

Beam-Beam Effects in Colliders

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Hadron Collective Effects

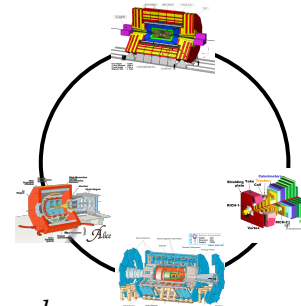
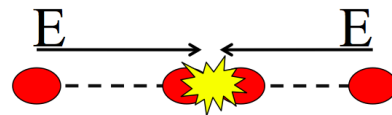


Hadron Colliders

$$E^* \approx 2 \times E$$

$$N_{event/s} = L \cdot \sigma_{event}$$

$$L \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b \cdot f_{rev}$$



Bunch intensity: $N_p = 1.15 - 1.65 \cdot 10^{11} \text{ ppb}$

Transverse Beam size: $\sigma_{x,y} = 16 - 30 \text{ } \mu m$

Number of bunches 1370 – 2808

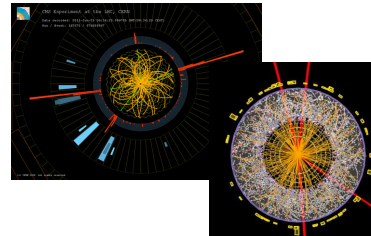
Revolution frequency 11 kHz

When do we have beam-beam effects?

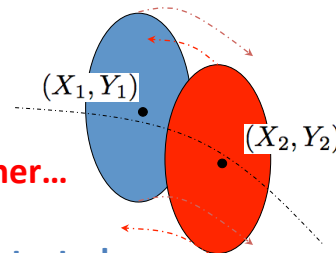
➤ They occur when two beams get closer and collide

➤ Two types

- High energy collisions between two particles (wanted)
- Distortions of beam by electromagnetic forces (unwanted)



- **Unfortunately: usually both go together...**
- 0.001% (or less) of particles collide
- 99.999% (or more) of particles are distorted



Beam-beam effects: overview

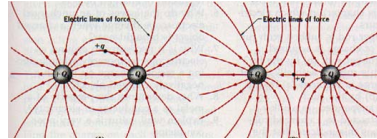
➤ **Circular Colliders:** interaction occurs at every turn

- Many effects and problems
- Try to understand some of them
- Overview of selected effects (single particle and multi-particle effects)
- Qualitative and physical picture of effects
- Observations
- Mathematical derivations and more info in References [1,3,4] or at

Beam-beam webpage <http://lhc-beam-beam.web.cern.ch/lhc-beam-beam/>
And CAS Proceedings

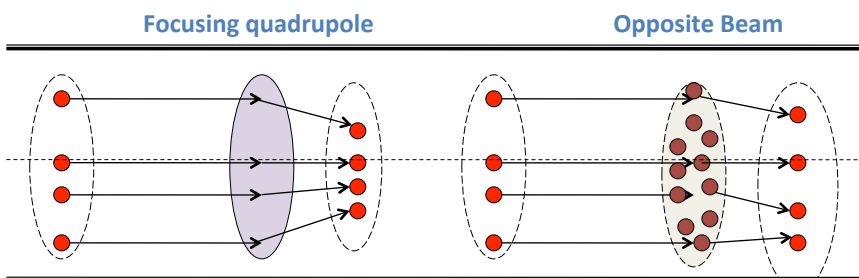
Beams EM potential

- Beam is a collection of charges
- Beam is an electromagnetic potential for other charges



Force on itself (**space charge**) and opposing beam (**beam-beam effects**)

Single particle motion and whole bunch motion **distorted**



A beam acts on particles like an electromagnetic lens, but...

Beam-beam Mathematics

General approach in electromagnetic problems Reference[5] already applied to beam-beam interactions in Reference[1,3, 4]

$$\Delta U = -\frac{1}{\epsilon_0} \rho(x, y, z)$$

Derive potential from Poisson equation for charges with distribution ρ

Solution of Poisson equation

$$U(x, y, z, \sigma_x, \sigma_y, \sigma_z) = \frac{1}{4\pi\epsilon_0} \iiint \frac{\rho(x_0, y_0, z_0) dx_0 dy_0 dz_0}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}}$$

$$\vec{E} = -\nabla U(x, y, z, \sigma_x, \sigma_y, \sigma_z)$$

Then compute the fields

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

From Lorentz force one calculates the force acting on test particle with charge q

Making some assumptions we can simplify the problem and derive analytical formula for the force...

Round Gaussian distribution:

Gaussian distribution for charges:

Round beams:

Very relativistic, Force has only radial component :

$$\sigma_x = \sigma_y = \sigma$$

$$\beta \approx 1 \quad r^2 = x^2 + y^2$$

$$F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}} \right]$$

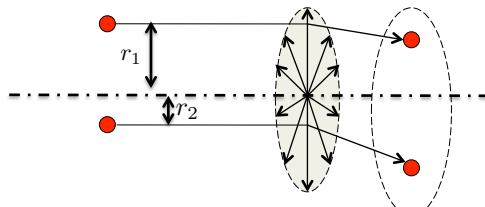
Beam-beam Force

$$\Delta r' = \frac{1}{mc\beta\gamma} \int F_r(r, s, t) dt$$

Beam-beam kick obtained
integrating the force over the
collision (i.e. time of passage)

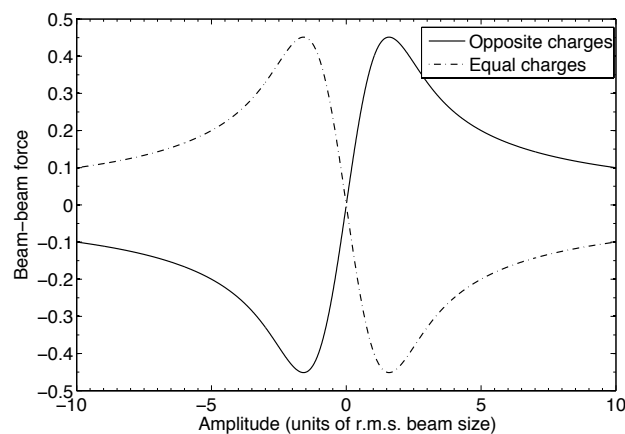
$$\Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2} [1 - e^{-\frac{r^2}{2\sigma^2}}]$$

Only radial component in
relativistic case



How does this force looks
like?

Beam-beam Force



$$F_r(r) = \pm \frac{ne^2(1 + \beta_{rel}^2)}{2\pi\epsilon_0} \frac{1}{r} \left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right) \right]$$

Why do we care?

Pushing for luminosity means stronger beam-beam effects

$$\mathcal{L} \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b$$

$$F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}} \right]$$

Physics fill lasts for many hours 10h – 24h

Strongest non-linearity in a collider YOU CANNOT AVOID!

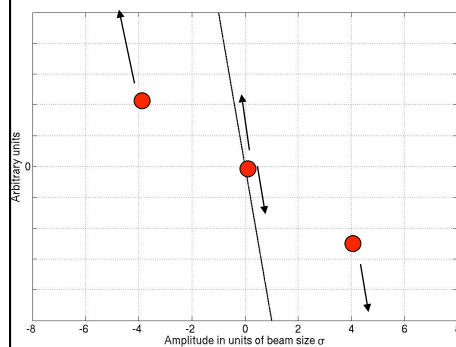


Two main questions:

What happens to a single particle?
What happens to the whole beam?

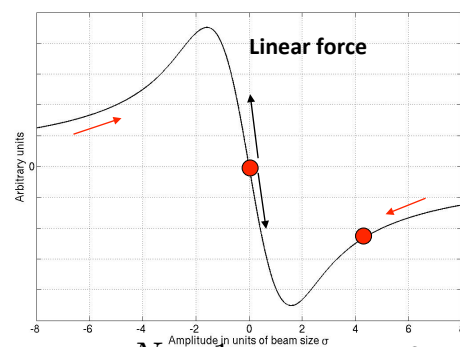
Beam-Beam Force: single particle...

Lattice defocusing quadrupole



$$F = -k \cdot r$$

Beam-beam force



$$F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}} \right]$$

For small amplitudes: linear force

For large amplitude: very non-linear

The beam will act as a strong non-linear electromagnetic lens!

