Black Hole Formation and Classicalization in Ultra-Planckian Graviton Scattering

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Outline:

I) Classicalization and the black Hole N-portrait

II) Large N graviton scattering amplitudes at high energies in FT and ST

III) Interpretation of high energy behavior in the light of the N-portrait

IV) Some final remarks and observations

I) Classicalization and the black Hole N-portrait

[G. Dvali, C. Gomez, ...]

Einstein gravity possesses many interesting features:

- Geometry
- Black Holes
- Relation to Yang-Mills
- .....

But there are still many unsolved problems:

Quantization ☞

- Perturbative renormalizibility
- Non-perturbative properties, space-time transitions, emergence of space & time, ..

In particular two questions and puzzles:

- What is the quantum nature of Black Holes - microscopic understanding of black hole entropy?

Two (interconnected ?) claims:
Solve these problems (partially) within Einstein gravity!

$\Rightarrow$ Classicalization & the black hole N-portrait.

- What is the high energy behavior of graviton scattering amplitudes ?

Unitarity at tree level ?

Classicalization & the black hole N-portrait:

- Are described by IR physics,

.. where there is no need to modify gravity in the IR

However there remain still some UV problems:

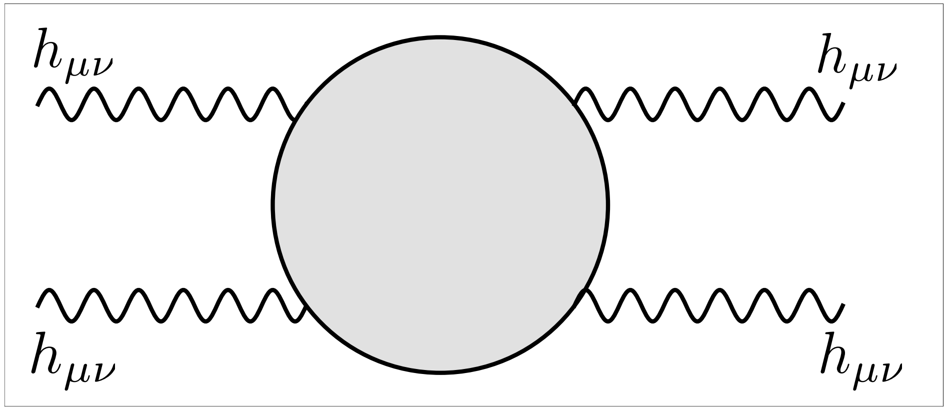
- Precise coefficient coefficient in black hole entropy:

$$S = \frac{1}{4} \frac{A}{L_P^2}$$

- Renormalization, UV finiteness of loop amplitudes

New UV degrees of freedom ➪ String theory !

Unitarity and classicalization:



$$\sim sL_P^2$$

It is known that tree level graviton scattering amplitudes grow like s (center of mass energy).

$\Rightarrow$ Violation of unitarity at $s = M_P^2$

One possible solution: Wilsonian approach:

Amplitude is unitarized by integrating in new weakly coupled degrees of freedom of shorter and shorter wave lengths (at higher and higher energies).

But: Gravity has smallest length scale: L_P

Beyond this length the Wilsonian approach breaks down:

It is expected that black holes will be produced in particle scattering processes with $\sqrt{s}^{-1} < R_s \equiv \sqrt{s} L_P^2$

[´t Hooft (1987); Antoniadis, Arakani-Hamed, Dimopoulos, Dvali (1998); Banks, Fischler (1999); Dimopoulos, Landsberg (2001); Yoshino, Nambu (2002); Giddings, Thomas (2002); Eardley, Giddings (2002); Giddings, Rychkov (2004); Rychkov (2004); ...]

Classicalization: Amplitudes get unitarized by classical black hole formation.

[G. Dvali, C. Gomez (2010); G. Dvali, G. Giudice, C. Gomez, A. Kehagias (2010)]

(Gravity protects itself at high energies by black hole formation.)

UV physics $\Leftrightarrow$ IR physics

So we need a better understanding of how black holes are formed in graviton scattering amplitudes.

