A Short Introduction to QCD

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Outline

- **QCD in Nutshell**
- **Elementary Particles**
- **Aces/Quarks Model**
- **Colors dof**
- **QED ⇔ QCD**
- **Asymptotic Freedom and Running Strong Coupling**
- **Lattice QCD (Thermodynamics)**

QCD in Nutshell

George Zweig (Erice 2014), Antonino Zichichi **and Murray Gell-Mann** (Erice 2012)

Development of QCD went through three main phases:

- **Discovery of Quarks (SU(3)) and/or Aces in early 1960's**
- **Determining Color Interactions (asymptotic freedom) in early 1970's**
- **Path Integral Descritization ➔ LQCD**

A. QCD is firmly established as QFT of strong interactions.

B. QCD is described by a remarkably simple Lagrangian,

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_f + \mathcal{L}_b \\ &= \sum_f \bar{\Psi}^f(r)(i\gamma_\mu D^\mu - m_f)\Psi^f(r) - \frac{1}{2}\mathrm{Tr}\,\mathcal{G}_{\mu\nu}(r)\mathcal{G}^{\mu\nu}(r)\end{aligned}$$

QCD in Nutshell

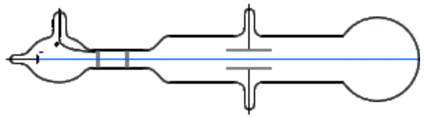
Despite its succeed in explaining and predicting various phenomena, it is a difficult theory to understand.

Nevertheless, different techniques have been developed, for instance:

- **perturbative techniques particularly suitable at very high-energy,**
- **numerical simulation techniques (on lattice) suitable for static or low-energy quantities**
- **special limits, such as large Nc-Limit (simpler QCD) and**
- **effective theories, LσM, NJL, etc. [second lecture tomorrow]**

Particle Discoveries in Atoms

Electron



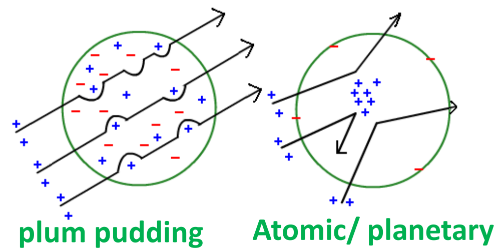
Nucleons:

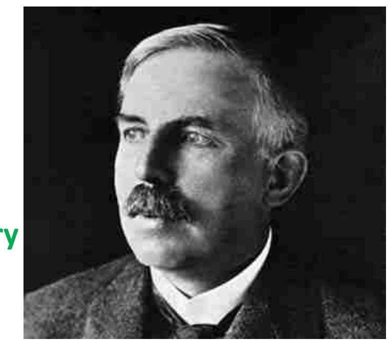
- Proton (nucleus of hydrogen)
- Neutron (nucleus of helium)

Photon (light!)

JJ Thomson: "discovering electrons and isotopes, and inventing the mass spectrometer". 1906 Nobel Prize in Physics for *the discovery of the electron and for his work on the conduction of electricity in gases*.

1909 Atomic Model: Corner-Stone experiment carried out by Hans Geiger and Ernest Marsden, under Ernest Rutherford at the Physical Laboratories of the University of Manchester





http://www.mhhe.com/physsci/chemistry/essentialchemistry/flash/ruther14.swf

In 1932, **James Chadwick** bombarded beryllium (Be) with alpha particles. The radiation emitted by beryllium is allowed to incident on a paraffin wax. Protons shot out form the paraffin wax were found.

H.Bethe: "beryllium radiations".

$${}^{9}_{4}\mathrm{Be} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{12}_{6}\mathrm{C} + {}^{1}_{0}\mathrm{n}$$

Elementary Particles!

Particle Discoveries beyond Atoms

Cloud Chamber: 1911, C.T.R. Wilson (Nobel Prize)

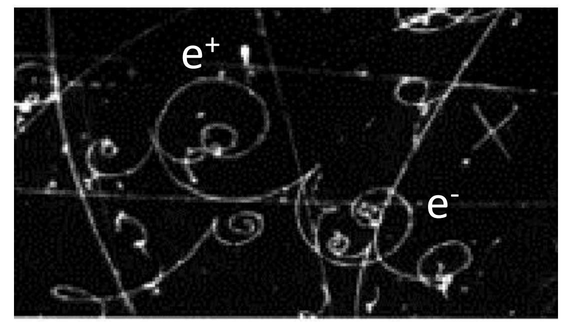
Vapors condensate into tiny droplets around ionized atoms along charged particle trajectories

Photo of α-particles emitted by radioactive source and seen in cloud chamber

Discovery of positron, Carl Anderson 1932

positively charged electrons detected in cosmic rays passing through a cloud chamber immersed in a magnetic field

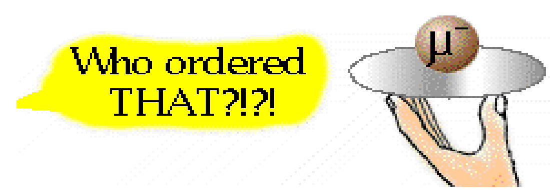
Photon conversions $\gamma \rightarrow e^+ e^-$ in a bubble chamber



- discovery of antimatter
- positrons predicted by Dirac in 1928 from relativistic theory of electrons.

Discovery of muon, Neddermeyer and Anderson 1937

- m_e < m_mu < m_p,
- No nuclear interactions (heavier versions of electrons)
- β decay: an electron and two invisible neutrinos: $\mu^- \rightarrow \nu_\mu e^- \nu_e$
- first encounter of the **generation problem**



Nobel laureate I.I. Rabi

Particle Discoveries beyond Atoms

Prediction of pion existence Yukawa 1935

- Nucleons (protons and neutrons) are held together by stronger force than electrostatic repulsion of protons
- In 1935 Yukawa predicted existence of a mediator of the strong interactions. Estimated its mass to be around 0.1 GeV.

Discovery of pions Cecil Powell 1947

- detected in cosmic rays captured in photographic emulsion
- Unlike muons they do interact with nuclei
- charged pions eventually decay to muons: $\pi^- \rightarrow \mu^- \nu_\mu$
- view of the particle world seemed complete for entire two months...

Discovery of strange meson (kaon) Rochester, Butler 1947

- cosmic ray particles with masses in between pions and protons which were just like pions except for strangely long lifetime (decay to pions or a muon and neutrino)
- Always produced in pairs
- Mass ~ 0.5 GeV

Periodic tables of particles (late 1950's)!

	Q= -1	Q= 0	Q=+1
S=+1		K^0	K^+
S= 0	π^+	π^0 ,η	π^+
S= -1	K^+	K^0	

Q - Electric Charge

Spin 0 Meson Octet

	Q= -1	Q= 0	Q=+1
S= 0		n	p
S= -1	Σ^-	Σ^0 ,Λ	Σ^+
S= -2	Ξ^+	Ξ^0	

Spin 1/2 Baryon Octet

	Q=-1	Q= 0	Q=+1	Q=+2
S= 0	Δ^-	Δ^0	Δ^+	Δ^{++}
S= -1	Σ^{*-}	Σ^{*0}	Σ^{*+}	
S= -2	Ξ^{*-}	Ξ^{*0}		
S= -3	Ω^-			

Spin 3/2 Baryon Decimet

S - Strangeness

Similar masses in each multiplet

Elementary Particles!

THE PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 76, NO. 12 — DECEMBER 15, 1949

Are Mesons Elementary Particles?

E. FERMI AND C. N. YANG*
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received August 24, 1949)

The hypothesis that π-mesons may be composite particles formed by the association of a nucleon with an anti-nucleon is discussed. From an extremely crude discussion of the model it appears that such a meson would have in most respects properties similar to those of the meson of the Yukawa theory.

"A motivation for the Aces Model": G. Zweig

Elementary Particles!