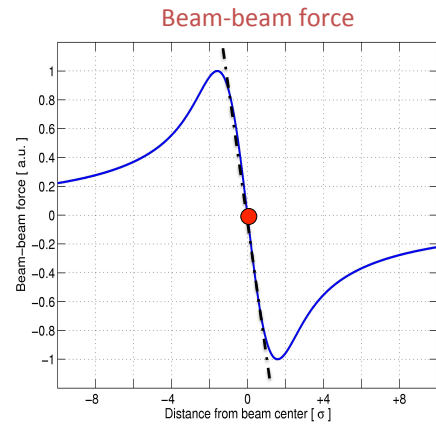
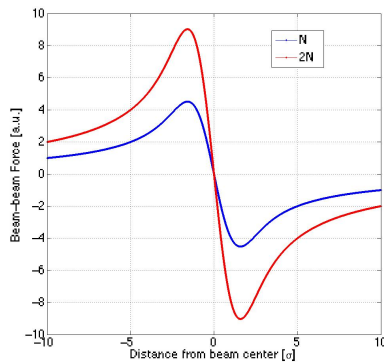
Can we quantify the beam-beam strenght?

Quantifies the strength of the force
but does NOT reflect the nonlinear
nature of the force

For small amplitudes: linear force

$$F \propto -\xi \cdot r$$

The slope of the force gives you
the beam-beam parameter ξ



$$\Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}} \right]$$

$$\Delta r' = \frac{2N_p r_0}{\gamma} \cdot \frac{1}{r} \cdot \left[1 - \left(1 - \frac{r^2}{2\sigma^2} + \dots \right) \right]$$

Colliders:

For round beams:

$$\xi = \frac{\beta^*}{4\pi} \cdot \frac{\delta(\Delta r')}{\delta r} = \frac{N r_0 \beta^*}{4\pi \gamma \sigma^2}$$

For non-round beams:

$$\xi_{x,y} = \frac{N r_0 \beta_{x,y}^*}{2\pi \gamma \sigma_{x,y} (\sigma_x + \sigma_y)}$$

Examples:

Parameters	LEP (e ⁺ e ⁻)	LHC(pp)
Intensity N _{p,e} /bunch	4 · 10 ¹¹	1.15 · 10 ¹¹
Energy GeV	100	7000
Beam size H	160-200 μm	16.6 μm
Beam size V	2-4 μm	16.6 μm
β _{x,y} * m	1.25-0.05	0.55-0.55
Crossing angle μrad	0	285
ξ_{bb}	0.07	0.0037

Linear Tune shift

For small amplitudes beam-beam can be approximated as linear force as a quadrupole

$$F \propto -\xi \cdot r$$

Focal length:
$$\frac{1}{f} = \frac{\Delta x'}{x} = \frac{Nr_0}{\gamma\sigma^2} = \frac{\xi \cdot 4\pi}{\beta^*}$$

Beam-beam matrix:
$$\begin{pmatrix} 1 & 0 \\ -\frac{\xi \cdot 4\pi}{\beta^*} & 1 \end{pmatrix}$$

Perturbed one turn matrix with perturbed tune ΔQ and beta function at the IP β^* :

$$\begin{pmatrix} \cos(2\pi(Q + \Delta Q)) & \beta^* \sin(2\pi(Q + \Delta Q)) \\ -\frac{1}{\beta^*} \sin(2\pi(Q + \Delta Q)) & \cos(2\pi(Q + \Delta Q)) \end{pmatrix} \\ = \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(2\pi Q) & \beta_0^* \sin(2\pi Q) \\ -\frac{1}{\beta_0^*} \sin(2\pi Q) & \cos(2\pi Q) \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix}$$

Linear tune

Solving the one turn matrix one can derive the tune shift ΔQ and the perturbed beta function at the IP β^* :

Tune is changed

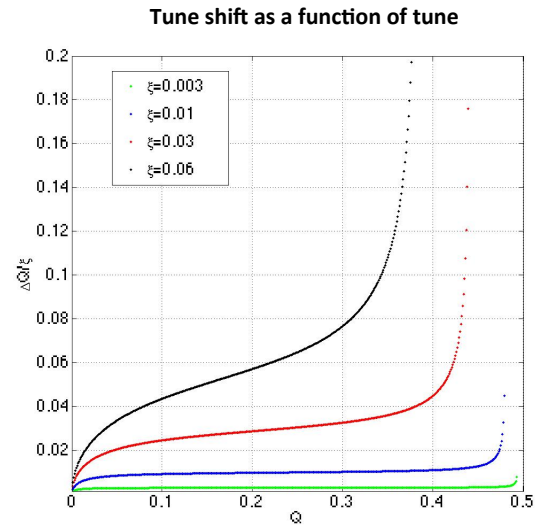
$$\cos(2\pi(Q + \Delta Q)) = \cos(2\pi Q) - \frac{\beta_0^* \cdot 4\pi\xi}{\beta^*} \sin(2\pi Q)$$

β -function is changed:

$$\frac{\beta^*}{\beta_0^*} = \frac{\sin(2\pi Q)}{\sin(2\pi(Q + \Delta Q))}$$

...how do they change?

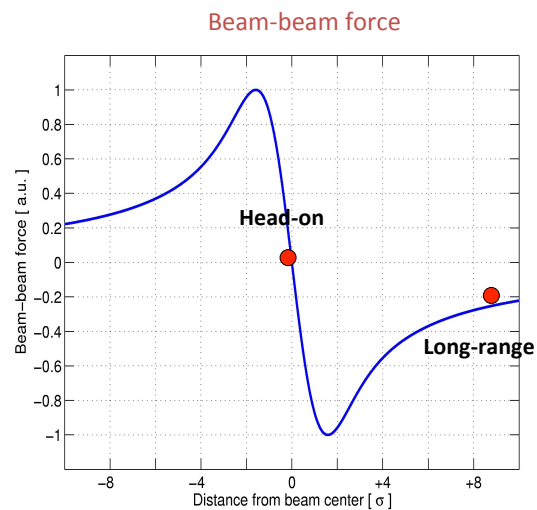
Tune dependence of tune shift and dynamic beta



Larger ξ ➔ Strongest variation with Q

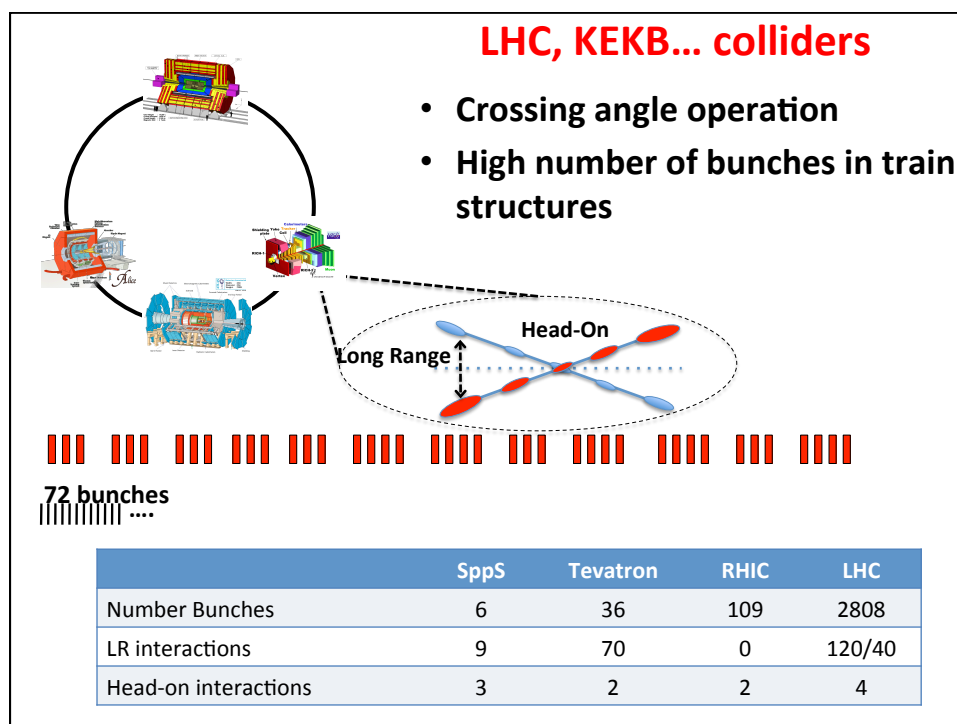
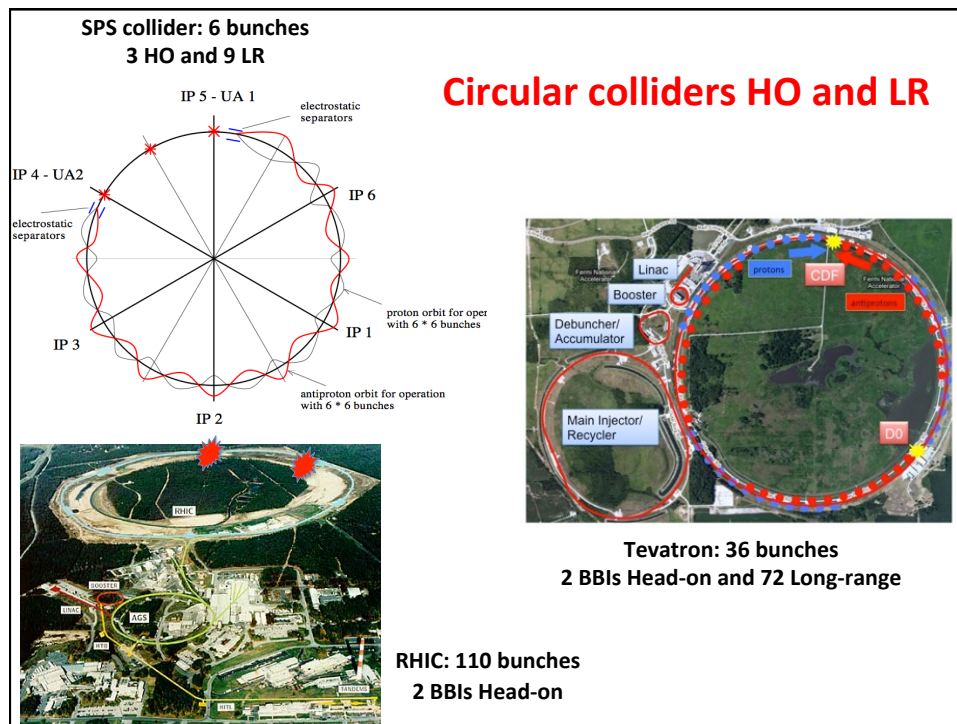
Head-on and Long-range interactions

