Finite N corrections to the second order cumulant of pressure via the Optimized Perturbation Theory on the Nambu–Jona-Lasinio model

Gabriel Neves Ferrari
Universidade Federal de Santa Catarina (UFSC)

ICTP-NCP School on LHC Physics, Islamabad - Pakistan

22^{th} November 2014

Índice

Florianópolis (Floripa) - Brazil

Florianópolis (Floripa) - Brazil

- 

Florianópolis (Floripa) - Brazil

Introduction

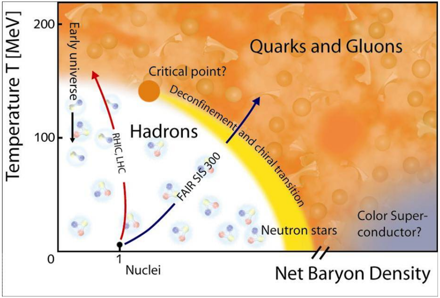


Figure: Sketch of the QCD phase diagram.

Introduction

- $\mu = 0$ and finite T: lattice QCD (LQCD), effective models (NJL, LSM) and experiments (LHC, RHIC).

Introduction

- $T = 0$ and finite μ: effective models and neutron stars.

Introduction

- First order phase transition $\rightarrow$ crossover $\Rightarrow$ critical end point (CEP).

Introduction

- Where is it? Answer might come from measure of quark number susceptibility (experimental), or cumulants of pressure (theoretical).

Introduction

- Effective models: in order to analyze compressed strongly interacting matter one should, at first, include vector channels [1, 2].

Introduction

- Effective models: in order to analyze compressed strongly interacting matter one should, at first, include vector channels [1, 2].
- Easily implemented in the NJL model: add a term such as $-G_V(\bar{\psi}\gamma^\mu\psi)^2$ to the original lagrangian density.

Introduction

- Effective models: in order to analyze compressed strongly interacting matter one should, at first, include vector channels [1, 2].
- Easily implemented in the NJL model: add a term such as $-G_V(\bar{\psi}\gamma^\mu\psi)^2$ to the original lagrangian density.
- However, G_V should be fixed using the ρ meson mass which, in general, is higher than the maximum energy scale set by the Λ cutoff. It cannot be determined from experiments and lattice QCD simulations and, theoretically, there is absolutely no consense about its fixation.

Introduction

- So, it would be convenient not to need an explicit imposition of such vectorial term.

Introduction

- So, it would be convenient not to need an explicit imposition of such vectorial term.
- **However, due to the Fierz identities, when going beyond mean field approximation (MFA) level one may induce quantum (loop) corrections which mimic the physical effects caused by a classical (tree) term such as** $-G_V(\bar{\psi}\gamma^\mu\psi)^2$.

Introduction

- This can come from the nonperturbative Optimized Perturbation Theory (OPT) method applied to the two flavor NJL model with vanishing G_V.

Introduction

- This can come from the nonperturbative Optimized Perturbation Theory (OPT) method applied to the two flavor NJL model with vanishing G_V.
- OPT $\simeq$ MFA + G_V. **Advantages: more loops calculated (more powerful method) and one doesn't need to fix** G_V.

Introduction

- This can come from the nonperturbative Optimized Perturbation Theory (OPT) method applied to the two flavor NJL model with vanishing G_V.
- OPT $\simeq$ MFA + G_V. **Advantages: more loops calculated (more powerful method) and one doesn't need to fix** G_V.
- OPT has been successful in Bose-Einstein condensation, evaluation of the critical density for polyacetylene, success in predicting QCD thermodynamical properties at the three-loop level, etc.

The Nambu–Jona-Lasinio Model and its effective potential

- Here we discuss the standard two flavor NJL model for quark matter.

The Nambu–Jona-Lasinio Model and its effective potential

- Here we discuss the standard two flavor NJL model for quark matter.
- The respective lagrangian density can be written as [4]

$$\mathcal{L}_{NJL} = \bar{\psi}\left(i\partial\!\!\!/ - \hat{m}_0\right)\psi + G_S\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi\right)^2\right], \quad (1)$$

The Nambu–Jona-Lasinio Model and its effective potential

- Here we discuss the standard two flavor NJL model for quark matter.
- The respective lagrangian density can be written as [4]

$$\mathcal{L}_{NJL} = \bar{\psi}\left(i\partial\!\!\!/ - \hat{m}_0\right)\psi + G_S\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi\right)^2\right], \quad (1)$$

- where ψ (sum over flavor and colour degrees of freedom is implicit) represents a isodoublet in flavor (u, d) and a N_c-plet quark field (N_c is the number of colors), $\boldsymbol{\tau}$ are the Pauli matrices and G_S represents the scalar coupling strength.