Black hole corpuscular N-portrait:

Quantum black hole = Bound state of N gravitons

(Bose-Einstein condensate)

[G. Dvali, C. Gomez (2011 - 2014); G. Dvali, C. Gomez, D.L. (2012)]

Relevant properties (for us):

- N is large and the gravitons are soft.
- Interaction strength among individual gravitons is small:

$$\alpha = \frac{L_P^2}{R^2} << 1 \qquad (R \text{ ... graviton wave length})$$

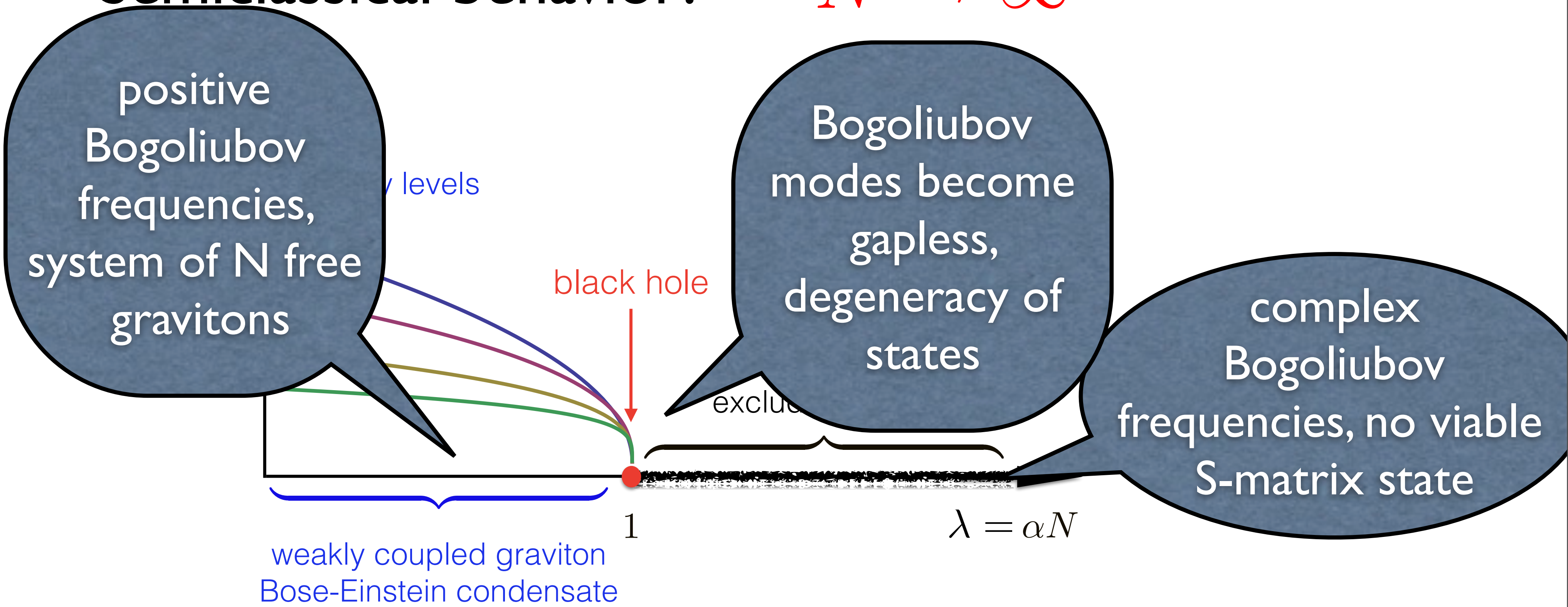
- Collective (\`t Hooft like) coupling: $\lambda = \alpha N$
- Black holes are formed at the quantum critical point:

$$\boxed{\lambda = 1} \qquad (R = \sqrt{N} L_P)$$

[G. Dvali, C. Gomez, arXiv:1207.4059;
Flassig, Pritzel, Wintergerst, arXiv:1212.3344]

Black hole bound state (at $\lambda = 1$):

- Mass and size: $M_{BH} = \sqrt{N} M_P \,, \quad R_{BH} = \sqrt{N} L_P$
- Exponential degeneracy, entropy: $S \sim N$
- Semiclassical behavior: $N \to \infty$



Can we reconcile this picture in graviton scattering processes (expressed in terms of N and λ)?

Interrelation between classicalization and b. h. N-portrait:

$2 \longrightarrow N$ graviton amplitude with high center of mass s:

$$N \to \infty\,, \quad s \to \infty \; (s >> M_P) \quad \text{with} \quad \lambda = \frac{s}{M_P^2 N} \neq 0$$