$$L \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b \cdot f_{rev}$$

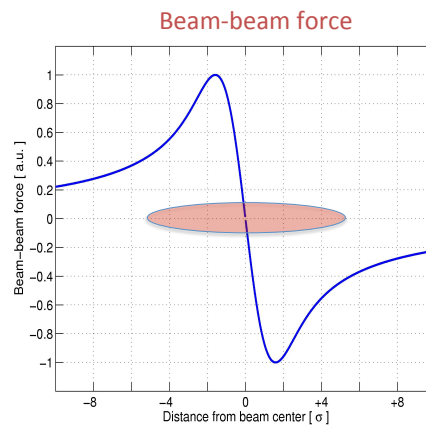


Other beam passing in the center force: **HEAD-ON** beam-beam interaction

Other beam passing at an offset of the force: **LONG-RANGE** beam-beam interaction



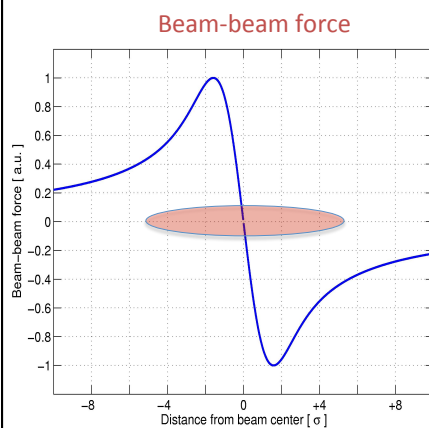
A beam is a collection of particles



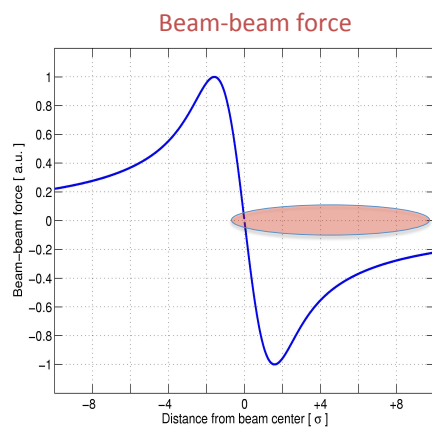
Beam 2 passing in the center of force produce by Beam 1
 Particles of Beam 2 will experience different ranges of the beam-beam forces

**Tune shift as a function of amplitude (detuning with amplitude or
 tune spread)**

A beam will experience all the force range



Second beam passing in the center
HEAD-ON beam-beam interaction

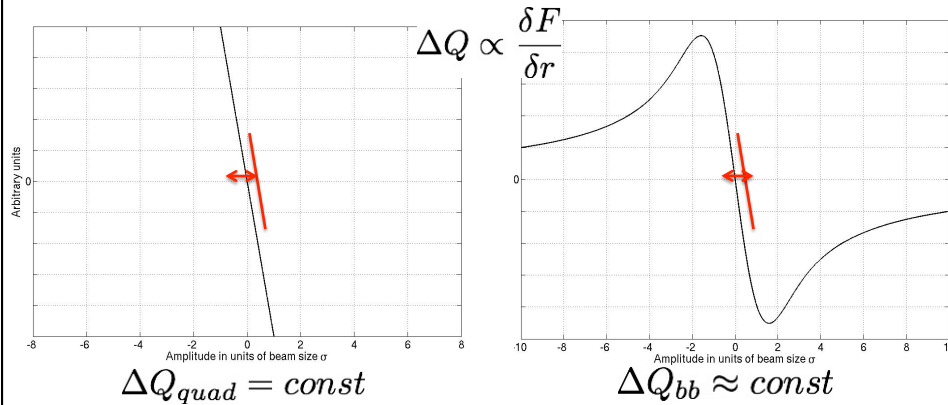


Second beam displaced offset
LONG-RANGE beam-beam interaction

Different particles will see different force

Detuning with Amplitude for head-on

Instantaneous tune shift of test particle when it crosses the other beam is related to the derivative of the force with respect to the amplitude

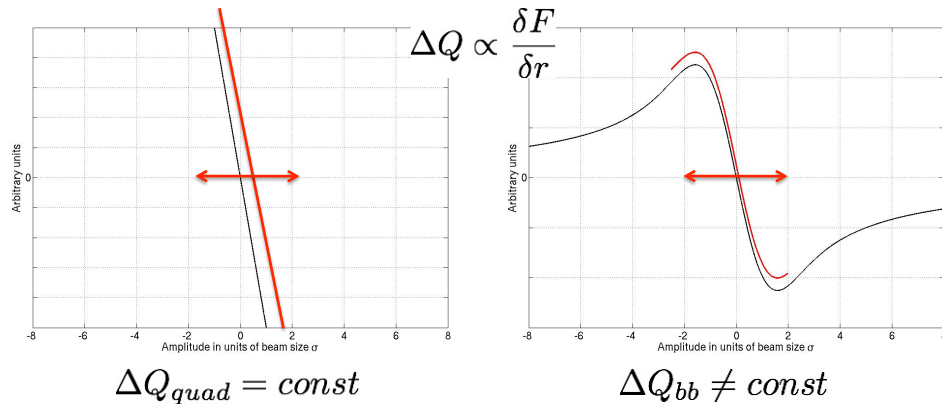


For small amplitude test particle
linear tune shift

$$\lim_{r \rightarrow 0} \Delta Q(r) = -\frac{Nr_0\beta^*}{4\pi\gamma\sigma^2} = \xi$$

Detuning with Amplitude for head-on

Beam with many particles this results in a tune spread

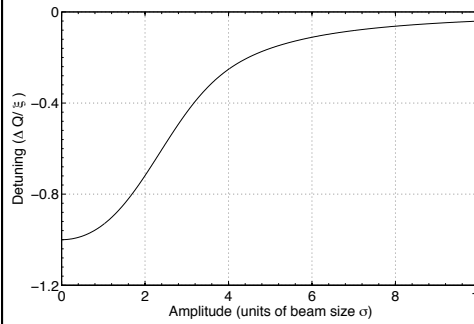


$$\Delta Q(x) = \frac{Nr_0\beta}{4\pi\gamma\sigma^2} \cdot \frac{1}{(\frac{x}{2})^2} \cdot (\exp -(\frac{x}{2})^2 I_0 (\frac{x}{2})^2 - 1)$$

Mathematical derivation in Ref [3] using Hamiltonian formalism and in
Ref [4] using Lie Algebra

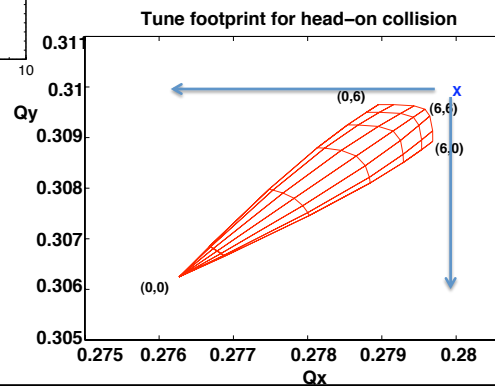
Head-on detuning with amplitude and footprints

1-D plot of detuning with amplitude

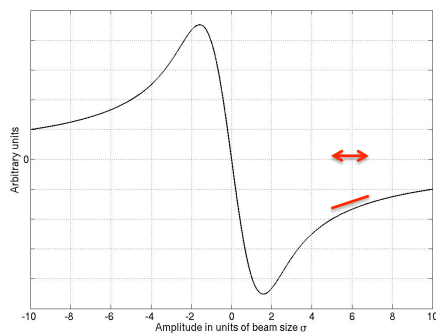


And in the other plane?
THE SAME DERIVATION
 same tune spread

FOOTPRINT
 2-D mapping of the detuning with
 amplitude of particles



And for long-range interactions?