The Nambu–Jona-Lasinio Model and its effective potential

- The whole thermodynamics of an effective model comes from its Landau's free energy, or the effective potential. To calculate it, one can use the path intgral formalism.

The Nambu–Jona-Lasinio Model and its effective potential

- The whole thermodynamics of an effective model comes from its Landau's free energy, or the effective potential. To calculate it, one can use the path intgral formalism.
- Within this approach the partition function, $\mathcal{Z}$, can be written in terms of the effective potential as follows (for details see Refs. [5, 1]):

$$\mathcal{Z} = \exp\left[-i \int d^4x \mathcal{F}\right]. \tag{2}$$

The Nambu–Jona-Lasinio Model and its effective potential

- The whole thermodynamics of an effective model comes from its Landau's free energy, or the effective potential. To calculate it, one can use the path intgral formalism.
- Within this approach the partition function, $\mathcal{Z}$, can be written in terms of the effective potential as follows (for details see Refs. [5, 1]):

$$\mathcal{Z} = \exp\left[-i\int d^4x\mathcal{F}\right]. \quad (2)$$

- Then, as within statistical mechanics, all the relevant thermodynamical quantities, such as the pressure, can easily be obtained once the free energy (or the partition function) is known.

The Nambu–Jona-Lasinio Model and its effective potential

- It is convenient to write down the Lagrangian density in terms of the auxiliary fields, σ and $\boldsymbol{\pi}$, by using the Hubbard-Stratanovich transformation,

The Nambu–Jona-Lasinio Model and its effective potential

- It is convenient to write down the Lagrangian density in terms of the auxiliary fields, σ and $\boldsymbol{\pi}$, by using the Hubbard-Stratanovich transformation,
- $$\mathcal{L}_{NJL} = \bar{\psi}\left(i\partial\!\!\!/ - m_0\right)\psi - \frac{1}{4G_S}\left(\sigma^2 + \boldsymbol{\pi}^2\right) - \bar{\psi}\left[\sigma + i\gamma_5\boldsymbol{\pi}\cdot\boldsymbol{\tau}\right]\psi. \tag{3}$$

The interpolated model

- To implement the OPT approximation, we modify the Lagrangian of a particular theory by introducing a dummy expansion parameter, δ.

The interpolated model

- To implement the OPT approximation, we modify the Lagrangian of a particular theory by introducing a dummy expansion parameter, δ.
- $$\mathcal{L}(\delta) = (1-\delta)\,\mathcal{L}_0 + \delta\mathcal{L} = \mathcal{L}_0 + \delta\,(\mathcal{L} - \mathcal{L}_0)\,. \tag{4}$$

The interpolated model

- To implement the OPT approximation, we modify the Lagrangian of a particular theory by introducing a dummy expansion parameter, δ.
-

$$\mathcal{L}(\delta) = (1-\delta)\,\mathcal{L}_0 + \delta\mathcal{L} = \mathcal{L}_0 + \delta\,(\mathcal{L} - \mathcal{L}_0)\,. \tag{4}$$

- $\mathcal{L}$ be the original Lagrangian density we want to solve, and $\mathcal{L}_0$ a Lagrangian density of a free theory.

The interpolated model

- One can fix the arbitrary mass parameter η by requiring that any physical quantity $\mathcal{P}(\eta)$, be at least locally η-independent. This optimization criterion translates into the following variational condition

$$\left.\frac{\partial \mathcal{P}(\eta)}{\partial \eta}\right|_{\bar{\eta}} = 0, \tag{5}$$

The interpolated model

- One can fix the arbitrary mass parameter η by requiring that any physical quantity $\mathcal{P}(\eta)$, be at least locally η-independent. This optimization criterion translates into the following variational condition

$$\left.\frac{\partial \mathcal{P}(\eta)}{\partial \eta}\right|_{\bar{\eta}} = 0, \tag{5}$$

The interpolated model

- One can fix the arbitrary mass parameter η by requiring that any physical quantity $\mathcal{P}(\eta)$, be at least locally η-independent. This optimization criterion translates into the following variational condition

$$\left.\frac{\partial \mathcal{P}(\eta)}{\partial \eta}\right|_{\bar{\eta}} = 0, \tag{5}$$

- which is known as the *principle of minimal sensitivity* (PMS).