Elementary Particles

An account of the abstract theoretical ideas which physicists use to help them understand the material world. These ideas begin to show some order in the jumble of subatomic particles

E. P. Rosenbaum and Murray Gell-Mann

Scientific American, 197, 72-86 (1957)

HYPERONS AND HEAVY MESONS (SYSTEMATICS AND DECAY[1])

By Murray Gell-Mann
Department of Physics, California Institute of Technology, Pasadena, California
and
Arthur H. Rosenfeld
Department of Physics and Radiation Laboratory, University of California Berkeley, California

Ann. Rev. Nucl. Sci, 7, 407-478 (1957).

Gell-Mann and Rosenfeld: *"summarize the information … available, both experimental and theoretical, on the classification and decays of hyperons and heavy mesons".*

Aces Model

Zweig, Erice 2014

Point particles

Spin 1/2 leptons	
Particle	Mass
e^-	1
μ^-	206.7
ν	0

Spin 1 photon	
Particle	Mass
γ	0

Spin 1/2 baryons		
Multiplet	Particle	Mass (m_e)
Ξ	Ξ^0	?
	Ξ^{-1}	2585
Σ	Σ^{-1}	2341
	Σ^+	2325
	Σ^0	2324
Λ	Λ	2182
N	n	1838.6
	p	1836.1

Spin 0 mesons		
Multiplet	Particle	Mass
π	π^+	273.2
	π^{-1}	273.2
	π^0	264.2
K	K^+	966.5
	K^-	966.5
	K_1^0	965
	K_2^0	965

19 Elementary particles reviewed by Rosenfeld and Gel-Mann

G. Zweig remarked that no resonances are mentioned in this list!

Aces Model

Zweig, Erice 2014

Theory related to experiment:

- Sakata model: Wrong baryons
- Particle classification (no dynamics): such as G(2) & SU(3)

Dynamics BUT no classification:

- Bootstrap, Fred Zacharisen (1961)

Experimental physics:

Additional particles were discovered since 1957 [previous slide]

- Point particles: the 4th lepton (μ_ν),
- Extended particles: Ξ^0 and η,
- Meson resonances: ρ, ω, K^*, ϕ, ...,

Aces Model: suppression of ϕ

G. Zweig: *"The Beginning of the End"*.

Zweig, Erice 2014

Zweig discussed this paper P.L. Connolly, et al., "Existence and Properties of the Meson", Phys. Rev. Lett. 10, 371 (1963) with Feynman

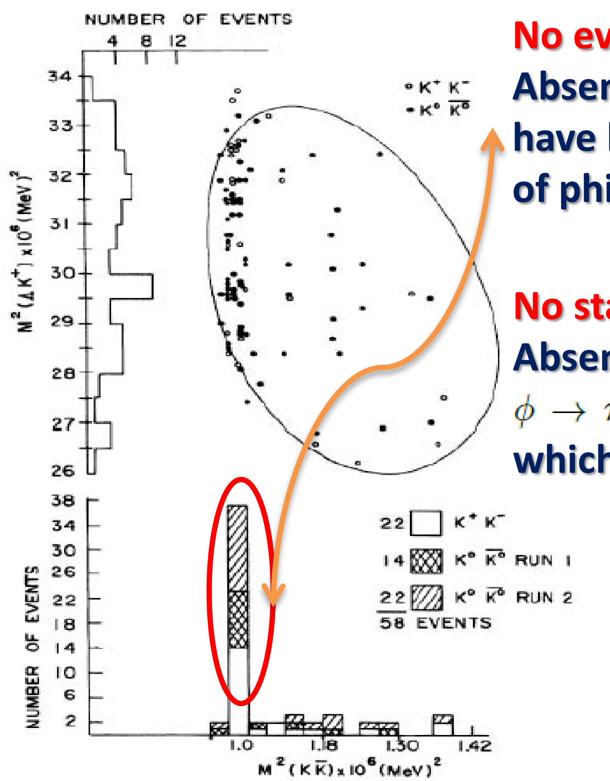


FIG. 1. Dalitz plot for the reaction $K^- + p \rightarrow \Lambda + K + \bar{K}$. The effective-mass distribution for $K\bar{K}$ and for ΛK^+ are projected on the abscissa and ordinate (see reference 7).

No evidence for phi!

Absence of decay should have broadened the width of phi

No statistical peak

Absence of the decay

$\phi \rightarrow \pi^+ + \pi^- + \pi^0$

which is though as

$$\phi \rightarrow \rho + \pi$$
$$\rho \rightarrow \pi + \pi$$

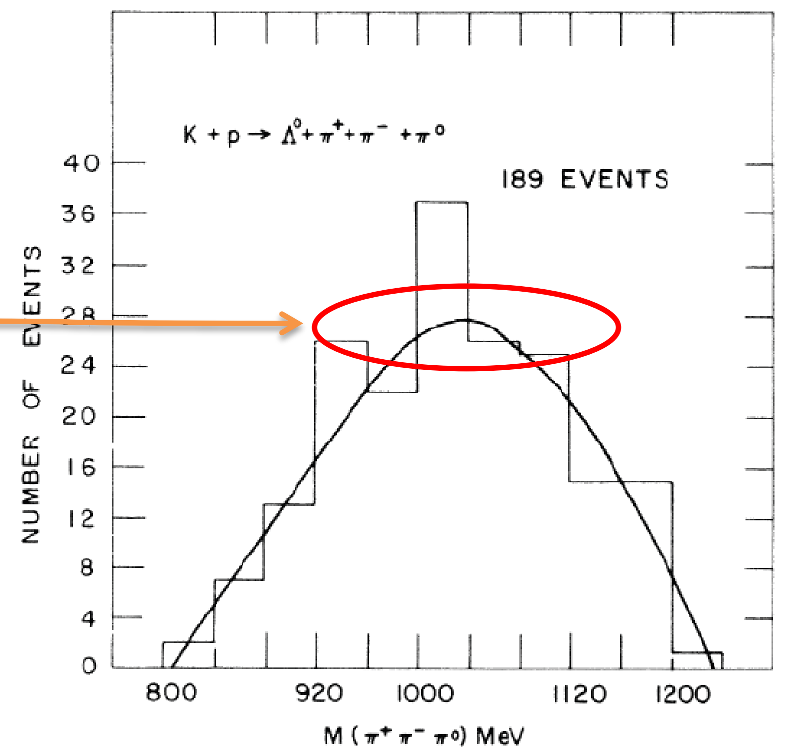


FIG. 4. The $M(\pi^+\pi^-\pi^0)$ distribution from the reaction $K^- + p \rightarrow \Lambda + \pi^+ + \pi^- + \pi^0$ after removing $Y_1{}^*$ production events (see text).

Aces Model

$$\frac{\Gamma_{K\bar{K}}}{\Gamma_{\rho\pi}} \sim \left(\frac{p_{K\bar{K}}}{p_{\rho\pi}}\right)^3 ,$$

p momentum
In rest frame
of phi

$$= \quad 1/4 \text{ (expected)},$$
$$\geqslant \quad 35 \quad \text{(observed)}.$$

"The observed rate [for $\phi \to \rho + \pi$] is lower than ... predicted values by one order of magnitude; however the above estimates are uncertain by at least this amount so that this discrepancy need not be disconcerting."

According to Zweig, An unexpected suppression in strong interactions

Suppression implies symmetry otherwise dynamics

Hadrons should have constituents obeying a simple dynamical rule when they decay.

Mesons are constructed out of fermion-antifermion pairs

- Assume hadrons have *point* constituents a (aces):

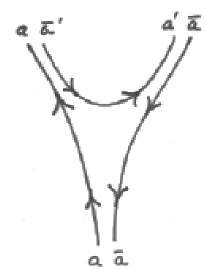
$$[\,N_0,\ \Lambda_0\,] \ \& \ [\,\bar{N}_0,\ \bar{\Lambda}_0\,]$$
$$[\,(p_0,\ n_0),\ \Lambda_0\,] \ \& \ [\,(\bar{p}_0,\ \bar{n}_0),\ \bar{\Lambda}_0\,]$$

d. 

Vector mesons as "deuces"

FIG. 2, CERN report TH-401, January 1964.

- A rule for *decay* (in modern notation):



Meson decay: a is an ace, $\bar{a}$ an antiace.

