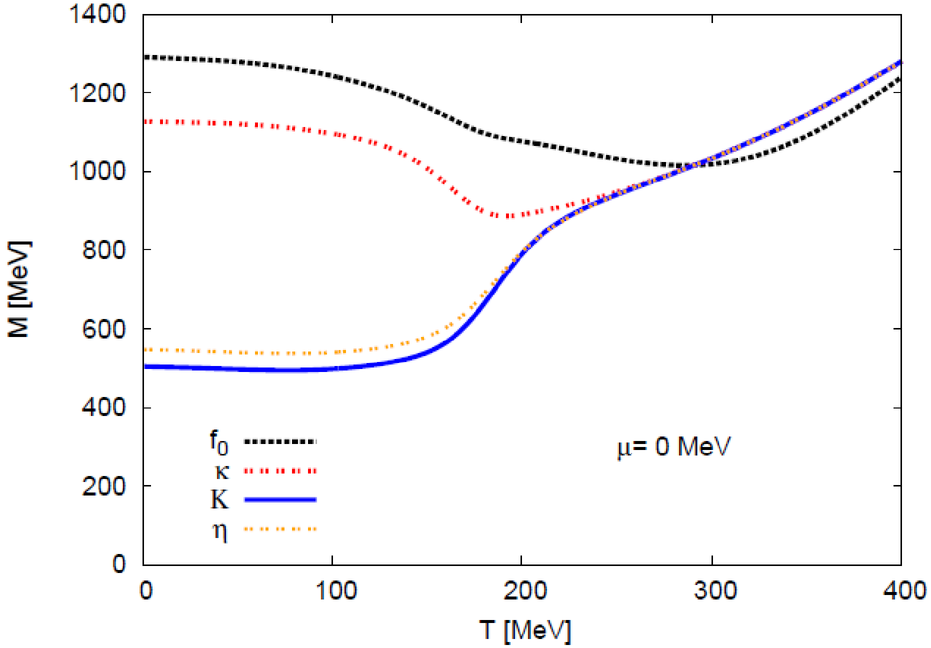


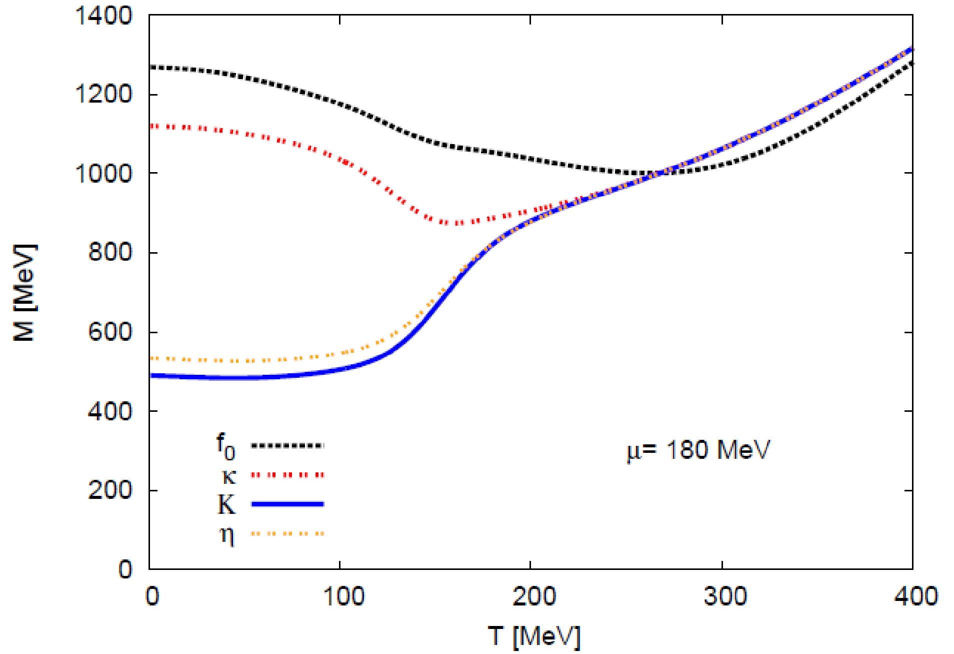


LSM: Scalars (top curves) / Pseudoscalars (bottom curves)