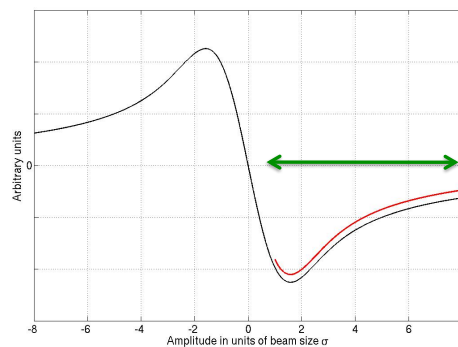


Second beam centered at d (i.e. 6σ)

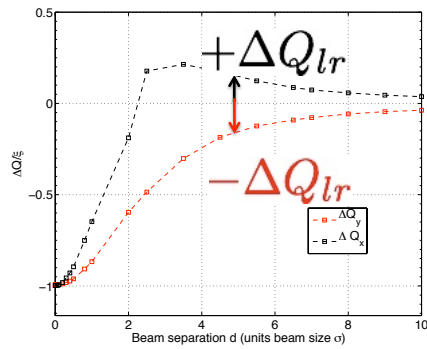
- Small amplitude particles **positive tune shifts**
- Large amplitude can go to **negative tune shifts**

Long range tune shift scaling for
 distances $d > 6\sigma$

$$\Delta Q_{lr} \propto -\frac{N}{d^2}$$

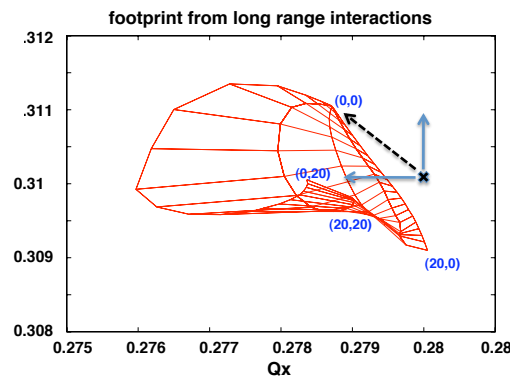


Long-range footprints

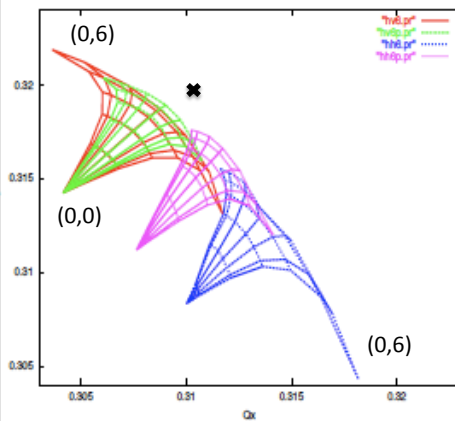


The picture is more complicated
now the **LARGE** amplitude particles
see the second beam and have
larger tune shift

Separation in vertical plane!
And in horizontal plane?
The test particle is centered with
the opposite beam
tune spread more like for head-on
at large amplitudes



Beam-beam tune shift and spread



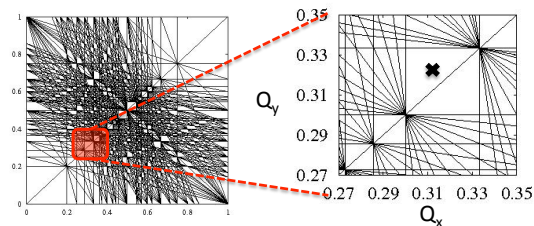
Footprints depend on:

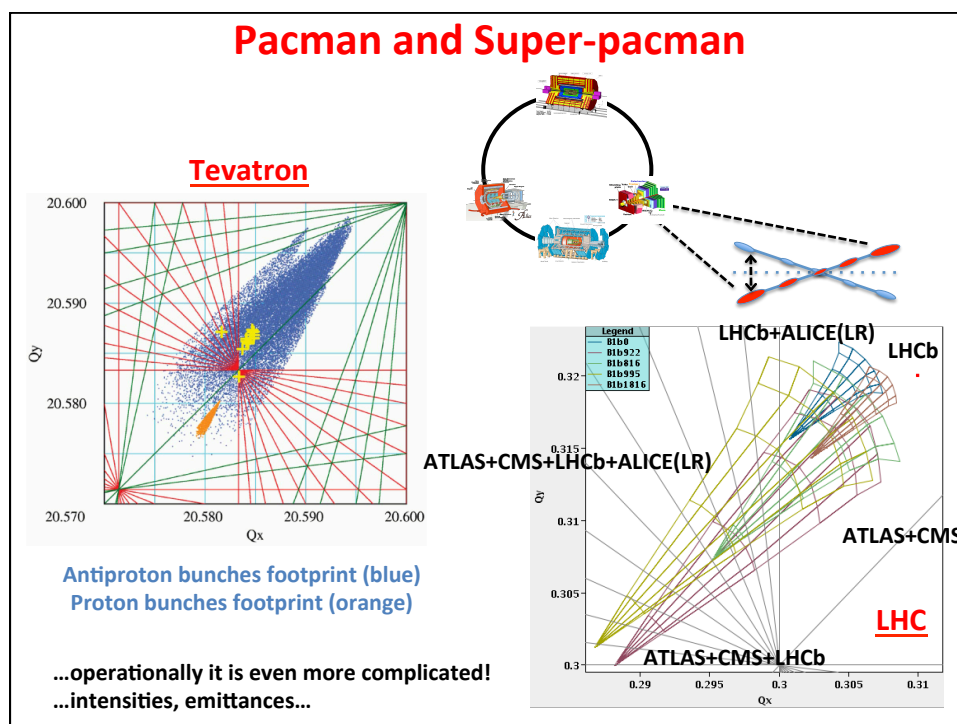
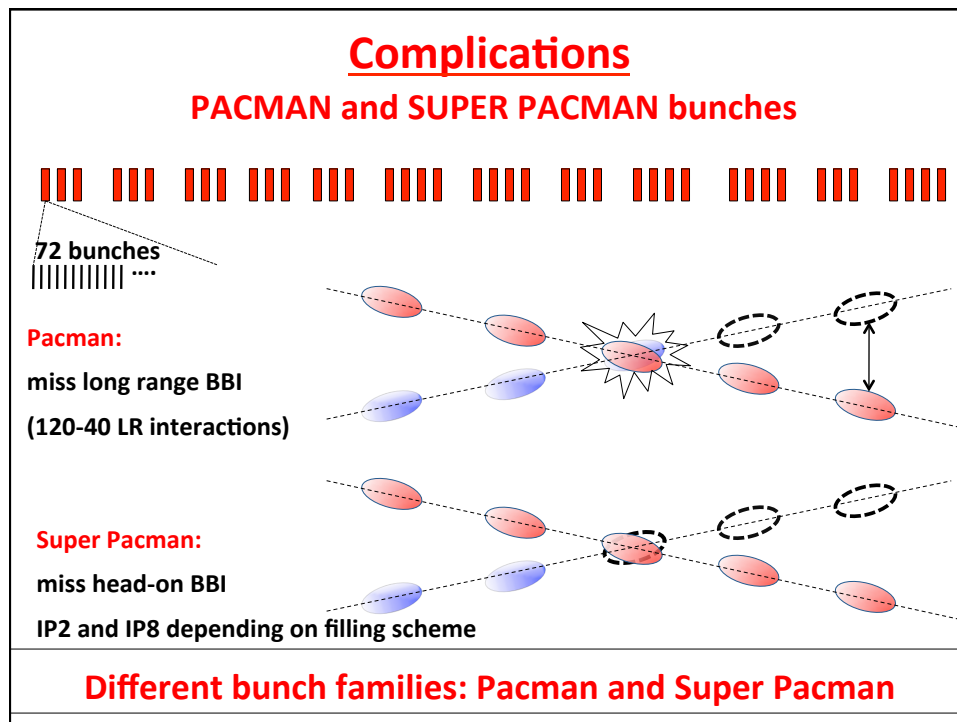
- number of interactions
- Type (Head-on and long-range)
- Plane of interaction

When long-range effects become
important footprint wings appear and
alternating crossing important

Aim to reduce the area as much as
possible!