– Implies $\phi \not\to \rho + \pi$

Aces Model

Zweig, Erice 2014

- A rule for meson masses:

  Mass $= \Sigma$ *constituent masses* $+$ *energies of interaction*, $|\Delta m| > |\Delta E|$.

  – Identical binding energies:

  $$m^2(\rho) \approx m^2(\omega) \; < \; m^2(K^*) \; < \; m^2(\phi).$$
  $$750^2 \qquad 784^2 \qquad 888^2 \qquad 1018^2$$

  – $\frac{1}{2}(E^{\bar{\Lambda}_0}_{\Lambda_0} + E^{\bar{N}_0}_{N_0}) \approx E^{\bar{N}_0}_{\Lambda_0} \approx E^{\bar{\Lambda}_0}_{N_0}, \;\; N_0 = p_0, n_0$:

  $$m^2(\phi) \approx 2m^2(K^*) - m^2(\rho).$$
  $$1018^2 \qquad 1007^2$$

- Make baryons from 3 aces aaa, not $aa\bar{a}$ (Sakata).

  $B = \frac{1}{3}$,

  $Q = e[I_z + \frac{B+S}{2}]$,

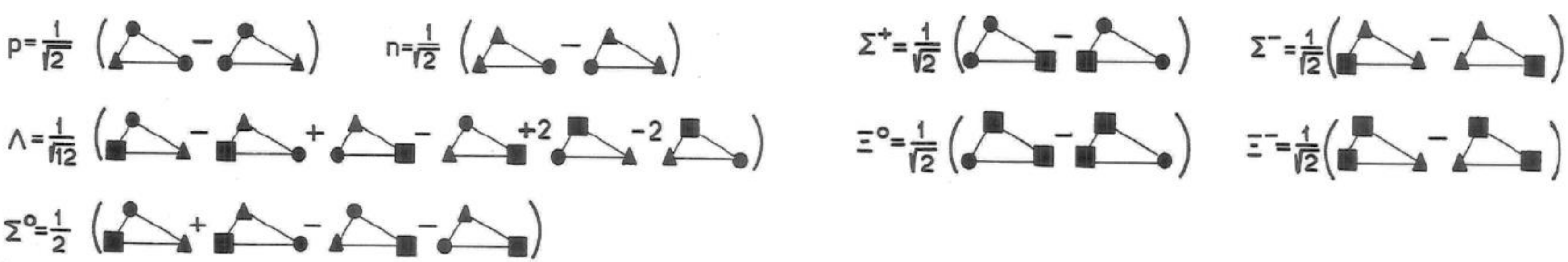
  $[\,(p_0,\; n_0),\; \Lambda_0\,] \rightarrow [\,(\frac{2}{3},\; -\frac{1}{3}),\; -\frac{1}{3}\,]$

  $3 \times 3 \times 3 = 1 + 8 + 8 + 10.$

- Mechanism for SU(3) & SU(2) symmetry breaking

  – SU(3): $m(p_0) = m(n_0) < m(\Lambda_0)$,

  – SU(2): $m(p_0) < m(n_0)$.



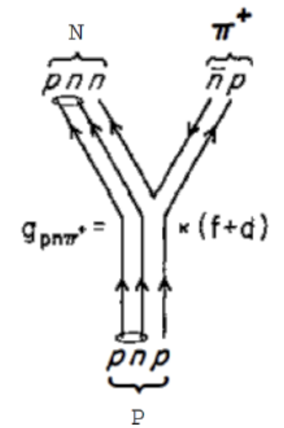
Aces Model

Zweig, Erice 2014

- Interactions: *Aces, not hadrons, interact.*
  - Strong interaction couplings: "Zweig's rule" (what's allowed!)
  - Electromagnetic and weak couplings:

$$a \to a + \gamma$$
$$a \to a' + e^- + \nu$$

(Identical to the "current-quark" model)



Graphical representation of the meson-baryon coupling. The "little loop" encloses antisymmetrized aces. The subscript "0" on aces is suppressed.

- Hadrons have point constituents called aces
- Aces $\leftrightarrow$ Leptons
- Origin of SU(3) symmetry
- Beyond SU(3) symmetry:

∗ Baryons only in 1, 8, 10, Mesons only in 1, 8, and 9.

∗ Hadrons have an $\vec{L}$ and an $\vec{S}$.

$L = 0$ mesons:

$(\uparrow\downarrow)\ J^{PC} = 0^{-+}$ and $(\uparrow\uparrow)\ 1^{--}$

∗ $L = 0$ baryons:

$(8,\ J^P = \frac{1}{2}^+)$, $(10,\ \frac{3}{2}^+)$, and $(1,\ \frac{1}{2}^-)$

∗ Higher L excitations.

∗ 0^{--}; 0^{+-}, $1^{-+}, \cdots$ forbidden for any L

Quarks Model

Key Assumption: Isospins of know *elementary* particles

Franz Muheim

For examples, P and N (baryons with almost same mass),

- having isospin ½, analog to ↑and ↓of spin ½ and
- strong potentials Vpp ≈ Vpn ≈ Vnn
- The Isospin is conserved in strong interaction allowing the calculation of
  - ratios and
  - cross-sections
- Thus they are useful for hadron-classification (isospin muliplet $|I, I_3>$)

$$\eta = |0,0\rangle$$

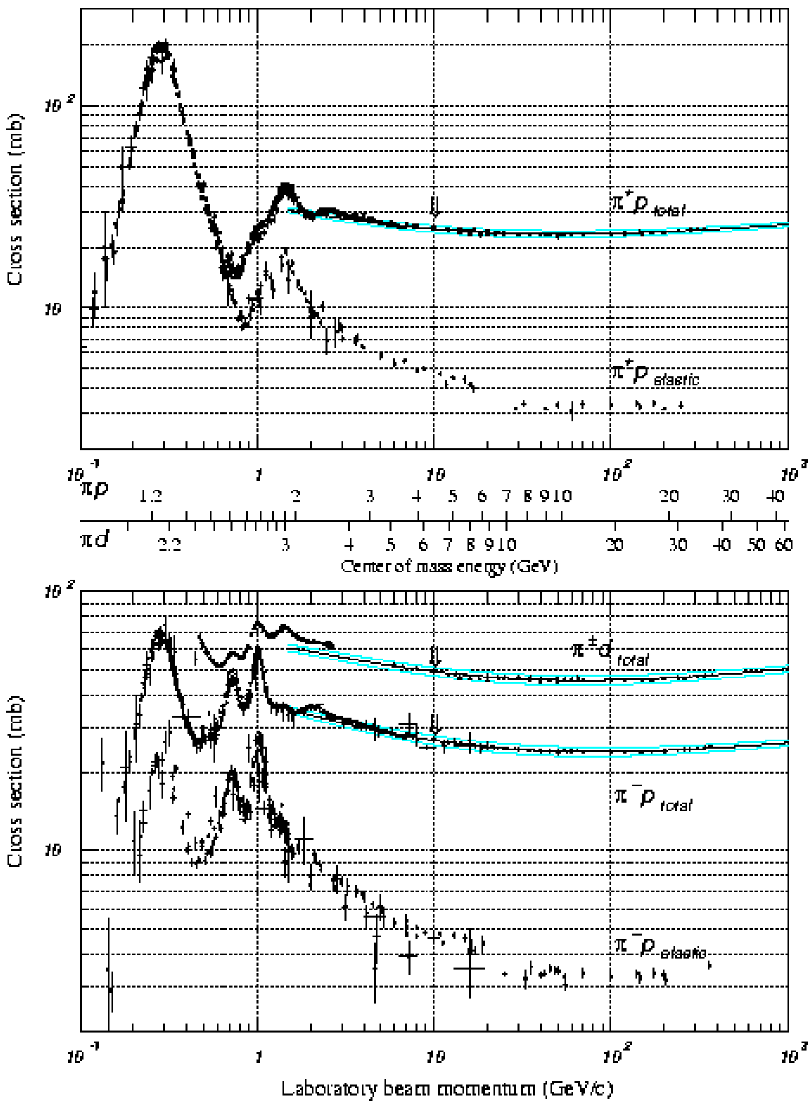
$$p = \left|\tfrac{1}{2}\,,\tfrac{1}{2}\right\rangle \qquad n = \left|\tfrac{1}{2}\,,-\tfrac{1}{2}\right\rangle$$

$$\pi^+ = |1,1\rangle \qquad \pi^0 = |1,0\rangle \qquad \pi^- = |1,-1\rangle$$

$$\Delta^{++} = \left|\tfrac{3}{2}\,,\tfrac{3}{2}\right\rangle \qquad \Delta^{+} = \left|\tfrac{3}{2}\,,\tfrac{1}{2}\right\rangle \qquad \Delta^{0} = \left|\tfrac{3}{2}\,,-\tfrac{1}{2}\right\rangle \qquad \Delta^{-} = \left|\tfrac{3}{2}\,,-\tfrac{3}{2}\right\rangle$$