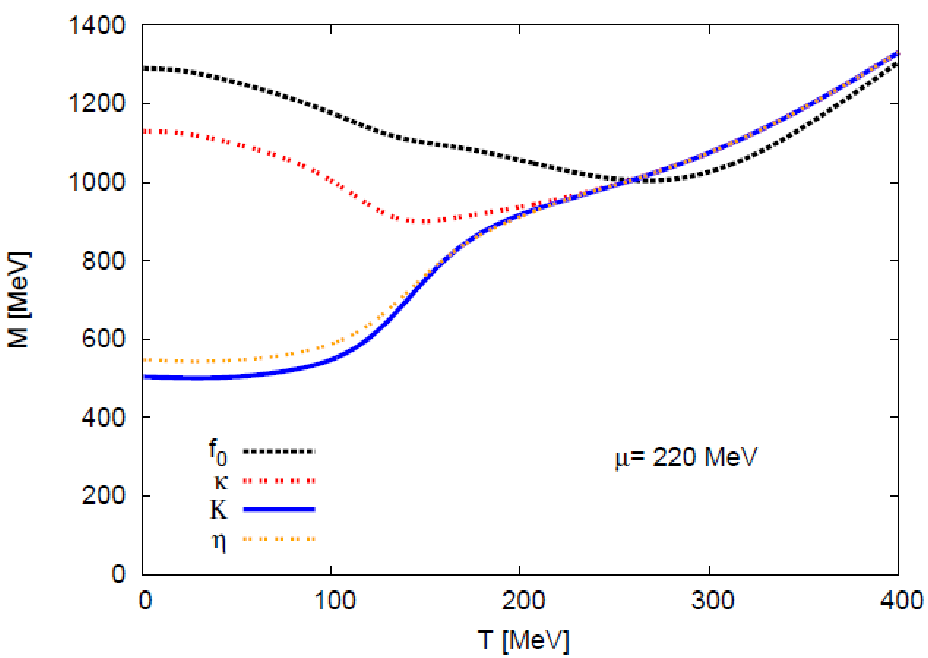


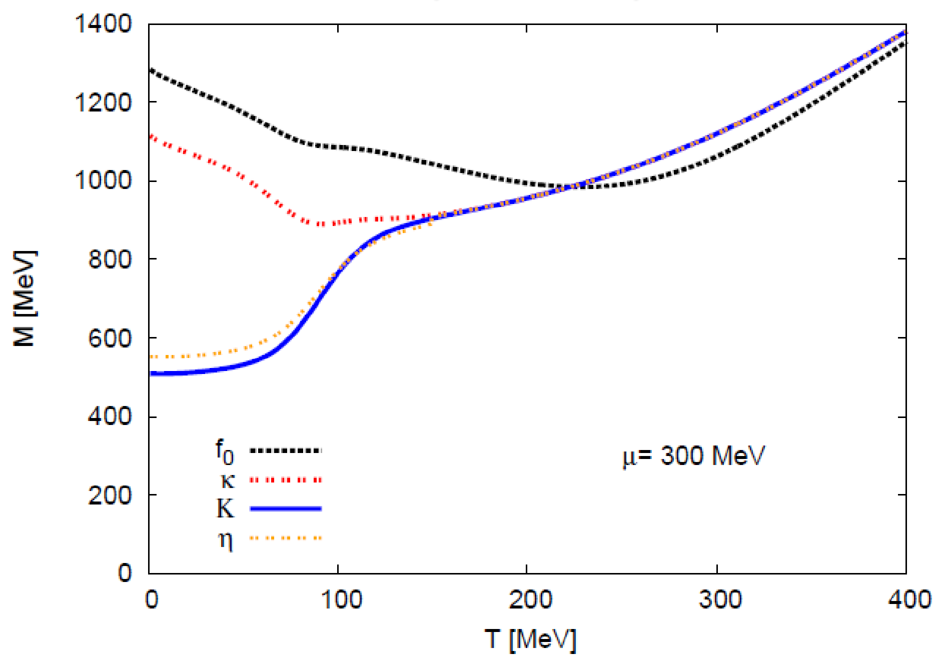


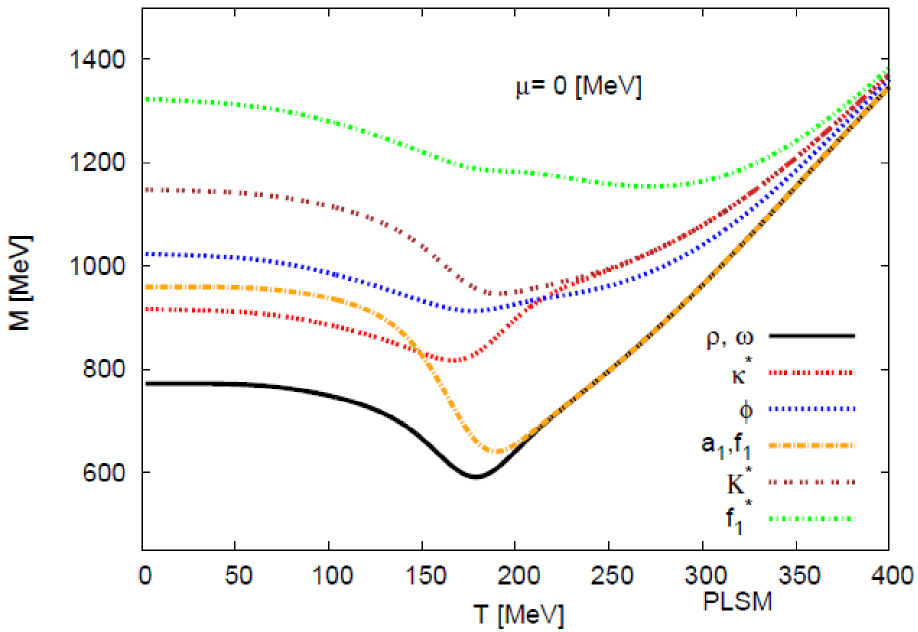


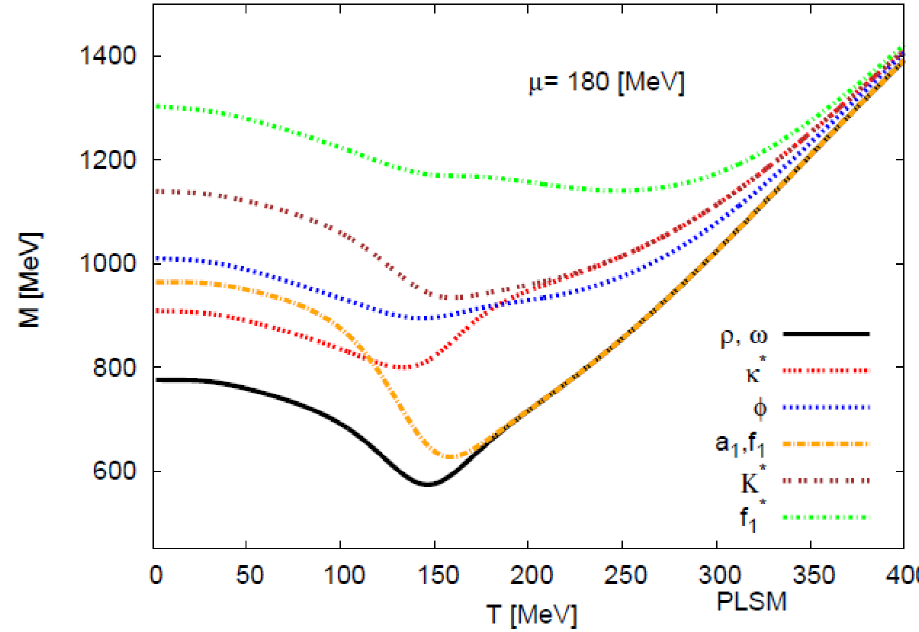


LSM: Scalars (top curves) /Pseudoscalars (bottom curves)