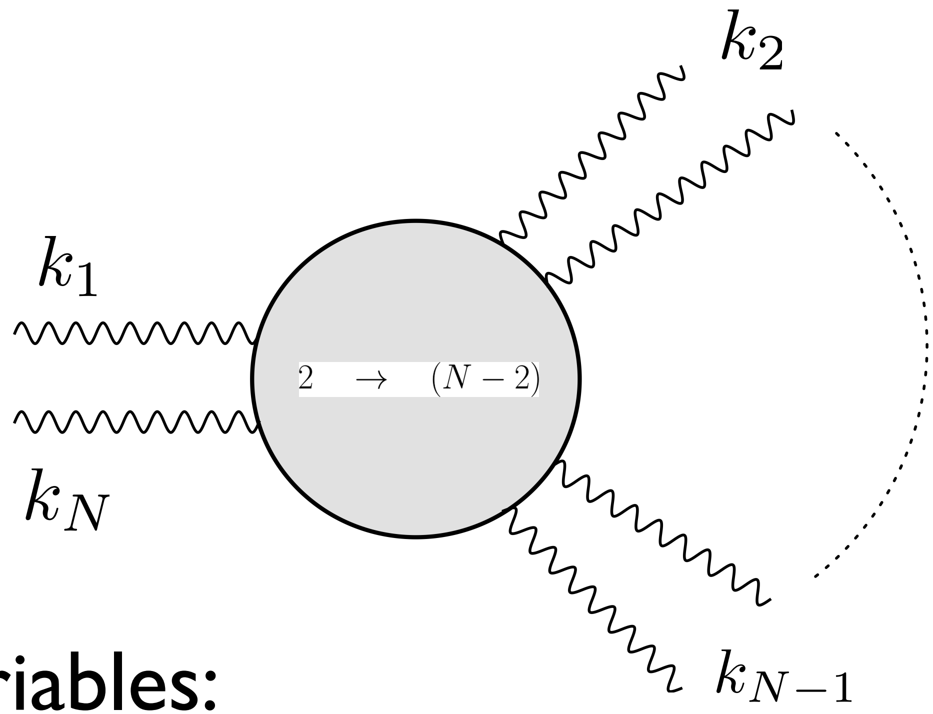
Results of the paper:

- Concrete technical computations of graviton scattering amplitudes in FT and ST in this kinematical regime - here only main results.
- Dependence on lambda shows interesting behavior that supports (at least in self-consistent way) the classicalization and black hole picture.
- Some interesting transition from FT with black holes to string theory with Regge modes.

II) Large N graviton scattering amplitudes at high energies in FT and ST

(i) Field theory

$$M_{2\to N-2}(N, s_{ij}) :$$



$\frac{1}{2}N(N-3)$ kinematic variables:

$$s_{ij} \equiv (k_i + k_j)^2 = 2\; k_i k_j$$

$$\#\text{constraints} = \frac{1}{2}(N-D)(N-D-1)$$

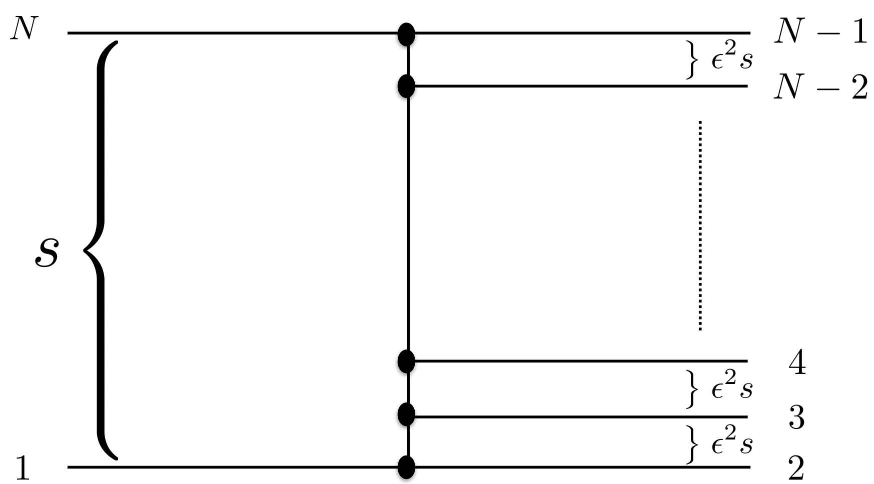
High energy regime: Eikonal Regge limit

Eikonal limit: two momenta are singled out

Regge limit: high center of mass energy

$$s_{ij} = (k_i + k_j)^2 \sim \begin{cases} s \ , & i,j \in \{1,N\} \ , \\ -\epsilon \ s \ , & i \in \{1,N\} \ , \ j \notin \{1,N\} \ , \\ \epsilon^2 \ s \ , & i,j \notin \{1,N\} \ , \end{cases}$$

$$s \to \infty \ , \qquad \epsilon \to 0$$



We want to have soft gravitons in the final state.