The Quark Model naturally explains the Isospin, $I_3 = \frac{1}{2}(n_u - n_d + n_{\bar{d}} - n_{\bar{u}})$

Example: Δ(1232) Resonance



Production:

$$\pi^+ p \to \Delta^{++} \to \pi^+ p$$
$$\pi^- p \to \Delta^{0} \to \pi^- p$$
$$\pi^- p \to \Delta^{0} \to \pi^0 n$$

Isospin:

$$\pi^+ p: \quad |1,1\rangle\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle = \left|\tfrac{3}{2},\tfrac{3}{2}\right\rangle$$
$$\pi^- p: \quad |1,-1\rangle\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle = \sqrt{\tfrac{1}{3}}\left|\tfrac{3}{2},-\tfrac{1}{2}\right\rangle - \sqrt{\tfrac{2}{3}}\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle$$
$$\pi^0 n: \quad |1,0\rangle\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle = \sqrt{\tfrac{2}{3}}\left|\tfrac{3}{2},-\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}}\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle$$

Matrix Element:

$$M\left(\pi^+ p \to \Delta^{++} \to \pi^+ p\right) = M_3$$
$$M\left(\pi^- p \to \Delta^{0} \to \pi^- p\right) = \tfrac{1}{3} M_3 + \tfrac{2}{3} M_1$$
$$M\left(\pi^- p \to \Delta^{0} \to \pi^0 n\right) = \tfrac{\sqrt{2}}{3} M_3 - \tfrac{\sqrt{2}}{3} M_1$$

$$M_3 = \left\langle \tfrac{3}{2}\right| H_3 \left|\tfrac{3}{2}\right\rangle$$
$$M_1 = \left\langle \tfrac{1}{2}\right| H_1 \left|\tfrac{1}{2}\right\rangle$$

Cross-Section:

$$\sigma\left(\pi^+ p \to \Delta^{++} \to \pi^+ p\right) \approx 200\ \text{mb} \approx 9\text{x}$$
$$\sigma\left(\pi^- p \to \Delta^{0} \to \text{all}\right) \approx 70\ \text{mb} \approx 3\text{x}$$
$$\sigma\left(\pi^- p \to \Delta^{0} \to \pi^- p\right) \approx 23\ \text{mb} \approx 1\text{x}$$

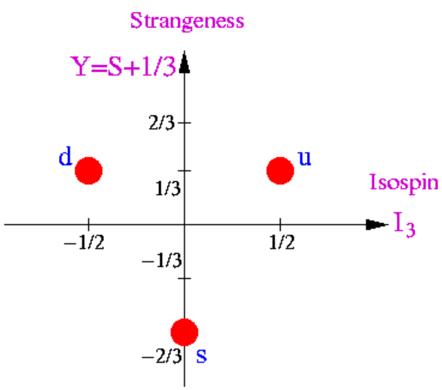
Quarks Model

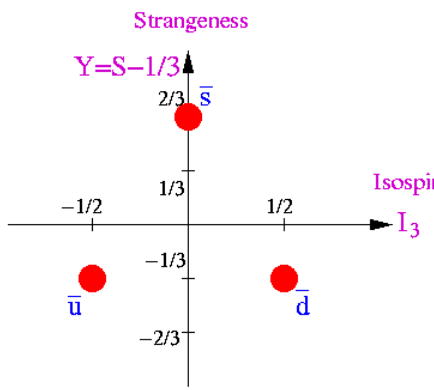
Quark	Charge Q [e]	Isospin $\|I, I_3\rangle$	Strange-ness S
up (u)	+2/3	$\|\frac{1}{2}, +\frac{1}{2}\rangle$	0
down (d)	-1/3	$\|\frac{1}{2}, -\frac{1}{2}\rangle$	0
strange (s)	-1/3	$\|0,0\rangle$	-1

Quarks	B	Q	I_3	Y
$q_1(up)$	$+\frac{1}{3}$	$+\frac{2}{3}$	$+\frac{1}{2}$	$+\frac{1}{3}$
$q_2(down)$	$+\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$+\frac{1}{3}$
$q_3(strange)$	$+\frac{1}{3}$	$-\frac{1}{3}$	0	$-\frac{2}{3}$

Antiquarks	B	Q	I_3	Y
$\bar{q}_1(anti-up)$	$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{2}$	$-\frac{1}{3}$
$\bar{q}_2(anti-down)$	$-\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{2}$	$-\frac{1}{3}$
$\bar{q}_3(anti-strange)$	$-\frac{1}{3}$	$+\frac{1}{3}$	0	$+\frac{2}{3}$

$$I_3 = Q - 2Y$$





$$Q=I_3+(B+S)/2$$

Additive quark qn are related (all independent)

Quarks Model

$$3 \times 3 \times 3 = 10 + 8 + 8 + 1$$

Baryons ($J^P = 1/2^+, 3/2^+$):

- spin-1/2 with $L = 0$,
- spin-3/2 with $L = 0$.

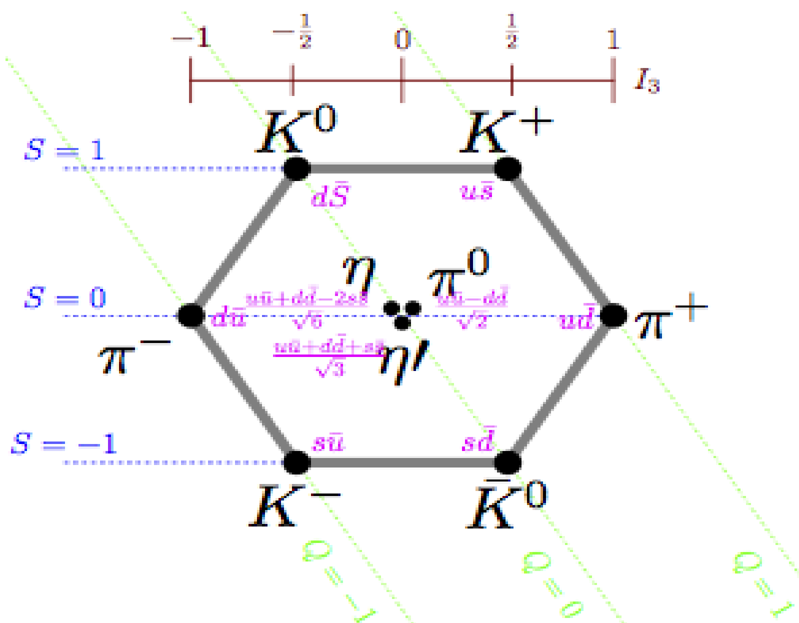
$$3 \times \bar{3} = 8 + 1$$

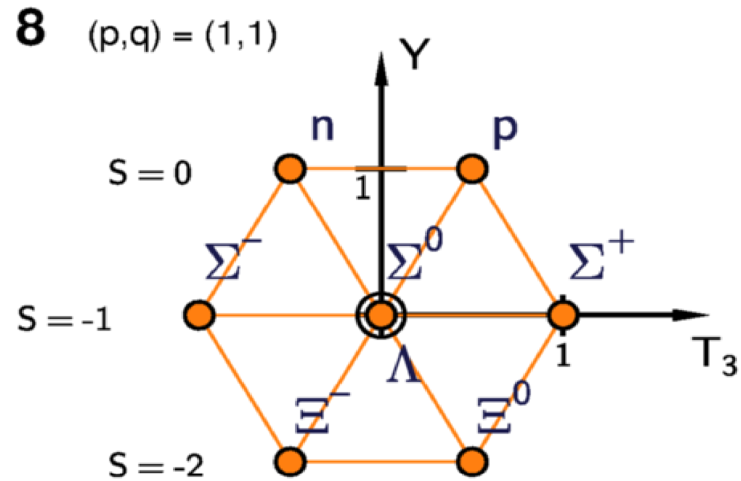
Mesons ($J^P = 0^-, 1^-$):