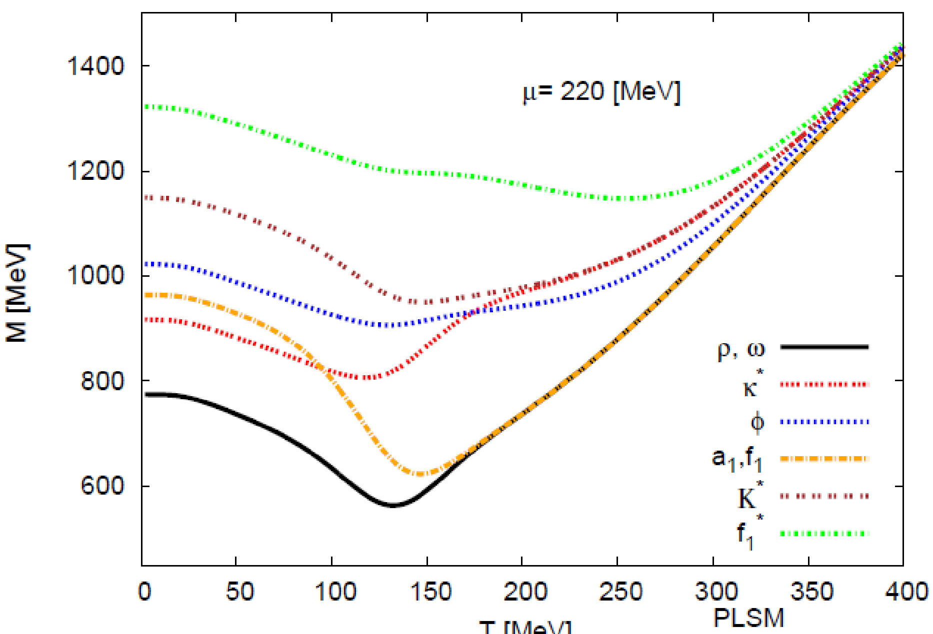


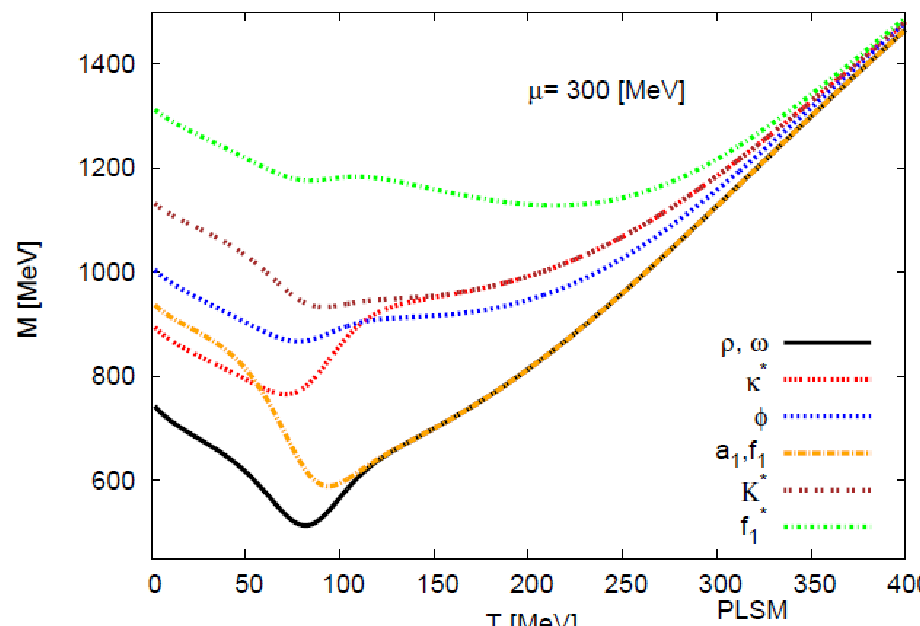


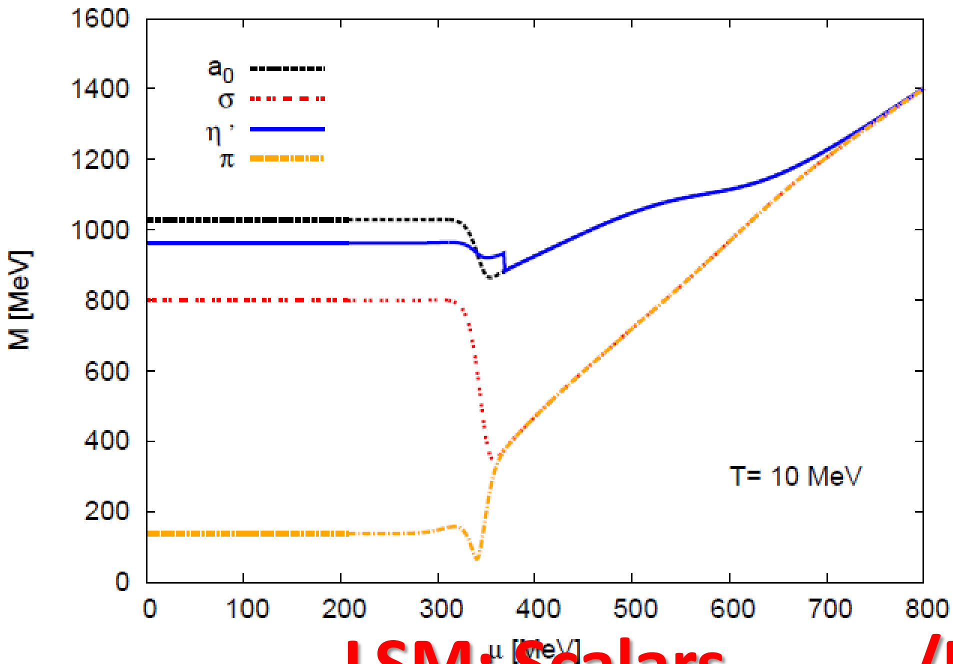


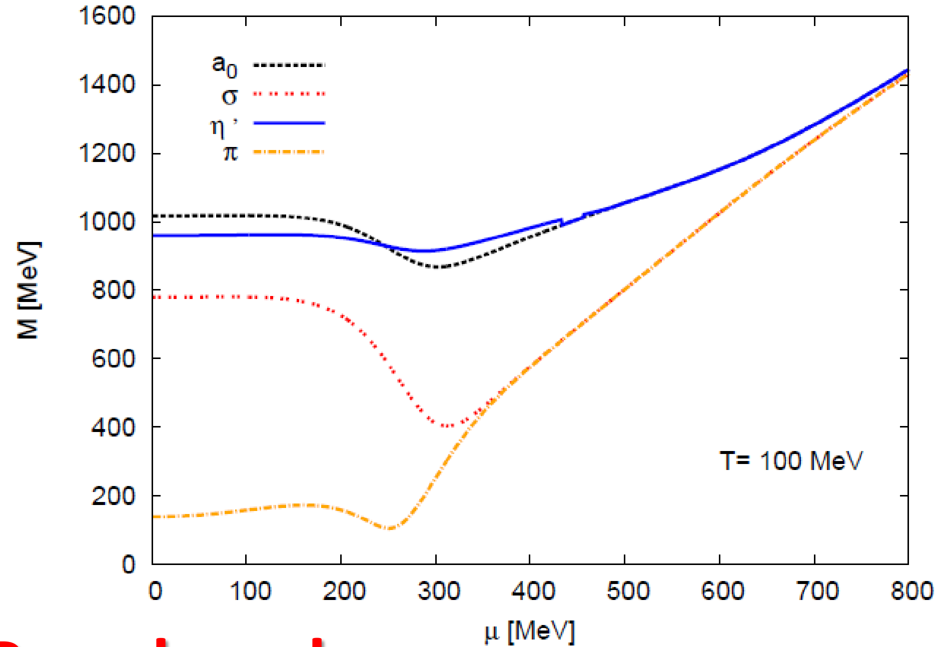


LSM: Vectors (top curves) /Axialvectors (bottom curves)

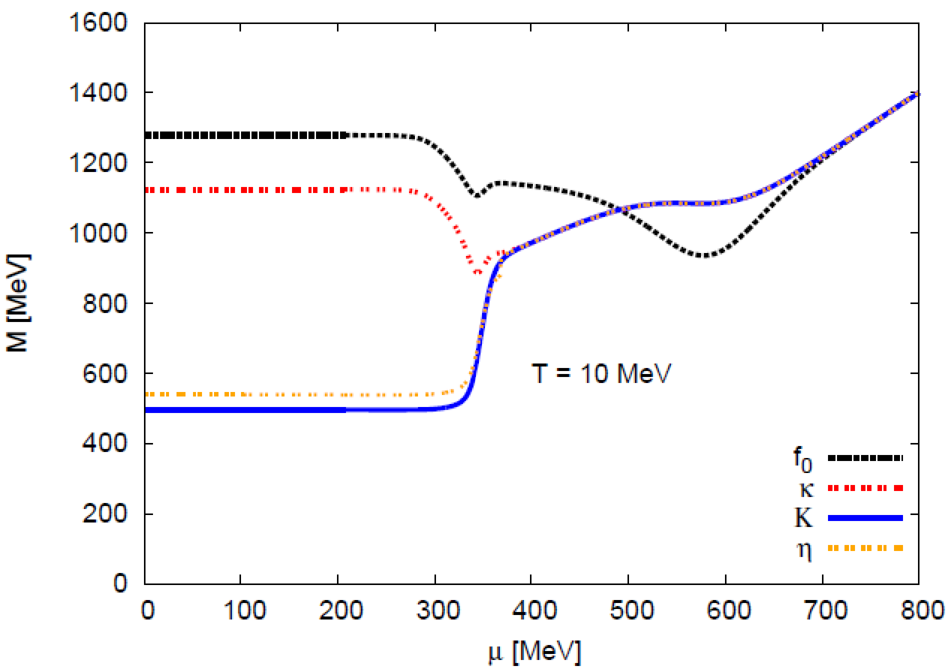


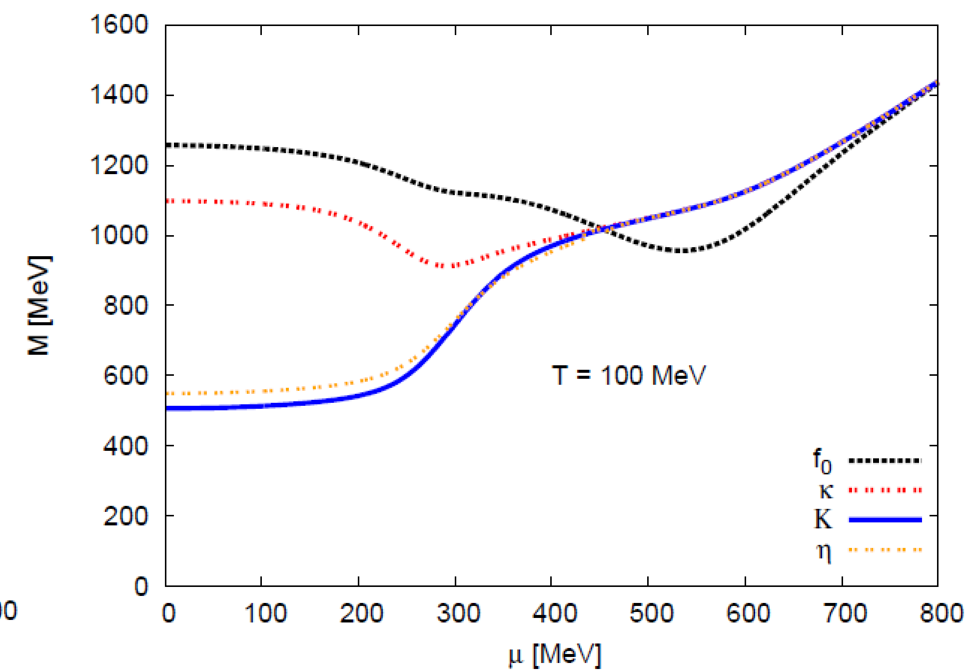


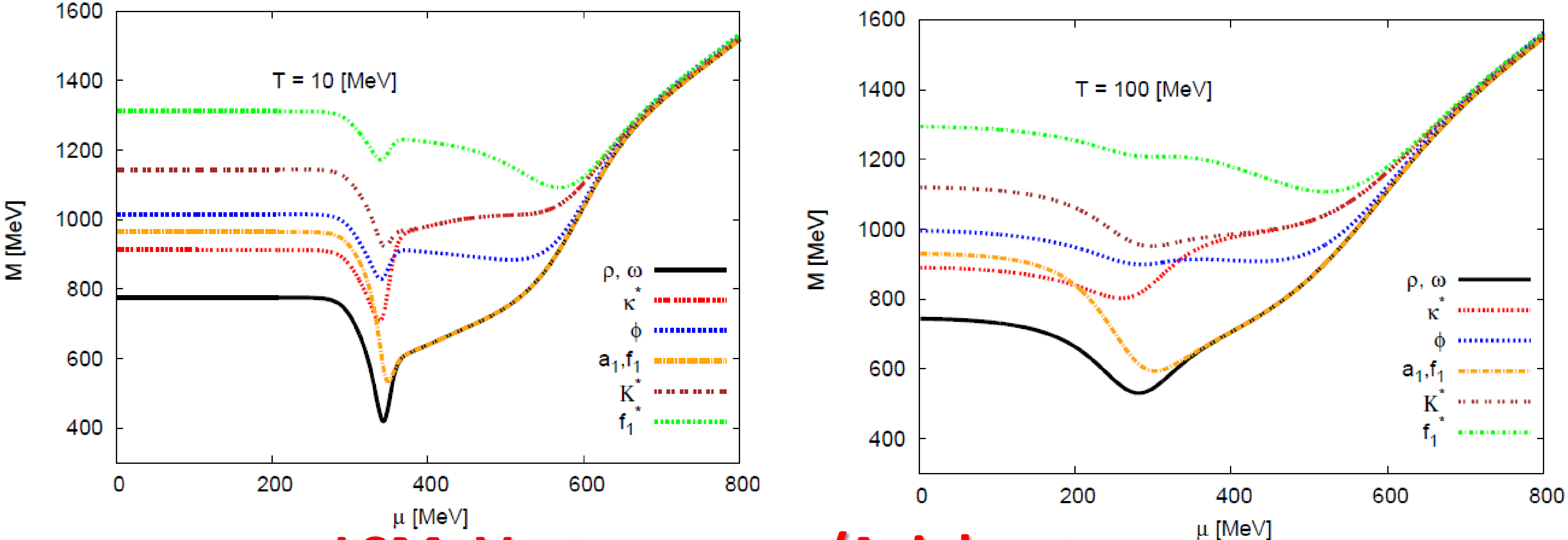




LSM: Scalars (top curves) /Pseudoscalars (bottom curves)







LSM: Vectors (top curves) /Axialvectors (bottom curves)