The interpolated model

- Then, following the OPT interpolation prescription, on the NJL model it gets

$$\begin{aligned}\mathcal{L}_{NJL}(\delta) =& \bar{\psi}\left(i\partial\!\!\!/ - m_0 - \eta\right)\psi \\ & + \delta\left[\eta\bar{\psi}\psi - \frac{1}{4G_S}\left(\sigma^2 + \boldsymbol{\pi}^2\right) - \left(\sigma\bar{\psi}\psi + \bar{\psi}i\gamma_5\boldsymbol{\pi}\cdot\boldsymbol{\tau}\psi\right)\right].\end{aligned} \tag{6}$$

The interpolated model

- To order δ, the effective potential is given by the these Feynman diagrams:

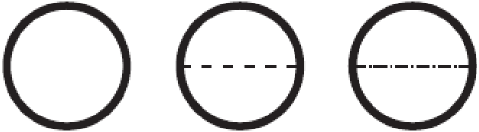


Figure: Diagrams contributing to $\mathcal{F}(\hat{\eta})$ at order δ. The thick continuous lines represent the OPT dressed fermionic propagators, the dashed line represents the σ propagator and the dashed-dotted line represents the π propagator. (Figure taken from Ref. [2]).

The interpolated model

- Applying the Feynman rules one then obtains, in finite temperature and chemical potential,

The interpolated model

- Applying the Feynman rules one then obtains, in finite temperature and chemical potential,
- $$\mathcal{F}(\eta,\sigma,\mu,T) = \frac{\sigma^2}{4G_S} - 2N_f N_c I_1 + 2\delta N_f N_c (\eta + m_0)(\eta - \sigma) I_2 + 4\delta G_S N_f N_c I_3^2 - 2\delta G_S N_f N_c (\eta + m_0)^2 I_2^2 \,, \quad (7)$$

where

The interpolated model

- $$I_1 = \int \frac{d^3p}{(2\pi)^3} \left\{ E_p + \frac{1}{\beta} \ln \left[1 + e^{-\beta(E_p+\mu)} \right] + \frac{1}{\beta} \ln \left[1 + e^{-\beta(E_p-\mu)} \right] \right\} \quad (8)$$

  $$I_2 = \int \frac{d^3p}{(2\pi)^3} \frac{1}{E_p} \left[1 - f^+ - f^- \right] \quad (9)$$

The interpolated model

- $$I_1 = \int \frac{d^3p}{(2\pi)^3} \left\{ E_p + \frac{1}{\beta} \ln \left[1 + e^{-\beta(E_p+\mu)} \right] + \frac{1}{\beta} \ln \left[1 + e^{-\beta(E_p-\mu)} \right] \right\} \quad (8)$$

  $$I_2 = \int \frac{d^3p}{(2\pi)^3} \frac{1}{E_p} \left[1 - f^+ - f^- \right] \quad (9)$$

- $$I_3 = \int \frac{d^3p}{(2\pi)^3} \left[f^+ - f^- \right], \quad (10)$$

  where $E_p^2 = \mathbf{p}^2 + (\eta + m_0)^2$ is the dispersion while

  $$f^+ = \frac{1}{e^{\beta(E_p-\mu)} + 1} \quad , \quad f^- = \frac{1}{e^{\beta(E_p+\mu)} + 1}, \quad (11)$$

  represent the fermion distribution functions for particles and antiparticles respectively.

The interpolated model

- There is a family of parameter sets tailored to reproduce the numerical values of these physical observables, respecting $M = m_0 + \Sigma \simeq 330$ MeV, where Σ represents the self energy. The parameter sets adopted in this work are given in table 1.