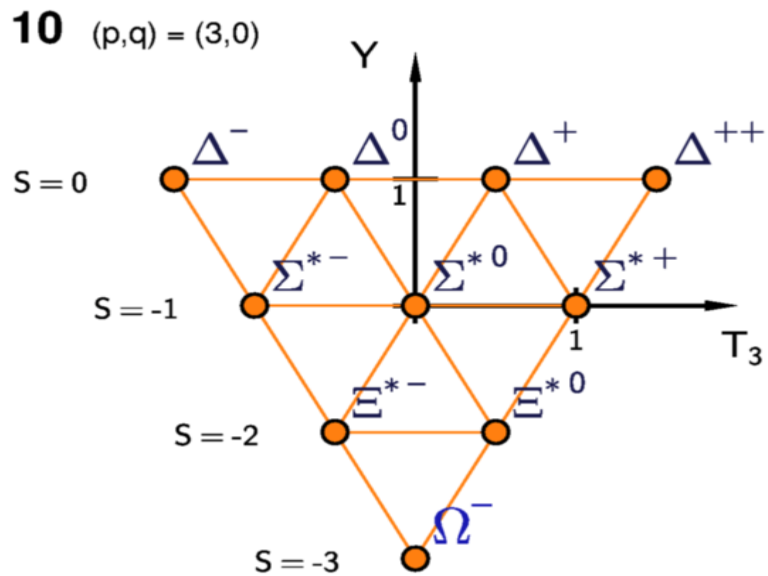
- spin-singlet ($\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$) with $L = 0$,
- spin-triplet ($\uparrow\uparrow$, $\frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$, $\downarrow\downarrow$) with $L = 0$

$J^P = 0^-$: $(\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, \eta)$

$J^P = 1^-$: $(\rho^\pm, \rho^0, K^{*\pm}, K^{*0}, \bar{K}^{*0}, \omega)$







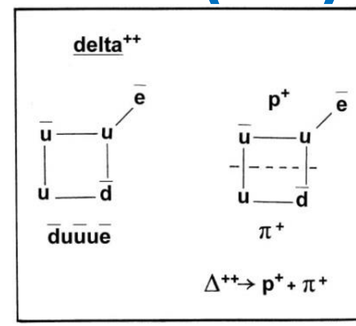
Baryon octet, $J^P = 1/2^+$: $(p, n, \Sigma^\pm, \Sigma^0, \Xi^-, \Xi^0, \Lambda)$.

Baryon decouplet, $J^P = 3/2^+$: $(\Delta^{++}, \Delta^+, \Delta^0, \Delta^-, \Sigma^\pm, \Sigma^0, \Xi^-, \Xi^0, \Omega^-)$

Colors dof

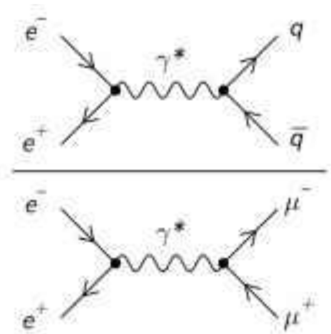
Motivations for Colors:

- **Imposition of Fermi Statistics on Baryon States (Δ^{++})**



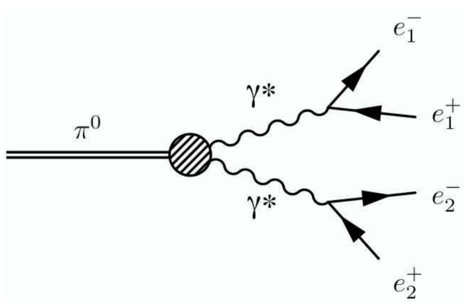
$\Delta^{++} = uu$, spin=3/2 at least 3 possible states

- ***$e^+ e^-$ –> hadrons*** $R = \frac{\sigma(e^- e^+ \to \text{hadrons})}{\sigma(e^- e^+ \to \mu^- \mu^+)} \approx$

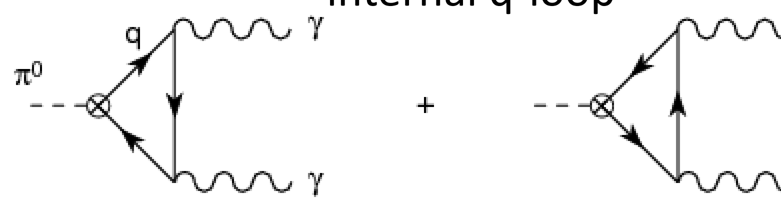


$= N_C \sum_q e_q^2$ with $N_C = 3$

- ***π^0 -> $\gamma\,\gamma$***



internal q-loop



$\Gamma \propto N_C^2 (Q_U^2 - Q_d^2)^2$

with $N_C = 3$

QED←→QCD

At least THREE colors are needed to explain some physical processes

In analogy to QED, QCD was developed:

- **Local gauge invariance (quark kinetic energy δμ and A_μ),**
- **Color is a continuous (rather than discrete) symmetry and**
- **Fermion(Boson) fields as 3(2)-vector in color-space** $\Psi\acute{(}x) = e^{i\lambda^i \alpha_i/2}$

$$\psi = \begin{pmatrix} \psi_r \\ \psi_b \\ \psi_g \end{pmatrix}$$

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \rightarrow U \begin{pmatrix} u \\ d \\ s \end{pmatrix} \qquad U \in SU(3)$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow U \begin{pmatrix} u \\ d \end{pmatrix} \qquad U \in SU(2)$$

$$s \rightarrow Us \qquad U \in U(1)$$

$$A\mu \rightarrow A\mu + \partial_\mu \alpha$$

$$\mathcal{D}_\mu \equiv \partial_\mu - i\frac{g}{2}\lambda^a A^a_\mu$$

where A_μ is a 3×3 matrix in color space formed from the 8 color fields $A_\mu \equiv \frac{1}{2}\lambda^i b^i_\mu$

The tensor field $G^i_{\mu\nu} = G_{\mu\nu}\lambda^i$ $\qquad G_{\mu\nu} = (ig)^-1[\mathcal{D}_\nu, \mathcal{D}_\mu] = \partial_\nu A_\mu \partial_\mu A_\mu + ig[A_\nu, A_\mu]$

with A_μ not commute (dislike QED)

Asymptotic Freedom and Running Coupling

- In electromagnetic interaction, quarks act as if they were free
- This would contradicts the *running* strong coupling α_s

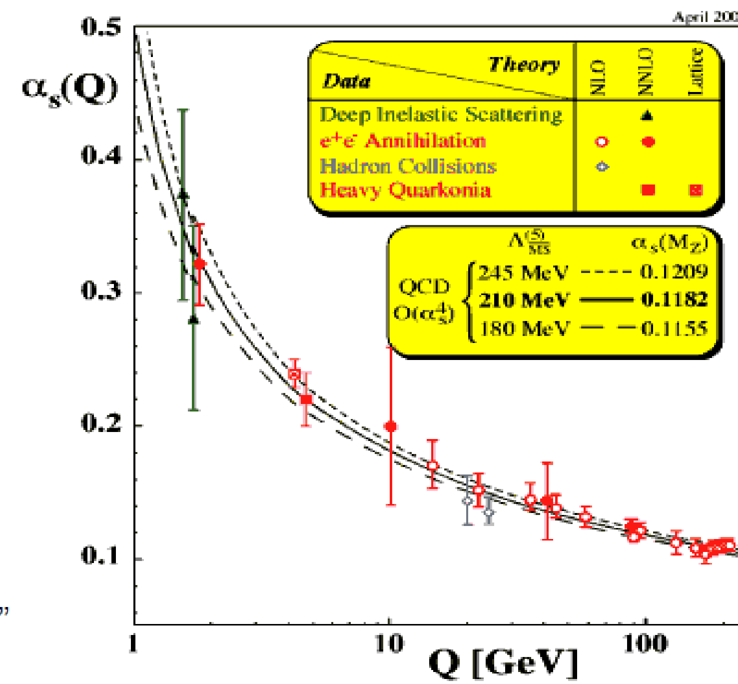
- **QCD's great breaking through: plausible explanation why**
  - **at low q^2 the interactions are very strong, while**
  - **at high q^2, quarks seem free**
  - **Asymptotic Freedom (Nobel Prize 2004)**