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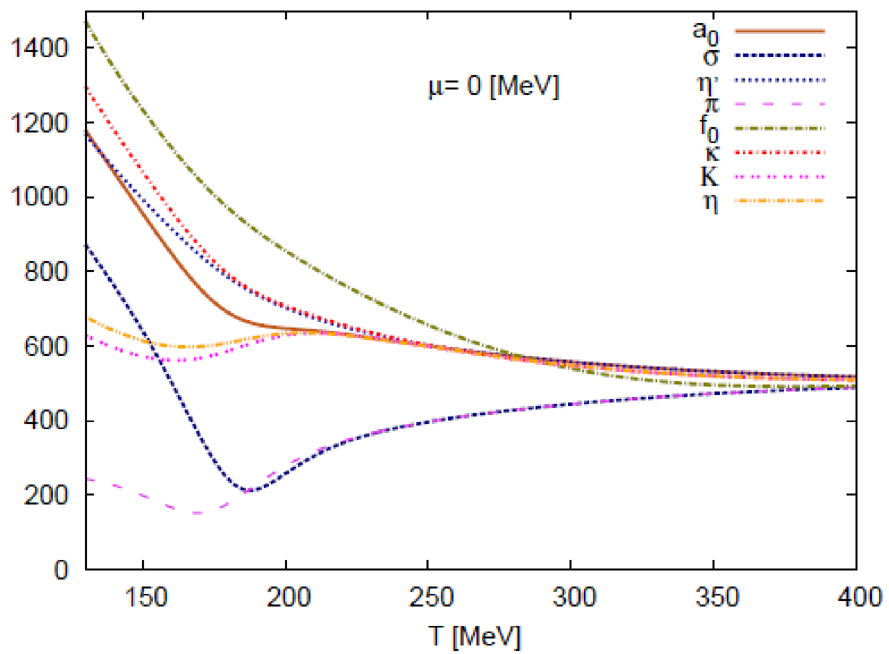
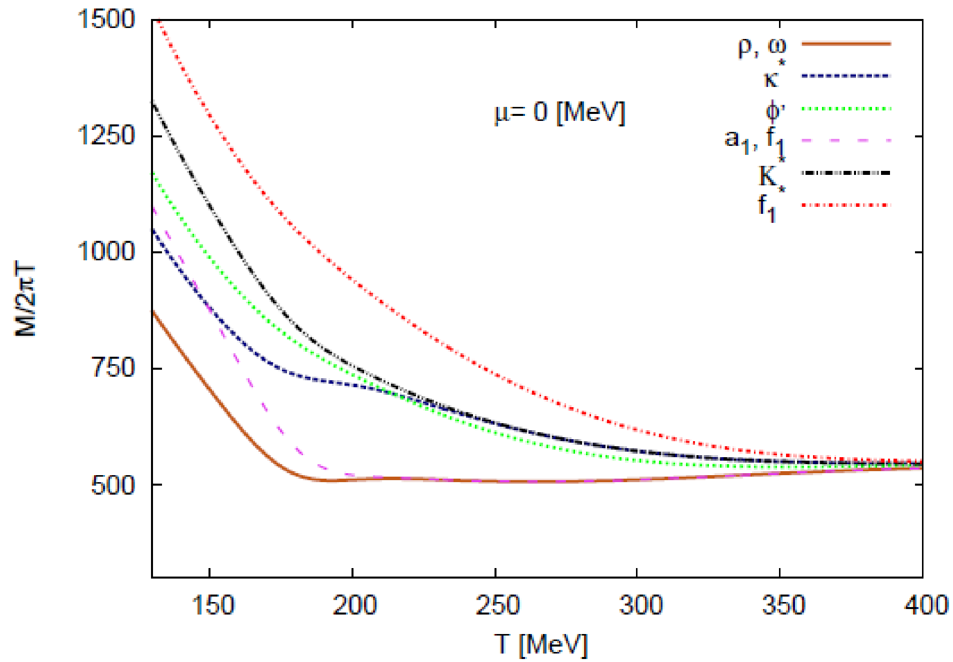
Dissolving Temperatures





Comparison	Scalar mesons	Pesudoscalar mesons	Vector mesons	Axial-vector mesons
meson	a_0 κ σ f_0	π K η η'	ρ K_0^* ω ϕ	a_1 K_1 f_1 f_1^*
$T^{Meson}_{Dissolving}$ [MeV]	200 250 320 320	320 230 235 300	195 300 195 300	205 250 205 350

The approximate dissolving temperature corresponding to meson sectors

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Effects of Polyakov-Loop Potential

WLCAPP World Laboratory Cosmology And Particle Physics

For chiral phase-structure, effects of Ployakov-loop potential should be taken into account

$$\delta m^2_{\alpha,ab} = \frac{\partial^2 \Omega_{\bar{q}q}(T,\mu)}{\partial \xi_{\alpha,a}\partial \xi_{\alpha,b}}\Big|_{min} = 3 \sum_{f=x,y} \int \frac{d^3p}{(2\pi)^3} \frac{1}{E_f}$$

$$\left[(A_f^+ + A_f^-) \left(m^2_{f,ab} - \frac{m^2_{f,a} m^2_{f,b}}{2E_f^2} \right) + (B_f^+ + B_f^-) \left(\frac{m^2_{f,a} m^2_{f,b}}{2E_f T} \right) \right]$$

$$A_f^+ = \frac{\Phi e^{-E_f^+/T} + 2\Phi^* e^{-2E_f^+/T} + e^{-3E_f^+/T}}{g_f^+} \qquad g_f^- = \left[1 + 3\Phi^* e^{-E_f^-/T} + 3\Phi e^{-2E_f^-/T} + e^{-3E_f^-/T} \right]$$

$$A_f^- = \frac{\Phi^* e^{-E_f^-/T} + 2\Phi e^{-2E_f^-/T} + e^{-3E_f^-/T}}{g_f^-} \qquad g_f^+ = \left[1 + 3\Phi e^{-E_f^+/T} + 3\Phi^* e^{-2E_f^+/T} + e^{-3E_f^+/T} \right]$$

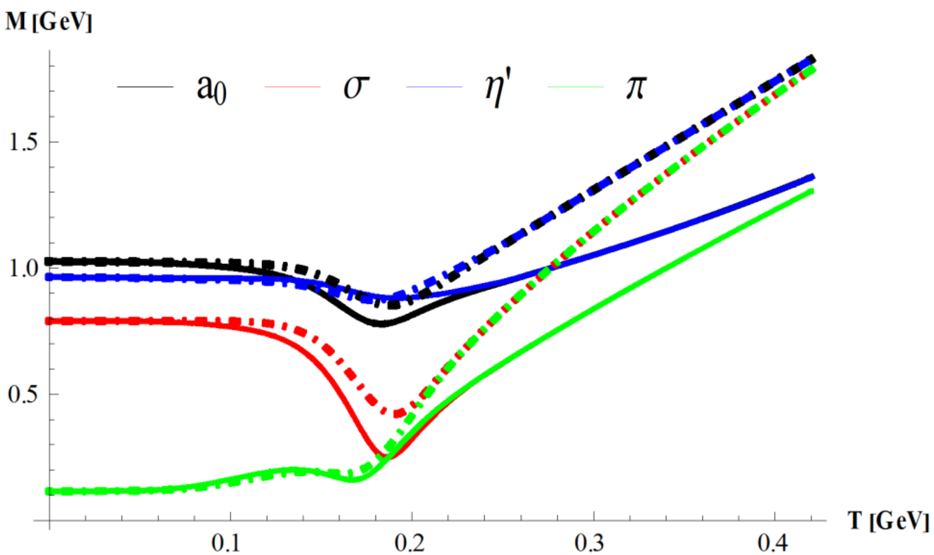
$$B_f^{\pm} = 3(A_f^{\pm})^2 - C_f^{\pm}, \qquad C_f^+ = \frac{\Phi e^{-E_f^+/T} + 4\Phi^* e^{-2E_f^+/T} + 3e^{-3E_f^+/T}}{g_f^+}$$

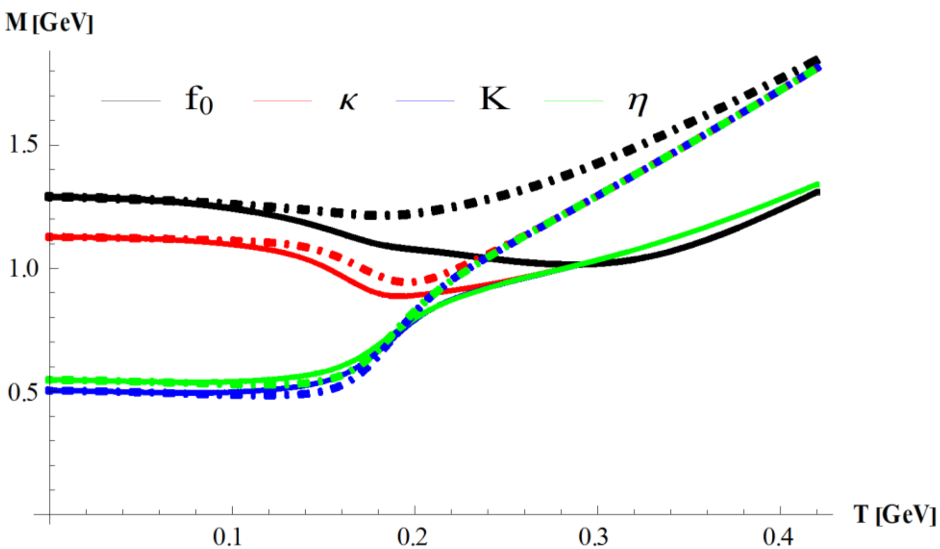
$$C_f^- = \frac{\Phi^* e^{-E_f^-/T} + 4\Phi e^{-2E_f^-/T} + 3e^{-3E_f^-/T}}{g_f^-}$$

0911.2464

With/Without Polyakov-Loop Potential

There figures shown that the comparison **with/ out** ploykov potential





Polyakov loop potential shown the chiral symmetry restoration and result that a sharper mass and degeneration as well as faster than without ploykov loop

Note:-
Solid lines represent to the LSM and dotted lines are the PLSM

QCD in an External Magnetic Field

- It is conjectured that the strongly interacting system can response to an external magnetic field with magnetization and magnetic Susceptibility.