Classicalization limit: $p_{in} \sim \sqrt{s}$ and $p_{out} \sim \frac{\sqrt{s}}{N-2}$

$$s_{ij} = (k_i + k_j)^2 \sim \begin{cases} s \; , \quad i,j \in \{1, N\} \; , \\ -\frac{s}{N-2} \; , \quad i \in \{1, N\} \; , \; j \notin \{1, N\} \; , \\ \frac{s}{(N-2)^2} \; , \quad i,j \notin \{1, N\} \; . \end{cases}$$

$$s \to \infty \; , \qquad \epsilon = \frac{1}{N-2} \to 0$$

To compute the graviton scattering amplitudes we use on-shell methods and KLT techniques. [Kawai, Lewellen, Tye (1986)]

N-point gravity tree level scattering via KLT:

$$M_N = \left(-\frac{\kappa}{2}\right)^{N-2} \sum_{\sigma,\gamma\in S_{(N-3)}} A_N(1,\sigma(2,\ldots,N-2),N-1,N)$$
$$S[\gamma(2,\ldots,N-2),\sigma(2,\ldots,N-2)]_{N-1} A_N(1,N-1,\sigma(2,\ldots,N-2),N)$$

[Bern,Dixon, Perelstein, Rozowsky (1998); .... Bjerrum-Bohr, Damgaard, Sondergaard,Vanhove (2010)]

- $S[\ldots,\ldots]$ is called momentum kernel, $S \sim s_{ij}^{N-3}$
- $A_N(\ldots)$ color ordered MHV Yang-Mills amplitude:

$$A_N(1^+,...,i^-,...,j^-,...,N^+) = \frac{\langle i\ j\rangle^4}{\langle 1\ 2\rangle\langle 2\ 3\rangle...\langle N-1\ N\rangle\langle N\ 1\rangle}$$

Spinor helicity brackets: $\langle ij\rangle = \sqrt{|s_{ij}|}e^{i\phi_{ij}}$ [Park,Taylor (1986)]

- Yang-Mills amplitude scales like:

$$A_N = s^{\frac{4-N}{2}} \; f(\phi) \; (N-2)^{N-4}$$

- Momentum kernel scales like:

$$S \sim \left(\frac{s}{(N-2)^2}\right)^{N-3}$$

- Thus gravity amplitude scales via KLT like:

$$M_N \sim \kappa^{N-2} \; \tilde{C}(N) \; s \; \times (N-2)^{-2}$$

with $\tilde{C}(N)$ double sum over phase factors.

It fixes the combinatorics of the amplitude.
We computed it using QFT methods and scattering equations in string theory:

$$\tilde{C}(N) = (N-1)!$$

To obtain the physical probability, i.e. the S-matrix element, we have to consider

$$d|\langle 2|S|N-2\rangle|^2 = \frac{1}{(N-2)!}\prod_{i=2}^{N-1} dp_i^4 |M_N|^2 \delta^4(P_{total})$$

Full cross section by integrating over momenta and summing over helicities: ($p_{in} \sim \sqrt{s}$, $p_{out} \sim \frac{\sqrt{s}}{N-2}$)

Physical $2 \to N-2$ scattering probability in classicalization regime:

$$|\langle 2|S|N-2\rangle|^2 = \left(\frac{L_P^2 s}{N^2}\right)^N N! = \left(\frac{\lambda}{N}\right)^N N! \sim e^{-N}\lambda^N$$

Collective coupling $\lambda \equiv \alpha N = s/M_P^2 N$

(ii) Closed string theory

High energy behavior of open/closed string amplitudes shows exponential fall off due to Regge modes.

[Veneziano (1968); Amati, Ciafaloni, Veneziano (1987); Gross, Mende (1987), Gross, Manes (1989)]

Example: 4-point graviton amplitude

$$\mathcal{M}_4 \sim K \, \frac{\Gamma(-\frac{\alpha'}{4}s)\Gamma(-\frac{\alpha'}{4}t)\Gamma(-\frac{\alpha'}{4}u)}{\Gamma(\frac{\alpha'}{4}s)\Gamma(\frac{\alpha'}{4}t)\Gamma(\frac{\alpha'}{4}u)}$$

$$\longrightarrow_{\alpha'\to\infty} \kappa^2 |A_4|^2 \times 4\pi\alpha' \frac{st}{u} \exp\left\{ \frac{\alpha'}{2}(s \ln|s| + t\ln|t| + u\ln|u|)\right\}$$

Square of YM-amplitude

Momentum kernel

String form factor

(Note: this was basically the state of the art before our paper.)

Generalization to arbitrary N:

[Stieberger, Taylor (2013, 2014)]

- KLT and Laplace saddle point methods.
- Recent work on scattering equations. [Cachazo, H, Yuan (2013)]

  → This fixes the combinatorics of string amplitude in the high energy limit $\alpha' \rightarrow \infty$.

$$\mathcal{M}_N = (-1)^{N-3}\ \kappa^{N-2}\ A^t_{YM}\ S_0\ \mathrm{sv}(\mathcal{A})$$

$\mathcal{A}$ is an $(N-3)!$ -dim. vector of independent open string amplitudes:

$$\mathcal{A}_N(1,\pi(2,\ldots,N-2),N-1,N) = g_{YM}^{N-2} \sum_{\sigma\in S_{N-3}} F_{\pi\sigma}\ A_{YM}(\sigma)\ , \quad \pi\in S_{N-3}$$