David J. Gross, H. David Politzer, Frank Wilczek

α_s runs:

Low q^2	α_s large	"confinement"
High q^2	α_s small	"asympotic freedom"



Asymptotic Freedom and Running Coupling

Running Coupling

QED
$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log\left(\frac{Q^2}{\mu^2}\right)}$$

QCD
$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi}(33 - 2n_f)\log\left(\frac{Q^2}{\mu^2}\right)}$$

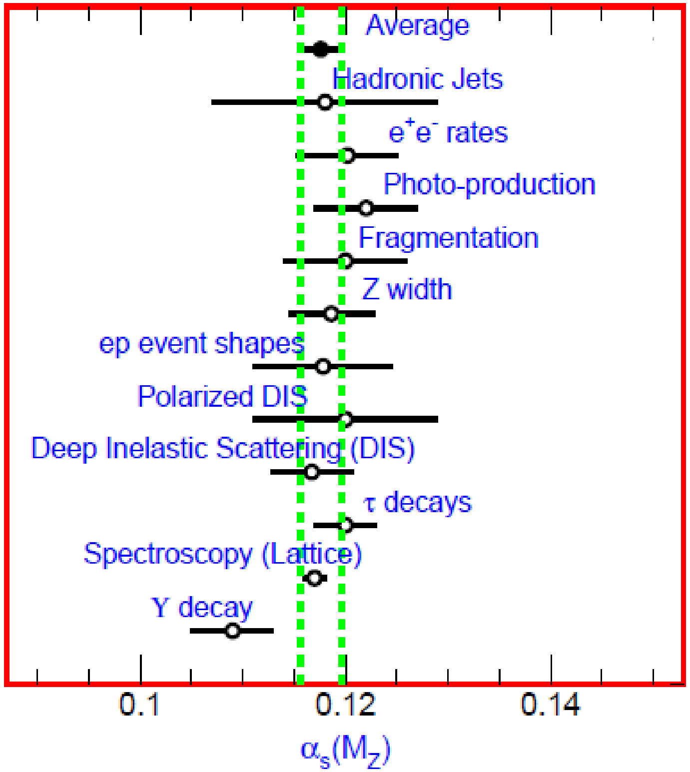
Measuring α_s in QED: $\mu^2 = Q^2 \to 0$

Measuring α_s in QCD: a cut-off (QCD scale)

$$\Lambda^2 \equiv \mu^2 \exp\left[\frac{-12\pi}{(33 - 2n_f)\,\alpha_s(\mu^2)}\right]$$

$\Lambda \sim$ few hundred MeV

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f)\log(Q^2/\Lambda^2)}$$



QCD Summary

- Physics is invented/devoted to describe and measure the Nature's phenomenon and finds out their universal laws.
- It can sometimes happen that a certain complex phenomena seems to be explainable by very simple laws, whereas other simple laws can lead to very complex consequences.
- The Lagrangian of QCD is a good example about the latter. A good example about the earlier case is the thermodynamics (in general).
- The laws of thermodynamics give essential information about the microscopic structure of the thermal system of interest.
- Its macroscopic structure is perfectly given by the statistics. Therefore, the entropy is playing an essential role relating micro- with macro-world.
- The QCD Lagrangian can entirely be expressed by one short equation, (about one line including various phenomenon)

$$\mathcal{L} = \sum_f \bar{\Psi}^f(r)(i\gamma_\mu D^\mu - m_f)\Psi^f(r) - \frac{1}{2}\mathrm{Tr}\,\mathcal{G}_{\mu\nu}(r)\mathcal{G}^{\mu\nu}(r).$$

Such a line includes among others the binding force of nuclear matter, the mass spectrum of all hadronic excitations, the high-energy behavior of hadronic scattering cross sections, the momentum and the spin distribution of partons which are contained inside nucleon, etc.

QCD Summary

- QCD is formulated as a non-Abelian $SU(3)$ gauge theory containing a set of n_f quark fermion fields $\Psi_1, \cdots \Psi_{n_f}$ with masses $m_1, \cdots, m_{n_f}$.
- The interactions with the gluonic gauge A^i_ν, where $i = 1 \cdots, 8$ as well as self interactions of the gluons are governed by an overall gauge coupling g.
- First principle calculations of all QCD phenomenon (included in Lagrangian) are still out of access. Therefore, we could not simultaneously understand the meaning of simplicity and efficiency of QCD Lagrangian. What would happen when one would like probe the QCD theory, two problems prohibit this;

  1. the coupling constant is strong and running, which makes a perturbative expansion impossible and
  2. the theory is non-Abelian. The latter leads - among others - to interactions of the gauge bosons which eventually are responsible for the confinement inside hadronic states.

  Therefore, the confinement is to be understood as the impossibility to observe the fundamental degrees of freedom in the Lagrangian as asymptotic states.
- Nevertheless, QCD has an asymptotic freedom, which means nearly deconfinement at large momentum transfers.

QCD

QCD has wo peculiar properties:

- Asymptotic freedom. It means that at very high-energy temperatures, quarks and gluons interact *very weakly*. This prediction of QCD was first discovered in the early 1970s by D. Politzer and by F. Wilczek and D. Gross. For this work they were awarded the 2004 Nobel Prize in Physics.
- Confinement It means that the force between quarks does not diminish as they are separated. Because of this, it would take an infinite amount of energy to separate two quarks; they are forever bound into hadrons such as the proton and the neutron. Although analytically unproven, confinement is widely believed to be true because it explains the consistent failure of free quark searches, and it is easy to demonstrate in lattice QCD.

We notice that these two properties are continuous. They have no phase-transition or any other singularity separating them.

The deconfinement can be explained as follows. Any color charge in vacuum is surrounded by a cloud of quantum fluctuations. In QCD, these quantum fields act in such a way as to enhance the original charge, anti-screening . At large momentum transfers, a small spatial region is resolved and as a consequence the anti-screening cloud is penetrated, the charge seen by the interaction appears weaker.

QCD

The QCD Lagrangian contains two parts

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_f + \mathcal{L}_b \\ &= \sum_f \bar{\Psi}^f(r)(i\gamma_\mu D^\mu - m_f)\Psi^f(r) - \frac{1}{2}\mathrm{Tr}\,\mathcal{G}_{\mu\nu}(r)\mathcal{G}^{\mu\nu}(r). \end{aligned} \quad (1)$$

The first term is the fermionic part (quarks). The gauge invariant gluon (bosonic) field strength tensor reads

$$\mathcal{G}_{\mu\nu} = \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A_{\nu,b} A_{\nu,c}\right) t_a,$$

where t^a stand for the generators of the SU(3) gauge group and a, b, c run over $(1 - n_c^2)$ in Lie Algebra. n_c is the color quantum number. $[t^a, t^b] = i f^{abc} t^c$ relates t^a to the structure constant of SU(3), f^{abc}. t^a are terms of the Gell-Mann matrices read $t^a = \lambda^a/2$. g is the strong coupling constant. It is related to the strong running coupling α_s.