- Both quantities characterize the magnetic properties of the system of interest, QGP.

- Also, the effects of the external magnetic field on the chiral condensates should be reflected in the chiral phase-transition, as well.

- The effects on the deconfinement phase-transition can be studied through their effects on Polyakov-loop.

QCD in an External Magnetic Field

- We have add some restrictions to quarks due to the existence of free charges in the plasma phase.
- To this end, we apply Landau theory (Landau quantization), which quantizes of the cyclotron orbits of charged particles in magnetic fields.

Findings:

- The proposed configuration requires additional temperature to derive the system through the chiral phase-transition.
- The value of the chiral condensates increase with increasing the external magnetic field.

Landau Levels (Quantizations)

- Consider 2D electron system in *x - y* plane with field $\mathbf{B} \parallel \hat{z}$
- Let us choose "Landau gauge" $\mathbf{A} = Bx\hat{y}$
- Then Hamiltonian is

$$H = \frac{1}{2m}(\hat{\mathbf{p}} + e\mathbf{A})^2$$
$$= \frac{1}{2m}\left(\hat{p}_x^2 + \hat{p}_y^2 + 2eBx\hat{p}_y + (eB)^2 x^2\right)$$

- We note that $[H, \hat{p}_y] = 0$, then eigenfuncs of *H* $\psi(x,y) = e^{ik_y y}X(x)$ satisfies

$$\frac{1}{2m}\left(-\hbar^2\nabla^2 + (eB)^2\left(x + \frac{\hbar k_y}{eB}\right)^2\right)X = EX$$

- Exact harmonic oscillator , with *x* shifted by $x_0 = \hbar k_y / eB$
- Then eigensolutions $\psi(x,y) = e^{ik_y y}u_n(x + x_0) = e^{ieBx_0 y/\hbar}u_n(x + x_0)$
- Und n-th eigenfuns with eigenvalues $E_n = \hbar\omega(n + 1/2)$
- Comparing with SHO, cyclotron frequency reads $\omega = \dfrac{eB}{m}$

Landau Levels (Quantizations)

- This is just classical frequency of orbital motion of charged particle in magnetic field.

- The energy levels labeled by *n* called *Landau levels*

- What is degeneracy of each level?

- If width of system in *y*-direction is *Ly*, assume periodic boundary conditions, $\psi(y) = \psi(y + L_y)$
where $k_y L_y = 2\pi\nu$ and ν is integer

- The upper bound of ν reads $0 \leq \nu \leq \frac{eB}{2\pi\hbar} L_x L_y \equiv \nu_{max}$

- The max. # of e occupying *Landau levels* $\nu_{max} = \frac{L_x L_y}{2\pi \ell_B^2}$

Landau Levels (Quantizations)

- We assume that the direction of B goes along z-direction.
- From the magnetic catalysis and by using Landau quantization, we find that when the system is affected by a strong magnetic field, the quark dispersion relation will be modified to be quantized by Landau quantum number, $n \geq 0$, and therefore the concept of dimensional reduction will be applied.

$$E_u = \sqrt{p_z^2 + m_q^2 + |q_u|(2n+1-\sigma)B},$$

$$E_d = \sqrt{p_z^2 + m_q^2 + |q_d|(2n+1-\sigma)B},$$

$$E_s = \sqrt{p_z^2 + m_s^2 + |q_s|(2n+1-\sigma)B},$$

ν

$$\sigma = \pm S/2$$

$$m_q = g\frac{\sigma_x}{2},$$

$$m_s = g\frac{\sigma_y}{\sqrt{2}}$$

From magnetic catalysis the dimensional is reduced

$$T\int \frac{d^3p}{(2\pi)^3} \longrightarrow \frac{|q_f|BT}{2\pi}\sum_{\nu=0}^{\infty}\int\frac{dp}{2\pi}(2-1\delta_{0n})$$

degenerate Landau level

The upper Landau levels $\nu_{max} = \frac{\Lambda_{QCD}^2}{2|q_f|B}$

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LσM in external magnetic field

For the Abelian gauge field, the influence of the external magnetic field, A_μ^M is given by the covariant derivative

$$D_\mu = \partial_\mu - i\,A_\mu - i\,Q\,A_\mu^M,$$

$Q = \mathtt{diag}(q_u, q_d, q_s)$ is a matrix defined by the quark electric charges

The coupling between the effective gluon field and quarks, and between the magnetic field, B, and the quarks is implemented through the covariant derivative

$$\mathcal{L}_q = \sum_f \overline{\psi}_f \big(i\gamma^\mu D_\mu - g T_a (\sigma_a + i\gamma_5 \pi_a)\big)\psi_f$$

A_μ coupling of the quarks to the Euclidean gauge field

T_a Gell-Man matrices

g flavor-blind Yukawa coupling

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PLσM in external Magnetic Field

The coupling between the Polyakov loop and the quarks is given by the covariant derivative of

$$D_\mu = \partial_\mu - iA_\mu$$

in PLSM Lagrangian

The quarks and antiquark contribution based on Landau quantization and magnetic catalysis concepts

$$\Omega_{\bar{\psi}\psi}(T,B) = -2\sum_f \frac{|q_f|BT}{2\pi} \sum_{\nu=0}^{\infty} \int \frac{dp}{2\pi} (2 - 1\delta_{0n}) \left\{ \ln\left[1 + 3\left(\phi + \phi^* e^{-\frac{(E_f-\mu)}{T}}\right) e^{-\frac{(E_f-\mu)}{T}} + e^{-3\frac{(E_f-\mu)}{T}}\right] + \ln\left[1 + 3\left(\phi^* + \phi e^{-\frac{(E_f+\mu)}{T}}\right) e^{-\frac{(E_f+\mu)}{T}} + e^{-3\frac{(E_f+\mu)}{T}}\right]\right\}. \quad (12)$$

The fermionic vacuum loop contribution

$$\Omega_{q\bar{q}}^{vac}(\sigma_x, \sigma_y) = -2Nc \sum_f \int \frac{d^3p}{(2\pi)^3} E_f = \frac{-Nc}{8\pi^2} \sum_f m_f^4 \ln\left(\frac{m_f}{\Lambda_{QCD}}\right)$$

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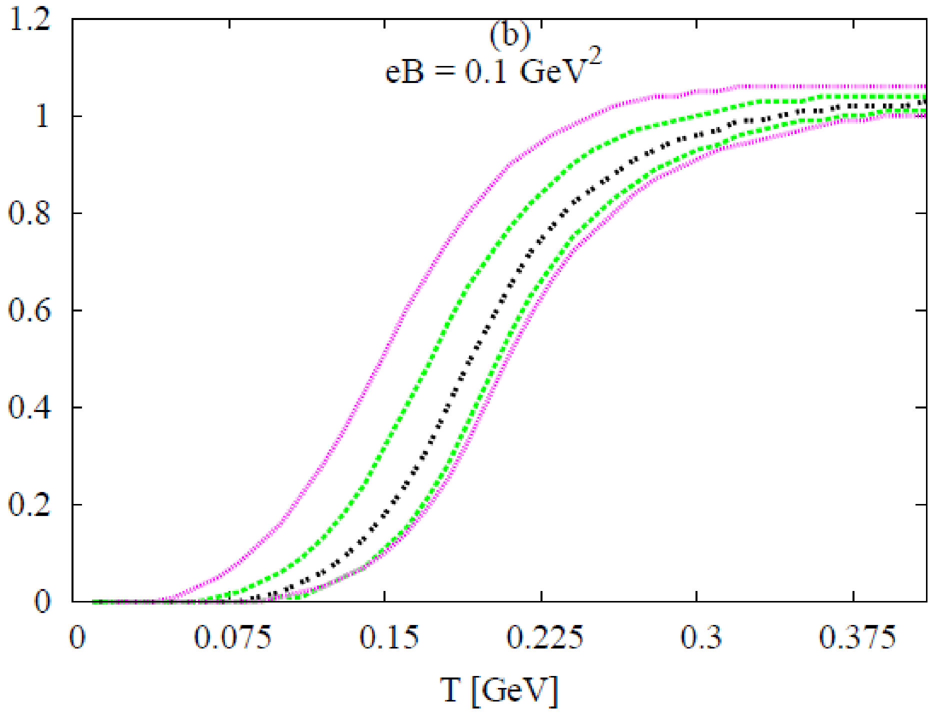
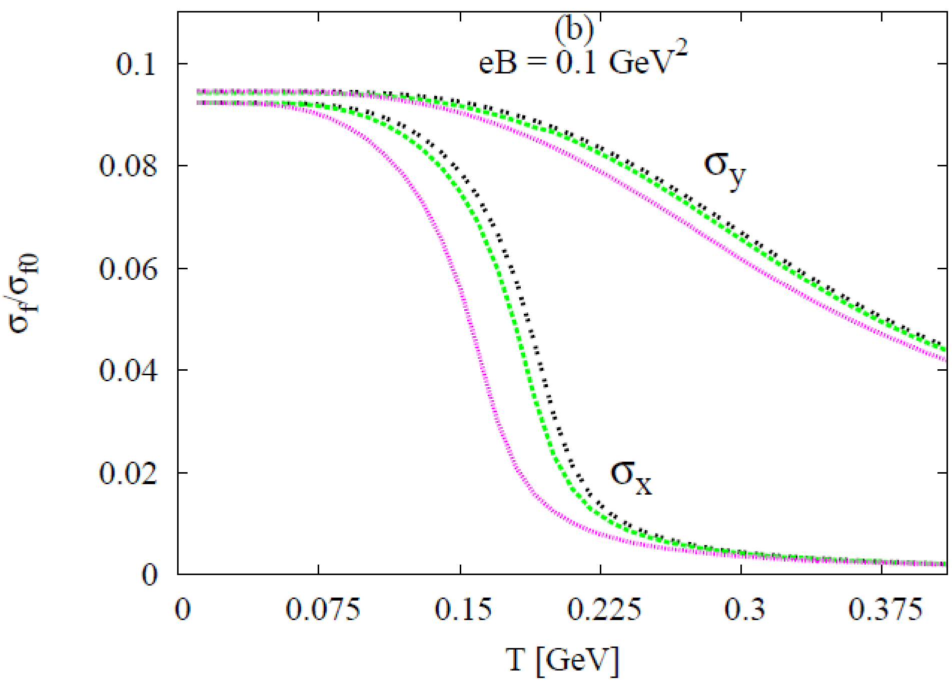
Condensates and Order Parameters