Passive compensation of tune shift Ref[7]

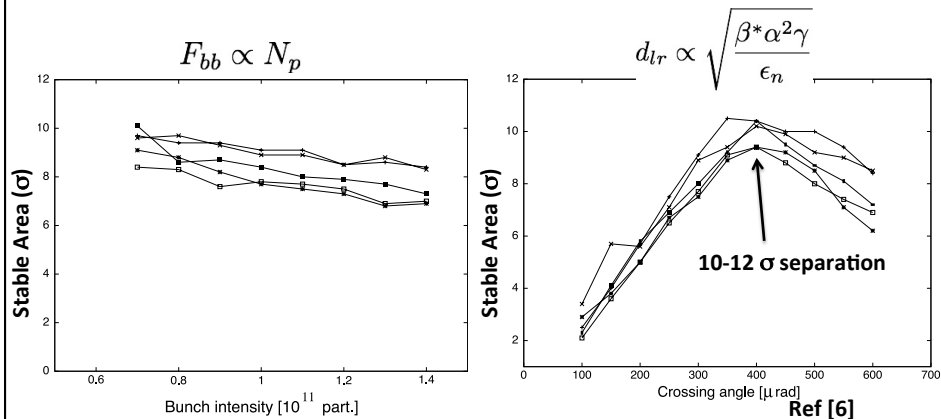




Particle Losses

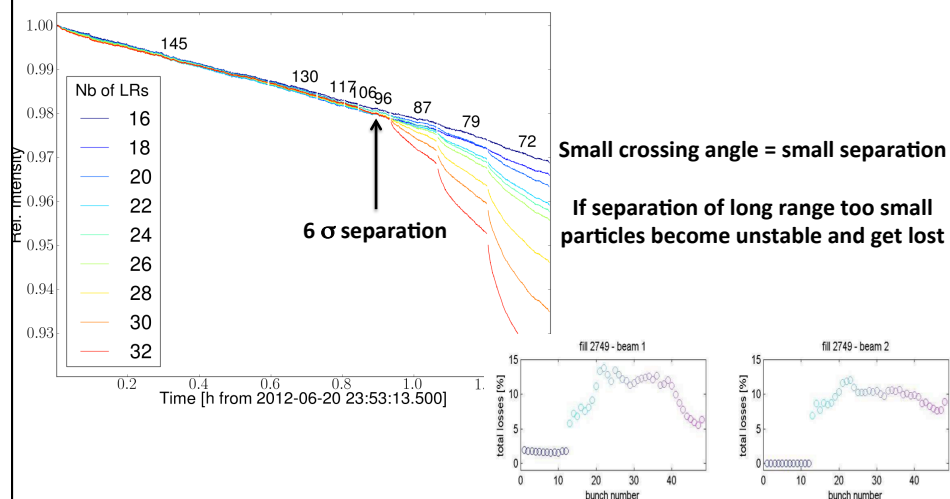
Dynamic Aperture: area in amplitude space with stable motion

Stable area of particles depends on beam intensity and crossing angle



Stable area depends on beam-beam interactions therefore the choice of running parameters (crossing angles, β^* , intensity) is the result of careful study of different effects!

DO we see the effects of LR in the LHC?



Particle losses follow number of Long range interactions
Nominal LHC will have twice the number of interactions

[Ref]

Observations in Leptons:

From our known formulas:

$$L = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \quad \xi_{x,y} = \frac{Nr_0\beta_{x,y}^*}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

Increasing bunch population N_1 and N_2 :

- luminosity should increase N_2
- beam-beam parameter linearly

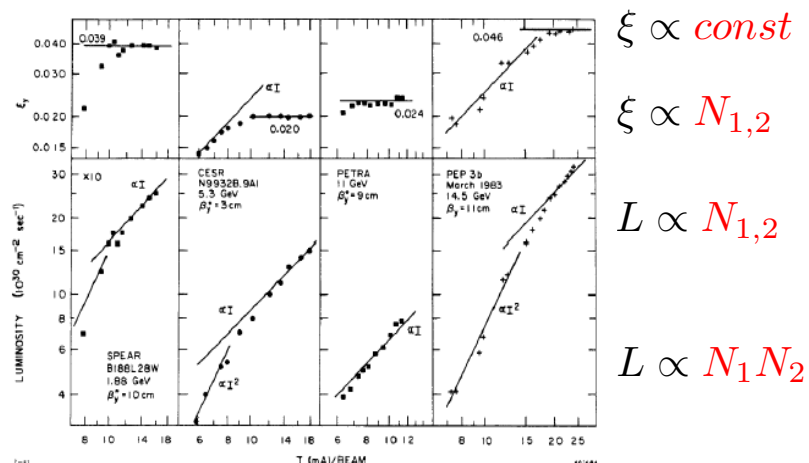
$$L \propto N_1 N_2$$

$$\xi \propto N_{1,2}$$

But...

Leptons beam-beam limit

First beam-beam limit (J. Seeman, 1983)



Luminosity and vertical tune shift parameter vs. beam current for SPEAR, CESR, PETRA & PEP.

What is happening?

Again....

$$\xi_{x,y} = \frac{Nr_0\beta_{x,y}^*}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

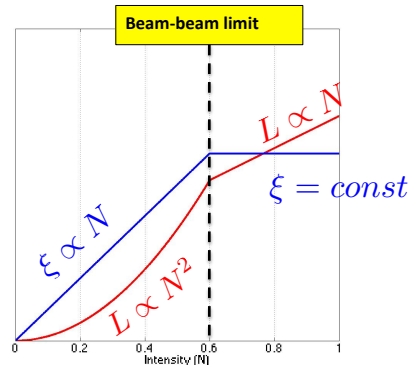
$$L = \frac{N^2 f n_b}{4\pi\sigma_x\sigma_y}$$

Lepton colliders $\sigma_x \gg \sigma_y$

$$\xi_y \approx \frac{r_0\beta_y^*}{2\pi\gamma\sigma_x} \left(\frac{N}{\sigma_y} \right)$$

$$L = \frac{N f n_b}{4\pi\sigma_x} \left(\frac{N}{\sigma_y} \right)$$

As to be constant!



Above beam-beam limit:

σ_y increases when N increases to keep ξ constant

Equilibrium emittance

1. Synchrotron radiation: vertical plane damped, horizontal plane excited!
2. Horizontal beam size normally much larger than vertical (LEP 200 - 4 μm)
3. Vertical beam-beam effect depends on horizontal (larger) amplitude
4. Coupling from horizontal to vertical plane

$$\xi_{x,y} = \frac{Nr_0\beta_{x,y}^*}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

Equilibrium between horizontal excitation and vertical damping determines ξ_{limit}

Long-range BB and Orbit Effects

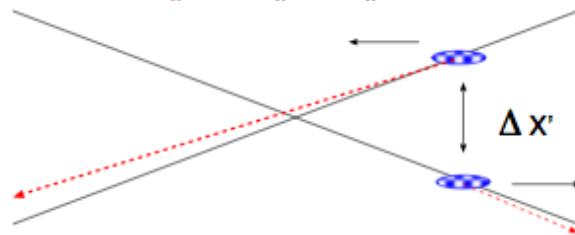
Long Range Beam-beam interactions lead to orbit effects

Long range kick
$$\Delta x'(x+d, y, r) = -\frac{2Nr_0}{\gamma} \frac{(x+d)}{r^2} [1 - \exp(-\frac{r^2}{2\sigma^2})]$$