The Lagrangian, Eq. (1), is invariant under gauge transformation $G(r) = \exp[i\omega_i(r) t^a]$,

$$\begin{aligned}\Psi(r) &\rightarrow \Psi'(r) = G(r)\Psi(r) \\ \bar{\Psi}(r) &\rightarrow \bar{\Psi}'(r) = G^\dagger(r)\Psi(r) \\ A_\mu(r) &\rightarrow A'_\mu(r) = G(r)\, A_\mu(r)\, G^\dagger(r) - \frac{i}{g} G(r)\, \partial_\mu(r)\, G^\dagger(r)\end{aligned}$$

QCD

If you are interested on thermodynamic features of QCD around T_c, then $n_f = 3$ and $T_c \approx 200$ MeV. In this limit

$$\Psi(r) = \begin{pmatrix} u_\alpha(r) \\ d_\alpha(r) \\ s_\alpha(r) \end{pmatrix}, \qquad m_q = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix},$$

where u, d and s stand of the quark flavors, up-, down- and strange-quark, respectively. The index $\alpha = 1 \cdots 3$ runs over the color index.

The QCD bare parameters, g and m_q depend on the energy scale, at which the theory is probed. The observable quantities should not depend on the normalization scale, E. This defines the normalization scale.

$$E \frac{d}{dE} \alpha_s(E) = \beta_0(\alpha_s) = -\frac{\beta_0}{6\pi} \alpha_s^2 - \frac{\beta_1}{24\pi^2} \alpha_s^3 - \mathcal{O}(\alpha_s^4),$$

where

$$\beta_0 = 11 n_c - 2 n_f \qquad \beta_1 = 34 n_c^2 - \left(10 n_c + \frac{2}{n_c}(n_c^2 - 1)\right) n_f.$$

The solution for α_s with the energy scale E shows asymptotic freedom,

$$\alpha_s(E) = \frac{12\pi}{\beta_0 \log(E^2/\Lambda^2)} - \frac{36\pi\beta_1 \log(\log(E^2/\Lambda^2))}{\beta_0^3 \log^2(E^2/\Lambda^2)} + \mathcal{O}\left(\frac{\log^2(\log E)}{\log^3(E)}\right). \tag{2}$$

Lattice QCD

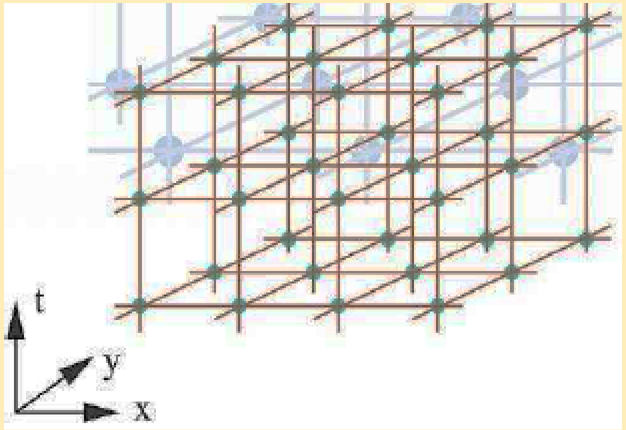
- It gives non-perturbative (mainly numerical) solutions for the QCD.
- If the interaction would be just an arbitrary small correction of the free Hamiltonian, then a perturbative expansion in terms/powers of the coupling constant can help.
- In this regard, the time evolution operator $\exp(iHt)$ and the statistical density operator $\exp(-\beta H)$ look similar.
- The latter can be interpreted as the evolution from 0 to β in imaginary time $\tau = it$.
- Therefore, the partition function can be given in terms of the Euclidean path integral. For instance, for field scalar, it reads

$$Z(T,V) = \oint \mathcal{D}A_\mu \mathcal{D}\bar{\Psi}\mathcal{D}\Psi \exp^{-\int_0^\beta d\tau \int_V d^3r \{\mathcal{L}_f^{equ} + \mathcal{L}_b^{equ}\}}, \tag{3}$$

  where $\mathcal{L}_{equ}$ is the QCD Lagrangian in Euclidean space *in-equilibrium*. The integral is restricted to periodic field configuration. For fermionic fields Ψ, antiperiodic boundary conditions are required, $\Psi(0,r) = -\Psi(\beta,r)$.

- At the energy scale of current heavy-ion collision, the coupling of QCD theory is not small (compare α_s).
- Thus, a perturbative treatment of the QCD partition function, Eq. (3), is not exactly possible.

Lattice QCD



- This is possible since one is dealing with an Euclidean path integral, where the dominant region can easily be identified since strong fluctuations are exponentially damped, unlike in Minkowski space, where the dominant contribution to the path integral emerges from the interference pattern of oscillating amplitudes.

Remark:
Introducing a lattice spacing a, automatically produces an ultraviolet cutoff and thus regularizing any divergences appearing in the continuum.
At the same time, Lorentz invariance is broken. The total extension of the lattice is given by the number of spatial N_σ^3 and temporal N_τ sites. The volume and (inverse) temperature of the simulation can then be identified as

$$V = N_{\sigma a}^3, \qquad \beta = N_\tau a.$$

Lattice QCD

Fermion fields are defined by points and gauge fields live on the links connecting adjacent lattice sites x and $x + a\hat{\epsilon}$, where $\hat{\epsilon}$ is a unit vector along one of the three spatial or the temporal axis.

$$U_\mu(x) = \mathcal{P} \exp^{ig \int_x^{x+a\hat{\epsilon}} dx'_\mu A^\mu(x')},$$

where $\mathcal{P}$ is the path ordering along the integration contour. These links transform homogeneously under a gauge transformation

$$U_\mu(x) \quad = \quad G(x) U_\mu(x) G^\dagger(x + a\hat{\epsilon}).$$

Then, the simplest gauge invariant object that can be constructed is a closed set of four links around a lattice plaquette reads

$$U_{\mu\nu} \quad = \quad \mathtt{Tr} \left[U_\mu(na) U_\nu(na + a\hat{\epsilon}_1) U_\mu^\dagger(na - a\hat{\epsilon}_2) U_\nu^\dagger(na) \right].$$

This is already a discretized approximation to the gauge field action (up to errors in powers of the lattice spacing a which vanish in the continuum limit), since

$$\mathtt{Re}\, U_{\mu\nu} \quad = \quad n_c - \frac{n_c g^2 a^4}{2} \mathcal{G}_{\mu\nu} \mathcal{G}^{\mu\nu} + \mathcal{O}(a^6).$$

Summing over all lattice sites, the so-called Wilson action for the gauge fields is obtained

$$S_{\mathcal{G}}^E = \frac{2 n_c}{g^2} \sum_{n, 0 \leq \mu < \nu \leq 3} 1 - \frac{1}{n_c} \mathrm{Re} U_{\mu\nu}$$

Lattice QCD

Gauge fields:

$$U_\mu(n) = \exp^{igaA_\mu(x)}.$$

$U_\mu(n)$ connects the site n to the nearest neighbour $n+\hat{\mu}$ in the μ-direction.

$$S_E = S_b + S_f,$$

where the Wilson gauge action S_b is the discretized gluonic (bosonic) action

$$S_b = \beta \sum_{n,\mu<\nu} \left(1 - \frac{1}{n_c} \mathrm{Re\ Tr}\, U_{\mu,\nu}(n)\right),$$

where $\beta = 2n_c/g^2$ stands for the coupling.

Fermionic fields:

$$S_b = \sum_{n,m} \Psi_n K_{n,m} \Psi_m,$$

where the sum extends over all lattice sites n and m. For vanishing chemical potentials,

$$K_{n,m} = \frac{1}{2} \sum_\mu \gamma_\mu \left(\delta_{n+\hat{\mu},m} U_\mu(n) - \delta_{n-\hat{\mu},m} U_\mu^\dagger(n-\hat{\mu})\right) + M\delta_{n,m},$$

with $M = a \cdot m$ is the dimensionless bare mass parameter.