S_0 is the momentum kernel:
$F_{\pi\sigma}$ is the string form factor:
} $(N-3)! \times (N-3)!$ matrices

sv: single valued map [Stieberger (2014)]

High energy limit:

$$s >> L_P^2\,,\;\; g_s^{-2} = \frac{L_s^2}{L_P^2} >> 1 \;\Rightarrow\; \alpha' s >> \frac{L_s^2}{L_P^2} >> 1$$

Eikonal limit & Classicalization regime:

$$\mathcal{M}_N = \kappa^{N-2}\, |A_{YM}(1,\ldots,N)|^2\, \mathcal{F}_N$$

String form factor, comprises all stringy physics

$$\mathcal{F}_N \;\sim\; (4\pi\alpha')^{N-3}\left(\frac{s_{12}\; s_{23}}{s_{2N}}\right)\, \exp\{\frac{\alpha'}{2}(\, s_{12}\ln s_{12} + s_{23}\ln s_{23} + s_{2N}\ln s_{2N}\,)\}$$

$$\times\;\; \prod_{l=1}^{N-4}\left(\frac{x_l\; y_l}{z_l}\right)\, \exp\{\frac{\alpha'}{2}(\, x_l\ln x_l + y_l\ln y_l + z_l\ln z_l\,)\}\;,$$

Two different energy regimes:

$$(i) \quad \frac{\sqrt{s}}{N} < M_s \; :$$ „infrared“, field theory regime

Field and ST theory amplitudes agree.

This was already conjectured for the MHV case up to 5 points by [Cheung, O´Connell, Wecht (2010)]

$$F_N = 1 \quad \Rightarrow \quad \mathcal{M}_N = M_N^{FT}$$

$$(ii) \quad \frac{\sqrt{s}}{N} > M_s \; :$$ „ultraviolet“, string theory regime

$$\mathcal{M}_N \sim \kappa^{N-2} \; {\alpha'}^{N-3} \; s \; e^{-\frac{\alpha'}{2}(N-3) \; s \ln(\alpha' s)}$$

Amplitude gets tamed by string states (Regge modes).

III) Interpretation of high energy behavior in the light of the N-portrait

What makes us believe that our results support the idea of classicalization and black hole formation?

(i) Field theory

Perturbative amplitude:

$$|\langle 2|S|N-2\rangle|^2 \sim e^{-N}\lambda^N\,, \quad \lambda = \frac{s}{M_P^2 N}$$

Unitarity threshold: amplitude changes behavior at $\lambda = 1$.

$\lambda < 1$ weak coupling: unitary behavior.

$\lambda > 1$ strong coupling: non-unitary behavior.

These regions precisely correspond to the 3 regimes of the black hole N-portrait.

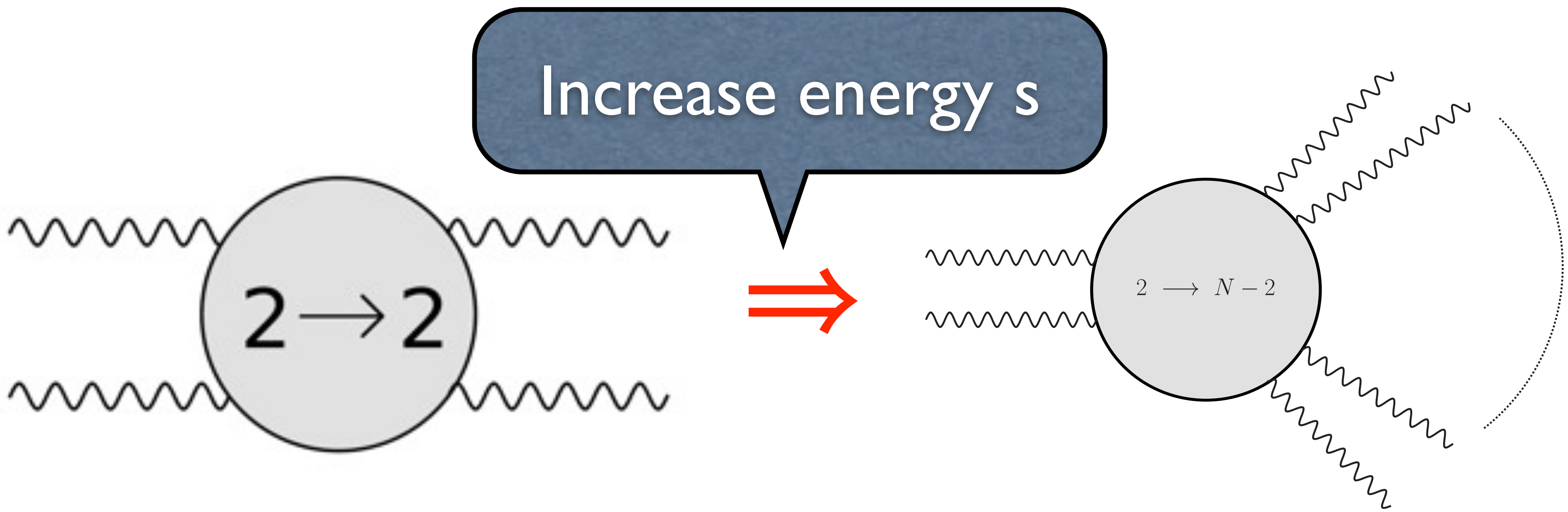
For large s unitarization occurs if N increases appropriately:

$$\frac{\sigma_{2\to N+1}}{\sigma_{2\to N}} \lesssim 1$$

This bound implies that $N \gtrsim N_{crit} = sL_P^2$

This is the core of the idea of classicalization!

N should be larger than the corresponding entropy of a black hole with mass equal to the center of mass energy.



Connection to non-perturbative black hole bound state:

The perturbative amplitude is suppressed by e^{-N} .

This is just the inverse of the degeneracy of states of a black hole with entropy $S \sim N$.

Therefore this suppression factor is compensated at the critical point $\lambda = 1$ by e^{N} from the degeneracy of black hole states:

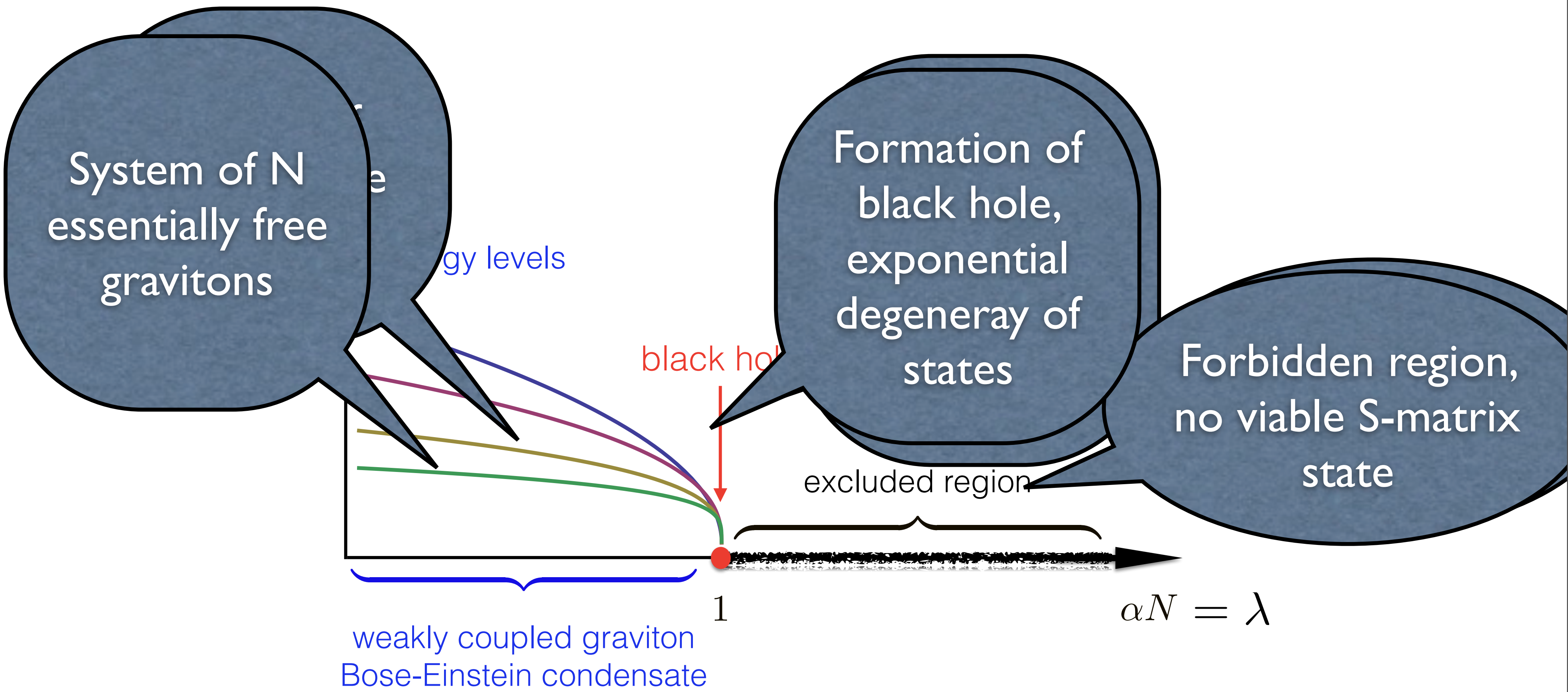
$$A_{BH} \sim \sum_j |\langle 2|S|N\rangle|_p^2 \; |\langle N|BH\rangle_j|_{np}^2 \sim \lambda^N e^{-N}|_p \times e^N|_{np}$$

pert. amplitude

projection on black hole bound state

(This was cross checked by a semiclassical calculation.)