For well separated beams $d \gg \sigma$

The force has an amplitude independent contribution: **ORBIT KICK**

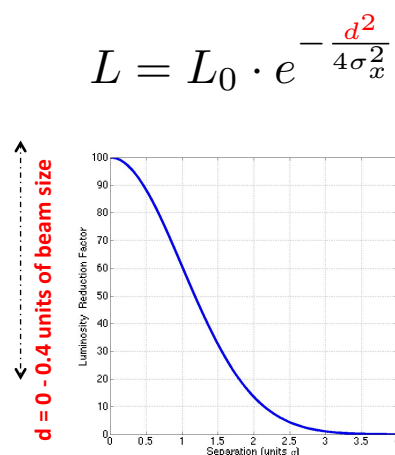
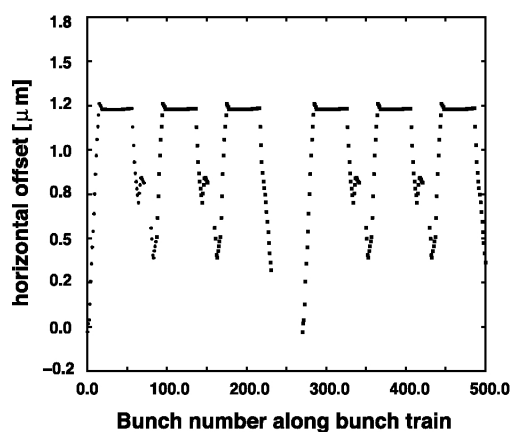
$$\Delta x' = \frac{\text{const}}{d} [1 - \frac{x}{d} + O(\frac{x^2}{d^2}) + \dots]$$



Orbit can be corrected but we should remember PACMAN effects

LHC orbit effects

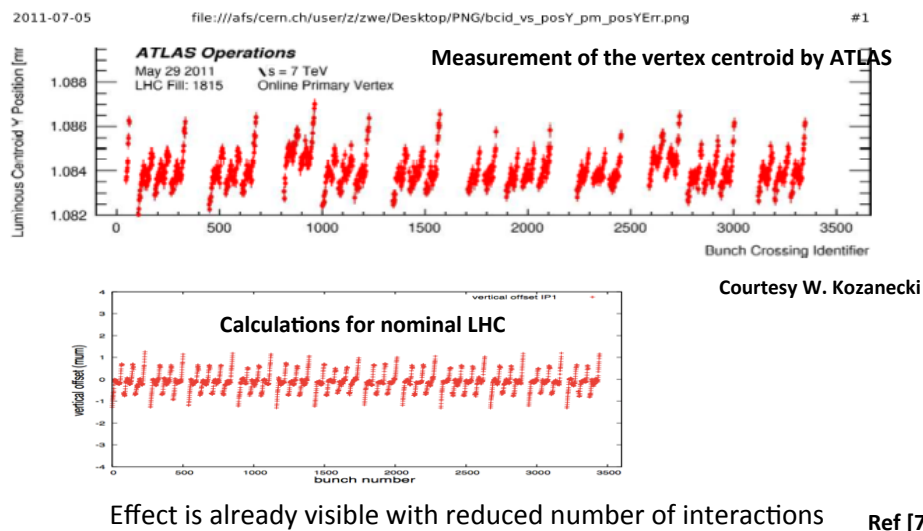
Orbit effects different due to Pacman effects and the many long-range add up giving a non negligible effect



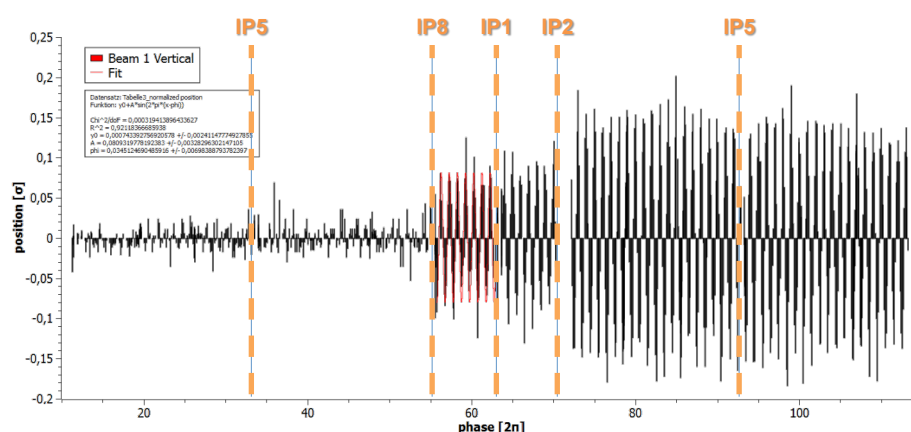
Ref [7]

Long range orbit effect

Long range interactions leads to orbit offsets at the experiment a direct consequence is deterioration of the luminosity

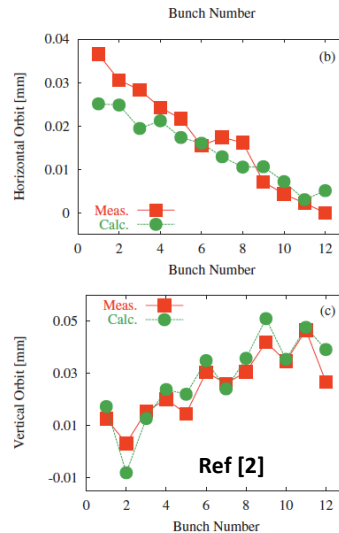


Long range orbit effect observations:



Vertical oscillation starts when one beam is ejected and dumped

Tevatron orbit effects



Beam-beam single bunch orbit can be well reproduced and measured also in LEP

Effects can become important (1σ offset not impossible)

LUMINOSITY Deterioration

Coherent dipolar beam-beam modes

Coherent beam-beam effects arise from the forces which an exciting bunch exerts on a **whole test bunch** during collision

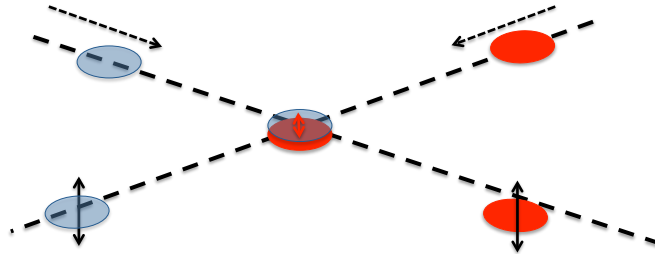
We study the **collective behaviour** of all particles of a bunch

Coherent motion requires an **organized behaviour** of all particles of the bunch

Coherent beam-beam force

- Beam distributions Ψ_1 and Ψ_2 mutually changed by interaction
- Interaction depends on distributions
 - Beam 1 Ψ_1 solution depends on beam 2 Ψ_2
 - Beam 2 Ψ_2 solution depends on beam 1 Ψ_1
- Need a **self-consistent** solution

Coherent beam-beam effects



- Whole bunch sees a kick as an entity (**coherent kick**)
- Coherent **kick seen by full bunch** different from single particle kick
- Requires **integration** of individual kick over particle distribution

$$\Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2} \cdot \left[1 - e^{-\frac{r^2}{4\sigma^2}} \right]$$

- Coherent kick of separated beams can excite coherent **dipolar oscillations**
- All bunches couple because each bunch “sees” many opposing bunches(LR): **many coherent modes possible!**

Coherent effects

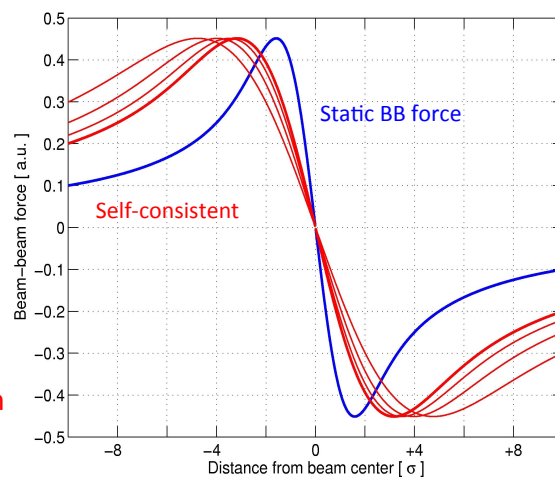
Self-consistent treatment needed

Perturbative methods

static source of distortion:
example magnet

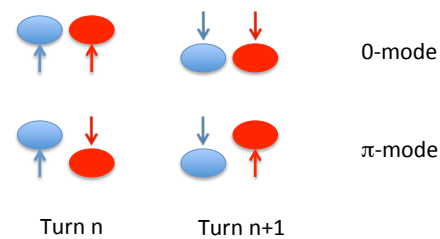
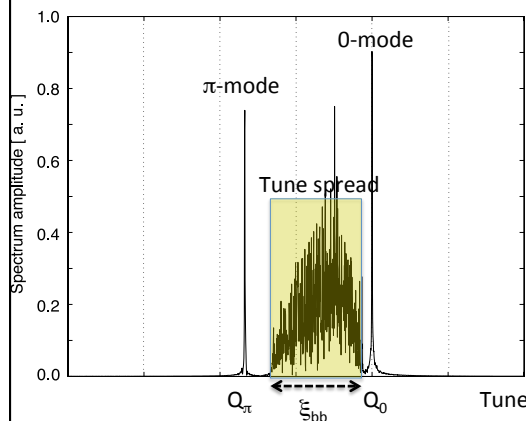
Self-consistent method

source of distortion changes
as a result of the distortion



For a complete understanding of BB effect a self-consistent treatment should be used