The interpolated model

- There is a family of parameter sets tailored to reproduce the numerical values of these physical observables, respecting $M = m_0 + \Sigma \simeq 330$ MeV, where Σ represents the self energy. The parameter sets adopted in this work are given in table 1.

  Table: Parameter set for the OPT and for the MFA approximation as given in Ref. [2]. These values were obtained to reproduce $m_\pi = 135$ MeV, $f_\pi = 92.4$ MeV and $-\langle\bar{\psi}\psi\rangle^{1/3} = 250$ MeV.

	Λ [MeV]	m_0 [MeV]	$G_S\Lambda^2$
OPT	640	4.9	1.99
MFA	640	5.2	2.14

Repulsive vector interaction in the NJL model

- Within the NJL model such a term can be of the form $-G_V\left(\bar{\psi}\gamma_\mu\psi\right)^2$ with $G_V > 0$ describing repulsion which is the case here and $G_V < 0$ describing attraction. Then the standard NJL lagrangian density becomes

Repulsive vector interaction in the NJL model

- Within the NJL model such a term can be of the form $-G_V\left(\bar{\psi}\gamma_\mu\psi\right)^2$ with $G_V > 0$ describing repulsion which is the case here and $G_V < 0$ describing attraction. Then the standard NJL lagrangian density becomes
-

$$\mathcal{L}_V = \bar{\psi}\left(i\partial\!\!\!/ - \hat{m}_0\right)\psi + G_S\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi\right)^2\right] - G_V\left(\bar{\psi}\gamma^\mu\psi\right)^2, \quad (12)$$

Repulsive vector interaction in the NJL model

- Within the NJL model such a term can be of the form $-G_V\left(\bar{\psi}\gamma_\mu\psi\right)^2$ with $G_V > 0$ describing repulsion which is the case here and $G_V < 0$ describing attraction. Then the standard NJL lagrangian density becomes
- $$\mathcal{L}_V = \bar{\psi}\left(i\partial\!\!\!/ - \hat{m}_0\right)\psi + G_S\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi\right)^2\right] - G_V\left(\bar{\psi}\gamma^\mu\psi\right)^2, \quad (12)$$
- and the effective potential in the MFA approximation reads [1]

Repulsive vector interaction in the NJL model

- Within the NJL model such a term can be of the form $-G_V\left(\bar{\psi}\gamma_\mu\psi\right)^2$ with $G_V > 0$ describing repulsion which is the case here and $G_V < 0$ describing attraction. Then the standard NJL lagrangian density becomes
- $$\mathcal{L}_V = \bar{\psi}\left(i\partial\!\!\!/ - \hat{m}_0\right)\psi + G_S\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi\right)^2\right] - G_V\left(\bar{\psi}\gamma^\mu\psi\right)^2, \quad (12)$$
- and the effective potential in the MFA approximation reads [1]
- $$\mathcal{F}_{MFA}^{G_V} = \frac{\sigma^2}{4G_S} - 2N_f N_c I_1\left(\tilde{\mu}, T\right) - 4G_V N_f^2 N_c^2 I_3^2\left(\tilde{\mu}, T\right), \quad (13)$$

Repulsive vector interaction in the NJL model

- Fukushima [3] has shown that the *combined* effect of $\tilde{\mu}$ and $-4G_V N_c^2 N_f^2 I_3^2$ in the above equation is to produce a *net* effect similar to $+4G_V N_c^2 N_f^2 I_3^2$.

Repulsive vector interaction in the NJL model

- Fukushima [3] has shown that the *combined* effect of $\tilde{\mu}$ and $-4G_V N_c^2 N_f^2 I_3^2$ in the above equation is to produce a *net* effect similar to $+4G_V N_c^2 N_f^2 I_3^2$.
- This interesting result allows us to better understand the type of $1/N_c$ contributions radiatively generated by the OPT.

Repulsive vector interaction in the NJL model

- Fukushima [3] has shown that the *combined* effect of $\tilde{\mu}$ and $-4G_V N_c^2 N_f^2 I_3^2$ in the above equation is to produce a *net* effect similar to $+4G_V N_c^2 N_f^2 I_3^2$.
- This interesting result allows us to better understand the type of $1/N_c$ contributions radiatively generated by the OPT.
- An inspection of Eq. (7) reveals that this approximation generates a $+4G_V N_c^2 N_f^2 I_3^2$ term which is similar to the $+4G_S N_c N_f I_3^2$ term appearing in the OPT result.