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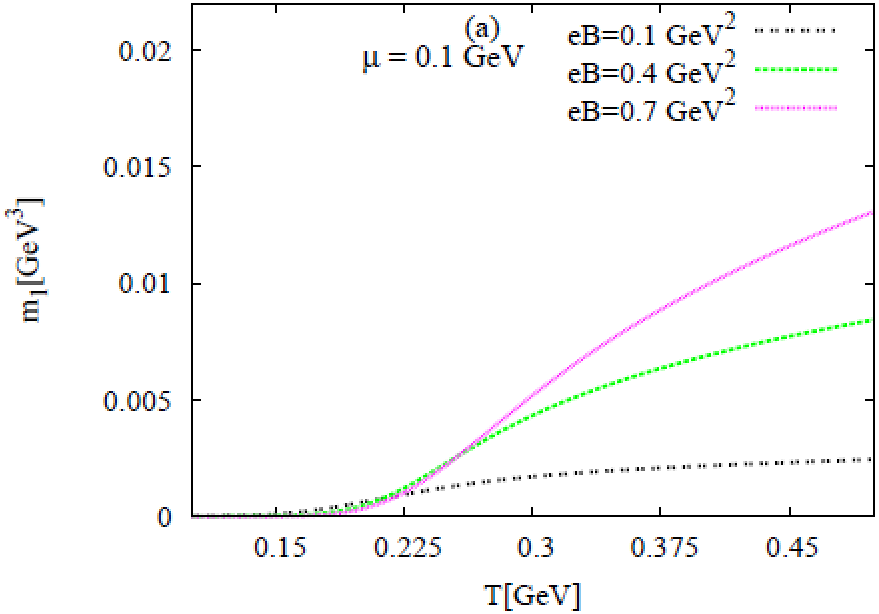
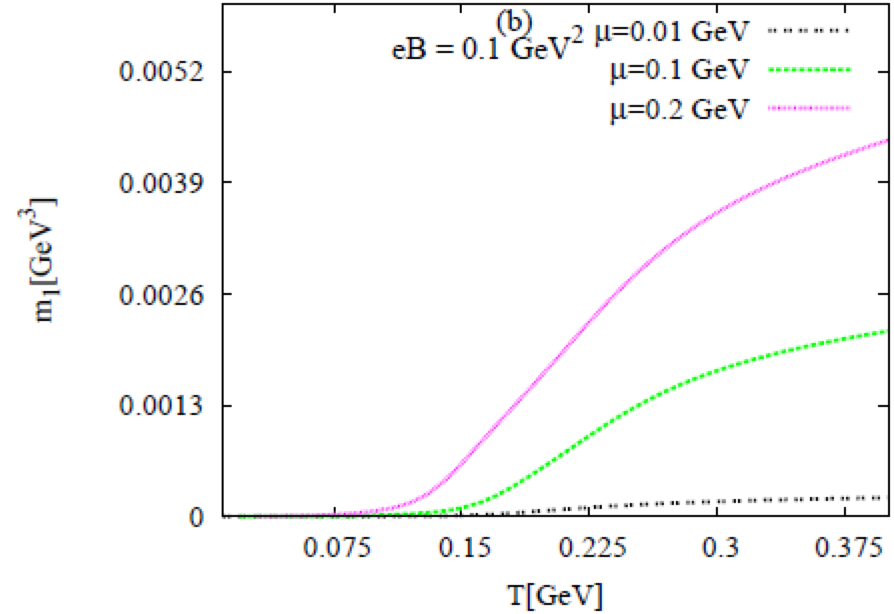
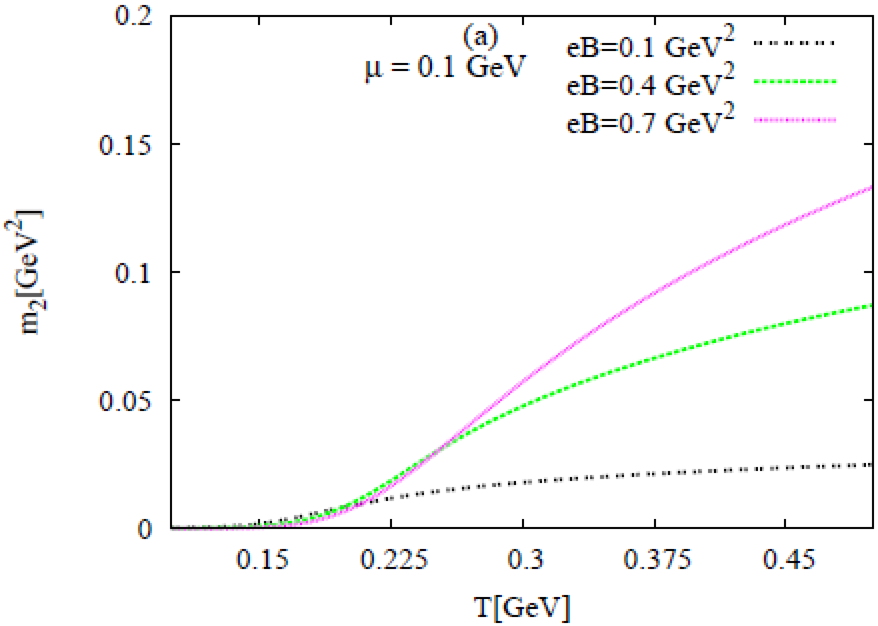
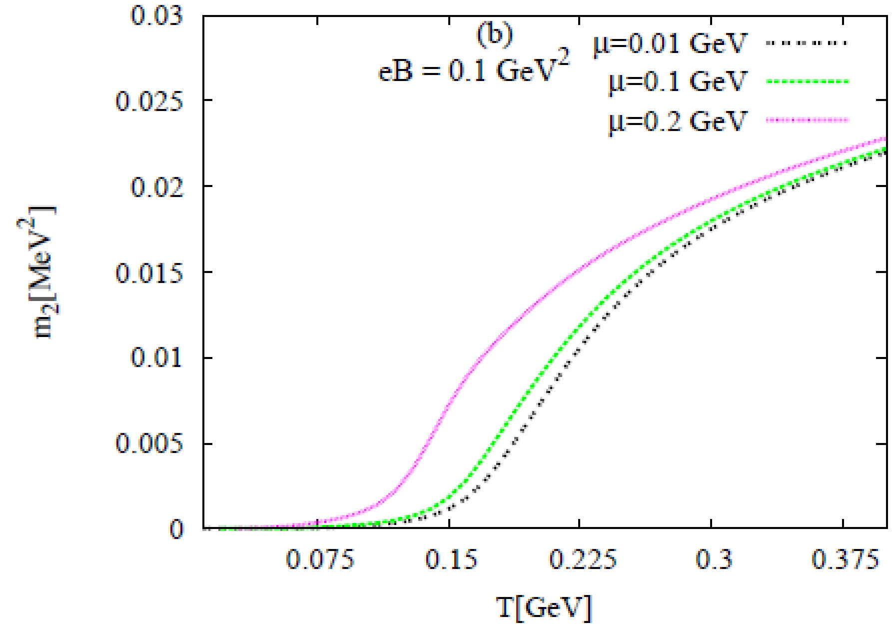
Higher Moments