Simple case: one bunch per beam



MOVIE

0-mode at unperturbed tune Q_0

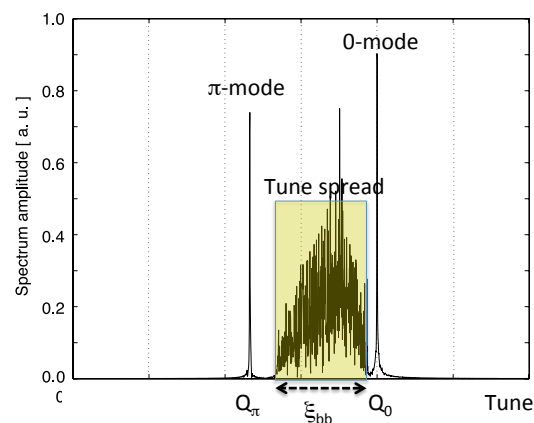
π -mode is shifted at $Q_\pi = 1.1-1.3 \xi_{bb}$

Incoherent tune spread range $[0, -\xi]$

$$\Delta Q = Y \cdot \xi$$

- Coherent mode: two bunches are “locked” in a coherent oscillation
- 0-mode is stable (mode with NO tune shift)
- π -mode can become unstable (mode with largest tune shift)

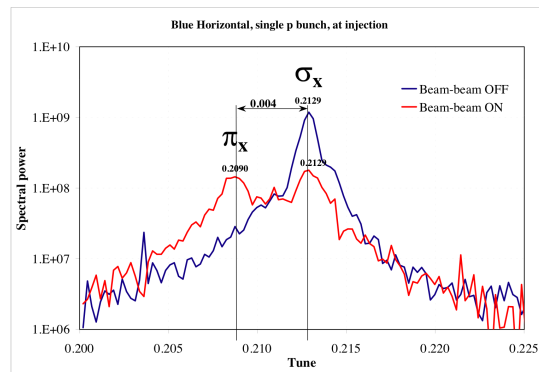
Simple case: one bunch per beam and Landau damping



Incoherent tune spread is the Landau damping region any mode with frequency laying in this range should not develop

- π -mode has frequency out of tune spread (Y) so it is not damped!

Coherent modes at RHIC

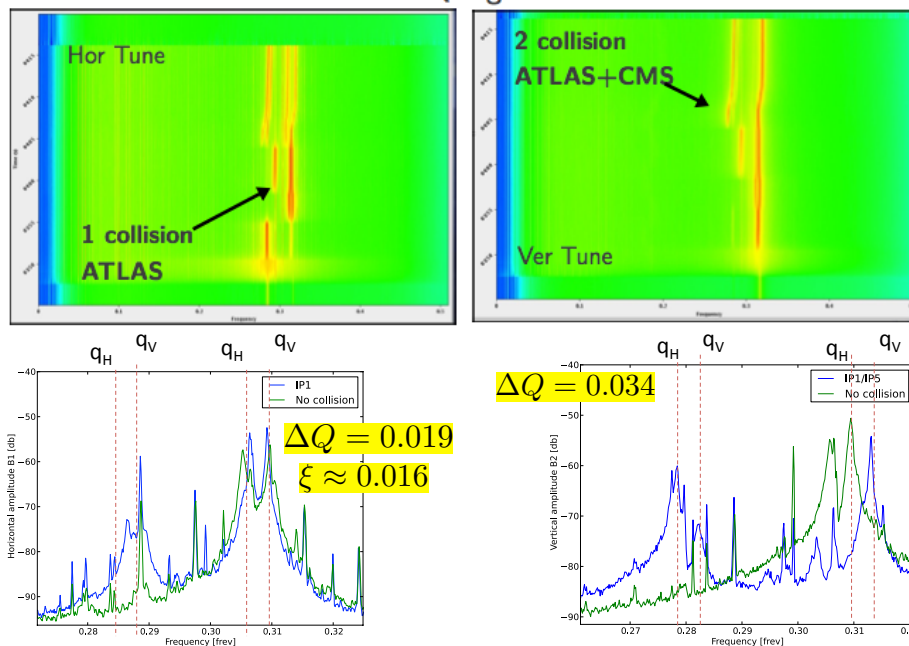


Courtesy W. Fischer (BNL)

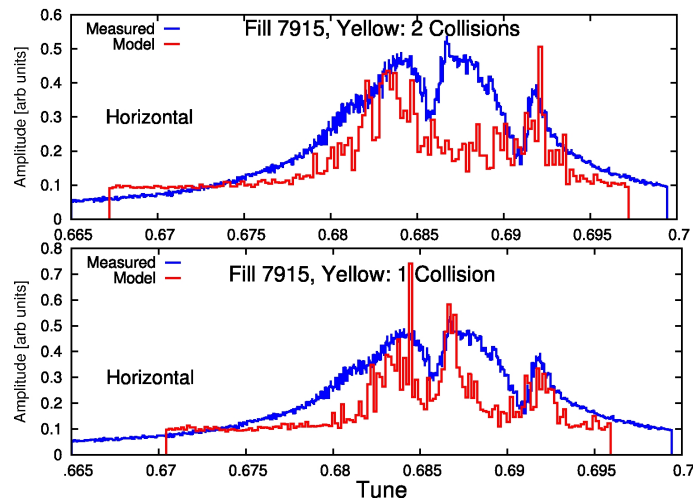
Tune spectra before collision and in collision two modes visible

Head-on beam-beam coherent mode: LHC

BBQ Signals

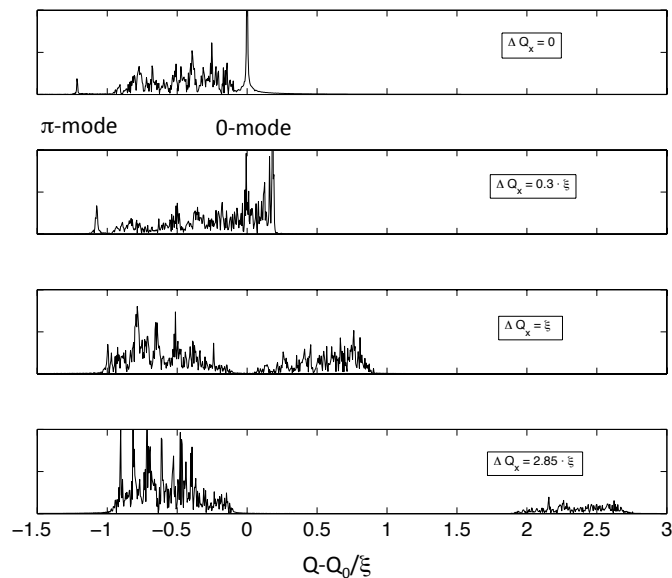


Beam-beam coherent modes and Landau Damping



Pacman effect on coherent modes
Single bunch diagnostic so important

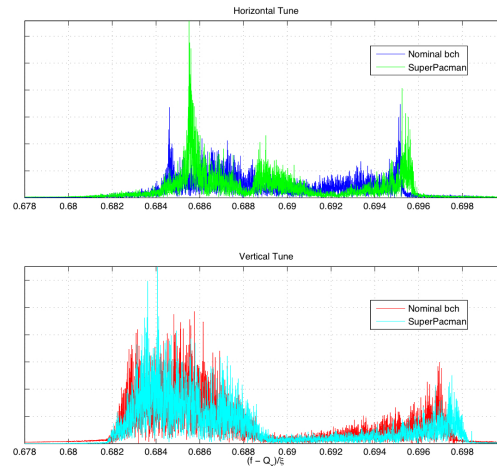
Different Tunes



Tune split breaks symmetry and coherent modes disappear
 Analytical calculations in Reference [8]