Lattice QCD

In the free case. vanishing coupling, i.,e $g = 0$, the fermion propagator calculated from this action is

$$\langle \Psi_n \otimes \Psi_m \rangle \;=\; \frac{1}{K_{n,m}} \int_{-\pi}^{+\pi} \frac{d^4p}{(2\pi)^4} \frac{-i\sum_\mu \gamma_\mu sin(P_\mu) + M}{\sum_\mu sin^2(P_\mu) + M^2} \exp^{ip(n-m)}.$$

While in continuum, this propagator gets poles at $P_\mu \sim \pm im$ only, there are now two poles in every direction μ at the corners of the Brillouin zone. Therefore in the continuum limit, this action describes $2^{\mathtt{dim}} = 2^4 = 16$ independent quarks instead of just one. There are several ways to deal with this doubling problem. The introduction of quark chemical potential μ can be transferred to the lattice gauge links. In the action the gauge links in the temporal direction have to be replaced by

$$\begin{aligned} U_4(n) &= \exp^{+igA_4(x)} \rightarrow \exp^{+igaA_4(x)-a\mu} = \exp^{-a\mu} U_4(n) \\ U_4^\dagger(n) &= \exp^{+igA_4(x)} \rightarrow \exp^{+igaA_4(x)-a\mu} = \exp^{-a\mu} U_4^\dagger(n) \end{aligned}$$

Lattice QCD

$$\mathcal{Z}(T,V,\mu) = \mathrm{Tr}\exp^{-\left(\beta H - \sum_{i=1}^{n_f}\mu_i n_{q,i}\right)}, \tag{4}$$

where, $H = \int dx^3 \mathcal{H}$ is the Hamiltonian. It is related to the Minkowsian QCD Lagrangian $L_M = \int dx^3 \mathcal{L}_M$ through the Legendre transformation. $n_{q,i}$ give the conserved charges which in lattice QCD simulations are likely taken as the quark numbers of the corresponding flavors.

$$n_{q,i} = \int d^3r J_0^{(i)}(r) = \int d^3r \bar{\Psi}_i \gamma_0 \Psi_i.$$

In this ensemble, the statistical average of any operator $\mathcal{O}$ is

$$< \mathcal{O} >_\beta = \frac{1}{\mathcal{Z}} \mathrm{Tr}\mathcal{O}\exp^{-\left(\beta H - \sum_{i=1}^{n_f}\mu_i n_{q,i}\right)}.$$

The partition function $\mathcal{Z}$ can then be translated into the path integral expression, Eq. (3)

$$\mathcal{Z}(T,V,\mu) = \int \mathcal{D}A\mathcal{D}\Psi\mathcal{D}\Psi \exp^{-S_E},$$

where the action S_E has same structure as in the Euclidean continuum case apart from the fact that the temporal integral is limited to the finite interval $[0,\beta]$, namely

$$S_E = \int_0^\beta d\tau \int_V d^3r \left(\mathcal{L}_E - \sum_{i=1}^{n_f}\mu_i \bar{\Psi}_i\gamma_0\Psi_i\right).$$

Lattice QCD

- In order to keep numerical expressions finite, we consider the case of a finite spatial volume V instead of integrating over infinite intervals.
- The (anti-)commutation relations for the bosonic and fermionic fields imply that all field configurations covered by path integral have to fulfil periodic boundary conditions for gluons and anti-periodic boundary conditions for fermions in the temporal direction.
- There are no restrictions on the boundary conditions in the spatial directions. We always like to choose them to be periodic.

$$S_E = \int_0^\beta d\tau \int_V d^3 r \mathcal{L}_E(\mu)$$

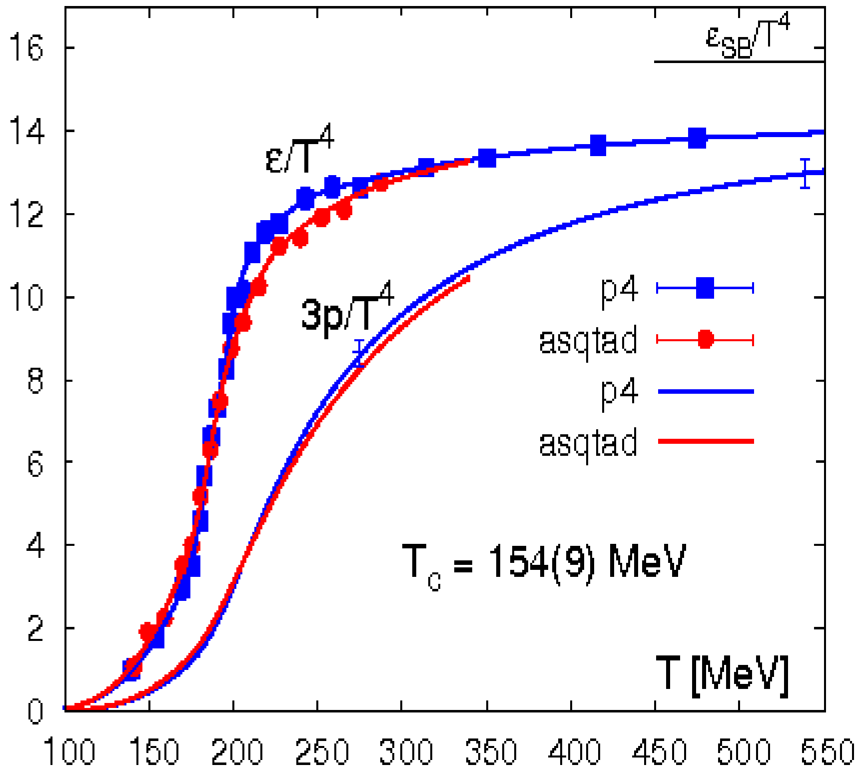
At finite μ, $\mathcal{L}_E(\mu)$ should have the same functional form as $\mathcal{L}_E$. Therefore,

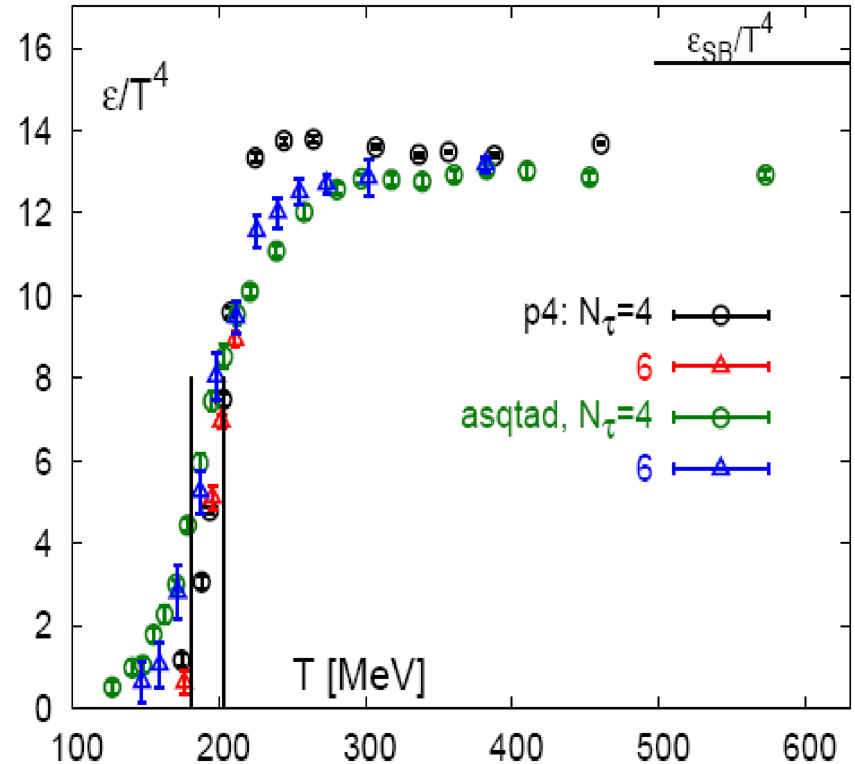
$$igA_0 \rightarrow igA_0 - \mu_i$$

As introduced above, bulk thermodynamical quantities $\mathcal{O}(V,T,\mu))$ can be calculated as ensemble averages

$$\mathcal{O}(V,T,\mu)) \;\; = \;\; \langle\mathcal{O}\rangle_{V,T,\mu} = \frac{1}{\mathcal{Z}} \int \mathcal{D}A\mathcal{D}\Psi\mathcal{D}\Psi\mathcal{O}(A,\bar{\psi},\psi,V,T,\mu) \exp^{-S_E}$$

Lattice QCD: Thermodynamics





شكرا جزيلا لكريم لاهتمامكم!

Thanks for your Attention!
Vielen Dank für Ihre Aufmerksamkeit!

http://atawfik.net/