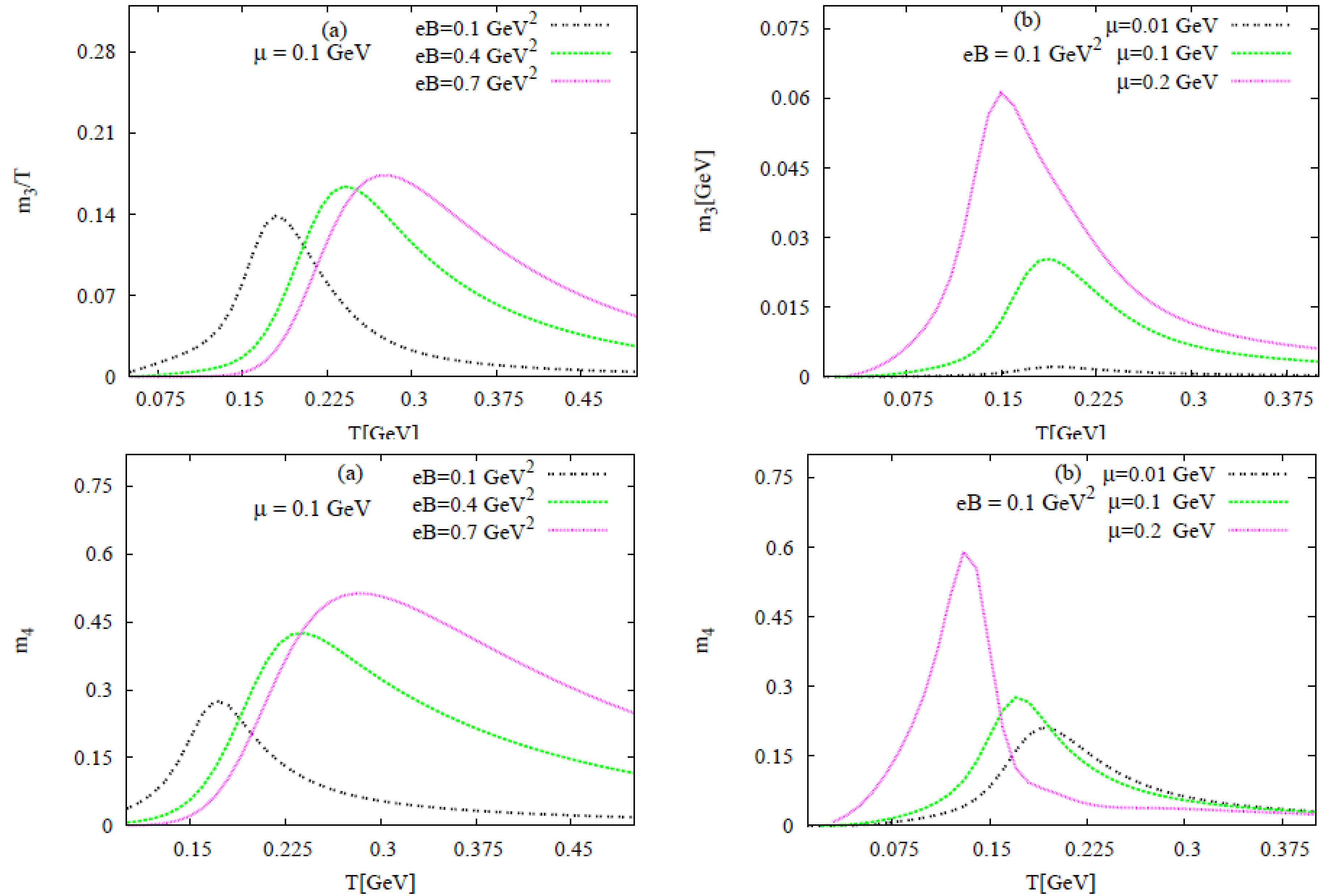








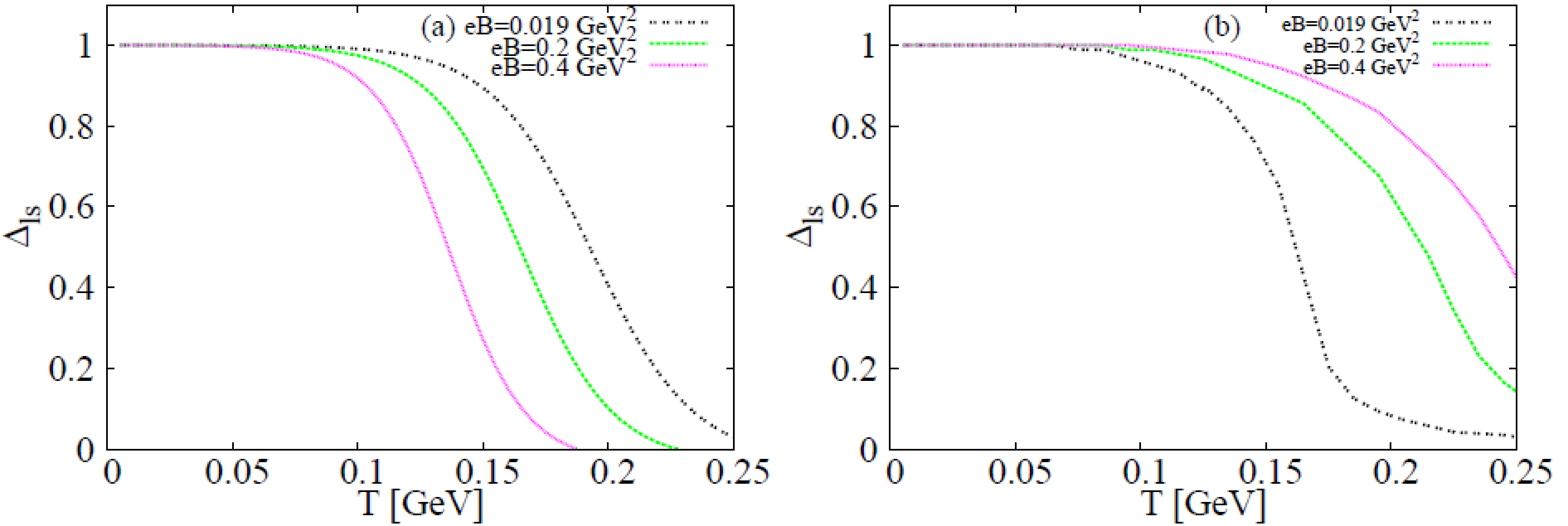
Higher Moments



Subtracted Chiral-Condensate

Is a dimensionless quantity reflecting the difference between non-strange and strange condensates

$$\Delta_{q,s}(T) = \frac{\langle \bar{q}q \rangle - \frac{m_q}{m_s}\langle \bar{s}s \rangle}{\langle \bar{q}q \rangle_0 - \frac{m_q}{m_s}\langle \bar{s}s \rangle_0},$$



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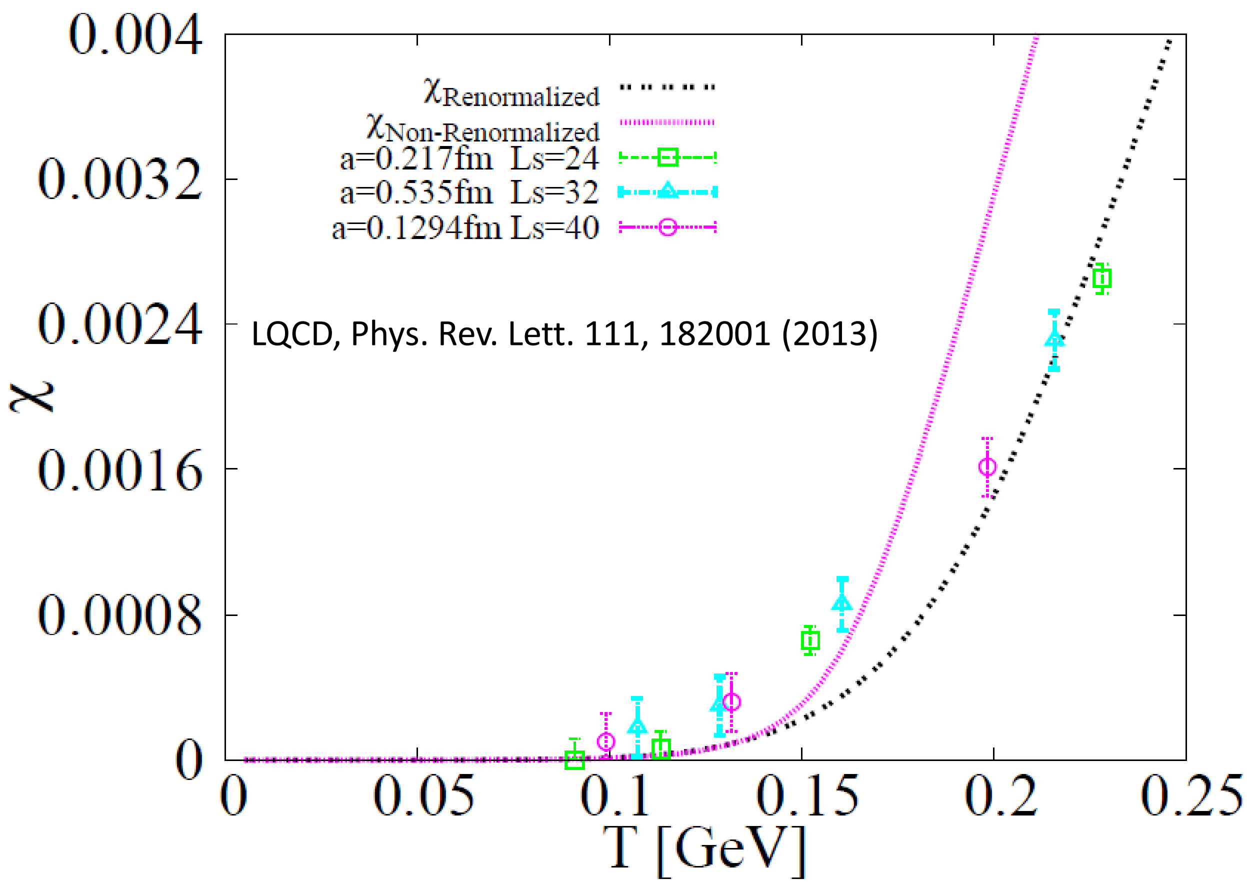
Magnetic Susceptibility