Repulsive vector interaction in the NJL model

- $$\mathcal{F}_{MFA} \sim \frac{\sigma^2}{4G_S} - 2N_f N_c I_1 \quad (14)$$

Repulsive vector interaction in the NJL model

- $$\mathcal{F}_{MFA} \sim \frac{\sigma^2}{4G_S} - 2N_f N_c I_1 \tag{14}$$

- $$\mathcal{F}_{OPT} \sim \frac{\sigma^2}{4G_S} - 2N_f N_c I_1 + 4G_S N_f N_c I_3^2 \tag{15}$$

Repulsive vector interaction in the NJL model

- $$\mathcal{F}_{MFA} \sim \frac{\sigma^2}{4G_S} - 2N_f N_c I_1 \tag{14}$$

- $$\mathcal{F}_{OPT} \sim \frac{\sigma^2}{4G_S} - 2N_f N_c I_1 + 4G_S N_f N_c I_3^2 \tag{15}$$

- $$\mathcal{F}_{MFA}^{G_V} \sim \frac{\sigma^2}{4G_S} - 2N_f N_c I_1 + 4G_V N_f^2 N_c^2 I_3^2 \tag{16}$$

Repulsive vector interaction in the NJL model

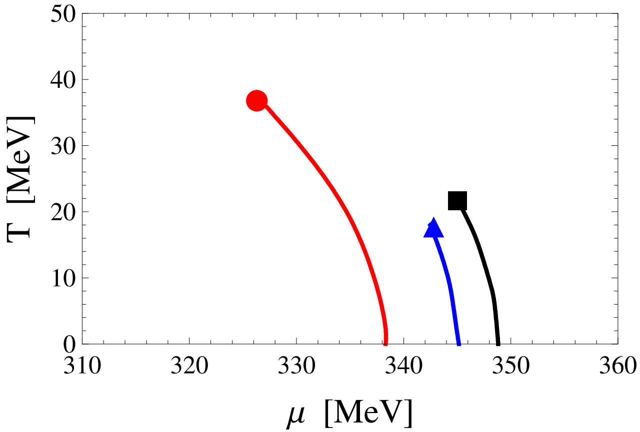


Figure: Phase diagram in the T–μ plane for the NJL model. MFA with $G_V = 0$, circle; MFAG_V with $G_V = G_S/(N_f N_c)$, triangle; OPT, square.

Repulsive vector interaction in the NJL model

- As we have just shown the OPT seems to reproduce the same effects without the need to explicitly introduce such a term (and one more parameter!) at the tree level.

Repulsive vector interaction in the NJL model

- As we have just shown the OPT seems to reproduce the same effects without the need to explicitly introduce such a term (and one more parameter!) at the tree level.
- So, the OPT can be seen as a powerful alternative to investigate the low-T/high-μ part of the QCD phase diagram which is currently non-accessible to the LQCD simulations.

Second order cumulant of the pressure

- Once the pressure has been evaluated within a given model approximation the coefficients can be obtained from

Second order cumulant of the pressure

- Once the pressure has been evaluated within a given model approximation the coefficients can be obtained from
- $$c_n(T) = \frac{1}{n!} \frac{\partial^n P(T,\mu)/T^4}{\partial(\mu/T)^n}\bigg|_{\mu=0} . \tag{17}$$

Second order cumulant of the pressure

- Once the pressure has been evaluated within a given model approximation the coefficients can be obtained from
- $$c_n(T) = \frac{1}{n!} \frac{\partial^n P(T,\mu)/T^4}{\partial(\mu/T)^n}\bigg|_{\mu=0} . \qquad (17)$$
- Nowadays the evaluation of these cumulants is receiving a lot of attention from LQCD researchers and the considerable amount of data already available can be used to check the reliability of other non-perturbative techniques such as the OPT and the MFA approximation.

Second order cumulant of the pressure

- Figure 4 and 6 show the coefficient c_2 obtained respectively with the OPT and with $G_v \neq 0$, both compared to the pure large-N approximation for the NJL model.