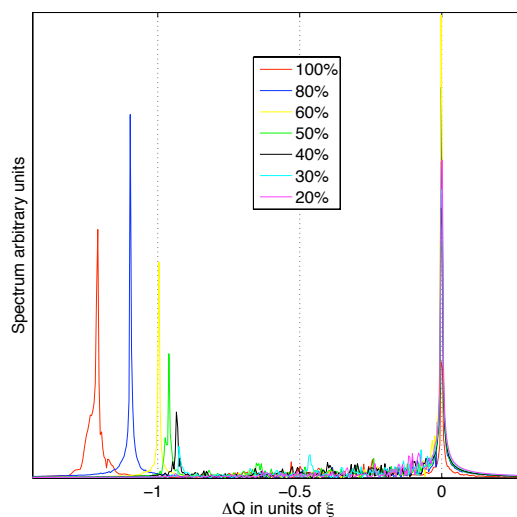
Different tunes or intensities

RHIC running with mirrored tune for years to break coherent oscillations



LHC has used a tune split to suppress coherent BB modes
2010 Physics Run

Different bunch intensities



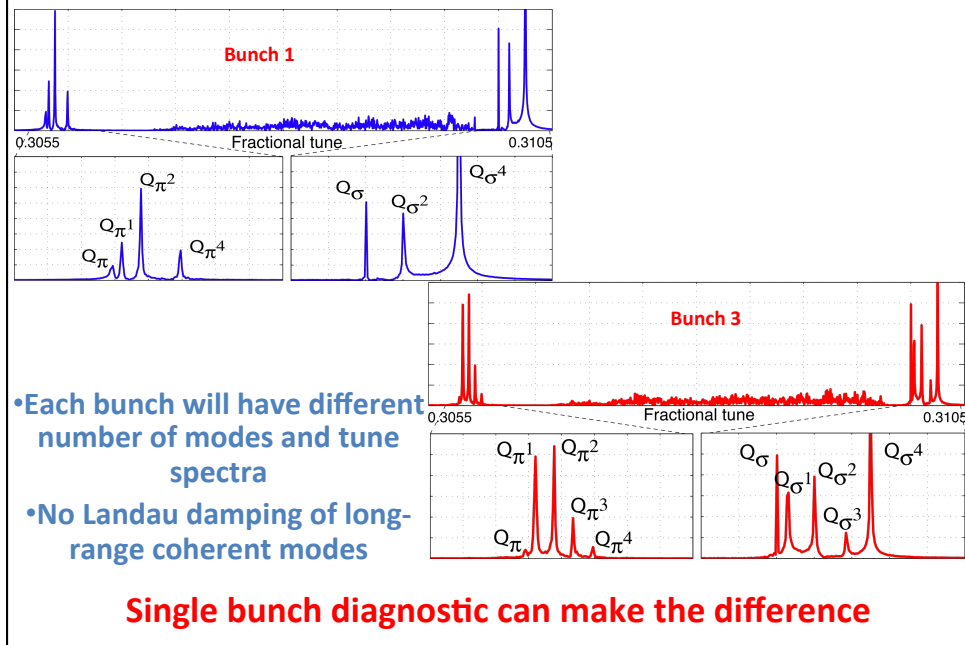
For two bunches colliding head-on in one IP the coherent mode disappears if intensity ratio between bunches is 55% [Reference\[9\]](#)

We assumed:

- equal emittances
- equal tunes
- NO PACMAN effects
(bunches will have different tunes)

For coherent modes the key is to break the symmetry in your coupled system...(tunes, intensities, collision patterns...)

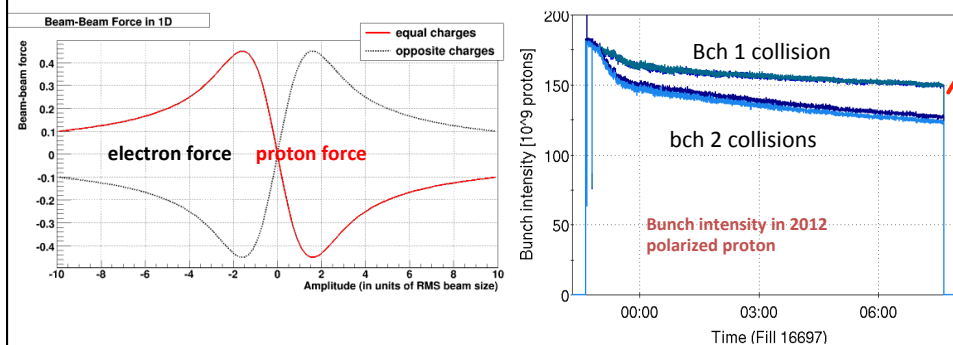
And Long range interactions?



Beam-beam compensations:

Head-on

- Linear e-lens, suppress shift
- Non-linear e-lens, suppress tune spread

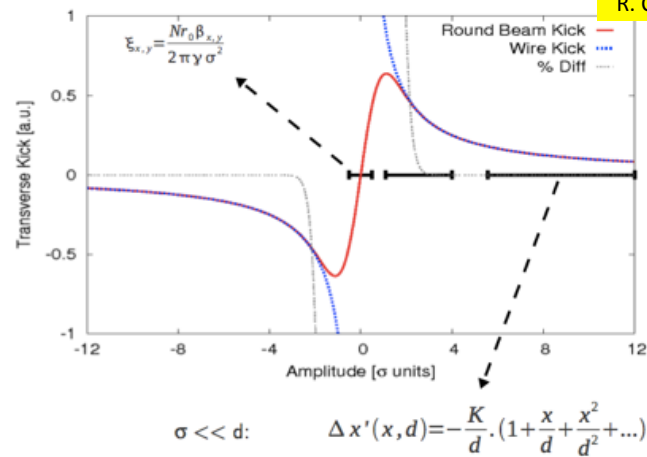


- Past experience: at Tevatron linear and non-linear e-lenses, also hollow....
- Present: test for half compensation at RHIC with non-linear e-lens

Beam-beam compensations: long-range

Beam-beam wire compensation

R. Calaga



- Past experience: at RHIC several tests till 2009...
- Present: simulation studies on-going for possible use in HL-LHC...

...not covered here...

- *Linear colliders special issues*
- *Asymmetric beams effects*
- *Coasting beams*
- *Beamstrahlung*
- *Synchrotron coupling*
- *Beam-beam experiments*
- *Beam-beam and impedance*
- ...

...some comments

- ❑ Beam-beam effects are very important for a collider
- ❑ Past experience and studies explain many features and observations but the LHC will still reserve some surprises (PACMAN effects)
- ❑ Single bunch diagnostic is at the basis of a correct interpretation of the collider performances
- ❑ Beam-beam has many effects and they depend on different parameters. Improving one can make others worse. That's way a one solution to the problem does NOT always exist!
- ❑ Careful choice of beam parameters if we know the limits will define the best operating scenario

References:

- [1] http://cern.ch/Werner.Herr/CAS2009/proceedings/bb_proc.pdf
- [2] V. Shiltsev et al, "Beam beam effects in the Tevatron", *Phys. Rev. ST Accel. Beams* 8, 101001 (2005)
- [3] Lyn Evans "The beam-beam interaction", CERN 84-15 (1984)
- [4] Alex Chao "Lie Algebra Techniques for Nonlinear Dynamics" SLAC-PUB-9574 (2002)
- [5] J. D. Jackson, "Classical Electrodynamics", John Wiley & Sons, NY, 1962.
- [6] H. Grote, F. Schmidt, L. H. A. Leunissen, "LHC Dynamic Aperture at Collision", LHC-Project-Note 197, (1999).
- [7] W. Herr, "Features and implications of different LHC crossing schemes", LHC-Project-Note 628, (2003).
- [8] A. Hofmann, "Beam-beam modes for two beams with unequal tunes", CERN-SL-99-039 (AP) (1999) p. 56.
- [9] Y. Alexahin, "On the Landau damping and decoherence of transverse dipole oscillations in colliding beams", *Part. Acc.* 59, 43 (1996).
- [10] R. Assmann et al., "Results of long-range beam-beam studies - scaling with beam separation and intensity "

...much more on the LHC Beam-beam webpage:

<http://lhc-beam-beam.web.cern.ch/lhc-beam-beam/>