2nd derivative for free energy density f with respect to magnetic field

$$\chi_m = -\frac{\partial^2 f}{\partial(eB)^2}\bigg|_{eB=0}$$



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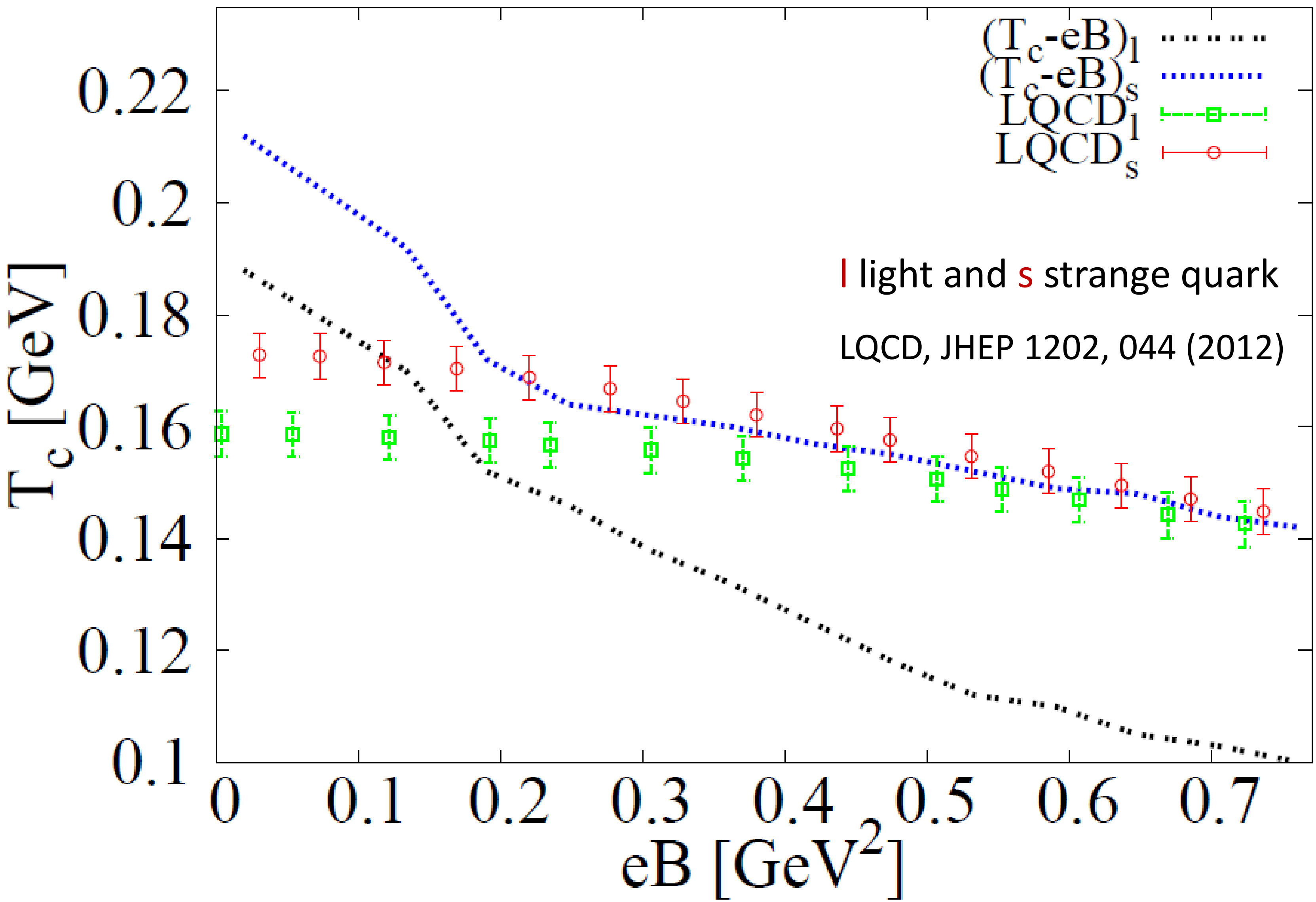
Critical Temperature



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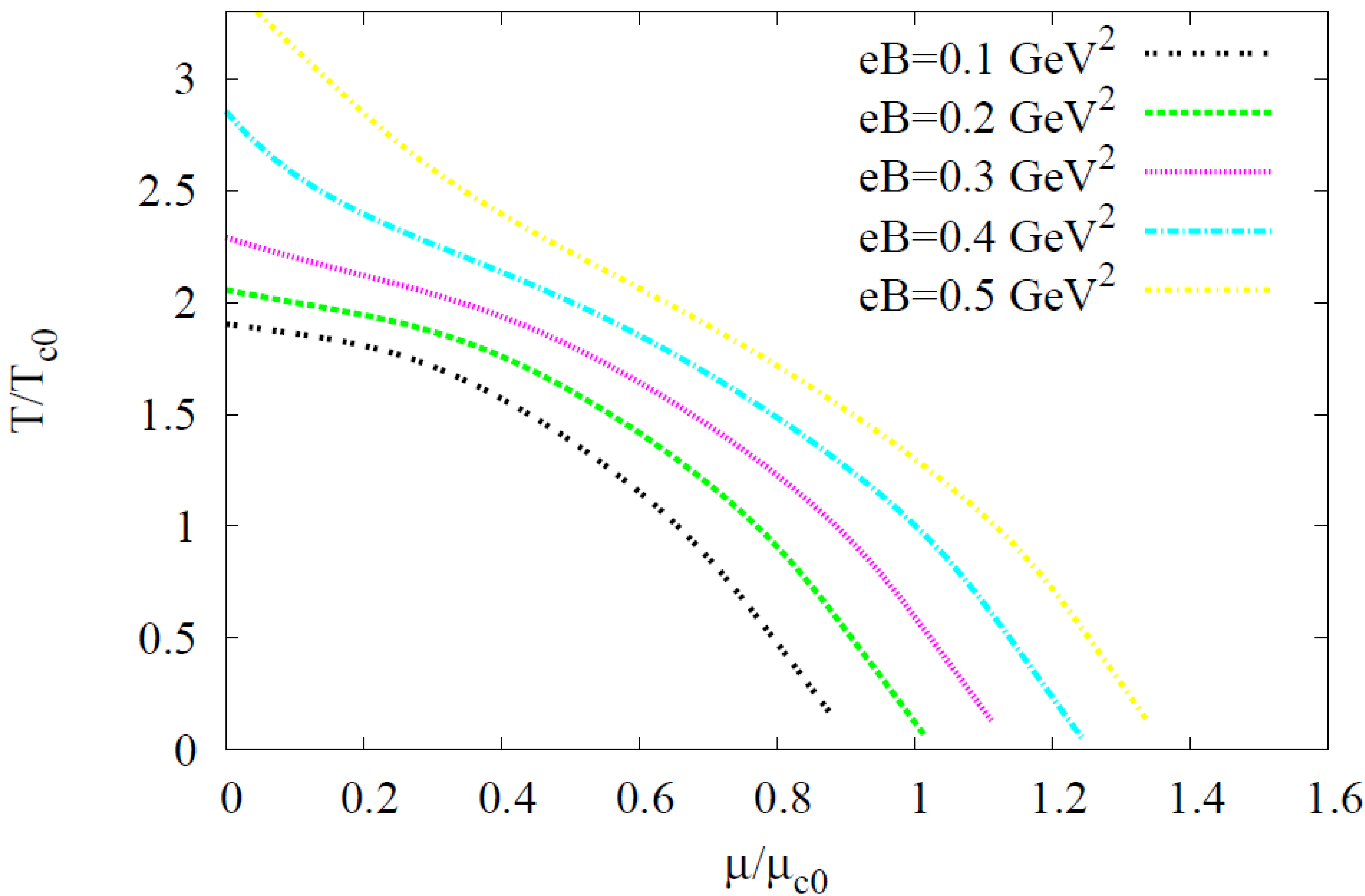
Chiral Phase-Diagram



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شكرا جزيلا لكريم لاهتمامكم!

Thanks for your Attention!
Vielen Dank für Ihre Aufmerksamkeit!
Grazie per la vostra attenzione!

http://atawfik.net/