Second order cumulant of the pressure

- Figure 4 and 6 show the coefficient c_2 obtained respectively with the OPT and with $G_v \neq 0$, both compared to the pure large-N approximation for the NJL model.
-

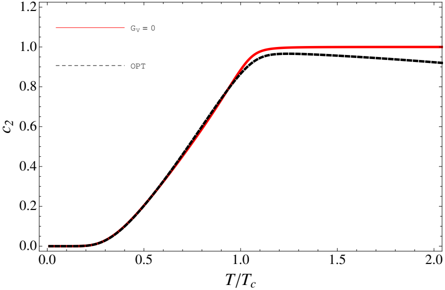


Figure: Taylor expansion coefficient c_2 as a function of T/T_σ obtained with the MFA approximation (continuous line) and with the OPT (dashed line).

Second order cumulant of the pressure

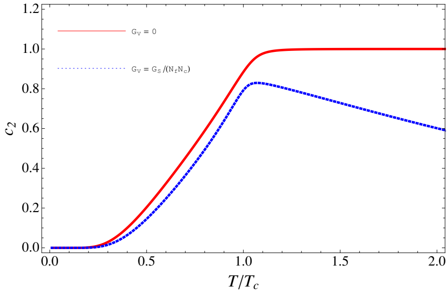


Figure: Taylor expansion coefficient c_2 as a function of T/T_σ obtained for the MFA approximation with $G_V = 0$ (continuous line) and $G_V \neq 0$ (dotted line).

Second order cumulant of the pressure

- We can see from the figures that the improvements do not behave as expected, since the value of c_2 is decreasing for $T > T_\sigma$ and is moving away from the Stefan–Boltzmann limit, which takes place at sufficiently high temperatures when the thermal fluctuations overcome the interparticle interactions and the system behaves as a free gas.

Second order cumulant of the pressure

- We can see from the figures that the improvements do not behave as expected, since the value of c_2 is decreasing for $T > T_\sigma$ and is moving away from the Stefan–Boltzmann limit, which takes place at sufficiently high temperatures when the thermal fluctuations overcome the interparticle interactions and the system behaves as a free gas.
- The cumulants can be identified with the quark number susceptibilities via

$$\frac{\chi_n}{T^2} = n! c_n(T) . \tag{18}$$

Conclusions

- Best methods, worse results. How? Why?

Conclusions

- Best methods, worse results. How? Why?
- 

Conclusions

- Best methods, worse results. How? Why?
- 

Hight T means vanishing G.

Conclusions

- The main source of this behavior at $T > T_\sigma$ is the medium term that appears in Eqs. like 7, for both OPT and the $G_v \neq 0$ case. This integral goes to highly negative values at high temperatures, and because of the pure large-N limit does not take it into account, the Stefan-Boltazmann limit is reached in this approximation.

Conclusions

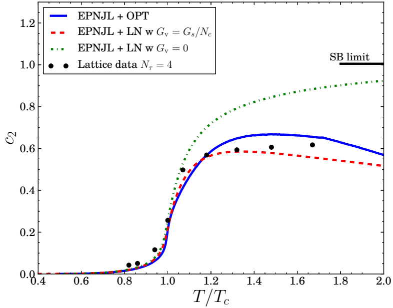
- Entangled Polyakov-loop extended Nambu–Jona-Lasinio model (EPNJL):
- 

Figure: Taylor expansion coefficient c_2 as a function of T/T_σ obtained for the EPNJL model with OPT.

References

- M. Buballa, *Phys. Rept.* **407**, 205 (2005).
- K. Masuda, T. Hatsuda and T. Takatsuda, Prog. Ther. Exp. Phys., 073D01 (2013).
- J. Steinheimer, S. Schramm, Phys.Lett. B **736** 241-245 (2014).
- Nambu, Y. & Jona-Lasinio, Phys. Rev., American Physical Society, 1961, 122, 345-358
- Kapusta, J. I., Finite-temperature field theory, Cambridge University Press, 1989

References

- Bailin, D. & Love, A. Brewer, D. F. Introduction to Gauge Field Theory, Taylor & Francis Group, 1993
- Kneur, J.-L.; Pinto, M. B. & Ramos, R. O., Phys. Rev. C, American Physical Society, 2010, 81, 065205.
- Fukushima, K., Phys. Rev. D, American Physical Society, 2008, 78, 114019

Swat Valley

- آپ کا شکریہ