So, black hole is exactly dominating at $\lambda = 1$.


System of N essentially free gravitons
gy levels
black ho
Formation of black hole, exponential degeneray of states
Forbidden region, no viable S-matrix state
excluded region
1
$\alpha N = \lambda$
weakly coupled graviton Bose-Einstein condensate

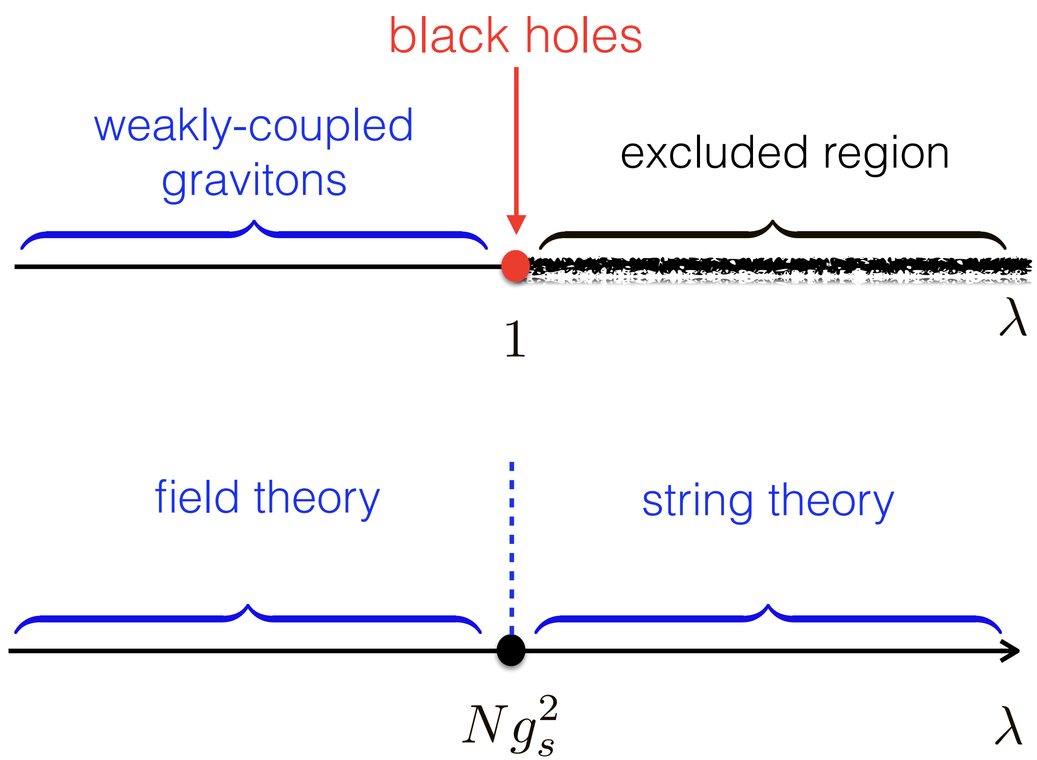
(ii) String theory: there are additional regimes:

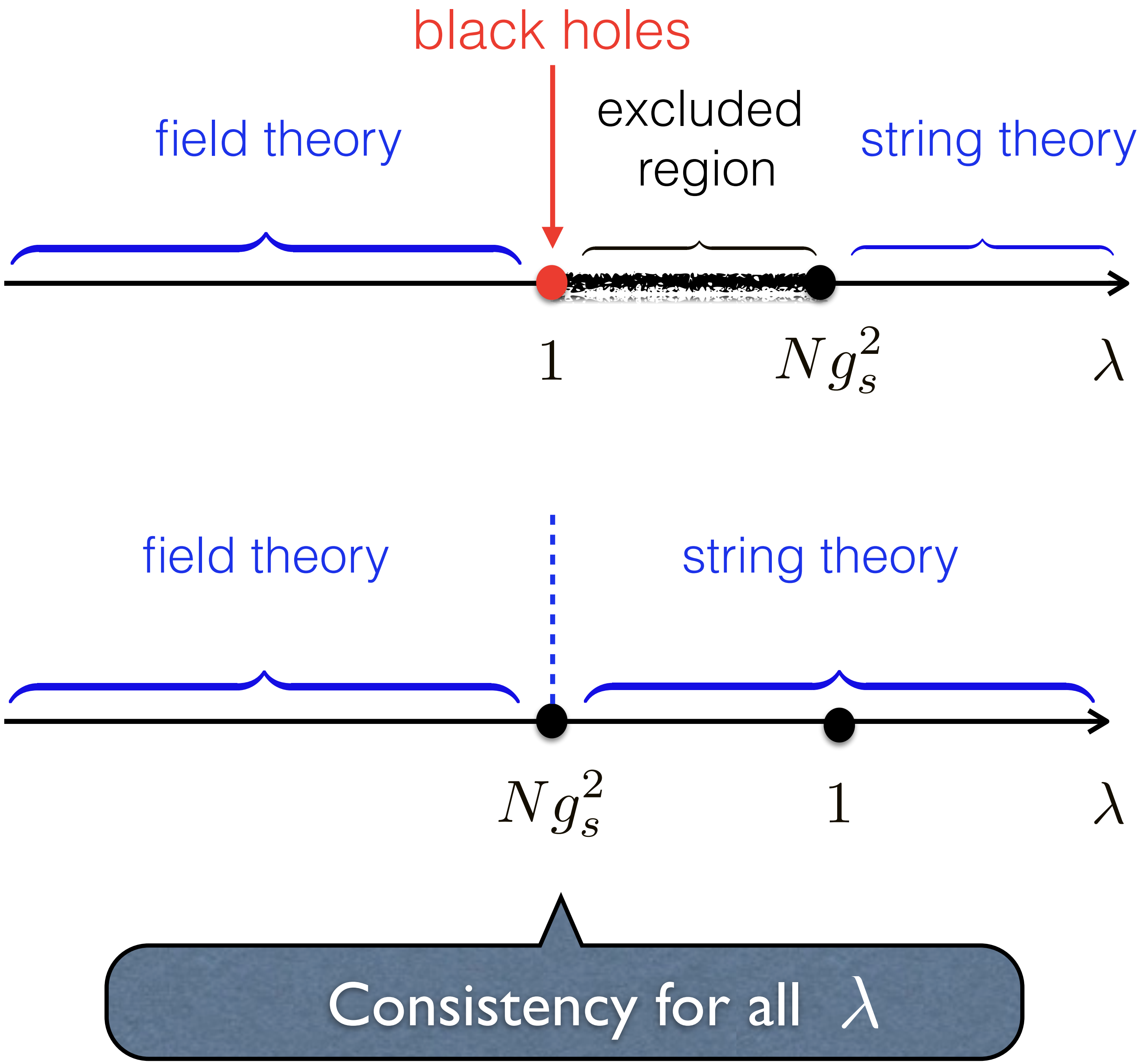
$$(i) \quad \frac{\sqrt{s}}{N} < M_s : \iff \lambda < N g_s^2$$

FT amplitude = ST amplitude, black hole dominance

$$(ii) \quad \frac{\sqrt{s}}{N} > M_s : \iff \lambda > N g_s^2$$

String states dominate.




black holes
field theory
excluded
region
string theory
1
Ng_s^2
λ
field theory
string theory
Ng_s^2
1
λ
Consistency for all λ

What is happening at the point $\lambda = N g_s^2 = 1$?

Here the F.T. amplitude agrees with the string amplitude at the critical point $\lambda = 1$.

This the point where the string effects match the amplitude from the F.T. black hole formation.

$g_s = \frac{1}{\sqrt{N}}$ $\Rightarrow$ String - black hole correspondence:

black hole can be described by a state of strings.

[Horowitz, Polchinski (1996); Dvali, D.L. (2009); Dvali, Gomez (2010)]

Here the IR is meeting the UV.

IV) Some final remarks and observations

As we have seen, the gravity amplitudes can be expressed as sums over Yang-Mills amplitudes.

But we never used the information about the number of colors N_c .

- Relation between open and closed string coupling:

$$g_s = g_{open}^2$$

- At point of string-bh correspondence: $g_s = 1/\sqrt{N}$
- Planar limit of gauge theory: $g_{open}^2 = 1/N_c$

So naively we get: $N = N_c^2$

What is the interpretation of this relation?

Summary:

- Exact computation of N-point gravity (string) amplitudes in transplanckian energy region in closed form.
- We found evidence for classicalization and black hole production (black hole N-portrait):
  - dependence on N
  - dependence on λ
  - dependence on entropy S
- We found an interesting transition between field theory: string - black hole correspondence.
- The limit of large N_c in Yang-Mills apparently corresponds to the limit of large number of constituent gravitons in scattering process.

Next steps:

[Stieberger (2013/14); Cachazo, He, Yuan (2014)]

- Mixed gauge boson (open)/gravity (closed) amplitudes:
  Bh N-portrait with matter
  [Dvali, Gomez, D.L. (2013)]

- Bh N-portrait beyond tree level First steps in [Kuhnel, Sundborg (2014)]

Thank you very much!